# Computer algebra independent integration tests

## 1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.9-P-x-d+e-x-^m-a+b-x+c-x^2-^p

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# Contents

# Chapter 1

# Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [ 400 ]. This is test number [ 38 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 100.00 ( 400 ) | % 0.00 ( 0 ) |
| Mathematica | % 98.50 ( 394 ) | % 1.50 ( 6 ) |
| Maple | % 97.00 ( 388 ) | % 3.00 ( 12 ) |
| Maxima | % 72.50 ( 290 ) | % 27.50 ( 110 ) |
| Fricas | % 83.00 ( 332 ) | % 17.00 ( 68 ) |
| Sympy | % 35.25 ( 141 ) | % 64.75 ( 259 ) |
| Giac | % 83.75 ( 335 ) | % 16.25 ( 65 ) |
| Mupad | % 48.75 ( 195 ) | % 51.25 ( 205 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

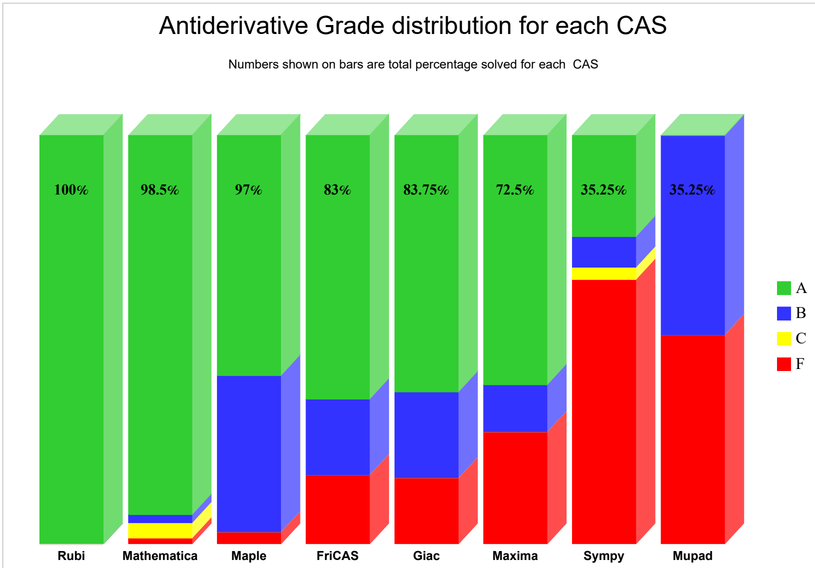
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

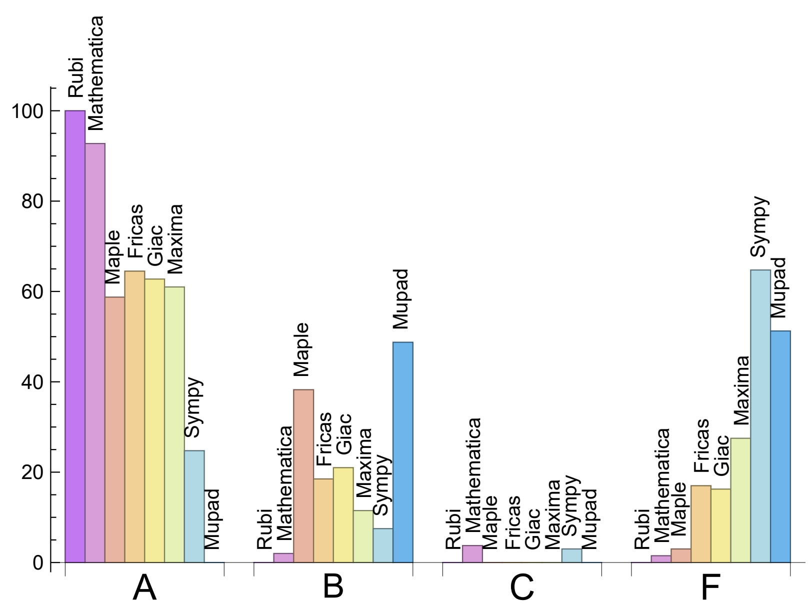
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100.00 | 0.00 | 0.00 | 0.00 |
| Mathematica | 92.75 | 2.00 | 3.75 | 1.50 |
| Maple | 58.75 | 38.25 | 0.00 | 3.00 |
| Maxima | 61.00 | 11.50 | 0.00 | 27.50 |
| Fricas | 64.50 | 18.50 | 0.00 | 17.00 |
| Sympy | 24.75 | 7.50 | 3.00 | 64.75 |
| Giac | 62.75 | 21.00 | 0.00 | 16.25 |
| Mupad | 0.00 | 48.75 | 0.00 | 51.25 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 % | 0.00 % | 0.00 % |
| Mathematica | 6 | 100.00 % | 0.00 % | 0.00 % |
| Maple | 12 | 100.00 % | 0.00 % | 0.00 % |
| Maxima | 110 | 31.82 % | 1.82 % | 66.36 % |
| Fricas | 68 | 36.76 % | 63.24 % | 0.00 % |
| Sympy | 259 | 74.90 % | 24.71 % | 0.39 % |
| Giac | 65 | 40.00 % | 21.54 % | 38.46 % |
| Mupad | 205 | 99.51 % | 0.49 % | 0.00 % |

Table 1.4: Time and leaf size performance for each CAS

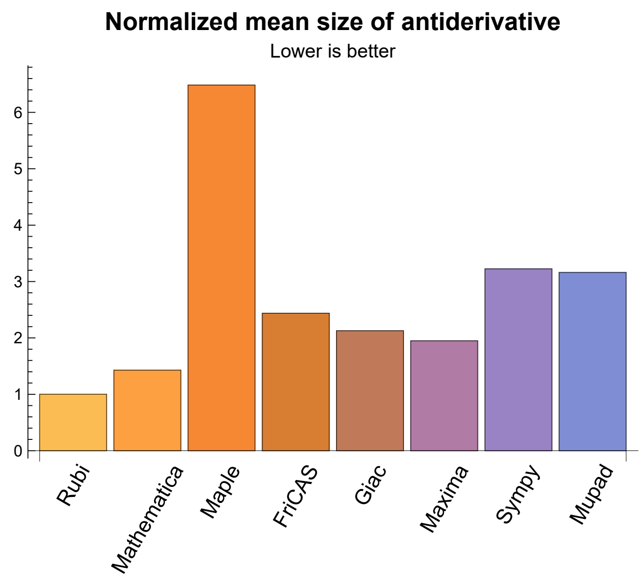
## 1.3 Performance

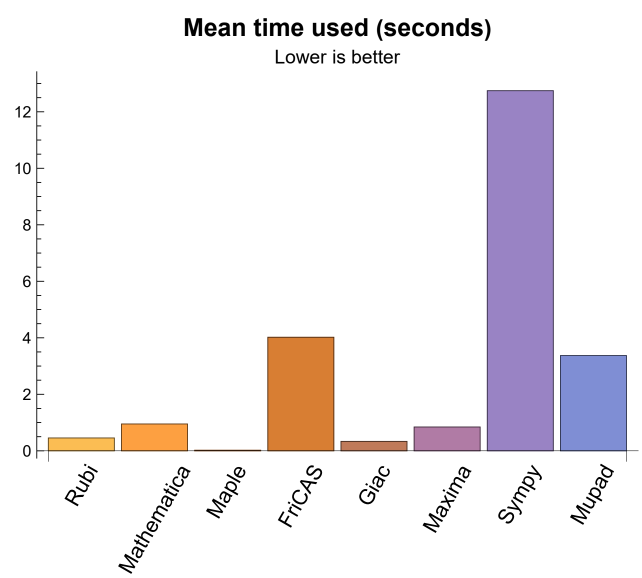
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.46 | 238.80 | 1.00 | 166.50 | 1.00 |
| Mathematica | 0.95 | 684.25 | 1.43 | 132.00 | 0.92 |
| Maple | 0.02 | 3404.39 | 6.48 | 209.00 | 1.46 |
| Maxima | 0.84 | 363.92 | 1.95 | 149.50 | 1.05 |
| Fricas | 4.02 | 455.78 | 2.44 | 172.00 | 1.40 |
| Sympy | 12.75 | 398.38 | 3.22 | 221.00 | 1.56 |
| Giac | 0.33 | 461.91 | 2.13 | 180.00 | 1.16 |
| Mupad | 3.37 | 773.44 | 3.16 | 185.00 | 1.29 |

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {259, 263, 264, 265, 271, 383, 398, 399, 400}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command AboluteTiming[] was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command Usage was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's time.time() call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is ValueError. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

### 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

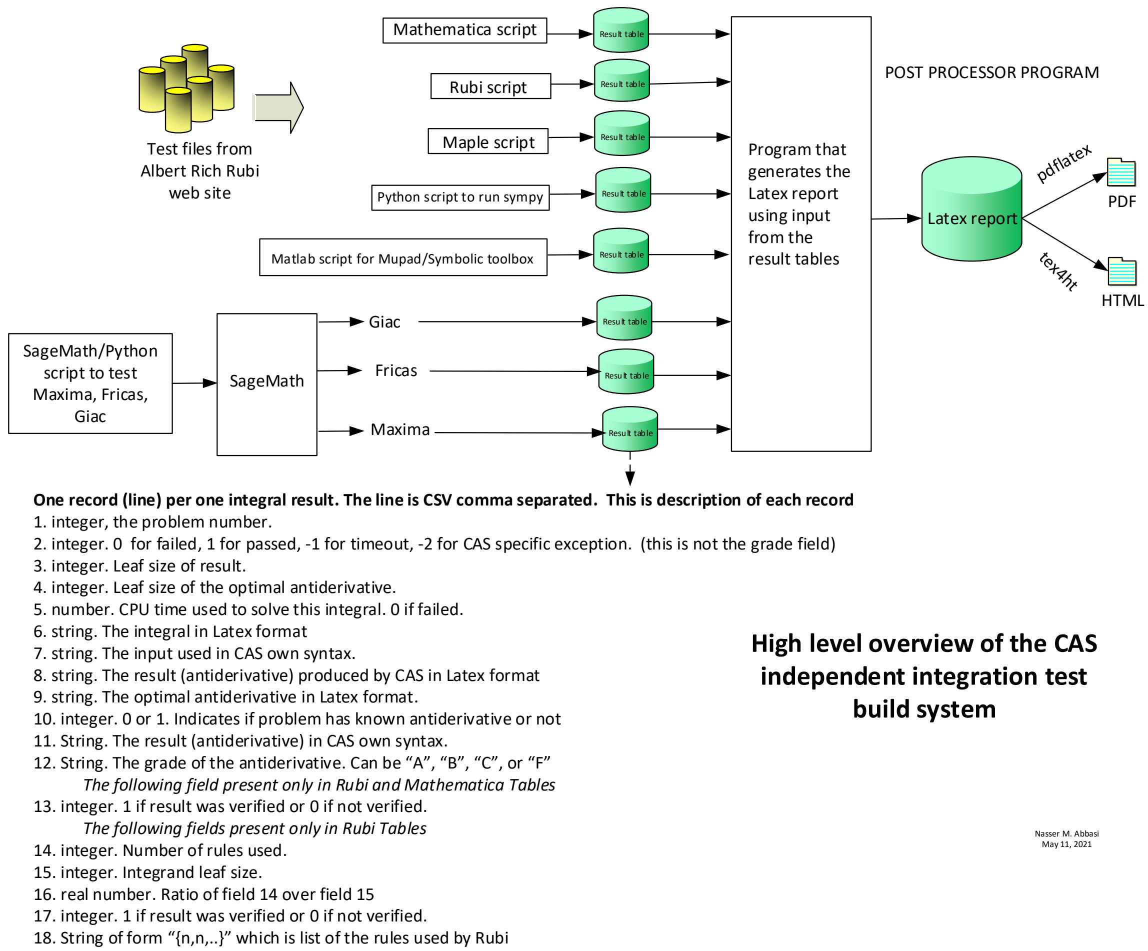
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



# Chapter 2

# detailed summary tables of results

## 2.1 List of integrals sorted by grade for each CAS

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

B grade: { 39, 40, 41, 42, 202, 203, 204, 278 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 275, 366 }

F grade: { 136, 137, 138, 272, 273, 274 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 134, 135, 140, 141, 142, 143, 144, 145, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 254, 256, 257, 258, 275, 276, 277, 279, 283, 284, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 312, 313, 314, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 389, 392, 393, 394 }

B grade: { 4, 5, 6, 7, 15, 38, 39, 40, 41, 42, 48, 49, 50, 51, 54, 55, 56, 57, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 127, 133, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 178, 179, 180, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 249, 252, 253, 255, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 278, 280, 281, 282, 286, 287, 288, 309, 310, 315, 316, 317, 322, 323, 358, 359, 360, 361, 362, 365, 366, 367, 368, 369, 372, 373, 377, 378, 379, 383, 384, 385, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 136, 137, 138, 139, 272, 273, 274, 370, 371, 398, 399, 400 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 88, 89, 90, 91, 93, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 275, 276, 277, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 361, 362, 363, 364, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 8, 9, 15, 16, 17, 39, 40, 41, 42, 56, 62, 63, 64, 84, 85, 86, 87, 92, 94, 95, 96, 97, 98, 106, 107, 112, 113, 114, 131, 177, 252, 253, 278, 317, 323, 343, 358, 359, 360, 367, 368, 369, 377, 383, 389, 395 }

C grade: { }

F grade: { 6, 7, 99, 136, 137, 138, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 365, 366, 370, 371, 372, 373, 378, 379, 384, 385, 390, 391, 396, 397, 398, 399, 400 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 52, 53, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 186, 187, 188, 189, 199, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 254, 255, 256, 257, 275, 276, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 312, 313, 314, 320, 321, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 362, 363, 364, 373, 374, 375, 376, 380, 381, 382, 386, 387, 388, 392, 393, 394 }

B grade: { 31, 38, 39, 40, 41, 42, 48, 50, 51, 54, 57, 58, 64, 65, 66, 67, 107, 112, 113, 114, 145, 146, 147, 155, 156, 157, 177, 178, 182, 183, 184, 185, 196, 197, 198, 232, 233, 234, 235, 236, 237, 238, 253, 258, 277, 278, 310, 311, 315, 316, 317, 318, 319, 322, 323, 360, 365, 366, 367, 368, 369, 372, 377, 378, 379, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

C grade: { }

F grade: { 49, 55, 56, 61, 62, 63, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 136, 137, 138, 139, 153, 154, 158, 159, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 230, 231, 239, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 370, 371, 398, 399, 400 }

### 2.1.6 Sympy

A grade: { 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 53, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 100, 101, 102, 103, 104, 110, 111, 115, 118, 119, 120, 140, 141, 142, 143, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 276, 279, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 314, 321 }

B grade: { 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 58, 116, 117, 126, 132, 144, 145, 146, 147, 148, 149, 150, 151, 156, 157, 177, 275, 277, 278 }

C grade: { 1, 2, 3, 304, 305, 306, 311, 312, 313, 318, 319, 320 }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 47, 48, 49, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 82, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 99, 105, 106, 107, 108, 109, 112, 113, 114, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 152, 153, 154, 155, 158, 159, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 285, 286, 287, 288, 308, 309, 310, 315, 316, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 88, 89, 90, 91, 92, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 186, 187, 188, 189, 199, 208, 209, 210, 211, 214, 215, 216, 217, 220, 221, 222, 223, 226, 227, 228, 229, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 252, 253, 254, 255, 279, 280, 281, 282, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 334, 335, 336, 338, 344, 345, 346, 350, 351, 352, 353, 354, 358, 359, 360, 361, 365, 366, 372, 373, 374, 375, 376, 377, 380, 381, 382, 383, 386, 387, 388, 389, 392, 393, 394, 395, 396 }

B grade: { 39, 40, 41, 42, 61, 63, 64, 65, 84, 85, 87, 94, 95, 97, 98, 99, 107, 112, 114, 121, 123, 128, 129, 134, 147, 158, 159, 178, 184, 185, 196, 197, 198, 203, 212, 218, 224, 232, 233, 234, 237, 239, 245, 250, 251, 256, 257, 275, 276, 277, 278, 285, 286, 287, 288, 327, 328, 329, 330, 331, 332, 333, 337, 339, 340, 341, 342, 343, 347, 348, 349, 355, 356, 357, 362, 363, 364, 367, 368, 369, 379, 390, 391, 397 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 83, 86, 93, 96, 106, 113, 136, 137, 138, 139, 190, 191, 192, 193, 194, 195, 200, 201, 202, 204, 205, 206, 207, 213, 219, 225, 230, 231, 238, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 283, 370, 371, 378, 384, 385, 398, 399, 400 }

### 2.1.8 Mupad

A grade: { }

B grade: { 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 103, 104, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 180, 182, 183, 184, 185, 186, 187, 188, 189, 208, 209, 210, 236, 254, 258, 275, 276, 277, 278, 279, 284, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 367, 368, 369, 372, 373, 374, 375, 376 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 14, 15, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 112, 113, 114, 136, 137, 138, 139, 178, 179, 181, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 280, 281, 282, 283, 285, 286, 287, 288, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 226 | 371 | 338 | 211 | 1231 | 197 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 1.57 | 1.43 | 0.89 | 5.22 | 0.83 | -0.00 |
| time (sec) | N/A | 0.470 | 0.506 | 0.059 | 0.996 | 0.681 | 22.934 | 0.224 | 0.000 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 190 | 304 | 202 | 173 | 670 | 160 | -1 |
| normalized size | 1 | 1.00 | 1.02 | 1.63 | 1.09 | 0.93 | 3.60 | 0.86 | -0.01 |
| time (sec) | N/A | 0.227 | 0.299 | 0.017 | 0.976 | 0.866 | 12.771 | 0.213 | 0.000 |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 121 | 154 | 116 | 108 | 343 | 85 | -1 |
| normalized size | 1 | 1.00 | 0.97 | 1.23 | 0.93 | 0.86 | 2.74 | 0.68 | -0.01 |
| time (sec) | N/A | 0.069 | 0.133 | 0.007 | 0.964 | 0.973 | 7.108 | 0.202 | 0.000 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 103 | 384 | 171 | 112 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.70 | 2.59 | 1.16 | 0.76 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.176 | 0.221 | 0.023 | 1.086 | 0.893 | 0.000 | 0.000 | 0.000 |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 109 | 439 | 197 | 190 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.64 | 2.58 | 1.16 | 1.12 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.203 | 0.227 | 0.028 | 1.014 | 0.675 | 0.000 | 0.000 | 0.000 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 114 | 318 | 0 | 258 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.77 | 2.13 | 0.00 | 1.73 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.189 | 0.221 | 0.018 | 0.000 | 0.852 | 0.000 | 0.000 | 0.000 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 112 | 453 | 0 | 304 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.57 | 2.31 | 0.00 | 1.55 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.184 | 0.298 | 0.017 | 0.000 | 0.832 | 0.000 | 0.000 | 0.000 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 109 | 116 | 945 | 320 | 0 | 0 | 601 |
| normalized size | 1 | 1.00 | 0.61 | 0.64 | 5.25 | 1.78 | 0.00 | 0.00 | 3.34 |
| time (sec) | N/A | 0.210 | 0.203 | 0.009 | 0.542 | 0.915 | 0.000 | 0.000 | 4.670 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F(-1) | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 144 | 152 | 1378 | 399 | 0 | 0 | 960 |
| normalized size | 1 | 1.00 | 0.62 | 0.65 | 5.89 | 1.71 | 0.00 | 0.00 | 4.10 |
| time (sec) | N/A | 0.264 | 0.225 | 0.011 | 0.575 | 1.072 | 0.000 | 0.000 | 5.243 |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 174 | 374 | 390 | 178 | 1268 | 166 | -1 |
| normalized size | 1 | 1.00 | 0.74 | 1.58 | 1.65 | 0.75 | 5.37 | 0.70 | -0.00 |
| time (sec) | N/A | 0.657 | 0.453 | 0.031 | 0.979 | 0.614 | 24.443 | 0.368 | 0.000 |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 139 | 301 | 253 | 145 | 891 | 131 | -1 |
| normalized size | 1 | 1.00 | 0.73 | 1.58 | 1.32 | 0.76 | 4.66 | 0.69 | -0.01 |
| time (sec) | N/A | 0.378 | 0.177 | 0.012 | 0.983 | 0.957 | 18.032 | 0.316 | 0.000 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 103 | 234 | 150 | 109 | 484 | 97 | 270 |
| normalized size | 1 | 1.00 | 0.72 | 1.64 | 1.05 | 0.76 | 3.38 | 0.68 | 1.89 |
| time (sec) | N/A | 0.199 | 0.107 | 0.011 | 0.979 | 0.971 | 10.172 | 0.292 | 5.012 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 67 | 108 | 70 | 71 | 262 | 52 | 148 |
| normalized size | 1 | 1.00 | 0.77 | 1.24 | 0.80 | 0.82 | 3.01 | 0.60 | 1.70 |
| time (sec) | N/A | 0.051 | 0.040 | 0.006 | 0.988 | 0.640 | 4.560 | 0.344 | 4.399 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 83 | 149 | 138 | 155 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 1.45 | 1.34 | 1.50 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.121 | 0.158 | 0.014 | 0.994 | 0.845 | 0.000 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 95 | 355 | 317 | 221 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.58 | 2.18 | 1.94 | 1.36 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.169 | 0.218 | 0.016 | 0.995 | 0.915 | 0.000 | 0.000 | 0.000 |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 103 | 116 | 608 | 244 | 0 | 0 | 109 |
| normalized size | 1 | 1.00 | 0.57 | 0.64 | 3.38 | 1.36 | 0.00 | 0.00 | 0.61 |
| time (sec) | N/A | 0.205 | 0.197 | 0.012 | 1.015 | 0.786 | 0.000 | 0.000 | 3.795 |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | F(-2) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 139 | 152 | 975 | 320 | 0 | 0 | 204 |
| normalized size | 1 | 1.00 | 0.59 | 0.65 | 4.17 | 1.37 | 0.00 | 0.00 | 0.87 |
| time (sec) | N/A | 0.249 | 0.221 | 0.010 | 1.061 | 0.923 | 0.000 | 0.000 | 3.776 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 173 | 208 | 217 | 202 | 248 | 257 | 242 | 206 |
| normalized size | 1 | 0.99 | 1.19 | 1.24 | 1.15 | 1.42 | 1.47 | 1.38 | 1.18 |
| time (sec) | N/A | 0.313 | 0.087 | 0.002 | 0.446 | 0.746 | 0.117 | 0.170 | 0.088 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 173 | 150 | 148 | 141 | 171 | 173 | 171 | 143 |
| normalized size | 1 | 0.99 | 0.86 | 0.85 | 0.81 | 0.98 | 0.99 | 0.98 | 0.82 |
| time (sec) | N/A | 0.216 | 0.060 | 0.001 | 0.448 | 0.741 | 0.095 | 0.152 | 3.614 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 86 | 79 | 80 | 94 | 97 | 100 | 80 |
| normalized size | 1 | 1.00 | 1.00 | 0.92 | 0.93 | 1.09 | 1.13 | 1.16 | 0.93 |
| time (sec) | N/A | 0.106 | 0.029 | 0.001 | 0.452 | 0.747 | 0.082 | 0.154 | 3.561 |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 39 | 38 | 40 | 42 | 40 | 39 |
| normalized size | 1 | 1.00 | 1.00 | 0.85 | 0.83 | 0.87 | 0.91 | 0.87 | 0.85 |
| time (sec) | N/A | 0.029 | 0.013 | 0.002 | 0.440 | 0.835 | 0.072 | 0.157 | 0.025 |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 143 | 136 | 210 | 159 | 161 | 148 | 170 | 175 |
| normalized size | 1 | 0.99 | 0.94 | 1.45 | 1.10 | 1.11 | 1.02 | 1.17 | 1.21 |
| time (sec) | N/A | 0.245 | 0.074 | 0.008 | 0.448 | 1.608 | 0.638 | 0.151 | 3.623 |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 151 | 142 | 234 | 169 | 250 | 185 | 240 | 192 |
| normalized size | 1 | 0.99 | 0.93 | 1.53 | 1.10 | 1.63 | 1.21 | 1.57 | 1.25 |
| time (sec) | N/A | 0.205 | 0.154 | 0.010 | 0.451 | 0.872 | 1.257 | 0.163 | 0.090 |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 154 | 176 | 257 | 177 | 273 | 206 | 167 | 185 |
| normalized size | 1 | 0.99 | 1.13 | 1.65 | 1.13 | 1.75 | 1.32 | 1.07 | 1.19 |
| time (sec) | N/A | 0.199 | 0.100 | 0.010 | 0.471 | 0.790 | 5.288 | 0.152 | 0.095 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 301 | 335 | 385 | 360 | 432 | 445 | 423 | 332 |
| normalized size | 1 | 0.99 | 1.10 | 1.27 | 1.18 | 1.42 | 1.46 | 1.39 | 1.09 |
| time (sec) | N/A | 0.535 | 0.131 | 0.002 | 0.450 | 0.756 | 0.134 | 0.159 | 0.140 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 216 | 241 | 268 | 257 | 302 | 311 | 302 | 244 |
| normalized size | 1 | 1.00 | 1.11 | 1.24 | 1.18 | 1.39 | 1.43 | 1.39 | 1.12 |
| time (sec) | N/A | 0.313 | 0.090 | 0.001 | 0.441 | 0.436 | 0.124 | 0.168 | 3.724 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 144 | 151 | 154 | 172 | 180 | 181 | 140 |
| normalized size | 1 | 1.00 | 1.12 | 1.18 | 1.20 | 1.34 | 1.41 | 1.41 | 1.09 |
| time (sec) | N/A | 0.159 | 0.051 | 0.001 | 0.447 | 0.806 | 0.097 | 0.152 | 3.695 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 69 | 75 | 74 | 76 | 83 | 76 | 74 |
| normalized size | 1 | 1.00 | 1.03 | 1.12 | 1.10 | 1.13 | 1.24 | 1.13 | 1.10 |
| time (sec) | N/A | 0.040 | 0.031 | 0.001 | 0.438 | 0.896 | 0.081 | 0.151 | 0.038 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 295 | 285 | 490 | 377 | 379 | 359 | 416 | 422 |
| normalized size | 1 | 0.99 | 0.96 | 1.65 | 1.27 | 1.28 | 1.21 | 1.40 | 1.42 |
| time (sec) | N/A | 0.640 | 0.169 | 0.008 | 0.485 | 0.616 | 0.937 | 0.158 | 3.685 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 289 | 272 | 527 | 392 | 553 | 416 | 497 | 575 |
| normalized size | 1 | 0.99 | 0.93 | 1.80 | 1.34 | 1.89 | 1.42 | 1.70 | 1.97 |
| time (sec) | N/A | 0.525 | 0.281 | 0.012 | 0.478 | 0.967 | 2.785 | 0.185 | 0.121 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 292 | 274 | 563 | 402 | 608 | 474 | 397 | 495 |
| normalized size | 1 | 0.99 | 0.93 | 1.91 | 1.36 | 2.06 | 1.61 | 1.35 | 1.68 |
| time (sec) | N/A | 0.495 | 0.122 | 0.014 | 0.494 | 0.909 | 14.200 | 0.160 | 3.825 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 400 | 459 | 553 | 512 | 618 | 646 | 606 | 490 |
| normalized size | 1 | 0.99 | 1.14 | 1.37 | 1.27 | 1.53 | 1.60 | 1.50 | 1.21 |
| time (sec) | N/A | 0.691 | 0.208 | 0.001 | 0.470 | 0.742 | 0.161 | 0.202 | 4.050 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 288 | 329 | 388 | 367 | 432 | 447 | 432 | 343 |
| normalized size | 1 | 1.00 | 1.14 | 1.34 | 1.27 | 1.49 | 1.55 | 1.49 | 1.19 |
| time (sec) | N/A | 0.424 | 0.130 | 0.001 | 0.454 | 0.750 | 0.147 | 0.155 | 3.938 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 196 | 223 | 222 | 249 | 265 | 261 | 187 |
| normalized size | 1 | 1.00 | 1.16 | 1.32 | 1.31 | 1.47 | 1.57 | 1.54 | 1.11 |
| time (sec) | N/A | 0.187 | 0.070 | 0.000 | 0.440 | 0.769 | 0.114 | 0.185 | 0.099 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 100 | 111 | 108 | 111 | 122 | 111 | 103 |
| normalized size | 1 | 1.00 | 1.15 | 1.28 | 1.24 | 1.28 | 1.40 | 1.28 | 1.18 |
| time (sec) | N/A | 0.058 | 0.031 | 0.001 | 0.432 | 0.769 | 0.086 | 0.186 | 0.057 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 490 | 487 | 498 | 880 | 672 | 674 | 685 | 764 | 741 |
| normalized size | 1 | 0.99 | 1.02 | 1.80 | 1.37 | 1.38 | 1.40 | 1.56 | 1.51 |
| time (sec) | N/A | 1.098 | 0.474 | 0.009 | 0.487 | 0.865 | 1.467 | 0.167 | 3.877 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 486 | 483 | 641 | 928 | 691 | 932 | 748 | 838 | 1511 |
| normalized size | 1 | 0.99 | 1.32 | 1.91 | 1.42 | 1.92 | 1.54 | 1.72 | 3.11 |
| time (sec) | N/A | 0.980 | 0.397 | 0.017 | 0.487 | 0.931 | 4.949 | 0.196 | 3.986 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 466 | 463 | 438 | 978 | 701 | 1025 | 816 | 727 | 1290 |
| normalized size | 1 | 0.99 | 0.94 | 2.10 | 1.50 | 2.20 | 1.75 | 1.56 | 2.77 |
| time (sec) | N/A | 0.967 | 0.225 | 0.017 | 0.530 | 0.926 | 25.279 | 0.174 | 3.936 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 62 | 76 | 82 | 78 | 73 | 111 | 85 |
| normalized size | 1 | 1.00 | 3.65 | 4.47 | 4.82 | 4.59 | 4.29 | 6.53 | 5.00 |
| time (sec) | N/A | 0.032 | 0.037 | 0.006 | 0.458 | 0.951 | 0.368 | 0.175 | 0.077 |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 62 | 76 | 82 | 78 | 73 | 111 | 85 |
| normalized size | 1 | 1.00 | 3.65 | 4.47 | 4.82 | 4.59 | 4.29 | 6.53 | 5.00 |
| time (sec) | N/A | 0.019 | 0.016 | 0.005 | 0.466 | 0.844 | 0.367 | 0.155 | 3.836 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 90 | 157 | 160 | 120 | 153 | 216 | 252 |
| normalized size | 1 | 1.00 | 5.29 | 9.24 | 9.41 | 7.06 | 9.00 | 12.71 | 14.82 |
| time (sec) | N/A | 0.046 | 0.042 | 0.007 | 0.452 | 0.872 | 0.587 | 0.183 | 3.777 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 90 | 157 | 160 | 120 | 153 | 216 | 252 |
| normalized size | 1 | 1.00 | 5.29 | 9.24 | 9.41 | 7.06 | 9.00 | 12.71 | 14.82 |
| time (sec) | N/A | 0.025 | 0.025 | 0.006 | 0.444 | 0.890 | 0.610 | 0.199 | 0.049 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 237 | 223 | 399 | 244 | 592 | 1008 | 279 | 277 |
| normalized size | 1 | 0.99 | 0.93 | 1.66 | 1.02 | 2.47 | 4.20 | 1.16 | 1.15 |
| time (sec) | N/A | 0.471 | 0.243 | 0.011 | 0.982 | 0.904 | 5.462 | 0.170 | 3.990 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 166 | 155 | 256 | 161 | 404 | 638 | 176 | 181 |
| normalized size | 1 | 0.99 | 0.92 | 1.52 | 0.96 | 2.40 | 3.80 | 1.05 | 1.08 |
| time (sec) | N/A | 0.262 | 0.167 | 0.007 | 0.990 | 0.791 | 3.150 | 0.157 | 3.899 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 86 | 133 | 86 | 206 | 337 | 91 | 97 |
| normalized size | 1 | 1.00 | 0.92 | 1.43 | 0.92 | 2.22 | 3.62 | 0.98 | 1.04 |
| time (sec) | N/A | 0.117 | 0.087 | 0.006 | 0.972 | 0.584 | 1.660 | 0.158 | 3.780 |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 56 | 59 | 48 | 125 | 156 | 48 | 56 |
| normalized size | 1 | 1.00 | 1.02 | 1.07 | 0.87 | 2.27 | 2.84 | 0.87 | 1.02 |
| time (sec) | N/A | 0.053 | 0.041 | 0.003 | 0.966 | 0.802 | 0.486 | 0.159 | 3.731 |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 133 | 120 | 247 | 123 | 262 | 0 | 125 | 840 |
| normalized size | 1 | 1.00 | 0.90 | 1.86 | 0.92 | 1.97 | 0.00 | 0.94 | 6.32 |
| time (sec) | N/A | 0.162 | 0.102 | 0.008 | 0.969 | 14.211 | 0.000 | 0.159 | 6.490 |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 214 | 188 | 462 | 255 | 904 | 0 | 270 | 1199 |
| normalized size | 1 | 1.00 | 0.88 | 2.16 | 1.19 | 4.22 | 0.00 | 1.26 | 5.60 |
| time (sec) | N/A | 0.355 | 0.312 | 0.010 | 1.035 | 69.906 | 0.000 | 0.170 | 6.773 |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 305 | 277 | 754 | 495 | 0 | 0 | 489 | 2980 |
| normalized size | 1 | 1.00 | 0.91 | 2.47 | 1.62 | 0.00 | 0.00 | 1.60 | 9.77 |
| time (sec) | N/A | 0.651 | 0.309 | 0.015 | 1.052 | 0.000 | 0.000 | 0.197 | 9.186 |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 233 | 484 | 287 | 931 | 952 | 289 | 303 |
| normalized size | 1 | 1.00 | 1.08 | 2.24 | 1.33 | 4.31 | 4.41 | 1.34 | 1.40 |
| time (sec) | N/A | 0.505 | 0.204 | 0.015 | 0.984 | 0.924 | 34.457 | 0.174 | 4.014 |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 175 | 323 | 188 | 631 | 593 | 184 | 195 |
| normalized size | 1 | 1.00 | 1.20 | 2.21 | 1.29 | 4.32 | 4.06 | 1.26 | 1.34 |
| time (sec) | N/A | 0.245 | 0.138 | 0.013 | 0.965 | 0.967 | 18.398 | 0.170 | 0.229 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 102 | 134 | 113 | 337 | 318 | 112 | 191 |
| normalized size | 1 | 1.00 | 1.05 | 1.38 | 1.16 | 3.47 | 3.28 | 1.15 | 1.97 |
| time (sec) | N/A | 0.082 | 0.095 | 0.009 | 0.972 | 0.970 | 6.411 | 0.218 | 0.139 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 68 | 76 | 62 | 195 | 116 | 60 | 60 |
| normalized size | 1 | 1.00 | 0.99 | 1.10 | 0.90 | 2.83 | 1.68 | 0.87 | 0.87 |
| time (sec) | N/A | 0.043 | 0.052 | 0.008 | 0.965 | 0.812 | 0.654 | 0.183 | 0.100 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 226 | 195 | 742 | 293 | 1024 | 0 | 350 | 1493 |
| normalized size | 1 | 1.00 | 0.86 | 3.28 | 1.30 | 4.53 | 0.00 | 1.55 | 6.61 |
| time (sec) | N/A | 0.435 | 0.224 | 0.017 | 1.002 | 64.284 | 0.000 | 0.193 | 7.675 |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 371 | 320 | 1036 | 604 | 0 | 0 | 608 | 2094 |
| normalized size | 1 | 0.99 | 0.86 | 2.77 | 1.61 | 0.00 | 0.00 | 1.63 | 5.60 |
| time (sec) | N/A | 0.950 | 0.413 | 0.023 | 1.039 | 0.000 | 0.000 | 0.190 | 9.909 |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 524 | 524 | 466 | 1588 | 1030 | 0 | 0 | 957 | 2828 |
| normalized size | 1 | 1.00 | 0.89 | 3.03 | 1.97 | 0.00 | 0.00 | 1.83 | 5.40 |
| time (sec) | N/A | 1.552 | 0.630 | 0.027 | 1.215 | 0.000 | 0.000 | 0.185 | 14.480 |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 209 | 281 | 402 | 379 | 1138 | 0 | 348 | 920 |
| normalized size | 1 | 1.00 | 1.34 | 1.92 | 1.81 | 5.44 | 0.00 | 1.67 | 4.40 |
| time (sec) | N/A | 0.304 | 0.247 | 0.013 | 1.001 | 1.434 | 0.000 | 0.269 | 1.767 |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 175 | 211 | 283 | 253 | 806 | 391 | 254 | 230 |
| normalized size | 1 | 1.12 | 1.35 | 1.81 | 1.62 | 5.17 | 2.51 | 1.63 | 1.47 |
| time (sec) | N/A | 0.231 | 0.139 | 0.010 | 0.995 | 1.216 | 141.179 | 0.160 | 3.959 |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 137 | 157 | 160 | 470 | 240 | 152 | 128 |
| normalized size | 1 | 1.00 | 1.05 | 1.21 | 1.23 | 3.62 | 1.85 | 1.17 | 0.98 |
| time (sec) | N/A | 0.108 | 0.101 | 0.009 | 0.983 | 1.197 | 32.420 | 0.185 | 0.150 |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 90 | 96 | 98 | 314 | 156 | 84 | 88 |
| normalized size | 1 | 1.00 | 0.92 | 0.98 | 1.00 | 3.20 | 1.59 | 0.86 | 0.90 |
| time (sec) | N/A | 0.063 | 0.070 | 0.007 | 0.971 | 1.255 | 1.234 | 0.162 | 3.843 |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 321 | 1598 | 655 | 0 | 0 | 715 | 2392 |
| normalized size | 1 | 1.00 | 0.91 | 4.53 | 1.86 | 0.00 | 0.00 | 2.03 | 6.78 |
| time (sec) | N/A | 0.734 | 0.424 | 0.023 | 1.097 | 0.000 | 0.000 | 0.177 | 9.900 |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 571 | 566 | 498 | 2159 | 1196 | 0 | 0 | 1107 | 6848 |
| normalized size | 1 | 0.99 | 0.87 | 3.78 | 2.09 | 0.00 | 0.00 | 1.94 | 11.99 |
| time (sec) | N/A | 1.925 | 0.764 | 0.031 | 1.239 | 0.000 | 0.000 | 0.235 | 6.657 |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 753 | 753 | 672 | 2737 | 1835 | 0 | 0 | 1532 | 8774 |
| normalized size | 1 | 1.00 | 0.89 | 3.63 | 2.44 | 0.00 | 0.00 | 2.03 | 11.65 |
| time (sec) | N/A | 3.143 | 1.078 | 0.037 | 1.274 | 0.000 | 0.000 | 0.201 | 7.244 |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 437 | 647 | 599 | 1864 | 0 | 636 | 669 |
| normalized size | 1 | 1.00 | 1.87 | 2.76 | 2.56 | 7.97 | 0.00 | 2.72 | 2.86 |
| time (sec) | N/A | 0.291 | 0.306 | 0.013 | 1.042 | 1.457 | 0.000 | 0.178 | 4.382 |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 288 | 350 | 464 | 457 | 1378 | 0 | 475 | 402 |
| normalized size | 1 | 1.13 | 1.38 | 1.83 | 1.80 | 5.43 | 0.00 | 1.87 | 1.58 |
| time (sec) | N/A | 0.542 | 0.297 | 0.011 | 1.018 | 0.833 | 0.000 | 0.166 | 4.069 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 266 | 333 | 323 | 1062 | 0 | 328 | 287 |
| normalized size | 1 | 1.00 | 1.18 | 1.48 | 1.44 | 4.72 | 0.00 | 1.46 | 1.28 |
| time (sec) | N/A | 0.398 | 0.161 | 0.012 | 1.005 | 0.913 | 0.000 | 0.166 | 0.227 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 171 | 182 | 208 | 636 | 298 | 194 | 164 |
| normalized size | 1 | 1.00 | 1.04 | 1.10 | 1.26 | 3.85 | 1.81 | 1.18 | 0.99 |
| time (sec) | N/A | 0.136 | 0.134 | 0.010 | 0.981 | 1.202 | 139.971 | 0.157 | 3.936 |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 112 | 113 | 133 | 430 | 196 | 109 | 116 |
| normalized size | 1 | 1.00 | 0.89 | 0.90 | 1.06 | 3.41 | 1.56 | 0.87 | 0.92 |
| time (sec) | N/A | 0.078 | 0.088 | 0.010 | 0.976 | 0.675 | 2.078 | 0.180 | 3.895 |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 29 | 30 | 29 | 46 | 29 | 29 | 30 |
| normalized size | 1 | 1.00 | 0.67 | 0.70 | 0.67 | 1.07 | 0.67 | 0.67 | 0.70 |
| time (sec) | N/A | 0.050 | 0.019 | 0.008 | 0.953 | 0.929 | 0.129 | 0.148 | 0.035 |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 27 | 24 | 23 | 40 | 20 | 23 | 23 |
| normalized size | 1 | 1.00 | 0.90 | 0.80 | 0.77 | 1.33 | 0.67 | 0.77 | 0.77 |
| time (sec) | N/A | 0.040 | 0.021 | 0.008 | 0.959 | 0.970 | 0.124 | 0.211 | 0.035 |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 23 | 24 | 23 | 33 | 20 | 23 | 25 |
| normalized size | 1 | 1.00 | 0.79 | 0.83 | 0.79 | 1.14 | 0.69 | 0.79 | 0.86 |
| time (sec) | N/A | 0.025 | 0.009 | 0.005 | 0.963 | 0.962 | 0.127 | 0.153 | 0.031 |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 13 | 12 | 20 | 10 | 12 | 14 |
| normalized size | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 1.43 | 0.71 | 0.86 | 1.00 |
| time (sec) | N/A | 0.011 | 0.007 | 0.005 | 0.955 | 0.866 | 0.114 | 0.148 | 3.797 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 28 | 26 | 25 | 41 | 24 | 26 | 32 |
| normalized size | 1 | 1.00 | 0.90 | 0.84 | 0.81 | 1.32 | 0.77 | 0.84 | 1.03 |
| time (sec) | N/A | 0.039 | 0.010 | 0.008 | 0.956 | 1.023 | 0.159 | 0.154 | 0.042 |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 30 | 34 | 49 | 31 | 35 | 38 |
| normalized size | 1 | 1.00 | 1.00 | 0.91 | 1.03 | 1.48 | 0.94 | 1.06 | 1.15 |
| time (sec) | N/A | 0.045 | 0.016 | 0.009 | 0.952 | 0.711 | 0.161 | 0.184 | 3.808 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 39 | 38 | 41 | 61 | 42 | 43 | 47 |
| normalized size | 1 | 1.00 | 0.87 | 0.84 | 0.91 | 1.36 | 0.93 | 0.96 | 1.04 |
| time (sec) | N/A | 0.063 | 0.016 | 0.010 | 0.971 | 1.068 | 0.176 | 0.162 | 0.038 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 22 | 12 | 13 | 12 | 18 | 8 | 12 | 12 |
| normalized size | 1 | 1.83 | 1.00 | 1.08 | 1.00 | 1.50 | 0.67 | 1.00 | 1.00 |
| time (sec) | N/A | 0.007 | 0.012 | 0.004 | 0.951 | 1.037 | 0.115 | 0.150 | 0.031 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 21 | 21 | 25 | 20 | 21 | 21 |
| normalized size | 1 | 1.00 | 1.00 | 0.78 | 0.78 | 0.93 | 0.74 | 0.78 | 0.78 |
| time (sec) | N/A | 0.014 | 0.011 | 0.006 | 0.965 | 0.833 | 0.127 | 0.183 | 3.830 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 387 | 362 | 661 | 436 | 855 | 1088 | 475 | -1 |
| normalized size | 1 | 0.99 | 0.93 | 1.69 | 1.12 | 2.19 | 2.79 | 1.22 | -0.00 |
| time (sec) | N/A | 0.830 | 0.457 | 0.018 | 0.466 | 0.968 | 28.367 | 0.227 | 0.000 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 279 | 256 | 446 | 305 | 595 | 738 | 321 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 1.59 | 1.09 | 2.12 | 2.64 | 1.15 | -0.00 |
| time (sec) | N/A | 0.498 | 0.646 | 0.009 | 0.463 | 1.046 | 21.001 | 0.206 | 0.000 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 153 | 230 | 169 | 329 | 384 | 180 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 1.31 | 0.97 | 1.88 | 2.19 | 1.03 | -0.01 |
| time (sec) | N/A | 0.268 | 0.417 | 0.007 | 0.449 | 1.003 | 11.876 | 0.208 | 0.000 |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 98 | 111 | 96 | 190 | 170 | 87 | -1 |
| normalized size | 1 | 1.00 | 0.92 | 1.05 | 0.91 | 1.79 | 1.60 | 0.82 | -0.01 |
| time (sec) | N/A | 0.064 | 0.204 | 0.005 | 0.451 | 0.946 | 6.903 | 0.184 | 0.000 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 224 | 1265 | 362 | 0 | 0 | 278 | -1 |
| normalized size | 1 | 1.00 | 1.09 | 6.14 | 1.76 | 0.00 | 0.00 | 1.35 | -0.00 |
| time (sec) | N/A | 0.392 | 0.436 | 0.018 | 0.606 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 303 | 264 | 2818 | 478 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.98 | 0.86 | 9.15 | 1.55 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.509 | 0.258 | 0.016 | 0.647 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 295 | 318 | 4432 | 927 | 0 | 0 | 923 | -1 |
| normalized size | 1 | 1.00 | 1.07 | 14.97 | 3.13 | 0.00 | 0.00 | 3.12 | -0.00 |
| time (sec) | N/A | 0.546 | 0.561 | 0.016 | 0.703 | 0.000 | 0.000 | 0.353 | 0.000 |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 314 | 314 | 382 | 5565 | 1772 | 0 | 0 | 1719 | -1 |
| normalized size | 1 | 1.00 | 1.22 | 17.72 | 5.64 | 0.00 | 0.00 | 5.47 | -0.00 |
| time (sec) | N/A | 0.505 | 0.825 | 0.015 | 0.841 | 0.000 | 0.000 | 0.503 | 0.000 |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 312 | 439 | 7237 | 3404 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.40 | 23.12 | 10.88 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.429 | 1.305 | 0.020 | 1.140 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 433 | 432 | 583 | 8546 | 5793 | 0 | 0 | 4212 | -1 |
| normalized size | 1 | 1.00 | 1.35 | 19.74 | 13.38 | 0.00 | 0.00 | 9.73 | -0.00 |
| time (sec) | N/A | 0.743 | 1.619 | 0.025 | 1.525 | 0.000 | 0.000 | 0.656 | 0.000 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 462 | 462 | 481 | 794 | 525 | 1177 | 1916 | 652 | -1 |
| normalized size | 1 | 1.00 | 1.04 | 1.72 | 1.14 | 2.55 | 4.15 | 1.41 | -0.00 |
| time (sec) | N/A | 1.134 | 0.541 | 0.018 | 0.456 | 1.482 | 72.414 | 0.274 | 0.000 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 345 | 346 | 552 | 380 | 831 | 1304 | 452 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 1.60 | 1.10 | 2.40 | 3.77 | 1.31 | -0.00 |
| time (sec) | N/A | 0.524 | 1.095 | 0.010 | 0.452 | 1.344 | 54.496 | 0.264 | 0.000 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 212 | 209 | 287 | 211 | 477 | 768 | 264 | -1 |
| normalized size | 1 | 1.00 | 0.98 | 1.35 | 0.99 | 2.24 | 3.61 | 1.24 | -0.00 |
| time (sec) | N/A | 0.271 | 0.633 | 0.005 | 0.447 | 1.022 | 27.893 | 0.254 | 0.000 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 125 | 146 | 131 | 262 | 348 | 129 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 1.07 | 0.96 | 1.91 | 2.54 | 0.94 | -0.01 |
| time (sec) | N/A | 0.083 | 0.257 | 0.006 | 0.448 | 1.195 | 17.013 | 0.234 | 0.000 |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 348 | 2420 | 632 | 0 | 0 | 551 | -1 |
| normalized size | 1 | 1.00 | 1.07 | 7.42 | 1.94 | 0.00 | 0.00 | 1.69 | -0.00 |
| time (sec) | N/A | 0.766 | 1.211 | 0.013 | 0.789 | 0.000 | 0.000 | 0.293 | 0.000 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 428 | 392 | 5121 | 708 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 0.91 | 11.85 | 1.64 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.901 | 0.525 | 0.017 | 0.783 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 488 | 480 | 435 | 7817 | 1299 | 0 | 0 | 1036 | -1 |
| normalized size | 1 | 0.98 | 0.89 | 16.02 | 2.66 | 0.00 | 0.00 | 2.12 | -0.00 |
| time (sec) | N/A | 0.921 | 0.655 | 0.017 | 0.878 | 0.000 | 0.000 | 0.433 | 0.000 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 475 | 469 | 517 | 9835 | 2415 | 0 | 0 | 1900 | -1 |
| normalized size | 1 | 0.99 | 1.09 | 20.71 | 5.08 | 0.00 | 0.00 | 4.00 | -0.00 |
| time (sec) | N/A | 0.845 | 1.233 | 0.021 | 1.095 | 0.000 | 0.000 | 0.612 | 0.000 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 511 | 511 | 575 | 12481 | 4326 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.13 | 24.42 | 8.47 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.092 | 2.154 | 0.025 | 1.503 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 507 | 507 | 639 | 14169 | 6650 | 0 | 0 | 4408 | -1 |
| normalized size | 1 | 1.00 | 1.26 | 27.95 | 13.12 | 0.00 | 0.00 | 8.69 | -0.00 |
| time (sec) | N/A | 0.860 | 2.268 | 0.029 | 1.893 | 0.000 | 0.000 | 1.327 | 0.000 |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 404 | 403 | 696 | 17026 | 10724 | 0 | 0 | 6122 | -1 |
| normalized size | 1 | 1.00 | 1.72 | 42.14 | 26.54 | 0.00 | 0.00 | 15.15 | -0.00 |
| time (sec) | N/A | 0.552 | 2.472 | 0.038 | 2.729 | 0.000 | 0.000 | 1.123 | 0.000 |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-1) | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 532 | 531 | 863 | 19093 | 0 | 0 | 0 | 7936 | -1 |
| normalized size | 1 | 1.00 | 1.62 | 35.89 | 0.00 | 0.00 | 0.00 | 14.92 | -0.00 |
| time (sec) | N/A | 0.889 | 2.495 | 0.051 | 0.000 | 0.000 | 0.000 | 1.118 | 0.000 |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 150 | 181 | 166 | 333 | 510 | 168 | -1 |
| normalized size | 1 | 1.00 | 0.89 | 1.08 | 0.99 | 1.98 | 3.04 | 1.00 | -0.01 |
| time (sec) | N/A | 0.102 | 0.311 | 0.009 | 0.453 | 1.317 | 32.916 | 0.213 | 0.000 |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 323 | 252 | 528 | 349 | 559 | 796 | 314 | -1 |
| normalized size | 1 | 0.99 | 0.78 | 1.62 | 1.07 | 1.72 | 2.45 | 0.97 | -0.00 |
| time (sec) | N/A | 0.664 | 0.355 | 0.015 | 0.452 | 1.221 | 22.204 | 0.255 | 0.000 |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 222 | 164 | 339 | 230 | 381 | 518 | 206 | -1 |
| normalized size | 1 | 1.00 | 0.74 | 1.52 | 1.03 | 1.71 | 2.32 | 0.92 | -0.00 |
| time (sec) | N/A | 0.372 | 0.233 | 0.010 | 0.453 | 0.684 | 15.782 | 0.230 | 0.000 |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 135 | 96 | 172 | 126 | 199 | 282 | 110 | 227 |
| normalized size | 1 | 0.99 | 0.71 | 1.26 | 0.93 | 1.46 | 2.07 | 0.81 | 1.67 |
| time (sec) | N/A | 0.179 | 0.105 | 0.006 | 0.436 | 0.873 | 9.024 | 0.211 | 5.171 |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 63 | 76 | 61 | 124 | 150 | 58 | 107 |
| normalized size | 1 | 1.00 | 0.85 | 1.03 | 0.82 | 1.68 | 2.03 | 0.78 | 1.45 |
| time (sec) | N/A | 0.048 | 0.038 | 0.006 | 0.435 | 0.602 | 3.496 | 0.197 | 4.559 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | F(-1) | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 125 | 453 | 218 | 0 | 0 | 138 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 3.48 | 1.68 | 0.00 | 0.00 | 1.06 | -0.01 |
| time (sec) | N/A | 0.174 | 0.219 | 0.012 | 0.564 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 218 | 923 | 419 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.30 | 5.49 | 2.49 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.235 | 0.363 | 0.015 | 0.578 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 224 | 254 | 1574 | 896 | 1088 | 0 | 848 | -1 |
| normalized size | 1 | 1.00 | 1.13 | 7.00 | 3.98 | 4.84 | 0.00 | 3.77 | -0.00 |
| time (sec) | N/A | 0.292 | 0.455 | 0.016 | 0.672 | 22.087 | 0.000 | 0.263 | 0.000 |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 228 | 246 | 516 | 346 | 758 | 0 | 339 | -1 |
| normalized size | 1 | 1.00 | 1.07 | 2.25 | 1.51 | 3.31 | 0.00 | 1.48 | -0.00 |
| time (sec) | N/A | 0.325 | 0.450 | 0.016 | 0.457 | 1.223 | 0.000 | 0.253 | 0.000 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 177 | 327 | 227 | 530 | 0 | 219 | -1 |
| normalized size | 1 | 1.00 | 1.19 | 2.19 | 1.52 | 3.56 | 0.00 | 1.47 | -0.01 |
| time (sec) | N/A | 0.184 | 0.284 | 0.010 | 0.448 | 0.891 | 0.000 | 0.247 | 0.000 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 102 | 163 | 126 | 278 | 209 | 116 | 151 |
| normalized size | 1 | 1.00 | 1.02 | 1.63 | 1.26 | 2.78 | 2.09 | 1.16 | 1.51 |
| time (sec) | N/A | 0.087 | 0.137 | 0.005 | 0.437 | 1.143 | 18.840 | 0.250 | 5.280 |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 74 | 69 | 61 | 181 | 87 | 63 | 68 |
| normalized size | 1 | 1.00 | 1.21 | 1.13 | 1.00 | 2.97 | 1.43 | 1.03 | 1.11 |
| time (sec) | N/A | 0.036 | 0.062 | 0.006 | 0.430 | 0.634 | 8.859 | 0.201 | 4.332 |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 137 | 862 | 453 | 721 | 0 | 294 | -1 |
| normalized size | 1 | 1.00 | 0.99 | 6.25 | 3.28 | 5.22 | 0.00 | 2.13 | -0.01 |
| time (sec) | N/A | 0.141 | 0.177 | 0.015 | 0.622 | 4.004 | 0.000 | 0.294 | 0.000 |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 239 | 285 | 1663 | 1085 | 1573 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.19 | 6.96 | 4.54 | 6.58 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.418 | 0.608 | 0.017 | 0.767 | 7.179 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 372 | 404 | 2584 | 2254 | 2853 | 0 | 1440 | -1 |
| normalized size | 1 | 0.99 | 1.08 | 6.91 | 6.03 | 7.63 | 0.00 | 3.85 | -0.00 |
| time (sec) | N/A | 1.028 | 1.184 | 0.018 | 1.017 | 33.904 | 0.000 | 0.394 | 0.000 |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 50 | 47 | 83 | 68 | 194 | 48 | 59 |
| normalized size | 1 | 1.00 | 0.75 | 0.70 | 1.24 | 1.01 | 2.90 | 0.72 | 0.88 |
| time (sec) | N/A | 0.042 | 0.032 | 0.004 | 0.435 | 0.788 | 17.131 | 0.211 | 4.222 |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 71 | 72 | 118 | 103 | 638 | 80 | 93 |
| normalized size | 1 | 1.00 | 0.73 | 0.74 | 1.22 | 1.06 | 6.58 | 0.82 | 0.96 |
| time (sec) | N/A | 0.057 | 0.050 | 0.004 | 0.441 | 0.895 | 37.219 | 0.264 | 4.281 |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 92 | 96 | 153 | 137 | 1880 | 112 | 115 |
| normalized size | 1 | 1.00 | 0.72 | 0.76 | 1.20 | 1.08 | 14.80 | 0.88 | 0.91 |
| time (sec) | N/A | 0.087 | 0.065 | 0.006 | 0.451 | 0.855 | 78.297 | 0.266 | 4.375 |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 54 | 79 | 78 | 60 | 94 | 54 | 45 |
| normalized size | 1 | 1.00 | 0.51 | 0.75 | 0.74 | 0.57 | 0.89 | 0.51 | 0.42 |
| time (sec) | N/A | 0.114 | 0.065 | 0.013 | 0.948 | 1.009 | 2.203 | 0.226 | 0.050 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 48 | 65 | 64 | 54 | 75 | 48 | 40 |
| normalized size | 1 | 1.00 | 0.59 | 0.79 | 0.78 | 0.66 | 0.91 | 0.59 | 0.49 |
| time (sec) | N/A | 0.088 | 0.043 | 0.005 | 0.959 | 0.899 | 1.185 | 0.202 | 4.099 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 44 | 51 | 50 | 49 | 63 | 44 | 35 |
| normalized size | 1 | 1.00 | 0.71 | 0.82 | 0.81 | 0.79 | 1.02 | 0.71 | 0.56 |
| time (sec) | N/A | 0.052 | 0.027 | 0.008 | 0.962 | 0.802 | 0.552 | 0.194 | 0.034 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 60 | 55 | 58 | 88 | 0 | 99 | 61 |
| normalized size | 1 | 1.00 | 0.90 | 0.82 | 0.87 | 1.31 | 0.00 | 1.48 | 0.91 |
| time (sec) | N/A | 0.081 | 0.028 | 0.009 | 0.962 | 0.914 | 0.000 | 0.235 | 0.189 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 64 | 65 | 65 | 106 | 0 | 48 | 68 |
| normalized size | 1 | 1.00 | 0.90 | 0.92 | 0.92 | 1.49 | 0.00 | 0.68 | 0.96 |
| time (sec) | N/A | 0.070 | 0.101 | 0.011 | 0.969 | 1.001 | 0.000 | 0.320 | 0.115 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 55 | 74 | 76 | 89 | 0 | 180 | 77 |
| normalized size | 1 | 1.00 | 0.71 | 0.96 | 0.99 | 1.16 | 0.00 | 2.34 | 1.00 |
| time (sec) | N/A | 0.067 | 0.064 | 0.012 | 0.971 | 0.775 | 0.000 | 0.257 | 0.111 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 58 | 79 | 78 | 76 | 0 | 54 | 110 |
| normalized size | 1 | 1.00 | 0.67 | 0.91 | 0.90 | 0.87 | 0.00 | 0.62 | 1.26 |
| time (sec) | N/A | 0.104 | 0.055 | 0.014 | 0.959 | 0.852 | 0.000 | 0.214 | 0.059 |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 53 | 65 | 64 | 72 | 0 | 49 | 105 |
| normalized size | 1 | 1.00 | 0.75 | 0.92 | 0.90 | 1.01 | 0.00 | 0.69 | 1.48 |
| time (sec) | N/A | 0.084 | 0.050 | 0.006 | 0.963 | 0.865 | 0.000 | 0.196 | 4.067 |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 48 | 51 | 50 | 67 | 114 | 44 | 100 |
| normalized size | 1 | 1.00 | 0.87 | 0.93 | 0.91 | 1.22 | 2.07 | 0.80 | 1.82 |
| time (sec) | N/A | 0.044 | 0.029 | 0.005 | 0.959 | 0.764 | 15.959 | 0.235 | 0.037 |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 51 | 88 | 58 | 83 | 0 | 82 | 106 |
| normalized size | 1 | 1.00 | 0.96 | 1.66 | 1.09 | 1.57 | 0.00 | 1.55 | 2.00 |
| time (sec) | N/A | 0.060 | 0.023 | 0.009 | 0.963 | 0.732 | 0.000 | 0.208 | 0.136 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 71 | 98 | 84 | 103 | 0 | 168 | 157 |
| normalized size | 1 | 1.00 | 0.95 | 1.31 | 1.12 | 1.37 | 0.00 | 2.24 | 2.09 |
| time (sec) | N/A | 0.077 | 0.042 | 0.012 | 0.965 | 1.120 | 0.000 | 0.274 | 4.145 |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 78 | 107 | 124 | 119 | 0 | 196 | 180 |
| normalized size | 1 | 1.00 | 0.80 | 1.10 | 1.28 | 1.23 | 0.00 | 2.02 | 1.86 |
| time (sec) | N/A | 0.123 | 0.083 | 0.011 | 0.981 | 0.820 | 0.000 | 0.245 | 4.169 |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 63 | 91 | 105 | 87 | 0 | 53 | 212 |
| normalized size | 1 | 1.00 | 0.86 | 1.25 | 1.44 | 1.19 | 0.00 | 0.73 | 2.90 |
| time (sec) | N/A | 0.085 | 0.068 | 0.014 | 0.951 | 0.851 | 0.000 | 0.184 | 0.057 |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 58 | 77 | 91 | 83 | 0 | 48 | 200 |
| normalized size | 1 | 1.00 | 0.97 | 1.28 | 1.52 | 1.38 | 0.00 | 0.80 | 3.33 |
| time (sec) | N/A | 0.075 | 0.051 | 0.008 | 0.962 | 0.780 | 0.000 | 0.194 | 0.050 |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 30 | 27 | 50 | 40 | 180 | 25 | 185 |
| normalized size | 1 | 1.00 | 0.73 | 0.66 | 1.22 | 0.98 | 4.39 | 0.61 | 4.51 |
| time (sec) | N/A | 0.053 | 0.017 | 0.005 | 0.425 | 0.771 | 77.502 | 0.336 | 4.107 |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 58 | 133 | 81 | 103 | 0 | 91 | 218 |
| normalized size | 1 | 1.00 | 0.79 | 1.82 | 1.11 | 1.41 | 0.00 | 1.25 | 2.99 |
| time (sec) | N/A | 0.085 | 0.048 | 0.009 | 0.972 | 0.578 | 0.000 | 0.214 | 0.135 |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 91 | 143 | 107 | 134 | 0 | 233 | 270 |
| normalized size | 1 | 1.00 | 0.96 | 1.51 | 1.13 | 1.41 | 0.00 | 2.45 | 2.84 |
| time (sec) | N/A | 0.169 | 0.055 | 0.013 | 0.977 | 0.592 | 0.000 | 0.517 | 4.312 |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 75 | 140 | 147 | 149 | 0 | 183 | 301 |
| normalized size | 1 | 1.00 | 0.64 | 1.20 | 1.26 | 1.27 | 0.00 | 1.56 | 2.57 |
| time (sec) | N/A | 0.206 | 0.100 | 0.014 | 0.987 | 0.903 | 0.000 | 0.295 | 4.190 |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 417 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.604 | 1.148 | 0.114 | 0.000 | 1.009 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 403 | 401 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.453 | 0.692 | 0.028 | 0.000 | 0.839 | 0.000 | 0.000 | 0.000 |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 474 | 474 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.519 | 2.850 | 0.046 | 0.000 | 1.519 | 0.000 | 0.000 | 0.000 |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 165 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.407 | 0.319 | 0.177 | 0.000 | 0.952 | 0.000 | 0.000 | 0.000 |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 256 | 343 | 263 | 308 | 320 | 308 | 244 |
| normalized size | 1 | 1.00 | 1.01 | 1.35 | 1.04 | 1.21 | 1.26 | 1.21 | 0.96 |
| time (sec) | N/A | 0.326 | 0.094 | 0.001 | 0.436 | 0.710 | 0.136 | 0.155 | 0.126 |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 163 | 223 | 165 | 187 | 197 | 187 | 149 |
| normalized size | 1 | 1.00 | 1.01 | 1.39 | 1.02 | 1.16 | 1.22 | 1.16 | 0.93 |
| time (sec) | N/A | 0.193 | 0.046 | 0.001 | 0.429 | 0.651 | 0.108 | 0.169 | 0.071 |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 96 | 90 | 87 | 99 | 102 | 99 | 88 |
| normalized size | 1 | 1.00 | 1.00 | 0.94 | 0.91 | 1.03 | 1.06 | 1.03 | 0.92 |
| time (sec) | N/A | 0.099 | 0.023 | 0.001 | 0.430 | 0.834 | 0.089 | 0.155 | 4.086 |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 39 | 38 | 40 | 42 | 40 | 39 |
| normalized size | 1 | 1.00 | 1.00 | 0.85 | 0.83 | 0.87 | 0.91 | 0.87 | 0.85 |
| time (sec) | N/A | 0.030 | 0.009 | 0.000 | 0.430 | 0.735 | 0.069 | 0.180 | 0.026 |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 84 | 140 | 0 | 265 | 413 | 78 | 224 |
| normalized size | 1 | 1.00 | 1.04 | 1.73 | 0.00 | 3.27 | 5.10 | 0.96 | 2.77 |
| time (sec) | N/A | 0.100 | 0.092 | 0.006 | 0.000 | 0.772 | 1.213 | 0.154 | 0.194 |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 98 | 146 | 0 | 511 | 376 | 108 | 172 |
| normalized size | 1 | 1.00 | 0.98 | 1.46 | 0.00 | 5.11 | 3.76 | 1.08 | 1.72 |
| time (sec) | N/A | 0.068 | 0.081 | 0.008 | 0.000 | 1.263 | 1.212 | 0.159 | 4.533 |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 160 | 373 | 0 | 1199 | 774 | 217 | 401 |
| normalized size | 1 | 1.00 | 0.99 | 2.32 | 0.00 | 7.45 | 4.81 | 1.35 | 2.49 |
| time (sec) | N/A | 0.114 | 0.205 | 0.011 | 0.000 | 0.894 | 2.362 | 0.181 | 4.173 |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 206 | 204 | 643 | 0 | 2103 | 1224 | 407 | 698 |
| normalized size | 1 | 1.00 | 0.99 | 3.12 | 0.00 | 10.21 | 5.94 | 1.98 | 3.39 |
| time (sec) | N/A | 0.190 | 0.408 | 0.015 | 0.000 | 0.967 | 4.220 | 0.172 | 4.358 |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 591 | 591 | 585 | 1738 | 0 | 2150 | 4972 | 771 | 967 |
| normalized size | 1 | 1.00 | 0.99 | 2.94 | 0.00 | 3.64 | 8.41 | 1.30 | 1.64 |
| time (sec) | N/A | 1.428 | 0.651 | 0.012 | 0.000 | 2.912 | 118.420 | 0.171 | 5.462 |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 348 | 345 | 1028 | 0 | 1273 | 2839 | 426 | 557 |
| normalized size | 1 | 1.00 | 0.99 | 2.95 | 0.00 | 3.66 | 8.16 | 1.22 | 1.60 |
| time (sec) | N/A | 0.677 | 0.380 | 0.009 | 0.000 | 1.110 | 47.193 | 0.161 | 4.684 |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 173 | 510 | 0 | 654 | 1265 | 201 | 273 |
| normalized size | 1 | 1.00 | 0.98 | 2.88 | 0.00 | 3.69 | 7.15 | 1.14 | 1.54 |
| time (sec) | N/A | 0.350 | 0.205 | 0.006 | 0.000 | 0.933 | 14.462 | 0.194 | 0.530 |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 95 | 196 | 0 | 302 | 488 | 89 | 132 |
| normalized size | 1 | 1.00 | 1.03 | 2.13 | 0.00 | 3.28 | 5.30 | 0.97 | 1.43 |
| time (sec) | N/A | 0.156 | 0.073 | 0.004 | 0.000 | 0.771 | 2.139 | 0.156 | 0.253 |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 196 | 193 | 622 | 0 | 625 | 0 | 204 | 2467 |
| normalized size | 1 | 1.00 | 0.98 | 3.17 | 0.00 | 3.19 | 0.00 | 1.04 | 12.59 |
| time (sec) | N/A | 0.349 | 0.206 | 0.009 | 0.000 | 83.612 | 0.000 | 0.163 | 10.450 |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 281 | 1125 | 0 | 0 | 0 | 449 | 3991 |
| normalized size | 1 | 1.00 | 0.89 | 3.56 | 0.00 | 0.00 | 0.00 | 1.42 | 12.63 |
| time (sec) | N/A | 0.761 | 0.536 | 0.013 | 0.000 | 0.000 | 0.000 | 0.196 | 14.713 |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 509 | 509 | 504 | 1945 | 0 | 0 | 0 | 1002 | 12784 |
| normalized size | 1 | 1.00 | 0.99 | 3.82 | 0.00 | 0.00 | 0.00 | 1.97 | 25.12 |
| time (sec) | N/A | 1.251 | 0.766 | 0.019 | 0.000 | 0.000 | 0.000 | 0.206 | 6.819 |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 288 | 288 | 398 | 1712 | 0 | 2771 | 0 | 540 | 742 |
| normalized size | 1 | 1.00 | 1.38 | 5.94 | 0.00 | 9.62 | 0.00 | 1.88 | 2.58 |
| time (sec) | N/A | 0.700 | 0.811 | 0.016 | 0.000 | 2.482 | 0.000 | 0.184 | 5.779 |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 225 | 500 | 0 | 1413 | 1535 | 285 | 376 |
| normalized size | 1 | 1.00 | 1.26 | 2.81 | 0.00 | 7.94 | 8.62 | 1.60 | 2.11 |
| time (sec) | N/A | 0.266 | 0.462 | 0.012 | 0.000 | 1.270 | 60.262 | 0.167 | 5.039 |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 114 | 194 | 0 | 632 | 459 | 125 | 203 |
| normalized size | 1 | 1.00 | 0.97 | 1.64 | 0.00 | 5.36 | 3.89 | 1.06 | 1.72 |
| time (sec) | N/A | 0.098 | 0.106 | 0.008 | 0.000 | 0.936 | 2.236 | 0.160 | 3.896 |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 405 | 3202 | 0 | 0 | 0 | 860 | 13698 |
| normalized size | 1 | 1.00 | 1.00 | 7.87 | 0.00 | 0.00 | 0.00 | 2.11 | 33.66 |
| time (sec) | N/A | 1.087 | 0.905 | 0.023 | 0.000 | 0.000 | 0.000 | 0.193 | 6.700 |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 673 | 673 | 650 | 4716 | 0 | 0 | 0 | 1437 | 26278 |
| normalized size | 1 | 1.00 | 0.97 | 7.01 | 0.00 | 0.00 | 0.00 | 2.14 | 39.05 |
| time (sec) | N/A | 2.559 | 2.206 | 0.038 | 0.000 | 0.000 | 0.000 | 0.327 | 8.926 |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 60 | 53 | 51 | 75 | 60 | 51 | 55 |
| normalized size | 1 | 1.00 | 0.97 | 0.85 | 0.82 | 1.21 | 0.97 | 0.82 | 0.89 |
| time (sec) | N/A | 0.074 | 0.036 | 0.008 | 0.955 | 1.360 | 0.158 | 0.158 | 0.041 |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 55 | 46 | 46 | 70 | 54 | 46 | 48 |
| normalized size | 1 | 1.00 | 1.00 | 0.84 | 0.84 | 1.27 | 0.98 | 0.84 | 0.87 |
| time (sec) | N/A | 0.066 | 0.026 | 0.007 | 0.950 | 0.805 | 0.151 | 0.159 | 0.043 |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 52 | 45 | 43 | 60 | 53 | 43 | 59 |
| normalized size | 1 | 1.00 | 1.00 | 0.87 | 0.83 | 1.15 | 1.02 | 0.83 | 1.13 |
| time (sec) | N/A | 0.052 | 0.021 | 0.005 | 0.956 | 1.345 | 0.148 | 0.151 | 3.838 |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 39 | 34 | 32 | 41 | 41 | 32 | 35 |
| normalized size | 1 | 1.00 | 0.95 | 0.83 | 0.78 | 1.00 | 1.00 | 0.78 | 0.85 |
| time (sec) | N/A | 0.033 | 0.022 | 0.005 | 0.945 | 1.268 | 0.137 | 0.157 | 3.833 |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 48 | 47 | 72 | 54 | 48 | 58 |
| normalized size | 1 | 1.00 | 1.00 | 0.86 | 0.84 | 1.29 | 0.96 | 0.86 | 1.04 |
| time (sec) | N/A | 0.089 | 0.026 | 0.009 | 0.953 | 1.091 | 0.184 | 0.151 | 0.100 |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 61 | 55 | 54 | 85 | 65 | 55 | 68 |
| normalized size | 1 | 1.00 | 1.00 | 0.90 | 0.89 | 1.39 | 1.07 | 0.90 | 1.11 |
| time (sec) | N/A | 0.128 | 0.023 | 0.011 | 0.958 | 1.199 | 0.198 | 0.150 | 4.132 |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 66 | 60 | 63 | 98 | 71 | 63 | 75 |
| normalized size | 1 | 1.00 | 0.97 | 0.88 | 0.93 | 1.44 | 1.04 | 0.93 | 1.10 |
| time (sec) | N/A | 0.110 | 0.030 | 0.011 | 0.954 | 0.923 | 0.217 | 0.158 | 0.098 |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 10 | 10 | 7 | 8 | 10 |
| normalized size | 1 | 1.00 | 1.00 | 1.10 | 1.00 | 1.00 | 0.70 | 0.80 | 1.00 |
| time (sec) | N/A | 0.011 | 0.006 | 0.004 | 0.430 | 0.822 | 0.095 | 0.149 | 0.045 |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 28 | 27 | 27 | 36 | 27 | 29 |
| normalized size | 1 | 1.00 | 1.00 | 0.90 | 0.87 | 0.87 | 1.16 | 0.87 | 0.94 |
| time (sec) | N/A | 0.032 | 0.008 | 0.003 | 0.953 | 1.031 | 0.117 | 0.173 | 0.032 |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 22 | 21 | 21 | 22 | 21 | 21 |
| normalized size | 1 | 1.00 | 1.00 | 0.96 | 0.91 | 0.91 | 0.96 | 0.91 | 0.91 |
| time (sec) | N/A | 0.035 | 0.005 | 0.005 | 0.953 | 0.655 | 0.110 | 0.151 | 0.042 |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 19 | 18 | 17 | 25 | 14 | 18 | 17 |
| normalized size | 1 | 1.00 | 0.90 | 0.86 | 0.81 | 1.19 | 0.67 | 0.86 | 0.81 |
| time (sec) | N/A | 0.013 | 0.009 | 0.007 | 0.420 | 0.785 | 0.087 | 0.166 | 0.042 |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 14 | 14 | 14 | 16 | 14 |
| normalized size | 1 | 1.00 | 1.00 | 0.83 | 0.78 | 0.78 | 0.78 | 0.89 | 0.78 |
| time (sec) | N/A | 0.018 | 0.005 | 0.006 | 0.427 | 0.691 | 0.111 | 0.149 | 3.922 |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 13 | 12 | 12 | 12 | 14 | 12 |
| normalized size | 1 | 1.00 | 1.00 | 0.93 | 0.86 | 0.86 | 0.86 | 1.00 | 0.86 |
| time (sec) | N/A | 0.017 | 0.006 | 0.006 | 0.426 | 0.861 | 0.114 | 0.156 | 3.851 |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 31 | 22 | 21 | 21 | 22 | 21 | 17 |
| normalized size | 1 | 1.00 | 1.15 | 0.81 | 0.78 | 0.78 | 0.81 | 0.78 | 0.63 |
| time (sec) | N/A | 0.028 | 0.005 | 0.004 | 0.943 | 0.607 | 0.123 | 0.151 | 3.800 |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 48 | 30 | 36 | 55 | 46 | 45 | 35 |
| normalized size | 1 | 1.00 | 1.00 | 0.62 | 0.75 | 1.15 | 0.96 | 0.94 | 0.73 |
| time (sec) | N/A | 0.045 | 0.075 | 0.003 | 0.956 | 0.898 | 0.123 | 0.190 | 0.111 |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 21 | 17 | 19 | 19 | 15 | 19 | 17 |
| normalized size | 1 | 1.00 | 1.00 | 0.81 | 0.90 | 0.90 | 0.71 | 0.90 | 0.81 |
| time (sec) | N/A | 0.012 | 0.008 | 0.005 | 0.425 | 0.777 | 0.119 | 0.157 | 3.841 |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 39 | 34 | 30 | 39 | 37 | 30 | 36 |
| normalized size | 1 | 1.00 | 1.00 | 0.87 | 0.77 | 1.00 | 0.95 | 0.77 | 0.92 |
| time (sec) | N/A | 0.023 | 0.027 | 0.006 | 0.958 | 0.644 | 0.136 | 0.165 | 3.835 |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 11 | 11 | 11 | 12 | 33 | 33 | 31 | 11 | 11 |
| normalized size | 1 | 1.00 | 1.00 | 1.09 | 3.00 | 3.00 | 2.82 | 1.00 | 1.00 |
| time (sec) | N/A | 0.008 | 0.006 | 0.005 | 0.436 | 0.833 | 0.132 | 0.151 | 3.800 |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 344 | 997 | 0 | 953 | 0 | 482 | -1 |
| normalized size | 1 | 1.00 | 1.29 | 3.73 | 0.00 | 3.57 | 0.00 | 1.81 | -0.00 |
| time (sec) | N/A | 0.238 | 0.861 | 0.011 | 0.000 | 1.201 | 0.000 | 0.275 | 0.000 |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 267 | 613 | 0 | 605 | 0 | 297 | -1 |
| normalized size | 1 | 1.00 | 1.26 | 2.89 | 0.00 | 2.85 | 0.00 | 1.40 | -0.00 |
| time (sec) | N/A | 0.183 | 0.581 | 0.010 | 0.000 | 1.094 | 0.000 | 0.282 | 0.000 |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 144 | 327 | 0 | 355 | 0 | 160 | 240 |
| normalized size | 1 | 1.00 | 0.92 | 2.08 | 0.00 | 2.26 | 0.00 | 1.02 | 1.53 |
| time (sec) | N/A | 0.124 | 0.208 | 0.008 | 0.000 | 0.880 | 0.000 | 0.217 | 4.262 |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 86 | 136 | 0 | 203 | 0 | 84 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.31 | 0.00 | 1.95 | 0.00 | 0.81 | -0.01 |
| time (sec) | N/A | 0.080 | 0.146 | 0.010 | 0.000 | 1.022 | 0.000 | 0.247 | 0.000 |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 104 | 169 | 0 | 403 | 0 | 110 | 108 |
| normalized size | 1 | 1.00 | 1.06 | 1.72 | 0.00 | 4.11 | 0.00 | 1.12 | 1.10 |
| time (sec) | N/A | 0.077 | 0.728 | 0.008 | 0.000 | 1.218 | 0.000 | 0.270 | 4.210 |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 107 | 137 | 0 | 242 | 0 | 193 | 127 |
| normalized size | 1 | 1.00 | 0.94 | 1.20 | 0.00 | 2.12 | 0.00 | 1.69 | 1.11 |
| time (sec) | N/A | 0.093 | 0.894 | 0.008 | 0.000 | 2.335 | 0.000 | 0.258 | 4.142 |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 148 | 316 | 0 | 563 | 0 | 452 | 578 |
| normalized size | 1 | 1.00 | 0.89 | 1.89 | 0.00 | 3.37 | 0.00 | 2.71 | 3.46 |
| time (sec) | N/A | 0.107 | 1.933 | 0.009 | 0.000 | 17.769 | 0.000 | 0.284 | 4.525 |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 199 | 555 | 0 | 978 | 0 | 805 | 1018 |
| normalized size | 1 | 1.00 | 0.90 | 2.52 | 0.00 | 4.45 | 0.00 | 3.66 | 4.63 |
| time (sec) | N/A | 0.142 | 1.738 | 0.012 | 0.000 | 52.069 | 0.000 | 0.313 | 5.063 |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 930 | 927 | 1093 | 3543 | 0 | 2817 | 0 | 1702 | 3262 |
| normalized size | 1 | 1.00 | 1.18 | 3.81 | 0.00 | 3.03 | 0.00 | 1.83 | 3.51 |
| time (sec) | N/A | 3.013 | 2.418 | 0.024 | 0.000 | 2.454 | 0.000 | 0.306 | 14.702 |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 584 | 581 | 436 | 2179 | 0 | 1791 | 0 | 1012 | 1881 |
| normalized size | 1 | 0.99 | 0.75 | 3.73 | 0.00 | 3.07 | 0.00 | 1.73 | 3.22 |
| time (sec) | N/A | 1.440 | 0.966 | 0.016 | 0.000 | 1.755 | 0.000 | 0.312 | 7.911 |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 322 | 258 | 1117 | 0 | 1009 | 0 | 495 | 877 |
| normalized size | 1 | 1.00 | 0.80 | 3.47 | 0.00 | 3.13 | 0.00 | 1.54 | 2.72 |
| time (sec) | N/A | 0.504 | 0.476 | 0.010 | 0.000 | 1.162 | 0.000 | 0.237 | 5.624 |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 173 | 453 | 0 | 465 | 0 | 212 | 320 |
| normalized size | 1 | 1.00 | 0.99 | 2.59 | 0.00 | 2.66 | 0.00 | 1.21 | 1.83 |
| time (sec) | N/A | 0.167 | 0.298 | 0.008 | 0.000 | 0.801 | 0.000 | 0.236 | 4.240 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 331 | 2549 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.03 | 7.94 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.778 | 0.786 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 453 | 486 | 6218 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 1.06 | 13.55 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.100 | 1.561 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 448 | 446 | 645 | 12139 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.44 | 27.10 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.875 | 3.584 | 0.018 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 603 | 601 | 439 | 19321 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.73 | 32.04 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.449 | 1.925 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 499 | 447 | 29161 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 58.67 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.855 | 3.901 | 0.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 824 | 826 | 1128 | 40336 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.37 | 48.95 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.335 | 6.333 | 0.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1169 | 1166 | 721 | 5881 | 0 | 4751 | 0 | 2977 | -1 |
| normalized size | 1 | 1.00 | 0.62 | 5.03 | 0.00 | 4.06 | 0.00 | 2.55 | -0.00 |
| time (sec) | N/A | 3.698 | 2.707 | 0.029 | 0.000 | 10.427 | 0.000 | 0.438 | 0.000 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 753 | 749 | 468 | 3769 | 0 | 3145 | 0 | 1852 | -1 |
| normalized size | 1 | 0.99 | 0.62 | 5.01 | 0.00 | 4.18 | 0.00 | 2.46 | -0.00 |
| time (sec) | N/A | 2.104 | 1.598 | 0.017 | 0.000 | 4.348 | 0.000 | 0.367 | 0.000 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 418 | 285 | 2026 | 0 | 1833 | 0 | 955 | -1 |
| normalized size | 1 | 1.00 | 0.68 | 4.85 | 0.00 | 4.39 | 0.00 | 2.28 | -0.00 |
| time (sec) | N/A | 0.645 | 0.785 | 0.011 | 0.000 | 1.034 | 0.000 | 0.294 | 0.000 |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 392 | 862 | 0 | 839 | 0 | 417 | -1 |
| normalized size | 1 | 1.00 | 1.66 | 3.65 | 0.00 | 3.56 | 0.00 | 1.77 | -0.00 |
| time (sec) | N/A | 0.242 | 0.691 | 0.007 | 0.000 | 0.514 | 0.000 | 0.264 | 0.000 |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 660 | 660 | 635 | 6715 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 10.17 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.825 | 2.210 | 0.017 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 754 | 750 | 756 | 14734 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 1.00 | 19.54 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.503 | 4.158 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 824 | 819 | 4162 | 26596 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 5.05 | 32.28 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.141 | 6.274 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | F(-1) | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 833 | 829 | 7806 | 40092 | 0 | 0 | 0 | 7319 | -1 |
| normalized size | 1 | 1.00 | 9.37 | 48.13 | 0.00 | 0.00 | 0.00 | 8.79 | -0.00 |
| time (sec) | N/A | 2.264 | 6.492 | 0.029 | 0.000 | 0.000 | 0.000 | 26.172 | 0.000 |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1097 | 1096 | 46895 | 57957 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 42.75 | 52.83 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.123 | 6.622 | 0.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1226 | 1223 | 1111 | 76693 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.91 | 62.56 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.997 | 6.296 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 657 | 660 | 766 | 100754 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.17 | 153.35 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.216 | 6.243 | 0.133 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1062 | 1062 | 1221 | 126612 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.15 | 119.22 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 3.004 | 6.419 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 70 | 115 | 126 | 83 | 0 | 78 | 170 |
| normalized size | 1 | 1.00 | 0.49 | 0.80 | 0.88 | 0.58 | 0.00 | 0.55 | 1.19 |
| time (sec) | N/A | 0.138 | 0.050 | 0.016 | 0.951 | 0.745 | 0.000 | 0.263 | 5.541 |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 118 | 65 | 98 | 109 | 78 | 0 | 73 | 153 |
| normalized size | 1 | 1.00 | 0.55 | 0.83 | 0.92 | 0.66 | 0.00 | 0.62 | 1.30 |
| time (sec) | N/A | 0.112 | 0.038 | 0.008 | 0.957 | 0.948 | 0.000 | 0.303 | 5.151 |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 60 | 81 | 92 | 73 | 0 | 68 | 136 |
| normalized size | 1 | 1.00 | 0.65 | 0.87 | 0.99 | 0.78 | 0.00 | 0.73 | 1.46 |
| time (sec) | N/A | 0.066 | 0.028 | 0.007 | 0.957 | 0.894 | 0.000 | 0.263 | 4.879 |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 86 | 95 | 96 | 115 | 0 | 126 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 0.94 | 0.95 | 1.14 | 0.00 | 1.25 | -0.01 |
| time (sec) | N/A | 0.117 | 0.054 | 0.010 | 0.963 | 0.907 | 0.000 | 0.394 | 0.000 |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 92 | 123 | 103 | 133 | 0 | 380 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.14 | 0.95 | 1.23 | 0.00 | 3.52 | -0.01 |
| time (sec) | N/A | 0.116 | 0.086 | 0.014 | 0.974 | 0.909 | 0.000 | 0.719 | 0.000 |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 93 | 125 | 114 | 149 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 1.09 | 0.99 | 1.30 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.117 | 0.085 | 0.013 | 0.986 | 0.847 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 80 | 134 | 155 | 93 | 0 | 88 | -1 |
| normalized size | 1 | 1.00 | 0.51 | 0.85 | 0.98 | 0.59 | 0.00 | 0.56 | -0.01 |
| time (sec) | N/A | 0.201 | 0.050 | 0.016 | 0.978 | 0.912 | 0.000 | 0.259 | 0.000 |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 75 | 117 | 138 | 88 | 0 | 83 | -1 |
| normalized size | 1 | 1.00 | 0.53 | 0.83 | 0.98 | 0.62 | 0.00 | 0.59 | -0.01 |
| time (sec) | N/A | 0.121 | 0.046 | 0.006 | 0.927 | 0.906 | 0.000 | 0.270 | 0.000 |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 70 | 100 | 121 | 83 | 0 | 78 | -1 |
| normalized size | 1 | 1.00 | 0.60 | 0.86 | 1.04 | 0.72 | 0.00 | 0.67 | -0.01 |
| time (sec) | N/A | 0.082 | 0.037 | 0.005 | 0.966 | 0.927 | 0.000 | 0.211 | 0.000 |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 96 | 151 | 125 | 125 | 0 | 136 | -1 |
| normalized size | 1 | 1.00 | 0.77 | 1.22 | 1.01 | 1.01 | 0.00 | 1.10 | -0.01 |
| time (sec) | N/A | 0.144 | 0.062 | 0.008 | 0.988 | 0.913 | 0.000 | 0.283 | 0.000 |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 103 | 179 | 132 | 143 | 0 | 570 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 1.37 | 1.01 | 1.09 | 0.00 | 4.35 | -0.01 |
| time (sec) | N/A | 0.140 | 0.092 | 0.011 | 0.978 | 0.881 | 0.000 | 0.841 | 0.000 |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 103 | 162 | 143 | 159 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.75 | 1.17 | 1.04 | 1.15 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.139 | 0.107 | 0.013 | 0.984 | 0.889 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 90 | 153 | 184 | 103 | 0 | 98 | -1 |
| normalized size | 1 | 1.00 | 0.48 | 0.81 | 0.97 | 0.54 | 0.00 | 0.52 | -0.01 |
| time (sec) | N/A | 0.156 | 0.062 | 0.017 | 0.978 | 0.886 | 0.000 | 0.201 | 0.000 |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 85 | 136 | 167 | 98 | 0 | 93 | -1 |
| normalized size | 1 | 1.00 | 0.52 | 0.83 | 1.02 | 0.60 | 0.00 | 0.57 | -0.01 |
| time (sec) | N/A | 0.133 | 0.056 | 0.007 | 0.971 | 0.954 | 0.000 | 0.212 | 0.000 |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 80 | 119 | 150 | 93 | 0 | 88 | -1 |
| normalized size | 1 | 1.00 | 0.58 | 0.86 | 1.08 | 0.67 | 0.00 | 0.63 | -0.01 |
| time (sec) | N/A | 0.092 | 0.048 | 0.006 | 0.967 | 0.895 | 0.000 | 0.187 | 0.000 |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 106 | 207 | 154 | 135 | 0 | 146 | -1 |
| normalized size | 1 | 1.00 | 0.72 | 1.41 | 1.05 | 0.92 | 0.00 | 0.99 | -0.01 |
| time (sec) | N/A | 0.160 | 0.075 | 0.010 | 0.972 | 0.877 | 0.000 | 0.307 | 0.000 |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 154 | 113 | 235 | 161 | 153 | 0 | 760 | -1 |
| normalized size | 1 | 1.00 | 0.73 | 1.53 | 1.05 | 0.99 | 0.00 | 4.94 | -0.01 |
| time (sec) | N/A | 0.163 | 0.116 | 0.012 | 0.999 | 0.962 | 0.000 | 1.097 | 0.000 |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 113 | 199 | 172 | 169 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.70 | 1.24 | 1.07 | 1.05 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.163 | 0.114 | 0.015 | 0.994 | 0.940 | 0.000 | 0.000 | 0.000 |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 693 | 692 | 588 | 1869 | 0 | 1435 | 0 | 822 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 2.70 | 0.00 | 2.07 | 0.00 | 1.19 | -0.00 |
| time (sec) | N/A | 2.102 | 1.251 | 0.019 | 0.000 | 1.733 | 0.000 | 0.323 | 0.000 |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 418 | 343 | 1069 | 0 | 861 | 0 | 457 | -1 |
| normalized size | 1 | 1.00 | 0.82 | 2.55 | 0.00 | 2.05 | 0.00 | 1.09 | -0.00 |
| time (sec) | N/A | 1.011 | 0.649 | 0.013 | 0.000 | 1.329 | 0.000 | 0.296 | 0.000 |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 215 | 505 | 0 | 461 | 0 | 210 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 2.26 | 0.00 | 2.07 | 0.00 | 0.94 | -0.00 |
| time (sec) | N/A | 0.303 | 0.234 | 0.008 | 0.000 | 1.107 | 0.000 | 0.269 | 0.000 |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 96 | 185 | 0 | 227 | 0 | 98 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.59 | 0.00 | 1.96 | 0.00 | 0.84 | -0.01 |
| time (sec) | N/A | 0.106 | 0.152 | 0.007 | 0.000 | 0.863 | 0.000 | 0.249 | 0.000 |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 172 | 599 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 3.35 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.293 | 0.281 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 239 | 227 | 1671 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 0.94 | 6.93 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.371 | 0.348 | 0.015 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 336 | 336 | 367 | 3615 | 0 | 2034 | 0 | 2307 | -1 |
| normalized size | 1 | 1.00 | 1.09 | 10.76 | 0.00 | 6.05 | 0.00 | 6.87 | -0.00 |
| time (sec) | N/A | 0.656 | 1.100 | 0.017 | 0.000 | 174.731 | 0.000 | 0.545 | 0.000 |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 504 | 502 | 715 | 2780 | 0 | 2937 | 0 | 1054 | -1 |
| normalized size | 1 | 1.00 | 1.42 | 5.52 | 0.00 | 5.83 | 0.00 | 2.09 | -0.00 |
| time (sec) | N/A | 1.176 | 1.595 | 0.020 | 0.000 | 28.674 | 0.000 | 0.350 | 0.000 |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 288 | 412 | 1557 | 0 | 1769 | 0 | 580 | -1 |
| normalized size | 1 | 1.00 | 1.43 | 5.39 | 0.00 | 6.12 | 0.00 | 2.01 | -0.00 |
| time (sec) | N/A | 0.392 | 0.827 | 0.013 | 0.000 | 20.322 | 0.000 | 0.322 | 0.000 |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 205 | 735 | 0 | 905 | 0 | 271 | -1 |
| normalized size | 1 | 1.00 | 1.10 | 3.95 | 0.00 | 4.87 | 0.00 | 1.46 | -0.01 |
| time (sec) | N/A | 0.227 | 0.734 | 0.009 | 0.000 | 14.447 | 0.000 | 0.284 | 0.000 |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 113 | 249 | 0 | 429 | 0 | 122 | 143 |
| normalized size | 1 | 1.00 | 1.02 | 2.24 | 0.00 | 3.86 | 0.00 | 1.10 | 1.29 |
| time (sec) | N/A | 0.066 | 0.295 | 0.006 | 0.000 | 1.367 | 0.000 | 0.267 | 4.535 |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 271 | 2079 | 0 | 1905 | 0 | 719 | -1 |
| normalized size | 1 | 1.00 | 1.20 | 9.24 | 0.00 | 8.47 | 0.00 | 3.20 | -0.00 |
| time (sec) | N/A | 0.266 | 0.493 | 0.014 | 0.000 | 34.395 | 0.000 | 0.286 | 0.000 |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 421 | 418 | 487 | 4930 | 0 | 5098 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 1.16 | 11.71 | 0.00 | 12.11 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.797 | 2.460 | 0.019 | 0.000 | 107.700 | 0.000 | 0.000 | 0.000 |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F(-1) | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 713 | 707 | 762 | 9126 | 0 | 0 | 0 | 5637 | -1 |
| normalized size | 1 | 0.99 | 1.07 | 12.80 | 0.00 | 0.00 | 0.00 | 7.91 | -0.00 |
| time (sec) | N/A | 2.670 | 5.291 | 0.024 | 0.000 | 0.000 | 0.000 | 0.882 | 0.000 |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 60 | 96 | 97 | 73 | 0 | 68 | -1 |
| normalized size | 1 | 1.00 | 0.50 | 0.80 | 0.81 | 0.61 | 0.00 | 0.57 | -0.01 |
| time (sec) | N/A | 0.135 | 0.045 | 0.015 | 0.954 | 0.841 | 0.000 | 0.244 | 0.000 |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 55 | 79 | 80 | 68 | 0 | 63 | -1 |
| normalized size | 1 | 1.00 | 0.58 | 0.83 | 0.84 | 0.72 | 0.00 | 0.66 | -0.01 |
| time (sec) | N/A | 0.099 | 0.031 | 0.008 | 0.972 | 0.757 | 0.000 | 0.221 | 0.000 |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 50 | 62 | 63 | 63 | 0 | 58 | -1 |
| normalized size | 1 | 1.00 | 0.71 | 0.89 | 0.90 | 0.90 | 0.00 | 0.83 | -0.01 |
| time (sec) | N/A | 0.058 | 0.021 | 0.007 | 0.960 | 0.755 | 0.000 | 0.264 | 0.000 |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 78 | 60 | 67 | 105 | 0 | 116 | -1 |
| normalized size | 1 | 1.00 | 1.00 | 0.77 | 0.86 | 1.35 | 0.00 | 1.49 | -0.01 |
| time (sec) | N/A | 0.097 | 0.046 | 0.009 | 0.973 | 0.766 | 0.000 | 0.572 | 0.000 |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 83 | 82 | 67 | 74 | 123 | 0 | 48 | -1 |
| normalized size | 1 | 1.00 | 0.99 | 0.81 | 0.89 | 1.48 | 0.00 | 0.58 | -0.01 |
| time (sec) | N/A | 0.095 | 0.053 | 0.012 | 0.979 | 0.874 | 0.000 | 0.277 | 0.000 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 69 | 74 | 82 | 96 | 0 | 204 | -1 |
| normalized size | 1 | 1.00 | 0.78 | 0.83 | 0.92 | 1.08 | 0.00 | 2.29 | -0.01 |
| time (sec) | N/A | 0.088 | 0.039 | 0.035 | 0.977 | 0.685 | 0.000 | 0.328 | 0.000 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 69 | 115 | 97 | 97 | 0 | 67 | -1 |
| normalized size | 1 | 1.00 | 0.67 | 1.12 | 0.94 | 0.94 | 0.00 | 0.65 | -0.01 |
| time (sec) | N/A | 0.124 | 0.041 | 0.014 | 0.965 | 0.707 | 0.000 | 0.210 | 0.000 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 61 | 98 | 80 | 92 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.74 | 1.20 | 0.98 | 1.12 | 0.00 | 0.76 | -0.01 |
| time (sec) | N/A | 0.108 | 0.033 | 0.009 | 0.969 | 0.595 | 0.000 | 0.271 | 0.000 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 50 | 81 | 63 | 87 | 0 | 57 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 1.29 | 1.00 | 1.38 | 0.00 | 0.90 | -0.02 |
| time (sec) | N/A | 0.060 | 0.119 | 0.006 | 0.943 | 0.652 | 0.000 | 0.272 | 0.000 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 73 | 102 | 64 | 96 | 0 | 91 | -1 |
| normalized size | 1 | 1.00 | 1.18 | 1.65 | 1.03 | 1.55 | 0.00 | 1.47 | -0.02 |
| time (sec) | N/A | 0.074 | 0.023 | 0.009 | 0.965 | 0.606 | 0.000 | 0.581 | 0.000 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 74 | 109 | 96 | 106 | 0 | 168 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.25 | 1.10 | 1.22 | 0.00 | 1.93 | -0.01 |
| time (sec) | N/A | 0.092 | 0.044 | 0.012 | 0.955 | 0.692 | 0.000 | 0.297 | 0.000 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 79 | 111 | 145 | 126 | 0 | 223 | -1 |
| normalized size | 1 | 1.00 | 0.71 | 0.99 | 1.29 | 1.12 | 0.00 | 1.99 | -0.01 |
| time (sec) | N/A | 0.155 | 0.056 | 0.013 | 0.967 | 0.637 | 0.000 | 0.313 | 0.000 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 71 | 163 | 202 | 117 | 0 | 67 | -1 |
| normalized size | 1 | 1.00 | 0.83 | 1.90 | 2.35 | 1.36 | 0.00 | 0.78 | -0.01 |
| time (sec) | N/A | 0.113 | 0.061 | 0.014 | 0.963 | 0.594 | 0.000 | 0.208 | 0.000 |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 66 | 146 | 185 | 112 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.97 | 2.15 | 2.72 | 1.65 | 0.00 | 0.91 | -0.01 |
| time (sec) | N/A | 0.095 | 0.126 | 0.009 | 0.967 | 0.865 | 0.000 | 0.237 | 0.000 |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 33 | 30 | 76 | 51 | 0 | 28 | 49 |
| normalized size | 1 | 1.00 | 0.70 | 0.64 | 1.62 | 1.09 | 0.00 | 0.60 | 1.04 |
| time (sec) | N/A | 0.048 | 0.059 | 0.005 | 0.437 | 0.804 | 0.000 | 0.196 | 4.196 |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 72 | 158 | 93 | 126 | 0 | 101 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.86 | 1.09 | 1.48 | 0.00 | 1.19 | -0.01 |
| time (sec) | N/A | 0.094 | 0.051 | 0.010 | 0.962 | 0.681 | 0.000 | 0.434 | 0.000 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 111 | 165 | 125 | 141 | 0 | 233 | -1 |
| normalized size | 1 | 1.00 | 1.01 | 1.50 | 1.14 | 1.28 | 0.00 | 2.12 | -0.01 |
| time (sec) | N/A | 0.151 | 0.064 | 0.011 | 0.974 | 0.744 | 0.000 | 0.363 | 0.000 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 89 | 148 | 174 | 156 | 0 | 233 | -1 |
| normalized size | 1 | 1.00 | 0.66 | 1.10 | 1.29 | 1.16 | 0.00 | 1.73 | -0.01 |
| time (sec) | N/A | 0.213 | 0.075 | 0.013 | 0.991 | 0.829 | 0.000 | 0.349 | 0.000 |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 219 | 324 | 0 | 465 | 0 | 0 | 1089 |
| normalized size | 1 | 1.00 | 1.05 | 1.56 | 0.00 | 2.24 | 0.00 | 0.00 | 5.24 |
| time (sec) | N/A | 0.424 | 0.505 | 0.013 | 0.000 | 33.082 | 0.000 | 0.000 | 5.748 |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 906 | 905 | 15669 | 19955 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 17.29 | 22.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.700 | 14.990 | 0.192 | 0.000 | 0.802 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 668 | 668 | 9965 | 12761 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 14.92 | 19.10 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.188 | 14.410 | 0.071 | 0.000 | 0.962 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 749 | 746 | 13240 | 8221 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 17.68 | 10.98 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.518 | 14.039 | 0.085 | 0.000 | 0.730 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 712 | 711 | 8456 | 21038 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 11.88 | 29.55 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.271 | 14.268 | 0.129 | 0.000 | 0.710 | 0.000 | 0.000 | 0.000 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F(-2) | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 992 | 989 | 12997 | 48427 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 13.10 | 48.82 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.916 | 14.666 | 0.252 | 0.000 | 0.613 | 0.000 | 0.000 | 0.000 |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1363 | 1363 | 19853 | 88790 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 14.57 | 65.14 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 4.200 | 16.015 | 0.428 | 0.000 | 0.753 | 0.000 | 0.000 | 0.000 |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1904 | 1904 | 29140 | 153623 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 15.30 | 80.68 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 6.243 | 18.387 | 0.804 | 0.000 | 0.822 | 0.000 | 0.000 | 0.000 |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 724 | 724 | 9972 | 14084 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 13.77 | 19.45 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.779 | 14.550 | 0.079 | 0.000 | 0.699 | 0.000 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 557 | 557 | 992 | 8161 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.78 | 14.65 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.889 | 11.588 | 0.061 | 0.000 | 0.565 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 471 | 470 | 1080 | 4251 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 2.29 | 9.03 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.482 | 12.500 | 0.054 | 0.000 | 0.745 | 0.000 | 0.000 | 0.000 |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 508 | 506 | 772 | 6053 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.52 | 11.92 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.649 | 7.280 | 0.072 | 0.000 | 0.649 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 684 | 680 | 1194 | 20481 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 0.99 | 1.75 | 29.94 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 1.192 | 12.156 | 0.132 | 0.000 | 0.691 | 0.000 | 0.000 | 0.000 |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 944 | 942 | 12295 | 46697 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 13.02 | 49.47 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 2.256 | 15.038 | 0.261 | 0.000 | 0.681 | 0.000 | 0.000 | 0.000 |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 510 | 508 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.833 | 2.277 | 0.155 | 0.000 | 0.928 | 0.000 | 0.000 | 0.000 |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 496 | 494 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.672 | 1.462 | 0.046 | 0.000 | 0.820 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | F | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 590 | 588 | 0 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.755 | 3.464 | 0.069 | 0.000 | 0.947 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 119 | 36 | 59 | 75 | 221 | 141 | 58 |
| normalized size | 1 | 1.00 | 2.90 | 0.88 | 1.44 | 1.83 | 5.39 | 3.44 | 1.41 |
| time (sec) | N/A | 0.052 | 0.106 | 0.005 | 0.576 | 0.581 | 13.248 | 0.181 | 4.246 |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 34 | 39 | 66 | 83 | 280 | 191 | 78 |
| normalized size | 1 | 1.00 | 0.74 | 0.85 | 1.43 | 1.80 | 6.09 | 4.15 | 1.70 |
| time (sec) | N/A | 0.072 | 0.130 | 0.005 | 0.576 | 0.631 | 173.947 | 0.244 | 4.395 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 43 | 51 | 98 | 123 | 483 | 314 | 120 |
| normalized size | 1 | 1.00 | 0.75 | 0.89 | 1.72 | 2.16 | 8.47 | 5.51 | 2.11 |
| time (sec) | N/A | 0.121 | 0.306 | 0.006 | 0.598 | 0.518 | 171.168 | 0.306 | 4.457 |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 167 | 8419 | 1779 | 2467 | 2281 | 2383 | 2026 |
| normalized size | 1 | 1.00 | 8.35 | 420.95 | 88.95 | 123.35 | 114.05 | 119.15 | 101.30 |
| time (sec) | N/A | 0.420 | 0.449 | 0.004 | 0.500 | 0.437 | 1.527 | 0.230 | 4.865 |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 19 | 18 | 18 | 20 | 20 | 18 |
| normalized size | 1 | 1.00 | 1.00 | 0.73 | 0.69 | 0.69 | 0.77 | 0.77 | 0.69 |
| time (sec) | N/A | 0.025 | 0.005 | 0.005 | 0.426 | 0.540 | 0.244 | 0.165 | 0.051 |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 346 | 346 | 282 | 783 | 0 | 701 | 0 | 330 | -1 |
| normalized size | 1 | 1.00 | 0.82 | 2.26 | 0.00 | 2.03 | 0.00 | 0.95 | -0.00 |
| time (sec) | N/A | 0.812 | 0.739 | 0.014 | 0.000 | 1.045 | 0.000 | 0.314 | 0.000 |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 245 | 245 | 199 | 532 | 0 | 499 | 0 | 228 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 2.17 | 0.00 | 2.04 | 0.00 | 0.93 | -0.00 |
| time (sec) | N/A | 0.438 | 0.443 | 0.010 | 0.000 | 0.710 | 0.000 | 0.386 | 0.000 |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 177 | 141 | 333 | 0 | 341 | 0 | 149 | -1 |
| normalized size | 1 | 1.00 | 0.80 | 1.88 | 0.00 | 1.93 | 0.00 | 0.84 | -0.01 |
| time (sec) | N/A | 0.235 | 0.264 | 0.009 | 0.000 | 0.757 | 0.000 | 0.268 | 0.000 |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 155 | 134 | 220 | 0 | 733 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 1.42 | 0.00 | 4.73 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.256 | 0.369 | 0.010 | 0.000 | 5.780 | 0.000 | 0.000 | 0.000 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 127 | 173 | 0 | 703 | 0 | 171 | 166 |
| normalized size | 1 | 1.00 | 0.91 | 1.24 | 0.00 | 5.06 | 0.00 | 1.23 | 1.19 |
| time (sec) | N/A | 0.235 | 0.399 | 0.012 | 0.000 | 3.717 | 0.000 | 0.372 | 4.460 |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 137 | 241 | 0 | 783 | 0 | 352 | -1 |
| normalized size | 1 | 1.00 | 0.86 | 1.52 | 0.00 | 4.92 | 0.00 | 2.21 | -0.01 |
| time (sec) | N/A | 0.244 | 0.359 | 0.012 | 0.000 | 4.813 | 0.000 | 0.361 | 0.000 |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 150 | 375 | 0 | 365 | 0 | 689 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 2.02 | 0.00 | 1.96 | 0.00 | 3.70 | -0.01 |
| time (sec) | N/A | 0.320 | 0.310 | 0.014 | 0.000 | 5.552 | 0.000 | 0.292 | 0.000 |

| Problem 287 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 212 | 591 | 0 | 525 | 0 | 1448 | -1 |
| normalized size | 1 | 1.00 | 0.79 | 2.19 | 0.00 | 1.94 | 0.00 | 5.36 | -0.00 |
| time (sec) | N/A | 0.488 | 0.522 | 0.013 | 0.000 | 10.830 | 0.000 | 0.300 | 0.000 |

| Problem 288 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 371 | 299 | 859 | 0 | 727 | 0 | 2177 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 2.32 | 0.00 | 1.96 | 0.00 | 5.87 | -0.00 |
| time (sec) | N/A | 0.817 | 0.731 | 0.018 | 0.000 | 42.469 | 0.000 | 0.508 | 0.000 |

| Problem 289 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 212 | 208 | 206 | 237 | 230 | 230 | 196 |
| normalized size | 1 | 1.00 | 0.82 | 0.81 | 0.80 | 0.92 | 0.89 | 0.89 | 0.76 |
| time (sec) | N/A | 0.257 | 0.040 | 0.002 | 0.434 | 0.739 | 0.518 | 0.168 | 4.204 |

| Problem 290 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 157 | 136 | 146 | 145 | 160 | 158 | 160 | 137 |
| normalized size | 1 | 1.00 | 0.87 | 0.93 | 0.92 | 1.02 | 1.01 | 1.02 | 0.87 |
| time (sec) | N/A | 0.166 | 0.032 | 0.002 | 0.432 | 0.670 | 0.153 | 0.157 | 4.108 |

| Problem 291 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 93 | 84 | 79 | 83 | 87 | 90 | 77 |
| normalized size | 1 | 1.00 | 1.00 | 0.90 | 0.85 | 0.89 | 0.94 | 0.97 | 0.83 |
| time (sec) | N/A | 0.108 | 0.013 | 0.001 | 0.428 | 0.608 | 1.911 | 0.151 | 0.047 |

| Problem 292 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 35 | 34 | 34 | 37 | 34 | 34 |
| normalized size | 1 | 1.00 | 1.00 | 0.83 | 0.81 | 0.81 | 0.88 | 0.81 | 0.81 |
| time (sec) | N/A | 0.025 | 0.002 | 0.002 | 0.423 | 0.642 | 0.288 | 0.148 | 0.026 |

| Problem 293 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 179 | 286 | 228 | 230 | 235 | 228 | 260 |
| normalized size | 1 | 1.00 | 0.79 | 1.25 | 1.00 | 1.01 | 1.03 | 1.00 | 1.14 |
| time (sec) | N/A | 0.193 | 0.059 | 0.006 | 0.433 | 0.840 | 1.174 | 0.155 | 4.139 |

| Problem 294 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 223 | 313 | 234 | 319 | 238 | 308 | 363 |
| normalized size | 1 | 1.00 | 0.98 | 1.37 | 1.03 | 1.40 | 1.04 | 1.35 | 1.59 |
| time (sec) | N/A | 0.191 | 0.087 | 0.012 | 0.435 | 0.781 | 1.134 | 0.173 | 4.181 |

| Problem 295 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 204 | 336 | 240 | 360 | 248 | 216 | 297 |
| normalized size | 1 | 1.00 | 0.88 | 1.45 | 1.04 | 1.56 | 1.07 | 0.94 | 1.29 |
| time (sec) | N/A | 0.204 | 0.066 | 0.011 | 0.444 | 0.570 | 2.627 | 0.180 | 0.092 |

| Problem 296 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 391 | 391 | 277 | 264 | 263 | 305 | 298 | 296 | 251 |
| normalized size | 1 | 1.00 | 0.71 | 0.68 | 0.67 | 0.78 | 0.76 | 0.76 | 0.64 |
| time (sec) | N/A | 0.386 | 0.043 | 0.002 | 0.435 | 0.701 | 0.203 | 0.157 | 4.247 |

| Problem 297 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 201 | 186 | 185 | 206 | 206 | 206 | 175 |
| normalized size | 1 | 1.00 | 1.00 | 0.93 | 0.92 | 1.02 | 1.02 | 1.02 | 0.87 |
| time (sec) | N/A | 0.240 | 0.026 | 0.001 | 0.429 | 0.700 | 0.146 | 0.161 | 0.108 |

| Problem 298 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 121 | 108 | 105 | 107 | 112 | 116 | 101 |
| normalized size | 1 | 1.00 | 1.00 | 0.89 | 0.87 | 0.88 | 0.93 | 0.96 | 0.83 |
| time (sec) | N/A | 0.160 | 0.016 | 0.001 | 0.431 | 0.673 | 0.136 | 0.156 | 4.173 |

| Problem 299 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 60 | 45 | 44 | 44 | 56 | 44 | 44 |
| normalized size | 1 | 1.00 | 1.00 | 0.75 | 0.73 | 0.73 | 0.93 | 0.73 | 0.73 |
| time (sec) | N/A | 0.036 | 0.001 | 0.000 | 0.423 | 0.723 | 0.155 | 0.149 | 0.035 |

| Problem 300 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 352 | 352 | 262 | 465 | 366 | 368 | 372 | 378 | 434 |
| normalized size | 1 | 1.00 | 0.74 | 1.32 | 1.04 | 1.05 | 1.06 | 1.07 | 1.23 |
| time (sec) | N/A | 0.316 | 0.122 | 0.006 | 0.436 | 0.870 | 0.997 | 0.170 | 0.079 |

| Problem 301 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 342 | 500 | 372 | 490 | 393 | 459 | 939 |
| normalized size | 1 | 1.00 | 0.97 | 1.42 | 1.05 | 1.39 | 1.11 | 1.30 | 2.66 |
| time (sec) | N/A | 0.328 | 0.139 | 0.012 | 0.438 | 1.004 | 2.311 | 0.180 | 4.217 |

| Problem 302 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 311 | 531 | 378 | 545 | 394 | 354 | 771 |
| normalized size | 1 | 1.00 | 0.88 | 1.50 | 1.07 | 1.54 | 1.11 | 1.00 | 2.18 |
| time (sec) | N/A | 0.343 | 0.102 | 0.014 | 0.449 | 0.807 | 4.872 | 0.161 | 0.127 |

| Problem 303 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 360 | 360 | 344 | 558 | 390 | 587 | 401 | 345 | 560 |
| normalized size | 1 | 1.00 | 0.96 | 1.55 | 1.08 | 1.63 | 1.11 | 0.96 | 1.56 |
| time (sec) | N/A | 0.358 | 0.123 | 0.014 | 0.462 | 0.651 | 8.089 | 0.159 | 4.284 |

| Problem 304 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 178 | 291 | 206 | 206 | 450 | 212 | 397 |
| normalized size | 1 | 1.00 | 0.81 | 1.32 | 0.93 | 0.93 | 2.04 | 0.96 | 1.80 |
| time (sec) | N/A | 0.190 | 0.121 | 0.007 | 0.964 | 0.844 | 2.579 | 0.167 | 4.182 |

| Problem 305 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 156 | 130 | 191 | 141 | 141 | 303 | 145 | 223 |
| normalized size | 1 | 1.00 | 0.83 | 1.22 | 0.90 | 0.90 | 1.94 | 0.93 | 1.43 |
| time (sec) | N/A | 0.162 | 0.085 | 0.006 | 0.961 | 0.788 | 1.719 | 0.159 | 0.099 |

| Problem 306 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 86 | 102 | 84 | 84 | 163 | 88 | 107 |
| normalized size | 1 | 1.00 | 0.87 | 1.03 | 0.85 | 0.85 | 1.65 | 0.89 | 1.08 |
| time (sec) | N/A | 0.108 | 0.054 | 0.005 | 0.956 | 0.778 | 0.849 | 0.155 | 0.070 |

| Problem 307 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 50 | 44 | 43 | 43 | 61 | 43 | 45 |
| normalized size | 1 | 1.00 | 0.89 | 0.79 | 0.77 | 0.77 | 1.09 | 0.77 | 0.80 |
| time (sec) | N/A | 0.049 | 0.018 | 0.003 | 0.967 | 0.649 | 0.232 | 0.162 | 0.042 |

| Problem 308 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 146 | 298 | 160 | 171 | 0 | 158 | 713 |
| normalized size | 1 | 1.00 | 0.87 | 1.77 | 0.95 | 1.02 | 0.00 | 0.94 | 4.24 |
| time (sec) | N/A | 0.194 | 0.110 | 0.012 | 0.964 | 1.001 | 0.000 | 0.216 | 6.389 |

| Problem 309 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 233 | 538 | 294 | 416 | 0 | 355 | 312 |
| normalized size | 1 | 1.00 | 1.00 | 2.31 | 1.26 | 1.79 | 0.00 | 1.52 | 1.34 |
| time (sec) | N/A | 0.251 | 0.160 | 0.016 | 0.983 | 1.051 | 0.000 | 0.177 | 4.669 |

| Problem 310 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 278 | 819 | 498 | 698 | 0 | 406 | 493 |
| normalized size | 1 | 1.00 | 0.88 | 2.58 | 1.57 | 2.20 | 0.00 | 1.28 | 1.56 |
| time (sec) | N/A | 0.290 | 0.434 | 0.015 | 0.998 | 1.356 | 0.000 | 0.179 | 4.763 |

| Problem 311 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 209 | 283 | 212 | 350 | 444 | 206 | 333 |
| normalized size | 1 | 1.00 | 1.11 | 1.50 | 1.12 | 1.85 | 2.35 | 1.09 | 1.76 |
| time (sec) | N/A | 0.255 | 0.158 | 0.016 | 0.958 | 0.872 | 2.772 | 0.162 | 0.148 |

| Problem 312 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 150 | 189 | 147 | 245 | 298 | 145 | 211 |
| normalized size | 1 | 1.00 | 1.07 | 1.35 | 1.05 | 1.75 | 2.13 | 1.04 | 1.51 |
| time (sec) | N/A | 0.208 | 0.113 | 0.012 | 0.960 | 0.803 | 1.961 | 0.156 | 0.114 |

| Problem 313 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 96 | 106 | 90 | 147 | 165 | 94 | 115 |
| normalized size | 1 | 1.00 | 0.99 | 1.09 | 0.93 | 1.52 | 1.70 | 0.97 | 1.19 |
| time (sec) | N/A | 0.193 | 0.066 | 0.010 | 0.961 | 0.825 | 1.023 | 0.156 | 4.152 |

| Problem 314 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 59 | 51 | 52 | 78 | 65 | 52 | 52 |
| normalized size | 1 | 1.00 | 0.94 | 0.81 | 0.83 | 1.24 | 1.03 | 0.83 | 0.83 |
| time (sec) | N/A | 0.077 | 0.037 | 0.007 | 0.952 | 0.769 | 0.192 | 0.153 | 4.148 |

| Problem 315 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 186 | 691 | 289 | 479 | 0 | 284 | 330 |
| normalized size | 1 | 1.00 | 0.83 | 3.08 | 1.29 | 2.14 | 0.00 | 1.27 | 1.47 |
| time (sec) | N/A | 0.340 | 0.164 | 0.020 | 0.982 | 0.937 | 0.000 | 0.170 | 4.612 |

| Problem 316 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 313 | 270 | 986 | 548 | 910 | 0 | 571 | 601 |
| normalized size | 1 | 1.00 | 0.86 | 3.15 | 1.75 | 2.91 | 0.00 | 1.82 | 1.92 |
| time (sec) | N/A | 0.498 | 0.262 | 0.023 | 1.015 | 1.124 | 0.000 | 0.204 | 4.835 |

| Problem 317 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 412 | 412 | 363 | 1314 | 851 | 1499 | 0 | 595 | 887 |
| normalized size | 1 | 1.00 | 0.88 | 3.19 | 2.07 | 3.64 | 0.00 | 1.44 | 2.15 |
| time (sec) | N/A | 0.715 | 0.396 | 0.027 | 1.090 | 1.464 | 0.000 | 0.204 | 4.940 |

| Problem 318 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 209 | 267 | 222 | 441 | 469 | 201 | 299 |
| normalized size | 1 | 1.00 | 1.22 | 1.56 | 1.30 | 2.58 | 2.74 | 1.18 | 1.75 |
| time (sec) | N/A | 0.336 | 0.201 | 0.015 | 0.968 | 0.837 | 8.078 | 0.208 | 0.152 |

| Problem 319 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 146 | 179 | 155 | 302 | 304 | 144 | 203 |
| normalized size | 1 | 1.00 | 1.09 | 1.34 | 1.16 | 2.25 | 2.27 | 1.07 | 1.51 |
| time (sec) | N/A | 0.239 | 0.182 | 0.013 | 0.961 | 0.832 | 3.963 | 0.165 | 4.214 |

| Problem 320 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | C | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 107 | 102 | 101 | 172 | 163 | 97 | 125 |
| normalized size | 1 | 1.00 | 1.04 | 0.99 | 0.98 | 1.67 | 1.58 | 0.94 | 1.21 |
| time (sec) | N/A | 0.145 | 0.085 | 0.011 | 0.956 | 0.760 | 2.311 | 0.185 | 0.120 |

| Problem 321 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 53 | 47 | 56 | 75 | 61 | 46 | 55 |
| normalized size | 1 | 1.00 | 0.83 | 0.73 | 0.88 | 1.17 | 0.95 | 0.72 | 0.86 |
| time (sec) | N/A | 0.050 | 0.039 | 0.007 | 0.958 | 0.735 | 0.201 | 0.155 | 0.049 |

| Problem 322 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 329 | 282 | 1437 | 571 | 1052 | 0 | 460 | 641 |
| normalized size | 1 | 1.00 | 0.86 | 4.37 | 1.74 | 3.20 | 0.00 | 1.40 | 1.95 |
| time (sec) | N/A | 0.496 | 0.305 | 0.025 | 1.030 | 1.128 | 0.000 | 0.247 | 4.793 |

| Problem 323 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 443 | 443 | 389 | 1850 | 916 | 1734 | 0 | 762 | 965 |
| normalized size | 1 | 1.00 | 0.88 | 4.18 | 2.07 | 3.91 | 0.00 | 1.72 | 2.18 |
| time (sec) | N/A | 0.893 | 0.526 | 0.031 | 1.134 | 1.562 | 0.000 | 0.256 | 4.989 |

| Problem 324 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 70 | 115 | 126 | 83 | 0 | 78 | 170 |
| normalized size | 1 | 1.00 | 0.49 | 0.80 | 0.88 | 0.58 | 0.00 | 0.55 | 1.19 |
| time (sec) | N/A | 0.155 | 0.154 | 0.017 | 0.965 | 0.808 | 0.000 | 0.204 | 1.716 |

| Problem 325 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 65 | 98 | 109 | 78 | 0 | 73 | 153 |
| normalized size | 1 | 1.00 | 0.52 | 0.79 | 0.88 | 0.63 | 0.00 | 0.59 | 1.23 |
| time (sec) | N/A | 0.092 | 0.095 | 0.006 | 0.954 | 0.847 | 0.000 | 0.199 | 0.767 |

| Problem 326 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 91 | 127 | 128 | 125 | 0 | 129 | -1 |
| normalized size | 1 | 1.00 | 0.61 | 0.85 | 0.86 | 0.84 | 0.00 | 0.87 | -0.01 |
| time (sec) | N/A | 0.240 | 0.147 | 0.010 | 1.006 | 0.887 | 0.000 | 0.224 | 0.000 |

| Problem 327 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 98 | 152 | 132 | 143 | 0 | 531 | -1 |
| normalized size | 1 | 1.00 | 0.66 | 1.02 | 0.89 | 0.96 | 0.00 | 3.56 | -0.01 |
| time (sec) | N/A | 0.237 | 0.162 | 0.013 | 1.003 | 0.919 | 0.000 | 0.531 | 0.000 |

| Problem 328 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 151 | 98 | 158 | 143 | 159 | 0 | 258 | -1 |
| normalized size | 1 | 1.00 | 0.65 | 1.05 | 0.95 | 1.05 | 0.00 | 1.71 | -0.01 |
| time (sec) | N/A | 0.228 | 0.148 | 0.015 | 1.002 | 0.839 | 0.000 | 0.244 | 0.000 |

| Problem 329 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 98 | 165 | 160 | 173 | 0 | 304 | -1 |
| normalized size | 1 | 1.00 | 0.62 | 1.04 | 1.01 | 1.09 | 0.00 | 1.92 | -0.01 |
| time (sec) | N/A | 0.226 | 0.159 | 0.013 | 0.980 | 0.656 | 0.000 | 0.258 | 0.000 |

| Problem 330 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 98 | 167 | 181 | 189 | 0 | 327 | -1 |
| normalized size | 1 | 1.00 | 0.59 | 1.01 | 1.10 | 1.15 | 0.00 | 1.98 | -0.01 |
| time (sec) | N/A | 0.234 | 0.176 | 0.013 | 1.022 | 0.839 | 0.000 | 0.373 | 0.000 |

| Problem 331 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 98 | 188 | 222 | 203 | 0 | 387 | -1 |
| normalized size | 1 | 1.00 | 0.59 | 1.14 | 1.35 | 1.23 | 0.00 | 2.35 | -0.01 |
| time (sec) | N/A | 0.229 | 0.200 | 0.014 | 1.044 | 0.888 | 0.000 | 0.278 | 0.000 |

| Problem 332 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 91 | 195 | 250 | 156 | 0 | 405 | -1 |
| normalized size | 1 | 1.00 | 0.54 | 1.15 | 1.48 | 0.92 | 0.00 | 2.40 | -0.01 |
| time (sec) | N/A | 0.218 | 0.169 | 0.016 | 1.050 | 0.825 | 0.000 | 0.264 | 0.000 |

| Problem 333 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 96 | 216 | 301 | 171 | 0 | 456 | -1 |
| normalized size | 1 | 1.00 | 0.49 | 1.11 | 1.55 | 0.88 | 0.00 | 2.35 | -0.01 |
| time (sec) | N/A | 0.268 | 0.198 | 0.019 | 1.039 | 0.709 | 0.000 | 0.293 | 0.000 |

| Problem 334 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 80 | 134 | 155 | 93 | 0 | 88 | -1 |
| normalized size | 1 | 1.00 | 0.48 | 0.81 | 0.93 | 0.56 | 0.00 | 0.53 | -0.01 |
| time (sec) | N/A | 0.194 | 0.184 | 0.018 | 0.977 | 0.806 | 0.000 | 0.191 | 0.000 |

| Problem 335 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 75 | 117 | 138 | 88 | 0 | 83 | -1 |
| normalized size | 1 | 1.00 | 0.51 | 0.80 | 0.94 | 0.60 | 0.00 | 0.56 | -0.01 |
| time (sec) | N/A | 0.121 | 0.124 | 0.005 | 1.003 | 0.816 | 0.000 | 0.334 | 0.000 |

| Problem 336 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 101 | 183 | 157 | 135 | 0 | 139 | -1 |
| normalized size | 1 | 1.00 | 0.59 | 1.06 | 0.91 | 0.78 | 0.00 | 0.81 | -0.01 |
| time (sec) | N/A | 0.268 | 0.183 | 0.010 | 1.002 | 0.615 | 0.000 | 0.231 | 0.000 |

| Problem 337 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 108 | 208 | 161 | 153 | 0 | 707 | -1 |
| normalized size | 1 | 1.00 | 0.63 | 1.21 | 0.94 | 0.89 | 0.00 | 4.11 | -0.01 |
| time (sec) | N/A | 0.282 | 0.210 | 0.013 | 1.013 | 0.877 | 0.000 | 0.451 | 0.000 |

| Problem 338 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 108 | 214 | 172 | 169 | 0 | 268 | -1 |
| normalized size | 1 | 1.00 | 0.62 | 1.23 | 0.99 | 0.97 | 0.00 | 1.54 | -0.01 |
| time (sec) | N/A | 0.273 | 0.216 | 0.015 | 1.001 | 0.924 | 0.000 | 0.303 | 0.000 |

| Problem 339 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 108 | 221 | 189 | 183 | 0 | 314 | -1 |
| normalized size | 1 | 1.00 | 0.60 | 1.22 | 1.04 | 1.01 | 0.00 | 1.73 | -0.01 |
| time (sec) | N/A | 0.267 | 0.219 | 0.017 | 1.036 | 0.974 | 0.000 | 0.269 | 0.000 |

| Problem 340 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 108 | 204 | 210 | 199 | 0 | 503 | -1 |
| normalized size | 1 | 1.00 | 0.57 | 1.09 | 1.12 | 1.06 | 0.00 | 2.68 | -0.01 |
| time (sec) | N/A | 0.265 | 0.228 | 0.017 | 1.042 | 0.956 | 0.000 | 0.421 | 0.000 |

| Problem 341 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 108 | 225 | 251 | 213 | 0 | 406 | -1 |
| normalized size | 1 | 1.00 | 0.55 | 1.15 | 1.29 | 1.09 | 0.00 | 2.08 | -0.01 |
| time (sec) | N/A | 0.263 | 0.237 | 0.017 | 1.060 | 1.034 | 0.000 | 0.324 | 0.000 |

| Problem 342 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 108 | 246 | 297 | 229 | 0 | 452 | -1 |
| normalized size | 1 | 1.00 | 0.55 | 1.26 | 1.52 | 1.17 | 0.00 | 2.32 | -0.01 |
| time (sec) | N/A | 0.268 | 0.253 | 0.017 | 1.072 | 1.056 | 0.000 | 0.345 | 0.000 |

| Problem 343 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 108 | 267 | 348 | 243 | 0 | 489 | -1 |
| normalized size | 1 | 1.00 | 0.55 | 1.37 | 1.78 | 1.25 | 0.00 | 2.51 | -0.01 |
| time (sec) | N/A | 0.262 | 0.259 | 0.019 | 1.074 | 0.909 | 0.000 | 0.332 | 0.000 |

| Problem 344 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 120 | 60 | 95 | 96 | 73 | 0 | 68 | -1 |
| normalized size | 1 | 1.00 | 0.50 | 0.79 | 0.80 | 0.61 | 0.00 | 0.57 | -0.01 |
| time (sec) | N/A | 0.136 | 0.117 | 0.012 | 0.974 | 0.875 | 0.000 | 0.194 | 0.000 |

| Problem 345 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 55 | 79 | 80 | 68 | 0 | 63 | -1 |
| normalized size | 1 | 1.00 | 0.54 | 0.78 | 0.79 | 0.67 | 0.00 | 0.62 | -0.01 |
| time (sec) | N/A | 0.080 | 0.073 | 0.006 | 0.963 | 0.783 | 0.000 | 0.196 | 0.000 |

| Problem 346 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 81 | 92 | 99 | 115 | 0 | 119 | -1 |
| normalized size | 1 | 1.00 | 0.64 | 0.73 | 0.79 | 0.91 | 0.00 | 0.94 | -0.01 |
| time (sec) | N/A | 0.210 | 0.102 | 0.008 | 0.981 | 0.739 | 0.000 | 0.232 | 0.000 |

| Problem 347 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 88 | 96 | 103 | 133 | 0 | 339 | -1 |
| normalized size | 1 | 1.00 | 0.70 | 0.76 | 0.82 | 1.06 | 0.00 | 2.69 | -0.01 |
| time (sec) | N/A | 0.202 | 0.115 | 0.012 | 0.980 | 0.969 | 0.000 | 0.402 | 0.000 |

| Problem 348 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 88 | 102 | 114 | 149 | 0 | 248 | -1 |
| normalized size | 1 | 1.00 | 0.69 | 0.80 | 0.89 | 1.16 | 0.00 | 1.94 | -0.01 |
| time (sec) | N/A | 0.208 | 0.129 | 0.012 | 0.984 | 0.964 | 0.000 | 0.255 | 0.000 |

| Problem 349 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 88 | 109 | 131 | 163 | 0 | 285 | -1 |
| normalized size | 1 | 1.00 | 0.65 | 0.81 | 0.97 | 1.21 | 0.00 | 2.11 | -0.01 |
| time (sec) | N/A | 0.205 | 0.149 | 0.014 | 1.003 | 0.854 | 0.000 | 0.257 | 0.000 |

| Problem 350 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 81 | 116 | 149 | 125 | 0 | 164 | -1 |
| normalized size | 1 | 1.00 | 0.58 | 0.83 | 1.07 | 0.90 | 0.00 | 1.18 | -0.01 |
| time (sec) | N/A | 0.191 | 0.135 | 0.013 | 1.024 | 0.946 | 0.000 | 0.283 | 0.000 |

| Problem 351 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 74 | 132 | 114 | 102 | 0 | 72 | -1 |
| normalized size | 1 | 1.00 | 0.60 | 1.06 | 0.92 | 0.82 | 0.00 | 0.58 | -0.01 |
| time (sec) | N/A | 0.152 | 0.470 | 0.017 | 0.972 | 0.987 | 0.000 | 0.225 | 0.000 |

| Problem 352 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 60 | 115 | 97 | 97 | 0 | 67 | -1 |
| normalized size | 1 | 1.00 | 0.58 | 1.12 | 0.94 | 0.94 | 0.00 | 0.65 | -0.01 |
| time (sec) | N/A | 0.102 | 0.185 | 0.007 | 0.970 | 0.862 | 0.000 | 0.216 | 0.000 |

| Problem 353 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 55 | 98 | 80 | 92 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.67 | 1.20 | 0.98 | 1.12 | 0.00 | 0.76 | -0.01 |
| time (sec) | N/A | 0.055 | 0.131 | 0.008 | 0.952 | 0.858 | 0.000 | 0.219 | 0.000 |

| Problem 354 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 86 | 148 | 99 | 149 | 0 | 118 | -1 |
| normalized size | 1 | 1.00 | 0.85 | 1.47 | 0.98 | 1.48 | 0.00 | 1.17 | -0.01 |
| time (sec) | N/A | 0.151 | 0.354 | 0.011 | 0.977 | 1.007 | 0.000 | 0.396 | 0.000 |

| Problem 355 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 104 | 152 | 116 | 157 | 0 | 225 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 1.41 | 1.07 | 1.45 | 0.00 | 2.08 | -0.01 |
| time (sec) | N/A | 0.153 | 0.323 | 0.011 | 1.007 | 0.965 | 0.000 | 0.423 | 0.000 |

| Problem 356 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 84 | 144 | 149 | 126 | 0 | 220 | -1 |
| normalized size | 1 | 1.00 | 0.75 | 1.29 | 1.33 | 1.12 | 0.00 | 1.96 | -0.01 |
| time (sec) | N/A | 0.146 | 0.298 | 0.013 | 0.985 | 0.919 | 0.000 | 0.247 | 0.000 |

| Problem 357 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 95 | 151 | 217 | 141 | 0 | 271 | -1 |
| normalized size | 1 | 1.00 | 0.69 | 1.10 | 1.58 | 1.03 | 0.00 | 1.98 | -0.01 |
| time (sec) | N/A | 0.204 | 0.158 | 0.014 | 1.007 | 0.878 | 0.000 | 0.292 | 0.000 |

| Problem 358 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 65 | 180 | 219 | 122 | 0 | 71 | -1 |
| normalized size | 1 | 1.00 | 0.62 | 1.71 | 2.09 | 1.16 | 0.00 | 0.68 | -0.01 |
| time (sec) | N/A | 0.131 | 0.687 | 0.023 | 0.983 | 0.863 | 0.000 | 0.216 | 0.000 |

| Problem 359 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 60 | 163 | 202 | 117 | 0 | 66 | -1 |
| normalized size | 1 | 1.00 | 0.70 | 1.90 | 2.35 | 1.36 | 0.00 | 0.77 | -0.01 |
| time (sec) | N/A | 0.082 | 0.253 | 0.008 | 0.967 | 0.719 | 0.000 | 0.206 | 0.000 |

| Problem 360 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 55 | 146 | 185 | 112 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.81 | 2.15 | 2.72 | 1.65 | 0.00 | 0.91 | -0.01 |
| time (sec) | N/A | 0.052 | 0.229 | 0.007 | 0.983 | 0.891 | 0.000 | 0.304 | 0.000 |

| Problem 361 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 80 | 190 | 110 | 126 | 0 | 92 | -1 |
| normalized size | 1 | 1.00 | 0.94 | 2.24 | 1.29 | 1.48 | 0.00 | 1.08 | -0.01 |
| time (sec) | N/A | 0.128 | 0.470 | 0.009 | 0.968 | 0.842 | 0.000 | 0.229 | 0.000 |

| Problem 362 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 92 | 194 | 127 | 141 | 0 | 206 | -1 |
| normalized size | 1 | 1.00 | 0.84 | 1.76 | 1.15 | 1.28 | 0.00 | 1.87 | -0.01 |
| time (sec) | N/A | 0.153 | 0.408 | 0.011 | 1.006 | 0.855 | 0.000 | 0.399 | 0.000 |

| Problem 363 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 135 | 97 | 200 | 178 | 155 | 0 | 228 | -1 |
| normalized size | 1 | 1.00 | 0.72 | 1.48 | 1.32 | 1.15 | 0.00 | 1.69 | -0.01 |
| time (sec) | N/A | 0.221 | 0.310 | 0.013 | 0.998 | 0.890 | 0.000 | 0.265 | 0.000 |

| Problem 364 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 89 | 207 | 246 | 170 | 0 | 279 | -1 |
| normalized size | 1 | 1.00 | 0.56 | 1.29 | 1.54 | 1.06 | 0.00 | 1.74 | -0.01 |
| time (sec) | N/A | 0.283 | 0.236 | 0.014 | 1.013 | 0.916 | 0.000 | 0.280 | 0.000 |

| Problem 365 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 316 | 1406 | 0 | 1373 | 0 | 465 | -1 |
| normalized size | 1 | 1.00 | 0.89 | 3.97 | 0.00 | 3.88 | 0.00 | 1.31 | -0.00 |
| time (sec) | N/A | 0.377 | 1.145 | 0.015 | 0.000 | 88.993 | 0.000 | 0.306 | 0.000 |

| Problem 366 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F(-1) | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 319 | 1453 | 0 | 1385 | 0 | 488 | -1 |
| normalized size | 1 | 1.00 | 0.90 | 4.12 | 0.00 | 3.92 | 0.00 | 1.38 | -0.00 |
| time (sec) | N/A | 0.386 | 1.111 | 0.018 | 0.000 | 95.777 | 0.000 | 0.448 | 0.000 |

| Problem 367 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 588 | 588 | 537 | 5924 | 2292 | 4795 | 0 | 10960 | 4341 |
| normalized size | 1 | 1.00 | 0.91 | 10.07 | 3.90 | 8.15 | 0.00 | 18.64 | 7.38 |
| time (sec) | N/A | 0.364 | 0.382 | 0.076 | 0.745 | 1.202 | 0.000 | 0.621 | 8.392 |

| Problem 368 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 432 | 391 | 3222 | 1414 | 2796 | 0 | 6223 | 2625 |
| normalized size | 1 | 1.00 | 0.91 | 7.46 | 3.27 | 6.47 | 0.00 | 14.41 | 6.08 |
| time (sec) | N/A | 0.243 | 0.242 | 0.035 | 0.641 | 0.971 | 0.000 | 0.392 | 6.050 |

| Problem 369 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 292 | 261 | 1504 | 788 | 1448 | 0 | 3098 | 1425 |
| normalized size | 1 | 1.00 | 0.89 | 5.15 | 2.70 | 4.96 | 0.00 | 10.61 | 4.88 |
| time (sec) | N/A | 0.189 | 0.169 | 0.016 | 0.548 | 0.608 | 0.000 | 0.266 | 5.087 |

| Problem 370 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 221 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.481 | 0.731 | 0.258 | 0.000 | 0.883 | 0.000 | 0.000 | 0.000 |

| Problem 371 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 377 | 377 | 441 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.900 | 1.803 | 0.125 | 0.000 | 0.711 | 0.000 | 0.000 | 0.000 |

| Problem 372 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 528 | 528 | 488 | 1244 | 0 | 3480 | 0 | 657 | 1027 |
| normalized size | 1 | 1.00 | 0.92 | 2.36 | 0.00 | 6.59 | 0.00 | 1.24 | 1.95 |
| time (sec) | N/A | 1.312 | 1.079 | 0.019 | 0.000 | 0.986 | 0.000 | 0.220 | 6.175 |

| Problem 373 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 765 | 765 | 754 | 1960 | 0 | 2643 | 0 | 982 | 2779 |
| normalized size | 1 | 1.00 | 0.99 | 2.56 | 0.00 | 3.45 | 0.00 | 1.28 | 3.63 |
| time (sec) | N/A | 5.825 | 0.731 | 0.013 | 0.000 | 1.838 | 0.000 | 0.177 | 7.261 |

| Problem 374 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 85 | 166 | 177 | 97 | 0 | 92 | 221 |
| normalized size | 1 | 1.00 | 0.41 | 0.80 | 0.85 | 0.47 | 0.00 | 0.44 | 1.06 |
| time (sec) | N/A | 0.352 | 0.305 | 0.033 | 0.990 | 0.864 | 0.000 | 0.210 | 6.306 |

| Problem 375 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 75 | 132 | 143 | 87 | 0 | 82 | 187 |
| normalized size | 1 | 1.00 | 0.45 | 0.80 | 0.86 | 0.52 | 0.00 | 0.49 | 1.13 |
| time (sec) | N/A | 0.203 | 0.189 | 0.010 | 0.973 | 0.895 | 0.000 | 0.203 | 6.009 |

| Problem 376 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 65 | 98 | 109 | 77 | 0 | 72 | 153 |
| normalized size | 1 | 1.00 | 0.52 | 0.79 | 0.88 | 0.62 | 0.00 | 0.58 | 1.23 |
| time (sec) | N/A | 0.116 | 0.100 | 0.007 | 0.959 | 0.872 | 0.000 | 0.189 | 5.378 |

| Problem 377 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 187 | 187 | 189 | 403 | 500 | 304 | 0 | 144 | -1 |
| normalized size | 1 | 1.00 | 1.01 | 2.16 | 2.67 | 1.63 | 0.00 | 0.77 | -0.01 |
| time (sec) | N/A | 0.355 | 0.895 | 0.090 | 1.173 | 0.905 | 0.000 | 0.265 | 0.000 |

| Problem 378 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 199 | 354 | 1084 | 0 | 378 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.78 | 5.45 | 0.00 | 1.90 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.250 | 1.276 | 0.032 | 0.000 | 0.990 | 0.000 | 0.000 | 0.000 |

| Problem 379 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 334 | 2342 | 0 | 390 | 0 | 378 | -1 |
| normalized size | 1 | 1.00 | 1.57 | 11.00 | 0.00 | 1.83 | 0.00 | 1.77 | -0.00 |
| time (sec) | N/A | 0.237 | 1.397 | 0.031 | 0.000 | 0.961 | 0.000 | 0.262 | 0.000 |

| Problem 380 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 95 | 185 | 206 | 107 | 0 | 102 | -1 |
| normalized size | 1 | 1.00 | 0.41 | 0.80 | 0.89 | 0.46 | 0.00 | 0.44 | -0.00 |
| time (sec) | N/A | 0.364 | 0.418 | 0.036 | 1.005 | 0.878 | 0.000 | 0.235 | 0.000 |

| Problem 381 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 85 | 151 | 172 | 97 | 0 | 92 | -1 |
| normalized size | 1 | 1.00 | 0.45 | 0.80 | 0.91 | 0.51 | 0.00 | 0.49 | -0.01 |
| time (sec) | N/A | 0.227 | 0.251 | 0.010 | 0.984 | 0.698 | 0.000 | 0.216 | 0.000 |

| Problem 382 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 75 | 117 | 138 | 87 | 0 | 82 | -1 |
| normalized size | 1 | 1.00 | 0.51 | 0.80 | 0.94 | 0.59 | 0.00 | 0.56 | -0.01 |
| time (sec) | N/A | 0.130 | 0.148 | 0.008 | 0.957 | 0.944 | 0.000 | 0.204 | 0.000 |

| Problem 383 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 210 | 202 | 730 | 535 | 326 | 0 | 154 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 3.48 | 2.55 | 1.55 | 0.00 | 0.73 | -0.00 |
| time (sec) | N/A | 0.304 | 0.938 | 0.020 | 1.310 | 0.985 | 0.000 | 0.282 | 0.000 |

| Problem 384 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 354 | 1828 | 0 | 378 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.59 | 8.23 | 0.00 | 1.70 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.322 | 1.833 | 0.023 | 0.000 | 0.897 | 0.000 | 0.000 | 0.000 |

| Problem 385 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | F(-2) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 376 | 3828 | 0 | 447 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.61 | 16.36 | 0.00 | 1.91 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.319 | 2.239 | 0.022 | 0.000 | 0.993 | 0.000 | 0.000 | 0.000 |

| Problem 386 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 75 | 147 | 148 | 87 | 0 | 82 | -1 |
| normalized size | 1 | 1.00 | 0.41 | 0.79 | 0.80 | 0.47 | 0.00 | 0.44 | -0.01 |
| time (sec) | N/A | 0.312 | 0.252 | 0.026 | 0.994 | 0.808 | 0.000 | 0.245 | 0.000 |

| Problem 387 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 65 | 113 | 114 | 77 | 0 | 72 | -1 |
| normalized size | 1 | 1.00 | 0.45 | 0.79 | 0.80 | 0.54 | 0.00 | 0.50 | -0.01 |
| time (sec) | N/A | 0.203 | 0.143 | 0.010 | 0.974 | 0.870 | 0.000 | 0.392 | 0.000 |

| Problem 388 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 55 | 79 | 80 | 67 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.54 | 0.78 | 0.79 | 0.66 | 0.00 | 0.61 | -0.01 |
| time (sec) | N/A | 0.145 | 0.080 | 0.009 | 0.964 | 0.729 | 0.000 | 0.220 | 0.000 |

| Problem 389 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 164 | 164 | 157 | 204 | 465 | 297 | 0 | 125 | -1 |
| normalized size | 1 | 1.00 | 0.96 | 1.24 | 2.84 | 1.81 | 0.00 | 0.76 | -0.01 |
| time (sec) | N/A | 0.230 | 0.467 | 0.019 | 1.138 | 0.857 | 0.000 | 0.275 | 0.000 |

| Problem 390 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 313 | 510 | 0 | 330 | 0 | 276 | -1 |
| normalized size | 1 | 1.00 | 1.76 | 2.87 | 0.00 | 1.85 | 0.00 | 1.55 | -0.01 |
| time (sec) | N/A | 0.196 | 0.999 | 0.021 | 0.000 | 0.865 | 0.000 | 0.279 | 0.000 |

| Problem 391 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 371 | 1194 | 0 | 390 | 0 | 378 | -1 |
| normalized size | 1 | 1.00 | 1.63 | 5.26 | 0.00 | 1.72 | 0.00 | 1.67 | -0.00 |
| time (sec) | N/A | 0.271 | 1.175 | 0.021 | 0.000 | 1.099 | 0.000 | 0.322 | 0.000 |

| Problem 392 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 75 | 166 | 148 | 112 | 0 | 81 | -1 |
| normalized size | 1 | 1.00 | 0.45 | 1.00 | 0.89 | 0.67 | 0.00 | 0.49 | -0.01 |
| time (sec) | N/A | 0.241 | 0.433 | 0.030 | 0.989 | 0.827 | 0.000 | 0.261 | 0.000 |

| Problem 393 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 65 | 132 | 114 | 102 | 0 | 71 | -1 |
| normalized size | 1 | 1.00 | 0.52 | 1.06 | 0.92 | 0.82 | 0.00 | 0.57 | -0.01 |
| time (sec) | N/A | 0.161 | 0.266 | 0.008 | 0.968 | 0.692 | 0.000 | 0.248 | 0.000 |

| Problem 394 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 55 | 98 | 80 | 92 | 0 | 62 | -1 |
| normalized size | 1 | 1.00 | 0.67 | 1.20 | 0.98 | 1.12 | 0.00 | 0.76 | -0.01 |
| time (sec) | N/A | 0.087 | 0.150 | 0.007 | 0.958 | 0.991 | 0.000 | 0.215 | 0.000 |

| Problem 395 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 174 | 489 | 777 | 333 | 0 | 112 | -1 |
| normalized size | 1 | 1.00 | 1.05 | 2.95 | 4.68 | 2.01 | 0.00 | 0.67 | -0.01 |
| time (sec) | N/A | 0.216 | 1.135 | 0.018 | 1.155 | 0.780 | 0.000 | 0.248 | 0.000 |

| Problem 396 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 215 | 351 | 1214 | 0 | 392 | 0 | 295 | -1 |
| normalized size | 1 | 1.00 | 1.63 | 5.65 | 0.00 | 1.82 | 0.00 | 1.37 | -0.00 |
| time (sec) | N/A | 0.316 | 1.138 | 0.020 | 0.000 | 0.881 | 0.000 | 0.275 | 0.000 |

| Problem 397 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 381 | 2600 | 0 | 452 | 0 | 397 | -1 |
| normalized size | 1 | 1.00 | 1.52 | 10.40 | 0.00 | 1.81 | 0.00 | 1.59 | -0.00 |
| time (sec) | N/A | 0.324 | 1.586 | 0.024 | 0.000 | 1.329 | 0.000 | 0.323 | 0.000 |

| Problem 398 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 242 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.46 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.147 | 0.401 | 0.075 | 0.000 | 0.893 | 0.000 | 0.000 | 0.000 |

| Problem 399 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 236 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 |
| time (sec) | N/A | 0.153 | 0.318 | 0.064 | 0.000 | 0.865 | 0.000 | 0.000 | 0.000 |

| Problem 400 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 252 | 302 | 0 | 0 | 0 | 0 | 0 | -1 |
| normalized size | 1 | 1.00 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.00 |
| time (sec) | N/A | 0.481 | 0.517 | 0.070 | 0.000 | 0.893 | 0.000 | 0.000 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [168] had the largest ratio of [.3571]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 7 | 5 | 1.00 | 34 | 0.147 |
| 2 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 3 | A | 5 | 5 | 1.00 | 27 | 0.185 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 4 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 5 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 6 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 7 | A | 8 | 6 | 1.00 | 34 | 0.176 |
| 8 | A | 4 | 4 | 1.00 | 34 | 0.118 |
| 9 | A | 5 | 4 | 1.00 | 34 | 0.118 |
| 10 | A | 7 | 4 | 1.00 | 34 | 0.118 |
| 11 | A | 6 | 4 | 1.00 | 34 | 0.118 |
| 12 | A | 5 | 4 | 1.00 | 32 | 0.125 |
| 13 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 14 | A | 4 | 4 | 1.00 | 34 | 0.118 |
| 15 | A | 7 | 5 | 1.00 | 34 | 0.147 |
| 16 | A | 4 | 4 | 1.00 | 34 | 0.118 |
| 17 | A | 5 | 4 | 1.00 | 34 | 0.118 |
| 18 | A | 2 | 1 | 0.99 | 25 | 0.040 |
| 19 | A | 2 | 1 | 0.99 | 25 | 0.040 |
| 20 | A | 2 | 1 | 1.00 | 23 | 0.043 |
| 21 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 22 | A | 2 | 1 | 0.99 | 25 | 0.040 |
| 23 | A | 2 | 1 | 0.99 | 25 | 0.040 |
| 24 | A | 2 | 1 | 0.99 | 25 | 0.040 |
| 25 | A | 3 | 2 | 0.99 | 27 | 0.074 |
| 26 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 27 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 28 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 29 | A | 2 | 1 | 0.99 | 27 | 0.037 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 30 | A | 2 | 1 | 0.99 | 27 | 0.037 |
| 31 | A | 2 | 1 | 0.99 | 27 | 0.037 |
| 32 | A | 3 | 2 | 0.99 | 27 | 0.074 |
| 33 | A | 3 | 2 | 1.00 | 27 | 0.074 |
| 34 | A | 3 | 2 | 1.00 | 25 | 0.080 |
| 35 | A | 3 | 2 | 1.00 | 20 | 0.100 |
| 36 | A | 2 | 1 | 0.99 | 27 | 0.037 |
| 37 | A | 2 | 1 | 0.99 | 27 | 0.037 |
| 38 | A | 2 | 1 | 0.99 | 27 | 0.037 |
| 39 | A | 1 | 1 | 1.00 | 32 | 0.031 |
| 40 | A | 1 | 1 | 1.00 | 31 | 0.032 |
| 41 | A | 1 | 1 | 1.00 | 34 | 0.029 |
| 42 | A | 1 | 1 | 1.00 | 33 | 0.030 |
| 43 | A | 5 | 4 | 0.99 | 27 | 0.148 |
| 44 | A | 5 | 4 | 0.99 | 27 | 0.148 |
| 45 | A | 5 | 4 | 1.00 | 25 | 0.160 |
| 46 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 47 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 48 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 49 | A | 5 | 4 | 1.00 | 27 | 0.148 |
| 50 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 51 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 52 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 53 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 54 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 55 | A | 6 | 5 | 0.99 | 27 | 0.185 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 56 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 57 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 58 | A | 3 | 3 | 1.12 | 27 | 0.111 |
| 59 | A | 3 | 3 | 1.00 | 25 | 0.120 |
| 60 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 61 | A | 7 | 6 | 1.00 | 27 | 0.222 |
| 62 | A | 7 | 5 | 0.99 | 27 | 0.185 |
| 63 | A | 7 | 5 | 1.00 | 27 | 0.185 |
| 64 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 65 | A | 4 | 4 | 1.13 | 27 | 0.148 |
| 66 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 67 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 68 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 69 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 70 | A | 5 | 5 | 1.00 | 17 | 0.294 |
| 71 | A | 4 | 4 | 1.00 | 15 | 0.267 |
| 72 | A | 3 | 3 | 1.00 | 14 | 0.214 |
| 73 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 74 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 75 | A | 6 | 5 | 1.00 | 17 | 0.294 |
| 76 | A | 3 | 3 | 1.83 | 16 | 0.188 |
| 77 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 78 | A | 7 | 6 | 0.99 | 29 | 0.207 |
| 79 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 80 | A | 5 | 5 | 1.00 | 27 | 0.185 |
| 81 | A | 5 | 5 | 1.00 | 22 | 0.227 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 82 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 83 | A | 7 | 6 | 0.98 | 29 | 0.207 |
| 84 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 85 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 86 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 87 | A | 6 | 6 | 1.00 | 29 | 0.207 |
| 88 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 89 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 90 | A | 6 | 5 | 1.00 | 27 | 0.185 |
| 91 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 92 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 93 | A | 8 | 6 | 0.99 | 29 | 0.207 |
| 94 | A | 8 | 7 | 0.98 | 29 | 0.241 |
| 95 | A | 8 | 6 | 0.99 | 29 | 0.207 |
| 96 | A | 8 | 7 | 1.00 | 29 | 0.241 |
| 97 | A | 8 | 6 | 1.00 | 29 | 0.207 |
| 98 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 99 | A | 7 | 6 | 1.00 | 29 | 0.207 |
| 100 | A | 7 | 5 | 1.00 | 22 | 0.227 |
| 101 | A | 6 | 5 | 0.99 | 29 | 0.172 |
| 102 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 103 | A | 4 | 4 | 0.99 | 27 | 0.148 |
| 104 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 105 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 106 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 107 | A | 4 | 4 | 1.00 | 29 | 0.138 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 108 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 109 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 110 | A | 4 | 4 | 1.00 | 27 | 0.148 |
| 111 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 112 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 113 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 114 | A | 5 | 5 | 0.99 | 29 | 0.172 |
| 115 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 116 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 117 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 118 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 119 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 120 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 121 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 122 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 123 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 124 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 125 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 126 | A | 3 | 3 | 1.00 | 27 | 0.111 |
| 127 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 128 | A | 4 | 4 | 1.00 | 29 | 0.138 |
| 129 | A | 5 | 5 | 1.00 | 29 | 0.172 |
| 130 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 131 | A | 4 | 3 | 1.00 | 29 | 0.103 |
| 132 | A | 2 | 2 | 1.00 | 27 | 0.074 |
| 133 | A | 5 | 5 | 1.00 | 29 | 0.172 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 134 | A | 5 | 4 | 1.00 | 29 | 0.138 |
| 135 | A | 6 | 5 | 1.00 | 29 | 0.172 |
| 136 | A | 6 | 4 | 0.99 | 27 | 0.148 |
| 137 | A | 6 | 4 | 1.00 | 29 | 0.138 |
| 138 | A | 5 | 5 | 1.00 | 31 | 0.161 |
| 139 | A | 6 | 6 | 1.00 | 70 | 0.086 |
| 140 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 141 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 142 | A | 2 | 1 | 1.00 | 20 | 0.050 |
| 143 | A | 2 | 1 | 1.00 | 18 | 0.056 |
| 144 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 145 | A | 4 | 4 | 1.00 | 20 | 0.200 |
| 146 | A | 5 | 5 | 1.00 | 20 | 0.250 |
| 147 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 148 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 149 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 150 | A | 6 | 5 | 1.00 | 28 | 0.179 |
| 151 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 152 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 153 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 154 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 155 | A | 6 | 6 | 1.00 | 30 | 0.200 |
| 156 | A | 5 | 5 | 1.00 | 28 | 0.179 |
| 157 | A | 4 | 4 | 1.00 | 23 | 0.174 |
| 158 | A | 7 | 6 | 1.00 | 30 | 0.200 |
| 159 | A | 7 | 6 | 1.00 | 30 | 0.200 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 160 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 161 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 162 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 163 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 164 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 165 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 166 | A | 7 | 6 | 1.00 | 20 | 0.300 |
| 167 | A | 1 | 1 | 1.00 | 16 | 0.062 |
| 168 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 169 | A | 6 | 5 | 1.00 | 16 | 0.312 |
| 170 | A | 3 | 2 | 1.00 | 18 | 0.111 |
| 171 | A | 5 | 3 | 1.00 | 16 | 0.188 |
| 172 | A | 5 | 3 | 1.00 | 19 | 0.158 |
| 173 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 174 | A | 5 | 3 | 1.00 | 19 | 0.158 |
| 175 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 176 | A | 4 | 4 | 1.00 | 17 | 0.235 |
| 177 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 178 | A | 7 | 5 | 1.00 | 22 | 0.227 |
| 179 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 180 | A | 5 | 5 | 1.00 | 22 | 0.227 |
| 181 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 182 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 183 | A | 3 | 3 | 1.00 | 22 | 0.136 |
| 184 | A | 4 | 4 | 1.00 | 22 | 0.182 |
| 185 | A | 5 | 4 | 1.00 | 22 | 0.182 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 186 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 187 | A | 6 | 6 | 0.99 | 32 | 0.188 |
| 188 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 189 | A | 5 | 5 | 1.00 | 25 | 0.200 |
| 190 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 191 | A | 7 | 6 | 0.99 | 32 | 0.188 |
| 192 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 193 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 194 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 195 | A | 6 | 6 | 1.00 | 32 | 0.188 |
| 196 | A | 8 | 6 | 1.00 | 32 | 0.188 |
| 197 | A | 7 | 6 | 0.99 | 32 | 0.188 |
| 198 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 199 | A | 6 | 5 | 1.00 | 25 | 0.200 |
| 200 | A | 8 | 6 | 1.00 | 32 | 0.188 |
| 201 | A | 8 | 6 | 0.99 | 32 | 0.188 |
| 202 | A | 8 | 7 | 0.99 | 32 | 0.219 |
| 203 | A | 8 | 6 | 1.00 | 32 | 0.188 |
| 204 | A | 8 | 7 | 1.00 | 32 | 0.219 |
| 205 | A | 8 | 6 | 1.00 | 32 | 0.188 |
| 206 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 207 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 208 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 209 | A | 6 | 6 | 1.00 | 32 | 0.188 |
| 210 | A | 5 | 5 | 1.00 | 30 | 0.167 |
| 211 | A | 7 | 7 | 1.00 | 32 | 0.219 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 212 | A | 7 | 7 | 1.00 | 32 | 0.219 |
| 213 | A | 7 | 7 | 1.00 | 32 | 0.219 |
| 214 | A | 9 | 6 | 1.00 | 32 | 0.188 |
| 215 | A | 7 | 6 | 1.00 | 32 | 0.188 |
| 216 | A | 6 | 5 | 1.00 | 30 | 0.167 |
| 217 | A | 8 | 7 | 1.00 | 32 | 0.219 |
| 218 | A | 8 | 7 | 1.00 | 32 | 0.219 |
| 219 | A | 8 | 8 | 1.00 | 32 | 0.250 |
| 220 | A | 9 | 6 | 1.00 | 32 | 0.188 |
| 221 | A | 8 | 6 | 1.00 | 32 | 0.188 |
| 222 | A | 7 | 5 | 1.00 | 30 | 0.167 |
| 223 | A | 9 | 7 | 1.00 | 32 | 0.219 |
| 224 | A | 9 | 7 | 1.00 | 32 | 0.219 |
| 225 | A | 9 | 8 | 1.00 | 32 | 0.250 |
| 226 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 227 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 228 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 229 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 230 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 231 | A | 6 | 5 | 0.99 | 32 | 0.156 |
| 232 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 233 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 234 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 235 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 236 | A | 4 | 4 | 1.00 | 25 | 0.160 |
| 237 | A | 4 | 4 | 1.00 | 32 | 0.125 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 238 | A | 4 | 4 | 0.99 | 32 | 0.125 |
| 239 | A | 5 | 5 | 0.99 | 32 | 0.156 |
| 240 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 241 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 242 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 243 | A | 6 | 6 | 1.00 | 32 | 0.188 |
| 244 | A | 6 | 6 | 1.00 | 32 | 0.188 |
| 245 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 246 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 247 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 248 | A | 4 | 4 | 1.00 | 30 | 0.133 |
| 249 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 250 | A | 4 | 4 | 1.00 | 32 | 0.125 |
| 251 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 252 | A | 5 | 4 | 1.00 | 32 | 0.125 |
| 253 | A | 5 | 4 | 1.00 | 32 | 0.125 |
| 254 | A | 2 | 2 | 1.00 | 30 | 0.067 |
| 255 | A | 5 | 5 | 1.00 | 32 | 0.156 |
| 256 | A | 5 | 4 | 1.00 | 32 | 0.125 |
| 257 | A | 6 | 5 | 1.00 | 32 | 0.156 |
| 258 | A | 3 | 3 | 1.00 | 47 | 0.064 |
| 259 | A | 8 | 7 | 1.00 | 34 | 0.206 |
| 260 | A | 7 | 6 | 1.00 | 34 | 0.176 |
| 261 | A | 7 | 6 | 1.00 | 34 | 0.176 |
| 262 | A | 7 | 6 | 1.00 | 34 | 0.176 |
| 263 | A | 7 | 6 | 1.00 | 34 | 0.176 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 264 | A | 8 | 7 | 1.00 | 34 | 0.206 |
| 265 | A | 9 | 7 | 1.00 | 34 | 0.206 |
| 266 | A | 8 | 6 | 1.00 | 34 | 0.176 |
| 267 | A | 7 | 6 | 1.00 | 34 | 0.176 |
| 268 | A | 6 | 5 | 1.00 | 34 | 0.147 |
| 269 | A | 6 | 5 | 1.00 | 34 | 0.147 |
| 270 | A | 7 | 6 | 0.99 | 34 | 0.176 |
| 271 | A | 8 | 6 | 1.00 | 34 | 0.176 |
| 272 | A | 6 | 4 | 1.00 | 30 | 0.133 |
| 273 | A | 6 | 4 | 1.00 | 32 | 0.125 |
| 274 | A | 5 | 5 | 1.00 | 34 | 0.147 |
| 275 | A | 3 | 3 | 1.00 | 42 | 0.071 |
| 276 | A | 2 | 2 | 1.00 | 46 | 0.043 |
| 277 | A | 2 | 2 | 1.00 | 69 | 0.029 |
| 278 | A | 2 | 2 | 1.00 | 75 | 0.027 |
| 279 | A | 6 | 4 | 1.00 | 16 | 0.250 |
| 280 | A | 6 | 5 | 1.00 | 33 | 0.152 |
| 281 | A | 5 | 4 | 1.00 | 31 | 0.129 |
| 282 | A | 5 | 4 | 1.00 | 30 | 0.133 |
| 283 | A | 7 | 5 | 1.00 | 33 | 0.152 |
| 284 | A | 7 | 6 | 1.00 | 33 | 0.182 |
| 285 | A | 7 | 5 | 1.00 | 33 | 0.152 |
| 286 | A | 5 | 4 | 1.00 | 33 | 0.121 |
| 287 | A | 6 | 5 | 1.00 | 33 | 0.152 |
| 288 | A | 7 | 5 | 1.00 | 33 | 0.152 |
| 289 | A | 2 | 1 | 1.00 | 36 | 0.028 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 290 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 291 | A | 2 | 1 | 1.00 | 34 | 0.029 |
| 292 | A | 2 | 1 | 1.00 | 29 | 0.034 |
| 293 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 294 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 295 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 296 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 297 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 298 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 299 | A | 2 | 1 | 1.00 | 31 | 0.032 |
| 300 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 301 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 302 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 303 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 304 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 305 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 306 | A | 6 | 5 | 1.00 | 36 | 0.139 |
| 307 | A | 6 | 5 | 1.00 | 31 | 0.161 |
| 308 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 309 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 310 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 311 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 312 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 313 | A | 7 | 6 | 1.00 | 36 | 0.167 |
| 314 | A | 7 | 6 | 1.00 | 31 | 0.194 |
| 315 | A | 7 | 6 | 1.00 | 38 | 0.158 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 316 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 317 | A | 7 | 6 | 1.00 | 38 | 0.158 |
| 318 | A | 8 | 6 | 1.00 | 38 | 0.158 |
| 319 | A | 8 | 6 | 1.00 | 38 | 0.158 |
| 320 | A | 6 | 6 | 1.00 | 36 | 0.167 |
| 321 | A | 5 | 4 | 1.00 | 31 | 0.129 |
| 322 | A | 8 | 6 | 1.00 | 38 | 0.158 |
| 323 | A | 8 | 6 | 1.00 | 38 | 0.158 |
| 324 | A | 7 | 5 | 1.00 | 38 | 0.132 |
| 325 | A | 7 | 5 | 1.00 | 33 | 0.152 |
| 326 | A | 9 | 7 | 1.00 | 40 | 0.175 |
| 327 | A | 9 | 8 | 1.00 | 40 | 0.200 |
| 328 | A | 9 | 8 | 1.00 | 40 | 0.200 |
| 329 | A | 9 | 7 | 1.00 | 40 | 0.175 |
| 330 | A | 9 | 7 | 1.00 | 40 | 0.175 |
| 331 | A | 9 | 7 | 1.00 | 40 | 0.175 |
| 332 | A | 7 | 5 | 1.00 | 40 | 0.125 |
| 333 | A | 8 | 6 | 1.00 | 40 | 0.150 |
| 334 | A | 8 | 5 | 1.00 | 38 | 0.132 |
| 335 | A | 8 | 5 | 1.00 | 33 | 0.152 |
| 336 | A | 10 | 7 | 1.00 | 40 | 0.175 |
| 337 | A | 10 | 8 | 1.00 | 40 | 0.200 |
| 338 | A | 10 | 8 | 1.00 | 40 | 0.200 |
| 339 | A | 10 | 7 | 1.00 | 40 | 0.175 |
| 340 | A | 10 | 8 | 1.00 | 40 | 0.200 |
| 341 | A | 10 | 7 | 1.00 | 40 | 0.175 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 342 | A | 10 | 8 | 1.00 | 40 | 0.200 |
| 343 | A | 10 | 7 | 1.00 | 40 | 0.175 |
| 344 | A | 6 | 4 | 1.00 | 38 | 0.105 |
| 345 | A | 6 | 4 | 1.00 | 33 | 0.121 |
| 346 | A | 8 | 6 | 1.00 | 40 | 0.150 |
| 347 | A | 8 | 7 | 1.00 | 40 | 0.175 |
| 348 | A | 8 | 7 | 1.00 | 40 | 0.175 |
| 349 | A | 8 | 6 | 1.00 | 40 | 0.150 |
| 350 | A | 6 | 4 | 1.00 | 40 | 0.100 |
| 351 | A | 7 | 5 | 1.00 | 40 | 0.125 |
| 352 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 353 | A | 5 | 5 | 1.00 | 33 | 0.152 |
| 354 | A | 7 | 7 | 1.00 | 40 | 0.175 |
| 355 | A | 7 | 7 | 1.00 | 40 | 0.175 |
| 356 | A | 5 | 5 | 1.00 | 40 | 0.125 |
| 357 | A | 6 | 5 | 1.00 | 40 | 0.125 |
| 358 | A | 6 | 5 | 1.00 | 40 | 0.125 |
| 359 | A | 5 | 4 | 1.00 | 38 | 0.105 |
| 360 | A | 5 | 4 | 1.00 | 33 | 0.121 |
| 361 | A | 5 | 4 | 1.00 | 40 | 0.100 |
| 362 | A | 5 | 4 | 1.00 | 40 | 0.100 |
| 363 | A | 6 | 5 | 1.00 | 40 | 0.125 |
| 364 | A | 7 | 5 | 1.00 | 40 | 0.125 |
| 365 | A | 5 | 4 | 1.00 | 35 | 0.114 |
| 366 | A | 5 | 4 | 1.00 | 36 | 0.111 |
| 367 | A | 2 | 1 | 1.00 | 38 | 0.026 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 368 | A | 2 | 1 | 1.00 | 38 | 0.026 |
| 369 | A | 2 | 1 | 1.00 | 36 | 0.028 |
| 370 | A | 4 | 2 | 1.00 | 38 | 0.053 |
| 371 | A | 5 | 3 | 1.00 | 38 | 0.079 |
| 372 | A | 6 | 5 | 1.00 | 38 | 0.132 |
| 373 | A | 6 | 5 | 1.00 | 53 | 0.094 |
| 374 | A | 11 | 5 | 1.00 | 35 | 0.143 |
| 375 | A | 9 | 5 | 1.00 | 35 | 0.143 |
| 376 | A | 7 | 5 | 1.00 | 33 | 0.152 |
| 377 | A | 9 | 7 | 1.00 | 35 | 0.200 |
| 378 | A | 9 | 7 | 1.00 | 35 | 0.200 |
| 379 | A | 7 | 5 | 1.00 | 35 | 0.143 |
| 380 | A | 12 | 5 | 1.00 | 35 | 0.143 |
| 381 | A | 10 | 5 | 1.00 | 35 | 0.143 |
| 382 | A | 8 | 5 | 1.00 | 33 | 0.152 |
| 383 | A | 10 | 7 | 1.00 | 35 | 0.200 |
| 384 | A | 10 | 8 | 1.00 | 35 | 0.229 |
| 385 | A | 10 | 7 | 1.00 | 35 | 0.200 |
| 386 | A | 10 | 4 | 1.00 | 35 | 0.114 |
| 387 | A | 8 | 4 | 1.00 | 35 | 0.114 |
| 388 | A | 6 | 4 | 1.00 | 33 | 0.121 |
| 389 | A | 8 | 6 | 1.00 | 35 | 0.171 |
| 390 | A | 6 | 4 | 1.00 | 35 | 0.114 |
| 391 | A | 7 | 4 | 1.00 | 35 | 0.114 |
| 392 | A | 9 | 5 | 1.00 | 35 | 0.143 |
| 393 | A | 7 | 5 | 1.00 | 35 | 0.143 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 394 | A | 5 | 5 | 1.00 | 33 | 0.152 |
| 395 | A | 6 | 4 | 1.00 | 35 | 0.114 |
| 396 | A | 7 | 4 | 1.00 | 35 | 0.114 |
| 397 | A | 8 | 4 | 1.00 | 35 | 0.114 |
| 398 | A | 7 | 5 | 1.00 | 26 | 0.192 |
| 399 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 400 | A | 11 | 8 | 1.00 | 29 | 0.276 |

# Chapter 3

# Listing of integrals

## 3.1 $\int (d+ex)^2 \left(A+Bx+Cx^2\right) \sqrt{d^2-e^2x^2}\, dx$

**Optimal**. Leaf size=236

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+3Cd^2\right)}{8e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7B\right.}{15e^3}$$

```
[Out] -1/15*d*(4*C*d^2+e*(10*A*e+7*B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/8*(3*C*d^2+2*
e*(A*e+2*B*d))*x*(-e^2*x^2+d^2)^(3/2)/e^2-1/5*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2
)^(3/2)/e-1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)+1/16*d^4*(10*A*e^2+4*B*d*e+3*C*d^2
)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/16*d^2*(10*A*e^2+4*B*d*e+3*C*d^2)*
x*(-e^2*x^2+d^2)^(1/2)/e^2
```

**Rubi [A]** time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1815, 641, 195, 217, 203}

$$\frac{d^2x\sqrt{d^2-e^2x^2}\left(10Ae^2+4Bde+3Cd^2\right)}{16e^2}-\frac{d\left(d^2-e^2x^2\right)^{3/2}\left(e(10Ae+7Bd)+4Cd^2\right)}{15e^3}-\frac{x\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2B\right.}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d^2*(3*C*d^2 + 4*B*d*e + 10*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(16*e^2) - (d*(4
*C*d^2 + e*(7*B*d + 10*A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((3*C*d^2 +
2*e*(2*B*d + A*e))*x*(d^2 - e^2*x^2)^(3/2))/(8*e^2) - ((2*C*d + B*e)*x^2*(d
^2 - e^2*x^2)^(3/2))/(5*e) - (C*x^3*(d^2 - e^2*x^2)^(3/2))/6 + (d^4*(3*C*d^
2 + 4*B*d*e + 10*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} - \frac{\int \sqrt{d^2-e^2x^2}\left(-6Ad^2e^2-6de^2(Bd+2Ae\right.}{}$$

$$= -\frac{(2Cd+Be)x^2\left(d^2-e^2x^2\right)^{3/2}}{5e} - \frac{1}{6}Cx^3\left(d^2-e^2x^2\right)^{3/2} + \frac{\int \sqrt{d^2-e^2}}{}$$

$$= -\frac{\left(3Cd^2+2e(2Bd+Ae)\right)x\left(d^2-e^2x^2\right)^{3/2}}{8e^2} - \frac{(2Cd+Be)x^2\left(d^2-e^2\right.}{5e}$$

$$= -\frac{d\left(4Cd^2+e(7Bd+10Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3} - \frac{\left(3Cd^2+2e(2Bd+A\right.}{8e^2}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

$$= \frac{d^2\left(3Cd^2+4Bde+10Ae^2\right)x\sqrt{d^2-e^2x^2}}{16e^2} - \frac{d\left(4Cd^2+e(7Bd+10A\right.}{15e^3}$$

**Mathematica [A]** time = 0.51, size = 226, normalized size = 0.96

$$\frac{\sqrt{d^2-e^2x^2}\left(15\sin^{-1}\left(\frac{ex}{d}\right)\left(2d^3e(5Ae+2Bd)+3Cd^5\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\left(2e\left(5Ae\left(-16d^3+9d^2ex+16de^2x^2+6e^3x^3\right)\right.\right.\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-64*d^5 - 45*d^4*e*x - 32
*d^3*e^2*x^2 + 50*d^2*e^3*x^3 + 96*d*e^4*x^4 + 40*e^5*x^5) + 2*e*(5*A*e*(-1
6*d^3 + 9*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3) + B*(-56*d^4 - 30*d^3*e*x + 3
2*d^2*e^2*x^2 + 60*d*e^3*x^3 + 24*e^4*x^4))) + 15*(3*C*d^5 + 2*d^3*e*(2*B*d
 + 5*A*e))*ArcSin[(e*x)/d]))/(240*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

**fricas [A]** time = 0.68, size = 211, normalized size = 0.89

$$-\frac{30\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(40\,Ce^5x^5-64\,Cd^5-112\,Bd^4e-160\,Ad^3e^2+48\left(2\,C\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/240*(30*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arctan(-(d - sqrt(-e^2*x^2
+ d^2))/(e*x)) - (40*C*e^5*x^5 - 64*C*d^5 - 112*B*d^4*e - 160*A*d^3*e^2 + 4
8*(2*C*d*e^4 + B*e^5)*x^4 + 10*(5*C*d^2*e^3 + 12*B*d*e^4 + 6*A*e^5)*x^3 - 3
2*(C*d^3*e^2 - 2*B*d^2*e^3 - 5*A*d*e^4)*x^2 - 15*(3*C*d^4*e + 4*B*d^3*e^2 -
 6*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

**giac [A]** time = 0.22, size = 197, normalized size = 0.83

$$\frac{1}{16}\left(3\,Cd^6+4\,Bd^5e+10\,Ad^4e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{240}\sqrt{-x^2e^2+d^2}\left(\left(2\left(\left(4\left(5\,Cxe^2+6\left(2\,Cde^9+Be^{10}\right)e^{(-\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*C*d^6 + 4*B*d^5*e + 10*A*d^4*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/2
40*sqrt(-x^2*e^2 + d^2)*((2*((4*(5*C*x*e^2 + 6*(2*C*d*e^9 + B*e^10)*e^(-8))
*x + 5*(5*C*d^2*e^8 + 12*B*d*e^9 + 6*A*e^10)*e^(-8))*x - 16*(C*d^3*e^7 - 2*
B*d^2*e^8 - 5*A*d*e^9)*e^(-8))*x - 15*(3*C*d^4*e^6 + 4*B*d^3*e^7 - 6*A*d^2*
e^8)*e^(-8))*x - 16*(4*C*d^5*e^5 + 7*B*d^4*e^6 + 10*A*d^3*e^7)*e^(-8))
```

**maple [A]** time = 0.06, size = 371, normalized size = 1.57

$$\frac{5A\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}}+\frac{B\,d^5\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{4\sqrt{e^2}\,e}+\frac{3C\,d^6\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{16\sqrt{e^2}\,e^2}+\frac{5\sqrt{-e^2x^2+d^2}\;A\,d^2x}{8}+\frac{\sqrt{-e^2x^2+\cdots}}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/6*C*x^3*(-e^2*x^2+d^2)^(3/2)-3/8/e^2*C*d^2*x*(-e^2*x^2+d^2)^(3/2)+3/16/e
^2*C*d^4*x*(-e^2*x^2+d^2)^(1/2)+3/16/e^2*C*d^6/(e^2)^(1/2)*arctan((e^2)^(1/
2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*x^2*(-e^2*x^2+d^2)^(3/2)*B-2/5*x^2*(-e^2*x^2
+d^2)^(3/2)/e*d*C-7/15*d^2/e^2*(-e^2*x^2+d^2)^(3/2)*B-4/15*d^3/e^3*(-e^2*x^
2+d^2)^(3/2)*C-1/4*x*(-e^2*x^2+d^2)^(3/2)*A-1/2*x*(-e^2*x^2+d^2)^(3/2)/e*B*
d+5/8*d^2*x*(-e^2*x^2+d^2)^(1/2)*A+1/4*d^3/e*x*(-e^2*x^2+d^2)^(1/2)*B+5/8*d
^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*A+1/4*d^5/e/(e^2)
^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*B-2/3*(-e^2*x^2+d^2)^(3/2
)/e*A*d
```

**maxima [A]** time = 1.00, size = 338, normalized size = 1.43

$$-\frac{1}{6}\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^3+\frac{Cd^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^3}+\frac{Ad^4\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{1}{2}\sqrt{-e^2x^2+d^2}\,Ad^2x+\frac{\sqrt{-e^2x^2+d^2}\,Cd^4x}{16\,e^2}-\frac{\left(-e^2x^2+\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/6*(-e^2*x^2 + d^2)^(3/2)*C*x^3 + 1/16*C*d^6*arcsin(e*x/d)/e^3 + 1/2*A*d^
4*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2 + d^2)*A*d^2*x + 1/16*sqrt(-e^2*x^2 +
 d^2)*C*d^4*x/e^2 - 1/8*(-e^2*x^2 + d^2)^(3/2)*C*d^2*x/e^2 + 1/8*(C*d^2 + 2
*B*d*e + A*e^2)*d^4*arcsin(e*x/d)/e^3 - 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d^2/e^
2 - 2/3*(-e^2*x^2 + d^2)^(3/2)*A*d/e + 1/8*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*
B*d*e + A*e^2)*d^2*x/e^2 - 1/5*(-e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*x^2
/e^2 - 1/4*(-e^2*x^2 + d^2)^(3/2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/15*(-
e^2*x^2 + d^2)^(3/2)*(2*C*d*e + B*e^2)*d^2/e^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int\sqrt{d^2-e^2\,x^2}\;(d+e\,x)^2\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2*(A + B*x + C*x^2), x)
```

**sympy [C]** time = 22.93, size = 1231, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2
/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + 2*
A*d*e*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/
2)/(3*e**2), True)) + A*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d
**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**
2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
 > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)
```

```
) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x*
*2/d**2)), True)) + B*d**2*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d
**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + 2*B*d*e*Piecewise((-I*d**4*acosh
(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3
/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sq
rt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5
/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*e**2*Piecewise((-2*d**4*sqrt(d*
*2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x*
*4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)) + C*d**2
*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2
*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*
sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*
e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**
2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 2*C*d*e*
Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
 e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqr
t(d**2)/4, True)) + C*e**2*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d*
*5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e
**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(
6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)
/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2
*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True))
```

## 3.2 $\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

**Optimal**. Leaf size=186

$$\frac{dx\sqrt{d^2-e^2x^2}\,\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)+\right.}{8e^3}$$

```
[Out] -1/15*(2*C*d^2+5*e*(A*e+B*d))*(-e^2*x^2+d^2)^(3/2)/e^3-1/4*(B*e+C*d)*x*(-e^
2*x^2+d^2)^(3/2)/e^2-1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e+1/8*d^3*(C*d^2+e*(4*A
*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*d*(C*d^2+e*(4*A*e+B*d))*x
*(-e^2*x^2+d^2)^(1/2)/e^2
```

**Rubi [A]** time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1815, 641, 195, 217, 203}

$$\frac{dx\sqrt{d^2-e^2x^2}\,\left(e(4Ae+Bd)+Cd^2\right)}{8e^2}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(5e(Ae+Bd)+2Cd^2\right)}{15e^3}+\frac{d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(4Ae+Bd)+\right.}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (d*(C*d^2 + e*(B*d + 4*A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d^2 + 5
*e*(B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(15*e^3) - ((C*d + B*e)*x*(d^2 - e^2
*x^2)^(3/2))/(4*e^2) - (C*x^2*(d^2 - e^2*x^2)^(3/2))/(5*e) + (d^3*(C*d^2 +
e*(B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int (d+ex)\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx = -\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}-\frac{\int \sqrt{d^2-e^2x^2}\,\left(-5Ade^2-e\left(2Cd^2+5e(Bd+\right.\right.}{5e^2}$$

$$= -\frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{Cx^2\left(d^2-e^2x^2\right)^{3/2}}{5e}+\frac{\int \left(5de^2\left(Cd^2+e\right.\right.}{}$$

$$= -\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-e^2x^2\right)^{3/2}}{15e^3}-\frac{(Cd+Be)x\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$

$$= \frac{d\left(Cd^2+e(Bd+4Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{\left(2Cd^2+5e(Bd+Ae)\right)\left(d^2-\right.}{15e^3}$$

**Mathematica [A]** time = 0.30, size = 190, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}\left(15\sin^{-1}\left(\frac{ex}{d}\right)\left(d^2e(4Ae+Bd)+Cd^4\right)+\sqrt{1-\frac{e^2x^2}{d^2}}\,\left(5e\left(4Ae\left(-2d^2+3dex+2e^2x^2\right)+B\left(-8d^3-3d^2e\right.\right.\right.\right.}{120e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(Sqrt[1 - (e^2*x^2)/d^2]*(C*(-16*d^4 - 15*d^3*e*x - 8*
d^2*e^2*x^2 + 30*d*e^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(-2*d^2 + 3*d*e*x + 2
*e^2*x^2) + B*(-8*d^3 - 3*d^2*e*x + 8*d*e^2*x^2 + 6*e^3*x^3))) + 15*(C*d^4
+ d^2*e*(B*d + 4*A*e))*ArcSin[(e*x)/d]))/(120*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

**fricas [A]** time = 0.87, size = 173, normalized size = 0.93

$$-\frac{30\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(24\,Ce^4x^4-16\,Cd^4-40\,Bd^3e-40\,Ad^2e^2+30\left(Cde^3+B\right.\right.}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(30*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2
))/(e*x)) - (24*C*e^4*x^4 - 16*C*d^4 - 40*B*d^3*e - 40*A*d^2*e^2 + 30*(C*d*
e^3 + B*e^4)*x^3 - 8*(C*d^2*e^2 - 5*B*d*e^3 - 5*A*e^4)*x^2 - 15*(C*d^3*e +
B*d^2*e^2 - 4*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

**giac [A]** time = 0.21, size = 160, normalized size = 0.86

$$\frac{1}{8}\left(Cd^5+Bd^4e+4\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)+\frac{1}{120}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\left(4\,Cxe+5\left(Cde^6+Be^7\right)e^{(-6)}\right)x-4\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^5 + B*d^4*e + 4*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) + 1/120*sqr
t(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(C*d*e^6 + B*e^7)*e^(-6))*x - 4*(C*d^
2*e^5 - 5*B*d*e^6 - 5*A*e^7)*e^(-6))*x - 15*(C*d^3*e^4 + B*d^2*e^5 - 4*A*d*
e^6)*e^(-6))*x - 8*(2*C*d^4*e^3 + 5*B*d^3*e^4 + 5*A*d^2*e^5)*e^(-6))
```

**maple [A]** time = 0.02, size = 304, normalized size = 1.63

$$\frac{A\,d^3\arctan\left(\frac{\sqrt{e^2}\;x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{B\,d^4\arctan\left(\frac{\sqrt{e^2}\;x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\;e}+\frac{C\,d^5\arctan\left(\frac{\sqrt{e^2}\;x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\;e^2}+\frac{\sqrt{-e^2x^2+d^2}\;Adx}{2}+\frac{\sqrt{-e^2x^2+d^2}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*x^2*(-e^2*x^2+d^2)^(3/2)/e-2/15/e^3*C*d^2*(-e^2*x^2+d^2)^(3/2)-1/4*x
*(-e^2*x^2+d^2)^(3/2)/e*B-1/4*x*(-e^2*x^2+d^2)^(3/2)/e^2*C*d+1/8*d^2/e*x*(-
e^2*x^2+d^2)^(1/2)*B+1/8*d^3/e^2*x*(-e^2*x^2+d^2)^(1/2)*C+1/8*d^4/e/(e^2)^(
1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*B+1/8*d^5/e^2/(e^2)^(1/2)*a
rctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)*C-1/3*(-e^2*x^2+d^2)^(3/2)/e*A-1/
3*(-e^2*x^2+d^2)^(3/2)/e^2*B*d+1/2*d*A*x*(-e^2*x^2+d^2)^(1/2)+1/2*d^3*A/(e^
2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

**maxima [A]** time = 0.98, size = 202, normalized size = 1.09

$$\frac{Ad^3\arcsin\left(\frac{ex}{d}\right)}{2e}+\frac{1}{2}\sqrt{-e^2x^2+d^2}\,Adx-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx^2}{5e}+\frac{(Cd+Be)d^4\arcsin\left(\frac{ex}{d}\right)}{8e^3}+\frac{\sqrt{-e^2x^2+d^2}\,(Cd+Be)d^2x}{8e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*d^3*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2 + d^2)*A*d*x - 1/5*(-e^2*x^2
+ d^2)^(3/2)*C*x^2/e + 1/8*(C*d + B*e)*d^4*arcsin(e*x/d)/e^3 + 1/8*sqrt(-e^
2*x^2 + d^2)*(C*d + B*e)*d^2*x/e^2 - 2/15*(-e^2*x^2 + d^2)^(3/2)*C*d^2/e^3
- 1/3*(-e^2*x^2 + d^2)^(3/2)*B*d/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*A/e - 1/4
*(-e^2*x^2 + d^2)^(3/2)*(C*d + B*e)*x/e^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(d + e*x)*(A + B*x + C*x^2), x)
```

**sympy [C]** time = 12.77, size = 670, normalized size = 3.60

$$Ad\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right)+Ae\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)+Bd\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} \\ -\frac{\left(d^2-e^2x^2\right)}{3e^2} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d*
*2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1
), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + A*e*P
iecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e
**2), True)) + B*d*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e*
*2*x**2)**(3/2)/(3*e**2), True)) + B*e*Piecewise((-I*d**4*acosh(e*x/d)/(8*e
**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*
x**2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*
x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1
 - e**2*x**2/d**2)), True)) + C*d*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3)
+ I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**
2*x**2/d**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/
d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/
d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e*
*2*x**2/d**2)), True)) + C*e*Piecewise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*
e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*
x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True))
```

## 3.3 $\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx$

**Optimal**. Leaf size=125

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

```
[Out] -1/3*B*(-e^2*x^2+d^2)^(3/2)/e^2-1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*d^2*(4
*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/8*(4*A+C*d^2/e^2)*x*(-
e^2*x^2+d^2)^(1/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1815, 641, 195, 217, 203}

$$\frac{1}{8}x\sqrt{d^2-e^2x^2}\left(4A+\frac{Cd^2}{e^2}\right)+\frac{d^2\left(4Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] ((4*A + (C*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2])/8 - (B*(d^2 - e^2*x^2)^(3/2))/(
3*e^2) - (C*x*(d^2 - e^2*x^2)^(3/2))/(4*e^2) + (d^2*(C*d^2 + 4*A*e^2)*ArcTa
n[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}\,dx &= -\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\int\left(-Cd^2-4Ae^2-4Be^2x\right)\sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= -\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(-Cd^2-4Ae^2\right)\int\sqrt{d^2-e^2x^2}\,dx}{4e^2}\\
&= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(d^2\left(-\right.\right.}{}\\
&= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}-\frac{\left(d^2\left(-\right.\right.}{}\\
&= \frac{\left(Cd^2+4Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2}-\frac{B\left(d^2-e^2x^2\right)^{3/2}}{3e^2}-\frac{Cx\left(d^2-e^2x^2\right)^{3/2}}{4e^2}+\frac{d^2\left(C\right.}{}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 121, normalized size = 0.97

$$\frac{\sqrt{d^2-e^2x^2}\left(e\sqrt{1-\frac{e^2x^2}{d^2}}\left(12Ae^2x-8Bd^2+8Be^2x^2-3Cd^2x+6Ce^2x^3\right)+3\left(4Ade^2+Cd^3\right)\sin^{-1}\left(\frac{ex}{d}\right)\right)}{24e^3\sqrt{1-\frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(e*Sqrt[1 - (e^2*x^2)/d^2]*(-8*B*d^2 - 3*C*d^2*x + 12*
A*e^2*x + 8*B*e^2*x^2 + 6*C*e^2*x^3) + 3*(C*d^3 + 4*A*d*e^2)*ArcSin[(e*x)/d
]))/(24*e^3*Sqrt[1 - (e^2*x^2)/d^2])
```

**fricas [A]** time = 0.97, size = 108, normalized size = 0.86

$$-\frac{6\left(Cd^4+4\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)-\left(6\,Ce^3x^3+8\,Be^3x^2-8\,Bd^2e-3\left(Cd^2e-4\,Ae^3\right)x\right)\sqrt{-e^2x^2+d^2}}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*(C*d^4 + 4*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) -
(6*C*e^3*x^3 + 8*B*e^3*x^2 - 8*B*d^2*e - 3*(C*d^2*e - 4*A*e^3)*x)*sqrt(-e^2
*x^2 + d^2))/e^3
```

**giac [A]** time = 0.20, size = 85, normalized size = 0.68

$$\frac{1}{8}\left(Cd^4+4\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\left(8\,Bd^2e^{(-2)}-\left(2\,(3\,Cx+4\,B)x-3\left(Cd^2e^2-4\,Ae^4\right)e^{(-4)}\right)x\right)\sqrt{-x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(C*d^4 + 4*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24*(8*B*d^2*e^(-2
) - (2*(3*C*x + 4*B)*x - 3*(C*d^2*e^2 - 4*A*e^4)*e^(-4))*x)*sqrt(-x^2*e^2 +
 d^2)
```

**maple [A]** time = 0.01, size = 154, normalized size = 1.23

$$\frac{A\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{C\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}+\frac{\sqrt{-e^2x^2+d^2}\,Ax}{2}+\frac{\sqrt{-e^2x^2+d^2}\,C\,d^2x}{8e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/8*C*d^2/e^2*x*(-e^2*x^2+d^2)^(1/2)+1/8*
C*d^4/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3*B*(-e^
2*x^2+d^2)^(3/2)/e^2+1/2*A*x*(-e^2*x^2+d^2)^(1/2)+1/2*A*d^2/(e^2)^(1/2)*arc
tan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

**maxima [A]** time = 0.96, size = 116, normalized size = 0.93

$$\frac{Cd^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{Ad^2\arcsin\left(\frac{ex}{d}\right)}{2\,e}+\frac{1}{2}\,\sqrt{-e^2x^2+d^2}\,Ax+\frac{\sqrt{-e^2x^2+d^2}\,Cd^2x}{8\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}Cx}{4\,e^2}-\frac{\left(-e^2x^2+d^2\right)^{\frac{3}{2}}B}{3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*C*d^4*arcsin(e*x/d)/e^3 + 1/2*A*d^2*arcsin(e*x/d)/e + 1/2*sqrt(-e^2*x^2
 + d^2)*A*x + 1/8*sqrt(-e^2*x^2 + d^2)*C*d^2*x/e^2 - 1/4*(-e^2*x^2 + d^2)^(
3/2)*C*x/e^2 - 1/3*(-e^2*x^2 + d^2)^(3/2)*B/e^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2), x)
```

**sympy [C]** time = 7.11, size = 343, normalized size = 2.74

$$A\left(\begin{cases} -\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e}-\frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}}+\frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for}\ \left|\frac{e^2x^2}{d^2}\right|>1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e}+\frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases}\right)+B\left(\begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for}\ e^2=0 \\ -\frac{\left(d^2-e^2x^2\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}\right)+C\left(\begin{cases} -\frac{id^4\operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} \\ \frac{d^4\operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3}- \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2
)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1),
 (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + B*Piece
wise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2)
, True)) + C*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**
2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asi
n(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**3/(8*
sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)
)
```

## 3.4 $\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx$

**Optimal**. Leaf size=148

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2*(-B*e+C*d)*(-e^2*x^2+d^2)^(3/2)/e^3/(e*
x+d)+1/2*d*(C*d^2-e*(-2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+1/2*
(C*d^2-e*(-2*A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3
```

**Rubi [A]** time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 795, 665, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(Cd^2-e(Bd-2Ae)\right)}{2e^3}+\frac{\left(d^2-e^2x^2\right)^{3/2}(Cd-Be)}{2e^3(d+ex)}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] ((C*d^2 - e*(B*d - 2*A*e))*Sqrt[d^2 - e^2*x^2])/(2*e^3) - (C*(d^2 - e^2*x^2
)^(3/2))/(3*e^3) + ((C*d - B*e)*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)) +
(d*(C*d^2 - e*(B*d - 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
```

```
|| EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}-\frac{\int \frac{\left(-3Ae^4+3e^3(Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{d+ex}\,dx}{3e^4}$$

$$= -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)}+\frac{\left(Cd^2-e(Bd-2Ae)\right)\int \frac{\sqrt{d^2 \ldots}}{d \ldots}}{2e^2}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x \ldots\right.}{2e^3(d+ex)}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x \ldots\right.}{2e^3(d+ex)}$$

$$= \frac{\left(Cd^2-e(Bd-2Ae)\right)\sqrt{d^2-e^2x^2}}{2e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{3e^3}+\frac{(Cd-Be)\left(d^2-e^2x \ldots\right.}{2e^3(d+ex)}$$

**Mathematica [A]** time = 0.22, size = 103, normalized size = 0.70

$$\frac{\sqrt{d^2-e^2x^2}\left(3e(2Ae-2Bd+Bex)+C\left(4d^2-3dex+2e^2x^2\right)\right)+3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae-Bd)+Cd^2\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(3*e*(-2*B*d + 2*A*e + B*e*x) + C*(4*d^2 - 3*d*e*x + 2
*e^2*x^2)) + 3*d*(C*d^2 + e*(-(B*d) + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(6*e^3)
```

**fricas [A]** time = 0.89, size = 112, normalized size = 0.76

$$-\frac{6\left(Cd^3 - Bd^2e + 2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - \left(2\,Ce^2x^2 + 4\,Cd^2 - 6\,Bde + 6\,Ae^2 - 3\left(Cde - Be^2\right)x\right)\sqrt{-e^2x^2 + d^2}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 - B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) - (2*C*e^2*x^2 + 4*C*d^2 - 6*B*d*e + 6*A*e^2 - 3*(C*d*e - B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
4*A*d*exp(2)^3-4*A*d*exp(1)^4*exp(2)+4*B*d^2*exp(1)^3*exp(2)-4*B*d^2*exp(1)
*exp(2)^2)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))
/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/exp(1)^4/exp(1)-1/4*(-2
*C*d^3-4*A*d*exp(2)+2*B*d^2*exp(1))*sign(d)*asin(x*exp(2)/d/exp(1))/exp(1)/
exp(2)+2*((16*exp(1)^4*C*1/96/exp(1)^5*x-(-24*exp(1)^4*B+24*exp(1)^3*C*d)*1
/96/exp(1)^5)*x-(-48*exp(1)^4*A+48*exp(1)^3*d*B-32*exp(1)^2*C*d^2)*1/96/exp
(1)^5)*sqrt(d^2-x^2*exp(2))
```

**maple [B]** time = 0.02, size = 384, normalized size = 2.59

$$\frac{Ad\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}} - \frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e} + \frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e} + \frac{C\,d^3\arctan\left(\frac{}{\sqrt{2\left(x\right.}}\right)}{\sqrt{e^2}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)
```

```
[Out] -1/3*C*(-e^2*x^2+d^2)^(3/2)/e^3+1/2/e*B*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*B*d^2/
(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/2/e^2*C*d*x*(-e^2*
x^2+d^2)^(1/2)-1/2/e^2*C*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^
(1/2)*x)+1/e*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*A-1/e^2*(-(x+d/e)^2*e^2+2
*d*e*(x+d/e))^(1/2)*B*d+1/e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*C*d^2+d/
(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))*A-1/
e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)
)*B+1/e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^(1/2))*C
```

**maxima** [A] time = 1.09, size = 171, normalized size = 1.16

$$\frac{Cd^3 \arcsin\left(\frac{ex}{d}\right)}{2\,e^3} - \frac{Bd^2 \arcsin\left(\frac{ex}{d}\right)}{2\,e^2} + \frac{Ad \arcsin\left(\frac{ex}{d}\right)}{e} - \frac{\sqrt{-e^2x^2+d^2}\,Cdx}{2\,e^2} + \frac{\sqrt{-e^2x^2+d^2}\,Bx}{2\,e} + \frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{e^3} - \sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^3*arcsin(e*x/d)/e^3 - 1/2*B*d^2*arcsin(e*x/d)/e^2 + A*d*arcsin(e*x/
d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*C*d*x/e^2 + 1/2*sqrt(-e^2*x^2 + d^2)*B*x/e
+ sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 + d^2)*B*d/e^2 + sqrt(-e^2
*x^2 + d^2)*A/e - 1/3*(-e^2*x^2 + d^2)^(3/2)*C/e^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x), x)
```

## 3.5 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^2} \, dx$

**Optimal**. Leaf size=170

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(5Cd^2-2e(2Bd-Ae))}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae)}{2e^3}$$

```
[Out] -(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^2-1/2*C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*arctan(e*x/(-e^2*x^2+d
^2)^(1/2))/e^3-1/2*(5*C*d^2-2*e*(-A*e+2*B*d))*(-e^2*x^2+d^2)^(1/2)/d/e^3
```

**Rubi [A]** time = 0.20, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 793, 665, 217, 203}

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{de^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(5Cd^2-2e(2Bd-Ae))}{2de^3}-\frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(5Cd^2-2e(2Bd-Ae)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] -((5*C*d^2 - 2*e*(2*B*d - A*e))*Sqrt[d^2 - e^2*x^2])/(2*d*e^3) - ((C*d^2 -
B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(d*e^3*(d + e*x)^2) - (C*(d^2 - e^2*x
^2)^(3/2))/(2*e^3*(d + e*x)) - ((5*C*d^2 - 2*e*(2*B*d - A*e))*ArcTan[(e*x)/
Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)} - \frac{\int \frac{\left(e^2\left(Cd^2-2Ae^2\right)+e^3(3Cd-2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2} - \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)} - \frac{\left(-3e^5\left(Cd^2-2Ae\right.\right.}{}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

$$= -\frac{\left(5Cd^2-2e(2Bd-Ae)\right)\sqrt{d^2-e^2x^2}}{2de^3} - \frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{de^3(d+ex)^2}$$

**Mathematica [A]** time = 0.23, size = 109, normalized size = 0.64

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(2e(-2Ae+3Bd+Bex)+C\left(-8d^2-3dex+e^2x^2\right)\right)}{d+ex}-\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(2e(Ae-2Bd)+5Cd^2\right)}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^2,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(2*e*(3*B*d - 2*A*e + B*e*x) + C*(-8*d^2 - 3*d*e*x +
e^2*x^2)))/(d + e*x) - (5*C*d^2 + 2*e*(-2*B*d + A*e))*ArcTan[(e*x)/Sqrt[d^2
 - e^2*x^2]])/(2*e^3)
```

**fricas [A]** time = 0.68, size = 190, normalized size = 1.12

$$-\frac{8\,Cd^3-6\,Bd^2e+4\,Ade^2+2\left(4\,Cd^2e-3\,Bde^2+2\,Ae^3\right)x-2\left(5\,Cd^3-4\,Bd^2e+2\,Ade^2+\left(5\,Cd^2e-4\,Bde^2+2\,A\right.\right.}{2\left(e^4x+de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(8*C*d^3 - 6*B*d^2*e + 4*A*d*e^2 + 2*(4*C*d^2*e - 3*B*d*e^2 + 2*A*e^3)
*x - 2*(5*C*d^3 - 4*B*d^2*e + 2*A*d*e^2 + (5*C*d^2*e - 4*B*d*e^2 + 2*A*e^3)
*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (C*e^2*x^2 - 8*C*d^2 + 6*B*
d*e - 4*A*e^2 - (3*C*d*e - 2*B*e^2)*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x + d*e^3
)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

**maple [B]** time = 0.03, size = 439, normalized size = 2.58

$$-\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}}+\frac{2Bd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e}-\frac{3C\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^2}+\frac{C\,d^2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x)
```

```
[Out] 1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/2*C/e^2*d^2/(e^2)^(1/2)*arctan((e^2)^(1/
2)/(-e^2*x^2+d^2)^(1/2)*x)-1/e^3/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)*A+1/e^4/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B-1/e^5*d/(x+d/
e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/e/d*(2*(x+d/e)*d*e-(x+d/e)^2*e
^2)^(1/2)*A+2/e^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*B-3/e^3*d*(2*(x+d/e)*
d*e-(x+d/e)^2*e^2)^(1/2)*C-1/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e-
(x+d/e)^2*e^2)^(1/2)*x)*A+2/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x)*B*d-3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e
)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*C*d^2
```

**maxima [A]** time = 1.01, size = 197, normalized size = 1.16

$$-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{e^4x+de^3}+\frac{2\sqrt{-e^2x^2+d^2}\,Bd}{e^3x+de^2}-\frac{5\,Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{2\,Bd\arcsin\left(\frac{ex}{d}\right)}{e^2}-\frac{A\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{2\sqrt{-e^2x^2+d^2}\,A}{e^2x+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^2,x, algorithm="maxima
")
```

```
[Out] -2*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^4*x + d*e^3) + 2*sqrt(-e^2*x^2 + d^2)*B*d/
(e^3*x + d*e^2) - 5/2*C*d^2*arcsin(e*x/d)/e^3 + 2*B*d*arcsin(e*x/d)/e^2 - A
*arcsin(e*x/d)/e - 2*sqrt(-e^2*x^2 + d^2)*A/(e^2*x + d*e) + 1/2*sqrt(-e^2*x
^2 + d^2)*C*x/e^2 - 2*sqrt(-e^2*x^2 + d^2)*C*d/e^3 + sqrt(-e^2*x^2 + d^2)*B
/e^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**2, x)
```

## 3.6 $\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx$

**Optimal**. Leaf size=149

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}\,(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

```
[Out] -1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^2+(-B*e+3*C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3+2
*(-B*e+3*C*d)*(-e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

**Rubi [A]** time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1639, 793, 663, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{3de^3(d+ex)^3}+\frac{2\sqrt{d^2-e^2x^2}\,(3Cd-Be)}{e^3(d+ex)}+\frac{(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] (2*(3*C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A
*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*d*e^3*(d + e*x)^3) - (C*(d^2 - e^2*x^2)^(3/
2))/(e^3*(d + e*x)^2) + ((3*C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e
^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
```

```
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx = -\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}-\frac{\int \frac{\left(e^2\left(2Cd^2-Ae^2\right)+e^3(3Cd-Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}-\frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^2}-\frac{(3Cd-Be)\int \frac{\sqrt{d^2-e}}{(d+ex}}{e^2}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}-\frac{C\left(d^2-e^2x^2\right.}{e^3(d+ex)}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}-\frac{C\left(d^2-e^2x^2\right.}{e^3(d+ex)}$$

$$= \frac{2(3Cd-Be)\sqrt{d^2-e^2x^2}}{e^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{3de^3(d+ex)^3}-\frac{C\left(d^2-e^2x^2\right.}{e^3(d+ex)}$$

**Mathematica [A]** time = 0.22, size = 114, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(e(Ae(ex-d)-Bd(5d+7ex))+Cd\left(14d^2+19dex+3e^2x^2\right)\right)}{d(d+ex)^2}+3(3Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^3,x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d*(14*d^2 + 19*d*e*x + 3*e^2*x^2) + e*(A*e*(-d + e
*x) - B*d*(5*d + 7*e*x))))/(d*(d + e*x)^2) + 3*(3*C*d - B*e)*ArcTan[(e*x)/S
qrt[d^2 - e^2*x^2]])/(3*e^3)
```

**fricas [A]** time = 0.85, size = 258, normalized size = 1.73

$$14\,Cd^4-5\,Bd^3e-Ad^2e^2+\left(14\,Cd^2e^2-5\,Bde^3-Ae^4\right)x^2+2\left(14\,Cd^3e-5\,Bd^2e^2-Ade^3\right)x-6\left(3\,Cd^4-Bd^3e+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="fricas
")
```

```
[Out] 1/3*(14*C*d^4 - 5*B*d^3*e - A*d^2*e^2 + (14*C*d^2*e^2 - 5*B*d*e^3 - A*e^4)*
x^2 + 2*(14*C*d^3*e - 5*B*d^2*e^2 - A*d*e^3)*x - 6*(3*C*d^4 - B*d^3*e + (3*
C*d^2*e^2 - B*d*e^3)*x^2 + 2*(3*C*d^3*e - B*d^2*e^2)*x)*arctan(-(d - sqrt(-
e^2*x^2 + d^2))/(e*x)) + (3*C*d*e^2*x^2 + 14*C*d^3 - 5*B*d^2*e - A*d*e^2 +
(19*C*d^2*e - 7*B*d*e^2 + A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^2 + 2*d^
2*e^4*x + d^3*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-8*A
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp
(2)^2-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^10*exp(2)-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^3*exp(1)^8*exp(2)^2-4*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^3*exp(1)^6*exp(2)^3-A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
```

```
*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^3+A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^4-4*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^4-A*exp(1)^6*exp(2)^3-
B*d*exp(1)^5*exp(2)^3-4*A*exp(1)^4*exp(2)^4+4*B*d*exp(1)^3*exp(2)^4-8*C*d^2
*exp(2)^5+2*B*d*exp(1)*exp(2)^5+2*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^2+8*B*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^2+4*B*d*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)^3-8*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(
2)^2-4*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^4*exp(2)^3+3*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^5*exp(2)^3+B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^3*exp(2)^4-5*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^3-3*C*d^2*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^5+4*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^4+3*C*d^2*e
xp(1)^4*exp(2)^3-8*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^2*exp(2)^5+1/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^4*exp(2)^4/x/exp(2)+6*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(
1)^6*exp(2)^3/x/exp(2)+A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^8*exp(2)^2/x/exp(2)-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^2*exp(1)^9*exp(2)+2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)*exp(2)^5+6*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)+13/2*C*d^2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)-7/2*B*d*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)-6*B*d*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^3/x/exp(2)+2*B*d*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2)+6*C*d^2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^3/x/exp(2)-5*C*d^2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d*exp(1)^9+d*exp(1)^5*exp(2)^2-d
*exp(1)^7*exp(2)+d*exp(1)*exp(2)^4)+1/2*(4*A*exp(1)^8*exp(2)^2-12*B*d*exp(1
)^7*exp(2)^2+20*C*d^2*exp(1)^6*exp(2)^2+2*A*exp(1)^6*exp(2)^3-6*B*d*exp(1)^
5*exp(2)^3+18*C*d^2*exp(1)^4*exp(2)^3+4*A*exp(1)^4*exp(2)^4+4*B*d*exp(1)^3*
exp(2)^4-24*C*d^2*exp(2)^5+4*B*d*exp(1)*exp(2)^5-4*C*d^2*exp(1)^8*exp(2))*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d*exp(1)^11-d*exp(1)^7*exp(2)^2-d*
exp(1)^5*exp(2)^3+d*exp(1)^9*exp(2))-1/4*(4*B*exp(1)-12*C*d)*sign(d)*asin(x
*exp(2)/d/exp(1))/exp(1)/exp(2)+4*exp(1)^2*C*1/4/exp(1)^5*sqrt(d^2-x^2*exp(
2))
```

**maple [B]** time = 0.02, size = 318, normalized size = 2.13

$$-\frac{B\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e}+\frac{3Cd\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}}\right)}{\sqrt{e^2}\,e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2e^2}\,B}{d\,e^2}+\frac{3\sqrt{2\left(x+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x)
```

```
[Out] -1/e^4/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B+2/e^5/(x+d/e)^2*(2
*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/e^2/d*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(
1/2)*B+3/e^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*C-1/e/(e^2)^(1/2)*arctan((
e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*B+3/e^2/(e^2)^(1/2)*arcta
n((e^2)^(1/2)/(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*x)*C*d-1/3*(A*e^2-B*d*e+C
*d^2)/e^6/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3,x, algorithm="maxima
")
```

```
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2-e^2\,x^2}\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**3, x)
```

## 3.7 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx$

**Optimal**. Leaf size=196

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{15d^2e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^4}+\frac{(d^2-e^2x^2)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^4+1/3*(-B*e+2*C
*d)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^3-1/15*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2
+d^2)^(3/2)/d^2/e^3/(e*x+d)^3-C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-2*C*(-
e^2*x^2+d^2)^(1/2)/e^3/(e*x+d)
```

**Rubi [A]** time = 0.18, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1637, 659, 651, 663, 217, 203}

$$-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{15d^2e^3(d+ex)^3}-\frac{(d^2-e^2x^2)^{3/2}(Ae^2-Bde+Cd^2)}{5de^3(d+ex)^4}+\frac{(d^2-e^2x^2)^{3/2}(2Cd-Be)}{3de^3(d+ex)^3}-\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] (-2*C*Sqrt[d^2 - e^2*x^2])/(e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2)*(d^2
- e^2*x^2)^(3/2))/(5*d*e^3*(d + e*x)^4) + ((2*C*d - B*e)*(d^2 - e^2*x^2)^(3
/2))/(3*d*e^3*(d + e*x)^3) - ((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2)
)/(15*d^2*e^3*(d + e*x)^3) - (C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 663

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + p + 1)), x] - Dist[(c*p)/(e^2*(m
+ p + 1)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c
, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m +
 2*p + 1, 0]) && NeQ[m + p + 1, 0] && IntegerQ[2*p]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^4}\,dx = \int\left(\frac{(Cd^2-Bde+Ae^2)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^4} + \frac{(-2Cd+Be)\sqrt{d^2-e^2x^2}}{e^2(d+ex)^3} + \frac{C\sqrt{d^2-}}{e^2(d+}\right.$$

$$= \frac{C\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^2}\,dx}{e^2} - \frac{(2Cd-Be)\int \frac{\sqrt{d^2-e^2x^2}}{(d+ex)^3}\,dx}{e^2} + \frac{(Cd^2-Bde+Ae^2)\int \frac{\sqrt{d^2-e}}{(d+ex}}{e^2}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e}{3de^3(d+ex)}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e}{3de^3(d+ex)}$$

$$= -\frac{2C\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{(Cd^2-Bde+Ae^2)(d^2-e^2x^2)^{3/2}}{5de^3(d+ex)^4} + \frac{(2Cd-Be)(d^2-e}{3de^3(d+ex)}$$

**Mathematica [A]** time = 0.30, size = 112, normalized size = 0.57

$$-\frac{\frac{\sqrt{d^2-e^2x^2}\left(e(d-ex)(Ae(4d+ex)+Bd(d+4ex))+3Cd^2\left(8d^2+19dex+13e^2x^2\right)\right)}{d^2(d+ex)^3}+15C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^4,x]
```

```
[Out] -1/15*((Sqrt[d^2 - e^2*x^2]*(3*C*d^2*(8*d^2 + 19*d*e*x + 13*e^2*x^2) + e*(d
 - e*x)*(A*e*(4*d + e*x) + B*d*(d + 4*e*x))))/(d^2*(d + e*x)^3) + 15*C*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

**fricas [A]** time = 0.83, size = 304, normalized size = 1.55

$$-\frac{24\,Cd^5+Bd^4e+4\,Ad^3e^2+\left(24\,Cd^2e^3+Bde^4+4\,Ae^5\right)x^3+3\left(24\,Cd^3e^2+Bd^2e^3+4\,Ade^4\right)x^2+3\left(24\,Cd^4e+B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="fricas
")
```

```
[Out] -1/15*(24*C*d^5 + B*d^4*e + 4*A*d^3*e^2 + (24*C*d^2*e^3 + B*d*e^4 + 4*A*e^5
)*x^3 + 3*(24*C*d^3*e^2 + B*d^2*e^3 + 4*A*d*e^4)*x^2 + 3*(24*C*d^4*e + B*d^
```

```
3*e^2 + 4*A*d^2*e^3)*x - 30*(C*d^2*e^3*x^3 + 3*C*d^3*e^2*x^2 + 3*C*d^4*e*x
+ C*d^5)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (24*C*d^4 + B*d^3*e +
4*A*d^2*e^2 + (39*C*d^2*e^2 - 4*B*d*e^3 - A*e^4)*x^2 + 3*(19*C*d^3*e + B*d^
2*e^2 - A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*
d^4*e^4*x + d^5*e^3)
```

**giac** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (8*A*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^16*exp
(2)+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^14*exp(2)^2+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^5*exp(1)^12*exp(2)^3+3/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^4*exp(2)^7/x/exp(2)+42*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^6*exp(2)^6/x/exp(2)+9*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^8*exp(2)^5/x/exp(2)+3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
*exp(1)^10*exp(2)^4/x/exp(2)-3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)*exp(1)^12*exp(2)^3/x/exp(2)+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^13*exp(2)^2+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp(2)^3+6*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^10*exp(2)^4+48*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^13*exp(2)^2
+36*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1
)^11*exp(2)^3+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^9*exp(2)^4-96*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^3*exp(1)^12*exp(2)^2-84*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^3-18*C*d^2*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^4+12*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(
2)^2-8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(
1)^12*exp(2)^3-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^4*exp(1)^10*exp(2)^4-12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^5*exp(1)^8*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^2+14*B*d*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^3+3*B*d*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^4+3*B
*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*e
xp(2)^5+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)^12*exp(2)^2-32*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^3*exp(1)^10*exp(2)^3-24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
```

2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^4-6*C*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5-12*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^3
-72*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
10*exp(2)^4-84*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^4*exp(1)^8*exp(2)^5-24*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^5*exp(1)^6*exp(2)^6+108*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^2*exp(1)^11*exp(2)^3+96*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^4+48*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^5+12*B*
d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*ex
p(2)^6-204*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^2*exp(1)^10*exp(2)^3-120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^4-12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^5-30*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)^4-30*A*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^5-3*
A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*ex
p(2)^6+3*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*ex
p(1)^4*exp(2)^7+24*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^9*exp(2)^4+12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^7*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^5*exp(2)^6-102*C*d^2*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4-42*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(
2)^5+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*
exp(1)^4*exp(2)^6-132*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^8*exp(2)^5-108*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^6-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^7+3*C*d^2*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(2)^8+60*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^5+36*B*d*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^5*exp(2)
^6+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^3*exp(2)^7+12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^2*exp(1)^6*exp(2)^5+36*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^6+2*A*exp(1)^10*exp(2)^4-12*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^6+
B*d*exp(1)^9*exp(2)^4+2*C*d^2*exp(1)^8*exp(2)^4+12*C*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^8+36*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-36*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^
7+12*B*d*exp(1)^7*exp(2)^5-24*C*d^2*exp(1)^6*exp(2)^5+36*C*d^2*(-1/2*(-2*d*
exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^8+12*B*d*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^7-5*A*

```
exp(1)^6*exp(2)^6+2*B*d*exp(1)^5*exp(2)^6-11*C*d^2*exp(1)^4*exp(2)^6+24*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^8-1
8*A*exp(1)^4*exp(2)^7+6*B*d*exp(1)^3*exp(2)^7+12*C*d^2*exp(2)^8+4*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)+8
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)
^14*exp(2)-33/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/
x/exp(2)-12*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)
^6/x/exp(2)-9/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*ex
p(2)^5/x/exp(2)-33*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9
*exp(2)^4/x/exp(2)-3*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^11*exp(2)^3/x/exp(2)-18*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*
exp(1)^4*exp(2)^6/x/exp(2)+30*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))*exp(1)^6*exp(2)^5/x/exp(2)+63*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))*exp(1)^8*exp(2)^4/x/exp(2)-6*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))*exp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x+exp(2))^3/(3*d^2*exp(1)^13-6*d^2*exp(1)^9*exp(2)^2-6*d^2*exp(1)^7*
exp(2)^3+3*d^2*exp(1)^5*exp(2)^4+3*d^2*exp(1)^11*exp(2)+3*d^2*exp(1)*exp(2)
^6)+1/2*(-4*B*d*exp(1)^11*exp(2)^2+16*C*d^2*exp(1)^10*exp(2)^2-2*B*d*exp(1)
^9*exp(2)^3+8*C*d^2*exp(1)^8*exp(2)^3+8*A*exp(1)^8*exp(2)^4-8*B*d*exp(1)^7*
exp(2)^4-8*C*d^2*exp(1)^6*exp(2)^4+2*A*exp(1)^6*exp(2)^5-10*C*d^2*exp(1)^4*
exp(2)^5+4*A*exp(1)^4*exp(2)^6+8*C*d^2*exp(2)^7)*atan((-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(
1)^4+exp(2)^2)/(-d^2*exp(1)^15+2*d^2*exp(1)^11*exp(2)^2+2*d^2*exp(1)^9*exp(
2)^3-d^2*exp(1)^7*exp(2)^4-d^2*exp(1)^5*exp(2)^5-d^2*exp(1)^13*exp(2))-C*si
gn(d)*asin(x*exp(2)/d/exp(1))/exp(1)/exp(2)
```

**maple [B]** time = 0.02, size = 453, normalized size = 2.31

$$-\frac{C \arctan\left(\frac{\sqrt{e^2}\, x}{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}}\right)}{\sqrt{e^2}\, e^2}-\frac{\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2}\, C}{d\, e^3}-\frac{\left(2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^2 e^2\right)^{\frac{3}{2}} A}{5\left(x+\frac{d}{e}\right)^4 d\, e^5}-\frac{\left(2\left(x+\frac{d}{e}\right)de \cdots}{15\left(x \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)
```

```
[Out] -C/e^5/d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)-C/e^3/d*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)-C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(2*(x+d/e)*d*e
-(x+d/e)^2*e^2)^(1/2)*x)-1/5/e^5/d/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(3/2)*A+1/5/e^6/(x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*B-1/5/e^7*d/(
x+d/e)^4*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1/15/e^4/d^2/(x+d/e)^3*(2*(x
+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*A+1/15/e^5/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e
```

```
)^2*e^2)^(3/2)*B-1/15/e^6/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)*C-1
/3*(B*e-2*C*d)/e^6/d/(x+d/e)^3*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(3/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-e^2x^2 + d^2}\left(Cx^2 + Bx + A\right)}{(ex + d)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-e^2*x^2 + d^2)*(C*x^2 + B*x + A)/(e*x + d)^4, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d^2 - e^2\,x^2}\,\left(C\,x^2 + B\,x + A\right)}{(d + e\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4,x)
```

```
[Out] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}\,\left(A + Bx + Cx^2\right)}{(d + ex)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**4, x)
```

## 3.8 $\int \frac{(A+Bx+Cx^2)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx$

**Optimal**. Leaf size=180

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{35d^2e^3(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{7de^3(d+ex)^5}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23C\right.}{105d^3e^3(d+ex)^3}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^5+C*(-e^2*x^2+d
^2)^(3/2)/e^3/(e*x+d)^4-1/35*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2
)/d^2/e^3/(e*x+d)^4-1/105*(23*C*d^2+e*(2*A*e+5*B*d))*(-e^2*x^2+d^2)^(3/2)/d
^3/e^3/(e*x+d)^3
```

**Rubi [A]** time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{105d^3e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(e(2Ae+5Bd)+23Cd^2\right)}{35d^2e^3(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+C\right.}{7de^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(7*d*e^3*(d + e*x)^5) + (C
*(d^2 - e^2*x^2)^(3/2))/(e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d + 2*A*e))
*(d^2 - e^2*x^2)^(3/2))/(35*d^2*e^3*(d + e*x)^4) - ((23*C*d^2 + e*(5*B*d +
2*A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\int \frac{\left(e^2\left(4Cd^2+Ae^2\right)+e^3(3Cd+Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^5}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} + \frac{\left(23Cd^2+e(5Bd\right.}{}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd\right.}{35d^2e}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{7de^3(d+ex)^5} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{e^3(d+ex)^4} - \frac{\left(23Cd^2+e(5Bd\right.}{35d^2e}$$

**Mathematica [A]** time = 0.20, size = 109, normalized size = 0.61

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(23d^2+10dex+2e^2x^2\right)+5Bd\left(d^2+5dex+e^2x^2\right)\right)+Cd^2\left(2d^2+10dex+23e^2x^2\right)\right)}{105d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^5,x]
```

```
[Out] -1/105*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 10*d*e*x + 23*e^2*x^2
) + e*(5*B*d*(d^2 + 5*d*e*x + e^2*x^2) + A*e*(23*d^2 + 10*d*e*x + 2*e^2*x^2
))))/(d^3*e^3*(d + e*x)^4)
```

**fricas [A]** time = 0.92, size = 320, normalized size = 1.78

$$-\frac{2\,Cd^6+5\,Bd^5e+23\,Ad^4e^2+\left(2\,Cd^2e^4+5\,Bde^5+23\,Ae^6\right)x^4+4\left(2\,Cd^3e^3+5\,Bd^2e^4+23\,Ade^5\right)x^3+6\left(2\,Cd^4e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="fricas
")
```

```
[Out] -1/105*(2*C*d^6 + 5*B*d^5*e + 23*A*d^4*e^2 + (2*C*d^2*e^4 + 5*B*d*e^5 + 23*
A*e^6)*x^4 + 4*(2*C*d^3*e^3 + 5*B*d^2*e^4 + 23*A*d*e^5)*x^3 + 6*(2*C*d^4*e^
2 + 5*B*d^3*e^3 + 23*A*d^2*e^4)*x^2 + 4*(2*C*d^5*e + 5*B*d^4*e^2 + 23*A*d^3
*e^3)*x + (2*C*d^5 + 5*B*d^4*e + 23*A*d^3*e^2 - (23*C*d^2*e^3 + 5*B*d*e^4 +
 2*A*e^5)*x^3 + (13*C*d^3*e^2 - 20*B*d^2*e^3 - 8*A*d*e^4)*x^2 + (8*C*d^4*e
+ 20*B*d^3*e^2 - 13*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^7*x^4 + 4*d^
4*e^6*x^3 + 6*d^5*e^5*x^2 + 4*d^6*e^4*x + d^7*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{2,[2,0]%%%},0,%%%{1,[4,0]%%%}+%%%
{-4,[2,1]%%%}+%%%{4,[0,2]%%%}] at parameters values [86,-97]Limit: Max orde
r reached or unable to make series expansion Error: Bad Argument Value
```

**maple [A]** time = 0.01, size = 116, normalized size = 0.64

$$-\frac{(-ex+d)\left(2A\,e^4x^2+5Bd\,e^3x^2+23C\,d^2e^2x^2+10Ad\,e^3x+25B\,d^2e^2x+10C\,d^3ex+23A\,d^2e^2+5B\,d^3e+2C\,d^4\right)}{105\,(ex+d)^4\,d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x)
```

```
[Out] -1/105*(-e*x+d)*(2*A*e^4*x^2+5*B*d*e^3*x^2+23*C*d^2*e^2*x^2+10*A*d*e^3*x+25
*B*d^2*e^2*x+10*C*d^3*e*x+23*A*d^2*e^2+5*B*d^3*e+2*C*d^4)*(-e^2*x^2+d^2)^(1
/2)/(e*x+d)^4/d^3/e^3
```

**maxima [B]** time = 0.54, size = 945, normalized size = 5.25

$$-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{7\left(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{35\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}+\frac{2\sqrt{-e^2x^2+d^2}\,C}{105\left(d^2e^5x^2+2\,d^3e^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5,x, algorithm="maxima
")
```

```
[Out] -2/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^7*x^4 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*
d^3*e^4*x + d^4*e^3) + 1/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e
^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5
*x^2 + 2*d^3*e^4*x + d^4*e^3) + 2/105*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x
 + d^4*e^3) + 2/7*sqrt(-e^2*x^2 + d^2)*B*d/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e
^4*x^2 + 4*d^3*e^3*x + d^4*e^2) - 1/35*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^5*x^3
+ 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(
d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) - 2/105*sqrt(-e^2*x^2 + d^2)*B*d/(d^3*
e^3*x + d^4*e^2) + 4/5*sqrt(-e^2*x^2 + d^2)*C*d/(e^6*x^3 + 3*d*e^5*x^2 + 3*
d^2*e^4*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^5*x^2 + 2*d^2*e^4
*x + d^3*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4*x + d^3*e^3) - 2/7*s
qrt(-e^2*x^2 + d^2)*A/(e^5*x^4 + 4*d*e^4*x^3 + 6*d^2*e^3*x^2 + 4*d^3*e^2*x
+ d^4*e) + 1/35*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d^3*e
^2*x + d^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x + d
^4*e) + 2/105*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 2/5*sqrt(-e^2*x^
2 + d^2)*B/(e^5*x^3 + 3*d*e^4*x^2 + 3*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2
*x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/15*sqrt(-e^2*x^2 + d^
2)*B/(d^2*e^3*x + d^3*e^2) - 2/3*sqrt(-e^2*x^2 + d^2)*C/(e^5*x^2 + 2*d*e^4*
x + d^2*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

**mupad [B]** time = 4.67, size = 601, normalized size = 3.34

$$\frac{B\sqrt{d^2-e^2\,x^2}}{21\,\left(d^3\,e^2+x\,d^2\,e^3\right)}-\frac{3\,B\sqrt{d^2-e^2\,x^2}}{7\,\left(d^3\,e^2+3\,d^2\,e^3\,x+3\,d\,e^4\,x^2+e^5\,x^3\right)}+\frac{2\,A\sqrt{d^2-e^2\,x^2}}{105\,\left(d^4\,e+2\,d^3\,e^2\,x+d^2\,e^3\,x^2\right)}+\frac{B\sqrt{d^2-e}}{21\,\left(d^3\,e^2+2\,d^2\,e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^5,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(21*(d^3*e^2 + d^2*e^3*x)) - (3*B*(d^2 - e^2*x^2)
^(1/2))/(7*(d^3*e^2 + e^5*x^3 + 3*d^2*e^3*x + 3*d*e^4*x^2)) + (2*A*(d^2 - e
```

```
^2*x^2)^(1/2))/(105*(d^4*e + 2*d^3*e^2*x + d^2*e^3*x^2)) + (B*(d^2 - e^2*x^
2)^(1/2))/(21*(d^3*e^2 + 2*d^2*e^3*x + d*e^4*x^2)) - (82*C*(d^2 - e^2*x^2)^
(1/2))/(105*(d^2*e^3 + e^5*x^2 + 2*d*e^4*x)) + (2*A*(d^2 - e^2*x^2)^(1/2))/
(105*(d^4*e + d^3*e^2*x)) + (23*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^2*e^3 + d*
e^4*x)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(7*(d^4*e + e^5*x^4 + 4*d^3*e^2*x + 4
*d*e^4*x^3 + 6*d^2*e^3*x^2)) + (A*(d^2 - e^2*x^2)^(1/2))/(35*(d^4*e + 3*d^3
*e^2*x + d*e^4*x^3 + 3*d^2*e^3*x^2)) - (2*C*d^2*(d^2 - e^2*x^2)^(1/2))/(7*(
d^4*e^3 + e^7*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) + (2*B*d*(d
^2 - e^2*x^2)^(1/2))/(7*(d^4*e^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*
d^2*e^4*x^2)) + (29*C*d*(d^2 - e^2*x^2)^(1/2))/(35*(d^3*e^3 + e^6*x^3 + 3*d
^2*e^4*x + 3*d*e^5*x^2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\left(A+Bx+Cx^2\right)}{\left(d+ex\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**5,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))*(A + B*x + C*x**2)/(d + e*x)**5, x)
```

## 3.9 $\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx$

**Optimal**. Leaf size=234

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+11Cd^2\right)}{42d^2e^3(d+ex)^5}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(Ae^2-Bde+Cd^2\right)}{9de^3(d+ex)^6}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+11C\right.}{315d^4e^3(d+ex)^3}$$

```
[Out] -1/9*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(3/2)/d/e^3/(e*x+d)^6+1/2*C*(-e^2*x
^2+d^2)^(3/2)/e^3/(e*x+d)^5-1/42*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^
(3/2)/d^2/e^3/(e*x+d)^5-1/105*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/
2)/d^3/e^3/(e*x+d)^4-1/315*(11*C*d^2+2*e*(A*e+2*B*d))*(-e^2*x^2+d^2)^(3/2)/
d^4/e^3/(e*x+d)^3
```

**Rubi [A]** time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+11Cd^2\right)}{315d^4e^3(d+ex)^3}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)+11Cd^2\right)}{105d^3e^3(d+ex)^4}-\frac{\left(d^2-e^2x^2\right)^{3/2}\left(2e(Ae+2Bd)\right.}{42d^2e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*(d^2 - e^2*x^2)^(3/2))/(9*d*e^3*(d + e*x)^6) + (C
*(d^2 - e^2*x^2)^(3/2))/(2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*d + A*e
))*(d^2 - e^2*x^2)^(3/2))/(42*d^2*e^3*(d + e*x)^5) - ((11*C*d^2 + 2*e*(2*B*
d + A*e))*(d^2 - e^2*x^2)^(3/2))/(105*d^3*e^3*(d + e*x)^4) - ((11*C*d^2 + 2
*e*(2*B*d + A*e))*(d^2 - e^2*x^2)^(3/2))/(315*d^4*e^3*(d + e*x)^3)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
```

```
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx = \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} + \frac{\int \frac{\left(e^2\left(5Cd^2+2Ae^2\right)+e^3(3Cd+2Be)x\right)\sqrt{d^2-e^2x^2}}{(d+ex)^6}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} + \frac{\left(11Cd^2+2e(2Bd+\cdots\right.}{6\cdots}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+\cdots\right.}{42d^2e^3\cdots}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+\cdots\right.}{42d^2e^3\cdots}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\left(d^2-e^2x^2\right)^{3/2}}{9de^3(d+ex)^6} + \frac{C\left(d^2-e^2x^2\right)^{3/2}}{2e^3(d+ex)^5} - \frac{\left(11Cd^2+2e(2Bd+\cdots\right.}{42d^2e^3\cdots}$$

**Mathematica [A]** time = 0.22, size = 144, normalized size = 0.62

$$-\frac{(d-ex)\sqrt{d^2-e^2x^2}\,\left(e\left(Ae\left(58d^3+33d^2ex+12de^2x^2+2e^3x^3\right)+Bd\left(11d^3+66d^2ex+24de^2x^2+4e^3x^3\right)\right)+Cd}{315d^4e^3(d+ex)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*Sqrt[d^2 - e^2*x^2])/(d + e*x)^6,x]
```

```
[Out] -1/315*((d - e*x)*Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d^3 + 24*d^2*e*x + 66*d*e^2
*x^2 + 11*e^3*x^3) + e*(A*e*(58*d^3 + 33*d^2*e*x + 12*d*e^2*x^2 + 2*e^3*x^3
) + B*d*(11*d^3 + 66*d^2*e*x + 24*d*e^2*x^2 + 4*e^3*x^3))))/(d^4*e^3*(d + e
*x)^5)
```

**fricas [A]** time = 1.07, size = 399, normalized size = 1.71

$$-\frac{4\,Cd^7+11\,Bd^6e+58\,Ad^5e^2+\left(4\,Cd^2e^5+11\,Bde^6+58\,Ae^7\right)x^5+5\left(4\,Cd^3e^4+11\,Bd^2e^5+58\,Ade^6\right)x^4+10\left(4\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="fricas
")
```

```
[Out] -1/315*(4*C*d^7 + 11*B*d^6*e + 58*A*d^5*e^2 + (4*C*d^2*e^5 + 11*B*d*e^6 + 5
8*A*e^7)*x^5 + 5*(4*C*d^3*e^4 + 11*B*d^2*e^5 + 58*A*d*e^6)*x^4 + 10*(4*C*d^
4*e^3 + 11*B*d^3*e^4 + 58*A*d^2*e^5)*x^3 + 10*(4*C*d^5*e^2 + 11*B*d^4*e^3 +
 58*A*d^3*e^4)*x^2 + 5*(4*C*d^6*e + 11*B*d^5*e^2 + 58*A*d^4*e^3)*x + (4*C*d
^6 + 11*B*d^5*e + 58*A*d^4*e^2 - (11*C*d^2*e^4 + 4*B*d*e^5 + 2*A*e^6)*x^4 -
 5*(11*C*d^3*e^3 + 4*B*d^2*e^4 + 2*A*d*e^5)*x^3 + 21*(2*C*d^4*e^2 - 2*B*d^3
*e^3 - A*d^2*e^4)*x^2 + 5*(4*C*d^5*e + 11*B*d^4*e^2 - 5*A*d^3*e^3)*x)*sqrt(
-e^2*x^2 + d^2))/(d^4*e^8*x^5 + 5*d^5*e^7*x^4 + 10*d^6*e^6*x^3 + 10*d^7*e^5
*x^2 + 5*d^8*e^4*x + d^9*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-960
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^24*
exp(2)^2-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^24*exp(2)^2-384*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
```

```
)/x/exp(2))^5*exp(1)^26*exp(2)-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^7*exp(1)^22*exp(2)^3-480*A*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^20*exp(2)^4-120*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^18*exp(2)^5-800*A*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^22*exp(2
)^3-800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp
(1)^20*exp(2)^4-480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^18*exp(2)^5-120*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^16*exp(2)^6-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^24*exp(2)^2-352*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^22*exp(2)^3+2480*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^20*exp(2)^
4+3440*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^18*exp(2)^5+1920*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^16*exp(2)^6+480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^14*exp(2)^7-800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^22*exp(2)^3-160*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^20*exp(2)^4+1680*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^18*exp(2)^
5+2640*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^16*exp(2)^6+1920*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^8*exp(1)^14*exp(2)^7+480*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^9*exp(1)^12*exp(2)^8-960*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^22*exp(2)^3+2480*A*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^20*exp(2)^4+4736*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^18*exp(2)
^5-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(
1)^16*exp(2)^6-4160*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^7*exp(1)^14*exp(2)^7-2880*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^8*exp(1)^12*exp(2)^8-720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^10*exp(2)^9-800*A*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^4+3120*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp(2)
^5+12680*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*ex
p(1)^16*exp(2)^6+15140*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*exp(1)^14*exp(2)^7+5900*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^7*exp(1)^12*exp(2)^8-450*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^10*exp(2)^9-450*A*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^8*exp(2)^10-480*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^20*exp(
2)^4+3440*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*e
xp(1)^18*exp(2)^5-80*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^4*exp(1)^16*exp(2)^6-6100*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^5*exp(1)^14*exp(2)^7-2010*A*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^12*exp(2)^8+2850*A*(-1/2*(-2*d*e
```

xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^10*exp(2)^9+2325*A*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^8*exp(
2)^10+525*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*e
xp(1)^6*exp(2)^11-320*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^2*exp(1)^18*exp(2)^5+3760*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^16*exp(2)^6+25860*A*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^7+42700*A*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^8+3678
0*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^10
*exp(2)^9+21780*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^7*exp(1)^8*exp(2)^10+7860*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^8*exp(1)^6*exp(2)^11+1140*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^4*exp(2)^12+1880*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^16*exp(2)^6-3840*A*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)^7
-490*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^12*exp(2)^8+9930*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^5*exp(1)^10*exp(2)^9+9030*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^6*exp(1)^8*exp(2)^10+2730*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^6*exp(2)^11+60*A*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^4*exp(2)^12-60*A*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(2)^14+1580*A*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)
^7+28100*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*ex
p(1)^12*exp(2)^8+57720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(1)^10*exp(2)^9+59400*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^8*exp(2)^10+33000*A*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^6*exp(2)^11+8280*A*(-1/2*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^4*exp(2)^12-24*
A*exp(1)^16*exp(2)^6+600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^8*exp(2)^14-2590*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^2*exp(1)^12*exp(2)^8+5590*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*exp(2)^9+11080*A*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^8*exp(2)^10+5400*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^1
1+600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1
)^4*exp(2)^12-6*B*d*exp(1)^15*exp(2)^6-20*A*exp(1)^14*exp(2)^7-120*A*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(2)^14+16100*A*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(
2)^9+41420*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*
exp(1)^8*exp(2)^10+42800*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^4*exp(1)^6*exp(2)^11+18000*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^12+98*A*exp(1)^12*exp(2)^8+2400
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(2)^14+
4130*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)

```
^8*exp(2)^10+4470*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^6*exp(2)^11+1200*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^4*exp(1)^4*exp(2)^12+27*B*d*exp(1)^11*exp(2)^8+90*A*exp(1
)^10*exp(2)^9+18040*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^6*exp(2)^11+15720*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^12-170*B*d*exp(1)^9*exp(2)^9-149*A*e
xp(1)^8*exp(2)^10+3600*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^4*exp(2)^14+840*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^4*exp(2)^12-86*B*d*exp(1)^7*exp(2)^10+380*A*exp(1)^6*
exp(2)^11+120*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^3*exp(2)^14-30*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^14/x/e
xp(2)-760*B*d*exp(1)^5*exp(2)^11+180*A*exp(1)^4*exp(2)^12+2400*A*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^14-40*B*d*exp(1)
^3*exp(2)^12+600*A*exp(2)^14-120*B*d*exp(1)*exp(2)^13-2430*A*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^12/x/exp(2)-1275/2*A*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^11/x/exp(2)-2125*A*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^10/x/exp(2)+385*A*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^9/x/exp(2)-210*
A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^8/x/exp(2)-2
50*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^14*exp(2)^7/x/exp(2
)+40*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^16*exp(2)^6/x/exp
(2)+60*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^18*exp(2)^5/x/e
xp(2)-480*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2
*exp(1)*exp(2)^13-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^6*exp(1)^23*exp(2)^2-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))/x/exp(2))^7*exp(1)^21*exp(2)^3-120*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^19*exp(2)^4-160*C*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^22*exp(2)^2
-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*ex
p(1)^20*exp(2)^3-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^7*exp(1)^19*exp(2)^4-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^8*exp(1)^17*exp(2)^5+320*C*d^2*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^22*exp(2)^2-160*C*d^2*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^20*exp(2)^
3-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*e
xp(1)^18*exp(2)^4-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^4*exp(1)^23*exp(2)^2-48*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^5*exp(1)^21*exp(2)^3+720*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^19*exp(2)^4+960*B*d*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^17*exp(2)^5+48
0*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^
15*exp(2)^6-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^22*exp(2)^2+128*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^20*exp(2)^3+640*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^18*exp(2)^4+640*C*d^2*(-1/2
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*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^16*exp(2)^5
-2720*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp
(1)^19*exp(2)^4-3560*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^6*exp(1)^17*exp(2)^5-920*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^7*exp(1)^15*exp(2)^6-60*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^13*exp(2)^7-60*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^11*exp(2)^8+1760
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)
^20*exp(2)^3+11360*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(1)^18*exp(2)^4+19120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^16*exp(2)^5+11920*C*d^2*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^14*exp(2)^6+3240*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^12*
exp(2)^7+360*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^9*exp(1)^10*exp(2)^8-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^3*exp(1)^21*exp(2)^3+720*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^19*exp(2)^4+604*B*d*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^17*exp(2)^5-1250*B*d
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^15*ex
p(2)^6-1910*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^7*exp(1)^13*exp(2)^7-855*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp
(1))/x/exp(2))^8*exp(1)^11*exp(2)^8-15*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^9*exp(2)^9-160*C*d^2*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^20*exp(2)^3+1120*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^18*exp
(2)^4+3416*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2)
)^5*exp(1)^16*exp(2)^5+3660*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))
*exp(1))/x/exp(2))^6*exp(1)^14*exp(2)^6+1860*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^12*exp(2)^7+810*C*d^2*(-1/2*(-
2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^10*exp(2)^8+90
*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)
^8*exp(2)^9+240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^19*exp(2)^4-10040*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^4*exp(1)^17*exp(2)^5-25760*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^15*exp(2)^6-32380*B*d*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^13*exp(2)^7-2
1460*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(
1)^11*exp(2)^8-6390*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^8*exp(1)^9*exp(2)^9-630*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^9*exp(1)^7*exp(2)^10+1760*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^18*exp(2)^4+32080*C*d^2*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^16*exp(2)
^5+56120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^
5*exp(1)^14*exp(2)^6+43140*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*
exp(1))/x/exp(2))^6*exp(1)^12*exp(2)^7+20460*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr

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t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^10*exp(2)^8+5670*C*d^2*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^8*exp(2)^9+63
0*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1
)^6*exp(2)^10-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^19*exp(2)^4+960*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^3*exp(1)^17*exp(2)^5-2450*B*d*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^15*exp(2)^6-720*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^13-6710*B
*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^13*
exp(2)^7-96*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^5*exp(1)^25*exp(2)-5590*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^6*exp(1)^11*exp(2)^8-1930*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^9*exp(2)^9-330*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^7*exp(2)^10-90*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^5*exp(2)^11
-160*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*ex
p(1)^18*exp(2)^4+1440*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^16*exp(2)^5+6140*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^6+7320*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^7+4350*C*
d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^10
*exp(2)^8+1530*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^7*exp(1)^8*exp(2)^9+135*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^8*exp(1)^6*exp(2)^10+15*C*d^2*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(1)^4*exp(2)^11+120*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^17*exp(2)^5-1268
0*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^
15*exp(2)^6-48820*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^13*exp(2)^7-67820*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^11*exp(2)^8-41940*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^9*exp(2)^9-13260*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^7*exp(2)^10-
2760*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(
1)^5*exp(2)^11-360*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^9*exp(1)^3*exp(2)^12+800*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^2*exp(1)^16*exp(2)^5+37680*C*d^2*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^14*exp(2)^6+63860*C*d^2*
(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^12*exp
(2)^7+51900*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2
))^5*exp(1)^10*exp(2)^8+24300*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^6*exp(1)^8*exp(2)^9+5460*C*d^2*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^6*exp(2)^10+540*C*d^2*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^8*exp(1)^4*exp(2)^11+6
0*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^9*exp(2
)^13+570*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*
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exp(1)^15*exp(2)^6-3890*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^13*exp(2)^7-7660*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^11*exp(2)^8-5780*B*d*(-1/2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^9*exp(2)^9-2220*B*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^7*exp(2)^
10-480*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*ex
p(1)*exp(2)^13-660*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/
exp(2))^7*exp(1)^5*exp(2)^11-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^8*exp(1)^3*exp(2)^12-6740*B*d*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^7-46140*B*d*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^8-6
0560*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(
1)^9*exp(2)^9-32400*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x
/exp(2))^5*exp(1)^7*exp(2)^10-9840*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*ex
p(2))*exp(1))/x/exp(2))^6*exp(1)^5*exp(2)^11-1920*B*d*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^7*exp(1)^3*exp(2)^12-4*C*d^2*exp(1)^
14*exp(2)^6+120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^7*exp(2)^13-2450*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^2*exp(1)^11*exp(2)^8-4710*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^
2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^9*exp(2)^9-3440*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^7*exp(2)^10-1200*B*d*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^5*exp(2)^
11-240*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*ex
p(1)^3*exp(2)^12+20*C*d^2*exp(1)^12*exp(2)^7-24860*B*d*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^9*exp(2)^9-26740*B*d*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^7*exp(2)^10
-12160*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*ex
p(1)^5*exp(2)^11-3600*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^5*exp(1)^3*exp(2)^12+28*C*d^2*exp(1)^10*exp(2)^8-1700*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^7*exp(2)^10-
940*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1
)^5*exp(2)^11-160*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^4*exp(1)^3*exp(2)^12+610*C*d^2*exp(1)^8*exp(2)^9-5840*B*d*(-1/2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2)^11-2880
*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^3
*exp(2)^12+81*C*d^2*exp(1)^6*exp(2)^10-120*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^13-80*B*d*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^3*exp(2)^12+420*C*d^2*exp(1)
^4*exp(2)^11-120*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^8*exp(1)*exp(2)^13+1100*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)^6+5740*C*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*exp(2)^7+5650*C*d^2*(-1/2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^10*exp(2)^8
+2430*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*e
xp(1)^8*exp(2)^9+330*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)

)/x/exp(2))^6*exp(1)^6*exp(2)^10-90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2
*exp(2))*exp(1))/x/exp(2))^7*exp(1)^4*exp(2)^11+21260*C*d^2*(-1/2*(-2*d*exp
(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^12*exp(2)^7+37540*C*d
^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^10*
exp(2)^8+32200*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/ex
p(2))^4*exp(1)^8*exp(2)^9+12600*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp
(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^10+1800*C*d^2*(-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^6*exp(1)^4*exp(2)^11+2570*C*d^2*(-1
/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^10*exp(2)
^8+1710*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3
*exp(1)^8*exp(2)^9+480*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^4*exp(1)^6*exp(2)^10+14180*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^
2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^9+11340*C*d^2*(-1/2*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^10+2400*
C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^
4*exp(2)^11+270*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/e
xp(2))^2*exp(1)^6*exp(2)^10+90*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(
2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^11+1560*C*d^2*(-1/2*(-2*d*exp(1)-2*
sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^11-64*C*d^2*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^24*exp(2)+30
*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^13/x/exp(2)+420*B
*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^12/x/exp(2)+
155*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^5*exp(2)^11/x/ex
p(2)+3485*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^1
0/x/exp(2)+845/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^9*e
xp(2)^9/x/exp(2)+820*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)
^11*exp(2)^8/x/exp(2)-135*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
xp(1)^13*exp(2)^7/x/exp(2)+30*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))*exp(1)^17*exp(2)^5/x/exp(2)+15/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))*exp(1)^4*exp(2)^11/x/exp(2)-1785*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))*exp(1)^6*exp(2)^10/x/exp(2)-360*C*d^2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^9/x/exp(2)-2870*C*d^2*(-2*d*exp(1
)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^10*exp(2)^8/x/exp(2)-140*C*d^2*(-2*
d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^12*exp(2)^7/x/exp(2)-100*C*d
^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^14*exp(2)^6/x/exp(2)+
20*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^16*exp(2)^5/x/e
xp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2
)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^5/(-60*d^4*exp(1)^1
9+240*d^4*exp(1)^15*exp(2)^2+240*d^4*exp(1)^13*exp(2)^3-360*d^4*exp(1)^11*e
xp(2)^4-360*d^4*exp(1)^9*exp(2)^5+240*d^4*exp(1)^7*exp(2)^6+240*d^4*exp(1)^
5*exp(2)^7-60*d^4*exp(1)^17*exp(2)-120*d^4*exp(1)*exp(2)^9)+1/2*(-4*B*d*exp
(1)^11*exp(2)^4+24*C*d^2*exp(1)^10*exp(2)^4-B*d*exp(1)^9*exp(2)^5+6*C*d^2*e
xp(1)^8*exp(2)^5+18*A*exp(1)^8*exp(2)^6-42*B*d*exp(1)^7*exp(2)^6+42*C*d^2*e
xp(1)^6*exp(2)^6+3*A*exp(1)^6*exp(2)^7-6*B*d*exp(1)^5*exp(2)^7+C*d^2*exp(1)
^4*exp(2)^7+44*A*exp(1)^4*exp(2)^8-24*B*d*exp(1)^3*exp(2)^8+4*C*d^2*exp(2)^

```
9+12*A*exp(2)^10)*atan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+
exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-2*d^4*exp(1)^1
9+8*d^4*exp(1)^15*exp(2)^2+8*d^4*exp(1)^13*exp(2)^3-12*d^4*exp(1)^11*exp(2)
^4-12*d^4*exp(1)^9*exp(2)^5+8*d^4*exp(1)^7*exp(2)^6+8*d^4*exp(1)^5*exp(2)^7
-2*d^4*exp(1)^17*exp(2)-4*d^4*exp(1)*exp(2)^9)
```

**maple [A]** time = 0.01, size = 152, normalized size = 0.65

$$-\frac{(-ex+d)\left(2A\,e^5x^3+4Bd\,e^4x^3+11C\,d^2e^3x^3+12Ad\,e^4x^2+24B\,d^2e^3x^2+66C\,d^3e^2x^2+33A\,d^2e^3x+66B\,d^3e^2x\right.}{315\,(ex+d)^5\,d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x)
```

```
[Out] -1/315*(-e*x+d)*(2*A*e^5*x^3+4*B*d*e^4*x^3+11*C*d^2*e^3*x^3+12*A*d*e^4*x^2+
24*B*d^2*e^3*x^2+66*C*d^3*e^2*x^2+33*A*d^2*e^3*x+66*B*d^3*e^2*x+24*C*d^4*e*
x+58*A*d^3*e^2+11*B*d^4*e+4*C*d^5)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^5/d^4/e^3
```

**maxima [B]** time = 0.58, size = 1378, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^6,x, algorithm="maxima
")
```

```
[Out] -2/9*sqrt(-e^2*x^2 + d^2)*C*d^2/(e^8*x^5 + 5*d*e^7*x^4 + 10*d^2*e^6*x^3 + 1
0*d^3*e^5*x^2 + 5*d^4*e^4*x + d^5*e^3) + 1/63*sqrt(-e^2*x^2 + d^2)*C*d^2/(d
*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2 + 4*d^4*e^4*x + d^5*e^3) + 1/105*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*
e^3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^
3) + 2/315*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e^4*x + d^5*e^3) + 2/9*sqrt(-e^2
*x^2 + d^2)*B*d/(e^7*x^5 + 5*d*e^6*x^4 + 10*d^2*e^5*x^3 + 10*d^3*e^4*x^2 +
5*d^4*e^3*x + d^5*e^2) - 1/63*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e
^5*x^3 + 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) - 1/105*sqrt(-e^2*x^2 + d^2
)*B*d/(d^2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e
^2*x^2 + d^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) - 2/315*sqrt(-e^2*x
^2 + d^2)*B*d/(d^4*e^3*x + d^5*e^2) + 4/7*sqrt(-e^2*x^2 + d^2)*C*d/(e^7*x^4
 + 4*d*e^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3) - 2/35*sqrt(-e^2*x^
2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) - 4/105*sq
rt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4*e^3) - 4/105*sqrt(-
e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^4*e^3) - 2/9*sqrt(-e^2*x^2 + d^2)*A/(e^6*
x^5 + 5*d*e^5*x^4 + 10*d^2*e^4*x^3 + 10*d^3*e^3*x^2 + 5*d^4*e^2*x + d^5*e)
+ 1/63*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*x^2 +
```

```
4*d^4*e^2*x + d^5*e) + 1/105*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d^3*e^
3*x^2 + 3*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^2 +
2*d^4*e^2*x + d^5*e) + 2/315*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e) - 2
/7*sqrt(-e^2*x^2 + d^2)*B/(e^6*x^4 + 4*d*e^5*x^3 + 6*d^2*e^4*x^2 + 4*d^3*e^
3*x + d^4*e^2) + 1/35*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3
*d^3*e^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e
^3*x + d^4*e^2) + 2/105*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 2/5*
sqrt(-e^2*x^2 + d^2)*C/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3) + 1/
15*sqrt(-e^2*x^2 + d^2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 1/15*sqrt(-
e^2*x^2 + d^2)*C/(d^2*e^4*x + d^3*e^3)
```

**mupad [B]** time = 5.24, size = 960, normalized size = 4.10

$$\frac{B\,\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+x\,d^3\,e^3\right)}+\frac{C\,\sqrt{d^2-e^2\,x^2}}{135\,\left(d^3\,e^3+x\,d^2\,e^4\right)}-\frac{19\,B\,\sqrt{d^2-e^2\,x^2}}{63\,\left(d^4\,e^2+4\,d^3\,e^3\,x+6\,d^2\,e^4\,x^2+4\,d\,e^5\,x^3+e^6\,x^4\right)}+\frac{A}{105\,\left(d^5\,e+3\,d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d^2 - e^2*x^2)^(1/2)*(A + B*x + C*x^2))/(d + e*x)^6,x)
```

```
[Out] (B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^2 + d^3*e^3*x)) + (C*(d^2 - e^2*x^2)^(
1/2))/(135*(d^3*e^3 + d^2*e^4*x)) - (19*B*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e
^2 + e^6*x^4 + 4*d^3*e^3*x + 4*d*e^5*x^3 + 6*d^2*e^4*x^2)) + (A*(d^2 - e^2*
x^2)^(1/2))/(105*(d^5*e + 3*d^4*e^2*x + 3*d^3*e^3*x^2 + d^2*e^4*x^3)) + (2*
B*(d^2 - e^2*x^2)^(1/2))/(105*(d^4*e^2 + 3*d^3*e^3*x + d*e^5*x^3 + 3*d^2*e^
4*x^2)) - (47*C*(d^2 - e^2*x^2)^(1/2))/(105*(d^3*e^3 + e^6*x^3 + 3*d^2*e^4*
x + 3*d*e^5*x^2)) + (2*A*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e + 2*d^4*e^2*x +
 d^3*e^3*x^2)) + (11*C*(d^2 - e^2*x^2)^(1/2))/(315*(d^3*e^3 + 2*d^2*e^4*x +
 d*e^5*x^2)) - (2*A*(d^2 - e^2*x^2)^(1/2))/(9*(d^5*e + e^6*x^5 + 5*d^4*e^2*
x + 5*d*e^5*x^4 + 10*d^3*e^3*x^2 + 10*d^2*e^4*x^3)) + (A*(d^2 - e^2*x^2)^(1
/2))/(63*(d^5*e + 4*d^4*e^2*x + d*e^5*x^4 + 6*d^3*e^3*x^2 + 4*d^2*e^4*x^3))
 - (2*A*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e + d^4*e^2*x)) + (4*B*(d^2 - e^2*
x^2)^(1/2))/(315*(d^4*e^2 + 2*d^3*e^3*x + d^2*e^4*x^2)) + (2*B*d*(d^2 - e^2
*x^2)^(1/2))/(9*(d^5*e^2 + e^7*x^5 + 5*d^4*e^3*x + 5*d*e^6*x^4 + 10*d^3*e^4
*x^2 + 10*d^2*e^5*x^3)) + (37*C*d*(d^2 - e^2*x^2)^(1/2))/(63*(d^4*e^3 + e^7
*x^4 + 4*d^3*e^4*x + 4*d*e^6*x^3 + 6*d^2*e^5*x^2)) - (2*C*d^2*(d^2 - e^2*x^
2)^(1/2))/(9*(d^5*e^3 + e^8*x^5 + 5*d^4*e^4*x + 5*d*e^7*x^4 + 10*d^3*e^5*x^
2 + 10*d^2*e^6*x^3)) + (8*A*e^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*
e^4*x)) + (26*C*d^2*(d^2 - e^2*x^2)^(1/2))/(945*(d^5*e^3 + d^4*e^4*x)) - (B
*d*e*(d^2 - e^2*x^2)^(1/2))/(315*(d^5*e^3 + d^4*e^4*x))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**6,x)
```

```
[Out] Timed out
```

## 3.10 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=236

$$-\frac{x^2\sqrt{d^2-e^2x^2}\,\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\,\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\,\left(55Ae^2+45B\right.}{15e^3}$$

```
[Out] 1/8*d^3*(20*A*e^2+15*B*d*e+13*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1
/15*d^2*(55*A*e^2+45*B*d*e+38*C*d^2)*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*d*(12*A*e
^2+15*B*d*e+13*C*d^2)*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/15*(19*C*d^2+5*e*(A*e+3*
B*d))*x^2*(-e^2*x^2+d^2)^(1/2)/e-1/4*(B*e+3*C*d)*x^3*(-e^2*x^2+d^2)^(1/2)-1
/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)
```

**Rubi [A]** time = 0.66, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x^2\sqrt{d^2-e^2x^2}\,\left(5e(Ae+3Bd)+19Cd^2\right)}{15e}-\frac{dx\sqrt{d^2-e^2x^2}\,\left(12Ae^2+15Bde+13Cd^2\right)}{8e^2}-\frac{d^2\sqrt{d^2-e^2x^2}\,\left(55Ae^2+45B\right.}{15e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(d^2*(38*C*d^2 + 45*B*d*e + 55*A*e^2)*Sqrt[d^2 - e^2*x^2])/(15*e^3) - (d*(
13*C*d^2 + 15*B*d*e + 12*A*e^2)*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((19*C*d^2
 + 5*e*(3*B*d + A*e))*x^2*Sqrt[d^2 - e^2*x^2])/(15*e) - ((3*C*d + B*e)*x^3*
Sqrt[d^2 - e^2*x^2])/4 - (C*e*x^4*Sqrt[d^2 - e^2*x^2])/5 + (d^3*(13*C*d^2 +
 15*B*d*e + 20*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} - \frac{\int \frac{-5Ad^3e^2-5d^2e^2(Bd+3Ae)x-5de^2\left(Cd^2+3e(Bd+Ae)\right)x^2-e^3\left(19Cd^2+5e\right.}{\sqrt{d^2-e^2x^2}}}{5e^2}$$

$$= -\frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Cex^4\sqrt{d^2-e^2x^2} + \frac{\int \frac{20Ad^3e^4+20d^2e^4(Bd+3Ae)x+}{}}{}$$

$$= -\frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2-e^2x^2}}{15e} - \frac{1}{4}(3Cd+Be)x^3\sqrt{d^2-e^2x^2} - \frac{1}{5}Ce$$

$$= -\frac{d\left(13Cd^2+15Bde+12Ae^2\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{\left(19Cd^2+5e(3Bd+Ae)\right)x^2\sqrt{d^2}}{15e}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x}{8e^2}$$

$$= -\frac{d^2\left(38Cd^2+45Bde+55Ae^2\right)\sqrt{d^2-e^2x^2}}{15e^3} - \frac{d\left(13Cd^2+15Bde+12Ae^2\right)x}{8e^2}$$

**Mathematica [A]** time = 0.45, size = 174, normalized size = 0.74

$$\frac{15d^3\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(5e(4Ae+3Bd)+13Cd^2\right)-\sqrt{d^2-e^2x^2}\left(5e\left(4Ae\left(22d^2+9dex+2e^2x^2\right)+3B\left(24d^3+15\right.\right.\right.}{120e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(304*d^4 + 195*d^3*e*x + 152*d^2*e^2*x^2 + 90*d*e
^3*x^3 + 24*e^4*x^4) + 5*e*(4*A*e*(22*d^2 + 9*d*e*x + 2*e^2*x^2) + 3*B*(24*
d^3 + 15*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3)))) + 15*d^3*(13*C*d^2 + 5*e*(3*
B*d + 4*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(120*e^3)
```

**fricas [A]** time = 0.61, size = 178, normalized size = 0.75

$$-\frac{30\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(24\,Ce^4x^4+304\,Cd^4+360\,Bd^3e+440\,Ad^2e^2+30\,(}{120\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/120*(30*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arctan(-(d - sqrt(-e^2*x^
2 + d^2))/(e*x)) + (24*C*e^4*x^4 + 304*C*d^4 + 360*B*d^3*e + 440*A*d^2*e^2
+ 30*(3*C*d*e^3 + B*e^4)*x^3 + 8*(19*C*d^2*e^2 + 15*B*d*e^3 + 5*A*e^4)*x^2
+ 15*(13*C*d^3*e + 15*B*d^2*e^2 + 12*A*d*e^3)*x)*sqrt(-e^2*x^2 + d^2))/e^3
```

**giac [A]** time = 0.37, size = 166, normalized size = 0.70

$$\frac{1}{8}\left(13\,Cd^5+15\,Bd^4e+20\,Ad^3e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{120}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\left(4\,Cxe+5\left(3\,Cde^6+Be^7\right)e^{(-6)}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*C*d^5 + 15*B*d^4*e + 20*A*d^3*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/
120*sqrt(-x^2*e^2 + d^2)*((2*(3*(4*C*x*e + 5*(3*C*d*e^6 + B*e^7)*e^(-6))*x
+ 4*(19*C*d^2*e^5 + 15*B*d*e^6 + 5*A*e^7)*e^(-6))*x + 15*(13*C*d^3*e^4 + 15
*B*d^2*e^5 + 12*A*d*e^6)*e^(-6))*x + 8*(38*C*d^4*e^3 + 45*B*d^3*e^4 + 55*A*
d^2*e^5)*e^(-6))
```

**maple [A]** time = 0.03, size = 374, normalized size = 1.58

$$-\frac{\sqrt{-e^2x^2+d^2}\;Ce\,x^4}{5}+\frac{5A\,d^3\arctan\left(\frac{\sqrt{e^2}\;x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{15B\,d^4\arctan\left(\frac{\sqrt{e^2}\;x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\;e}-\frac{\sqrt{-e^2x^2+d^2}\;Be\,x^3}{4}+\frac{13C\,d^5\arctan}{8\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/5*C*e*x^4*(-e^2*x^2+d^2)^(1/2)-19/15/e*C*d^2*x^2*(-e^2*x^2+d^2)^(1/2)-38
/15/e^3*C*d^4*(-e^2*x^2+d^2)^(1/2)-1/4*x^3*e*(-e^2*x^2+d^2)^(1/2)*B-3/4*x^3
*(-e^2*x^2+d^2)^(1/2)*d*C-15/8*(-e^2*x^2+d^2)^(1/2)*B*d^2/e*x-13/8*(-e^2*x^
2+d^2)^(1/2)*C*d^3/e^2*x+15/8/(e^2)^(1/2)*B*d^4/e*arctan((e^2)^(1/2)/(-e^2*
x^2+d^2)^(1/2)*x)+13/8/(e^2)^(1/2)*C*d^5/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)-1/3*x^2*e*(-e^2*x^2+d^2)^(1/2)*A-x^2*(-e^2*x^2+d^2)^(1/2)*d*B-
11/3*d^2/e*(-e^2*x^2+d^2)^(1/2)*A-3*d^3/e^2*(-e^2*x^2+d^2)^(1/2)*B-3/2*(-e^
2*x^2+d^2)^(1/2)*A*d*x+5/2/(e^2)^(1/2)*A*d^3*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)
```

**maxima** [A] time = 0.98, size = 390, normalized size = 1.65

$$-\frac{1}{5}\sqrt{-e^2x^2+d^2}\,Cex^4-\frac{4\sqrt{-e^2x^2+d^2}\,Cd^2x^2}{15\,e}+\frac{Ad^3\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{8\sqrt{-e^2x^2+d^2}\,Cd^4}{15\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,Bd^3}{e^2}-\frac{3\sqrt{-e^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*e*x^4 - 4/15*sqrt(-e^2*x^2 + d^2)*C*d^2*x^2/e +
 A*d^3*arcsin(e*x/d)/e - 8/15*sqrt(-e^2*x^2 + d^2)*C*d^4/e^3 - sqrt(-e^2*x^
2 + d^2)*B*d^3/e^2 - 3*sqrt(-e^2*x^2 + d^2)*A*d^2/e - 1/4*(3*C*d*e^2 + B*e^
3)*sqrt(-e^2*x^2 + d^2)*x^3/e^2 - 1/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(
-e^2*x^2 + d^2)*x^2/e^2 + 3/8*(3*C*d*e^2 + B*e^3)*d^4*arcsin(e*x/d)/e^5 + 1
/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*d^2*arcsin(e*x/d)/e^3 - 3/8*(3*C*d*e^2 +
 B*e^3)*sqrt(-e^2*x^2 + d^2)*d^2*x/e^4 - 1/2*(C*d^3 + 3*B*d^2*e + 3*A*d*e^2
)*sqrt(-e^2*x^2 + d^2)*x/e^2 - 2/3*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*sqrt(-e^
2*x^2 + d^2)*d^2/e^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^3*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

**sympy** [A] time = 24.44, size = 1268, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**3*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 3*A*d**2*e*Piecewise((x**2/(2*sqrt(d**2
)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + 3*A*d*e**2*Piecew
ise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**
2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt
(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + A*e**
3*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)) + B*d**3*Piecewis
e((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True))
 + 3*B*d**2*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e*
*2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d*
*2)), True)) + 3*B*d*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4
) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)),
True)) + B*e**3*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*
e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2
)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d
**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x
**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2))
, True)) + C*d**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1
+ e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*
e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**
2/d**2)), True)) + 3*C*d**2*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*
e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2
)), True)) + 3*C*d*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d*
*3*x/(8*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x
**2/d**2)) - I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) >
1), (3*d**4*asin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2
)) + d*x**3/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) + C*e**3*Piecewise((-8*d**4*sqrt(d**2 - e**2*x**2)/(15*e**
6) - 4*d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**4) - x**4*sqrt(d**2 - e**2*x
**2)/(5*e**2), Ne(e, 0)), (x**6/(6*sqrt(d**2)), True))
```

# 3.11 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=191

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8B\right.}{8e^3}$$

```
[Out] 1/8*d^2*(12*A*e^2+8*B*d*e+7*C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3
*d*(4*C*d^2+e*(6*A*e+5*B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/8*(7*C*d^2+4*e*(A*e
+2*B*d))*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/3*(B*e+2*C*d)*x^2*(-e^2*x^2+d^2)^(1/2
)/e-1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)
```

**Rubi [A]** time = 0.38, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1815, 641, 217, 203}

$$-\frac{x\sqrt{d^2-e^2x^2}\left(4e(Ae+2Bd)+7Cd^2\right)}{8e^2}-\frac{d\sqrt{d^2-e^2x^2}\left(e(6Ae+5Bd)+4Cd^2\right)}{3e^3}+\frac{d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(12Ae^2+8B\right.}{8e^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -(d*(4*C*d^2 + e*(5*B*d + 6*A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((7*C*d^2
+ 4*e*(2*B*d + A*e))*x*Sqrt[d^2 - e^2*x^2])/(8*e^2) - ((2*C*d + B*e)*x^2*Sq
rt[d^2 - e^2*x^2])/(3*e) - (C*x^3*Sqrt[d^2 - e^2*x^2])/4 + (d^2*(7*C*d^2 +
8*B*d*e + 12*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(8*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} - \frac{\int \frac{-4Ad^2e^2-4de^2(Bd+2Ae)x-e^2\left(7Cd^2+4e(2Bd+Ae)\right)x^2-4e^3(2Cd+Be)x^3}{\sqrt{d^2-e^2x^2}}\,d}{4e^2}$$

$$= -\frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d^2-e^2x^2} + \frac{\int \frac{12Ad^2e^4+4de^3\left(4Cd^2+e(5Bd+6Ae)\right.}{\sqrt{d^2-e^2}}}{12e}$$

$$= -\frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2} - \frac{(2Cd+Be)x^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}Cx^3\sqrt{d}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

$$= -\frac{d\left(4Cd^2+e(5Bd+6Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{\left(7Cd^2+4e(2Bd+Ae)\right)x\sqrt{d^2-e^2x^2}}{8e^2}$$

**Mathematica [A]** time = 0.18, size = 139, normalized size = 0.73

$$\frac{3d^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(4e(3Ae+2Bd)+7Cd^2\right)-\sqrt{d^2-e^2x^2}\,\left(4e\left(3Ae(4d+ex)+2B\left(5d^2+3dex+e^2x^2\right)\right)+C\left(3\right.\right.}{24e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(C*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)
+ 4*e*(3*A*e*(4*d + e*x) + 2*B*(5*d^2 + 3*d*e*x + e^2*x^2)))) + 3*d^2*(7*C*
d^2 + 4*e*(2*B*d + 3*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(24*e^3)
```

**fricas [A]** time = 0.96, size = 145, normalized size = 0.76

$$-\frac{6\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(6\,Ce^3x^3+32\,Cd^3+40\,Bd^2e+48\,Ade^2+8\left(2\,Cde^2+\right.\right.}{24\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/24*(6*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arctan(-(d - sqrt(-e^2*x^2 +
d^2))/(e*x)) + (6*C*e^3*x^3 + 32*C*d^3 + 40*B*d^2*e + 48*A*d*e^2 + 8*(2*C*d
*e^2 + B*e^3)*x^2 + 3*(7*C*d^2*e + 8*B*d*e^2 + 4*A*e^3)*x)*sqrt(-e^2*x^2 +
d^2))/e^3
```

**giac [A]** time = 0.32, size = 131, normalized size = 0.69

$$\frac{1}{8}\left(7\,Cd^4+8\,Bd^3e+12\,Ad^2e^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{24}\sqrt{-x^2e^2+d^2}\left(\left(2\left(3\,Cx+4\left(2\,Cde^4+Be^5\right)e^{(-5)}\right)x+3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(7*C*d^4 + 8*B*d^3*e + 12*A*d^2*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/24
*sqrt(-x^2*e^2 + d^2)*((2*(3*C*x + 4*(2*C*d*e^4 + B*e^5)*e^(-5))*x + 3*(7*C
*d^2*e^3 + 8*B*d*e^4 + 4*A*e^5)*e^(-5))*x + 8*(4*C*d^3*e^2 + 5*B*d^2*e^3 +
6*A*d*e^4)*e^(-5))
```

**maple [A]** time = 0.01, size = 301, normalized size = 1.58

$$\frac{3A\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}}+\frac{B\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e}+\frac{7C\,d^4\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{8\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,C\,x^3}{4}-\frac{\sqrt{-e^2x^2+}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/4*C*x^3*(-e^2*x^2+d^2)^(1/2)-7/8*(-e^2*x^2+d^2)^(1/2)*C*d^2/e^2*x+7/8/(e
^2)^(1/2)*C*d^4/e^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3*x^2*(-e^
2*x^2+d^2)^(1/2)*B-2/3*x^2/e*(-e^2*x^2+d^2)^(1/2)*d*C-5/3*d^2/e^2*(-e^2*x^2
+d^2)^(1/2)*B-4/3*d^3/e^3*(-e^2*x^2+d^2)^(1/2)*C-1/2*(-e^2*x^2+d^2)^(1/2)*A
*x-x/e*(-e^2*x^2+d^2)^(1/2)*B*d+3/2/(e^2)^(1/2)*A*d^2*arctan((e^2)^(1/2)/(-
e^2*x^2+d^2)^(1/2)*x)+d^3/e/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(
1/2)*x)*B-2/e*(-e^2*x^2+d^2)^(1/2)*A*d
```

**maxima** [A] time = 0.98, size = 253, normalized size = 1.32

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}\,Cx^3+\frac{3\,Cd^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^3}+\frac{Ad^2\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{3\sqrt{-e^2x^2+d^2}\,Cd^2x}{8\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,Bd^2}{e^2}-\frac{2\sqrt{-e^2x^2+}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*C*x^3 + 3/8*C*d^4*arcsin(e*x/d)/e^3 + A*d^2*arcsi
n(e*x/d)/e - 3/8*sqrt(-e^2*x^2 + d^2)*C*d^2*x/e^2 - sqrt(-e^2*x^2 + d^2)*B*
d^2/e^2 - 2*sqrt(-e^2*x^2 + d^2)*A*d/e - 1/3*sqrt(-e^2*x^2 + d^2)*(2*C*d*e
+ B*e^2)*x^2/e^2 + 1/2*(C*d^2 + 2*B*d*e + A*e^2)*d^2*arcsin(e*x/d)/e^3 - 1/
2*sqrt(-e^2*x^2 + d^2)*(C*d^2 + 2*B*d*e + A*e^2)*x/e^2 - 2/3*sqrt(-e^2*x^2
+ d^2)*(2*C*d*e + B*e^2)*d^2/e^4
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x)^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d^2-e^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^2*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2), x)
```

**sympy** [A] time = 18.03, size = 891, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d**2*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2
 > 0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2)
, (d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(
-d**2), (d**2 < 0) & (e**2 < 0))) + 2*A*d*e*Piecewise((x**2/(2*sqrt(d**2)),
 Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + A*e**2*Piecewise((-I
*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs
(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e*
*2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + B*d**2*Piece
wise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, Tru
e)) + 2*B*d*e*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e*
```

```
*2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3
) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d*
*2)), True)) + B*e**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) -
x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True
)) + C*d**2*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2
*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3)
- d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2
)), True)) + 2*C*d*e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x
**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True)
) + C*e**2*Piecewise((-3*I*d**4*acosh(e*x/d)/(8*e**5) + 3*I*d**3*x/(8*e**4*
sqrt(-1 + e**2*x**2/d**2)) - I*d*x**3/(8*e**2*sqrt(-1 + e**2*x**2/d**2)) -
I*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (3*d**4*a
sin(e*x/d)/(8*e**5) - 3*d**3*x/(8*e**4*sqrt(1 - e**2*x**2/d**2)) + d*x**3/(
8*e**2*sqrt(1 - e**2*x**2/d**2)) + x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), Tru
e))
```

## 3.12 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=143

$$-\frac{\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2 \cdots}}{3e \cdots}$$

```
[Out] 1/2*d*(C*d^2+e*(2*A*e+B*d))*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(2*C*d
^2+3*e*(A*e+B*d))*(-e^2*x^2+d^2)^(1/2)/e^3-1/2*(B*e+C*d)*x*(-e^2*x^2+d^2)^(
1/2)/e^2-1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e
```

**Rubi [A]** time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1815, 641, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\left(3e(Ae+Bd)+2Cd^2\right)}{3e^3}+\frac{d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}\,(Be+Cd)}{2e^2}-\frac{Cx^2\sqrt{d^2 \cdots}}{3e \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((2*C*d^2 + 3*e*(B*d + A*e))*Sqrt[d^2 - e^2*x^2])/(3*e^3) - ((C*d + B*e)*x
*Sqrt[d^2 - e^2*x^2])/(2*e^2) - (C*x^2*Sqrt[d^2 - e^2*x^2])/(3*e) + (d*(C*d
^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\sqrt{d^2-e^2x^2}}\,dx = -\frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{\int \frac{-3Ade^2-e\left(2Cd^2+3e(Bd+Ae)\right)x-3e^2(Cd+Be)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{3e^2}$$

$$= -\frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2x^2}}{3e} + \frac{\int \frac{3de^2\left(Cd^2+e(Bd+2Ae)\right)+2e^3\left(2Cd^2+3e(\right.}{\sqrt{d^2-e^2x^2}}}{6e^4}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2}}{3e}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2}}{3e}$$

$$= -\frac{\left(2Cd^2+3e(Bd+Ae)\right)\sqrt{d^2-e^2x^2}}{3e^3} - \frac{(Cd+Be)x\sqrt{d^2-e^2x^2}}{2e^2} - \frac{Cx^2\sqrt{d^2-e^2}}{3e}$$

**Mathematica [A]** time = 0.11, size = 103, normalized size = 0.72

$$\frac{3d\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(e(2Ae+Bd)+Cd^2\right)-\sqrt{d^2-e^2x^2}\left(3e(2Ae+2Bd+Bex)+C\left(4d^2+3dex+2e^2x^2\right)\right)}{6e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(Sqrt[d^2 - e^2*x^2]*(3*e*(2*B*d + 2*A*e + B*e*x) + C*(4*d^2 + 3*d*e*x +
2*e^2*x^2))) + 3*d*(C*d^2 + e*(B*d + 2*A*e))*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^
2]])/(6*e^3)
```

**fricas [A]** time = 0.97, size = 109, normalized size = 0.76

$$-\frac{6\left(Cd^3+Bd^2e+2\,Ade^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\left(2\,Ce^2x^2+4\,Cd^2+6\,Bde+6\,Ae^2+3\left(Cde+Be^2\right)x\right)\sqrt{-e^2x}}{6\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e
*x)) + (2*C*e^2*x^2 + 4*C*d^2 + 6*B*d*e + 6*A*e^2 + 3*(C*d*e + B*e^2)*x)*sq
rt(-e^2*x^2 + d^2))/e^3
```

**giac** [A] time = 0.29, size = 97, normalized size = 0.68

$$\frac{1}{2}\left(Cd^3+Bd^2e+2\,Ade^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{6}\sqrt{-x^2e^2+d^2}\left(\left(2\,Cxe^{(-1)}+3\left(Cde^3+Be^4\right)e^{(-5)}\right)x+2\left(2\,Cd^2e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^3 + B*d^2*e + 2*A*d*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/6*sqrt(-x
^2*e^2 + d^2)*((2*C*x*e^(-1) + 3*(C*d*e^3 + B*e^4)*e^(-5))*x + 2*(2*C*d^2*e
^2 + 3*B*d*e^3 + 3*A*e^4)*e^(-5))
```

**maple** [A] time = 0.01, size = 234, normalized size = 1.64

$$\frac{Ad\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{B\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e}+\frac{C\,d^3\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,C\,x^2}{3e}-\frac{\sqrt{-e^2x^2+d^2}\,Bx}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/3*C*x^2*(-e^2*x^2+d^2)^(1/2)/e-2/3/e^3*C*d^2*(-e^2*x^2+d^2)^(1/2)-1/2*(-
e^2*x^2+d^2)^(1/2)*B/e*x-1/2*(-e^2*x^2+d^2)^(1/2)*C*d/e^2*x+1/2/(e^2)^(1/2)
*B*d^2/e*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)+1/2/(e^2)^(1/2)*C*d^3/e
^2*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/e*(-e^2*x^2+d^2)^(1/2)*A-1/
e^2*(-e^2*x^2+d^2)^(1/2)*B*d+A*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d
^2)^(1/2)*x)
```

**maxima** [A] time = 0.98, size = 150, normalized size = 1.05

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cx^2}{3\,e}+\frac{Ad\arcsin\left(\frac{ex}{d}\right)}{e}+\frac{(Cd+Be)d^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}-\frac{2\,\sqrt{-e^2x^2+d^2}\,Cd^2}{3\,e^3}-\frac{\sqrt{-e^2x^2+d^2}\,Bd}{e^2}-\frac{\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*x^2/e + A*d*arcsin(e*x/d)/e + 1/2*(C*d + B*e)*d
^2*arcsin(e*x/d)/e^3 - 2/3*sqrt(-e^2*x^2 + d^2)*C*d^2/e^3 - sqrt(-e^2*x^2 +
 d^2)*B*d/e^2 - sqrt(-e^2*x^2 + d^2)*A/e - 1/2*sqrt(-e^2*x^2 + d^2)*(C*d +
B*e)*x/e^2
```

**mupad [B]** time = 5.01, size = 270, normalized size = 1.89

$$\begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{d^2}} \\ \frac{A\,d\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{A\,\sqrt{d^2-e^2\,x^2}}{e}-\frac{B\,d\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{B\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e}-\frac{C\,\sqrt{d^2-e^2\,x^2}\,\left(2\,d^2+e^2\,x^2\right)}{3\,e^3}-\frac{C\,d^3\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2 \dots\right.}{2\,{\left(-e^2\right)}^{3/2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*(d^2)^(1/2)), e ~= 0
, - (A*(d^2 - e^2*x^2)^(1/2))/e + (A*d*log(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)
^(1/2)))/(-e^2)^(1/2) - (B*d*(d^2 - e^2*x^2)^(1/2))/e^2 - (B*x*(d^2 - e^2*x
^2)^(1/2))/(2*e) - (C*(d^2 - e^2*x^2)^(1/2)*(2*d^2 + e^2*x^2))/(3*e^3) - (C
*d^3*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) - (B
*d^2*e*log(2*x*(-e^2)^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)) -
(C*d*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2))
```

**sympy [A]** time = 10.17, size = 484, normalized size = 3.38

$$Ad\left(\begin{cases} \frac{\sqrt{\frac{d^2}{e^2}}\,\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0 \wedge e^2>0 \\ \frac{\sqrt{-\frac{d^2}{e^2}}\,\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0 \wedge e^2<0 \\ \frac{\sqrt{\frac{d^2}{e^2}}\,\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0 \wedge e^2<0 \end{cases}\right)+Ae\left(\begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0 \\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise} \end{cases}\right)+Bd\left(\begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0 \\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise} \end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*d*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 >
0) & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (
d**2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d*
*2), (d**2 < 0) & (e**2 < 0))) + A*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**
```

```
2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*d*Piecewise((x**2/(2*sqrt
(d**2)), Eq(e**2, 0)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + B*e*Piecewis
e((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2)
, Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1
 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*d*Pie
cewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*
e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*s
qrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + C*
e*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**
2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), True))
```

# 3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=87

$$\frac{\left(2Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

```
[Out] 1/2*(2*A*e^2+C*d^2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-B*(-e^2*x^2+d^2)^(
1/2)/e^2-1/2*C*x*(-e^2*x^2+d^2)^(1/2)/e^2
```

**Rubi [A]** time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1815, 641, 217, 203}

$$\frac{\left(2Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] -((B*Sqrt[d^2 - e^2*x^2])/e^2) - (C*x*Sqrt[d^2 - e^2*x^2])/(2*e^2) + ((C*d^
2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{d^2-e^2x^2}}\,dx &= -\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\int \frac{-Cd^2-2Ae^2-2Be^2x}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\ &= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{2e^2}\\ &= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}-\frac{\left(-Cd^2-2Ae^2\right)\text{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\frac{x}{\sqrt{d^2-e^2x^2}}\right)}{2e^2}\\ &= -\frac{B\sqrt{d^2-e^2x^2}}{e^2}-\frac{Cx\sqrt{d^2-e^2x^2}}{2e^2}+\frac{\left(Cd^2+2Ae^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 67, normalized size = 0.77

$$\frac{\left(2Ae^2+Cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-e(2B+Cx)\sqrt{d^2-e^2x^2}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/Sqrt[d^2 - e^2*x^2],x]
```

```
[Out] (-(e*(2*B + C*x)*Sqrt[d^2 - e^2*x^2]) + (C*d^2 + 2*A*e^2)*ArcTan[(e*x)/Sqrt
[d^2 - e^2*x^2]])/(2*e^3)
```

**fricas [A]** time = 0.64, size = 71, normalized size = 0.82

$$-\frac{2\left(Cd^2+2\,Ae^2\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\sqrt{-e^2x^2+d^2}\,(Cex+2\,Be)}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(C*d^2 + 2*A*e^2)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + sqrt(
-e^2*x^2 + d^2)*(C*e*x + 2*B*e))/e^3
```

**giac [A]** time = 0.34, size = 52, normalized size = 0.60

$$\frac{1}{2}\left(Cd^2+2\,Ae^2\right)\arcsin\left(\frac{xe}{d}\right)e^{(-3)}\mathrm{sgn}(d)-\frac{1}{2}\sqrt{-x^2e^2+d^2}\left(Cxe^{(-2)}+2\,Be^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(C*d^2 + 2*A*e^2)*arcsin(x*e/d)*e^(-3)*sgn(d) - 1/2*sqrt(-x^2*e^2 + d^2
)*(C*x*e^(-2) + 2*B*e^(-2))
```

**maple [A]** time = 0.01, size = 108, normalized size = 1.24

$$\frac{A\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}}+\frac{C\,d^2\arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{2\sqrt{e^2}\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,Cx}{2e^2}-\frac{\sqrt{-e^2x^2+d^2}\,B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/2*(-e^2*x^2+d^2)^(1/2)*C/e^2*x+1/2/(e^2)^(1/2)*C*d^2/e^2*arctan((e^2)^(1
/2)/(-e^2*x^2+d^2)^(1/2)*x)-B*(-e^2*x^2+d^2)^(1/2)/e^2+A/(e^2)^(1/2)*arctan
((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)
```

**maxima [A]** time = 0.99, size = 70, normalized size = 0.80

$$\frac{Cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{A\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{\sqrt{-e^2x^2+d^2}\,Cx}{2\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,B}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*C*d^2*arcsin(e*x/d)/e^3 + A*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*
C*x/e^2 - sqrt(-e^2*x^2 + d^2)*B/e^2
```

**mupad [B]** time = 4.40, size = 148, normalized size = 1.70

$$\begin{cases}\frac{2\,C\,x^3+3\,B\,x^2+6\,A\,x}{6\,\sqrt{d^2}} & \text{if } e=0\\ \frac{A\,\ln\left(x\,\sqrt{-e^2}+\sqrt{d^2-e^2\,x^2}\right)}{\sqrt{-e^2}}-\frac{B\,\sqrt{d^2-e^2\,x^2}}{e^2}-\frac{C\,x\,\sqrt{d^2-e^2\,x^2}}{2\,e^2}-\frac{C\,d^2\,\ln\left(2\,x\,\sqrt{-e^2}+2\,\sqrt{d^2-e^2\,x^2}\right)}{2\,{\left(-e^2\right)}^{3/2}} & \text{if } e\neq 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(d^2 - e^2*x^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, (6*A*x + 3*B*x^2 + 2*C*x^3)/(6*(d^2)^(1/2)), e ~= 0, (A*l
og(x*(-e^2)^(1/2) + (d^2 - e^2*x^2)^(1/2)))/(-e^2)^(1/2) - (B*(d^2 - e^2*x^
2)^(1/2))/e^2 - (C*x*(d^2 - e^2*x^2)^(1/2))/(2*e^2) - (C*d^2*log(2*x*(-e^2)
^(1/2) + 2*(d^2 - e^2*x^2)^(1/2)))/(2*(-e^2)^(3/2)))
```

**sympy [A]** time = 4.56, size = 262, normalized size = 3.01

$$A\left(\begin{cases}\frac{\sqrt{\frac{d^2}{e^2}}\operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2>0\\ \frac{\sqrt{-\frac{d^2}{e^2}}\operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2>0\wedge e^2<0\\ \frac{\sqrt{\frac{d^2}{e^2}}\operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } d^2<0\wedge e^2<0\end{cases}\right)+B\left(\begin{cases}\frac{x^2}{2\sqrt{d^2}} & \text{for } e^2=0\\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise}\end{cases}\right)+C\left(\begin{cases}-\frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3}-\frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2}\\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3}-\frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}}+\frac{}{2d\sqrt{}}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(d**2/e**2)*asin(x*sqrt(e**2/d**2))/sqrt(d**2), (d**2 > 0)
 & (e**2 > 0)), (sqrt(-d**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d*
*2 > 0) & (e**2 < 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2
), (d**2 < 0) & (e**2 < 0))) + B*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0
)), (-sqrt(d**2 - e**2*x**2)/e**2, True)) + C*Piecewise((-I*d**2*acosh(e*x/
d)/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2)
 > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) +
 x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))
```

# 3.14 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

```
[Out] -(-B*e+C*d)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-C*(-e^2*x^2+d^2)^(1/2)/e^3
-(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)
```

**Rubi [A]** time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 217, 203}

$$-\frac{\sqrt{d^2-e^2x^2}\left(Ae^2-Bde+Cd^2\right)}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}-\frac{C\sqrt{d^2-e^2x^2}}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*Sqrt[d^2 - e^2*x^2])/e^3) - ((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^
2])/(d*e^3*(d + e*x)) - ((C*d - B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
```

```
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx &= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\int \frac{-Ae^4+e^3(Cd-Be)x}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^4}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\,\mathrm{Subst}\left(\int \frac{1}{1+e^2x^2}\,dx,x,\right.}{e^2}\\
&= -\frac{C\sqrt{d^2-e^2x^2}}{e^3}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{(Cd-Be)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.16, size = 83, normalized size = 0.81

$$
\frac{(Be-Cd)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)-\frac{\sqrt{d^2-e^2x^2}\,(e(Ae-Bd)+Cd(2d+ex))}{d(d+ex)}}{e^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] (-((Sqrt[d^2 - e^2*x^2]*(e*(-(B*d) + A*e) + C*d*(2*d + e*x)))/(d*(d + e*x))
) + (-(C*d) + B*e)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^3
```

**fricas [A]** time = 0.85, size = 155, normalized size = 1.50

$$-\frac{2\,Cd^{3}-Bd^{2}e+Ade^{2}+\left(2\,Cd^{2}e-Bde^{2}+Ae^{3}\right)x-2\,\left(Cd^{3}-Bd^{2}e+\left(Cd^{2}e-Bde^{2}\right)x\right)\arctan\left(-\frac{d-\sqrt{-e^{2}x^{2}+d^{2}}}{ex}\right)+}{de^{4}x+d^{2}e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*C*d^3 - B*d^2*e + A*d*e^2 + (2*C*d^2*e - B*d*e^2 + A*e^3)*x - 2*(C*d^3
- B*d^2*e + (C*d^2*e - B*d*e^2)*x)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)
) + (C*d*e*x + 2*C*d^2 - B*d*e + A*e^2)*sqrt(-e^2*x^2 + d^2))/(d*e^4*x + d^
2*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: 1/2*(
-4*A*exp(2)^2-4*C*d^2*exp(2)+4*B*d*exp(1)*exp(2))*atan((-1/2*(-2*d*exp(1)-2
*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(2)^2))/sqrt(-exp
(1)^4+exp(2)^2)/d/exp(1)/exp(2)-1/4*(-4*B*exp(1)+4*C*d)*sign(d)*asin(x*exp(
2)/d/exp(1))/exp(1)/exp(2)-4*exp(1)^2*C*1/4/exp(1)^5*sqrt(-exp(2)*x^2+d^2)
```

**maple [A]** time = 0.01, size = 149, normalized size = 1.45

$$\frac{B\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e}-\frac{Cd\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)}{\sqrt{e^{2}}\,e^{2}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,C}{e^{3}}-\frac{\left(A\,e^{2}-Bde+C\,d^{2}\right)\sqrt{2\left(x+\frac{d}{e}\right)de-\left(x+\frac{d}{e}\right)^{2}}}{\left(x+\frac{d}{e}\right)d\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -C*(-e^2*x^2+d^2)^(1/2)/e^3+1/e*B/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+
d^2)^(1/2)*x)-1/e^2*C*d/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)
*x)-(A*e^2-B*d*e+C*d^2)/e^4/d/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)
```

**maxima [A]** time = 0.99, size = 138, normalized size = 1.34

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,Cd}{e^{4}x+de^{3}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,A}{de^{2}x+d^{2}e}+\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,B}{e^{3}x+de^{2}}-\frac{Cd\arcsin\left(\frac{ex}{d}\right)}{e^{3}}+\frac{B\arcsin\left(\frac{ex}{d}\right)}{e^{2}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,C}{e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*C*d/(e^4*x + d*e^3) - sqrt(-e^2*x^2 + d^2)*A/(d*e^2*x
 + d^2*e) + sqrt(-e^2*x^2 + d^2)*B/(e^3*x + d*e^2) - C*d*arcsin(e*x/d)/e^3
+ B*arcsin(e*x/d)/e^2 - sqrt(-e^2*x^2 + d^2)*C/e^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-(-d+ex)(d+ex)}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

## 3.15 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=163

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}\,(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

```
[Out] C*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^3-1/3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^
2)^(1/2)/d/e^3/(e*x+d)^2+(-B*e+2*C*d)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)-1/
3*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d^2/e^3/(e*x+d)
```

**Rubi [A]** time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1637, 217, 203, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3d^2e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{3de^3(d+ex)^2}+\frac{\sqrt{d^2-e^2x^2}\,(2Cd-Be)}{de^3(d+ex)}+\frac{C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(3*d*e^3*(d + e*x)^2) + ((2*
C*d - B*e)*Sqrt[d^2 - e^2*x^2])/(d*e^3*(d + e*x)) - ((C*d^2 - B*d*e + A*e^2
)*Sqrt[d^2 - e^2*x^2])/(3*d^2*e^3*(d + e*x)) + (C*ArcTan[(e*x)/Sqrt[d^2 - e
^2*x^2]])/e^3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
```

```
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 1637

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + c*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, c,
d, e}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^2\sqrt{d^2-e^2x^2}}\,dx &= \int\left(\frac{C}{e^2\sqrt{d^2-e^2x^2}}+\frac{Cd^2-Bde+Ae^2}{e^2(d+ex)^2\sqrt{d^2-e^2x^2}}+\frac{-2Cd+Be}{e^2(d+ex)\sqrt{d^2-e^2x^2}}\right)dx\\ &= \frac{C\int\frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{e^2}-\frac{(2Cd-Be)\int\frac{1}{(d+ex)\sqrt{d^2-e^2x^2}}\,dx}{e^2}+\frac{\left(Cd^2-Bde+Ae^2\right)\int\frac{1}{(d+ex)^2\sqrt{d^2\ldots}}}{e^2}\\ &= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}+\frac{C\,\text{Subst}\left(\int\frac{1}{1+e^2x^2}\,dx,\ldots\right.}{e^2}\\ &= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{3de^3(d+ex)^2}+\frac{(2Cd-Be)\sqrt{d^2-e^2x^2}}{de^3(d+ex)}-\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{\ldots}}{3d^2e^3(d+ex)\ldots}\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 95, normalized size = 0.58

$$\frac{\frac{\sqrt{d^2-e^2x^2}\left(Cd^2(4d+5ex)-e(Ae(2d+ex)+Bd(d+2ex))\right)}{d^2(d+ex)^2}+3C\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(C*d^2*(4*d + 5*e*x) - e*(A*e*(2*d + e*x) + B*d*(d +
2*e*x))))/(d^2*(d + e*x)^2) + 3*C*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(3*e^3
)
```

**fricas [A]** time = 0.92, size = 221, normalized size = 1.36

$$\frac{4\,Cd^4 - Bd^3e - 2\,Ad^2e^2 + \left(4\,Cd^2e^2 - Bde^3 - 2\,Ae^4\right)x^2 + 2\left(4\,Cd^3e - Bd^2e^2 - 2\,Ade^3\right)x - 6\left(Cd^2e^2x^2 + 2\,Cd^3ex \cdots\right.}{3\left(d^2e^5x^2 + 2\,d^3e^4x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/3*(4*C*d^4 - B*d^3*e - 2*A*d^2*e^2 + (4*C*d^2*e^2 - B*d*e^3 - 2*A*e^4)*x^
2 + 2*(4*C*d^3*e - B*d^2*e^2 - 2*A*d*e^3)*x - 6*(C*d^2*e^2*x^2 + 2*C*d^3*e*
x + C*d^4)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (4*C*d^3 - B*d^2*e -
 2*A*d*e^2 + (5*C*d^2*e - 2*B*d*e^2 - A*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^2*
e^5*x^2 + 2*d^3*e^4*x + d^4*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 355, normalized size = 2.18

$$\frac{C \arctan\left(\frac{\sqrt{e^2}\,x}{\sqrt{-e^2x^2+d^2}}\right)}{\sqrt{e^2}\,e^2} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\,A}{3\left(x+\frac{d}{e}\right)^2 d\,e^3} - \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+\frac{d}{e}\right)^2 e^2}\,A}{3\left(x+\frac{d}{e}\right)d^2e^2} + \frac{\sqrt{2\left(x+\frac{d}{e}\right)de - \left(x+ \cdots\right.}}{3\left(x+\frac{d}{e}\right)d\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] C/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)/(-e^2*x^2+d^2)^(1/2)*x)-1/3/e^3/d/(x+d
/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*A+1/3/e^4/(x+d/e)^2*(2*(x+d/e)*d*
```

```
e-(x+d/e)^2*e^2)^(1/2)*B-1/3/e^5*d/(x+d/e)^2*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^
(1/2)*C-1/3/e^2/d^2/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*A+1/3/e^3/d
/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e)^2*e^2)^(1/2)*B-1/3/e^4/(x+d/e)*(2*(x+d/e)*d
*e-(x+d/e)^2*e^2)^(1/2)*C-1/e^4*(B*e-2*C*d)/d/(x+d/e)*(2*(x+d/e)*d*e-(x+d/e
)^2*e^2)^(1/2)
```

**maxima [B]** time = 1.00, size = 317, normalized size = 1.94

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\left(de^5x^2+2\,d^2e^4x+d^3e^3\right)}-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{3\left(d^2e^4x+d^3e^3\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\left(de^4x^2+2\,d^2e^3x+d^3e^2\right)}+\frac{\sqrt{-e^2x^2+d^2}\,Bd}{3\left(d^2e^3x+d^3e^2\right)}-\frac{\sqrt{-e^2x^2+d^2}}{3\left(de^3x^2+2\,d^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*s
qrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^4*x + d^3*e^3) + 1/3*sqrt(-e^2*x^2 + d^2)*
B*d/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) + 1/3*sqrt(-e^2*x^2 + d^2)*B*d/(d^2
*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d*e^3*x^2 + 2*d^2*e^2*x + d
^3*e) - 1/3*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^2*x + d^3*e) - sqrt(-e^2*x^2 + d^
2)*B/(d*e^3*x + d^2*e^2) + 2*sqrt(-e^2*x^2 + d^2)*C/(e^4*x + d*e^3) + C*arc
sin(e*x/d)/e^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+B\,x+A}{\sqrt{d^2-e^2\,x^2}\,\left(d+e\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**2), x)
```

## 3.16 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx$

**Optimal**. Leaf size=180

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}+\frac{C}{e}$$

```
[Out] -1/5*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^3+C*(-e^2*x^2+d
^2)^(1/2)/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)
/d^2/e^3/(e*x+d)^2-1/15*(7*C*d^2+e*(2*A*e+3*B*d))*(-e^2*x^2+d^2)^(1/2)/d^3/
e^3/(e*x+d)
```

**Rubi [A]** time = 0.21, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^3e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,\left(e(2Ae+3Bd)+7Cd^2\right)}{15d^2e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,\left(Ae^2-Bde+Cd^2\right)}{5de^3(d+ex)^3}+\frac{C}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(5*d*e^3*(d + e*x)^3) + (C*S
qrt[d^2 - e^2*x^2])/(e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*Sqrt
[d^2 - e^2*x^2])/(15*d^2*e^3*(d + e*x)^2) - ((7*C*d^2 + e*(3*B*d + 2*A*e))*
Sqrt[d^2 - e^2*x^2])/(15*d^3*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\int \frac{e^2\left(2Cd^2+Ae^2\right)+e^3(Cd+Be)x}{(d+ex)^3\sqrt{d^2-e^2x^2}}\,dx}{e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} + \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\int \frac{}{(d+ex}}{5de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} - \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2-}}{15d^2e^3(d+ex)^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{5de^3(d+ex)^3} + \frac{C\sqrt{d^2-e^2x^2}}{e^3(d+ex)^2} - \frac{\left(7Cd^2+e(3Bd+2Ae)\right)\sqrt{d^2-}}{15d^2e^3(d+ex)^2}$$

**Mathematica [A]** time = 0.20, size = 103, normalized size = 0.57

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(Ae\left(7d^2+6dex+2e^2x^2\right)+3Bd\left(d^2+3dex+e^2x^2\right)\right)+Cd^2\left(2d^2+6dex+7e^2x^2\right)\right)}{15d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/15*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(2*d^2 + 6*d*e*x + 7*e^2*x^2) + e*(3*B*d*
(d^2 + 3*d*e*x + e^2*x^2) + A*e*(7*d^2 + 6*d*e*x + 2*e^2*x^2))))/(d^3*e^3*(
d + e*x)^3)
```

**fricas [A]** time = 0.79, size = 244, normalized size = 1.36

$$-\frac{2\,Cd^5+3\,Bd^4e+7\,Ad^3e^2+\left(2\,Cd^2e^3+3\,Bde^4+7\,Ae^5\right)x^3+3\left(2\,Cd^3e^2+3\,Bd^2e^3+7\,Ade^4\right)x^2+3\left(2\,Cd^4e+3\right.}{15\left(d^3e^6x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/15*(2*C*d^5 + 3*B*d^4*e + 7*A*d^3*e^2 + (2*C*d^2*e^3 + 3*B*d*e^4 + 7*A*e
^5)*x^3 + 3*(2*C*d^3*e^2 + 3*B*d^2*e^3 + 7*A*d*e^4)*x^2 + 3*(2*C*d^4*e + 3*
B*d^3*e^2 + 7*A*d^2*e^3)*x + (2*C*d^4 + 3*B*d^3*e + 7*A*d^2*e^2 + (7*C*d^2*
e^2 + 3*B*d*e^3 + 2*A*e^4)*x^2 + 3*(2*C*d^3*e + 3*B*d^2*e^2 + 2*A*d*e^3)*x)
*sqrt(-e^2*x^2 + d^2))/(d^3*e^6*x^3 + 3*d^4*e^5*x^2 + 3*d^5*e^4*x + d^6*e^3
)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (-2*A
*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp
(2)^2+7*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^6*exp(2)^3-2*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)^10*exp(2)-2*B*d*exp(1)*exp(2)^5+2*C*d^2*(-1/2*(-2*d*exp(1)-2*s
qrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)^2+5*A*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^4-11/2*A*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^4*exp(2)^4/x/exp(2)+A*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^8*exp(2)^2/x/exp(2)-5*B*d*(
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2
)^3-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp
(1)^9*exp(2)-2*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(
2))^2*exp(1)*exp(2)^5-3*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1
))/x/exp(2))^3*exp(1)^3*exp(2)^4+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^3+C*d^2*(-1/2*(-2*d*exp(1)-2*sq
rt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(2)^5-A*exp(1)^6*exp(2)^3-B*d*exp
(1)^5*exp(2)^3+3*C*d^2*exp(1)^4*exp(2)^3+4*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
```

```
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)^6+4*A*exp(2)^6+6*C*d^2*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)+1/2*C*d^2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^5/x/exp(2)+5/2*B*d*(-2*d
*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^4/x/exp(2)+2*B*d*(-2
*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(2)^2/x/exp(2)-5*C*d^2
*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^6*exp(2)^2/x/exp(2))/((
-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(2)-(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))^2/(-d^3*exp(1)^9+2*d^3*exp(1
)^5*exp(2)^2-d^3*exp(1)*exp(2)^4)+1/2*(-2*A*exp(1)^4*exp(2)^3+6*B*d*exp(1)^
3*exp(2)^3-2*C*d^2*exp(2)^4-4*A*exp(2)^5-4*C*d^2*exp(1)^6*exp(2))*atan((-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1)^4+exp(
2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(d^3*exp(1)^9-2*d^3*exp(1)^5*exp(2)^2+d^3*e
xp(1)*exp(2)^4)
```

**maple [A]** time = 0.01, size = 116, normalized size = 0.64

$$-\frac{(-ex+d)\left(2A\,e^4x^2+3Bd\,e^3x^2+7C\,d^2e^2x^2+6Ad\,e^3x+9B\,d^2e^2x+6C\,d^3ex+7A\,d^2e^2+3B\,d^3e+2C\,d^4\right)}{15\,(ex+d)^2\,\sqrt{-e^2x^2+d^2}\;d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/15*(-e*x+d)*(2*A*e^4*x^2+3*B*d*e^3*x^2+7*C*d^2*e^2*x^2+6*A*d*e^3*x+9*B*d
^2*e^2*x+6*C*d^3*e*x+7*A*d^2*e^2+3*B*d^3*e+2*C*d^4)/(e*x+d)^2/d^3/e^3/(-e^2
*x^2+d^2)^(1/2)
```

**maxima [B]** time = 1.02, size = 608, normalized size = 3.38

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{5\left(de^6x^3+3\,d^2e^5x^2+3\,d^3e^4x+d^4e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{15\left(d^2e^5x^2+2\,d^3e^4x+d^4e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}\,Cd^2}{15\left(d^3e^4x+d^4e^3\right)}+\frac{\sqrt{-e^2x^2+}}{5\left(de^5x^3+3\,d^2e^4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^6*x^3 + 3*d^2*e^5*x^2 + 3*d^3*e^4*x +
d^4*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^5*x^2 + 2*d^3*e^4*x + d^4
*e^3) - 2/15*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^3*e^4*x + d^4*e^3) + 1/5*sqrt(-e
^2*x^2 + d^2)*B*d/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4*e^2) + 2/1
5*sqrt(-e^2*x^2 + d^2)*B*d/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) + 2/15*sqr
t(-e^2*x^2 + d^2)*B*d/(d^3*e^3*x + d^4*e^2) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/
(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) + 2/3*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^4
*x + d^3*e^3) - 1/5*sqrt(-e^2*x^2 + d^2)*A/(d*e^4*x^3 + 3*d^2*e^3*x^2 + 3*d
```

```
^3*e^2*x + d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^3*x^2 + 2*d^3*e^2*x
+ d^4*e) - 2/15*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^2*x + d^4*e) - 1/3*sqrt(-e^2*
x^2 + d^2)*B/(d*e^4*x^2 + 2*d^2*e^3*x + d^3*e^2) - 1/3*sqrt(-e^2*x^2 + d^2)
*B/(d^2*e^3*x + d^3*e^2) - sqrt(-e^2*x^2 + d^2)*C/(d*e^4*x + d^2*e^3)
```

**mupad [B]** time = 3.80, size = 109, normalized size = 0.61

$$-\frac{\sqrt{d^2-e^2\,x^2}\,\left(2\,C\,d^4+6\,C\,d^3\,e\,x+3\,B\,d^3\,e+7\,C\,d^2\,e^2\,x^2+9\,B\,d^2\,e^2\,x+7\,A\,d^2\,e^2+3\,B\,d\,e^3\,x^2+6\,A\,d\,e^3\,x+2\right.}{15\,d^3\,e^3\,{\left(d+e\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^3),x)
```

```
[Out] -((d^2 - e^2*x^2)^(1/2)*(2*C*d^4 + 7*A*d^2*e^2 + 2*A*e^4*x^2 + 3*B*d^3*e +
7*C*d^2*e^2*x^2 + 6*A*d*e^3*x + 6*C*d^3*e*x + 9*B*d^2*e^2*x + 3*B*d*e^3*x^2
))/(15*d^3*e^3*(d + e*x)^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

# 3.17 $\int \frac{A+Bx+Cx^2}{(d+ex)^4 \sqrt{d^2-e^2x^2}} \, dx$

**Optimal**. Leaf size=234

$$-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{70d^2e^3(d+ex)^3}-\frac{\sqrt{d^2-e^2x^2}\,(Ae^2-Bde+Cd^2)}{7de^3(d+ex)^4}-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{105d^4e^3(d+ex)}-\frac{\sqrt{d}}{}$$

```
[Out] -1/7*(A*e^2-B*d*e+C*d^2)*(-e^2*x^2+d^2)^(1/2)/d/e^3/(e*x+d)^4+1/2*C*(-e^2*x
^2+d^2)^(1/2)/e^3/(e*x+d)^3-1/70*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^
(1/2)/d^2/e^3/(e*x+d)^3-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/
2)/d^3/e^3/(e*x+d)^2-1/105*(6*A*e^2+8*B*d*e+13*C*d^2)*(-e^2*x^2+d^2)^(1/2)/
d^4/e^3/(e*x+d)
```

**Rubi [A]** time = 0.25, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1639, 793, 659, 651}

$$-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{105d^4e^3(d+ex)}-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{105d^3e^3(d+ex)^2}-\frac{\sqrt{d^2-e^2x^2}\,(6Ae^2+8Bde+13Cd^2)}{70d^2e^3(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)*Sqrt[d^2 - e^2*x^2])/(7*d*e^3*(d + e*x)^4) + (C*S
qrt[d^2 - e^2*x^2])/(2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e^2)*S
qrt[d^2 - e^2*x^2])/(70*d^2*e^3*(d + e*x)^3) - ((13*C*d^2 + 8*B*d*e + 6*A*e
^2)*Sqrt[d^2 - e^2*x^2])/(105*d^3*e^3*(d + e*x)^2) - ((13*C*d^2 + 8*B*d*e +
 6*A*e^2)*Sqrt[d^2 - e^2*x^2])/(105*d^4*e^3*(d + e*x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rule 659

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(e*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(m + p + 1)), x] + Dist[Simplif
y[m + 2*p + 2]/(2*d*(m + p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x],
x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p
] && ILtQ[Simplify[m + 2*p + 2], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
p + q)*(d + e*x)^(q - 2)*(a*e - c*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
0]] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[c*d^2 + a*e^2, 0
] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx = \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\int \frac{e^2\left(3Cd^2+2Ae^2\right)+e^3(Cd+2Be)x}{(d+ex)^4\sqrt{d^2-e^2x^2}}\,dx}{2e^4}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} + \frac{\left(13Cd^2+8Bde+6Ae^2\right)\int \frac{}{(d+}}{14de^2}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-}}{70d^2e^3(d+ex)^3}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-}}{70d^2e^3(d+ex)^3}$$

$$= -\frac{\left(Cd^2-Bde+Ae^2\right)\sqrt{d^2-e^2x^2}}{7de^3(d+ex)^4} + \frac{C\sqrt{d^2-e^2x^2}}{2e^3(d+ex)^3} - \frac{\left(13Cd^2+8Bde+6Ae^2\right)\sqrt{d^2-}}{70d^2e^3(d+ex)^3}$$

**Mathematica [A]** time = 0.22, size = 139, normalized size = 0.59

$$-\frac{\sqrt{d^2-e^2x^2}\left(e\left(3Ae\left(12d^3+13d^2ex+8de^2x^2+2e^3x^3\right)+Bd\left(13d^3+52d^2ex+32de^2x^2+8e^3x^3\right)\right)+Cd^2\left(8d^3+\right.\right.}{105d^4e^3(d+ex)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^4*Sqrt[d^2 - e^2*x^2]),x]
```

```
[Out] -1/105*(Sqrt[d^2 - e^2*x^2]*(C*d^2*(8*d^3 + 32*d^2*e*x + 52*d*e^2*x^2 + 13*
e^3*x^3) + e*(3*A*e*(12*d^3 + 13*d^2*e*x + 8*d*e^2*x^2 + 2*e^3*x^3) + B*d*(
13*d^3 + 52*d^2*e*x + 32*d*e^2*x^2 + 8*e^3*x^3))))/(d^4*e^3*(d + e*x)^4)
```

**fricas [A]** time = 0.92, size = 320, normalized size = 1.37

$$-\frac{8\,Cd^6+13\,Bd^5e+36\,Ad^4e^2+\left(8\,Cd^2e^4+13\,Bde^5+36\,Ae^6\right)x^4+4\left(8\,Cd^3e^3+13\,Bd^2e^4+36\,Ade^5\right)x^3+6\left(8\,Cd^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/105*(8*C*d^6 + 13*B*d^5*e + 36*A*d^4*e^2 + (8*C*d^2*e^4 + 13*B*d*e^5 + 3
6*A*e^6)*x^4 + 4*(8*C*d^3*e^3 + 13*B*d^2*e^4 + 36*A*d*e^5)*x^3 + 6*(8*C*d^4
*e^2 + 13*B*d^3*e^3 + 36*A*d^2*e^4)*x^2 + 4*(8*C*d^5*e + 13*B*d^4*e^2 + 36*
A*d^3*e^3)*x + (8*C*d^5 + 13*B*d^4*e + 36*A*d^3*e^2 + (13*C*d^2*e^3 + 8*B*d
*e^4 + 6*A*e^5)*x^3 + 4*(13*C*d^3*e^2 + 8*B*d^2*e^3 + 6*A*d*e^4)*x^2 + (32*
C*d^4*e + 52*B*d^3*e^2 + 39*A*d^2*e^3)*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^7*x^
4 + 4*d^5*e^6*x^3 + 6*d^6*e^5*x^2 + 4*d^7*e^4*x + d^8*e^3)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: (64*C
*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
0*exp(2)^3-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))
^2*exp(1)^10*exp(2)^4+8*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))
/x/exp(2))^3*exp(1)^16*exp(2)+12*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2)
)*exp(1))/x/exp(2))^4*exp(1)^14*exp(2)^2+6*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^12*exp(2)^3+6*B*d*(-1/2*(-2*d*exp(1)
-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^13*exp(2)^2+12*A*(-1/2*(
-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^14*exp(2)^2-8
*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^12*
exp(2)^3-36*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4
*exp(1)^10*exp(2)^4-18*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
```

```
x/exp(2))^5*exp(1)^8*exp(2)^5+6*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2
))*exp(1))/x/exp(2))^2*exp(1)^13*exp(2)^2-34*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(
d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^11*exp(2)^3-33*B*d*(-1/2*(-2*d*e
xp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^9*exp(2)^4-3*B*d*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^7*exp(2)
^5+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*e
xp(1)^12*exp(2)^2+60*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1)
)/x/exp(2))^4*exp(1)^8*exp(2)^4+12*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*
exp(2))*exp(1))/x/exp(2))^5*exp(1)^6*exp(2)^5+42*A*(-1/2*(-2*d*exp(1)-2*sqr
t(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^8*exp(2)^5+81*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)^6*exp(2)^6+27*A*(-1/
2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^4*exp(2)^7
-84*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1
)^9*exp(2)^4-84*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp
(2))^3*exp(1)^7*exp(2)^5-42*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*e
xp(1))/x/exp(2))^4*exp(1)^5*exp(2)^6-12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x
^2*exp(2))*exp(1))/x/exp(2))^5*exp(1)^3*exp(2)^7+102*C*d^2*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^8*exp(2)^4+78*C*d^2*(-
1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^6*exp(2)
^5+15*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*e
xp(1)^4*exp(2)^6+3*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/
x/exp(2))^5*exp(2)^8+2*A*exp(1)^10*exp(2)^4+120*A*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^6*exp(2)^6+108*A*(-1/2*(-2*d*ex
p(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^4*exp(2)^7+B*d*exp(1
)^9*exp(2)^4+2*C*d^2*exp(1)^8*exp(2)^4-60*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2
-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^5*exp(2)^6+18*A*(-1/2*(-2*d*exp(1)-
2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(2)^9-10*B*d*exp(1)^5*exp(2)^
6-6*B*d*exp(1)*exp(2)^8-36*B*d*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*ex
p(1))/x/exp(2))^3*exp(1)^3*exp(2)^7+24*C*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-
x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)^4*exp(2)^6-5*A*exp(1)^6*exp(2)^6+13*
C*d^2*exp(1)^4*exp(2)^6+36*A*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)^9+18*A*exp(2)^9-81/2*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^4*exp(2)^7/x/exp(2)+6*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^8*exp(2)^5/x/exp(2)-3*A*(-2*d*exp(1)-2*sqrt(d^2-x^2*e
xp(2))*exp(1))*exp(1)^12*exp(2)^3/x/exp(2)-12*B*d*(-1/2*(-2*d*exp(1)-2*sqrt
(d^2-x^2*exp(2))*exp(1))/x/exp(2))^2*exp(1)*exp(2)^8+4*B*d*(-1/2*(-2*d*exp(
1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^15*exp(2)-6*B*d*(-1/2*
(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^4*exp(1)*exp(2)^8+8*C
*d^2*(-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x/exp(2))^3*exp(1)^1
4*exp(2)+3/2*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(2)^8/x/e
xp(2)+12*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^3*exp(2)^7/
x/exp(2)+57/2*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^7*exp(
2)^5/x/exp(2)-3*B*d*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1)^11*e
xp(2)^3/x/exp(2)-33*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*exp(1
)^6*exp(2)^5/x/exp(2)-6*C*d^2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))*e
```

```
xp(1)^10*exp(2)^3/x/exp(2))/((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(
1))/x/exp(2))^2*exp(2)-(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2)
)^3/(3*d^4*exp(1)^13-9*d^4*exp(1)^9*exp(2)^2+9*d^4*exp(1)^5*exp(2)^4-3*d^4*
exp(1)*exp(2)^6)+1/2*(2*B*d*exp(1)^7*exp(2)^3-8*C*d^2*exp(1)^6*exp(2)^3-6*A
*exp(1)^4*exp(2)^5+8*B*d*exp(1)^3*exp(2)^5-2*C*d^2*exp(2)^6-4*A*exp(2)^7)*a
tan((-1/2*(-2*d*exp(1)-2*sqrt(d^2-x^2*exp(2))*exp(1))/x+exp(2))/sqrt(-exp(1
)^4+exp(2)^2))/sqrt(-exp(1)^4+exp(2)^2)/(-d^4*exp(1)^13+3*d^4*exp(1)^9*exp(
2)^2-3*d^4*exp(1)^5*exp(2)^4+d^4*exp(1)*exp(2)^6)
```

**maple [A]** time = 0.01, size = 152, normalized size = 0.65

$$-\frac{(-ex+d)\left(6A\,e^5x^3+8Bd\,e^4x^3+13C\,d^2e^3x^3+24Ad\,e^4x^2+32B\,d^2e^3x^2+52C\,d^3e^2x^2+39A\,d^2e^3x+52B\,d^3e^2x+\cdots\right)}{105\,(ex+d)^3\sqrt{-e^2x^2+d^2}\;d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x)
```

```
[Out] -1/105*(-e*x+d)*(6*A*e^5*x^3+8*B*d*e^4*x^3+13*C*d^2*e^3*x^3+24*A*d*e^4*x^2+
32*B*d^2*e^3*x^2+52*C*d^3*e^2*x^2+39*A*d^2*e^3*x+52*B*d^3*e^2*x+32*C*d^4*e*
x+36*A*d^3*e^2+13*B*d^4*e+8*C*d^5)/(e*x+d)^3/d^4/e^3/(-e^2*x^2+d^2)^(1/2)
```

**maxima [B]** time = 1.06, size = 975, normalized size = 4.17

$$-\frac{\sqrt{-e^2x^2+d^2}\,Cd^2}{7\left(de^7x^4+4\,d^2e^6x^3+6\,d^3e^5x^2+4\,d^4e^4x+d^5e^3\right)}-\frac{3\sqrt{-e^2x^2+d^2}\,Cd^2}{35\left(d^2e^6x^3+3\,d^3e^5x^2+3\,d^4e^4x+d^5e^3\right)}-\frac{2\sqrt{-e^2x^2+d^2}\,\cdots}{35\left(d^3e^5x^2+2\,d^4e^4x\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^4/(-e^2*x^2+d^2)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*C*d^2/(d*e^7*x^4 + 4*d^2*e^6*x^3 + 6*d^3*e^5*x^2
+ 4*d^4*e^4*x + d^5*e^3) - 3/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^2*e^6*x^3 + 3
*d^3*e^5*x^2 + 3*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^
3*e^5*x^2 + 2*d^4*e^4*x + d^5*e^3) - 2/35*sqrt(-e^2*x^2 + d^2)*C*d^2/(d^4*e
^4*x + d^5*e^3) + 1/7*sqrt(-e^2*x^2 + d^2)*B*d/(d*e^6*x^4 + 4*d^2*e^5*x^3 +
 6*d^3*e^4*x^2 + 4*d^4*e^3*x + d^5*e^2) + 3/35*sqrt(-e^2*x^2 + d^2)*B*d/(d^
2*e^5*x^3 + 3*d^3*e^4*x^2 + 3*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d
^2)*B*d/(d^3*e^4*x^2 + 2*d^4*e^3*x + d^5*e^2) + 2/35*sqrt(-e^2*x^2 + d^2)*B
*d/(d^4*e^3*x + d^5*e^2) + 2/5*sqrt(-e^2*x^2 + d^2)*C*d/(d*e^6*x^3 + 3*d^2*
e^5*x^2 + 3*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^2*e^5*x
^2 + 2*d^3*e^4*x + d^4*e^3) + 4/15*sqrt(-e^2*x^2 + d^2)*C*d/(d^3*e^4*x + d^
4*e^3) - 1/7*sqrt(-e^2*x^2 + d^2)*A/(d*e^5*x^4 + 4*d^2*e^4*x^3 + 6*d^3*e^3*
x^2 + 4*d^4*e^2*x + d^5*e) - 3/35*sqrt(-e^2*x^2 + d^2)*A/(d^2*e^4*x^3 + 3*d
```

```
^3*e^3*x^2 + 3*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^3*e^3*x^
2 + 2*d^4*e^2*x + d^5*e) - 2/35*sqrt(-e^2*x^2 + d^2)*A/(d^4*e^2*x + d^5*e)
- 1/5*sqrt(-e^2*x^2 + d^2)*B/(d*e^5*x^3 + 3*d^2*e^4*x^2 + 3*d^3*e^3*x + d^4
*e^2) - 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^2*e^4*x^2 + 2*d^3*e^3*x + d^4*e^2) -
 2/15*sqrt(-e^2*x^2 + d^2)*B/(d^3*e^3*x + d^4*e^2) - 1/3*sqrt(-e^2*x^2 + d^
2)*C/(d*e^5*x^2 + 2*d^2*e^4*x + d^3*e^3) - 1/3*sqrt(-e^2*x^2 + d^2)*C/(d^2*
e^4*x + d^3*e^3)
```

**mupad** [B] time = 3.78, size = 204, normalized size = 0.87

$$\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{C}{5\,e^3}-\frac{-4\,C\,d^2+4\,B\,d\,e+3\,A\,e^2}{35\,d^2\,e^3}\right)}{{\left(d+e\,x\right)}^3}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(\frac{A}{7\,d\,e}+\frac{d\,\left(\frac{C}{7\,e^2}-\frac{B}{7\,d\,e}\right)}{e}\right)}{{\left(d+e\,x\right)}^4}-\frac{\sqrt{d^2-e^2\,x^2}\,\left(13\,C\,d^2+8\,B\,d\,e+6\,A\,e\right.}{105\,d^3\,e^3\,{\left(d+e\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d^2 - e^2*x^2)^(1/2)*(d + e*x)^4),x)
```

```
[Out] ((d^2 - e^2*x^2)^(1/2)*(C/(5*e^3) - (3*A*e^2 - 4*C*d^2 + 4*B*d*e)/(35*d^2*e
^3)))/(d + e*x)^3 - ((d^2 - e^2*x^2)^(1/2)*(A/(7*d*e) + (d*(C/(7*e^2) - B/(
7*d*e)))/e))/(d + e*x)^4 - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d^2 + 8*B
*d*e))/(105*d^3*e^3*(d + e*x)^2) - ((d^2 - e^2*x^2)^(1/2)*(6*A*e^2 + 13*C*d
^2 + 8*B*d*e))/(105*d^4*e^3*(d + e*x))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-d+ex\right)\left(d+ex\right)}\,\left(d+ex\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**4/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**4), x)
```

# 3.18 $\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=175

$$\frac{(d+ex)^6 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{5e^5}+\frac{(d+ex)^4 \left(a\right.}{}$$

```
[Out] 1/4*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^4/e^5-1/5*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^5/e^5+1/6*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^6/e^5-1/7*c*(-B*e+4*C*d)*(e*x+d)^7/e^5+1/8*c*C*(e*x+d)
^8/e^5
```

**Rubi [A]** time = 0.31, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{(d+ex)^6 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{6e^5}-\frac{(d+ex)^5 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{5e^5}+\frac{(d+ex)^4 \left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^4)/(4*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^5)/(5*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^6)/(6*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^7)/(7*e^5) + (c*C*(d + e*x)^8)/(8*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) dx = \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{e^4}+\frac{\left(-4cCd^3+cde(3Bd-\right.}{}\right.$$

$$=\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^4}{4e^5}-\frac{\left(4cCd^3-cde(3Bd-2Ae)\right.}{5}$$

**Mathematica [A]** time = 0.09, size = 208, normalized size = 1.19

$$\frac{1}{5}x^5\left(ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{6}ex^6\left(aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{3}dx^3\left(A\left(3ae^2+cd^2\right)+a\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^3*x + (a*d^2*(B*d + 3*A*e)*x^2)/2 + (d*(a*d*(C*d + 3*B*e) + A*(c*d^2
+ 3*a*e^2))*x^3)/3 + ((B*c*d^3 + 3*A*c*d^2*e + 3*a*C*d^2*e + 3*a*B*d*e^2 +
a*A*e^3)*x^4)/4 + ((c*C*d^3 + 3*c*d*e*(B*d + A*e) + a*e^2*(3*C*d + B*e))*x^
5)/5 + (e*(3*c*C*d^2 + a*C*e^2 + c*e*(3*B*d + A*e))*x^6)/6 + (c*e^2*(3*C*d
+ B*e)*x^7)/7 + (c*C*e^3*x^8)/8
```

**fricas [A]** time = 0.75, size = 248, normalized size = 1.42

$$\frac{1}{8}x^8e^3cC+\frac{3}{7}x^7e^2dcC+\frac{1}{7}x^7e^3cB+\frac{1}{2}x^6ed^2cC+\frac{1}{6}x^6e^3aC+\frac{1}{2}x^6e^2dcB+\frac{1}{6}x^6e^3cA+\frac{1}{5}x^5d^3cC+\frac{3}{5}x^5e^2daC+\frac{3}{5}x^5ed^2cB+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e^3*c*C + 3/7*x^7*e^2*d*c*C + 1/7*x^7*e^3*c*B + 1/2*x^6*e*d^2*c*C +
 1/6*x^6*e^3*a*C + 1/2*x^6*e^2*d*c*B + 1/6*x^6*e^3*c*A + 1/5*x^5*d^3*c*C +
3/5*x^5*e^2*d*a*C + 3/5*x^5*e*d^2*c*B + 1/5*x^5*e^3*a*B + 3/5*x^5*e^2*d*c*A
 + 3/4*x^4*e*d^2*a*C + 1/4*x^4*d^3*c*B + 3/4*x^4*e^2*d*a*B + 3/4*x^4*e*d^2*
c*A + 1/4*x^4*e^3*a*A + 1/3*x^3*d^3*a*C + x^3*e*d^2*a*B + 1/3*x^3*d^3*c*A +
 x^3*e^2*d*a*A + 1/2*x^2*d^3*a*B + 3/2*x^2*e*d^2*a*A + x*d^3*a*A
```

**giac [A]** time = 0.17, size = 242, normalized size = 1.38

$$\frac{1}{8}Ccx^8e^3+\frac{3}{7}Ccdx^7e^2+\frac{1}{2}Ccd^2x^6e+\frac{1}{5}Ccd^3x^5+\frac{1}{7}Bcx^7e^3+\frac{1}{2}Bcdx^6e^2+\frac{3}{5}Bcd^2x^5e+\frac{1}{4}Bcd^3x^4+\frac{1}{6}Cax^6e^3+\frac{1}{6}Acx^6e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8*e^3 + 3/7*C*c*d*x^7*e^2 + 1/2*C*c*d^2*x^6*e + 1/5*C*c*d^3*x^5 +
 1/7*B*c*x^7*e^3 + 1/2*B*c*d*x^6*e^2 + 3/5*B*c*d^2*x^5*e + 1/4*B*c*d^3*x^4
+ 1/6*C*a*x^6*e^3 + 1/6*A*c*x^6*e^3 + 3/5*C*a*d*x^5*e^2 + 3/5*A*c*d*x^5*e^2
 + 3/4*C*a*d^2*x^4*e + 3/4*A*c*d^2*x^4*e + 1/3*C*a*d^3*x^3 + 1/3*A*c*d^3*x^
3 + 1/5*B*a*x^5*e^3 + 3/4*B*a*d*x^4*e^2 + B*a*d^2*x^3*e + 1/2*B*a*d^3*x^2 +
 1/4*A*a*x^4*e^3 + A*a*d*x^3*e^2 + 3/2*A*a*d^2*x^2*e + A*a*d^3*x
```

**maple [A]** time = 0.00, size = 217, normalized size = 1.24

$$\frac{Cc\,e^3x^8}{8}+\frac{\left(e^3cB+3d\,e^2cC\right)x^7}{7}+Aa\,d^3x+\frac{\left(Ac\,e^3+3Bcd\,e^2+\left(e^3a+3d^2ec\right)C\right)x^6}{6}+\frac{\left(3Acd\,e^2+\left(e^3a+3d^2ec\right)B+\left(3\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*e^3*c*C*x^8+1/7*(B*c*e^3+3*C*c*d*e^2)*x^7+1/6*((a*e^3+3*c*d^2*e)*C+3*d*
e^2*c*B+e^3*c*A)*x^6+1/5*((3*a*d*e^2+c*d^3)*C+(a*e^3+3*c*d^2*e)*B+3*d*e^2*c
*A)*x^5+1/4*(3*d^2*e*a*C+(3*a*d*e^2+c*d^3)*B+(a*e^3+3*c*d^2*e)*A)*x^4+1/3*(
d^3*a*C+3*d^2*e*a*B+(3*a*d*e^2+c*d^3)*A)*x^3+1/2*(3*A*a*d^2*e+B*a*d^3)*x^2+
d^3*a*A*x
```

**maxima [A]** time = 0.45, size = 202, normalized size = 1.15

$$\frac{1}{8}\,Cce^3x^8+\frac{1}{7}\left(3\,Ccde^2+Bce^3\right)x^7+\frac{1}{6}\left(3\,Ccd^2e+3\,Bcde^2+(Ca+Ac)e^3\right)x^6+Aad^3x+\frac{1}{5}\left(Ccd^3+3\,Bcd^2e+Bae^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*e^3*x^8 + 1/7*(3*C*c*d*e^2 + B*c*e^3)*x^7 + 1/6*(3*C*c*d^2*e + 3*B*
c*d*e^2 + (C*a + A*c)*e^3)*x^6 + A*a*d^3*x + 1/5*(C*c*d^3 + 3*B*c*d^2*e + B
*a*e^3 + 3*(C*a + A*c)*d*e^2)*x^5 + 1/4*(B*c*d^3 + 3*B*a*d*e^2 + A*a*e^3 +
3*(C*a + A*c)*d^2*e)*x^4 + 1/3*(3*B*a*d^2*e + 3*A*a*d*e^2 + (C*a + A*c)*d^3
)*x^3 + 1/2*(B*a*d^3 + 3*A*a*d^2*e)*x^2
```

**mupad [B]** time = 0.09, size = 206, normalized size = 1.18

$$x^3\,\left(\frac{A\,c\,d^3}{3}+\frac{C\,a\,d^3}{3}+A\,a\,d\,e^2+B\,a\,d^2\,e\right)+x^6\,\left(\frac{A\,c\,e^3}{6}+\frac{C\,a\,e^3}{6}+\frac{B\,c\,d\,e^2}{2}+\frac{C\,c\,d^2\,e}{2}\right)+x^4\,\left(\frac{A\,a\,e^3}{4}+\frac{B\,c\,d^3}{4}+\frac{3\,B\,a}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d^3)/3 + (C*a*d^3)/3 + A*a*d*e^2 + B*a*d^2*e) + x^6*((A*c*e^3)/6
+ (C*a*e^3)/6 + (B*c*d*e^2)/2 + (C*c*d^2*e)/2) + x^4*((A*a*e^3)/4 + (B*c*d^
3)/4 + (3*B*a*d*e^2)/4 + (3*A*c*d^2*e)/4 + (3*C*a*d^2*e)/4) + x^5*((B*a*e^3
)/5 + (C*c*d^3)/5 + (3*A*c*d*e^2)/5 + (3*C*a*d*e^2)/5 + (3*B*c*d^2*e)/5) +
A*a*d^3*x + (C*c*e^3*x^8)/8 + (a*d^2*x^2*(3*A*e + B*d))/2 + (c*e^2*x^7*(B*e
 + 3*C*d))/7
```

**sympy [A]** time = 0.12, size = 257, normalized size = 1.47

$$Aad^3x+\frac{Cce^3x^8}{8}+x^7\left(\frac{Bce^3}{7}+\frac{3Ccde^2}{7}\right)+x^6\left(\frac{Ace^3}{6}+\frac{Bcde^2}{2}+\frac{Cae^3}{6}+\frac{Ccd^2e}{2}\right)+x^5\left(\frac{3Acde^2}{5}+\frac{Bae^3}{5}+\frac{3Bcd^2e}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**3*x + C*c*e**3*x**8/8 + x**7*(B*c*e**3/7 + 3*C*c*d*e**2/7) + x**6*(A
*c*e**3/6 + B*c*d*e**2/2 + C*a*e**3/6 + C*c*d**2*e/2) + x**5*(3*A*c*d*e**2/
5 + B*a*e**3/5 + 3*B*c*d**2*e/5 + 3*C*a*d*e**2/5 + C*c*d**3/5) + x**4*(A*a*
e**3/4 + 3*A*c*d**2*e/4 + 3*B*a*d*e**2/4 + B*c*d**3/4 + 3*C*a*d**2*e/4) + x
**3*(A*a*d*e**2 + A*c*d**3/3 + B*a*d**2*e + C*a*d**3/3) + x**2*(3*A*a*d**2*
e/2 + B*a*d**3/2)
```

## 3.19 $\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx$

**Optimal**. Leaf size=175

$$\frac{(d+ex)^5 \left(aCe^2+c\left(6Cd^2-e(3Bd-Ae)\right)\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{4e^5}+\frac{(d+ex)^3 \left(a\right.}{}$$

```
[Out] 1/3*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)*(e*x+d)^3/e^5-1/4*(a*e^2*(-B*e+2*C*d)
+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*(e*x+d)^4/e^5+1/5*(a*C*e^2+c*(6*C*d^2-e*(-
A*e+3*B*d)))*(e*x+d)^5/e^5-1/6*c*(-B*e+4*C*d)*(e*x+d)^6/e^5+1/7*c*C*(e*x+d)
^7/e^5
```

**Rubi [A]** time = 0.22, antiderivative size = 173, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{(d+ex)^5 \left(aCe^2-ce(3Bd-Ae)+6cCd^2\right)}{5e^5}-\frac{(d+ex)^4 \left(ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3\right)}{4e^5}+\frac{(d+ex)^3 \left(ae^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*(d + e*x)^3)/(3*e^5) - ((4*c*C*d^3
 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*(d + e*x)^4)/(4*e^5) + ((6*
c*C*d^2 + a*C*e^2 - c*e*(3*B*d - A*e))*(d + e*x)^5)/(5*e^5) - (c*(4*C*d - B
*e)*(d + e*x)^6)/(6*e^5) + (c*C*(d + e*x)^7)/(7*e^5)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right) \left(A+Bx+Cx^2\right) \, dx = \int \left(\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^2}{e^4}+\frac{\left(-4cCd^3+cde(3Bd-\right.}{}\right.$$

$$= \frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)(d+ex)^3}{3e^5}-\frac{\left(4cCd^3-cde(3Bd-2Ae)\right.}{4}$$

**Mathematica [A]** time = 0.06, size = 150, normalized size = 0.86

$$\frac{1}{5}x^5\left(aCe^2+Ace^2+2Bcde+cCd^2\right)+\frac{1}{4}x^4\left(aBe^2+2aCde+2Acde+Bcd^2\right)+\frac{1}{3}x^3\left(aAe^2+2aBde+aCd^2+Acd^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d^2*x + (a*d*(B*d + 2*A*e)*x^2)/2 + ((A*c*d^2 + a*C*d^2 + 2*a*B*d*e + a
*A*e^2)*x^3)/3 + ((B*c*d^2 + 2*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4)/4 + ((c*
C*d^2 + 2*B*c*d*e + A*c*e^2 + a*C*e^2)*x^5)/5 + (c*e*(2*C*d + B*e)*x^6)/6 +
 (c*C*e^2*x^7)/7
```

**fricas [A]** time = 0.74, size = 171, normalized size = 0.98

$$\frac{1}{7}x^7e^2cC+\frac{1}{3}x^6edcC+\frac{1}{6}x^6e^2cB+\frac{1}{5}x^5d^2cC+\frac{1}{5}x^5e^2aC+\frac{2}{5}x^5edcB+\frac{1}{5}x^5e^2cA+\frac{1}{2}x^4edaC+\frac{1}{4}x^4d^2cB+\frac{1}{4}x^4e^2aB+\frac{1}{2}x^4ed$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*e^2*c*C + 1/3*x^6*e*d*c*C + 1/6*x^6*e^2*c*B + 1/5*x^5*d^2*c*C + 1/5
*x^5*e^2*a*C + 2/5*x^5*e*d*c*B + 1/5*x^5*e^2*c*A + 1/2*x^4*e*d*a*C + 1/4*x^
4*d^2*c*B + 1/4*x^4*e^2*a*B + 1/2*x^4*e*d*c*A + 1/3*x^3*d^2*a*C + 2/3*x^3*e
*d*a*B + 1/3*x^3*d^2*c*A + 1/3*x^3*e^2*a*A + 1/2*x^2*d^2*a*B + x^2*e*d*a*A
+ x*d^2*a*A
```

**giac [A]** time = 0.15, size = 171, normalized size = 0.98

$$\frac{1}{7}Ccx^7e^2+\frac{1}{3}Ccdx^6e+\frac{1}{5}Ccd^2x^5+\frac{1}{6}Bcx^6e^2+\frac{2}{5}Bcdx^5e+\frac{1}{4}Bcd^2x^4+\frac{1}{5}Cax^5e^2+\frac{1}{5}Acx^5e^2+\frac{1}{2}Cadx^4e+\frac{1}{2}Acdx^4e+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7*e^2 + 1/3*C*c*d*x^6*e + 1/5*C*c*d^2*x^5 + 1/6*B*c*x^6*e^2 + 2/5
*B*c*d*x^5*e + 1/4*B*c*d^2*x^4 + 1/5*C*a*x^5*e^2 + 1/5*A*c*x^5*e^2 + 1/2*C*
a*d*x^4*e + 1/2*A*c*d*x^4*e + 1/3*C*a*d^2*x^3 + 1/3*A*c*d^2*x^3 + 1/4*B*a*x
^4*e^2 + 2/3*B*a*d*x^3*e + 1/2*B*a*d^2*x^2 + 1/3*A*a*x^3*e^2 + A*a*d*x^2*e
+ A*a*d^2*x
```

**maple [A]** time = 0.00, size = 148, normalized size = 0.85

$$\frac{Cce^2x^7}{7}+\frac{\left(ce^2B+2decC\right)x^6}{6}+Aad^2x+\frac{\left(Ace^2+2Bcde+\left(ae^2+cd^2\right)C\right)x^5}{5}+\frac{\left(2Acde+2Cade+\left(ae^2+cd^2\right)B\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c*e^2*C*x^7+1/6*(B*c*e^2+2*C*c*d*e)*x^6+1/5*((a*e^2+c*d^2)*C+2*d*e*c*B+
c*e^2*A)*x^5+1/4*(2*d*e*a*C+(a*e^2+c*d^2)*B+2*d*e*c*A)*x^4+1/3*(d^2*a*C+2*d
*e*a*B+A*(a*e^2+c*d^2))*x^3+1/2*(2*A*a*d*e+B*a*d^2)*x^2+d^2*a*A*x
```

**maxima [A]** time = 0.45, size = 141, normalized size = 0.81

$$\frac{1}{7}Cce^2x^7+\frac{1}{6}\left(2\,Ccde+Bce^2\right)x^6+\frac{1}{5}\left(Ccd^2+2\,Bcde+(Ca+Ac)e^2\right)x^5+Aad^2x+\frac{1}{4}\left(Bcd^2+Bae^2+2\,(Ca+Ac)de\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*e^2*x^7 + 1/6*(2*C*c*d*e + B*c*e^2)*x^6 + 1/5*(C*c*d^2 + 2*B*c*d*e
+ (C*a + A*c)*e^2)*x^5 + A*a*d^2*x + 1/4*(B*c*d^2 + B*a*e^2 + 2*(C*a + A*c)
*d*e)*x^4 + 1/3*(2*B*a*d*e + A*a*e^2 + (C*a + A*c)*d^2)*x^3 + 1/2*(B*a*d^2
+ 2*A*a*d*e)*x^2
```

**mupad [B]** time = 3.61, size = 143, normalized size = 0.82

$$x^3\left(\frac{A\,a\,e^2}{3}+\frac{A\,c\,d^2}{3}+\frac{C\,a\,d^2}{3}+\frac{2\,B\,a\,d\,e}{3}\right)+x^5\left(\frac{A\,c\,e^2}{5}+\frac{C\,a\,e^2}{5}+\frac{C\,c\,d^2}{5}+\frac{2\,B\,c\,d\,e}{5}\right)+x^4\left(\frac{B\,a\,e^2}{4}+\frac{B\,c\,d^2}{4}+\frac{A\,c\,d}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a*e^2)/3 + (A*c*d^2)/3 + (C*a*d^2)/3 + (2*B*a*d*e)/3) + x^5*((A*c*e
^2)/5 + (C*a*e^2)/5 + (C*c*d^2)/5 + (2*B*c*d*e)/5) + x^4*((B*a*e^2)/4 + (B*
c*d^2)/4 + (A*c*d*e)/2 + (C*a*d*e)/2) + A*a*d^2*x + (a*d*x^2*(2*A*e + B*d))
/2 + (c*e*x^6*(B*e + 2*C*d))/6 + (C*c*e^2*x^7)/7
```

**sympy [A]** time = 0.10, size = 173, normalized size = 0.99

$$Aad^2x+\frac{Cce^2x^7}{7}+x^6\left(\frac{Bce^2}{6}+\frac{Ccde}{3}\right)+x^5\left(\frac{Ace^2}{5}+\frac{2Bcde}{5}+\frac{Cae^2}{5}+\frac{Ccd^2}{5}\right)+x^4\left(\frac{Acde}{2}+\frac{Bae^2}{4}+\frac{Bcd^2}{4}+\frac{Cade}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d**2*x + C*c*e**2*x**7/7 + x**6*(B*c*e**2/6 + C*c*d*e/3) + x**5*(A*c*e*
*2/5 + 2*B*c*d*e/5 + C*a*e**2/5 + C*c*d**2/5) + x**4*(A*c*d*e/2 + B*a*e**2/
4 + B*c*d**2/4 + C*a*d*e/2) + x**3*(A*a*e**2/3 + A*c*d**2/3 + 2*B*a*d*e/3 +
 C*a*d**2/3) + x**2*(A*a*d*e + B*a*d**2/2)
```

## 3.20 $\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

**Optimal**. Leaf size=86

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

```
[Out] a*A*d*x+1/2*a*(A*e+B*d)*x^2+1/3*(A*c*d+B*a*e+C*a*d)*x^3+1/4*(A*c*e+B*c*d+C*
a*e)*x^4+1/5*c*(B*e+C*d)*x^5+1/6*c*C*e*x^6
```

**Rubi [A]** time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1628}

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx=\int \left(aAd+a(Bd+Ae)x+(Acd+aCd+aBe)x^2+(Bcd+Ace+aCe$$
$$=aAdx+\frac{1}{2}a(Bd+Ae)x^2+\frac{1}{3}(Acd+aCd+aBe)x^3+\frac{1}{4}(Bcd+Ace+$$

**Mathematica [A]** time = 0.03, size = 86, normalized size = 1.00

$$\frac{1}{4}x^4(aCe+Ace+Bcd)+\frac{1}{3}x^3(aBe+aCd+Acd)+\frac{1}{2}ax^2(Ae+Bd)+aAdx+\frac{1}{5}cx^5(Be+Cd)+\frac{1}{6}cCex^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*d*x + (a*(B*d + A*e)*x^2)/2 + ((A*c*d + a*C*d + a*B*e)*x^3)/3 + ((B*c*d
 + A*c*e + a*C*e)*x^4)/4 + (c*(C*d + B*e)*x^5)/5 + (c*C*e*x^6)/6
```

**fricas [A]** time = 0.75, size = 94, normalized size = 1.09

$$\frac{1}{6}x^6ecC+\frac{1}{5}x^5dcC+\frac{1}{5}x^5ecB+\frac{1}{4}x^4eaC+\frac{1}{4}x^4dcB+\frac{1}{4}x^4ecA+\frac{1}{3}x^3daC+\frac{1}{3}x^3eaB+\frac{1}{3}x^3dcA+\frac{1}{2}x^2daB+\frac{1}{2}x^2eaA+xdaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*e*c*C + 1/5*x^5*d*c*C + 1/5*x^5*e*c*B + 1/4*x^4*e*a*C + 1/4*x^4*d*c
*B + 1/4*x^4*e*c*A + 1/3*x^3*d*a*C + 1/3*x^3*e*a*B + 1/3*x^3*d*c*A + 1/2*x^
2*d*a*B + 1/2*x^2*e*a*A + x*d*a*A
```

**giac [A]** time = 0.15, size = 100, normalized size = 1.16

$$\frac{1}{6}Ccx^6e+\frac{1}{5}Ccdx^5+\frac{1}{5}Bcx^5e+\frac{1}{4}Bcdx^4+\frac{1}{4}Cax^4e+\frac{1}{4}Acx^4e+\frac{1}{3}Cadx^3+\frac{1}{3}Acdx^3+\frac{1}{3}Bax^3e+\frac{1}{2}Badx^2+\frac{1}{2}Aax^2e+Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6*e + 1/5*C*c*d*x^5 + 1/5*B*c*x^5*e + 1/4*B*c*d*x^4 + 1/4*C*a*x^4
*e + 1/4*A*c*x^4*e + 1/3*C*a*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*x^3*e + 1/2*B*
a*d*x^2 + 1/2*A*a*x^2*e + A*a*d*x
```

**maple [A]** time = 0.00, size = 79, normalized size = 0.92

$$\frac{Cce\,x^6}{6}+\frac{(ecB+cdC)\,x^5}{5}+Aadx+\frac{(Ace+Bcd+aCe)\,x^4}{4}+\frac{(Acd+Bae+Cad)\,x^3}{3}+\frac{(aeA+adB)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x)
```

```
[Out] 1/6*c*C*e*x^6+1/5*(B*c*e+C*c*d)*x^5+1/4*(A*c*e+B*c*d+C*a*e)*x^4+1/3*(A*c*d+
B*a*e+C*a*d)*x^3+1/2*(A*a*e+B*a*d)*x^2+a*A*d*x
```

**maxima [A]** time = 0.45, size = 80, normalized size = 0.93

$$\frac{1}{6}Ccex^6+\frac{1}{5}(Ccd+Bce)x^5+\frac{1}{4}(Bcd+(Ca+Ac)e)x^4+Aadx+\frac{1}{3}(Bae+(Ca+Ac)d)x^3+\frac{1}{2}(Bad+Aae)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*C*c*e*x^6 + 1/5*(C*c*d + B*c*e)*x^5 + 1/4*(B*c*d + (C*a + A*c)*e)*x^4 +
 A*a*d*x + 1/3*(B*a*e + (C*a + A*c)*d)*x^3 + 1/2*(B*a*d + A*a*e)*x^2
```

**mupad [B]** time = 3.56, size = 80, normalized size = 0.93

$$\frac{C\,c\,e\,x^6}{6}+\frac{c\,\left(B\,e+C\,d\right)\,x^5}{5}+\left(\frac{A\,c\,e}{4}+\frac{B\,c\,d}{4}+\frac{C\,a\,e}{4}\right)\,x^4+\left(\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}+\frac{C\,a\,d}{3}\right)\,x^3+\frac{a\,\left(A\,e+B\,d\right)\,x^2}{2}+A\,a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c*d)/3 + (B*a*e)/3 + (C*a*d)/3) + x^4*((A*c*e)/4 + (B*c*d)/4 + (C*a
*e)/4) + (a*x^2*(A*e + B*d))/2 + (c*x^5*(B*e + C*d))/5 + (C*c*e*x^6)/6 + A*
a*d*x
```

**sympy [A]** time = 0.08, size = 97, normalized size = 1.13

$$Aadx+\frac{Ccex^6}{6}+x^5\left(\frac{Bce}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ace}{4}+\frac{Bcd}{4}+\frac{Cae}{4}\right)+x^3\left(\frac{Acd}{3}+\frac{Bae}{3}+\frac{Cad}{3}\right)+x^2\left(\frac{Aae}{2}+\frac{Bad}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*c*d/5) + x**4*(A*c*e/4 + B*c*d/4
 + C*a*e/4) + x**3*(A*c*d/3 + B*a*e/3 + C*a*d/3) + x**2*(A*a*e/2 + B*a*d/2)
```

## 3.21 $\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx$

**Optimal**. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

**Rubi [A]** time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)\left(A+Bx+Cx^2\right)\,dx &= \int \left(aA+aBx+(Ac+aC)x^2+Bcx^3+cCx^4\right)\,dx\\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}aBx^2+\frac{1}{4}Bcx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)*(A + B*x + C*x^2),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*c + a*C)*x^3)/3 + (B*c*x^4)/4 + (c*C*x^5)/5
```

**fricas [A]** time = 0.84, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC + \frac{1}{4}x^4cB + \frac{1}{3}x^3aC + \frac{1}{3}x^3cA + \frac{1}{2}x^2aB + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="fricas")
[Out] 1/5*x^5*c*C + 1/4*x^4*c*B + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*a*B + x*a*A
```

**giac [A]** time = 0.16, size = 40, normalized size = 0.87

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Bcx^4 + \frac{1}{3}\,Cax^3 + \frac{1}{3}\,Acx^3 + \frac{1}{2}\,Bax^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="giac")
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*B*a*x^2 + A*a*x
```

**maple [A]** time = 0.00, size = 39, normalized size = 0.85

$$\frac{Cc\,x^5}{5} + \frac{Bc\,x^4}{4} + \frac{Ba\,x^2}{2} + Aax + \frac{(Ac + aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A),x)
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*c+C*a)*x^3+1/4*B*c*x^4+1/5*c*C*x^5
```

**maxima [A]** time = 0.44, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Bcx^4 + \frac{1}{2}\,Bax^2 + \frac{1}{3}\,(Ca + Ac)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A),x, algorithm="maxima")
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*a*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

**mupad [B]** time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5} + \frac{B\,c\,x^4}{4} + \left(\frac{A\,c}{3} + \frac{C\,a}{3}\right)\,x^3 + \frac{B\,a\,x^2}{2} + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (B*a*x^2)/2 + (B*c*x^4)/4 + (C*c*x^5)/5
```

**sympy [A]** time = 0.07, size = 42, normalized size = 0.91

$$Aax + \frac{Bax^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

## 3.22 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx$

**Optimal**. Leaf size=145

$$\frac{(ae^2+cd^2)\log(d+ex)(Ae^2-Bde+Cd^2)}{e^5}-\frac{x\left(ae^2(Cd-Be)+cd\left(Cd^2-e(Bd-Ae)\right)\right)}{e^4}+\frac{x^2\left(aCe^2+c\left(Cd^2-e(B\right.\right.}{2e^3}$$

```
[Out] -(a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))*x/e^4+1/2*(a*C*e^2+c*(C*d^2-e*
(-A*e+B*d)))*x^2/e^3-1/3*c*(-B*e+C*d)*x^3/e^2+1/4*c*C*x^4/e+(a*e^2+c*d^2)*(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^5
```

**Rubi [A]** time = 0.25, antiderivative size = 143, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{x^2\left(aCe^2-ce(Bd-Ae)+cCd^2\right)}{2e^3}-\frac{x\left(ae^2(Cd-Be)-cde(Bd-Ae)+cCd^3\right)}{e^4}+\frac{(ae^2+cd^2)\log(d+ex)\left(Ae^2-Bde\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*C*d^3 - c*d*e*(B*d - A*e) + a*e^2*(C*d - B*e))*x)/e^4) + ((c*C*d^2 +
a*C*e^2 - c*e*(B*d - A*e))*x^2)/(2*e^3) - (c*(C*d - B*e)*x^3)/(3*e^2) + (c*
C*x^4)/(4*e) + ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^2)(A+Bx+Cx^2)}{d+ex}\,dx &= \int \left(\frac{-ae^2(Cd-Be)-c\left(Cd^3-de(Bd-Ae)\right)}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right)}{e^3}\right.\\ &= -\frac{\left(cCd^3-cde(Bd-Ae)+ae^2(Cd-Be)\right)x}{e^4}+\frac{\left(cCd^2+aCe^2-ce(Bd-Ae)\right)x^2}{2e^3}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 136, normalized size = 0.94

$$\frac{12(ae^2+cd^2)\log(d+ex)\left(e(Ae-Bd)+Cd^2\right)+ex\left(6ae^2(2Be-2Cd+Cex)+2ce\left(3Ae(ex-2d)+B\left(6d^2-3de\right.\right.\right.}{12e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(6*a*e^2*(-2*C*d + 2*B*e + C*e*x) + c*C*(-12*d^3 + 6*d^2*e*x - 4*d*e^2
*x^2 + 3*e^3*x^3) + 2*c*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))) + 12*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/(12*e^
5)
```

**fricas [A]** time = 1.61, size = 161, normalized size = 1.11

$$\frac{3\,Cce^4x^4 - 4\left(Ccde^3 - Bce^4\right)x^3 + 6\left(Ccd^2e^2 - Bcde^3 + (Ca + Ac)e^4\right)x^2 - 12\left(Ccd^3e - Bcd^2e^2 - Bae^4 + (Ca + Ac)d}{12\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*e^4*x^4 - 4*(C*c*d*e^3 - B*c*e^4)*x^3 + 6*(C*c*d^2*e^2 - B*c*d*
e^3 + (C*a + A*c)*e^4)*x^2 - 12*(C*c*d^3*e - B*c*d^2*e^2 - B*a*e^4 + (C*a +
 A*c)*d*e^3)*x + 12*(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c
)*d^2*e^2)*log(e*x + d))/e^5
```

**giac [A]** time = 0.15, size = 170, normalized size = 1.17

$$\left(Ccd^4 - Bcd^3e + Cad^2e^2 + Acd^2e^2 - Bade^3 + Aae^4\right)e^{(-5)}\log\left(|xe + d|\right) + \frac{1}{12}\left(3\,Ccx^4e^3 - 4\,Ccdx^3e^2 + 6\,Ccd^2x^2e - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c*d^4 - B*c*d^3*e + C*a*d^2*e^2 + A*c*d^2*e^2 - B*a*d*e^3 + A*a*e^4)*e^(
-5)*log(abs(x*e + d)) + 1/12*(3*C*c*x^4*e^3 - 4*C*c*d*x^3*e^2 + 6*C*c*d^2*x
^2*e - 12*C*c*d^3*x + 4*B*c*x^3*e^3 - 6*B*c*d*x^2*e^2 + 12*B*c*d^2*x*e + 6*
C*a*x^2*e^3 + 6*A*c*x^2*e^3 - 12*C*a*d*x*e^2 - 12*A*c*d*x*e^2 + 12*B*a*x*e^
3)*e^(-4)
```

**maple [A]** time = 0.01, size = 210, normalized size = 1.45

$$\frac{Cc\,x^4}{4e} + \frac{Bc\,x^3}{3e} - \frac{Ccd\,x^3}{3e^2} + \frac{Ac\,x^2}{2e} - \frac{Bcd\,x^2}{2e^2} + \frac{Ca\,x^2}{2e} + \frac{Cc\,d^2x^2}{2e^3} + \frac{Aa\ln(ex+d)}{e} + \frac{Ac\,d^2\ln(ex+d)}{e^3} - \frac{Acdx}{e^2} - \frac{Bad\ln(ex+}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 1/4*c*C*x^4/e+1/3/e*B*x^3*c-1/3/e^2*C*x^3*c*d+1/2/e*A*x^2*c-1/2/e^2*B*x^2*c
*d+1/2/e*C*x^2*a+1/2/e^3*C*x^2*c*d^2-1/e^2*A*x*c*d+1/e*B*x*a+1/e^3*B*x*c*d^
```

```
2-1/e^2*C*x*a*d-1/e^4*C*x*c*d^3+1/e*ln(e*x+d)*A*a+1/e^3*ln(e*x+d)*A*c*d^2-1
/e^2*ln(e*x+d)*B*a*d-1/e^4*ln(e*x+d)*B*c*d^3+1/e^3*ln(e*x+d)*C*a*d^2+1/e^5*
ln(e*x+d)*C*c*d^4
```

**maxima [A]** time = 0.45, size = 159, normalized size = 1.10

$$\frac{3\,Cce^3x^4 - 4\left(Ccde^2 - Bce^3\right)x^3 + 6\left(Ccd^2e - Bcde^2 + (Ca + Ac)e^3\right)x^2 - 12\left(Ccd^3 - Bcd^2e - Bae^3 + (Ca + Ac)de^2\right.}{12\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/12*(3*C*c*e^3*x^4 - 4*(C*c*d*e^2 - B*c*e^3)*x^3 + 6*(C*c*d^2*e - B*c*d*e^
2 + (C*a + A*c)*e^3)*x^2 - 12*(C*c*d^3 - B*c*d^2*e - B*a*e^3 + (C*a + A*c)*
d*e^2)*x)/e^4 + (C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^
2*e^2)*log(e*x + d)/e^5
```

**mupad [B]** time = 3.62, size = 175, normalized size = 1.21

$$x^3\left(\frac{B\,c}{3\,e}-\frac{C\,c\,d}{3\,e^2}\right)-x\left(\frac{d\left(\frac{A\,c+C\,a}{e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{e}\right)}{e}-\frac{B\,a}{e}\right)+x^2\left(\frac{A\,c+C\,a}{2\,e}-\frac{d\left(\frac{B\,c}{e}-\frac{C\,c\,d}{e^2}\right)}{2\,e}\right)+\frac{\ln\left(d+e\,x\right)\,\left(A\,a\,e^4+C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^3*((B*c)/(3*e) - (C*c*d)/(3*e^2)) - x*((d*((A*c + C*a)/e - (d*((B*c)/e -
(C*c*d)/e^2))/e))/e - (B*a)/e) + x^2*((A*c + C*a)/(2*e) - (d*((B*c)/e - (C*
c*d)/e^2))/(2*e)) + (log(d + e*x)*(A*a*e^4 + C*c*d^4 - B*a*d*e^3 - B*c*d^3*
e + A*c*d^2*e^2 + C*a*d^2*e^2))/e^5 + (C*c*x^4)/(4*e)
```

**sympy [A]** time = 0.64, size = 148, normalized size = 1.02

$$\frac{Ccx^4}{4e}+x^3\left(\frac{Bc}{3e}-\frac{Ccd}{3e^2}\right)+x^2\left(\frac{Ac}{2e}-\frac{Bcd}{2e^2}+\frac{Ca}{2e}+\frac{Ccd^2}{2e^3}\right)+x\left(-\frac{Acd}{e^2}+\frac{Ba}{e}+\frac{Bcd^2}{e^3}-\frac{Cad}{e^2}-\frac{Ccd^3}{e^4}\right)+\frac{\left(ae^2+cd^2\right)\left(A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c*x**4/(4*e) + x**3*(B*c/(3*e) - C*c*d/(3*e**2)) + x**2*(A*c/(2*e) - B*c*
d/(2*e**2) + C*a/(2*e) + C*c*d**2/(2*e**3)) + x*(-A*c*d/e**2 + B*a/e + B*c*
d**2/e**3 - C*a*d/e**2 - C*c*d**3/e**4) + (a*e**2 + c*d**2)*(A*e**2 - B*d*e
 + C*d**2)*log(d + e*x)/e**5
```

## 3.23 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx$

**Optimal**. Leaf size=153

$$-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)\right)}{e^5}+\frac{x\left(aCe^2+c\left(3Cd^2-\right.\right.}{e^4}$$

```
[Out] (a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^4-1/2*c*(-B*e+2*C*d)*x^2/e^3+1/3*c
*C*x^3/e^2-(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)/e^5/(e*x+d)-(a*e^2*(-B*e+2*C*d
)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*ln(e*x+d)/e^5
```

**Rubi [A]** time = 0.20, antiderivative size = 151, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{x\left(aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{e^4}-\frac{(ae^2+cd^2)(Ae^2-Bde+Cd^2)}{e^5(d+ex)}-\frac{\log(d+ex)\left(ae^2(2Cd-Be)-cde(3Bd-2Ae\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((3*c*C*d^2 + a*C*e^2 - c*e*(2*B*d - A*e))*x)/e^4 - (c*(2*C*d - B*e)*x^2)/(
2*e^3) + (c*C*x^3)/(3*e^2) - ((c*d^2 + a*e^2)*(C*d^2 - B*d*e + A*e^2))/(e^5
*(d + e*x)) - ((4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + a*e^2*(2*C*d - B*e))*Lo
g[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^2}\,dx=\int\left(\frac{3cCd^2+aCe^2-ce(2Bd-Ae)}{e^4}+\frac{c(-2Cd+Be)x}{e^3}+\frac{cCx^2}{e^2}+\frac{(cd^2+ae^2)(C}{e^4(d}\right.$$

$$=\frac{\left(3cCd^2+aCe^2-ce(2Bd-Ae)\right)x}{e^4}-\frac{c(2Cd-Be)x^2}{2e^3}+\frac{cCx^3}{3e^2}-\frac{(cd^2+ae^2)(Cd}{e^5(d}$$

**Mathematica [A]** time = 0.15, size = 142, normalized size = 0.93

$$\frac{6\log(d+ex)\left(ae^2(Be-2Cd)+cde(3Bd-2Ae)-4cCd^3\right)+6ex\left(aCe^2+ce(Ae-2Bd)+3cCd^2\right)-\frac{6\left(ae^2+cd^2\right)\left(e(Ae\cdots\right.}{d+ex}}{6e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (6*e*(3*c*C*d^2 + a*C*e^2 + c*e*(-2*B*d + A*e))*x + 3*c*e^2*(-2*C*d + B*e)*
x^2 + 2*c*C*e^3*x^3 - (6*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e
*x) + 6*(-4*c*C*d^3 + c*d*e*(3*B*d - 2*A*e) + a*e^2*(-2*C*d + B*e))*Log[d +
 e*x])/(6*e^5)
```

**fricas [A]** time = 0.87, size = 250, normalized size = 1.63

$$\frac{2\,Cce^4x^4-6\,Ccd^4+6\,Bcd^3e+6\,Bade^3-6\,Aae^4-6\,(Ca+Ac)d^2e^2-\left(4\,Ccde^3-3\,Bce^4\right)x^3+3\left(4\,Ccd^2e^2-3\,Bc\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*e^4*x^4 - 6*C*c*d^4 + 6*B*c*d^3*e + 6*B*a*d*e^3 - 6*A*a*e^4 - 6*
(C*a + A*c)*d^2*e^2 - (4*C*c*d*e^3 - 3*B*c*e^4)*x^3 + 3*(4*C*c*d^2*e^2 - 3*
B*c*d*e^3 + 2*(C*a + A*c)*e^4)*x^2 + 6*(3*C*c*d^3*e - 2*B*c*d^2*e^2 + (C*a
+ A*c)*d*e^3)*x - 6*(4*C*c*d^4 - 3*B*c*d^3*e - B*a*d*e^3 + 2*(C*a + A*c)*d^
2*e^2 + (4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)*lo
g(e*x + d))/(e^6*x + d*e^5)
```

**giac [A]** time = 0.16, size = 240, normalized size = 1.57

$$\frac{1}{6}\left(2\,Cc-\frac{3\left(4\,Ccde-Bce^2\right)e^{(-1)}}{xe+d}+\frac{6\left(6\,Ccd^2e^2-3\,Bcde^3+Cae^4+Ace^4\right)e^{(-2)}}{(xe+d)^2}\right)(xe+d)^3e^{(-5)}+\left(4\,Ccd^3-3\,Bcd^2e\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*C*c - 3*(4*C*c*d*e - B*c*e^2)*e^(-1)/(x*e + d) + 6*(6*C*c*d^2*e^2 -
3*B*c*d*e^3 + C*a*e^4 + A*c*e^4)*e^(-2)/(x*e + d)^2)*(x*e + d)^3*e^(-5) + (
4*C*c*d^3 - 3*B*c*d^2*e + 2*C*a*d*e^2 + 2*A*c*d*e^2 - B*a*e^3)*e^(-5)*log(a
bs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c*d^4*e^3/(x*e + d) - B*c*d^3*e^4/(x*e
 + d) + C*a*d^2*e^5/(x*e + d) + A*c*d^2*e^5/(x*e + d) - B*a*d*e^6/(x*e + d)
 + A*a*e^7/(x*e + d))*e^(-8)
```

**maple [A]** time = 0.01, size = 234, normalized size = 1.53

$$\frac{Cc\,x^{3}}{3e^{2}}+\frac{Bc\,x^{2}}{2e^{2}}-\frac{Ccd\,x^{2}}{e^{3}}-\frac{Aa}{(ex+d)\,e}-\frac{Ac\,d^{2}}{(ex+d)\,e^{3}}-\frac{2Acd\ln(ex+d)}{e^{3}}+\frac{Acx}{e^{2}}+\frac{Bad}{(ex+d)\,e^{2}}+\frac{Ba\ln(ex+d)}{e^{2}}+\frac{Bc\,d^{3}}{(ex+d)\,e^{4}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 1/3*c*C*x^3/e^2+1/2/e^2*B*x^2*c-1/e^3*C*x^2*c*d+1/e^2*A*c*x-2/e^3*B*c*d*x+1
/e^2*a*C*x+3/e^4*C*c*d^2*x-1/e/(e*x+d)*A*a-1/e^3/(e*x+d)*A*c*d^2+1/e^2/(e*x
+d)*B*d*a+1/e^4/(e*x+d)*B*c*d^3-1/e^3/(e*x+d)*C*a*d^2-1/e^5/(e*x+d)*C*c*d^4
-2/e^3*ln(e*x+d)*A*c*d+1/e^2*ln(e*x+d)*B*a+3/e^4*ln(e*x+d)*B*c*d^2-2/e^3*ln
(e*x+d)*C*a*d-4/e^5*ln(e*x+d)*C*c*d^3
```

**maxima [A]** time = 0.45, size = 169, normalized size = 1.10

$$-\frac{Ccd^{4}-Bcd^{3}e-Bade^{3}+Aae^{4}+(Ca+Ac)d^{2}e^{2}}{e^{6}x+de^{5}}+\frac{2\,Cce^{2}x^{3}-3\left(2\,Ccde-Bce^{2}\right)x^{2}+6\left(3\,Ccd^{2}-2\,Bcde+(Ca+A\right.}{6\,e^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c*d^4 - B*c*d^3*e - B*a*d*e^3 + A*a*e^4 + (C*a + A*c)*d^2*e^2)/(e^6*x +
 d*e^5) + 1/6*(2*C*c*e^2*x^3 - 3*(2*C*c*d*e - B*c*e^2)*x^2 + 6*(3*C*c*d^2 -
 2*B*c*d*e + (C*a + A*c)*e^2)*x)/e^4 - (4*C*c*d^3 - 3*B*c*d^2*e - B*a*e^3 +
 2*(C*a + A*c)*d*e^2)*log(e*x + d)/e^5
```

**mupad [B]** time = 0.09, size = 192, normalized size = 1.25

$$x^{2}\left(\frac{B\,c}{2\,e^{2}}-\frac{C\,c\,d}{e^{3}}\right)-x\left(\frac{2\,d\,\left(\frac{B\,c}{e^{2}}-\frac{2\,C\,c\,d}{e^{3}}\right)}{e}-\frac{A\,c+C\,a}{e^{2}}+\frac{C\,c\,d^{2}}{e^{4}}\right)-\frac{\ln(d+e\,x)\,\left(4\,C\,c\,d^{3}-B\,a\,e^{3}+2\,A\,c\,d\,e^{2}+2\,C\,a\,d\right.}{e^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^2*((B*c)/(2*e^2) - (C*c*d)/e^3) - x*((2*d*((B*c)/e^2 - (2*C*c*d)/e^3))/e
- (A*c + C*a)/e^2 + (C*c*d^2)/e^4) - (log(d + e*x)*(4*C*c*d^3 - B*a*e^3 + 2
*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e))/e^5 - (A*a*e^4 + C*c*d^4 - B*a*d*e
^3 - B*c*d^3*e + A*c*d^2*e^2 + C*a*d^2*e^2)/(e*(d*e^4 + e^5*x)) + (C*c*x^3)
/(3*e^2)
```

**sympy [A]** time = 1.26, size = 185, normalized size = 1.21

$$\frac{Ccx^3}{3e^2}+x^2\left(\frac{Bc}{2e^2}-\frac{Ccd}{e^3}\right)+x\left(\frac{Ac}{e^2}-\frac{2Bcd}{e^3}+\frac{Ca}{e^2}+\frac{3Ccd^2}{e^4}\right)+\frac{-Aae^4-Acd^2e^2+Bade^3+Bcd^3e-Cad^2e^2-Ccd^4}{de^5+e^6x}-\text{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c*x**3/(3*e**2) + x**2*(B*c/(2*e**2) - C*c*d/e**3) + x*(A*c/e**2 - 2*B*c*
d/e**3 + C*a/e**2 + 3*C*c*d**2/e**4) + (-A*a*e**4 - A*c*d**2*e**2 + B*a*d*e
**3 + B*c*d**3*e - C*a*d**2*e**2 - C*c*d**4)/(d*e**5 + e**6*x) - (2*A*c*d*e
**2 - B*a*e**3 - 3*B*c*d**2*e + 2*C*a*d*e**2 + 4*C*c*d**3)*log(d + e*x)/e**
5
```

## 3.24 $\int \frac{(a+cx^2)(A+Bx+Cx^2)}{(d+ex)^3} dx$

**Optimal**. Leaf size=156

$$\frac{ae^2(2Cd-Be)+cd\left(4Cd^2-e(3Bd-2Ae)\right)}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2+c\left(6Cd^2-e(3B\right.\right.}{e^5}$$

```
[Out] -c*(-B*e+3*C*d)*x/e^4+1/2*c*C*x^2/e^3-1/2*(a*e^2+c*d^2)*(A*e^2-B*d*e+C*d^2)
/e^5/(e*x+d)^2+(a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))/e^5/(e*x
+d)+(a*C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*ln(e*x+d)/e^5
```

**Rubi [A]** time = 0.20, antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1628}

$$\frac{ae^2(2Cd-Be)-cde(3Bd-2Ae)+4cCd^3}{e^5(d+ex)}-\frac{\left(ae^2+cd^2\right)\left(Ae^2-Bde+Cd^2\right)}{2e^5(d+ex)^2}+\frac{\log(d+ex)\left(aCe^2-ce(3Bd-Ae)+\right.}{e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*C*d - B*e)*x)/e^4) + (c*C*x^2)/(2*e^3) - ((c*d^2 + a*e^2)*(C*d^2 -
B*d*e + A*e^2))/(2*e^5*(d + e*x)^2) + (4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) +
a*e^2*(2*C*d - B*e))/(e^5*(d + e*x)) + ((6*c*C*d^2 + a*C*e^2 - c*e*(3*B*d -
 A*e))*Log[d + e*x])/e^5
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)\left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx=\int \left(\frac{c(-3Cd+Be)}{e^4}+\frac{cCx}{e^3}+\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)}{e^4(d+ex)^3}+\frac{-4cCd^3+cde(3\right.$$

$$=-\frac{c(3Cd-Be)x}{e^4}+\frac{cCx^2}{2e^3}-\frac{\left(cd^2+ae^2\right)\left(Cd^2-Bde+Ae^2\right)}{2e^5(d+ex)^2}+\frac{4cCd^3-cde(3Bd}{e^5}$$

**Mathematica [A]** time = 0.10, size = 176, normalized size = 1.13

$$\frac{\log(d+ex)\left(aCe^2+Ace^2-3Bcde+6cCd^2\right)}{e^5}+\frac{-aBe^3+2aCde^2+2Acde^2-3Bcd^2e+4cCd^3}{e^5(d+ex)}+\frac{-aAe^4+aBde^3-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (c*(-3*C*d + B*e)*x)/e^4 + (c*C*x^2)/(2*e^3) + (-(c*C*d^4) + B*c*d^3*e - A*
c*d^2*e^2 - a*C*d^2*e^2 + a*B*d*e^3 - a*A*e^4)/(2*e^5*(d + e*x)^2) + (4*c*C
*d^3 - 3*B*c*d^2*e + 2*A*c*d*e^2 + 2*a*C*d*e^2 - a*B*e^3)/(e^5*(d + e*x)) +
 ((6*c*C*d^2 - 3*B*c*d*e + A*c*e^2 + a*C*e^2)*Log[d + e*x])/e^5
```

**fricas [A]** time = 0.79, size = 273, normalized size = 1.75

$$\frac{Cce^4x^4+7\,Ccd^4-5\,Bcd^3e-Bade^3-Aae^4+3\,(Ca+Ac)d^2e^2-2\left(2\,Ccde^3-Bce^4\right)x^3-\left(11\,Ccd^2e^2-4\,Bcde^3\right)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(C*c*e^4*x^4 + 7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a +
 A*c)*d^2*e^2 - 2*(2*C*c*d*e^3 - B*c*e^4)*x^3 - (11*C*c*d^2*e^2 - 4*B*c*d*e
^3)*x^2 + 2*(C*c*d^3*e - 2*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x +
 2*(6*C*c*d^4 - 3*B*c*d^3*e + (C*a + A*c)*d^2*e^2 + (6*C*c*d^2*e^2 - 3*B*c*
d*e^3 + (C*a + A*c)*e^4)*x^2 + 2*(6*C*c*d^3*e - 3*B*c*d^2*e^2 + (C*a + A*c)
*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

**giac [A]** time = 0.15, size = 167, normalized size = 1.07

$$\left(6\,Ccd^2-3\,Bcde+Cae^2+Ace^2\right)e^{(-5)}\log\left(|xe+d|\right)+\frac{1}{2}\left(Ccx^2e^3-6\,Ccdxe^2+2\,Bcxe^3\right)e^{(-6)}+\frac{\left(7\,Ccd^4-5\,Bcd^3e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (6*C*c*d^2 - 3*B*c*d*e + C*a*e^2 + A*c*e^2)*e^(-5)*log(abs(x*e + d)) + 1/2*
(C*c*x^2*e^3 - 6*C*c*d*x*e^2 + 2*B*c*x*e^3)*e^(-6) + 1/2*(7*C*c*d^4 - 5*B*c
*d^3*e + 3*C*a*d^2*e^2 + 3*A*c*d^2*e^2 - B*a*d*e^3 - A*a*e^4 + 2*(4*C*c*d^3
*e - 3*B*c*d^2*e^2 + 2*C*a*d*e^3 + 2*A*c*d*e^3 - B*a*e^4)*x)*e^(-5)/(x*e +
d)^2
```

**maple [A]** time = 0.01, size = 257, normalized size = 1.65

$$-\frac{Aa}{2\left(ex+d\right)^{2}e}-\frac{Ac\,d^{2}}{2\left(ex+d\right)^{2}e^{3}}+\frac{Bad}{2\left(ex+d\right)^{2}e^{2}}+\frac{Bc\,d^{3}}{2\left(ex+d\right)^{2}e^{4}}-\frac{Ca\,d^{2}}{2\left(ex+d\right)^{2}e^{3}}-\frac{Cc\,d^{4}}{2\left(ex+d\right)^{2}e^{5}}+\frac{Cc\,x^{2}}{2e^{3}}+\frac{2Acd}{\left(ex+d\right)e^{3}}+\frac{A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] 1/2*c*C*x^2/e^3+c/e^3*B*x-3*c/e^4*C*d*x+2/e^3/(e*x+d)*A*c*d-1/e^2/(e*x+d)*B
*a-3/e^4/(e*x+d)*B*c*d^2+2/e^3/(e*x+d)*C*a*d+4/e^5/(e*x+d)*C*c*d^3+1/e^3*ln
(e*x+d)*A*c-3/e^4*ln(e*x+d)*B*c*d+1/e^3*ln(e*x+d)*a*C+6/e^5*ln(e*x+d)*C*c*d
^2-1/2/e/(e*x+d)^2*A*a-1/2/e^3/(e*x+d)^2*A*d^2*c+1/2/e^2/(e*x+d)^2*B*d*a+1/
2/e^4/(e*x+d)^2*B*c*d^3-1/2/e^3/(e*x+d)^2*C*d^2*a-1/2/e^5/(e*x+d)^2*C*c*d^4
```

**maxima [A]** time = 0.47, size = 177, normalized size = 1.13

$$\frac{7\,Ccd^{4}-5\,Bcd^{3}e-Bade^{3}-Aae^{4}+3\,(Ca+Ac)d^{2}e^{2}+2\left(4\,Ccd^{3}e-3\,Bcd^{2}e^{2}-Bae^{4}+2\,(Ca+Ac)de^{3}\right)x}{2\left(e^{7}x^{2}+2\,de^{6}x+d^{2}e^{5}\right)}+\frac{Ccex^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*C*c*d^4 - 5*B*c*d^3*e - B*a*d*e^3 - A*a*e^4 + 3*(C*a + A*c)*d^2*e^2
+ 2*(4*C*c*d^3*e - 3*B*c*d^2*e^2 - B*a*e^4 + 2*(C*a + A*c)*d*e^3)*x)/(e^7*x
^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(C*c*e*x^2 - 2*(3*C*c*d - B*c*e)*x)/e^4 + (
6*C*c*d^2 - 3*B*c*d*e + (C*a + A*c)*e^2)*log(e*x + d)/e^5
```

**mupad [B]** time = 0.09, size = 185, normalized size = 1.19

$$\frac{x\,\left(4\,C\,c\,d^{3}-B\,a\,e^{3}+2\,A\,c\,d\,e^{2}+2\,C\,a\,d\,e^{2}-3\,B\,c\,d^{2}\,e\right)-\frac{A\,a\,e^{4}-7\,C\,c\,d^{4}+B\,a\,d\,e^{3}+5\,B\,c\,d^{3}\,e-3\,A\,c\,d^{2}\,e^{2}-3\,C\,a\,d^{2}\,e^{2}}{2\,e}}{d^{2}\,e^{4}+2\,d\,e^{5}\,x+e^{6}\,x^{2}}+x\,\left(\frac{B\,c}{e^{3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] (x*(4*C*c*d^3 - B*a*e^3 + 2*A*c*d*e^2 + 2*C*a*d*e^2 - 3*B*c*d^2*e) - (A*a*e
^4 - 7*C*c*d^4 + B*a*d*e^3 + 5*B*c*d^3*e - 3*A*c*d^2*e^2 - 3*C*a*d^2*e^2)/(
2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) + x*((B*c)/e^3 - (3*C*c*d)/e^4) + (lo
g(d + e*x)*(A*c*e^2 + C*a*e^2 + 6*C*c*d^2 - 3*B*c*d*e))/e^5 + (C*c*x^2)/(2*
e^3)
```

**sympy [A]** time = 5.29, size = 206, normalized size = 1.32

$$\frac{Ccx^2}{2e^3}+x\left(\frac{Bc}{e^3}-\frac{3Ccd}{e^4}\right)+\frac{-Aae^4+3Acd^2e^2-Bade^3-5Bcd^3e+3Cad^2e^2+7Ccd^4+x\left(4Acde^3-2Bae^4-6Bcd^2e\right.}{2d^2e^5+4de^6x+2e^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c*x**2/(2*e**3) + x*(B*c/e**3 - 3*C*c*d/e**4) + (-A*a*e**4 + 3*A*c*d**2*e
**2 - B*a*d*e**3 - 5*B*c*d**3*e + 3*C*a*d**2*e**2 + 7*C*c*d**4 + x*(4*A*c*d
*e**3 - 2*B*a*e**4 - 6*B*c*d**2*e**2 + 4*C*a*d*e**3 + 8*C*c*d**3*e))/(2*d**
2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (A*c*e**2 - 3*B*c*d*e + C*a*e**2 + 6*C
*c*d**2)*log(d + e*x)/e**5
```

## 3.25 $\int (d+ex)^3 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=304

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+cd\left(3\right.\right.$$

```
[Out] a^2*A*d^3*x+1/3*a*d*(a*d*(3*B*e+C*d)+A*(3*a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*(A*c*d*(6*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)+2*
c*d^2*(3*B*e+C*d)))*x^5+1/6*a*e*(a*C*e^2+2*c*(3*C*d^2+e*(A*e+3*B*d)))*x^6+1
/7*c*(2*a*e^2*(B*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^7+1/8*c*e*(2*a*C*e^2
+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*e^2*(B*e+3*C*d)*x^9+1/10*c^2*C*e^3*
x^10+1/6*d^2*(3*A*e+B*d)*(c*x^2+a)^3/c
```

**Rubi [A]** time = 0.53, antiderivative size = 301, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{4}a^2ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^2Ad^3x+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+3cCd^2\right)+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a*d*(a*d*(C*d + 3*B*e) + A*(2*c*d^2 + 3*a*e^2))*x^3)/3 + (a^
2*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e
^2*(3*C*d + B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*e*(6*c*C*d^2 + a*C*e
^2 + 2*c*e*(3*B*d + A*e))*x^6)/6 + (c*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 2*a*
e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(3*B*d + A*e)
)*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*x^10)/10 + (d^2*(B*d
+ 3*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-\left(Bd^3+3Ad^2e\right)x+(d$$

$$= \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad^3+ad\left(ad(Cd+3Be)+A\left(2c\right.\right.\right.$$

$$= a^2Ad^3x+\frac{1}{3}ad\left(ad(Cd+3Be)+A\left(2cd^2+3ae^2\right)\right)x^3+\frac{1}{4}a^2e\left(3Cd^2\right.$$

**Mathematica [A]** time = 0.13, size = 335, normalized size = 1.10

$$\frac{1}{2}a^2d^2x^2(3Ae+Bd)+a^2Ad^3x+\frac{1}{7}cx^7\left(2ae^2(Be+3Cd)+3cde(Ae+Bd)+cCd^3\right)+\frac{1}{8}cex^8\left(2aCe^2+ce(Ae+3Bd)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^3*x + (a^2*d^2*(B*d + 3*A*e)*x^2)/2 + (a*d*(a*d*(C*d + 3*B*e) + A*(
2*c*d^2 + 3*a*e^2))*x^3)/3 + (a*(2*B*c*d^3 + 6*A*c*d^2*e + 3*a*C*d^2*e + 3*
a*B*d*e^2 + a*A*e^3)*x^4)/4 + ((A*c*d*(c*d^2 + 6*a*e^2) + a*(a*e^2*(3*C*d +
 B*e) + 2*c*d^2*(C*d + 3*B*e)))*x^5)/5 + ((a*C*e*(6*c*d^2 + a*e^2) + A*c*e*
(3*c*d^2 + 2*a*e^2) + B*c*d*(c*d^2 + 6*a*e^2))*x^6)/6 + (c*(c*C*d^3 + 3*c*d
*e*(B*d + A*e) + 2*a*e^2*(3*C*d + B*e))*x^7)/7 + (c*e*(3*c*C*d^2 + 2*a*C*e^
2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*e^2*(3*C*d + B*e)*x^9)/9 + (c^2*C*e^3*
x^10)/10
```

**fricas [A]** time = 0.76, size = 432, normalized size = 1.42

$$\frac{1}{10}x^{10}e^3c^2C+\frac{1}{3}x^9e^2dc^2C+\frac{1}{9}x^9e^3c^2B+\frac{3}{8}x^8ed^2c^2C+\frac{1}{4}x^8e^3caC+\frac{3}{8}x^8e^2dc^2B+\frac{1}{8}x^8e^3c^2A+\frac{1}{7}x^7d^3c^2C+\frac{6}{7}x^7e^2dcaC+\frac{3}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^3*c^2*C + 1/3*x^9*e^2*d*c^2*C + 1/9*x^9*e^3*c^2*B + 3/8*x^8*e*d
^2*c^2*C + 1/4*x^8*e^3*c*a*C + 3/8*x^8*e^2*d*c^2*B + 1/8*x^8*e^3*c^2*A + 1/
7*x^7*d^3*c^2*C + 6/7*x^7*e^2*d*c*a*C + 3/7*x^7*e*d^2*c^2*B + 2/7*x^7*e^3*c
```

```
*a*B + 3/7*x^7*e^2*d*c^2*A + x^6*e*d^2*c*a*C + 1/6*x^6*e^3*a^2*C + 1/6*x^6*
d^3*c^2*B + x^6*e^2*d*c*a*B + 1/2*x^6*e*d^2*c^2*A + 1/3*x^6*e^3*c*a*A + 2/5
*x^5*d^3*c*a*C + 3/5*x^5*e^2*d*a^2*C + 6/5*x^5*e*d^2*c*a*B + 1/5*x^5*e^3*a^
2*B + 1/5*x^5*d^3*c^2*A + 6/5*x^5*e^2*d*c*a*A + 3/4*x^4*e*d^2*a^2*C + 1/2*x
^4*d^3*c*a*B + 3/4*x^4*e^2*d*a^2*B + 3/2*x^4*e*d^2*c*a*A + 1/4*x^4*e^3*a^2*
A + 1/3*x^3*d^3*a^2*C + x^3*e*d^2*a^2*B + 2/3*x^3*d^3*c*a*A + x^3*e^2*d*a^2
*A + 1/2*x^2*d^3*a^2*B + 3/2*x^2*e*d^2*a^2*A + x*d^3*a^2*A
```

**giac [A]** time = 0.16, size = 423, normalized size = 1.39

$$\frac{1}{10}\,Cc^2x^{10}e^3+\frac{1}{3}\,Cc^2dx^9e^2+\frac{3}{8}\,Cc^2d^2x^8e+\frac{1}{7}\,Cc^2d^3x^7+\frac{1}{9}\,Bc^2x^9e^3+\frac{3}{8}\,Bc^2dx^8e^2+\frac{3}{7}\,Bc^2d^2x^7e+\frac{1}{6}\,Bc^2d^3x^6+\frac{1}{4}\,Cacx^8e^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10*e^3 + 1/3*C*c^2*d*x^9*e^2 + 3/8*C*c^2*d^2*x^8*e + 1/7*C*c^2
*d^3*x^7 + 1/9*B*c^2*x^9*e^3 + 3/8*B*c^2*d*x^8*e^2 + 3/7*B*c^2*d^2*x^7*e +
1/6*B*c^2*d^3*x^6 + 1/4*C*a*c*x^8*e^3 + 1/8*A*c^2*x^8*e^3 + 6/7*C*a*c*d*x^7
*e^2 + 3/7*A*c^2*d*x^7*e^2 + C*a*c*d^2*x^6*e + 1/2*A*c^2*d^2*x^6*e + 2/5*C*
a*c*d^3*x^5 + 1/5*A*c^2*d^3*x^5 + 2/7*B*a*c*x^7*e^3 + B*a*c*d*x^6*e^2 + 6/5
*B*a*c*d^2*x^5*e + 1/2*B*a*c*d^3*x^4 + 1/6*C*a^2*x^6*e^3 + 1/3*A*a*c*x^6*e^
3 + 3/5*C*a^2*d*x^5*e^2 + 6/5*A*a*c*d*x^5*e^2 + 3/4*C*a^2*d^2*x^4*e + 3/2*A
*a*c*d^2*x^4*e + 1/3*C*a^2*d^3*x^3 + 2/3*A*a*c*d^3*x^3 + 1/5*B*a^2*x^5*e^3
+ 3/4*B*a^2*d*x^4*e^2 + B*a^2*d^2*x^3*e + 1/2*B*a^2*d^3*x^2 + 1/4*A*a^2*x^4
*e^3 + A*a^2*d*x^3*e^2 + 3/2*A*a^2*d^2*x^2*e + A*a^2*d^3*x
```

**maple [A]** time = 0.00, size = 385, normalized size = 1.27

$$\frac{C\,c^2e^3x^{10}}{10}+\frac{\left(e^3c^2B+3d\,e^2c^2C\right)x^9}{9}+\frac{\left(A\,c^2e^3+3B\,c^2d\,e^2+\left(2e^3ac+3d^2e\,c^2\right)C\right)x^8}{8}+A\,a^2d^3x+\frac{\left(3A\,c^2d\,e^2+\left(2e^3ac+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^2*C*e^3*x^10+1/9*(B*c^2*e^3+3*C*c^2*d*e^2)*x^9+1/8*((2*a*c*e^3+3*c^2
*d^2*e)*C+3*d*e^2*c^2*B+e^3*c^2*A)*x^8+1/7*((6*a*c*d*e^2+c^2*d^3)*C+(2*a*c*
e^3+3*c^2*d^2*e)*B+3*d*e^2*c^2*A)*x^7+1/6*((a^2*e^3+6*a*c*d^2*e)*C+(6*a*c*d
*e^2+c^2*d^3)*B+(2*a*c*e^3+3*c^2*d^2*e)*A)*x^6+1/5*((3*a^2*d*e^2+2*a*c*d^3)
*C+(a^2*e^3+6*a*c*d^2*e)*B+(6*a*c*d*e^2+c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^2*C+
(3*a^2*d*e^2+2*a*c*d^3)*B+(a^2*e^3+6*a*c*d^2*e)*A)*x^4+1/3*(d^3*a^2*C+3*d^2
*e*a^2*B+(3*a^2*d*e^2+2*a*c*d^3)*A)*x^3+1/2*(3*A*a^2*d^2*e+B*a^2*d^3)*x^2+a
^2*A*d^3*x
```

**maxima [A]** time = 0.45, size = 360, normalized size = 1.18

$$\frac{1}{10}\,Cc^2e^3x^{10}+\frac{1}{9}\left(3\,Cc^2de^2+Bc^2e^3\right)x^9+\frac{1}{8}\left(3\,Cc^2d^2e+3\,Bc^2de^2+\left(2\,Cac+Ac^2\right)e^3\right)x^8+\frac{1}{7}\left(Cc^2d^3+3\,Bc^2d^2e+2\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*e^3*x^10 + 1/9*(3*C*c^2*d*e^2 + B*c^2*e^3)*x^9 + 1/8*(3*C*c^2*d^
2*e + 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^8 + 1/7*(C*c^2*d^3 + 3*B*c^2
*d^2*e + 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x^7 + A*a^2*d^3*x + 1/6*(
B*c^2*d^3 + 6*B*a*c*d*e^2 + 3*(2*C*a*c + A*c^2)*d^2*e + (C*a^2 + 2*A*a*c)*e
^3)*x^6 + 1/5*(6*B*a*c*d^2*e + B*a^2*e^3 + (2*C*a*c + A*c^2)*d^3 + 3*(C*a^2
 + 2*A*a*c)*d*e^2)*x^5 + 1/4*(2*B*a*c*d^3 + 3*B*a^2*d*e^2 + A*a^2*e^3 + 3*(
C*a^2 + 2*A*a*c)*d^2*e)*x^4 + 1/3*(3*B*a^2*d^2*e + 3*A*a^2*d*e^2 + (C*a^2 +
 2*A*a*c)*d^3)*x^3 + 1/2*(B*a^2*d^3 + 3*A*a^2*d^2*e)*x^2
```

**mupad [B]** time = 0.14, size = 332, normalized size = 1.09

$$x^5\left(\frac{3\,C\,a^2\,d\,e^2}{5}+\frac{B\,a^2\,e^3}{5}+\frac{2\,C\,a\,c\,d^3}{5}+\frac{6\,B\,a\,c\,d^2\,e}{5}+\frac{6\,A\,a\,c\,d\,e^2}{5}+\frac{A\,c^2\,d^3}{5}\right)+x^6\left(\frac{C\,a^2\,e^3}{6}+C\,a\,c\,d^2\,e+B\,a\,c\,d\,e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((A*c^2*d^3)/5 + (B*a^2*e^3)/5 + (2*C*a*c*d^3)/5 + (3*C*a^2*d*e^2)/5 +
(6*A*a*c*d*e^2)/5 + (6*B*a*c*d^2*e)/5) + x^6*((B*c^2*d^3)/6 + (C*a^2*e^3)/6
 + (A*a*c*e^3)/3 + (A*c^2*d^2*e)/2 + B*a*c*d*e^2 + C*a*c*d^2*e) + (a*x^4*(A
*a*e^3 + 2*B*c*d^3 + 3*B*a*d*e^2 + 6*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c*x^7*(
2*B*a*e^3 + C*c*d^3 + 3*A*c*d*e^2 + 6*C*a*d*e^2 + 3*B*c*d^2*e))/7 + (C*c^2*
e^3*x^10)/10 + (a^2*d^2*x^2*(3*A*e + B*d))/2 + (c^2*e^2*x^9*(B*e + 3*C*d))/
9 + (a*d*x^3*(3*A*a*e^2 + 2*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (c*e*x^8*(A
*c*e^2 + 2*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/8 + A*a^2*d^3*x
```

**sympy [A]** time = 0.13, size = 445, normalized size = 1.46

$$Aa^2d^3x+\frac{Cc^2e^3x^{10}}{10}+x^9\left(\frac{Bc^2e^3}{9}+\frac{Cc^2de^2}{3}\right)+x^8\left(\frac{Ac^2e^3}{8}+\frac{3Bc^2de^2}{8}+\frac{Cace^3}{4}+\frac{3Cc^2d^2e}{8}\right)+x^7\left(\frac{3Ac^2de^2}{7}+\frac{2Bace^3}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**3*x + C*c**2*e**3*x**10/10 + x**9*(B*c**2*e**3/9 + C*c**2*d*e**2/
3) + x**8*(A*c**2*e**3/8 + 3*B*c**2*d*e**2/8 + C*a*c*e**3/4 + 3*C*c**2*d**2
```

```
*e/8) + x**7*(3*A*c**2*d*e**2/7 + 2*B*a*c*e**3/7 + 3*B*c**2*d**2*e/7 + 6*C*
a*c*d*e**2/7 + C*c**2*d**3/7) + x**6*(A*a*c*e**3/3 + A*c**2*d**2*e/2 + B*a*
c*d*e**2 + B*c**2*d**3/6 + C*a**2*e**3/6 + C*a*c*d**2*e) + x**5*(6*A*a*c*d*
e**2/5 + A*c**2*d**3/5 + B*a**2*e**3/5 + 6*B*a*c*d**2*e/5 + 3*C*a**2*d*e**2
/5 + 2*C*a*c*d**3/5) + x**4*(A*a**2*e**3/4 + 3*A*a*c*d**2*e/2 + 3*B*a**2*d*
e**2/4 + B*a*c*d**3/2 + 3*C*a**2*d**2*e/4) + x**3*(A*a**2*d*e**2 + 2*A*a*c*
d**3/3 + B*a**2*d**2*e + C*a**2*d**3/3) + x**2*(3*A*a**2*d**2*e/2 + B*a**2*
d**3/2)
```

# 3.26 $\int (d+ex)^2 \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=217

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+c\left(e(Ae+2Bd)+Cd^2\right)\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be\right.\right.$$

```
[Out] a^2*A*d^2*x+1/3*a*(a*d*(2*B*e+C*d)+A*(a*e^2+2*c*d^2))*x^3+1/4*a^2*e*(B*e+2*
C*d)*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+a*(a*C*e^2+2*c*d*(2*B*e+C*d)))*x^5+1/3*a*
c*e*(B*e+2*C*d)*x^6+1/7*c*(2*a*C*e^2+c*(C*d^2+e*(A*e+2*B*d)))*x^7+1/8*c^2*e
*(B*e+2*C*d)*x^8+1/9*c^2*C*e^2*x^9+1/6*d*(2*A*e+B*d)*(c*x^2+a)^3/c
```

**Rubi [A]** time = 0.31, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$a^2Ad^2x+\frac{1}{4}a^2ex^4(Be+2Cd)+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{5}x^5\left(Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a*(a*d*(C*d + 2*B*e) + A*(2*c*d^2 + a*e^2))*x^3)/3 + (a^2*e*
(2*C*d + B*e)*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d +
2*B*e)))*x^5)/5 + (a*c*e*(2*C*d + B*e)*x^6)/3 + (c*(c*C*d^2 + 2*a*C*e^2 + c
*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
+ (d*(B*d + 2*A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx=\frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a+cx^2\right)^2\left(-\left(Bd^2+2Ade\right)x+(d+ex$$
$$=\frac{d(Bd+2Ae)\left(a+cx^2\right)^3}{6c}+\int \left(a^2Ad^2+a\left(ad(Cd+2Be)+A\left(2cd^2+\right.\right.\right.$$
$$=a^2Ad^2x+\frac{1}{3}a\left(ad(Cd+2Be)+A\left(2cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^2e(2Cd+Be$$

**Mathematica [A]** time = 0.09, size = 241, normalized size = 1.11

$$\frac{1}{2}a^2dx^2(2Ae+Bd)+a^2Ad^2x+\frac{1}{7}cx^7\left(2aCe^2+ce(Ae+2Bd)+cCd^2\right)+\frac{1}{6}cx^6\left(2aBe^2+4aCde+2Acde+Bcd^2\right)+\frac{1}{5}x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d^2*x + (a^2*d*(B*d + 2*A*e)*x^2)/2 + (a*(a*d*(C*d + 2*B*e) + A*(2*c*
d^2 + a*e^2))*x^3)/3 + (a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2)*x^4
)/4 + ((A*c*(c*d^2 + 2*a*e^2) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)))*x^5)/5 +
 (c*(B*c*d^2 + 2*A*c*d*e + 4*a*C*d*e + 2*a*B*e^2)*x^6)/6 + (c*(c*C*d^2 + 2*
a*C*e^2 + c*e*(2*B*d + A*e))*x^7)/7 + (c^2*e*(2*C*d + B*e)*x^8)/8 + (c^2*C*
e^2*x^9)/9
```

**fricas [A]** time = 0.44, size = 302, normalized size = 1.39

$$\frac{1}{9}x^9e^2c^2C+\frac{1}{4}x^8edc^2C+\frac{1}{8}x^8e^2c^2B+\frac{1}{7}x^7d^2c^2C+\frac{2}{7}x^7e^2caC+\frac{2}{7}x^7edc^2B+\frac{1}{7}x^7e^2c^2A+\frac{2}{3}x^6edcaC+\frac{1}{6}x^6d^2c^2B+\frac{1}{3}x^6e^2caB$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c^2*C + 1/4*x^8*e*d*c^2*C + 1/8*x^8*e^2*c^2*B + 1/7*x^7*d^2*c^2
*C + 2/7*x^7*e^2*c*a*C + 2/7*x^7*e*d*c^2*B + 1/7*x^7*e^2*c^2*A + 2/3*x^6*e*
d*c*a*C + 1/6*x^6*d^2*c^2*B + 1/3*x^6*e^2*c*a*B + 1/3*x^6*e*d*c^2*A + 2/5*x
^5*d^2*c*a*C + 1/5*x^5*e^2*a^2*C + 4/5*x^5*e*d*c*a*B + 1/5*x^5*d^2*c^2*A +
2/5*x^5*e^2*c*a*A + 1/2*x^4*e*d*a^2*C + 1/2*x^4*d^2*c*a*B + 1/4*x^4*e^2*a^2
*B + x^4*e*d*c*a*A + 1/3*x^3*d^2*a^2*C + 2/3*x^3*e*d*a^2*B + 2/3*x^3*d^2*c*
a*A + 1/3*x^3*e^2*a^2*A + 1/2*x^2*d^2*a^2*B + x^2*e*d*a^2*A + x*d^2*a^2*A
```

**giac [A]** time = 0.17, size = 302, normalized size = 1.39

$$\frac{1}{9}Cc^2x^9e^2+\frac{1}{4}Cc^2dx^8e+\frac{1}{7}Cc^2d^2x^7+\frac{1}{8}Bc^2x^8e^2+\frac{2}{7}Bc^2dx^7e+\frac{1}{6}Bc^2d^2x^6+\frac{2}{7}Cacx^7e^2+\frac{1}{7}Ac^2x^7e^2+\frac{2}{3}Cacdx^6e+\frac{1}{3}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9*e^2 + 1/4*C*c^2*d*x^8*e + 1/7*C*c^2*d^2*x^7 + 1/8*B*c^2*x^8*e
^2 + 2/7*B*c^2*d*x^7*e + 1/6*B*c^2*d^2*x^6 + 2/7*C*a*c*x^7*e^2 + 1/7*A*c^2*
x^7*e^2 + 2/3*C*a*c*d*x^6*e + 1/3*A*c^2*d*x^6*e + 2/5*C*a*c*d^2*x^5 + 1/5*A
*c^2*d^2*x^5 + 1/3*B*a*c*x^6*e^2 + 4/5*B*a*c*d*x^5*e + 1/2*B*a*c*d^2*x^4 +
1/5*C*a^2*x^5*e^2 + 2/5*A*a*c*x^5*e^2 + 1/2*C*a^2*d*x^4*e + A*a*c*d*x^4*e +
 1/3*C*a^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 1/4*B*a^2*x^4*e^2 + 2/3*B*a^2*d*x^
3*e + 1/2*B*a^2*d^2*x^2 + 1/3*A*a^2*x^3*e^2 + A*a^2*d*x^2*e + A*a^2*d^2*x
```

**maple [A]** time = 0.00, size = 268, normalized size = 1.24

$$\frac{C\,c^2e^2x^9}{9}+\frac{\left(e^2c^2B+2de\,c^2C\right)x^8}{8}+\frac{\left(A\,c^2e^2+2B\,c^2de+\left(2e^2ac+c^2d^2\right)C\right)x^7}{7}+A\,a^2d^2x+\frac{\left(2A\,c^2de+4Cacde+\left(2e^2\right.\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^2*C*e^2*x^9+1/8*(B*c^2*e^2+2*C*c^2*d*e)*x^8+1/7*((2*a*c*e^2+c^2*d^2)*
C+2*d*e*c^2*B+e^2*c^2*A)*x^7+1/6*(4*d*e*a*c*C+(2*a*c*e^2+c^2*d^2)*B+2*d*e*c
^2*A)*x^6+1/5*((a^2*e^2+2*a*c*d^2)*C+4*d*e*a*c*B+(2*a*c*e^2+c^2*d^2)*A)*x^5
+1/4*(2*d*e*a^2*C+(a^2*e^2+2*a*c*d^2)*B+4*d*e*a*c*A)*x^4+1/3*(d^2*a^2*C+2*d
*e*a^2*B+(a^2*e^2+2*a*c*d^2)*A)*x^3+1/2*(2*A*a^2*d*e+B*a^2*d^2)*x^2+a^2*A*d
^2*x
```

**maxima [A]** time = 0.44, size = 257, normalized size = 1.18

$$\frac{1}{9}Cc^2e^2x^9+\frac{1}{8}\left(2\,Cc^2de+Bc^2e^2\right)x^8+\frac{1}{7}\left(Cc^2d^2+2\,Bc^2de+\left(2\,Cac+Ac^2\right)e^2\right)x^7+\frac{1}{6}\left(Bc^2d^2+2\,Bace^2+2\left(2\,Cac+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + B*c^2*e^2)*x^8 + 1/7*(C*c^2*d^2 + 2*
B*c^2*d*e + (2*C*a*c + A*c^2)*e^2)*x^7 + 1/6*(B*c^2*d^2 + 2*B*a*c*e^2 + 2*(
2*C*a*c + A*c^2)*d*e)*x^6 + A*a^2*d^2*x + 1/5*(4*B*a*c*d*e + (2*C*a*c + A*c
^2)*d^2 + (C*a^2 + 2*A*a*c)*e^2)*x^5 + 1/4*(2*B*a*c*d^2 + B*a^2*e^2 + 2*(C*
```

```
a^2 + 2*A*a*c)*d*e)*x^4 + 1/3*(2*B*a^2*d*e + A*a^2*e^2 + (C*a^2 + 2*A*a*c)*
d^2)*x^3 + 1/2*(B*a^2*d^2 + 2*A*a^2*d*e)*x^2
```

**mupad [B]** time = 3.72, size = 244, normalized size = 1.12

$$x^3\left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,A\,c\,a\,d^2}{3}\right)+x^7\left(\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right)+x^5\left(\frac{C\,a^2\,e^2}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^2*e^2)/3 + (C*a^2*d^2)/3 + (2*A*a*c*d^2)/3 + (2*B*a^2*d*e)/3) + x
^7*((A*c^2*e^2)/7 + (C*c^2*d^2)/7 + (2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7) + x^
5*((A*c^2*d^2)/5 + (C*a^2*e^2)/5 + (2*A*a*c*e^2)/5 + (2*C*a*c*d^2)/5 + (4*B
*a*c*d*e)/5) + (a*x^4*(B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e + 2*C*a*d*e))/4 + (c
*x^6*(2*B*a*e^2 + B*c*d^2 + 2*A*c*d*e + 4*C*a*d*e))/6 + (C*c^2*e^2*x^9)/9 +
 A*a^2*d^2*x + (a^2*d*x^2*(2*A*e + B*d))/2 + (c^2*e*x^8*(B*e + 2*C*d))/8
```

**sympy [A]** time = 0.12, size = 311, normalized size = 1.43

$$Aa^2d^2x+\frac{Cc^2e^2x^9}{9}+x^8\left(\frac{Bc^2e^2}{8}+\frac{Cc^2de}{4}\right)+x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bc^2de}{7}+\frac{2Cace^2}{7}+\frac{Cc^2d^2}{7}\right)+x^6\left(\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bc^2d^2}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*c**2*d*e/4) +
x**7*(A*c**2*e**2/7 + 2*B*c**2*d*e/7 + 2*C*a*c*e**2/7 + C*c**2*d**2/7) + x*
*6*(A*c**2*d*e/3 + B*a*c*e**2/3 + B*c**2*d**2/6 + 2*C*a*c*d*e/3) + x**5*(2*
A*a*c*e**2/5 + A*c**2*d**2/5 + 4*B*a*c*d*e/5 + C*a**2*e**2/5 + 2*C*a*c*d**2
/5) + x**4*(A*a*c*d*e + B*a**2*e**2/4 + B*a*c*d**2/2 + C*a**2*d*e/2) + x**3
*(A*a**2*e**2/3 + 2*A*a*c*d**2/3 + 2*B*a**2*d*e/3 + C*a**2*d**2/3) + x**2*(
A*a**2*d*e + B*a**2*d**2/2)
```

# 3.27 $\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)\,dx$

**Optimal**. Leaf size=128

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7(B$$

```
[Out] a^2*A*d*x+1/3*a*(2*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^2*C*e*x^4+1/5*c*(A*c*d+2*a*
(B*e+C*d))*x^5+1/3*a*c*C*e*x^6+1/7*c^2*(B*e+C*d)*x^7+1/8*c^2*C*e*x^8+1/6*(A
*e+B*d)*(c*x^2+a)^3/c
```

**Rubi** [A] time = 0.16, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$a^2Adx+\frac{1}{4}a^2Cex^4+\frac{1}{5}cx^5(2a(Be+Cd)+Acd)+\frac{1}{3}ax^3(aBe+aCd+2Acd)+\frac{\left(a+cx^2\right)^3(Ae+Bd)}{6c}+\frac{1}{3}acCex^6+\frac{1}{7}c^2x^7(B$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^2*C*e*x^4)/4 + (c*(A*c
*d + 2*a*(C*d + B*e))*x^5)/5 + (a*c*C*e*x^6)/3 + (c^2*(C*d + B*e)*x^7)/7 +
(c^2*C*e*x^8)/8 + ((B*d + A*e)*(a + c*x^2)^3)/(6*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)dx = \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2\left(-(Bd+Ae)x+(d+ex)\left(A+B\right.\right.$$
$$= \frac{(Bd+Ae)\left(a+cx^2\right)^3}{6c} + \int \left(a^2Ad + a(2Acd+aCd+aBe)x^2 + a^2Cex\right.$$
$$= a^2Adx + \frac{1}{3}a(2Acd+aCd+aBe)x^3 + \frac{1}{4}a^2Cex^4 + \frac{1}{5}c(Acd+2a(Cd+$$

**Mathematica [A]** time = 0.05, size = 144, normalized size = 1.12

$$\frac{1}{2}a^2x^2(Ae+Bd)+a^2Adx+\frac{1}{6}cx^6(2aCe+Ace+Bcd)+\frac{1}{5}cx^5(2aBe+2aCd+Acd)+\frac{1}{4}ax^4(aCe+2Ace+2Bcd)+\frac{1}{3}ax^3(aBe+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*d*x + (a^2*(B*d + A*e)*x^2)/2 + (a*(2*A*c*d + a*C*d + a*B*e)*x^3)/3 +
 (a*(2*B*c*d + 2*A*c*e + a*C*e)*x^4)/4 + (c*(A*c*d + 2*a*C*d + 2*a*B*e)*x^5
)/5 + (c*(B*c*d + A*c*e + 2*a*C*e)*x^6)/6 + (c^2*(C*d + B*e)*x^7)/7 + (c^2*
C*e*x^8)/8
```

**fricas [A]** time = 0.81, size = 172, normalized size = 1.34

$$\frac{1}{8}x^8ec^2C+\frac{1}{7}x^7dc^2C+\frac{1}{7}x^7ec^2B+\frac{1}{3}x^6ecaC+\frac{1}{6}x^6dc^2B+\frac{1}{6}x^6ec^2A+\frac{2}{5}x^5dcaC+\frac{2}{5}x^5ecaB+\frac{1}{5}x^5dc^2A+\frac{1}{4}x^4ea^2C+\frac{1}{2}x^4dca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*e*c^2*C + 1/7*x^7*d*c^2*C + 1/7*x^7*e*c^2*B + 1/3*x^6*e*c*a*C + 1/6
*x^6*d*c^2*B + 1/6*x^6*e*c^2*A + 2/5*x^5*d*c*a*C + 2/5*x^5*e*c*a*B + 1/5*x^
5*d*c^2*A + 1/4*x^4*e*a^2*C + 1/2*x^4*d*c*a*B + 1/2*x^4*e*c*a*A + 1/3*x^3*d
*a^2*C + 1/3*x^3*e*a^2*B + 2/3*x^3*d*c*a*A + 1/2*x^2*d*a^2*B + 1/2*x^2*e*a^
2*A + x*d*a^2*A
```

**giac [A]** time = 0.15, size = 181, normalized size = 1.41

$$\frac{1}{8}Cc^2x^8e+\frac{1}{7}Cc^2dx^7+\frac{1}{7}Bc^2x^7e+\frac{1}{6}Bc^2dx^6+\frac{1}{3}Cacx^6e+\frac{1}{6}Ac^2x^6e+\frac{2}{5}Cacdx^5+\frac{1}{5}Ac^2dx^5+\frac{2}{5}Bacx^5e+\frac{1}{2}Bacdx^4+\frac{1}{4}C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8*e + 1/7*C*c^2*d*x^7 + 1/7*B*c^2*x^7*e + 1/6*B*c^2*d*x^6 + 1/3
*C*a*c*x^6*e + 1/6*A*c^2*x^6*e + 2/5*C*a*c*d*x^5 + 1/5*A*c^2*d*x^5 + 2/5*B*
a*c*x^5*e + 1/2*B*a*c*d*x^4 + 1/4*C*a^2*x^4*e + 1/2*A*a*c*x^4*e + 1/3*C*a^2
*d*x^3 + 2/3*A*a*c*d*x^3 + 1/3*B*a^2*x^3*e + 1/2*B*a^2*d*x^2 + 1/2*A*a^2*x^
2*e + A*a^2*d*x
```

**maple [A]** time = 0.00, size = 151, normalized size = 1.18

$$\frac{C c^2 e x^8}{8}+\frac{\left(c^2 eB+c^2 dC\right)x^7}{7}+\frac{\left(c^2 eA+c^2 dB+2eacC\right)x^6}{6}+A a^2 dx+\frac{\left(c^2 dA+2eacB+2dacC\right)x^5}{5}+\frac{\left(2eacA+2dac\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*c^2*C*e*x^8+1/7*(B*c^2*e+C*c^2*d)*x^7+1/6*(A*c^2*e+B*c^2*d+2*C*a*c*e)*x
^6+1/5*(A*c^2*d+2*B*a*c*e+2*C*a*c*d)*x^5+1/4*(2*A*a*c*e+2*B*a*c*d+C*a^2*e)*
x^4+1/3*(2*A*a*c*d+B*a^2*e+C*a^2*d)*x^3+1/2*(A*a^2*e+B*a^2*d)*x^2+a^2*A*d*x
```

**maxima [A]** time = 0.45, size = 154, normalized size = 1.20

$$\frac{1}{8}Cc^2ex^8+\frac{1}{7}\left(Cc^2d+Bc^2e\right)x^7+\frac{1}{6}\left(Bc^2d+\left(2Cac+Ac^2\right)e\right)x^6+\frac{1}{5}\left(2Bace+\left(2Cac+Ac^2\right)d\right)x^5+Aa^2dx+\frac{1}{4}\left(2Bac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*e*x^8 + 1/7*(C*c^2*d + B*c^2*e)*x^7 + 1/6*(B*c^2*d + (2*C*a*c + A
*c^2)*e)*x^6 + 1/5*(2*B*a*c*e + (2*C*a*c + A*c^2)*d)*x^5 + A*a^2*d*x + 1/4*
(2*B*a*c*d + (C*a^2 + 2*A*a*c)*e)*x^4 + 1/3*(B*a^2*e + (C*a^2 + 2*A*a*c)*d)
*x^3 + 1/2*(B*a^2*d + A*a^2*e)*x^2
```

**mupad [B]** time = 3.69, size = 140, normalized size = 1.09

$$x^3\left(\frac{B a^2 e}{3}+\frac{C a^2 d}{3}+\frac{2 A a c d}{3}\right)+x^6\left(\frac{A c^2 e}{6}+\frac{B c^2 d}{6}+\frac{C a c e}{3}\right)+\frac{c x^5\left(A c d+2 B a e+2 C a d\right)}{5}+\frac{a x^4\left(2 A c e+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^2*e)/3 + (C*a^2*d)/3 + (2*A*a*c*d)/3) + x^6*((A*c^2*e)/6 + (B*c^2
*d)/6 + (C*a*c*e)/3) + (c*x^5*(A*c*d + 2*B*a*e + 2*C*a*d))/5 + (a*x^4*(2*A*
c*e + 2*B*c*d + C*a*e))/4 + (a^2*x^2*(A*e + B*d))/2 + (c^2*x^7*(B*e + C*d))
/7 + A*a^2*d*x + (C*c^2*e*x^8)/8
```

**sympy [A]** time = 0.10, size = 180, normalized size = 1.41

$$Aa^2dx+\frac{Cc^2ex^8}{8}+x^7\left(\frac{Bc^2e}{7}+\frac{Cc^2d}{7}\right)+x^6\left(\frac{Ac^2e}{6}+\frac{Bc^2d}{6}+\frac{Cace}{3}\right)+x^5\left(\frac{Ac^2d}{5}+\frac{2Bace}{5}+\frac{2Cacd}{5}\right)+x^4\left(\frac{Aace}{2}+\frac{Bac}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*d*x + C*c**2*e*x**8/8 + x**7*(B*c**2*e/7 + C*c**2*d/7) + x**6*(A*c**
2*e/6 + B*c**2*d/6 + C*a*c*e/3) + x**5*(A*c**2*d/5 + 2*B*a*c*e/5 + 2*C*a*c*
d/5) + x**4*(A*a*c*e/2 + B*a*c*d/2 + C*a**2*e/4) + x**3*(2*A*a*c*d/3 + B*a*
*2*e/3 + C*a**2*d/3) + x**2*(A*a**2*e/2 + B*a**2*d/2)
```

# 3.28 $\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=67

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+1/3*a*(2*A*c+C*a)*x^3+1/5*c*(A*c+2*C*a)*x^5+1/7*c^2*C*x^7+1/6*B*(c*
x^2+a)^3/c
```

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 373}

$$a^2Ax+\frac{1}{5}cx^5(2aC+Ac)+\frac{1}{3}ax^3(aC+2Ac)+\frac{B\left(a+cx^2\right)^3}{6c}+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] a^2*A*x + (a*(2*A*c + a*C)*x^3)/3 + (c*(A*c + 2*a*C)*x^5)/5 + (c^2*C*x^7)/7
 + (B*(a + c*x^2)^3)/(6*c)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int \left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right) dx = \frac{B\left(a+cx^2\right)^3}{6c} + \int \left(a+cx^2\right)^2 \left(A+Cx^2\right) dx$$
$$= \frac{B\left(a+cx^2\right)^3}{6c} + \int \left(a^2A + a(2Ac+aC)x^2 + c(Ac+2aC)x^4 + c^2Cx^6\right) dx$$
$$= a^2Ax + \frac{1}{3}a(2Ac+aC)x^3 + \frac{1}{5}c(Ac+2aC)x^5 + \frac{1}{7}c^2Cx^7 + \frac{B\left(a+cx^2\right)^3}{6c}$$

**Mathematica [A]** time = 0.03, size = 69, normalized size = 1.03

$$\frac{1}{210}x\left(35a^2(6A+x(3B+2Cx)) + 7acx^2(20A+3x(5B+4Cx)) + c^2x^4(42A+5x(7B+6Cx))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^2*(A + B*x + C*x^2),x]
```

```
[Out] (x*(35*a^2*(6*A + x*(3*B + 2*C*x)) + 7*a*c*x^2*(20*A + 3*x*(5*B + 4*C*x)) +
 c^2*x^4*(42*A + 5*x*(7*B + 6*C*x))))/210
```

**fricas [A]** time = 0.90, size = 76, normalized size = 1.13

$$\frac{1}{7}x^7c^2C + \frac{1}{6}x^6c^2B + \frac{2}{5}x^5caC + \frac{1}{5}x^5c^2A + \frac{1}{2}x^4caB + \frac{1}{3}x^3a^2C + \frac{2}{3}x^3caA + \frac{1}{2}x^2a^2B + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/6*x^6*c^2*B + 2/5*x^5*c*a*C + 1/5*x^5*c^2*A + 1/2*x^4*c*a
*B + 1/3*x^3*a^2*C + 2/3*x^3*c*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

**giac [A]** time = 0.15, size = 76, normalized size = 1.13

$$\frac{1}{7}Cc^2x^7 + \frac{1}{6}Bc^2x^6 + \frac{2}{5}Cacx^5 + \frac{1}{5}Ac^2x^5 + \frac{1}{2}Bacx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aacx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*a*c*x
^4 + 1/3*C*a^2*x^3 + 2/3*A*a*c*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

**maple** [A] time = 0.00, size = 75, normalized size = 1.12

$$\frac{C\,c^{2}x^{7}}{7}+\frac{B\,c^{2}x^{6}}{6}+\frac{Bac\,x^{4}}{2}+\frac{B\,a^{2}x^{2}}{2}+\frac{\left(A\,c^{2}+2acC\right)x^{5}}{5}+A\,a^{2}x+\frac{\left(2acA+a^{2}C\right)x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A),x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*c^2*B*x^6+1/5*(A*c^2+2*C*a*c)*x^5+1/2*a*c*B*x^4+1/3*(2*A*
a*c+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

**maxima** [A] time = 0.44, size = 74, normalized size = 1.10

$$\frac{1}{7}\,Cc^{2}x^{7}+\frac{1}{6}\,Bc^{2}x^{6}+\frac{1}{2}\,Bacx^{4}+\frac{1}{5}\left(2\,Cac+Ac^{2}\right)x^{5}+\frac{1}{2}\,Ba^{2}x^{2}+Aa^{2}x+\frac{1}{3}\left(Ca^{2}+2\,Aac\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*a*c*x^4 + 1/5*(2*C*a*c + A*c^2)*x^5 +
 1/2*B*a^2*x^2 + A*a^2*x + 1/3*(C*a^2 + 2*A*a*c)*x^3
```

**mupad** [B] time = 0.04, size = 74, normalized size = 1.10

$$x^{3}\left(\frac{C\,a^{2}}{3}+\frac{2\,A\,c\,a}{3}\right)+x^{5}\left(\frac{A\,c^{2}}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{B\,a^{2}\,x^{2}}{2}+\frac{B\,c^{2}\,x^{6}}{6}+\frac{C\,c^{2}\,x^{7}}{7}+A\,a^{2}\,x+\frac{B\,a\,c\,x^{4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (2*C*a*c)/5) + (B*a^2*x^2)
/2 + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + A*a^2*x + (B*a*c*x^4)/2
```

**sympy** [A] time = 0.08, size = 83, normalized size = 1.24

$$Aa^{2}x+\frac{Ba^{2}x^{2}}{2}+\frac{Bacx^{4}}{2}+\frac{Bc^{2}x^{6}}{6}+\frac{Cc^{2}x^{7}}{7}+x^{5}\left(\frac{Ac^{2}}{5}+\frac{2Cac}{5}\right)+x^{3}\left(\frac{2Aac}{3}+\frac{Ca^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x
**5*(A*c**2/5 + 2*C*a*c/5) + x**3*(2*A*a*c/3 + C*a**2/3)
```

## 3.29 $\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{d+ex}\,dx$

**Optimal**. Leaf size=297

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2d^2\left(Cd^2-e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)\right.}{e^6}$$

```
[Out] -(a^2*e^4*(-B*e+C*d)+c^2*d^3*(C*d^2-e*(-A*e+B*d))+2*a*c*d*e^2*(C*d^2-e*(-A*
e+B*d)))*x/e^6+1/2*(a^2*C*e^4+c^2*d^2*(C*d^2-e*(-A*e+B*d))+2*a*c*e^2*(C*d^2
-e*(-A*e+B*d)))*x^2/e^5-1/3*c*(2*a*e^2*(-B*e+C*d)+c*d*(C*d^2-e*(-A*e+B*d)))
*x^3/e^4+1/4*c*(2*a*C*e^2+c*(C*d^2-e*(-A*e+B*d)))*x^4/e^3-1/5*c^2*(-B*e+C*d
)*x^5/e^2+1/6*c^2*C*x^6/e+(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^7
```

**Rubi [A]** time = 0.64, antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{x^2\left(a^2Ce^4+2ace^2\left(Cd^2-e(Bd-Ae)\right)+c^2\left(Cd^4-d^2e(Bd-Ae)\right)\right)}{2e^5}-\frac{x\left(a^2e^4(Cd-Be)+2acde^2\left(Cd^2-e(Bd-Ae)\right)\right.}{e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((a^2*e^4*(C*d - B*e) + 2*a*c*d*e^2*(C*d^2 - e*(B*d - A*e)) + c^2*(C*d^5
- d^3*e*(B*d - A*e)))*x)/e^6) + ((a^2*C*e^4 + 2*a*c*e^2*(C*d^2 - e*(B*d - A
*e)) + c^2*(C*d^4 - d^2*e*(B*d - A*e)))*x^2)/(2*e^5) - (c*(c*C*d^3 - c*d*e*
(B*d - A*e) + 2*a*e^2*(C*d - B*e))*x^3)/(3*e^4) + (c*(c*C*d^2 + 2*a*C*e^2 -
 c*e*(B*d - A*e))*x^4)/(4*e^3) - (c^2*(C*d - B*e)*x^5)/(5*e^2) + (c^2*C*x^6
)/(6*e) + ((c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{d+ex}\,dx = \int \left( \frac{-a^2e^4(Cd-Be)-2acde^2(Cd^2-e(Bd-Ae))-c^2(Cd^5-d^3e(Bd-Ae)}{e^6} \right.$$

$$= -\frac{(a^2e^4(Cd-Be)+2acde^2(Cd^2-e(Bd-Ae))+c^2(Cd^5-d^3e(Bd-Ae)))x}{e^6}$$

**Mathematica** [A] time = 0.17, size = 285, normalized size = 0.96

$$\frac{ex(30a^2e^4(2Be-2Cd+Cex)+10ace^2(2e(3Ae(ex-2d)+B(6d^2-3dex+2e^2x^2))+C(-12d^3+6d^2ex-4de^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (e*x*(30*a^2*e^4*(-2*C*d + 2*B*e + C*e*x) + 10*a*c*e^2*(C*(-12*d^3 + 6*d^2*
e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d*e
*x + 2*e^2*x^2))) + c^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*
e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2*e*x - 4*d*
e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x
^3 + 12*e^4*x^4)))) + 60*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e))*Log[d
 + e*x])/(60*e^7)
```

**fricas** [A] time = 0.62, size = 379, normalized size = 1.28

$$\frac{10\,Cc^2e^6x^6-12\left(Cc^2de^5-Bc^2e^6\right)x^5+15\left(Cc^2d^2e^4-Bc^2de^5+\left(2\,Cac+Ac^2\right)e^6\right)x^4-20\left(Cc^2d^3e^3-Bc^2d^2e^4-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*e^6*x^6 - 12*(C*c^2*d*e^5 - B*c^2*e^6)*x^5 + 15*(C*c^2*d^2*e
^4 - B*c^2*d*e^5 + (2*C*a*c + A*c^2)*e^6)*x^4 - 20*(C*c^2*d^3*e^3 - B*c^2*d
^2*e^4 - 2*B*a*c*e^6 + (2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(C*c^2*d^4*e^2 - B
*c^2*d^3*e^3 - 2*B*a*c*d*e^5 + (2*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c
)*e^6)*x^2 - 60*(C*c^2*d^5*e - B*c^2*d^4*e^2 - 2*B*a*c*d^2*e^4 - B*a^2*e^6
+ (2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x + 60*(C*c^2*d^6 -
B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)
*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d))/e^7
```

**giac [A]** time = 0.16, size = 416, normalized size = 1.40

$$\left(Cc^2d^6 - Bc^2d^5e + 2\,Cacd^4e^2 + Ac^2d^4e^2 - 2\,Bacd^3e^3 + Ca^2d^2e^4 + 2\,Aacd^2e^4 - Ba^2de^5 + Aa^2e^6\right)e^{(-7)}\log\left(|xe + d|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^2*d^6 - B*c^2*d^5*e + 2*C*a*c*d^4*e^2 + A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^
3 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - B*a^2*d*e^5 + A*a^2*e^6)*e^(-7)*log(a
bs(x*e + d)) + 1/60*(10*C*c^2*x^6*e^5 - 12*C*c^2*d*x^5*e^4 + 15*C*c^2*d^2*x
^4*e^3 - 20*C*c^2*d^3*x^3*e^2 + 30*C*c^2*d^4*x^2*e - 60*C*c^2*d^5*x + 12*B*
c^2*x^5*e^5 - 15*B*c^2*d*x^4*e^4 + 20*B*c^2*d^2*x^3*e^3 - 30*B*c^2*d^3*x^2*
e^2 + 60*B*c^2*d^4*x*e + 30*C*a*c*x^4*e^5 + 15*A*c^2*x^4*e^5 - 40*C*a*c*d*x
^3*e^4 - 20*A*c^2*d*x^3*e^4 + 60*C*a*c*d^2*x^2*e^3 + 30*A*c^2*d^2*x^2*e^3 -
 120*C*a*c*d^3*x*e^2 - 60*A*c^2*d^3*x*e^2 + 40*B*a*c*x^3*e^5 - 60*B*a*c*d*x
^2*e^4 + 120*B*a*c*d^2*x*e^3 + 30*C*a^2*x^2*e^5 + 60*A*a*c*x^2*e^5 - 60*C*a
^2*d*x*e^4 - 120*A*a*c*d*x*e^4 + 60*B*a^2*x*e^5)*e^(-6)
```

**maple [A]** time = 0.01, size = 490, normalized size = 1.65

$$\frac{C\,c^2x^6}{6e}+\frac{B\,c^2x^5}{5e}-\frac{C\,c^2d\,x^5}{5e^2}+\frac{A\,c^2x^4}{4e}-\frac{B\,c^2d\,x^4}{4e^2}+\frac{Cac\,x^4}{2e}+\frac{C\,c^2d^2x^4}{4e^3}-\frac{A\,c^2d\,x^3}{3e^2}+\frac{2Bac\,x^3}{3e}+\frac{B\,c^2d^2x^3}{3e^3}-\frac{2Cacd\,x^3}{3e^2}-\frac{C\,c^2d}{3e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x)
```

```
[Out] 1/3/e^3*B*x^3*c^2*d^2-1/3/e^4*C*x^3*c^2*d^3-1/5/e^2*C*x^5*c^2*d-1/4/e^2*B*x
^4*c^2*d-1/e^2*ln(e*x+d)*B*a^2*d+1/e^5*B*x*c^2*d^4-1/e^2*C*x*a^2*d-1/e^4*A*
x*c^2*d^3+1/e*A*x^2*a*c-1/e^6*C*x*c^2*d^5+1/2/e^3*A*x^2*c^2*d^2-1/2/e^4*B*x
^2*c^2*d^3+1/2/e^5*C*x^2*c^2*d^4+1/e^7*ln(e*x+d)*C*c^2*d^6+1/e^5*ln(e*x+d)*
A*c^2*d^4+1/6*c^2*C*x^6/e-2/e^4*ln(e*x+d)*B*a*c*d^3+2/e^5*ln(e*x+d)*C*a*c*d
^4+1/e^3*C*x^2*a*c*d^2-1/e^2*B*x^2*a*c*d-2/3/e^2*C*x^3*a*c*d-2/e^4*C*x*a*c*
d^3-2/e^2*A*x*a*c*d+2/e^3*B*x*a*c*d^2+2/e^3*ln(e*x+d)*A*a*c*d^2+1/2/e*C*x^2
*a^2+1/5/e*B*x^5*c^2+1/4/e*A*x^4*c^2+1/e*B*x*a^2+1/e*ln(e*x+d)*A*a^2+2/3/e*
B*x^3*a*c-1/3/e^2*A*x^3*c^2*d+1/2/e*C*x^4*a*c+1/4/e^3*C*x^4*c^2*d^2+1/e^3*l
n(e*x+d)*C*a^2*d^2-1/e^6*ln(e*x+d)*B*c^2*d^5
```

**maxima [A]** time = 0.48, size = 377, normalized size = 1.27

$$\frac{10\,Cc^2e^5x^6 - 12\left(Cc^2de^4 - Bc^2e^5\right)x^5 + 15\left(Cc^2d^2e^3 - Bc^2de^4 + \left(2\,Cac + Ac^2\right)e^5\right)x^4 - 20\left(Cc^2d^3e^2 - Bc^2d^2e^3 - 2\,B\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/60*(10*C*c^2*e^5*x^6 - 12*(C*c^2*d*e^4 - B*c^2*e^5)*x^5 + 15*(C*c^2*d^2*e
^3 - B*c^2*d*e^4 + (2*C*a*c + A*c^2)*e^5)*x^4 - 20*(C*c^2*d^3*e^2 - B*c^2*d
^2*e^3 - 2*B*a*c*e^5 + (2*C*a*c + A*c^2)*d*e^4)*x^3 + 30*(C*c^2*d^4*e - B*c
^2*d^3*e^2 - 2*B*a*c*d*e^4 + (2*C*a*c + A*c^2)*d^2*e^3 + (C*a^2 + 2*A*a*c)*
e^5)*x^2 - 60*(C*c^2*d^5 - B*c^2*d^4*e - 2*B*a*c*d^2*e^3 - B*a^2*e^5 + (2*C
*a*c + A*c^2)*d^3*e^2 + (C*a^2 + 2*A*a*c)*d*e^4)*x)/e^6 + (C*c^2*d^6 - B*c^
2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*C*a*c + A*c^2)*d^4
*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)*log(e*x + d)/e^7
```

**mupad [B]** time = 3.68, size = 422, normalized size = 1.42

$$x^5\left(\frac{B\,c^2}{5\,e}-\frac{C\,c^2\,d}{5\,e^2}\right)-x\left(\frac{d\left(\frac{C\,a^2+2\,A\,c\,a}{e}+\frac{d\left(\frac{d\left(\frac{A\,c^2+2\,C\,a\,c}{e}-\frac{d\left(\frac{B\,c^2}{e}-\frac{C\,c^2\,d}{e^2}\right)}{e}\right)}{e}-\frac{2\,B\,a\,c}{e}\right)}{e}\right)}{e}-\frac{B\,a^2}{e}\right)+x^4\left(\frac{A\,c^2+2\,C\,a\,c}{4\,e}-\frac{d\left(\frac{B\,c^2}{e}-\right.}{4\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x),x)
```

```
[Out] x^5*((B*c^2)/(5*e) - (C*c^2*d)/(5*e^2)) - x*((d*((C*a^2 + 2*A*a*c)/e + (d*(
(d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*c)
/e))/e))/e - (B*a^2)/e) + x^4*((A*c^2 + 2*C*a*c)/(4*e) - (d*((B*c^2)/e - (C
*c^2*d)/e^2))/(4*e)) - x^3*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^
2*d)/e^2))/e))/(3*e) - (2*B*a*c)/(3*e)) + x^2*((C*a^2 + 2*A*a*c)/(2*e) + (d
*((d*((A*c^2 + 2*C*a*c)/e - (d*((B*c^2)/e - (C*c^2*d)/e^2))/e))/e - (2*B*a*
```

```
c)/e))/(2*e)) + (log(d + e*x)*(A*a^2*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*
d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 +
 2*C*a*c*d^4*e^2))/e^7 + (C*c^2*x^6)/(6*e)
```

**sympy [A]** time = 0.94, size = 359, normalized size = 1.21

$$\frac{Cc^2x^6}{6e}+x^5\left(\frac{Bc^2}{5e}-\frac{Cc^2d}{5e^2}\right)+x^4\left(\frac{Ac^2}{4e}-\frac{Bc^2d}{4e^2}+\frac{Cac}{2e}+\frac{Cc^2d^2}{4e^3}\right)+x^3\left(-\frac{Ac^2d}{3e^2}+\frac{2Bac}{3e}+\frac{Bc^2d^2}{3e^3}-\frac{2Cacd}{3e^2}-\frac{Cc^2d^3}{3e^4}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d),x)
```

```
[Out] C*c**2*x**6/(6*e) + x**5*(B*c**2/(5*e) - C*c**2*d/(5*e**2)) + x**4*(A*c**2/
(4*e) - B*c**2*d/(4*e**2) + C*a*c/(2*e) + C*c**2*d**2/(4*e**3)) + x**3*(-A*
c**2*d/(3*e**2) + 2*B*a*c/(3*e) + B*c**2*d**2/(3*e**3) - 2*C*a*c*d/(3*e**2)
 - C*c**2*d**3/(3*e**4)) + x**2*(A*a*c/e + A*c**2*d**2/(2*e**3) - B*a*c*d/e
**2 - B*c**2*d**3/(2*e**4) + C*a**2/(2*e) + C*a*c*d**2/e**3 + C*c**2*d**4/(
2*e**5)) + x*(-2*A*a*c*d/e**2 - A*c**2*d**3/e**4 + B*a**2/e + 2*B*a*c*d**2/
e**3 + B*c**2*d**4/e**5 - C*a**2*d/e**2 - 2*C*a*c*d**3/e**4 - C*c**2*d**5/e
**6) + (a*e**2 + c*d**2)**2*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**7
```

## 3.30 $\int \frac{(a+cx^2)^2 (A+Bx+Cx^2)}{(d+ex)^2} \, dx$

**Optimal**. Leaf size=292

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2d^2\left(5Cd^2-e(4Bd-3Ae)\right)\right)}{e^6}-\frac{\left(ae^2+cd^2\right)^2\left(Ae^2-Bde+Cd^2\right)}{e^7(d+ex)}-\frac{\left(ae^2\right.}{}$$

```
[Out] (a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+2*a*c*e^2*(3*C*d^2-e*(-A*e+2*
B*d)))*x/e^6-1/2*c*(2*a*e^2*(-B*e+2*C*d)+c*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^
2/e^5+1/3*c*(2*a*C*e^2+c*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^4-1/4*c^2*(-B*e+2*
C*d)*x^4/e^3+1/5*c^2*C*x^5/e^2-(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x
+d)-(a*e^2+c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))*ln(e*
x+d)/e^7
```

**Rubi** [A] time = 0.53, antiderivative size = 289, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{x\left(a^2Ce^4+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{e^6}+\frac{cx^3\left(2aCe^2-ce(2Bd-Ae)+3cCd^2\right)}{3e^4}-$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^2*C*e^4 + c^2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 2*a*c*e^2*(3*C*d^2 -
e*(2*B*d - A*e)))*x)/e^6 - (c*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 2*a*e^2*
(2*C*d - B*e))*x^2)/(2*e^5) + (c*(3*c*C*d^2 + 2*a*C*e^2 - c*e*(2*B*d - A*e)
)*x^3)/(3*e^4) - (c^2*(2*C*d - B*e)*x^4)/(4*e^3) + (c^2*C*x^5)/(5*e^2) - ((
c*d^2 + a*e^2)^2*(C*d^2 - B*d*e + A*e^2))/(e^7*(d + e*x)) - ((c*d^2 + a*e^2
)*(6*c*C*d^3 - c*d*e*(5*B*d - 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e
^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx = \int\left(\frac{a^2Ce^4+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)}{e^6}+\frac{c\,(\right.$$

$$= \frac{\left(a^2Ce^4+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)+2ace^2\left(3Cd^2-e(2Bd-Ae)\right)\right)x}{e^6}-\frac{c\,(}{}$$

**Mathematica** [A] time = 0.28, size = 272, normalized size = 0.93

$$\frac{60ex\left(a^2Ce^4+2ace^2\left(e(Ae-2Bd)+3Cd^2\right)+c^2\left(d^2e(3Ae-4Bd)+5Cd^4\right)\right)-30ce^2x^2\left(-2ae^2(Be-2Cd)+cde(2A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (60*e*(a^2*C*e^4 + 2*a*c*e^2*(3*C*d^2 + e*(-2*B*d + A*e)) + c^2*(5*C*d^4 +
d^2*e*(-4*B*d + 3*A*e)))*x - 30*c*e^2*(4*c*C*d^3 + c*d*e*(-3*B*d + 2*A*e) -
 2*a*e^2*(-2*C*d + B*e))*x^2 + 20*c*e^3*(3*c*C*d^2 + 2*a*C*e^2 + c*e*(-2*B*
d + A*e))*x^3 + 15*c^2*e^4*(-2*C*d + B*e)*x^4 + 12*c^2*C*e^5*x^5 - (60*(c*d
^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) - 60*(c*d^2 + a*e^2)*(6
*c*C*d^3 + c*d*e*(-5*B*d + 4*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/(60*
e^7)
```

**fricas** [A] time = 0.97, size = 553, normalized size = 1.89

$$\frac{12\,Cc^2e^6x^6-60\,Cc^2d^6+60\,Bc^2d^5e+120\,Bacd^3e^3+60\,Ba^2de^5-60\,Aa^2e^6-60\left(2\,Cac+Ac^2\right)d^4e^2-60\left(Ca^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*e^6*x^6 - 60*C*c^2*d^6 + 60*B*c^2*d^5*e + 120*B*a*c*d^3*e^3
+ 60*B*a^2*d*e^5 - 60*A*a^2*e^6 - 60*(2*C*a*c + A*c^2)*d^4*e^2 - 60*(C*a^2
+ 2*A*a*c)*d^2*e^4 - 3*(6*C*c^2*d*e^5 - 5*B*c^2*e^6)*x^5 + 5*(6*C*c^2*d^2*e
^4 - 5*B*c^2*d*e^5 + 4*(2*C*a*c + A*c^2)*e^6)*x^4 - 10*(6*C*c^2*d^3*e^3 - 5
*B*c^2*d^2*e^4 - 6*B*a*c*e^6 + 4*(2*C*a*c + A*c^2)*d*e^5)*x^3 + 30*(6*C*c^2
*d^4*e^2 - 5*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 4*(2*C*a*c + A*c^2)*d^2*e^4 +
2*(C*a^2 + 2*A*a*c)*e^6)*x^2 + 60*(5*C*c^2*d^5*e - 4*B*c^2*d^4*e^2 - 4*B*a*
c*d^2*e^4 + 3*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 + 2*A*a*c)*d*e^5)*x - 60*(
6*C*c^2*d^6 - 5*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 - B*a^2*d*e^5 + 4*(2*C*a*c +
```

```
A*c^2)*d^4*e^2 + 2*(C*a^2 + 2*A*a*c)*d^2*e^4 + (6*C*c^2*d^5*e - 5*B*c^2*d^4
*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^2)*d^3*e^3 + 2*(C*a^2
 + 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

**giac [A]** time = 0.18, size = 497, normalized size = 1.70

$$\frac{1}{60}\left(12\,Cc^2 - \frac{15\left(6\,Cc^2de - Bc^2e^2\right)e^{(-1)}}{xe+d} + \frac{20\left(15\,Cc^2d^2e^2 - 5\,Bc^2de^3 + 2\,Cace^4 + Ac^2e^4\right)e^{(-2)}}{(xe+d)^2} - \frac{60\left(10\,Cc^2d^3e^3 - \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/60*(12*C*c^2 - 15*(6*C*c^2*d*e - B*c^2*e^2)*e^(-1)/(x*e + d) + 20*(15*C*c
^2*d^2*e^2 - 5*B*c^2*d*e^3 + 2*C*a*c*e^4 + A*c^2*e^4)*e^(-2)/(x*e + d)^2 -
60*(10*C*c^2*d^3*e^3 - 5*B*c^2*d^2*e^4 + 4*C*a*c*d*e^5 + 2*A*c^2*d*e^5 - B*
a*c*e^6)*e^(-3)/(x*e + d)^3 + 60*(15*C*c^2*d^4*e^4 - 10*B*c^2*d^3*e^5 + 12*
C*a*c*d^2*e^6 + 6*A*c^2*d^2*e^6 - 6*B*a*c*d*e^7 + C*a^2*e^8 + 2*A*a*c*e^8)*
e^(-4)/(x*e + d)^4)*(x*e + d)^5*e^(-7) + (6*C*c^2*d^5 - 5*B*c^2*d^4*e + 8*C
*a*c*d^3*e^2 + 4*A*c^2*d^3*e^2 - 6*B*a*c*d^2*e^3 + 2*C*a^2*d*e^4 + 4*A*a*c*
d*e^4 - B*a^2*e^5)*e^(-7)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^2*d^6
*e^5/(x*e + d) - B*c^2*d^5*e^6/(x*e + d) + 2*C*a*c*d^4*e^7/(x*e + d) + A*c^
2*d^4*e^7/(x*e + d) - 2*B*a*c*d^3*e^8/(x*e + d) + C*a^2*d^2*e^9/(x*e + d) +
 2*A*a*c*d^2*e^9/(x*e + d) - B*a^2*d*e^10/(x*e + d) + A*a^2*e^11/(x*e + d))
*e^(-12)
```

**maple [A]** time = 0.01, size = 527, normalized size = 1.80

$$\frac{C\,c^2x^5}{5e^2} + \frac{B\,c^2x^4}{4e^2} - \frac{C\,c^2d\,x^4}{2e^3} + \frac{A\,c^2x^3}{3e^2} - \frac{2B\,c^2d\,x^3}{3e^3} + \frac{2Cac\,x^3}{3e^2} + \frac{C\,c^2d^2x^3}{e^4} - \frac{A\,c^2d\,x^2}{e^3} + \frac{Bac\,x^2}{e^2} + \frac{3B\,c^2d^2x^2}{2e^4} - \frac{2Cacd\,x^2}{e^3} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] -2/e^3*C*x^2*a*c*d-4/e^3*d*a*c*B*x+6/e^4*C*a*c*d^2*x-2/e^3/(e*x+d)*A*a*c*d^
2+2/e^4/(e*x+d)*B*a*c*d^3-2/e^5/(e*x+d)*C*a*c*d^4-4/e^3*ln(e*x+d)*A*a*c*d+6
/e^4*ln(e*x+d)*B*a*c*d^2-8/e^5*ln(e*x+d)*C*a*c*d^3+1/5*c^2*C*x^5/e^2-1/e/(e
*x+d)*A*a^2+1/e^2*ln(e*x+d)*B*a^2+1/4/e^2*B*x^4*c^2+1/3/e^2*A*x^3*c^2+1/e^2
*a^2*C*x-4/e^5*ln(e*x+d)*A*c^2*d^3+5/e^6*ln(e*x+d)*B*c^2*d^4-4/e^5*B*c^2*d^
3*x+5/e^6*C*c^2*d^4*x-1/e^5/(e*x+d)*A*c^2*d^4+1/e^2/(e*x+d)*B*d*a^2+1/e^6/(
e*x+d)*B*c^2*d^5-1/e^3/(e*x+d)*C*a^2*d^2-1/e^7/(e*x+d)*C*c^2*d^6+2/3/e^2*C*
x^3*a*c+1/e^4*C*x^3*c^2*d^2-1/e^3*A*x^2*c^2*d+1/e^2*B*x^2*a*c+3/2/e^4*B*x^2
*c^2*d^2-2/e^5*C*x^2*c^2*d^3+2/e^2*A*a*c*x+3/e^4*A*c^2*d^2*x-1/2/e^3*C*x^4*
c^2*d-2/3/e^3*B*x^3*c^2*d-2/e^3*ln(e*x+d)*C*a^2*d-6/e^7*ln(e*x+d)*C*c^2*d^5
```

**maxima [A]** time = 0.48, size = 392, normalized size = 1.34

$$-\frac{Cc^2d^6 - Bc^2d^5e - 2\,Bacd^3e^3 - Ba^2de^5 + Aa^2e^6 + \left(2\,Cac + Ac^2\right)d^4e^2 + \left(Ca^2 + 2\,Aac\right)d^2e^4}{e^8x + de^7} + \frac{12\,Cc^2e^4x^5 - 15\left(2\,C\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^2*d^6 - B*c^2*d^5*e - 2*B*a*c*d^3*e^3 - B*a^2*d*e^5 + A*a^2*e^6 + (2*
C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A*a*c)*d^2*e^4)/(e^8*x + d*e^7) + 1/60*
(12*C*c^2*e^4*x^5 - 15*(2*C*c^2*d*e^3 - B*c^2*e^4)*x^4 + 20*(3*C*c^2*d^2*e^
2 - 2*B*c^2*d*e^3 + (2*C*a*c + A*c^2)*e^4)*x^3 - 30*(4*C*c^2*d^3*e - 3*B*c^
2*d^2*e^2 - 2*B*a*c*e^4 + 2*(2*C*a*c + A*c^2)*d*e^3)*x^2 + 60*(5*C*c^2*d^4
- 4*B*c^2*d^3*e - 4*B*a*c*d*e^3 + 3*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2*
A*a*c)*e^4)*x)/e^6 - (6*C*c^2*d^5 - 5*B*c^2*d^4*e - 6*B*a*c*d^2*e^3 - B*a^2
*e^5 + 4*(2*C*a*c + A*c^2)*d^3*e^2 + 2*(C*a^2 + 2*A*a*c)*d*e^4)*log(e*x + d
)/e^7
```

**mupad [B]** time = 0.12, size = 575, normalized size = 1.97

$$x^4\left(\frac{B\,c^2}{4\,e^2}-\frac{C\,c^2\,d}{2\,e^3}\right)+x\left(\frac{C\,a^2+2\,A\,c\,a}{e^2}+\frac{d^2\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^2}+\frac{C\,c^2\,d^2}{e^4}\right)}{e^2}-\frac{2\,d\left(\frac{2\,d\left(\frac{2\,d\left(\frac{B\,c^2}{e^2}-\frac{2\,C\,c^2\,d}{e^3}\right)}{e}-\frac{A\,c^2+2\,C\,a}{e^2}\right.}{e}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x^4*((B*c^2)/(4*e^2) - (C*c^2*d)/(2*e^3)) + x*((C*a^2 + 2*A*a*c)/e^2 + (d^2
*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*
d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c
^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4))/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e
^3))/e^2 + (2*B*a*c)/e^2))/e) - x^3*((2*d*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/
(3*e) - (A*c^2 + 2*C*a*c)/(3*e^2) + (C*c^2*d^2)/(3*e^4)) + x^2*((d*((2*d*((
B*c^2)/e^2 - (2*C*c^2*d)/e^3))/e - (A*c^2 + 2*C*a*c)/e^2 + (C*c^2*d^2)/e^4)
)/e - (d^2*((B*c^2)/e^2 - (2*C*c^2*d)/e^3))/(2*e^2) + (B*a*c)/e^2) - (A*a^2
*e^6 + C*c^2*d^6 - B*a^2*d*e^5 - B*c^2*d^5*e + A*c^2*d^4*e^2 + C*a^2*d^2*e^
```

```
4 + 2*A*a*c*d^2*e^4 - 2*B*a*c*d^3*e^3 + 2*C*a*c*d^4*e^2)/(e*(d*e^6 + e^7*x)
) - (log(d + e*x)*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4 - 5*B*c^2*d^4*e
+ 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a*c*d^3*e^2))/e^7
 + (C*c^2*x^5)/(5*e^2)
```

**sympy [A]** time = 2.79, size = 416, normalized size = 1.42

$$\frac{Cc^2x^5}{5e^2}+x^4\left(\frac{Bc^2}{4e^2}-\frac{Cc^2d}{2e^3}\right)+x^3\left(\frac{Ac^2}{3e^2}-\frac{2Bc^2d}{3e^3}+\frac{2Cac}{3e^2}+\frac{Cc^2d^2}{e^4}\right)+x^2\left(-\frac{Ac^2d}{e^3}+\frac{Bac}{e^2}+\frac{3Bc^2d^2}{2e^4}-\frac{2Cacd}{e^3}-\frac{2Cc^2d}{e^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**2*x**5/(5*e**2) + x**4*(B*c**2/(4*e**2) - C*c**2*d/(2*e**3)) + x**3*(A
*c**2/(3*e**2) - 2*B*c**2*d/(3*e**3) + 2*C*a*c/(3*e**2) + C*c**2*d**2/e**4)
 + x**2*(-A*c**2*d/e**3 + B*a*c/e**2 + 3*B*c**2*d**2/(2*e**4) - 2*C*a*c*d/e
**3 - 2*C*c**2*d**3/e**5) + x*(2*A*a*c/e**2 + 3*A*c**2*d**2/e**4 - 4*B*a*c*
d/e**3 - 4*B*c**2*d**3/e**5 + C*a**2/e**2 + 6*C*a*c*d**2/e**4 + 5*C*c**2*d*
*4/e**6) + (-A*a**2*e**6 - 2*A*a*c*d**2*e**4 - A*c**2*d**4*e**2 + B*a**2*d*
e**5 + 2*B*a*c*d**3*e**3 + B*c**2*d**5*e - C*a**2*d**2*e**4 - 2*C*a*c*d**4*
e**2 - C*c**2*d**6)/(d*e**7 + e**8*x) - (a*e**2 + c*d**2)*(4*A*c*d*e**2 - B
*a*e**3 - 5*B*c*d**2*e + 2*C*a*d*e**2 + 6*C*c*d**3)*log(d + e*x)/e**7
```

## 3.31 $\int \frac{(a+cx^2)^2(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

**Optimal**. Leaf size=295

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(15Cd^2-2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{\left(ae^2+cd^2\right)\left(ae^2(2Cd-Be)\right.}{e^7(d}$$

```
[Out] -c*(2*a*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^6+1/2*c*(2*a*
C*e^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^5-1/3*c^2*(-B*e+3*C*d)*x^3/e^4+1/4*
c^2*C*x^4/e^3-1/2*(a*e^2+c*d^2)^2*(A*e^2-B*d*e+C*d^2)/e^7/(e*x+d)^2+(a*e^2+
c*d^2)*(a*e^2*(-B*e+2*C*d)+c*d*(6*C*d^2-e*(-4*A*e+5*B*d)))/e^7/(e*x+d)+(a^2
*C*e^4+c^2*d^2*(15*C*d^2-2*e*(-3*A*e+5*B*d))+2*a*c*e^2*(6*C*d^2-e*(-A*e+3*B
*d)))*ln(e*x+d)/e^7
```

**Rubi [A]** time = 0.49, antiderivative size = 292, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{\log(d+ex)\left(a^2Ce^4+2ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{e^7}+\frac{cx^2\left(2aCe^2-ce(3Bd-Ae)\right.}{2e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 2*a*e^2*(3*C*d - B*e))*x)/e^6) +
 (c*(6*c*C*d^2 + 2*a*C*e^2 - c*e*(3*B*d - A*e))*x^2)/(2*e^5) - (c^2*(3*C*d
- B*e)*x^3)/(3*e^4) + (c^2*C*x^4)/(4*e^3) - ((c*d^2 + a*e^2)^2*(C*d^2 - B*d
*e + A*e^2))/(2*e^7*(d + e*x)^2) + ((c*d^2 + a*e^2)*(6*c*C*d^3 - c*d*e*(5*B
*d - 4*A*e) + a*e^2*(2*C*d - B*e)))/(e^7*(d + e*x)) + ((a^2*C*e^4 + c^2*(15
*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 2*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e)))*
Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^2 \left(A+Bx+Cx^2\right)}{(d+ex)^3}\,dx = \int \left(\frac{c\left(-10cCd^3+3cde(2Bd-Ae)-2ae^2(3Cd-Be)\right)}{e^6} + \frac{c\left(6cCd^2+2aCe^2 -\right.}{e^5}\right.$$

$$= -\frac{c\left(10cCd^3-3cde(2Bd-Ae)+2ae^2(3Cd-Be)\right)x}{e^6} + \frac{c\left(6cCd^2+2aCe^2-c\right.}{2e^5}$$

**Mathematica** [A] time = 0.12, size = 274, normalized size = 0.93

$$\frac{12\log(d+ex)\left(a^2Ce^4+2ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)-12cex\left(-2ae^2(Be-3C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-12*c*e*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 2*a*e^2*(-3*C*d + B*e))*x +
 6*c*e^2*(6*c*C*d^2 + 2*a*C*e^2 + c*e*(-3*B*d + A*e))*x^2 + 4*c^2*e^3*(-3*C
*d + B*e)*x^3 + 3*c^2*C*e^4*x^4 - (6*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) +
 A*e)))/(d + e*x)^2 + (12*(c*d^2 + a*e^2)*(6*c*C*d^3 + c*d*e*(-5*B*d + 4*A*
e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 12*(a^2*C*e^4 + 2*a*c*e^2*(6*C*d^2 +
 e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B*d + 3*A*e)))*Log[d + e*x
])/(12*e^7)
```

**fricas** [B] time = 0.91, size = 608, normalized size = 2.06

$$\frac{3\,Cc^2e^6x^6+66\,Cc^2d^6-54\,Bc^2d^5e-60\,Bacd^3e^3-6\,Ba^2de^5-6\,Aa^2e^6+42\left(2\,Cac+Ac^2\right)d^4e^2+18\left(Ca^2+2\,Aa\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c^2*e^6*x^6 + 66*C*c^2*d^6 - 54*B*c^2*d^5*e - 60*B*a*c*d^3*e^3 -
6*B*a^2*d*e^5 - 6*A*a^2*e^6 + 42*(2*C*a*c + A*c^2)*d^4*e^2 + 18*(C*a^2 + 2*
A*a*c)*d^2*e^4 - 2*(3*C*c^2*d*e^5 - 2*B*c^2*e^6)*x^5 + (15*C*c^2*d^2*e^4 -
10*B*c^2*d*e^5 + 6*(2*C*a*c + A*c^2)*e^6)*x^4 - 4*(15*C*c^2*d^3*e^3 - 10*B*
c^2*d^2*e^4 - 6*B*a*c*e^6 + 6*(2*C*a*c + A*c^2)*d*e^5)*x^3 - 6*(34*C*c^2*d^
4*e^2 - 21*B*c^2*d^3*e^3 - 8*B*a*c*d*e^5 + 11*(2*C*a*c + A*c^2)*d^2*e^4)*x^
2 - 12*(4*C*c^2*d^5*e - B*c^2*d^4*e^2 + 4*B*a*c*d^2*e^4 + B*a^2*e^6 - (2*C*
a*c + A*c^2)*d^3*e^3 - 2*(C*a^2 + 2*A*a*c)*d*e^5)*x + 12*(15*C*c^2*d^6 - 10
*B*c^2*d^5*e - 6*B*a*c*d^3*e^3 + 6*(2*C*a*c + A*c^2)*d^4*e^2 + (C*a^2 + 2*A
```

```
*a*c)*d^2*e^4 + (15*C*c^2*d^4*e^2 - 10*B*c^2*d^3*e^3 - 6*B*a*c*d*e^5 + 6*(2
*C*a*c + A*c^2)*d^2*e^4 + (C*a^2 + 2*A*a*c)*e^6)*x^2 + 2*(15*C*c^2*d^5*e -
10*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 + 6*(2*C*a*c + A*c^2)*d^3*e^3 + (C*a^2 +
 2*A*a*c)*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

**giac** [**A**] time = 0.16, size = 397, normalized size = 1.35

$$\left(15\,Cc^2d^4 - 10\,Bc^2d^3e + 12\,Cacd^2e^2 + 6\,Ac^2d^2e^2 - 6\,Bacde^3 + Ca^2e^4 + 2\,Aace^4\right)e^{(-7)}\log\left(|xe+d|\right) + \frac{1}{12}\left(3\,Cc^2x^4e^9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (15*C*c^2*d^4 - 10*B*c^2*d^3*e + 12*C*a*c*d^2*e^2 + 6*A*c^2*d^2*e^2 - 6*B*a
*c*d*e^3 + C*a^2*e^4 + 2*A*a*c*e^4)*e^(-7)*log(abs(x*e + d)) + 1/12*(3*C*c^
2*x^4*e^9 - 12*C*c^2*d*x^3*e^8 + 36*C*c^2*d^2*x^2*e^7 - 120*C*c^2*d^3*x*e^6
 + 4*B*c^2*x^3*e^9 - 18*B*c^2*d*x^2*e^8 + 72*B*c^2*d^2*x*e^7 + 12*C*a*c*x^2
*e^9 + 6*A*c^2*x^2*e^9 - 72*C*a*c*d*x*e^8 - 36*A*c^2*d*x*e^8 + 24*B*a*c*x*e
^9)*e^(-12) + 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e + 14*C*a*c*d^4*e^2 + 7*A*c^
2*d^4*e^2 - 10*B*a*c*d^3*e^3 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - B*a^2*d*
e^5 - A*a^2*e^6 + 2*(6*C*c^2*d^5*e - 5*B*c^2*d^4*e^2 + 8*C*a*c*d^3*e^3 + 4*
A*c^2*d^3*e^3 - 6*B*a*c*d^2*e^4 + 2*C*a^2*d*e^5 + 4*A*a*c*d*e^5 - B*a^2*e^6
)*x)*e^(-7)/(x*e + d)^2
```

**maple** [**A**] time = 0.01, size = 563, normalized size = 1.91

$$\frac{C\,c^2x^4}{4e^3} + \frac{B\,c^2x^3}{3e^3} - \frac{C\,c^2d\,x^3}{e^4} - \frac{A\,a^2}{2\,(ex+d)^2\,e} - \frac{Aac\,d^2}{(ex+d)^2\,e^3} - \frac{A\,c^2d^4}{2\,(ex+d)^2\,e^5} + \frac{A\,c^2x^2}{2e^3} + \frac{B\,a^2d}{2\,(ex+d)^2\,e^2} + \frac{Bac\,d^3}{(ex+d)^2\,e^4} + \frac{B\,c}{2\,(ex+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] -6*c/e^4*C*x*a*d+4/e^3/(e*x+d)*A*a*c*d-6/e^4/(e*x+d)*B*a*c*d^2+8/e^5/(e*x+d
)*C*a*c*d^3-6/e^4*ln(e*x+d)*B*a*c*d+1/4*c^2*C*x^4/e^3+12/e^5*ln(e*x+d)*C*a*
c*d^2-1/e^3/(e*x+d)^2*A*d^2*a*c+1/e^4/(e*x+d)^2*B*a*c*d^3-1/e^5/(e*x+d)^2*C
*a*c*d^4+1/e^3*ln(e*x+d)*a^2*C-1/2/e/(e*x+d)^2*A*a^2+1/3*c^2/e^3*B*x^3+1/2*
c^2/e^3*A*x^2-1/e^2/(e*x+d)*B*a^2-c^2/e^4*C*x^3*d-3/2*c^2/e^4*B*x^2*d-10/e^
6*ln(e*x+d)*B*c^2*d^3+15/e^7*ln(e*x+d)*C*c^2*d^4-1/2/e^5/(e*x+d)^2*A*c^2*d^
4+1/2/e^2/(e*x+d)^2*B*d*a^2+1/2/e^6/(e*x+d)^2*B*c^2*d^5-1/2/e^3/(e*x+d)^2*C
*d^2*a^2-1/2/e^7/(e*x+d)^2*C*c^2*d^6+c/e^3*C*x^2*a+3*c^2/e^5*C*x^2*d^2-3*c^
2/e^4*A*x*d+2*c/e^3*B*x*a+6*c^2/e^5*B*x*d^2-10*c^2/e^6*C*x*d^3+4/e^5/(e*x+d
)*A*c^2*d^3-5/e^6/(e*x+d)*B*c^2*d^4+2/e^3/(e*x+d)*C*a^2*d+6/e^7/(e*x+d)*C*c
^2*d^5+2/e^3*ln(e*x+d)*A*a*c+6/e^5*ln(e*x+d)*A*c^2*d^2
```

**maxima [A]** time = 0.49, size = 402, normalized size = 1.36

$$\frac{11\,Cc^2d^6 - 9\,Bc^2d^5e - 10\,Bacd^3e^3 - Ba^2de^5 - Aa^2e^6 + 7\left(2\,Cac + Ac^2\right)d^4e^2 + 3\left(Ca^2 + 2\,Aac\right)d^2e^4 + 2\left(6\,Cc^2d^5e\right.}{2\left(e^9x^2 + 2\,de^8x + d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^2*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(11*C*c^2*d^6 - 9*B*c^2*d^5*e - 10*B*a*c*d^3*e^3 - B*a^2*d*e^5 - A*a^2*
e^6 + 7*(2*C*a*c + A*c^2)*d^4*e^2 + 3*(C*a^2 + 2*A*a*c)*d^2*e^4 + 2*(6*C*c^
2*d^5*e - 5*B*c^2*d^4*e^2 - 6*B*a*c*d^2*e^4 - B*a^2*e^6 + 4*(2*C*a*c + A*c^
2)*d^3*e^3 + 2*(C*a^2 + 2*A*a*c)*d*e^5)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
+ 1/12*(3*C*c^2*e^3*x^4 - 4*(3*C*c^2*d*e^2 - B*c^2*e^3)*x^3 + 6*(6*C*c^2*d^
2*e - 3*B*c^2*d*e^2 + (2*C*a*c + A*c^2)*e^3)*x^2 - 12*(10*C*c^2*d^3 - 6*B*c
^2*d^2*e - 2*B*a*c*e^3 + 3*(2*C*a*c + A*c^2)*d*e^2)*x)/e^6 + (15*C*c^2*d^4
- 10*B*c^2*d^3*e - 6*B*a*c*d*e^3 + 6*(2*C*a*c + A*c^2)*d^2*e^2 + (C*a^2 + 2
*A*a*c)*e^4)*log(e*x + d)/e^7
```

**mupad [B]** time = 3.82, size = 495, normalized size = 1.68

$$x\left(\frac{3\,d\left(\frac{3\,d\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e}-\frac{A\,c^2+2\,C\,a\,c}{e^3}+\frac{3\,C\,c^2\,d^2}{e^5}\right)}{e}-\frac{3\,d^2\left(\frac{B\,c^2}{e^3}-\frac{3\,C\,c^2\,d}{e^4}\right)}{e^2}+\frac{2\,B\,a\,c}{e^3}-\frac{C\,c^2\,d^3}{e^6}\right)+x^3\left(\frac{B\,c^2}{3\,e^3}-\frac{C\,c^2\,d}{e^4}\right)-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^2*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x*((3*d*((3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e - (A*c^2 + 2*C*a*c)/e^3 +
(3*C*c^2*d^2)/e^5))/e - (3*d^2*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/e^2 + (2*B*
a*c)/e^3 - (C*c^2*d^3)/e^6) + x^3*((B*c^2)/(3*e^3) - (C*c^2*d)/e^4) - x^2*(
(3*d*((B*c^2)/e^3 - (3*C*c^2*d)/e^4))/(2*e) - (A*c^2 + 2*C*a*c)/(2*e^3) + (
3*C*c^2*d^2)/(2*e^5)) + ((11*C*c^2*d^6 - A*a^2*e^6 - B*a^2*d*e^5 - 9*B*c^2*
d^5*e + 7*A*c^2*d^4*e^2 + 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 - 10*B*a*c*d^3*
e^3 + 14*C*a*c*d^4*e^2)/(2*e) + x*(6*C*c^2*d^5 - B*a^2*e^5 + 2*C*a^2*d*e^4
- 5*B*c^2*d^4*e + 4*A*c^2*d^3*e^2 + 4*A*a*c*d*e^4 - 6*B*a*c*d^2*e^3 + 8*C*a
*c*d^3*e^2))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) + (log(d + e*x)*(C*a^2*e^4 + 1
5*C*c^2*d^4 + 2*A*a*c*e^4 - 10*B*c^2*d^3*e + 6*A*c^2*d^2*e^2 - 6*B*a*c*d*e^
3 + 12*C*a*c*d^2*e^2))/e^7 + (C*c^2*x^4)/(4*e^3)
```

**sympy [A]** time = 14.20, size = 474, normalized size = 1.61

$$\frac{Cc^2x^4}{4e^3}+x^3\left(\frac{Bc^2}{3e^3}-\frac{Cc^2d}{e^4}\right)+x^2\left(\frac{Ac^2}{2e^3}-\frac{3Bc^2d}{2e^4}+\frac{Cac}{e^3}+\frac{3Cc^2d^2}{e^5}\right)+x\left(-\frac{3Ac^2d}{e^4}+\frac{2Bac}{e^3}+\frac{6Bc^2d^2}{e^5}-\frac{6Cacd}{e^4}-\frac{10Cc^2}{e^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**2*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**2*x**4/(4*e**3) + x**3*(B*c**2/(3*e**3) - C*c**2*d/e**4) + x**2*(A*c**
2/(2*e**3) - 3*B*c**2*d/(2*e**4) + C*a*c/e**3 + 3*C*c**2*d**2/e**5) + x*(-3
*A*c**2*d/e**4 + 2*B*a*c/e**3 + 6*B*c**2*d**2/e**5 - 6*C*a*c*d/e**4 - 10*C*
c**2*d**3/e**6) + (-A*a**2*e**6 + 6*A*a*c*d**2*e**4 + 7*A*c**2*d**4*e**2 -
B*a**2*d*e**5 - 10*B*a*c*d**3*e**3 - 9*B*c**2*d**5*e + 3*C*a**2*d**2*e**4 +
 14*C*a*c*d**4*e**2 + 11*C*c**2*d**6 + x*(8*A*a*c*d*e**5 + 8*A*c**2*d**3*e*
*3 - 2*B*a**2*e**6 - 12*B*a*c*d**2*e**4 - 10*B*c**2*d**4*e**2 + 4*C*a**2*d*
e**5 + 16*C*a*c*d**3*e**3 + 12*C*c**2*d**5*e))/(2*d**2*e**7 + 4*d*e**8*x +
2*e**9*x**2) + (2*A*a*c*e**4 + 6*A*c**2*d**2*e**2 - 6*B*a*c*d*e**3 - 10*B*c
**2*d**3*e + C*a**2*e**4 + 12*C*a*c*d**2*e**2 + 15*C*c**2*d**4)*log(d + e*x
)/e**7
```

## 3.32 $\int (d+ex)^3 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=404

$$\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3Cd^2\right)+a^3Ad^3x+\frac{1}{6}a^2ex^6\left(aCe^2+3c\left(e(Ae+3Bd)+3Cd^2\right)\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+\right.$$

```
[Out] a^3*A*d^3*x+1/3*a^2*d*(a*d*(3*B*e+C*d)+3*A*(a*e^2+c*d^2))*x^3+1/4*a^3*e*(3*
C*d^2+e*(A*e+3*B*d))*x^4+1/5*a*(3*A*c*d*(3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d
)+3*c*d^2*(3*B*e+C*d)))*x^5+1/6*a^2*e*(a*C*e^2+3*c*(3*C*d^2+e*(A*e+3*B*d)))
*x^6+1/7*c*(A*c*d*(9*a*e^2+c*d^2)+3*a*(a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*x^7+3/8*a*c*e*(a*C*e^2+c*(3*C*d^2+e*(A*e+3*B*d)))*x^8+1/9*c^2*(3*a*e^2*(B
*e+3*C*d)+c*d*(C*d^2+3*e*(A*e+B*d)))*x^9+1/10*c^2*e*(3*a*C*e^2+c*(3*C*d^2+e
*(A*e+3*B*d)))*x^10+1/11*c^3*e^2*(B*e+3*C*d)*x^11+1/12*c^3*C*e^3*x^12+1/8*d
^2*(3*A*e+B*d)*(c*x^2+a)^4/c
```

**Rubi [A]** time = 0.69, antiderivative size = 400, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{6}a^2ex^6\left(aCe^2+3ce(Ae+3Bd)+9cCd^2\right)+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^3ex^4\left(e(Ae+3Bd)+3C\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^2*d*(a*d*(C*d + 3*B*e) + 3*A*(c*d^2 + a*e^2))*x^3)/3 + (a^
3*e*(3*C*d^2 + e*(3*B*d + A*e))*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*
(a*e^2*(3*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a^2*e*(9*c*C*d^2 +
 a*C*e^2 + 3*c*e*(3*B*d + A*e))*x^6)/6 + (c*(A*c*d*(c*d^2 + 9*a*e^2) + 3*a*
(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/7 + (3*a*c*e*(3*c*C*d^2 +
 a*C*e^2 + c*e*(3*B*d + A*e))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e)
+ 3*a*e^2*(3*C*d + B*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*
d + A*e))*x^10)/10 + (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
+ (d^2*(B*d + 3*A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
```

```
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx = \frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a+cx^2\right)^3\left(-\left(Bd^3+3Ad^2e\right)x+(d+$$

$$=\frac{d^2(Bd+3Ae)\left(a+cx^2\right)^4}{8c}+\int \left(a^3Ad^3+a^2d\left(ad(Cd+3Be)+3A\left(c\right.\right.\right.$$

$$=a^3Ad^3x+\frac{1}{3}a^2d\left(ad(Cd+3Be)+3A\left(cd^2+ae^2\right)\right)x^3+\frac{1}{4}a^3e\left(3Cd^2+\right.$$

**Mathematica [A]** time = 0.21, size = 459, normalized size = 1.14

$$\frac{1}{2}a^3d^2x^2(3Ae+Bd)+a^3Ad^3x+\frac{1}{3}a^2dx^3\left(3A\left(ae^2+cd^2\right)+ad(3Be+Cd)\right)+\frac{1}{4}a^2x^4\left(aAe^3+3aBde^2+3aCd^2e+9Acd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^3*x + (a^3*d^2*(B*d + 3*A*e)*x^2)/2 + (a^2*d*(a*d*(C*d + 3*B*e) + 3
*A*(c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^3 + 9*A*c*d^2*e + 3*a*C*d^2*e +
3*a*B*d*e^2 + a*A*e^3)*x^4)/4 + (a*(3*A*c*d*(c*d^2 + 3*a*e^2) + a*(a*e^2*(3
*C*d + B*e) + 3*c*d^2*(C*d + 3*B*e)))*x^5)/5 + (a*(3*A*c*e*(3*c*d^2 + a*e^2
) + a*C*e*(9*c*d^2 + a*e^2) + 3*B*c*d*(c*d^2 + 3*a*e^2))*x^6)/6 + (c*(A*c*d
*(c*d^2 + 9*a*e^2) + 3*a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*x^7)/
7 + (c*(B*c*d*(c*d^2 + 9*a*e^2) + 3*e*(A*c*(c*d^2 + a*e^2) + a*C*(3*c*d^2 +
 a*e^2)))*x^8)/8 + (c^2*(c*C*d^3 + 3*c*d*e*(B*d + A*e) + 3*a*e^2*(3*C*d + B
*e))*x^9)/9 + (c^2*e*(3*c*C*d^2 + 3*a*C*e^2 + c*e*(3*B*d + A*e))*x^10)/10 +
 (c^3*e^2*(3*C*d + B*e)*x^11)/11 + (c^3*C*e^3*x^12)/12
```

**fricas [A]** time = 0.74, size = 618, normalized size = 1.53

$$\frac{1}{12}x^{12}e^3c^3C+\frac{3}{11}x^{11}e^2dc^3C+\frac{1}{11}x^{11}e^3c^3B+\frac{3}{10}x^{10}ed^2c^3C+\frac{3}{10}x^{10}e^3c^2aC+\frac{3}{10}x^{10}e^2dc^3B+\frac{1}{10}x^{10}e^3c^3A+\frac{1}{9}x^9d^3c^3C+x^9e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^3*c^3*C + 3/11*x^11*e^2*d*c^3*C + 1/11*x^11*e^3*c^3*B + 3/10*x^
10*e*d^2*c^3*C + 3/10*x^10*e^3*c^2*a*C + 3/10*x^10*e^2*d*c^3*B + 1/10*x^10*
e^3*c^3*A + 1/9*x^9*d^3*c^3*C + x^9*e^2*d*c^2*a*C + 1/3*x^9*e*d^2*c^3*B + 1
/3*x^9*e^3*c^2*a*B + 1/3*x^9*e^2*d*c^3*A + 9/8*x^8*e*d^2*c^2*a*C + 3/8*x^8*
e^3*c*a^2*C + 1/8*x^8*d^3*c^3*B + 9/8*x^8*e^2*d*c^2*a*B + 3/8*x^8*e*d^2*c^3
*A + 3/8*x^8*e^3*c^2*a*A + 3/7*x^7*d^3*c^2*a*C + 9/7*x^7*e^2*d*c*a^2*C + 9/
7*x^7*e*d^2*c^2*a*B + 3/7*x^7*e^3*c*a^2*B + 1/7*x^7*d^3*c^3*A + 9/7*x^7*e^2
*d*c^2*a*A + 3/2*x^6*e*d^2*c*a^2*C + 1/6*x^6*e^3*a^3*C + 1/2*x^6*d^3*c^2*a*
B + 3/2*x^6*e^2*d*c*a^2*B + 3/2*x^6*e*d^2*c^2*a*A + 1/2*x^6*e^3*c*a^2*A + 3
/5*x^5*d^3*c*a^2*C + 3/5*x^5*e^2*d*a^3*C + 9/5*x^5*e*d^2*c*a^2*B + 1/5*x^5*
e^3*a^3*B + 3/5*x^5*d^3*c^2*a*A + 9/5*x^5*e^2*d*c*a^2*A + 3/4*x^4*e*d^2*a^3
*C + 3/4*x^4*d^3*c*a^2*B + 3/4*x^4*e^2*d*a^3*B + 9/4*x^4*e*d^2*c*a^2*A + 1/
4*x^4*e^3*a^3*A + 1/3*x^3*d^3*a^3*C + x^3*e*d^2*a^3*B + x^3*d^3*c*a^2*A + x
^3*e^2*d*a^3*A + 1/2*x^2*d^3*a^3*B + 3/2*x^2*e*d^2*a^3*A + x*d^3*a^3*A
```

**giac [A]** time = 0.20, size = 606, normalized size = 1.50

$$\frac{1}{12}\,Cc^3x^{12}e^3+\frac{3}{11}\,Cc^3dx^{11}e^2+\frac{3}{10}\,Cc^3d^2x^{10}e+\frac{1}{9}\,Cc^3d^3x^9+\frac{1}{11}\,Bc^3x^{11}e^3+\frac{3}{10}\,Bc^3dx^{10}e^2+\frac{1}{3}\,Bc^3d^2x^9e+\frac{1}{8}\,Bc^3d^3x^8+\frac{3}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*C*c^3*x^12*e^3 + 3/11*C*c^3*d*x^11*e^2 + 3/10*C*c^3*d^2*x^10*e + 1/9*C
*c^3*d^3*x^9 + 1/11*B*c^3*x^11*e^3 + 3/10*B*c^3*d*x^10*e^2 + 1/3*B*c^3*d^2*
x^9*e + 1/8*B*c^3*d^3*x^8 + 3/10*C*a*c^2*x^10*e^3 + 1/10*A*c^3*x^10*e^3 + C
*a*c^2*d*x^9*e^2 + 1/3*A*c^3*d*x^9*e^2 + 9/8*C*a*c^2*d^2*x^8*e + 3/8*A*c^3*
d^2*x^8*e + 3/7*C*a*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 1/3*B*a*c^2*x^9*e^3 +
 9/8*B*a*c^2*d*x^8*e^2 + 9/7*B*a*c^2*d^2*x^7*e + 1/2*B*a*c^2*d^3*x^6 + 3/8*
C*a^2*c*x^8*e^3 + 3/8*A*a*c^2*x^8*e^3 + 9/7*C*a^2*c*d*x^7*e^2 + 9/7*A*a*c^2
*d*x^7*e^2 + 3/2*C*a^2*c*d^2*x^6*e + 3/2*A*a*c^2*d^2*x^6*e + 3/5*C*a^2*c*d^
3*x^5 + 3/5*A*a*c^2*d^3*x^5 + 3/7*B*a^2*c*x^7*e^3 + 3/2*B*a^2*c*d*x^6*e^2 +
 9/5*B*a^2*c*d^2*x^5*e + 3/4*B*a^2*c*d^3*x^4 + 1/6*C*a^3*x^6*e^3 + 1/2*A*a^
2*c*x^6*e^3 + 3/5*C*a^3*d*x^5*e^2 + 9/5*A*a^2*c*d*x^5*e^2 + 3/4*C*a^3*d^2*x
^4*e + 9/4*A*a^2*c*d^2*x^4*e + 1/3*C*a^3*d^3*x^3 + A*a^2*c*d^3*x^3 + 1/5*B*
a^3*x^5*e^3 + 3/4*B*a^3*d*x^4*e^2 + B*a^3*d^2*x^3*e + 1/2*B*a^3*d^3*x^2 + 1
/4*A*a^3*x^4*e^3 + A*a^3*d*x^3*e^2 + 3/2*A*a^3*d^2*x^2*e + A*a^3*d^3*x
```

**maple [A]** time = 0.00, size = 553, normalized size = 1.37

$$\frac{C\,c^3e^3x^{12}}{12}+\frac{\left(e^3c^3B+3d\,e^2c^3C\right)x^{11}}{11}+\frac{\left(A\,c^3e^3+3B\,c^3d\,e^2+\left(3e^3a\,c^2+3d^2e\,c^3\right)C\right)x^{10}}{10}+\frac{\left(3A\,c^3d\,e^2+\left(3e^3a\,c^2+3d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/12*c^3*C*e^3*x^12+1/11*(B*c^3*e^3+3*C*c^3*d*e^2)*x^11+1/10*((3*a*c^2*e^3+
3*c^3*d^2*e)*C+3*d*e^2*c^3*B+e^3*c^3*A)*x^10+1/9*((9*a*c^2*d*e^2+c^3*d^3)*C
+(3*a*c^2*e^3+3*c^3*d^2*e)*B+3*d*e^2*c^3*A)*x^9+1/8*((3*a^2*c*e^3+9*a*c^2*d
^2*e)*C+(9*a*c^2*d*e^2+c^3*d^3)*B+(3*a*c^2*e^3+3*c^3*d^2*e)*A)*x^8+1/7*((9*
a^2*c*d*e^2+3*a*c^2*d^3)*C+(3*a^2*c*e^3+9*a*c^2*d^2*e)*B+(9*a*c^2*d*e^2+c^3
*d^3)*A)*x^7+1/6*((a^3*e^3+9*a^2*c*d^2*e)*C+(9*a^2*c*d*e^2+3*a*c^2*d^3)*B+(
3*a^2*c*e^3+9*a*c^2*d^2*e)*A)*x^6+1/5*((3*a^3*d*e^2+3*a^2*c*d^3)*C+(a^3*e^3
+9*a^2*c*d^2*e)*B+(9*a^2*c*d*e^2+3*a*c^2*d^3)*A)*x^5+1/4*(3*d^2*e*a^3*C+(3*
a^3*d*e^2+3*a^2*c*d^3)*B+(a^3*e^3+9*a^2*c*d^2*e)*A)*x^4+1/3*(d^3*a^3*C+3*d^
2*e*a^3*B+(3*a^3*d*e^2+3*a^2*c*d^3)*A)*x^3+1/2*(3*A*a^3*d^2*e+B*a^3*d^3)*x^
2+a^3*A*d^3*x
```

**maxima [A]** time = 0.47, size = 512, normalized size = 1.27

$$\frac{1}{12}Cc^3e^3x^{12}+\frac{1}{11}\left(3\,Cc^3de^2+Bc^3e^3\right)x^{11}+\frac{1}{10}\left(3\,Cc^3d^2e+3\,Bc^3de^2+\left(3\,Cac^2+Ac^3\right)e^3\right)x^{10}+\frac{1}{9}\left(Cc^3d^3+3\,Bc^3d^2e+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*C*c^3*e^3*x^12 + 1/11*(3*C*c^3*d*e^2 + B*c^3*e^3)*x^11 + 1/10*(3*C*c^3
*d^2*e + 3*B*c^3*d*e^2 + (3*C*a*c^2 + A*c^3)*e^3)*x^10 + 1/9*(C*c^3*d^3 + 3
*B*c^3*d^2*e + 3*B*a*c^2*e^3 + 3*(3*C*a*c^2 + A*c^3)*d*e^2)*x^9 + 1/8*(B*c^
3*d^3 + 9*B*a*c^2*d*e^2 + 3*(3*C*a*c^2 + A*c^3)*d^2*e + 3*(C*a^2*c + A*a*c^
2)*e^3)*x^8 + A*a^3*d^3*x + 1/7*(9*B*a*c^2*d^2*e + 3*B*a^2*c*e^3 + (3*C*a*c
^2 + A*c^3)*d^3 + 9*(C*a^2*c + A*a*c^2)*d*e^2)*x^7 + 1/6*(3*B*a*c^2*d^3 + 9
*B*a^2*c*d*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2*e + (C*a^3 + 3*A*a^2*c)*e^3)*x^6
 + 1/5*(9*B*a^2*c*d^2*e + B*a^3*e^3 + 3*(C*a^2*c + A*a*c^2)*d^3 + 3*(C*a^3
+ 3*A*a^2*c)*d*e^2)*x^5 + 1/4*(3*B*a^2*c*d^3 + 3*B*a^3*d*e^2 + A*a^3*e^3 +
3*(C*a^3 + 3*A*a^2*c)*d^2*e)*x^4 + 1/3*(3*B*a^3*d^2*e + 3*A*a^3*d*e^2 + (C*
a^3 + 3*A*a^2*c)*d^3)*x^3 + 1/2*(B*a^3*d^3 + 3*A*a^3*d^2*e)*x^2
```

**mupad [B]** time = 4.05, size = 490, normalized size = 1.21

$$x^5\left(\frac{3\,C\,a^3\,d\,e^2}{5}+\frac{B\,a^3\,e^3}{5}+\frac{3\,C\,a^2\,c\,d^3}{5}+\frac{9\,B\,a^2\,c\,d^2\,e}{5}+\frac{9\,A\,a^2\,c\,d\,e^2}{5}+\frac{3\,A\,a\,c^2\,d^3}{5}\right)+x^8\left(\frac{3\,C\,a^2\,c\,e^3}{8}+\frac{9\,C\,a\,c^2\,d^2\,e}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^5*((B*a^3*e^3)/5 + (3*A*a*c^2*d^3)/5 + (3*C*a^2*c*d^3)/5 + (3*C*a^3*d*e^2
)/5 + (9*A*a^2*c*d*e^2)/5 + (9*B*a^2*c*d^2*e)/5) + x^8*((B*c^3*d^3)/8 + (3*
A*a*c^2*e^3)/8 + (3*C*a^2*c*e^3)/8 + (3*A*c^3*d^2*e)/8 + (9*B*a*c^2*d*e^2)/
```

```
8 + (9*C*a*c^2*d^2*e)/8) + x^6*((C*a^3*e^3)/6 + (A*a^2*c*e^3)/2 + (B*a*c^2*
d^3)/2 + (3*A*a*c^2*d^2*e)/2 + (3*B*a^2*c*d*e^2)/2 + (3*C*a^2*c*d^2*e)/2) +
 x^7*((A*c^3*d^3)/7 + (3*B*a^2*c*e^3)/7 + (3*C*a*c^2*d^3)/7 + (9*A*a*c^2*d*
e^2)/7 + (9*B*a*c^2*d^2*e)/7 + (9*C*a^2*c*d*e^2)/7) + (a^2*x^4*(A*a*e^3 + 3
*B*c*d^3 + 3*B*a*d*e^2 + 9*A*c*d^2*e + 3*C*a*d^2*e))/4 + (c^2*x^9*(3*B*a*e^
3 + C*c*d^3 + 3*A*c*d*e^2 + 9*C*a*d*e^2 + 3*B*c*d^2*e))/9 + (C*c^3*e^3*x^12
)/12 + (a^3*d^2*x^2*(3*A*e + B*d))/2 + (c^3*e^2*x^11*(B*e + 3*C*d))/11 + A*
a^3*d^3*x + (a^2*d*x^3*(3*A*a*e^2 + 3*A*c*d^2 + C*a*d^2 + 3*B*a*d*e))/3 + (
c^2*e*x^10*(A*c*e^2 + 3*C*a*e^2 + 3*C*c*d^2 + 3*B*c*d*e))/10
```

**sympy [A]** time = 0.16, size = 646, normalized size = 1.60

$$Aa^3d^3x+\frac{Cc^3e^3x^{12}}{12}+x^{11}\left(\frac{Bc^3e^3}{11}+\frac{3Cc^3de^2}{11}\right)+x^{10}\left(\frac{Ac^3e^3}{10}+\frac{3Bc^3de^2}{10}+\frac{3Cac^2e^3}{10}+\frac{3Cc^3d^2e}{10}\right)+x^9\left(\frac{Ac^3de^2}{3}+\frac{Bac}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**3*x + C*c**3*e**3*x**12/12 + x**11*(B*c**3*e**3/11 + 3*C*c**3*d*e
**2/11) + x**10*(A*c**3*e**3/10 + 3*B*c**3*d*e**2/10 + 3*C*a*c**2*e**3/10 +
 3*C*c**3*d**2*e/10) + x**9*(A*c**3*d*e**2/3 + B*a*c**2*e**3/3 + B*c**3*d**
2*e/3 + C*a*c**2*d*e**2 + C*c**3*d**3/9) + x**8*(3*A*a*c**2*e**3/8 + 3*A*c*
*3*d**2*e/8 + 9*B*a*c**2*d*e**2/8 + B*c**3*d**3/8 + 3*C*a**2*c*e**3/8 + 9*C
*a*c**2*d**2*e/8) + x**7*(9*A*a*c**2*d*e**2/7 + A*c**3*d**3/7 + 3*B*a**2*c*
e**3/7 + 9*B*a*c**2*d**2*e/7 + 9*C*a**2*c*d*e**2/7 + 3*C*a*c**2*d**3/7) + x
**6*(A*a**2*c*e**3/2 + 3*A*a*c**2*d**2*e/2 + 3*B*a**2*c*d*e**2/2 + B*a*c**2
*d**3/2 + C*a**3*e**3/6 + 3*C*a**2*c*d**2*e/2) + x**5*(9*A*a**2*c*d*e**2/5
+ 3*A*a*c**2*d**3/5 + B*a**3*e**3/5 + 9*B*a**2*c*d**2*e/5 + 3*C*a**3*d*e**2
/5 + 3*C*a**2*c*d**3/5) + x**4*(A*a**3*e**3/4 + 9*A*a**2*c*d**2*e/4 + 3*B*a
**3*d*e**2/4 + 3*B*a**2*c*d**3/4 + 3*C*a**3*d**2*e/4) + x**3*(A*a**3*d*e**2
 + A*a**2*c*d**3 + B*a**3*d**2*e + C*a**3*d**3/3) + x**2*(3*A*a**3*d**2*e/2
 + B*a**3*d**3/2)
```

# 3.33 $\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=289

$$a^3 A d^2 x+\frac{1}{4} a^3 e x^4 (Be+2Cd)+\frac{1}{3} a^2 x^3 \left(A \left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+\frac{1}{2} a^2 c e x^6 (Be+2Cd)+\frac{1}{9} c^2 x^9 \left(3aCe^2+c \left(e(Ae\right.\right.$$

```
[Out] a^3*A*d^2*x+1/3*a^2*(a*d*(2*B*e+C*d)+A*(a*e^2+3*c*d^2))*x^3+1/4*a^3*e*(B*e+
2*C*d)*x^4+1/5*a*(3*A*c*(a*e^2+c*d^2)+a*(a*C*e^2+3*c*d*(2*B*e+C*d)))*x^5+1/
2*a^2*c*e*(B*e+2*C*d)*x^6+1/7*c*(A*c*(3*a*e^2+c*d^2)+3*a*(a*C*e^2+c*d*(2*B*
e+C*d)))*x^7+3/8*a*c^2*e*(B*e+2*C*d)*x^8+1/9*c^2*(3*a*C*e^2+c*(C*d^2+e*(A*e
+2*B*d)))*x^9+1/10*c^3*e*(B*e+2*C*d)*x^10+1/11*c^3*C*e^2*x^11+1/8*d*(2*A*e+
B*d)*(c*x^2+a)^4/c
```

**Rubi [A]** time = 0.42, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1582, 1810}

$$\frac{1}{3} a^2 x^3 \left(A \left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+a^3 A d^2 x+\frac{1}{2} a^2 c e x^6 (Be+2Cd)+\frac{1}{4} a^3 e x^4 (Be+2Cd)+\frac{1}{9} c^2 x^9 \left(3aCe^2+ce(Ae+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^2*(a*d*(C*d + 2*B*e) + A*(3*c*d^2 + a*e^2))*x^3)/3 + (a^3*
e*(2*C*d + B*e)*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*
d + 2*B*e)))*x^5)/5 + (a^2*c*e*(2*C*d + B*e)*x^6)/2 + (c*(A*c*(c*d^2 + 3*a*
e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (3*a*c^2*e*(2*C*d + B*e)
*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*(2*B*d + A*e))*x^9)/9 + (c^3*e*(2
*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11 + (d*(B*d + 2*A*e)*(a + c*x^2)^4
)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2 \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx = \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(-\left(Bd^2+2Ade\right)x+(d+$$

$$= \frac{d(Bd+2Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad^2+a^2\left(ad(Cd+2Be)+A\left(3cd\right.\right.\right.$$

$$= a^3Ad^2x + \frac{1}{3}a^2\left(ad(Cd+2Be)+A\left(3cd^2+ae^2\right)\right)x^3 + \frac{1}{4}a^3e(2Cd+$$

**Mathematica [A]** time = 0.13, size = 329, normalized size = 1.14

$$\frac{1}{2}a^3dx^2(2Ae+Bd)+a^3Ad^2x+\frac{1}{4}a^2x^4\left(aBe^2+2aCde+6Acde+3Bcd^2\right)+\frac{1}{3}a^2x^3\left(A\left(ae^2+3cd^2\right)+ad(2Be+Cd)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d^2*x + (a^3*d*(B*d + 2*A*e)*x^2)/2 + (a^2*(a*d*(C*d + 2*B*e) + A*(3*
c*d^2 + a*e^2))*x^3)/3 + (a^2*(3*B*c*d^2 + 6*A*c*d*e + 2*a*C*d*e + a*B*e^2)
*x^4)/4 + (a*(3*A*c*(c*d^2 + a*e^2) + a*(a*C*e^2 + 3*c*d*(C*d + 2*B*e)))*x^
5)/5 + (a*c*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2))*x^6)/2 + (c*(A*c*(c*d^2
 + 3*a*e^2) + 3*a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x^7)/7 + (c^2*(B*c*d^2 + 2
*A*c*d*e + 6*a*C*d*e + 3*a*B*e^2)*x^8)/8 + (c^2*(c*C*d^2 + 3*a*C*e^2 + c*e*
(2*B*d + A*e))*x^9)/9 + (c^3*e*(2*C*d + B*e)*x^10)/10 + (c^3*C*e^2*x^11)/11
```

**fricas [A]** time = 0.75, size = 432, normalized size = 1.49

$$\frac{1}{11}x^{11}e^2c^3C+\frac{1}{5}x^{10}edc^3C+\frac{1}{10}x^{10}e^2c^3B+\frac{1}{9}x^9d^2c^3C+\frac{1}{3}x^9e^2c^2aC+\frac{2}{9}x^9edc^3B+\frac{1}{9}x^9e^2c^3A+\frac{3}{4}x^8edc^2aC+\frac{1}{8}x^8d^2c^3B+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c^3*C + 1/5*x^10*e*d*c^3*C + 1/10*x^10*e^2*c^3*B + 1/9*x^9*d^
2*c^3*C + 1/3*x^9*e^2*c^2*a*C + 2/9*x^9*e*d*c^3*B + 1/9*x^9*e^2*c^3*A + 3/4
*x^8*e*d*c^2*a*C + 1/8*x^8*d^2*c^3*B + 3/8*x^8*e^2*c^2*a*B + 1/4*x^8*e*d*c^
3*A + 3/7*x^7*d^2*c^2*a*C + 3/7*x^7*e^2*c*a^2*C + 6/7*x^7*e*d*c^2*a*B + 1/7
```

```
*x^7*d^2*c^3*A + 3/7*x^7*e^2*c^2*a*A + x^6*e*d*c*a^2*C + 1/2*x^6*d^2*c^2*a*
B + 1/2*x^6*e^2*c*a^2*B + x^6*e*d*c^2*a*A + 3/5*x^5*d^2*c*a^2*C + 1/5*x^5*e
^2*a^3*C + 6/5*x^5*e*d*c*a^2*B + 3/5*x^5*d^2*c^2*a*A + 3/5*x^5*e^2*c*a^2*A
+ 1/2*x^4*e*d*a^3*C + 3/4*x^4*d^2*c*a^2*B + 1/4*x^4*e^2*a^3*B + 3/2*x^4*e*d
*c*a^2*A + 1/3*x^3*d^2*a^3*C + 2/3*x^3*e*d*a^3*B + x^3*d^2*c*a^2*A + 1/3*x^
3*e^2*a^3*A + 1/2*x^2*d^2*a^3*B + x^2*e*d*a^3*A + x*d^2*a^3*A
```

**giac [A]** time = 0.15, size = 432, normalized size = 1.49

$$\frac{1}{11}Cc^3x^{11}e^2+\frac{1}{5}Cc^3dx^{10}e+\frac{1}{9}Cc^3d^2x^9+\frac{1}{10}Bc^3x^{10}e^2+\frac{2}{9}Bc^3dx^9e+\frac{1}{8}Bc^3d^2x^8+\frac{1}{3}Cac^2x^9e^2+\frac{1}{9}Ac^3x^9e^2+\frac{3}{4}Cac^2dx^8e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^3*x^11*e^2 + 1/5*C*c^3*d*x^10*e + 1/9*C*c^3*d^2*x^9 + 1/10*B*c^3*x
^10*e^2 + 2/9*B*c^3*d*x^9*e + 1/8*B*c^3*d^2*x^8 + 1/3*C*a*c^2*x^9*e^2 + 1/9
*A*c^3*x^9*e^2 + 3/4*C*a*c^2*d*x^8*e + 1/4*A*c^3*d*x^8*e + 3/7*C*a*c^2*d^2*
x^7 + 1/7*A*c^3*d^2*x^7 + 3/8*B*a*c^2*x^8*e^2 + 6/7*B*a*c^2*d*x^7*e + 1/2*B
*a*c^2*d^2*x^6 + 3/7*C*a^2*c*x^7*e^2 + 3/7*A*a*c^2*x^7*e^2 + C*a^2*c*d*x^6*
e + A*a*c^2*d*x^6*e + 3/5*C*a^2*c*d^2*x^5 + 3/5*A*a*c^2*d^2*x^5 + 1/2*B*a^2
*c*x^6*e^2 + 6/5*B*a^2*c*d*x^5*e + 3/4*B*a^2*c*d^2*x^4 + 1/5*C*a^3*x^5*e^2
+ 3/5*A*a^2*c*x^5*e^2 + 1/2*C*a^3*d*x^4*e + 3/2*A*a^2*c*d*x^4*e + 1/3*C*a^3
*d^2*x^3 + A*a^2*c*d^2*x^3 + 1/4*B*a^3*x^4*e^2 + 2/3*B*a^3*d*x^3*e + 1/2*B*
a^3*d^2*x^2 + 1/3*A*a^3*x^3*e^2 + A*a^3*d*x^2*e + A*a^3*d^2*x
```

**maple [A]** time = 0.00, size = 388, normalized size = 1.34

$$\frac{C c^3e^2x^{11}}{11}+\frac{\left(e^2c^3B+2de\,c^3C\right)x^{10}}{10}+\frac{\left(A\,c^3e^2+2B\,c^3de+\left(3e^2a\,c^2+d^2c^3\right)C\right)x^9}{9}+\frac{\left(2A\,c^3de+6Ca\,c^2de+\left(3e^2a\,c^2+\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/11*c^3*C*e^2*x^11+1/10*(B*c^3*e^2+2*C*c^3*d*e)*x^10+1/9*((3*a*c^2*e^2+c^3
*d^2)*C+2*d*e*c^3*B+e^2*c^3*A)*x^9+1/8*(6*d*e*a*c^2*C+(3*a*c^2*e^2+c^3*d^2)
*B+2*d*e*c^3*A)*x^8+1/7*((3*a^2*c*e^2+3*a*c^2*d^2)*C+6*d*e*a*c^2*B+(3*a*c^2
*e^2+c^3*d^2)*A)*x^7+1/6*(6*d*e*a^2*c*C+(3*a^2*c*e^2+3*a*c^2*d^2)*B+6*d*e*a
*c^2*A)*x^6+1/5*((a^3*e^2+3*a^2*c*d^2)*C+6*d*e*a^2*c*B+(3*a^2*c*e^2+3*a*c^2
*d^2)*A)*x^5+1/4*(2*d*e*a^3*C+(a^3*e^2+3*a^2*c*d^2)*B+6*d*e*a^2*c*A)*x^4+1/
3*(d^2*a^3*C+2*d*e*a^3*B+(a^3*e^2+3*a^2*c*d^2)*A)*x^3+1/2*(2*A*a^3*d*e+B*a^
3*d^2)*x^2+a^3*A*d^2*x
```

**maxima [A]** time = 0.45, size = 367, normalized size = 1.27

$$\frac{1}{11}Cc^3e^2x^{11}+\frac{1}{10}\left(2\,Cc^3de+Bc^3e^2\right)x^{10}+\frac{1}{9}\left(Cc^3d^2+2\,Bc^3de+\left(3\,Cac^2+Ac^3\right)e^2\right)x^9+\frac{1}{8}\left(Bc^3d^2+3\,Bac^2e^2+2\left(3\,C\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^3*e^2*x^11 + 1/10*(2*C*c^3*d*e + B*c^3*e^2)*x^10 + 1/9*(C*c^3*d^2
+ 2*B*c^3*d*e + (3*C*a*c^2 + A*c^3)*e^2)*x^9 + 1/8*(B*c^3*d^2 + 3*B*a*c^2*e
^2 + 2*(3*C*a*c^2 + A*c^3)*d*e)*x^8 + 1/7*(6*B*a*c^2*d*e + (3*C*a*c^2 + A*c
^3)*d^2 + 3*(C*a^2*c + A*a*c^2)*e^2)*x^7 + A*a^3*d^2*x + 1/2*(B*a*c^2*d^2 +
 B*a^2*c*e^2 + 2*(C*a^2*c + A*a*c^2)*d*e)*x^6 + 1/5*(6*B*a^2*c*d*e + 3*(C*a
^2*c + A*a*c^2)*d^2 + (C*a^3 + 3*A*a^2*c)*e^2)*x^5 + 1/4*(3*B*a^2*c*d^2 + B
*a^3*e^2 + 2*(C*a^3 + 3*A*a^2*c)*d*e)*x^4 + 1/3*(2*B*a^3*d*e + A*a^3*e^2 +
(C*a^3 + 3*A*a^2*c)*d^2)*x^3 + 1/2*(B*a^3*d^2 + 2*A*a^3*d*e)*x^2
```

**mupad [B]** time = 3.94, size = 343, normalized size = 1.19

$$x^3\left(\frac{C\,a^3\,d^2}{3}+\frac{2\,B\,a^3\,d\,e}{3}+\frac{A\,a^3\,e^2}{3}+A\,c\,a^2\,d^2\right)+x^9\left(\frac{C\,c^3\,d^2}{9}+\frac{2\,B\,c^3\,d\,e}{9}+\frac{A\,c^3\,e^2}{9}+\frac{C\,a\,c^2\,e^2}{3}\right)+x^5\left(\frac{C\,a^3\,e^2}{5}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)^2*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((A*a^3*e^2)/3 + (C*a^3*d^2)/3 + (2*B*a^3*d*e)/3 + A*a^2*c*d^2) + x^9*(
(A*c^3*e^2)/9 + (C*c^3*d^2)/9 + (2*B*c^3*d*e)/9 + (C*a*c^2*e^2)/3) + x^5*((
C*a^3*e^2)/5 + (3*A*a*c^2*d^2)/5 + (3*A*a^2*c*e^2)/5 + (3*C*a^2*c*d^2)/5 +
(6*B*a^2*c*d*e)/5) + x^7*((A*c^3*d^2)/7 + (3*A*a*c^2*e^2)/7 + (3*C*a*c^2*d^
2)/7 + (3*C*a^2*c*e^2)/7 + (6*B*a*c^2*d*e)/7) + (a^2*x^4*(B*a*e^2 + 3*B*c*d
^2 + 6*A*c*d*e + 2*C*a*d*e))/4 + (c^2*x^8*(3*B*a*e^2 + B*c*d^2 + 2*A*c*d*e
+ 6*C*a*d*e))/8 + (C*c^3*e^2*x^11)/11 + (a*c*x^6*(B*a*e^2 + B*c*d^2 + 2*A*c
*d*e + 2*C*a*d*e))/2 + A*a^3*d^2*x + (a^3*d*x^2*(2*A*e + B*d))/2 + (c^3*e*x
^10*(B*e + 2*C*d))/10
```

**sympy [A]** time = 0.15, size = 447, normalized size = 1.55

$$Aa^3d^2x+\frac{Cc^3e^2x^{11}}{11}+x^{10}\left(\frac{Bc^3e^2}{10}+\frac{Cc^3de}{5}\right)+x^9\left(\frac{Ac^3e^2}{9}+\frac{2Bc^3de}{9}+\frac{Cac^2e^2}{3}+\frac{Cc^3d^2}{9}\right)+x^8\left(\frac{Ac^3de}{4}+\frac{3Bac^2e^2}{8}+\frac{B}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d**2*x + C*c**3*e**2*x**11/11 + x**10*(B*c**3*e**2/10 + C*c**3*d*e/5
) + x**9*(A*c**3*e**2/9 + 2*B*c**3*d*e/9 + C*a*c**2*e**2/3 + C*c**3*d**2/9)
 + x**8*(A*c**3*d*e/4 + 3*B*a*c**2*e**2/8 + B*c**3*d**2/8 + 3*C*a*c**2*d*e/
4) + x**7*(3*A*a*c**2*e**2/7 + A*c**3*d**2/7 + 6*B*a*c**2*d*e/7 + 3*C*a**2*
c*e**2/7 + 3*C*a*c**2*d**2/7) + x**6*(A*a*c**2*d*e + B*a**2*c*e**2/2 + B*a*
```

```
c**2*d**2/2 + C*a**2*c*d*e) + x**5*(3*A*a**2*c*e**2/5 + 3*A*a*c**2*d**2/5 +
 6*B*a**2*c*d*e/5 + C*a**3*e**2/5 + 3*C*a**2*c*d**2/5) + x**4*(3*A*a**2*c*d
*e/2 + B*a**3*e**2/4 + 3*B*a**2*c*d**2/4 + C*a**3*d*e/2) + x**3*(A*a**3*e**
2/3 + A*a**2*c*d**2 + 2*B*a**3*d*e/3 + C*a**3*d**2/3) + x**2*(A*a**3*d*e +
B*a**3*d**2/2)
```

# 3.34 $\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)\,dx$

**Optimal**. Leaf size=169

$$a^3Adx+\frac{1}{4}a^3Cex^4+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{2}a^2cCex^6+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{(a\cdots}{}$$

```
[Out] a^3*A*d*x+1/3*a^2*(3*A*c*d+B*a*e+C*a*d)*x^3+1/4*a^3*C*e*x^4+3/5*a*c*(A*c*d+
B*a*e+C*a*d)*x^5+1/2*a^2*c*C*e*x^6+1/7*c^2*(A*c*d+3*a*(B*e+C*d))*x^7+3/8*a*
c^2*C*e*x^8+1/9*c^3*(B*e+C*d)*x^9+1/10*c^3*C*e*x^10+1/8*(A*e+B*d)*(c*x^2+a)
^4/c
```

**Rubi [A]** time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1582, 1810}

$$\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+a^3Adx+\frac{1}{2}a^2cCex^6+\frac{1}{4}a^3Cex^4+\frac{1}{7}c^2x^7(3a(Be+Cd)+Acd)+\frac{3}{5}acx^5(aBe+aCd+Acd)+\frac{(a\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3 + (a^3*C*e*x^4)/4 + (3*a*
c*(A*c*d + a*C*d + a*B*e)*x^5)/5 + (a^2*c*C*e*x^6)/2 + (c^2*(A*c*d + 3*a*(C
*d + B*e))*x^7)/7 + (3*a*c^2*C*e*x^8)/8 + (c^3*(C*d + B*e)*x^9)/9 + (c^3*C*
e*x^10)/10 + ((B*d + A*e)*(a + c*x^2)^4)/(8*c)
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)dx = \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3\left(-(Bd+Ae)x+(d+ex)\left(A+B\right.\right.$$
$$= \frac{(Bd+Ae)\left(a+cx^2\right)^4}{8c} + \int \left(a^3Ad + a^2(3Acd+aCd+aBe)x^2 + a^3Ce\right.$$
$$= a^3Adx + \frac{1}{3}a^2(3Acd+aCd+aBe)x^3 + \frac{1}{4}a^3Cex^4 + \frac{3}{5}ac(Acd+aCd+$$

**Mathematica [A]** time = 0.07, size = 196, normalized size = 1.16

$$\frac{1}{2}a^3x^2(Ae+Bd)+a^3Adx+\frac{1}{4}a^2x^4(aCe+3Ace+3Bcd)+\frac{1}{3}a^2x^3(aBe+aCd+3Acd)+\frac{1}{8}c^2x^8(3aCe+Ace+Bcd)+\frac{1}{7}c^2x^7(3a$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*d*x + (a^3*(B*d + A*e)*x^2)/2 + (a^2*(3*A*c*d + a*C*d + a*B*e)*x^3)/3
 + (a^2*(3*B*c*d + 3*A*c*e + a*C*e)*x^4)/4 + (3*a*c*(A*c*d + a*C*d + a*B*e)
*x^5)/5 + (a*c*(B*c*d + A*c*e + a*C*e)*x^6)/2 + (c^2*(A*c*d + 3*a*C*d + 3*a
*B*e)*x^7)/7 + (c^2*(B*c*d + A*c*e + 3*a*C*e)*x^8)/8 + (c^3*(C*d + B*e)*x^9
)/9 + (c^3*C*e*x^10)/10
```

**fricas [A]** time = 0.77, size = 249, normalized size = 1.47

$$\frac{1}{10}x^{10}ec^3C+\frac{1}{9}x^9dc^3C+\frac{1}{9}x^9ec^3B+\frac{3}{8}x^8ec^2aC+\frac{1}{8}x^8dc^3B+\frac{1}{8}x^8ec^3A+\frac{3}{7}x^7dc^2aC+\frac{3}{7}x^7ec^2aB+\frac{1}{7}x^7dc^3A+\frac{1}{2}x^6eca^2C+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c^3*C + 1/9*x^9*d*c^3*C + 1/9*x^9*e*c^3*B + 3/8*x^8*e*c^2*a*C +
 1/8*x^8*d*c^3*B + 1/8*x^8*e*c^3*A + 3/7*x^7*d*c^2*a*C + 3/7*x^7*e*c^2*a*B
+ 1/7*x^7*d*c^3*A + 1/2*x^6*e*c*a^2*C + 1/2*x^6*d*c^2*a*B + 1/2*x^6*e*c^2*a
*A + 3/5*x^5*d*c*a^2*C + 3/5*x^5*e*c*a^2*B + 3/5*x^5*d*c^2*a*A + 1/4*x^4*e*
a^3*C + 3/4*x^4*d*c*a^2*B + 3/4*x^4*e*c*a^2*A + 1/3*x^3*d*a^3*C + 1/3*x^3*e
*a^3*B + x^3*d*c*a^2*A + 1/2*x^2*d*a^3*B + 1/2*x^2*e*a^3*A + x*d*a^3*A
```

**giac [A]** time = 0.19, size = 261, normalized size = 1.54

$$\frac{1}{10}Cc^3x^{10}e+\frac{1}{9}Cc^3dx^9+\frac{1}{9}Bc^3x^9e+\frac{1}{8}Bc^3dx^8+\frac{3}{8}Cac^2x^8e+\frac{1}{8}Ac^3x^8e+\frac{3}{7}Cac^2dx^7+\frac{1}{7}Ac^3dx^7+\frac{3}{7}Bac^2x^7e+\frac{1}{2}Bac^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*C*c^3*x^10*e + 1/9*C*c^3*d*x^9 + 1/9*B*c^3*x^9*e + 1/8*B*c^3*d*x^8 + 3
/8*C*a*c^2*x^8*e + 1/8*A*c^3*x^8*e + 3/7*C*a*c^2*d*x^7 + 1/7*A*c^3*d*x^7 +
3/7*B*a*c^2*x^7*e + 1/2*B*a*c^2*d*x^6 + 1/2*C*a^2*c*x^6*e + 1/2*A*a*c^2*x^6
*e + 3/5*C*a^2*c*d*x^5 + 3/5*A*a*c^2*d*x^5 + 3/5*B*a^2*c*x^5*e + 3/4*B*a^2*
c*d*x^4 + 1/4*C*a^3*x^4*e + 3/4*A*a^2*c*x^4*e + 1/3*C*a^3*d*x^3 + A*a^2*c*d
*x^3 + 1/3*B*a^3*x^3*e + 1/2*B*a^3*d*x^2 + 1/2*A*a^3*x^2*e + A*a^3*d*x
```

**maple [A]** time = 0.00, size = 223, normalized size = 1.32

$$\frac{C c^3 e x^{10}}{10}+\frac{\left(e c^3 B+c^3 d C\right) x^9}{9}+\frac{\left(e c^3 A+c^3 d B+3 e a c^2 C\right) x^8}{8}+\frac{\left(c^3 d A+3 e a c^2 B+3 d a c^2 C\right) x^7}{7}+A a^3 d x+\frac{\left(3 e a c^2 A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/10*c^3*C*e*x^10+1/9*(B*c^3*e+C*c^3*d)*x^9+1/8*(A*c^3*e+B*c^3*d+3*C*a*c^2*
e)*x^8+1/7*(A*c^3*d+3*B*a*c^2*e+3*C*a*c^2*d)*x^7+1/6*(3*A*a*c^2*e+3*B*a*c^2
*d+3*C*a^2*c*e)*x^6+1/5*(3*A*a*c^2*d+3*B*a^2*c*e+3*C*a^2*c*d)*x^5+1/4*(3*A*
a^2*c*e+3*B*a^2*c*d+C*a^3*e)*x^4+1/3*(3*A*a^2*c*d+B*a^3*e+C*a^3*d)*x^3+1/2*
(A*a^3*e+B*a^3*d)*x^2+a^3*A*d*x
```

**maxima [A]** time = 0.44, size = 222, normalized size = 1.31

$$\frac{1}{10} C c^3 e x^{10}+\frac{1}{9} \left(C c^3 d+B c^3 e\right) x^9+\frac{1}{8} \left(B c^3 d+\left(3 C a c^2+A c^3\right) e\right) x^8+\frac{1}{7} \left(3 B a c^2 e+\left(3 C a c^2+A c^3\right) d\right) x^7+\frac{1}{2} \left(B a c^2 d+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*C*c^3*e*x^10 + 1/9*(C*c^3*d + B*c^3*e)*x^9 + 1/8*(B*c^3*d + (3*C*a*c^2
 + A*c^3)*e)*x^8 + 1/7*(3*B*a*c^2*e + (3*C*a*c^2 + A*c^3)*d)*x^7 + 1/2*(B*a
*c^2*d + (C*a^2*c + A*a*c^2)*e)*x^6 + A*a^3*d*x + 3/5*(B*a^2*c*e + (C*a^2*c
 + A*a*c^2)*d)*x^5 + 1/4*(3*B*a^2*c*d + (C*a^3 + 3*A*a^2*c)*e)*x^4 + 1/3*(B
*a^3*e + (C*a^3 + 3*A*a^2*c)*d)*x^3 + 1/2*(B*a^3*d + A*a^3*e)*x^2
```

**mupad [B]** time = 0.10, size = 187, normalized size = 1.11

$$x^3 \left(\frac{B a^3 e}{3}+\frac{C a^3 d}{3}+A a^2 c d\right)+x^8 \left(\frac{A c^3 e}{8}+\frac{B c^3 d}{8}+\frac{3 C a c^2 e}{8}\right)+\frac{a^3 x^2 (A e+B d)}{2}+\frac{c^3 x^9 (B e+C d)}{9}+\frac{c^2 x^7 (A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(d + e*x)*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((B*a^3*e)/3 + (C*a^3*d)/3 + A*a^2*c*d) + x^8*((A*c^3*e)/8 + (B*c^3*d)/
8 + (3*C*a*c^2*e)/8) + (a^3*x^2*(A*e + B*d))/2 + (c^3*x^9*(B*e + C*d))/9 +
(c^2*x^7*(A*c*d + 3*B*a*e + 3*C*a*d))/7 + (a^2*x^4*(3*A*c*e + 3*B*c*d + C*a
*e))/4 + A*a^3*d*x + (3*a*c*x^5*(A*c*d + B*a*e + C*a*d))/5 + (a*c*x^6*(A*c*
e + B*c*d + C*a*e))/2 + (C*c^3*e*x^10)/10
```

**sympy [A]** time = 0.11, size = 265, normalized size = 1.57

$$Aa^3dx+\frac{Cc^3ex^{10}}{10}+x^9\left(\frac{Bc^3e}{9}+\frac{Cc^3d}{9}\right)+x^8\left(\frac{Ac^3e}{8}+\frac{Bc^3d}{8}+\frac{3Cac^2e}{8}\right)+x^7\left(\frac{Ac^3d}{7}+\frac{3Bac^2e}{7}+\frac{3Cac^2d}{7}\right)+x^6\left(\frac{Aac^2e}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*d*x + C*c**3*e*x**10/10 + x**9*(B*c**3*e/9 + C*c**3*d/9) + x**8*(A*c
**3*e/8 + B*c**3*d/8 + 3*C*a*c**2*e/8) + x**7*(A*c**3*d/7 + 3*B*a*c**2*e/7
+ 3*C*a*c**2*d/7) + x**6*(A*a*c**2*e/2 + B*a*c**2*d/2 + C*a**2*c*e/2) + x**
5*(3*A*a*c**2*d/5 + 3*B*a**2*c*e/5 + 3*C*a**2*c*d/5) + x**4*(3*A*a**2*c*e/4
 + 3*B*a**2*c*d/4 + C*a**3*e/4) + x**3*(A*a**2*c*d + B*a**3*e/3 + C*a**3*d/
3) + x**2*(A*a**3*e/2 + B*a**3*d/2)
```

# 3.35 $\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=87

$$a^3Ax+\frac{1}{3}a^2x^3(aC+3Ac)+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*c+C*a)*x^3+3/5*a*c*(A*c+C*a)*x^5+1/7*c^2*(A*c+3*C*a)*x
^7+1/9*c^3*C*x^9+1/8*B*(c*x^2+a)^4/c
```

**Rubi [A]** time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1582, 373}

$$\frac{1}{3}a^2x^3(aC+3Ac)+a^3Ax+\frac{1}{7}c^2x^7(3aC+Ac)+\frac{3}{5}acx^5(aC+Ac)+\frac{B\left(a+cx^2\right)^4}{8c}+\frac{1}{9}c^3Cx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*c + a*C)*x^3)/3 + (3*a*c*(A*c + a*C)*x^5)/5 + (c^2*(A*c
 + 3*a*C)*x^7)/7 + (c^3*C*x^9)/9 + (B*(a + c*x^2)^4)/(8*c)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 1582

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(Coeff[Px, x, n
- 1]*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rubi steps

$$\int \left(a+cx^2\right)^3 \left(A+Bx+Cx^2\right) dx = \frac{B\left(a+cx^2\right)^4}{8c} + \int \left(a+cx^2\right)^3 \left(A+Cx^2\right) dx$$
$$= \frac{B\left(a+cx^2\right)^4}{8c} + \int \left(a^3A + a^2(3Ac+aC)x^2 + 3ac(Ac+aC)x^4 + c^2(Ac+3aC)\right.$$
$$= a^3Ax + \frac{1}{3}a^2(3Ac+aC)x^3 + \frac{3}{5}ac(Ac+aC)x^5 + \frac{1}{7}c^2(Ac+3aC)x^7 + \frac{1}{9}c^3Cx^9$$

**Mathematica** [**A**] time = 0.03, size = 100, normalized size = 1.15

$$\frac{1}{6}a^3x(6A+x(3B+2Cx))+\frac{1}{20}a^2cx^3(20A+3x(5B+4Cx))+\frac{1}{70}ac^2x^5(42A+5x(7B+6Cx))+\frac{1}{504}c^3x^7(72A+7x(9B+8C$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^3*(A + B*x + C*x^2),x]
```

```
[Out] (a^3*x*(6*A + x*(3*B + 2*C*x)))/6 + (a^2*c*x^3*(20*A + 3*x*(5*B + 4*C*x)))/
20 + (a*c^2*x^5*(42*A + 5*x*(7*B + 6*C*x)))/70 + (c^3*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/504
```

**fricas** [**A**] time = 0.77, size = 111, normalized size = 1.28

$$\frac{1}{9}x^9c^3C+\frac{1}{8}x^8c^3B+\frac{3}{7}x^7c^2aC+\frac{1}{7}x^7c^3A+\frac{1}{2}x^6c^2aB+\frac{3}{5}x^5ca^2C+\frac{3}{5}x^5c^2aA+\frac{3}{4}x^4ca^2B+\frac{1}{3}x^3a^3C+x^3ca^2A+\frac{1}{2}x^2a^3B+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 1/8*x^8*c^3*B + 3/7*x^7*c^2*a*C + 1/7*x^7*c^3*A + 1/2*x^6*c
^2*a*B + 3/5*x^5*c*a^2*C + 3/5*x^5*c^2*a*A + 3/4*x^4*c*a^2*B + 1/3*x^3*a^3*
C + x^3*c*a^2*A + 1/2*x^2*a^3*B + x*a^3*A
```

**giac** [**A**] time = 0.19, size = 111, normalized size = 1.28

$$\frac{1}{9}Cc^3x^9+\frac{1}{8}Bc^3x^8+\frac{3}{7}Cac^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bac^2x^6+\frac{3}{5}Ca^2cx^5+\frac{3}{5}Aac^2x^5+\frac{3}{4}Ba^2cx^4+\frac{1}{3}Ca^3x^3+Aa^2cx^3+\frac{1}{2}Ba^3x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 3/7*C*a*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*a*c
^2*x^6 + 3/5*C*a^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*B*a^2*c*x^4 + 1/3*C*a^3*x^
3 + A*a^2*c*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

**maple** [A] time = 0.00, size = 111, normalized size = 1.28

$$\frac{C c^3x^9}{9}+\frac{B c^3x^8}{8}+\frac{Ba c^2x^6}{2}+\frac{3B a^2c x^4}{4}+\frac{\left(c^3A+3a c^2C\right)x^7}{7}+\frac{B a^3x^2}{2}+A a^3x+\frac{\left(3a c^2A+3a^2cC\right)x^5}{5}+\frac{\left(3a^2cA+a^3C\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A),x)
```

```
[Out] 1/9*c^3*C*x^9+1/8*c^3*B*x^8+1/7*(A*c^3+3*C*a*c^2)*x^7+1/2*a*c^2*B*x^6+1/5*(
3*A*a*c^2+3*C*a^2*c)*x^5+3/4*a^2*c*B*x^4+1/3*(3*A*a^2*c+C*a^3)*x^3+1/2*a^3*
B*x^2+a^3*A*x
```

**maxima** [A] time = 0.43, size = 108, normalized size = 1.24

$$\frac{1}{9}Cc^3x^9+\frac{1}{8}Bc^3x^8+\frac{1}{2}Bac^2x^6+\frac{3}{4}Ba^2cx^4+\frac{1}{7}\left(3Cac^2+Ac^3\right)x^7+\frac{1}{2}Ba^3x^2+\frac{3}{5}\left(Ca^2c+Aac^2\right)x^5+Aa^3x+\frac{1}{3}\left(Ca^3+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 1/8*B*c^3*x^8 + 1/2*B*a*c^2*x^6 + 3/4*B*a^2*c*x^4 + 1/7*(3*
C*a*c^2 + A*c^3)*x^7 + 1/2*B*a^3*x^2 + 3/5*(C*a^2*c + A*a*c^2)*x^5 + A*a^3*
x + 1/3*(C*a^3 + 3*A*a^2*c)*x^3
```

**mupad** [B] time = 0.06, size = 103, normalized size = 1.18

$$x^3\left(\frac{C a^3}{3}+A c a^2\right)+x^7\left(\frac{A c^3}{7}+\frac{3 C a c^2}{7}\right)+\frac{B a^3x^2}{2}+\frac{B c^3x^8}{8}+\frac{C c^3x^9}{9}+A a^3x+\frac{3 a c x^5\left(A c+C a\right)}{5}+\frac{3 B a^2 c x^4}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^3*(A + B*x + C*x^2),x)
```

```
[Out] x^3*((C*a^3)/3 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7) + (B*a^3*x^2)/2
 + (B*c^3*x^8)/8 + (C*c^3*x^9)/9 + A*a^3*x + (3*a*c*x^5*(A*c + C*a))/5 + (3
*B*a^2*c*x^4)/4 + (B*a*c^2*x^6)/2
```

**sympy** [A] time = 0.09, size = 122, normalized size = 1.40

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{3Ba^2cx^4}{4}+\frac{Bac^2x^6}{2}+\frac{Bc^3x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}\right)+x^5\left(\frac{3Aac^2}{5}+\frac{3Ca^2c}{5}\right)+x^3\left(Aa^2c+\frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + 3*B*a**2*c*x**4/4 + B*a*c**2*x**6/2 + B*c**3*x**
8/8 + C*c**3*x**9/9 + x**7*(A*c**3/7 + 3*C*a*c**2/7) + x**5*(3*A*a*c**2/5 +
 3*C*a**2*c/5) + x**3*(A*a**2*c + C*a**3/3)
```

# 3.36 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{d+ex}\,dx$

**Optimal**. Leaf size=490

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2d^2\left(14Cd^2-e(7Bd-3Ae)\right)\right)}{4e^9}+\frac{(d+ex)^2\left(ae^2+cd^2\right)\left(a^2C\right.}{}$$

```
[Out] -(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*x/e^8+
1/2*(a*e^2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*
(17*C*d^2-3*e*(-A*e+3*B*d)))*(e*x+d)^2/e^9-1/3*c*(3*a^2*e^4*(-B*e+4*C*d)+c^
2*d^3*(56*C*d^2-5*e*(-4*A*e+7*B*d))+6*a*c*d*e^2*(10*C*d^2-e*(-2*A*e+5*B*d))
)*(e*x+d)^3/e^9+1/4*c*(3*a^2*C*e^4+5*c^2*d^2*(14*C*d^2-e*(-3*A*e+7*B*d))+3*
a*c*e^2*(15*C*d^2-e*(-A*e+5*B*d)))*(e*x+d)^4/e^9-1/5*c^2*(3*a*e^2*(-B*e+6*C
*d)+c*d*(56*C*d^2-3*e*(-2*A*e+7*B*d)))*(e*x+d)^5/e^9+1/6*c^2*(3*a*C*e^2+c*(
28*C*d^2-e*(-A*e+7*B*d)))*(e*x+d)^6/e^9-1/7*c^3*(-B*e+8*C*d)*(e*x+d)^7/e^9+
1/8*c^3*C*(e*x+d)^8/e^9+(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e^9
```

**Rubi [A]** time = 1.10, antiderivative size = 487, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{c(d+ex)^4\left(3a^2Ce^4+3ace^2\left(15Cd^2-e(5Bd-Ae)\right)+5c^2\left(14Cd^4-d^2e(7Bd-3Ae)\right)\right)}{4e^9}-\frac{c(d+ex)^3\left(3a^2e^4(4Cd-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] -(((c*d^2 + a*e^2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*
e))*x)/e^8) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d
- 5*A*e)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*(d + e*x)^2)/(2*e^9) -
(c*(3*a^2*e^4*(4*C*d - B*e) + c^2*(56*C*d^5 - 5*d^3*e*(7*B*d - 4*A*e)) + 6*
a*c*d*e^2*(10*C*d^2 - e*(5*B*d - 2*A*e)))*(d + e*x)^3)/(3*e^9) + (c*(3*a^2*
C*e^4 + 5*c^2*(14*C*d^4 - d^2*e*(7*B*d - 3*A*e)) + 3*a*c*e^2*(15*C*d^2 - e*
(5*B*d - A*e)))*(d + e*x)^4)/(4*e^9) - (c^2*(56*c*C*d^3 - 3*c*d*e*(7*B*d -
2*A*e) + 3*a*e^2*(6*C*d - B*e))*(d + e*x)^5)/(5*e^9) + (c^2*(28*c*C*d^2 + 3
*a*C*e^2 - c*e*(7*B*d - A*e))*(d + e*x)^6)/(6*e^9) - (c^3*(8*C*d - B*e)*(d
+ e*x)^7)/(7*e^9) + (c^3*C*(d + e*x)^8)/(8*e^9) + ((c*d^2 + a*e^2)^3*(C*d^2
 - B*d*e + A*e^2)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{d+ex}\,dx = \int \left(\frac{\left(cd^2+ae^2\right)^2\left(-8cCd^3+cde(7Bd-6Ae)-ae^2(2Cd-Be)\right)}{e^8}+\frac{\left(cd^2+ae^2\right)^3}{e\right.$$

$$= -\frac{\left(cd^2+ae^2\right)^2\left(8cCd^3-cde(7Bd-6Ae)+ae^2(2Cd-Be)\right)x}{e^8}+\frac{\left(cd^2+ae^2\right)\left(a\right.}{}$$

**Mathematica [A]** time = 0.47, size = 498, normalized size = 1.02

$$\frac{x\left(420a^3e^6(2Be-2Cd+Cex)+210a^2ce^4\left(2e\left(3Ae(ex-2d)+B\left(6d^2-3dex+2e^2x^2\right)\right)+C\left(-12d^3+6d^2ex-4de\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x]
```

```
[Out] (x*(420*a^3*e^6*(-2*C*d + 2*B*e + C*e*x) + 210*a^2*c*e^4*(C*(-12*d^3 + 6*d^
2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 2*e*(3*A*e*(-2*d + e*x) + B*(6*d^2 - 3*d
*e*x + 2*e^2*x^2))) + 42*a*c^2*e^2*(C*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^
2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + e*(5*A*e*(-12*d^3 + 6*d^2
*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + B*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 -
 15*d*e^3*x^3 + 12*e^4*x^4))) + c^3*(C*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^
2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6
 + 105*e^7*x^7) + 2*e*(7*A*e*(-60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^
2*e^3*x^3 - 12*d*e^4*x^4 + 10*e^5*x^5) + B*(420*d^6 - 210*d^5*e*x + 140*d^4
*e^2*x^2 - 105*d^3*e^3*x^3 + 84*d^2*e^4*x^4 - 70*d*e^5*x^5 + 60*e^6*x^6))))
)/(840*e^8) + ((c*d^2 + a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e))*Log[d + e*x])/e
^9
```

**fricas [A]** time = 0.87, size = 674, normalized size = 1.38

$$\frac{105\,Cc^3e^8x^8-120\left(Cc^3de^7-Bc^3e^8\right)x^7+140\left(Cc^3d^2e^6-Bc^3de^7+\left(3\,Cac^2+Ac^3\right)e^8\right)x^6-168\left(Cc^3d^3e^5-Bc^3d^2e^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^3*e^8*x^8 - 120*(C*c^3*d*e^7 - B*c^3*e^8)*x^7 + 140*(C*c^3*d
^2*e^6 - B*c^3*d*e^7 + (3*C*a*c^2 + A*c^3)*e^8)*x^6 - 168*(C*c^3*d^3*e^5 -
```

```
B*c^3*d^2*e^6 - 3*B*a*c^2*e^8 + (3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 210*(C*c^3
*d^4*e^4 - B*c^3*d^3*e^5 - 3*B*a*c^2*d*e^7 + (3*C*a*c^2 + A*c^3)*d^2*e^6 +
3*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 280*(C*c^3*d^5*e^3 - B*c^3*d^4*e^4 - 3*B*a
*c^2*d^2*e^6 - 3*B*a^2*c*e^8 + (3*C*a*c^2 + A*c^3)*d^3*e^5 + 3*(C*a^2*c + A
*a*c^2)*d*e^7)*x^3 + 420*(C*c^3*d^6*e^2 - B*c^3*d^5*e^3 - 3*B*a*c^2*d^3*e^5
 - 3*B*a^2*c*d*e^7 + (3*C*a*c^2 + A*c^3)*d^4*e^4 + 3*(C*a^2*c + A*a*c^2)*d^
2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 - 840*(C*c^3*d^7*e - B*c^3*d^6*e^2 - 3
*B*a*c^2*d^4*e^4 - 3*B*a^2*c*d^2*e^6 - B*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^5*
e^3 + 3*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)*x + 840*(C
*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^
7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4
 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d))/e^9
```

**giac [A]** time = 0.17, size = 764, normalized size = 1.56

$$\left(Cc^3d^8 - Bc^3d^7e + 3\,Cac^2d^6e^2 + Ac^3d^6e^2 - 3\,Bac^2d^5e^3 + 3\,Ca^2cd^4e^4 + 3\,Aac^2d^4e^4 - 3\,Ba^2cd^3e^5 + Ca^3d^2e^6 + 3\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="giac")
```

```
[Out] (C*c^3*d^8 - B*c^3*d^7*e + 3*C*a*c^2*d^6*e^2 + A*c^3*d^6*e^2 - 3*B*a*c^2*d^
5*e^3 + 3*C*a^2*c*d^4*e^4 + 3*A*a*c^2*d^4*e^4 - 3*B*a^2*c*d^3*e^5 + C*a^3*d
^2*e^6 + 3*A*a^2*c*d^2*e^6 - B*a^3*d*e^7 + A*a^3*e^8)*e^(-9)*log(abs(x*e +
d)) + 1/840*(105*C*c^3*x^8*e^7 - 120*C*c^3*d*x^7*e^6 + 140*C*c^3*d^2*x^6*e^
5 - 168*C*c^3*d^3*x^5*e^4 + 210*C*c^3*d^4*x^4*e^3 - 280*C*c^3*d^5*x^3*e^2 +
 420*C*c^3*d^6*x^2*e - 840*C*c^3*d^7*x + 120*B*c^3*x^7*e^7 - 140*B*c^3*d*x^
6*e^6 + 168*B*c^3*d^2*x^5*e^5 - 210*B*c^3*d^3*x^4*e^4 + 280*B*c^3*d^4*x^3*e
^3 - 420*B*c^3*d^5*x^2*e^2 + 840*B*c^3*d^6*x*e + 420*C*a*c^2*x^6*e^7 + 140*
A*c^3*x^6*e^7 - 504*C*a*c^2*d*x^5*e^6 - 168*A*c^3*d*x^5*e^6 + 630*C*a*c^2*d
^2*x^4*e^5 + 210*A*c^3*d^2*x^4*e^5 - 840*C*a*c^2*d^3*x^3*e^4 - 280*A*c^3*d^
3*x^3*e^4 + 1260*C*a*c^2*d^4*x^2*e^3 + 420*A*c^3*d^4*x^2*e^3 - 2520*C*a*c^2
*d^5*x*e^2 - 840*A*c^3*d^5*x*e^2 + 504*B*a*c^2*x^5*e^7 - 630*B*a*c^2*d*x^4*
e^6 + 840*B*a*c^2*d^2*x^3*e^5 - 1260*B*a*c^2*d^3*x^2*e^4 + 2520*B*a*c^2*d^4
*x*e^3 + 630*C*a^2*c*x^4*e^7 + 630*A*a*c^2*x^4*e^7 - 840*C*a^2*c*d*x^3*e^6
- 840*A*a*c^2*d*x^3*e^6 + 1260*C*a^2*c*d^2*x^2*e^5 + 1260*A*a*c^2*d^2*x^2*e
^5 - 2520*C*a^2*c*d^3*x*e^4 - 2520*A*a*c^2*d^3*x*e^4 + 840*B*a^2*c*x^3*e^7
- 1260*B*a^2*c*d*x^2*e^6 + 2520*B*a^2*c*d^2*x*e^5 + 420*C*a^3*x^2*e^7 + 126
0*A*a^2*c*x^2*e^7 - 840*C*a^3*d*x*e^6 - 2520*A*a^2*c*d*x*e^6 + 840*B*a^3*x*
e^7)*e^(-8)
```

**maple [A]** time = 0.01, size = 880, normalized size = 1.80

$$\frac{C\,c^3x^8}{8e}+\frac{B\,c^3x^7}{7e}-\frac{C\,c^3d\,x^7}{7e^2}+\frac{A\,c^3x^6}{6e}-\frac{B\,c^3d\,x^6}{6e^2}+\frac{Ca\,c^2x^6}{2e}+\frac{C\,c^3d^2x^6}{6e^3}-\frac{A\,c^3d\,x^5}{5e^2}+\frac{3Ba\,c^2x^5}{5e}+\frac{B\,c^3d^2x^5}{5e^3}-\frac{3Ca\,c^2d\,x^5}{5e^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x)

[Out] 3/e^3*ln(e*x+d)*A*a^2*c*d^2+3/e^5*ln(e*x+d)*A*a*c^2*d^4-3/e^4*ln(e*x+d)*B*a
^2*c*d^3-3/e^4*A*x*a*c^2*d^3-3/e^6*ln(e*x+d)*B*a*c^2*d^5+3/e^5*ln(e*x+d)*C*
a^2*c*d^4+3/e^7*ln(e*x+d)*C*a*c^2*d^6+3/e^3*B*x*a^2*c*d^2+3/e^5*B*x*a*c^2*d
^4-3/e^2*A*x*a^2*c*d-3/e^4*C*x*a^2*c*d^3-3/e^6*C*x*a*c^2*d^5-3/5/e^2*C*x^5*
a*c^2*d-3/4/e^2*B*x^4*a*c^2*d+3/4/e^3*C*x^4*a*c^2*d^2-1/e^2*A*x^3*a*c^2*d-1
/e^2*C*x^3*a^2*c*d-1/e^4*C*x^3*a*c^2*d^3+3/2/e^3*A*x^2*a*c^2*d^2-3/2/e^2*B*
x^2*a^2*c*d-3/2/e^4*B*x^2*a*c^2*d^3+3/2/e^3*C*x^2*a^2*c*d^2+3/2/e^5*C*x^2*a
*c^2*d^4+1/e^3*B*x^3*a*c^2*d^2+1/8/e*C*c^3*x^8+1/7/e*B*x^7*c^3+1/6/e*A*x^6*
c^3+1/e*B*x*a^3+1/e*ln(e*x+d)*A*a^3+1/2/e*C*x^2*a^3+1/2/e*C*x^6*a*c^2+1/4/e
^5*C*x^4*c^3*d^4-1/2/e^6*B*x^2*c^3*d^5+1/2/e^7*C*x^2*c^3*d^6+1/e*B*x^3*a^2*
c+3/5/e*B*x^5*a*c^2+1/5/e^3*B*x^5*c^3*d^2-1/5/e^4*C*x^5*c^3*d^3+3/4/e*A*x^4
*a*c^2+1/4/e^3*A*x^4*c^3*d^2-1/4/e^4*B*x^4*c^3*d^3+3/4/e*C*x^4*a^2*c+1/e^7*
ln(e*x+d)*A*c^3*d^6-1/e^2*ln(e*x+d)*B*a^3*d-1/e^8*ln(e*x+d)*B*c^3*d^7+1/e^3
*ln(e*x+d)*C*a^3*d^2+1/e^9*ln(e*x+d)*C*c^3*d^8-1/e^6*A*x*c^3*d^5+1/e^7*B*x*
c^3*d^6-1/6/e^2*B*x^6*c^3*d+1/6/e^3*C*x^6*c^3*d^2-1/5/e^2*A*x^5*c^3*d-1/7/e
^2*C*x^7*c^3*d-1/3/e^4*A*x^3*c^3*d^3+1/3/e^5*B*x^3*c^3*d^4-1/3/e^6*C*x^3*c^
3*d^5+3/2/e*A*x^2*a^2*c+1/2/e^5*A*x^2*c^3*d^4-1/e^2*C*x*a^3*d-1/e^8*C*x*c^3
*d^7

**maxima [A]** time = 0.49, size = 672, normalized size = 1.37

$$\frac{105\,Cc^3e^7x^8 - 120\left(Cc^3de^6 - Bc^3e^7\right)x^7 + 140\left(Cc^3d^2e^5 - Bc^3de^6 + \left(3\,Cac^2 + Ac^3\right)e^7\right)x^6 - 168\left(Cc^3d^3e^4 - Bc^3d^2e^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d),x, algorithm="maxima")

[Out] 1/840*(105*C*c^3*e^7*x^8 - 120*(C*c^3*d*e^6 - B*c^3*e^7)*x^7 + 140*(C*c^3*d
^2*e^5 - B*c^3*d*e^6 + (3*C*a*c^2 + A*c^3)*e^7)*x^6 - 168*(C*c^3*d^3*e^4 -
B*c^3*d^2*e^5 - 3*B*a*c^2*e^7 + (3*C*a*c^2 + A*c^3)*d*e^6)*x^5 + 210*(C*c^3
*d^4*e^3 - B*c^3*d^3*e^4 - 3*B*a*c^2*d*e^6 + (3*C*a*c^2 + A*c^3)*d^2*e^5 +
3*(C*a^2*c + A*a*c^2)*e^7)*x^4 - 280*(C*c^3*d^5*e^2 - B*c^3*d^4*e^3 - 3*B*a
*c^2*d^2*e^5 - 3*B*a^2*c*e^7 + (3*C*a*c^2 + A*c^3)*d^3*e^4 + 3*(C*a^2*c + A
*a*c^2)*d*e^6)*x^3 + 420*(C*c^3*d^6*e - B*c^3*d^5*e^2 - 3*B*a*c^2*d^3*e^4 -
3*B*a^2*c*d*e^6 + (3*C*a*c^2 + A*c^3)*d^4*e^3 + 3*(C*a^2*c + A*a*c^2)*d^2*
e^5 + (C*a^3 + 3*A*a^2*c)*e^7)*x^2 - 840*(C*c^3*d^7 - B*c^3*d^6*e - 3*B*a*c
^2*d^4*e^3 - 3*B*a^2*c*d^2*e^5 - B*a^3*e^7 + (3*C*a*c^2 + A*c^3)*d^5*e^2 +
3*(C*a^2*c + A*a*c^2)*d^3*e^4 + (C*a^3 + 3*A*a^2*c)*d*e^6)*x)/e^8 + (C*c^3*
d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + A
*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6)*log(e*x + d)/e^9

**mupad [B]** time = 3.88, size = 741, normalized size = 1.51

$$x\left(\frac{B a^3}{e}-\frac{d\left(\frac{C a^3+3 A c a^2}{e}+\frac{d\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{A c^3+3 C a c^2}{e}-\frac{d\left(\frac{B c^3}{e}-\frac{C c^3 d}{e^2}\right)}{e}\right)}{e}-\frac{3 B a c^2}{e}\right)}{e}+\frac{3 a c (A c+C a)}{e}\right)}{e}-\frac{3 B a^2 c}{e}\right)}{e}\right)}{e}\right)+x^7\left(\frac{B c^3}{7 e}-\frac{C c^3 d}{7 e^2}\right)-x^5\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x),x)

[Out] x*((B*a^3)/e - (d*((C*a^3 + 3*A*a^2*c)/e + (d*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/e))/e) + x^7*((B*c^3)/(7*e) - (C*c^3*d)/(7*e^2)) - x^5*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/(5*e) - (3*B*a*c^2)/(5*e)) + x^4*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/(4*e) + (3*a*c*(A*c + C*a))/(4*e)) + x^2*((C*a^3 + 3*A*a^2*c)/(2*e) + (d*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/e - (3*B*a^2*c)/e))/(2*e)) + x^6*((A*c^3 + 3*C*a*c^2)/(6*e) - (d*((B*c^3)/e - (C*c^3*d)/e^2))/(6*e)) - x^3*((d*((d*((d*((A*c^3 + 3*C*a*c^2)/e - (d*((B*c^3)/e - (C*c^3*d)/e^2))/e))/e - (3*B*a*c^2)/e))/e + (3*a*c*(A*c + C*a))/e))/(3*e) - (B*a^2*c)/e) + (log(d + e*x)*(A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4))/e^9 + (C*c^3*x^8)/(8*e)

**sympy [A]** time = 1.47, size = 685, normalized size = 1.40

$$\frac{Cc^3x^8}{8e}+x^7\left(\frac{Bc^3}{7e}-\frac{Cc^3d}{7e^2}\right)+x^6\left(\frac{Ac^3}{6e}-\frac{Bc^3d}{6e^2}+\frac{Cac^2}{2e}+\frac{Cc^3d^2}{6e^3}\right)+x^5\left(-\frac{Ac^3d}{5e^2}+\frac{3Bac^2}{5e}+\frac{Bc^3d^2}{5e^3}-\frac{3Cac^2d}{5e^2}-\frac{Cc^3d^3}{5e^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d),x)

[Out] C*c**3*x**8/(8*e) + x**7*(B*c**3/(7*e) - C*c**3*d/(7*e**2)) + x**6*(A*c**3/(6*e) - B*c**3*d/(6*e**2) + C*a*c**2/(2*e) + C*c**3*d**2/(6*e**3)) + x**5*(-A*c**3*d/(5*e**2) + 3*B*a*c**2/(5*e) + B*c**3*d**2/(5*e**3) - 3*C*a*c**2*d/(5*e**2) - C*c**3*d**3/(5*e**4)) + x**4*(3*A*a*c**2/(4*e) + A*c**3*d**2/(4*e**3) - 3*B*a*c**2*d/(4*e**2) - B*c**3*d**3/(4*e**4) + 3*C*a**2*c/(4*e) + 3*C*a*c**2*d**2/(4*e**3) + C*c**3*d**4/(4*e**5)) + x**3*(-A*a*c**2*d/e**2 - A*c**3*d**3/(3*e**4) + B*a**2*c/e + B*a*c**2*d**2/e**3 + B*c**3*d**4/(3*e**5) - C*a**2*c*d/e**2 - C*a*c**2*d**3/e**4 - C*c**3*d**5/(3*e**6)) + x**2*(3*A*a**2*c/(2*e) + 3*A*a*c**2*d**2/(2*e**3) + A*c**3*d**4/(2*e**5) - 3*B*a**2*c*d/(2*e**2) - 3*B*a*c**2*d**3/(2*e**4) - B*c**3*d**5/(2*e**6) + C*a**3/(2*e) + 3*C*a**2*c*d**2/(2*e**3) + 3*C*a*c**2*d**4/(2*e**5) + C*c**3*d**6/(2*e**7)) + x*(-3*A*a**2*c*d/e**2 - 3*A*a*c**2*d**3/e**4 - A*c**3*d**5/e**6 + B*a**3/e + 3*B*a**2*c*d**2/e**3 + 3*B*a*c**2*d**4/e**5 + B*c**3*d**6/e**7 - C*a**3*d/e**2 - 3*C*a**2*c*d**3/e**4 - 3*C*a*c**2*d**5/e**6 - C*c**3*d**7/e**8) + (a*e**2 + c*d**2)**3*(A*e**2 - B*d*e + C*d**2)*log(d + e*x)/e**9

# 3.37 $\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx$

**Optimal**. Leaf size=486

$$\frac{cx^3\left(3a^2Ce^4+3ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2d^2\left(5Cd^2-e(4Bd-3Ae)\right)\right)}{3e^6}-\frac{cx^2\left(3a^2e^4(2Cd-Be)+3acde^2\left(4C\right.\right.}{}$$

```
[Out] (a^3*C*e^6+c^3*d^4*(7*C*d^2-e*(-5*A*e+6*B*d))+3*a*c^2*d^2*e^2*(5*C*d^2-e*(-
3*A*e+4*B*d))+3*a^2*c*e^4*(3*C*d^2-e*(-A*e+2*B*d)))*x/e^8-1/2*c*(3*a^2*e^4*
(-B*e+2*C*d)+c^2*d^3*(6*C*d^2-e*(-4*A*e+5*B*d))+3*a*c*d*e^2*(4*C*d^2-e*(-2*
A*e+3*B*d)))*x^2/e^7+1/3*c*(3*a^2*C*e^4+c^2*d^2*(5*C*d^2-e*(-3*A*e+4*B*d))+
3*a*c*e^2*(3*C*d^2-e*(-A*e+2*B*d)))*x^3/e^6-1/4*c^2*(3*a*e^2*(-B*e+2*C*d)+c
*d*(4*C*d^2-e*(-2*A*e+3*B*d)))*x^4/e^5+1/5*c^2*(3*a*C*e^2+c*(3*C*d^2-e*(-A*
e+2*B*d)))*x^5/e^4-1/6*c^3*(-B*e+2*C*d)*x^6/e^3+1/7*c^3*C*x^7/e^2-(a*e^2+c*
d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)-(a*e^2+c*d^2)^2*(a*e^2*(-B*e+2*C*d)+
c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))*ln(e*x+d)/e^9
```

**Rubi [A]** time = 0.98, antiderivative size = 483, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{cx^3\left(3a^2Ce^4+3ace^2\left(3Cd^2-e(2Bd-Ae)\right)+c^2\left(5Cd^4-d^2e(4Bd-3Ae)\right)\right)}{3e^6}-\frac{cx^2\left(3a^2e^4(2Cd-Be)+3acde^2\left(4C\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] ((a^3*C*e^6 + c^3*(7*C*d^6 - d^4*e*(6*B*d - 5*A*e)) + 3*a*c^2*d^2*e^2*(5*C*
d^2 - e*(4*B*d - 3*A*e)) + 3*a^2*c*e^4*(3*C*d^2 - e*(2*B*d - A*e)))*x)/e^8
- (c*(3*a^2*e^4*(2*C*d - B*e) + c^2*(6*C*d^5 - d^3*e*(5*B*d - 4*A*e)) + 3*a
*c*d*e^2*(4*C*d^2 - e*(3*B*d - 2*A*e)))*x^2)/(2*e^7) + (c*(3*a^2*C*e^4 + c^
2*(5*C*d^4 - d^2*e*(4*B*d - 3*A*e)) + 3*a*c*e^2*(3*C*d^2 - e*(2*B*d - A*e))
)*x^3)/(3*e^6) - (c^2*(4*c*C*d^3 - c*d*e*(3*B*d - 2*A*e) + 3*a*e^2*(2*C*d -
 B*e))*x^4)/(4*e^5) + (c^2*(3*c*C*d^2 + 3*a*C*e^2 - c*e*(2*B*d - A*e))*x^5)
/(5*e^4) - (c^3*(2*C*d - B*e)*x^6)/(6*e^3) + (c^3*C*x^7)/(7*e^2) - ((c*d^2
+ a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(e^9*(d + e*x)) - ((c*d^2 + a*e^2)^2*(8
*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e))*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^3\left(A+Bx+Cx^2\right)}{(d+ex)^2}\,dx = \int \left(\frac{a^3Ce^6+c^3\left(7Cd^6-d^4e(6Bd-5Ae)\right)+3ac^2d^2e^2\left(5Cd^2-e(4Bd-3Ae)\right)}{e^8}\right.$$

$$= \frac{\left(a^3Ce^6+c^3\left(7Cd^6-d^4e(6Bd-5Ae)\right)+3ac^2d^2e^2\left(5Cd^2-e(4Bd-3Ae)\right)+3\right.}{e^8}$$

**Mathematica [A]** time = 0.40, size = 641, normalized size = 1.32

$$\frac{420a^3e^6\left(e(Bd-Ae)+C\left(-d^2+dex+e^2x^2\right)\right)+210a^2ce^4\left(3e\left(2Ae\left(-d^2+dex+e^2x^2\right)+B\left(2d^3-4d^2ex-3de^2x^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x]
```

```
[Out] (420*a^3*e^6*(e*(B*d - A*e) + C*(-d^2 + d*e*x + e^2*x^2)) + 210*a^2*c*e^4*(
2*C*(-3*d^4 + 9*d^3*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + 3*e*(2*A
*e*(-d^2 + d*e*x + e^2*x^2) + B*(2*d^3 - 4*d^2*e*x - 3*d*e^2*x^2 + e^3*x^3)
)) + 21*a*c^2*e^2*(-6*C*(10*d^6 - 50*d^5*e*x - 30*d^4*e^2*x^2 + 10*d^3*e^3*
x^3 - 5*d^2*e^4*x^4 + 3*d*e^5*x^5 - 2*e^6*x^6) + 5*e*(4*A*e*(-3*d^4 + 9*d^3
*e*x + 6*d^2*e^2*x^2 - 2*d*e^3*x^3 + e^4*x^4) + B*(12*d^5 - 48*d^4*e*x - 30
*d^3*e^2*x^2 + 10*d^2*e^3*x^3 - 5*d*e^4*x^4 + 3*e^5*x^5))) + c^3*(-4*C*(105
*d^8 - 735*d^7*e*x - 420*d^6*e^2*x^2 + 140*d^5*e^3*x^3 - 70*d^4*e^4*x^4 + 4
2*d^3*e^5*x^5 - 28*d^2*e^6*x^6 + 20*d*e^7*x^7 - 15*e^8*x^8) + 7*e*(6*A*e*(-
10*d^6 + 50*d^5*e*x + 30*d^4*e^2*x^2 - 10*d^3*e^3*x^3 + 5*d^2*e^4*x^4 - 3*d
*e^5*x^5 + 2*e^6*x^6) + B*(60*d^7 - 360*d^6*e*x - 210*d^5*e^2*x^2 + 70*d^4*
e^3*x^3 - 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 - 14*d*e^6*x^6 + 10*e^7*x^7))) -
420*(c*d^2 + a*e^2)^2*(8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d -
B*e))*(d + e*x)*Log[d + e*x])/(420*e^9*(d + e*x))
```

**fricas [A]** time = 0.93, size = 932, normalized size = 1.92

$$\frac{60\,Cc^3e^8x^8-420\,Cc^3d^8+420\,Bc^3d^7e+1260\,Bac^2d^5e^3+1260\,Ba^2cd^3e^5+420\,Ba^3de^7-420\,Aa^3e^8-420\left(3\,Cac^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/420*(60*C*c^3*e^8*x^8 - 420*C*c^3*d^8 + 420*B*c^3*d^7*e + 1260*B*a*c^2*d^
5*e^3 + 1260*B*a^2*c*d^3*e^5 + 420*B*a^3*d*e^7 - 420*A*a^3*e^8 - 420*(3*C*a
*c^2 + A*c^3)*d^6*e^2 - 1260*(C*a^2*c + A*a*c^2)*d^4*e^4 - 420*(C*a^3 + 3*A
*a^2*c)*d^2*e^6 - 10*(8*C*c^3*d*e^7 - 7*B*c^3*e^8)*x^7 + 14*(8*C*c^3*d^2*e^
6 - 7*B*c^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 21*(8*C*c^3*d^3*e^5 -
7*B*c^3*d^2*e^6 - 15*B*a*c^2*e^8 + 6*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 35*(8
*C*c^3*d^4*e^4 - 7*B*c^3*d^3*e^5 - 15*B*a*c^2*d*e^7 + 6*(3*C*a*c^2 + A*c^3)
*d^2*e^6 + 12*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 70*(8*C*c^3*d^5*e^3 - 7*B*c^3*
d^4*e^4 - 15*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 6*(3*C*a*c^2 + A*c^3)*d^3*e^
5 + 12*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 + 210*(8*C*c^3*d^6*e^2 - 7*B*c^3*d^5*
e^3 - 15*B*a*c^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^4*e^4
+ 12*(C*a^2*c + A*a*c^2)*d^2*e^6 + 2*(C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 420*(7*
C*c^3*d^7*e - 6*B*c^3*d^6*e^2 - 12*B*a*c^2*d^4*e^4 - 6*B*a^2*c*d^2*e^6 + 5*
(3*C*a*c^2 + A*c^3)*d^5*e^3 + 9*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*
a^2*c)*d*e^7)*x - 420*(8*C*c^3*d^8 - 7*B*c^3*d^7*e - 15*B*a*c^2*d^5*e^3 - 9
*B*a^2*c*d^3*e^5 - B*a^3*d*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 12*(C*a^2*
c + A*a*c^2)*d^4*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + (8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

**giac [A]** time = 0.20, size = 838, normalized size = 1.72

$$\frac{1}{420}\left(60\,Cc^3 - \frac{70\left(8\,Cc^3de - Bc^3e^2\right)e^{(-1)}}{xe+d} + \frac{84\left(28\,Cc^3d^2e^2 - 7\,Bc^3de^3 + 3\,Cac^2e^4 + Ac^3e^4\right)e^{(-2)}}{(xe+d)^2} - \frac{105\left(56\,Cc^3d^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/420*(60*C*c^3 - 70*(8*C*c^3*d*e - B*c^3*e^2)*e^(-1)/(x*e + d) + 84*(28*C*
c^3*d^2*e^2 - 7*B*c^3*d*e^3 + 3*C*a*c^2*e^4 + A*c^3*e^4)*e^(-2)/(x*e + d)^2
 - 105*(56*C*c^3*d^3*e^3 - 21*B*c^3*d^2*e^4 + 18*C*a*c^2*d*e^5 + 6*A*c^3*d*
e^5 - 3*B*a*c^2*e^6)*e^(-3)/(x*e + d)^3 + 140*(70*C*c^3*d^4*e^4 - 35*B*c^3*
d^3*e^5 + 45*C*a*c^2*d^2*e^6 + 15*A*c^3*d^2*e^6 - 15*B*a*c^2*d*e^7 + 3*C*a^
2*c*e^8 + 3*A*a*c^2*e^8)*e^(-4)/(x*e + d)^4 - 210*(56*C*c^3*d^5*e^5 - 35*B*
c^3*d^4*e^6 + 60*C*a*c^2*d^3*e^7 + 20*A*c^3*d^3*e^7 - 30*B*a*c^2*d^2*e^8 +
12*C*a^2*c*d*e^9 + 12*A*a*c^2*d*e^9 - 3*B*a^2*c*e^10)*e^(-5)/(x*e + d)^5 +
420*(28*C*c^3*d^6*e^6 - 21*B*c^3*d^5*e^7 + 45*C*a*c^2*d^4*e^8 + 15*A*c^3*d^
4*e^8 - 30*B*a*c^2*d^3*e^9 + 18*C*a^2*c*d^2*e^10 + 18*A*a*c^2*d^2*e^10 - 9*
B*a^2*c*d*e^11 + C*a^3*e^12 + 3*A*a^2*c*e^12)*e^(-6)/(x*e + d)^6)*(x*e + d)
^7*e^(-9) + (8*C*c^3*d^7 - 7*B*c^3*d^6*e + 18*C*a*c^2*d^5*e^2 + 6*A*c^3*d^5
*e^2 - 15*B*a*c^2*d^4*e^3 + 12*C*a^2*c*d^3*e^4 + 12*A*a*c^2*d^3*e^4 - 9*B*a
^2*c*d^2*e^5 + 2*C*a^3*d*e^6 + 6*A*a^2*c*d*e^6 - B*a^3*e^7)*e^(-9)*log(abs(
```

```
x*e + d)*e^(-1)/(x*e + d)^2) - (C*c^3*d^8*e^7/(x*e + d) - B*c^3*d^7*e^8/(x*
e + d) + 3*C*a*c^2*d^6*e^9/(x*e + d) + A*c^3*d^6*e^9/(x*e + d) - 3*B*a*c^2*
d^5*e^10/(x*e + d) + 3*C*a^2*c*d^4*e^11/(x*e + d) + 3*A*a*c^2*d^4*e^11/(x*e
 + d) - 3*B*a^2*c*d^3*e^12/(x*e + d) + C*a^3*d^2*e^13/(x*e + d) + 3*A*a^2*c
*d^2*e^13/(x*e + d) - B*a^3*d*e^14/(x*e + d) + A*a^3*e^15/(x*e + d))*e^(-16
)
```

**maple [A]** time = 0.02, size = 928, normalized size = 1.91

$$\frac{C c^3 x^7}{7e^2}+\frac{B c^3 x^6}{6e^2}-\frac{C c^3 d x^6}{3e^3}+\frac{A c^3 x^5}{5e^2}-\frac{2B c^3 d x^5}{5e^3}+\frac{3Ca c^2 x^5}{5e^2}+\frac{3C c^3 d^2 x^5}{5e^4}-\frac{A c^3 d x^4}{2e^3}+\frac{3Ba c^2 x^4}{4e^2}+\frac{3B c^3 d^2 x^4}{4e^4}-\frac{3Ca c^2 d x^4}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x)
```

```
[Out] 1/7*c^3*C*x^7/e^2-1/e/(e*x+d)*A*a^3+1/e^2*ln(e*x+d)*B*a^3+1/e^2*a^3*C*x+1/5
/e^2*A*x^5*c^3+1/6/e^2*B*x^6*c^3-3/e^3*A*x^2*a*c^2*d+9/2/e^4*B*x^2*a*c^2*d^
2-3/e^3*C*x^2*a^2*c*d-3/e^3/(e*x+d)*A*a^2*c*d^2-3/e^5/(e*x+d)*A*a*c^2*d^4+3
/e^4/(e*x+d)*B*a^2*c*d^3+3/e^6/(e*x+d)*B*a*c^2*d^5-3/e^5/(e*x+d)*C*a^2*c*d^
4-3/e^7/(e*x+d)*C*a*c^2*d^6-6/e^3*ln(e*x+d)*A*a^2*c*d-12/e^5*ln(e*x+d)*A*a*
c^2*d^3+9/e^4*ln(e*x+d)*B*a^2*c*d^2+15/e^6*ln(e*x+d)*B*a*c^2*d^4-12/e^5*ln(
e*x+d)*C*a^2*c*d^3-18/e^7*ln(e*x+d)*C*a*c^2*d^5-3/2/e^3*C*x^4*a*c^2*d-6/e^5
*C*x^2*a*c^2*d^3+9/e^4*A*a*c^2*d^2*x-6/e^3*d*a^2*c*B*x-12/e^5*B*a*c^2*d^3*x
+9/e^4*C*a^2*c*d^2*x+15/e^6*C*a*c^2*d^4*x-2/e^3*B*x^3*a*c^2*d+3/e^4*C*x^3*a
*c^2*d^2+3/5/e^2*C*x^5*a*c^2+3/5/e^4*C*x^5*c^3*d^2-1/2/e^3*A*x^4*c^3*d+3/4/
e^2*B*x^4*a*c^2+3/4/e^4*B*x^4*c^3*d^2-1/e^5*C*x^4*c^3*d^3-4/3/e^5*B*x^3*c^3
*d^3+5/3/e^6*C*x^3*c^3*d^4-2/e^5*A*x^2*c^3*d^3+3/2/e^2*B*x^2*a^2*c+5/2/e^6*
B*x^2*c^3*d^4-3/e^7*C*x^2*c^3*d^5+3/e^2*A*a^2*c*x+5/e^6*A*c^3*d^4*x-6/e^7*B
*c^3*d^5*x-1/3/e^3*C*x^6*c^3*d+1/e^2*C*x^3*a^2*c+1/e^2*A*x^3*a*c^2+1/e^4*A*
x^3*c^3*d^2-1/e^7/(e*x+d)*A*c^3*d^6+1/e^2/(e*x+d)*B*d*a^3+7/e^8*C*c^3*d^6*x
-2/5/e^3*B*x^5*c^3*d+1/e^8/(e*x+d)*B*c^3*d^7-1/e^3/(e*x+d)*C*a^3*d^2-1/e^9/
(e*x+d)*C*c^3*d^8-6/e^7*ln(e*x+d)*A*c^3*d^5+7/e^8*ln(e*x+d)*B*c^3*d^6-2/e^3
*ln(e*x+d)*C*a^3*d-8/e^9*ln(e*x+d)*C*c^3*d^7
```

**maxima [A]** time = 0.49, size = 691, normalized size = 1.42

$$-\frac{Cc^3d^8-Bc^3d^7e-3\,Bac^2d^5e^3-3\,Ba^2cd^3e^5-Ba^3de^7+Aa^3e^8+\left(3\,Cac^2+Ac^3\right)d^6e^2+3\left(Ca^2c+Aac^2\right)d^4e^4+\left(C\right.}{e^{10}x+de^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -(C*c^3*d^8 - B*c^3*d^7*e - 3*B*a*c^2*d^5*e^3 - 3*B*a^2*c*d^3*e^5 - B*a^3*d
*e^7 + A*a^3*e^8 + (3*C*a*c^2 + A*c^3)*d^6*e^2 + 3*(C*a^2*c + A*a*c^2)*d^4*
```

```
e^4 + (C*a^3 + 3*A*a^2*c)*d^2*e^6)/(e^10*x + d*e^9) + 1/420*(60*C*c^3*e^6*x
^7 - 70*(2*C*c^3*d*e^5 - B*c^3*e^6)*x^6 + 84*(3*C*c^3*d^2*e^4 - 2*B*c^3*d*e
^5 + (3*C*a*c^2 + A*c^3)*e^6)*x^5 - 105*(4*C*c^3*d^3*e^3 - 3*B*c^3*d^2*e^4
- 3*B*a*c^2*e^6 + 2*(3*C*a*c^2 + A*c^3)*d*e^5)*x^4 + 140*(5*C*c^3*d^4*e^2 -
 4*B*c^3*d^3*e^3 - 6*B*a*c^2*d*e^5 + 3*(3*C*a*c^2 + A*c^3)*d^2*e^4 + 3*(C*a
^2*c + A*a*c^2)*e^6)*x^3 - 210*(6*C*c^3*d^5*e - 5*B*c^3*d^4*e^2 - 9*B*a*c^2
*d^2*e^4 - 3*B*a^2*c*e^6 + 4*(3*C*a*c^2 + A*c^3)*d^3*e^3 + 6*(C*a^2*c + A*a
*c^2)*d*e^5)*x^2 + 420*(7*C*c^3*d^6 - 6*B*c^3*d^5*e - 12*B*a*c^2*d^3*e^3 -
6*B*a^2*c*d*e^5 + 5*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 9*(C*a^2*c + A*a*c^2)*d^2
*e^4 + (C*a^3 + 3*A*a^2*c)*e^6)*x)/e^8 - (8*C*c^3*d^7 - 7*B*c^3*d^6*e - 15*
B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 - B*a^3*e^7 + 6*(3*C*a*c^2 + A*c^3)*d^5
*e^2 + 12*(C*a^2*c + A*a*c^2)*d^3*e^4 + 2*(C*a^3 + 3*A*a^2*c)*d*e^6)*log(e*
x + d)/e^9
```

**mupad [B]** time = 3.99, size = 1511, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^2,x)
```

```
[Out] x*((C*a^3 + 3*A*a^2*c)/e^2 + (2*d*((2*d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3
*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*
((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3
*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2
))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c
^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^
3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e^2 - (3*B*a^2*c)/e^2))/
e - (d^2*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^
2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d
)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c^3)/e
^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/e + (3*a*c*(A*c + C*a))/e^2)
)/e^2) + x^4*((d*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*
c^2)/e^2 + (C*c^3*d^2)/e^4))/(2*e) - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/
(4*e^2) + (3*B*a*c^2)/(4*e^2)) - x^2*((d*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^
3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e^2 - (2*d*((2*d
*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^
3*d^2)/e^4))/e - (d^2*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^
2))/e + (3*a*c*(A*c + C*a))/e^2))/e + (d^2*((2*d*((2*d*((B*c^3)/e^2 - (2*C*
c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B*c
^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(2*e^2) - (3*B*a^2*c)/(
2*e^2)) + x^6*((B*c^3)/(6*e^2) - (C*c^3*d)/(3*e^3)) - x^5*((2*d*((B*c^3)/e^
2 - (2*C*c^3*d)/e^3))/(5*e) - (A*c^3 + 3*C*a*c^2)/(5*e^2) + (C*c^3*d^2)/(5*
e^4)) + x^3*((d^2*((2*d*((B*c^3)/e^2 - (2*C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a
*c^2)/e^2 + (C*c^3*d^2)/e^4))/(3*e^2) - (2*d*((2*d*((2*d*((B*c^3)/e^2 - (2*
```

```
C*c^3*d)/e^3))/e - (A*c^3 + 3*C*a*c^2)/e^2 + (C*c^3*d^2)/e^4))/e - (d^2*((B
*c^3)/e^2 - (2*C*c^3*d)/e^3))/e^2 + (3*B*a*c^2)/e^2))/(3*e) + (a*c*(A*c + C
*a))/e^2) - (A*a^3*e^8 + C*c^3*d^8 - B*a^3*d*e^7 - B*c^3*d^7*e + A*c^3*d^6*
e^2 + C*a^3*d^2*e^6 + 3*A*a*c^2*d^4*e^4 + 3*A*a^2*c*d^2*e^6 - 3*B*a*c^2*d^5
*e^3 - 3*B*a^2*c*d^3*e^5 + 3*C*a*c^2*d^6*e^2 + 3*C*a^2*c*d^4*e^4)/(e*(d*e^8
 + e^9*x)) - (log(d + e*x)*(8*C*c^3*d^7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c
^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*
a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 12*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))
/e^9 + (C*c^3*x^7)/(7*e^2)
```

**sympy [A]** time = 4.95, size = 748, normalized size = 1.54

$$\frac{Cc^3x^7}{7e^2}+x^6\left(\frac{Bc^3}{6e^2}-\frac{Cc^3d}{3e^3}\right)+x^5\left(\frac{Ac^3}{5e^2}-\frac{2Bc^3d}{5e^3}+\frac{3Cac^2}{5e^2}+\frac{3Cc^3d^2}{5e^4}\right)+x^4\left(-\frac{Ac^3d}{2e^3}+\frac{3Bac^2}{4e^2}+\frac{3Bc^3d^2}{4e^4}-\frac{3Cac^2d}{2e^3}-\frac{C\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**2,x)
```

```
[Out] C*c**3*x**7/(7*e**2) + x**6*(B*c**3/(6*e**2) - C*c**3*d/(3*e**3)) + x**5*(A
*c**3/(5*e**2) - 2*B*c**3*d/(5*e**3) + 3*C*a*c**2/(5*e**2) + 3*C*c**3*d**2/
(5*e**4)) + x**4*(-A*c**3*d/(2*e**3) + 3*B*a*c**2/(4*e**2) + 3*B*c**3*d**2/
(4*e**4) - 3*C*a*c**2*d/(2*e**3) - C*c**3*d**3/e**5) + x**3*(A*a*c**2/e**2
+ A*c**3*d**2/e**4 - 2*B*a*c**2*d/e**3 - 4*B*c**3*d**3/(3*e**5) + C*a**2*c/
e**2 + 3*C*a*c**2*d**2/e**4 + 5*C*c**3*d**4/(3*e**6)) + x**2*(-3*A*a*c**2*d
/e**3 - 2*A*c**3*d**3/e**5 + 3*B*a**2*c/(2*e**2) + 9*B*a*c**2*d**2/(2*e**4)
 + 5*B*c**3*d**4/(2*e**6) - 3*C*a**2*c*d/e**3 - 6*C*a*c**2*d**3/e**5 - 3*C*
c**3*d**5/e**7) + x*(3*A*a**2*c/e**2 + 9*A*a*c**2*d**2/e**4 + 5*A*c**3*d**4
/e**6 - 6*B*a**2*c*d/e**3 - 12*B*a*c**2*d**3/e**5 - 6*B*c**3*d**5/e**7 + C*
a**3/e**2 + 9*C*a**2*c*d**2/e**4 + 15*C*a*c**2*d**4/e**6 + 7*C*c**3*d**6/e*
*8) + (-A*a**3*e**8 - 3*A*a**2*c*d**2*e**6 - 3*A*a*c**2*d**4*e**4 - A*c**3*
d**6*e**2 + B*a**3*d*e**7 + 3*B*a**2*c*d**3*e**5 + 3*B*a*c**2*d**5*e**3 + B
*c**3*d**7*e - C*a**3*d**2*e**6 - 3*C*a**2*c*d**4*e**4 - 3*C*a*c**2*d**6*e*
*2 - C*c**3*d**8)/(d*e**9 + e**10*x) - (a*e**2 + c*d**2)**2*(6*A*c*d*e**2 -
 B*a*e**3 - 7*B*c*d**2*e + 2*C*a*d*e**2 + 8*C*c*d**3)*log(d + e*x)/e**9
```

# 3.38 $\int \frac{(a+cx^2)^3(A+Bx+Cx^2)}{(d+ex)^3}\,dx$

**Optimal**. Leaf size=466

$$\frac{(ae^2+cd^2)\log(d+ex)\left(a^2Ce^4+ace^2\left(17Cd^2-3e(3Bd-Ae)\right)+c^2d^2\left(28Cd^2-3e(7Bd-5Ae)\right)\right)}{e^9}+\frac{cx^2\left(3a^2Ce^4\right.}{}$$

```
[Out] -c*(3*a^2*e^4*(-B*e+3*C*d)+c^2*d^3*(21*C*d^2-5*e*(-2*A*e+3*B*d))+3*a*c*d*e^
2*(10*C*d^2-3*e*(-A*e+2*B*d)))*x/e^8+1/2*c*(3*a^2*C*e^4+c^2*d^2*(15*C*d^2-2
*e*(-3*A*e+5*B*d))+3*a*c*e^2*(6*C*d^2-e*(-A*e+3*B*d)))*x^2/e^7-1/3*c^2*(3*a
*e^2*(-B*e+3*C*d)+c*d*(10*C*d^2-3*e*(-A*e+2*B*d)))*x^3/e^6+1/4*c^2*(3*a*C*e
^2+c*(6*C*d^2-e*(-A*e+3*B*d)))*x^4/e^5-1/5*c^3*(-B*e+3*C*d)*x^5/e^4+1/6*c^3
*C*x^6/e^3-1/2*(a*e^2+c*d^2)^3*(A*e^2-B*d*e+C*d^2)/e^9/(e*x+d)^2+(a*e^2+c*d
^2)^2*(a*e^2*(-B*e+2*C*d)+c*d*(8*C*d^2-e*(-6*A*e+7*B*d)))/e^9/(e*x+d)+(a*e^
2+c*d^2)*(a^2*C*e^4+c^2*d^2*(28*C*d^2-3*e*(-5*A*e+7*B*d))+a*c*e^2*(17*C*d^2
-3*e*(-A*e+3*B*d)))*ln(e*x+d)/e^9
```

**Rubi** [A] time = 0.97, antiderivative size = 463, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1628}

$$\frac{cx^2\left(3a^2Ce^4+3ace^2\left(6Cd^2-e(3Bd-Ae)\right)+c^2\left(15Cd^4-2d^2e(5Bd-3Ae)\right)\right)}{2e^7}-\frac{cx\left(3a^2e^4(3Cd-Be)+3acde^2\left(10\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] -((c*(3*a^2*e^4*(3*C*d - B*e) + c^2*(21*C*d^5 - 5*d^3*e*(3*B*d - 2*A*e)) +
3*a*c*d*e^2*(10*C*d^2 - 3*e*(2*B*d - A*e)))*x)/e^8) + (c*(3*a^2*C*e^4 + c^2
*(15*C*d^4 - 2*d^2*e*(5*B*d - 3*A*e)) + 3*a*c*e^2*(6*C*d^2 - e*(3*B*d - A*e
)))*x^2)/(2*e^7) - (c^2*(10*c*C*d^3 - 3*c*d*e*(2*B*d - A*e) + 3*a*e^2*(3*C*
d - B*e))*x^3)/(3*e^6) + (c^2*(6*c*C*d^2 + 3*a*C*e^2 - c*e*(3*B*d - A*e))*x
^4)/(4*e^5) - (c^3*(3*C*d - B*e)*x^5)/(5*e^4) + (c^3*C*x^6)/(6*e^3) - ((c*d
^2 + a*e^2)^3*(C*d^2 - B*d*e + A*e^2))/(2*e^9*(d + e*x)^2) + ((c*d^2 + a*e^
2)^2*(8*c*C*d^3 - c*d*e*(7*B*d - 6*A*e) + a*e^2*(2*C*d - B*e)))/(e^9*(d + e
*x)) + ((c*d^2 + a*e^2)*(a^2*C*e^4 + c^2*(28*C*d^4 - 3*d^2*e*(7*B*d - 5*A*e
)) + a*c*e^2*(17*C*d^2 - 3*e*(3*B*d - A*e)))*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+cx^2)^3 (A+Bx+Cx^2)}{(d+ex)^3}\,dx = \int \left(\frac{c\left(-3a^2e^4(3Cd-Be)-c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)-3acde^2\left(10Cd^2-\right.\right.}{e^8}\right.$$

$$= -\frac{c\left(3a^2e^4(3Cd-Be)+c^2\left(21Cd^5-5d^3e(3Bd-2Ae)\right)+3acde^2\left(10Cd^2-3e(\right.\right.}{e^8}$$

**Mathematica [A]** time = 0.23, size = 438, normalized size = 0.94

$$\frac{30ce^2x^2\left(3a^2Ce^4+3ace^2\left(e(Ae-3Bd)+6Cd^2\right)+c^2\left(2d^2e(3Ae-5Bd)+15Cd^4\right)\right)+60\left(ae^2+cd^2\right)\log(d+ex)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x]
```

```
[Out] (-60*c*e*(-3*a^2*e^4*(-3*C*d + B*e) + 3*a*c*d*e^2*(10*C*d^2 + 3*e*(-2*B*d +
 A*e)) + c^2*(21*C*d^5 + 5*d^3*e*(-3*B*d + 2*A*e)))*x + 30*c*e^2*(3*a^2*C*e
^4 + 3*a*c*e^2*(6*C*d^2 + e*(-3*B*d + A*e)) + c^2*(15*C*d^4 + 2*d^2*e*(-5*B
*d + 3*A*e)))*x^2 - 20*c^2*e^3*(10*c*C*d^3 + 3*c*d*e*(-2*B*d + A*e) - 3*a*e
^2*(-3*C*d + B*e))*x^3 + 15*c^2*e^4*(6*c*C*d^2 + 3*a*C*e^2 + c*e*(-3*B*d +
A*e))*x^4 + 12*c^3*e^5*(-3*C*d + B*e)*x^5 + 10*c^3*C*e^6*x^6 - (30*(c*d^2 +
 a*e^2)^3*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 + (60*(c*d^2 + a*e^2)^2*(
8*c*C*d^3 + c*d*e*(-7*B*d + 6*A*e) + a*e^2*(2*C*d - B*e)))/(d + e*x) + 60*(
c*d^2 + a*e^2)*(a^2*C*e^4 + a*c*e^2*(17*C*d^2 + 3*e*(-3*B*d + A*e)) + c^2*(
28*C*d^4 + 3*d^2*e*(-7*B*d + 5*A*e)))*Log[d + e*x])/(60*e^9)
```

**fricas [B]** time = 0.93, size = 1025, normalized size = 2.20

$$\frac{10\,Cc^3e^8x^8+450\,Cc^3d^8-390\,Bc^3d^7e-810\,Bac^2d^5e^3-450\,Ba^2cd^3e^5-30\,Ba^3de^7-30\,Aa^3e^8+330\left(3\,Cac^2+Ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^3*e^8*x^8 + 450*C*c^3*d^8 - 390*B*c^3*d^7*e - 810*B*a*c^2*d^5*
e^3 - 450*B*a^2*c*d^3*e^5 - 30*B*a^3*d*e^7 - 30*A*a^3*e^8 + 330*(3*C*a*c^2
```

```
+ A*c^3)*d^6*e^2 + 630*(C*a^2*c + A*a*c^2)*d^4*e^4 + 90*(C*a^3 + 3*A*a^2*c)
*d^2*e^6 - 4*(4*C*c^3*d*e^7 - 3*B*c^3*e^8)*x^7 + (28*C*c^3*d^2*e^6 - 21*B*c
^3*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*e^8)*x^6 - 2*(28*C*c^3*d^3*e^5 - 21*B*c^3
*d^2*e^6 - 30*B*a*c^2*e^8 + 15*(3*C*a*c^2 + A*c^3)*d*e^7)*x^5 + 5*(28*C*c^3
*d^4*e^4 - 21*B*c^3*d^3*e^5 - 30*B*a*c^2*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^2
*e^6 + 18*(C*a^2*c + A*a*c^2)*e^8)*x^4 - 20*(28*C*c^3*d^5*e^3 - 21*B*c^3*d^
4*e^4 - 30*B*a*c^2*d^2*e^6 - 9*B*a^2*c*e^8 + 15*(3*C*a*c^2 + A*c^3)*d^3*e^5
 + 18*(C*a^2*c + A*a*c^2)*d*e^7)*x^3 - 30*(69*C*c^3*d^6*e^2 - 50*B*c^3*d^5*
e^3 - 63*B*a*c^2*d^3*e^5 - 12*B*a^2*c*d*e^7 + 34*(3*C*a*c^2 + A*c^3)*d^4*e^
4 + 33*(C*a^2*c + A*a*c^2)*d^2*e^6)*x^2 - 60*(13*C*c^3*d^7*e - 8*B*c^3*d^6*
e^2 - 3*B*a*c^2*d^4*e^4 + 6*B*a^2*c*d^2*e^6 + B*a^3*e^8 + 4*(3*C*a*c^2 + A*
c^3)*d^5*e^3 - 3*(C*a^2*c + A*a*c^2)*d^3*e^5 - 2*(C*a^3 + 3*A*a^2*c)*d*e^7)
*x + 60*(28*C*c^3*d^8 - 21*B*c^3*d^7*e - 30*B*a*c^2*d^5*e^3 - 9*B*a^2*c*d^3
*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 18*(C*a^2*c + A*a*c^2)*d^4*e^4 + (C
*a^3 + 3*A*a^2*c)*d^2*e^6 + (28*C*c^3*d^6*e^2 - 21*B*c^3*d^5*e^3 - 30*B*a*c
^2*d^3*e^5 - 9*B*a^2*c*d*e^7 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^4 + 18*(C*a^2*c
 + A*a*c^2)*d^2*e^6 + (C*a^3 + 3*A*a^2*c)*e^8)*x^2 + 2*(28*C*c^3*d^7*e - 21
*B*c^3*d^6*e^2 - 30*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 + 15*(3*C*a*c^2 + A
*c^3)*d^5*e^3 + 18*(C*a^2*c + A*a*c^2)*d^3*e^5 + (C*a^3 + 3*A*a^2*c)*d*e^7)
*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e^9)
```

**giac [A]** time = 0.17, size = 727, normalized size = 1.56

$$\Big(28\,Cc^3d^6 - 21\,Bc^3d^5e + 45\,Cac^2d^4e^2 + 15\,Ac^3d^4e^2 - 30\,Bac^2d^3e^3 + 18\,Ca^2cd^2e^4 + 18\,Aac^2d^2e^4 - 9\,Ba^2cde^5 + C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] (28*C*c^3*d^6 - 21*B*c^3*d^5*e + 45*C*a*c^2*d^4*e^2 + 15*A*c^3*d^4*e^2 - 30
*B*a*c^2*d^3*e^3 + 18*C*a^2*c*d^2*e^4 + 18*A*a*c^2*d^2*e^4 - 9*B*a^2*c*d*e^
5 + C*a^3*e^6 + 3*A*a^2*c*e^6)*e^(-9)*log(abs(x*e + d)) + 1/60*(10*C*c^3*x^
6*e^15 - 36*C*c^3*d*x^5*e^14 + 90*C*c^3*d^2*x^4*e^13 - 200*C*c^3*d^3*x^3*e^
12 + 450*C*c^3*d^4*x^2*e^11 - 1260*C*c^3*d^5*x*e^10 + 12*B*c^3*x^5*e^15 - 4
5*B*c^3*d*x^4*e^14 + 120*B*c^3*d^2*x^3*e^13 - 300*B*c^3*d^3*x^2*e^12 + 900*
B*c^3*d^4*x*e^11 + 45*C*a*c^2*x^4*e^15 + 15*A*c^3*x^4*e^15 - 180*C*a*c^2*d*
x^3*e^14 - 60*A*c^3*d*x^3*e^14 + 540*C*a*c^2*d^2*x^2*e^13 + 180*A*c^3*d^2*x
^2*e^13 - 1800*C*a*c^2*d^3*x*e^12 - 600*A*c^3*d^3*x*e^12 + 60*B*a*c^2*x^3*e
^15 - 270*B*a*c^2*d*x^2*e^14 + 1080*B*a*c^2*d^2*x*e^13 + 90*C*a^2*c*x^2*e^1
5 + 90*A*a*c^2*x^2*e^15 - 540*C*a^2*c*d*x*e^14 - 540*A*a*c^2*d*x*e^14 + 180
*B*a^2*c*x*e^15)*e^(-18) + 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e + 33*C*a*c^2*
d^6*e^2 + 11*A*c^3*d^6*e^2 - 27*B*a*c^2*d^5*e^3 + 21*C*a^2*c*d^4*e^4 + 21*A
*a*c^2*d^4*e^4 - 15*B*a^2*c*d^3*e^5 + 3*C*a^3*d^2*e^6 + 9*A*a^2*c*d^2*e^6 -
 B*a^3*d*e^7 - A*a^3*e^8 + 2*(8*C*c^3*d^7*e - 7*B*c^3*d^6*e^2 + 18*C*a*c^2*
```

```
d^5*e^3 + 6*A*c^3*d^5*e^3 - 15*B*a*c^2*d^4*e^4 + 12*C*a^2*c*d^3*e^5 + 12*A*
a*c^2*d^3*e^5 - 9*B*a^2*c*d^2*e^6 + 2*C*a^3*d*e^7 + 6*A*a^2*c*d*e^7 - B*a^3
*e^8)*x)*e^(-9)/(x*e + d)^2
```

**maple [B]** time = 0.02, size = 978, normalized size = 2.10

$$\frac{C c^3 x^6}{6e^3}+\frac{B c^3 x^5}{5e^3}-\frac{3C c^3 d x^5}{5e^4}+\frac{A c^3 x^4}{4e^3}-\frac{3B c^3 d x^4}{4e^4}+\frac{3Ca c^2 x^4}{4e^3}+\frac{3C c^3 d^2 x^4}{2e^5}-\frac{A c^3 d x^3}{e^4}+\frac{Ba c^2 x^3}{e^3}+\frac{2B c^3 d^2 x^3}{e^5}-\frac{3Ca c^2 d x^3}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x)
```

```
[Out] 1/6*c^3*C*x^6/e^3-1/e^2/(e*x+d)*B*a^3+1/e^3*ln(e*x+d)*a^3*C-1/2/e/(e*x+d)^2
*A*a^3+1/5*c^3/e^3*B*x^5+1/4*c^3/e^3*A*x^4-3/2/e^7/(e*x+d)^2*C*a*c^2*d^6-9*
c^2/e^4*A*x*a*d+18*c^2/e^5*B*x*a*d^2-30*c^2/e^6*C*x*a*d^3-3*c^2/e^4*C*x^3*a
*d-9/2*c^2/e^4*B*x^2*a*d-9*c/e^4*C*x*a^2*d+9*c^2/e^5*C*x^2*a*d^2+6/e^3/(e*x
+d)*A*a^2*c*d+12/e^5/(e*x+d)*A*a*c^2*d^3-9/e^4/(e*x+d)*B*a^2*c*d^2-15/e^6/(
e*x+d)*B*a*c^2*d^4+12/e^5/(e*x+d)*C*a^2*c*d^3+18/e^7/(e*x+d)*C*a*c^2*d^5+18
/e^5*ln(e*x+d)*A*a*c^2*d^2-9/e^4*ln(e*x+d)*B*a^2*c*d-30/e^6*ln(e*x+d)*B*a*c
^2*d^3+18/e^5*ln(e*x+d)*C*a^2*c*d^2+45/e^7*ln(e*x+d)*C*a*c^2*d^4-3/2/e^3/(e
*x+d)^2*A*d^2*a^2*c-3/2/e^5/(e*x+d)^2*A*a*c^2*d^4+3/2/e^4/(e*x+d)^2*B*a^2*c
*d^3+3/2/e^6/(e*x+d)^2*B*a*c^2*d^5-3/2/e^5/(e*x+d)^2*C*a^2*c*d^4-21/e^8*ln(
e*x+d)*B*c^3*d^5+28/e^9*ln(e*x+d)*C*c^3*d^6-1/2/e^7/(e*x+d)^2*A*c^3*d^6+1/2
/e^2/(e*x+d)^2*B*d*a^3+1/2/e^8/(e*x+d)^2*B*c^3*d^7-1/2/e^3/(e*x+d)^2*C*d^2*
a^3-1/2/e^9/(e*x+d)^2*C*c^3*d^8+3/2*c/e^3*C*x^2*a^2+15*c^3/e^7*B*x*d^4-10*c
^3/e^6*A*x*d^3+3/2*c^2/e^3*A*x^2*a+3*c/e^3*B*x*a^2-21*c^3/e^8*C*x*d^5+3*c^3
/e^5*A*x^2*d^2-5*c^3/e^6*B*x^2*d^3+15/2*c^3/e^7*C*x^2*d^4+3/4*c^2/e^3*C*x^4
*a+3/2*c^3/e^5*C*x^4*d^2-c^3/e^4*A*x^3*d+c^2/e^3*B*x^3*a+2*c^3/e^5*B*x^3*d^
2-10/3*c^3/e^6*C*x^3*d^3+6/e^7/(e*x+d)*A*c^3*d^5-7/e^8/(e*x+d)*B*c^3*d^6+2/
e^3/(e*x+d)*C*a^3*d+8/e^9/(e*x+d)*C*c^3*d^7-3/5*c^3/e^4*C*x^5*d-3/4*c^3/e^4
*B*x^4*d+3/e^3*ln(e*x+d)*A*a^2*c+15/e^7*ln(e*x+d)*A*c^3*d^4
```

**maxima [A]** time = 0.53, size = 701, normalized size = 1.50

$$\frac{15\,Cc^3d^8 - 13\,Bc^3d^7e - 27\,Bac^2d^5e^3 - 15\,Ba^2cd^3e^5 - Ba^3de^7 - Aa^3e^8 + 11\left(3\,Cac^2 + Ac^3\right)d^6e^2 + 21\left(Ca^2c + Aac^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^3*(C*x^2+B*x+A)/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(15*C*c^3*d^8 - 13*B*c^3*d^7*e - 27*B*a*c^2*d^5*e^3 - 15*B*a^2*c*d^3*e^
5 - B*a^3*d*e^7 - A*a^3*e^8 + 11*(3*C*a*c^2 + A*c^3)*d^6*e^2 + 21*(C*a^2*c
+ A*a*c^2)*d^4*e^4 + 3*(C*a^3 + 3*A*a^2*c)*d^2*e^6 + 2*(8*C*c^3*d^7*e - 7*B
*c^3*d^6*e^2 - 15*B*a*c^2*d^4*e^4 - 9*B*a^2*c*d^2*e^6 - B*a^3*e^8 + 6*(3*C*
```

```
a*c^2 + A*c^3)*d^5*e^3 + 12*(C*a^2*c + A*a*c^2)*d^3*e^5 + 2*(C*a^3 + 3*A*a^
2*c)*d*e^7)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/60*(10*C*c^3*e^5*x^6 -
 12*(3*C*c^3*d*e^4 - B*c^3*e^5)*x^5 + 15*(6*C*c^3*d^2*e^3 - 3*B*c^3*d*e^4 +
 (3*C*a*c^2 + A*c^3)*e^5)*x^4 - 20*(10*C*c^3*d^3*e^2 - 6*B*c^3*d^2*e^3 - 3*
B*a*c^2*e^5 + 3*(3*C*a*c^2 + A*c^3)*d*e^4)*x^3 + 30*(15*C*c^3*d^4*e - 10*B*
c^3*d^3*e^2 - 9*B*a*c^2*d*e^4 + 6*(3*C*a*c^2 + A*c^3)*d^2*e^3 + 3*(C*a^2*c
+ A*a*c^2)*e^5)*x^2 - 60*(21*C*c^3*d^5 - 15*B*c^3*d^4*e - 18*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 + 10*(3*C*a*c^2 + A*c^3)*d^3*e^2 + 9*(C*a^2*c + A*a*c^2)*
d*e^4)*x)/e^8 + (28*C*c^3*d^6 - 21*B*c^3*d^5*e - 30*B*a*c^2*d^3*e^3 - 9*B*a
^2*c*d*e^5 + 15*(3*C*a*c^2 + A*c^3)*d^4*e^2 + 18*(C*a^2*c + A*a*c^2)*d^2*e^
4 + (C*a^3 + 3*A*a^2*c)*e^6)*log(e*x + d)/e^9
```

**mupad [B]** time = 3.94, size = 1290, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^3*(A + B*x + C*x^2))/(d + e*x)^3,x)
```

```
[Out] x^3*((d*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3
+ (3*C*c^3*d^2)/e^5))/e - (d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (B*a*
c^2)/e^3 - (C*c^3*d^3)/(3*e^6)) + x*((3*d*((3*d*((3*d*((3*d*((B*c^3)/e^3 -
(3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e - (3*
d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*c^3*d^3)/e^
6))/e - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^
2)/e^3 + (3*C*c^3*d^2)/e^5))/e^2 + (d^3*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^
3 - (3*a*c*(A*c + C*a))/e^3))/e + (d^3*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4
))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/e^3 - (3*d^2*((3*d*((3
*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*
d^2)/e^5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^
3 - (C*c^3*d^3)/e^6))/e^2 + (3*B*a^2*c)/e^3) + x^5*((B*c^3)/(5*e^3) - (3*C*
c^3*d)/(5*e^4)) - x^4*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/(4*e) - (A*c^3
 + 3*C*a*c^2)/(4*e^3) + (3*C*c^3*d^2)/(4*e^5)) - x^2*((3*d*((3*d*((3*d*((B*
c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^
5))/e - (3*d^2*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e^2 + (3*B*a*c^2)/e^3 - (C*
c^3*d^3)/e^6))/(2*e) - (3*d^2*((3*d*((B*c^3)/e^3 - (3*C*c^3*d)/e^4))/e - (A
*c^3 + 3*C*a*c^2)/e^3 + (3*C*c^3*d^2)/e^5))/(2*e^2) + (d^3*((B*c^3)/e^3 - (
3*C*c^3*d)/e^4))/(2*e^3) - (3*a*c*(A*c + C*a))/(2*e^3)) + ((15*C*c^3*d^8 -
A*a^3*e^8 - B*a^3*d*e^7 - 13*B*c^3*d^7*e + 11*A*c^3*d^6*e^2 + 3*C*a^3*d^2*e
^6 + 21*A*a*c^2*d^4*e^4 + 9*A*a^2*c*d^2*e^6 - 27*B*a*c^2*d^5*e^3 - 15*B*a^2
*c*d^3*e^5 + 33*C*a*c^2*d^6*e^2 + 21*C*a^2*c*d^4*e^4)/(2*e) + x*(8*C*c^3*d^
7 - B*a^3*e^7 + 2*C*a^3*d*e^6 - 7*B*c^3*d^6*e + 6*A*c^3*d^5*e^2 + 12*A*a*c^
2*d^3*e^4 - 15*B*a*c^2*d^4*e^3 - 9*B*a^2*c*d^2*e^5 + 18*C*a*c^2*d^5*e^2 + 1
2*C*a^2*c*d^3*e^4 + 6*A*a^2*c*d*e^6))/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (l
og(d + e*x)*(C*a^3*e^6 + 28*C*c^3*d^6 + 3*A*a^2*c*e^6 - 21*B*c^3*d^5*e + 15
```

```
*A*c^3*d^4*e^2 + 18*A*a*c^2*d^2*e^4 - 30*B*a*c^2*d^3*e^3 + 45*C*a*c^2*d^4*e
^2 + 18*C*a^2*c*d^2*e^4 - 9*B*a^2*c*d*e^5))/e^9 + (C*c^3*x^6)/(6*e^3)
```

**sympy [A]** time = 25.28, size = 816, normalized size = 1.75

$$\frac{Cc^3x^6}{6e^3}+x^5\left(\frac{Bc^3}{5e^3}-\frac{3Cc^3d}{5e^4}\right)+x^4\left(\frac{Ac^3}{4e^3}-\frac{3Bc^3d}{4e^4}+\frac{3Cac^2}{4e^3}+\frac{3Cc^3d^2}{2e^5}\right)+x^3\left(-\frac{Ac^3d}{e^4}+\frac{Bac^2}{e^3}+\frac{2Bc^3d^2}{e^5}-\frac{3Cac^2d}{e^4}-\frac{10}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**3*(C*x**2+B*x+A)/(e*x+d)**3,x)
```

```
[Out] C*c**3*x**6/(6*e**3) + x**5*(B*c**3/(5*e**3) - 3*C*c**3*d/(5*e**4)) + x**4*
(A*c**3/(4*e**3) - 3*B*c**3*d/(4*e**4) + 3*C*a*c**2/(4*e**3) + 3*C*c**3*d**
2/(2*e**5)) + x**3*(-A*c**3*d/e**4 + B*a*c**2/e**3 + 2*B*c**3*d**2/e**5 - 3
*C*a*c**2*d/e**4 - 10*C*c**3*d**3/(3*e**6)) + x**2*(3*A*a*c**2/(2*e**3) + 3
*A*c**3*d**2/e**5 - 9*B*a*c**2*d/(2*e**4) - 5*B*c**3*d**3/e**6 + 3*C*a**2*c
/(2*e**3) + 9*C*a*c**2*d**2/e**5 + 15*C*c**3*d**4/(2*e**7)) + x*(-9*A*a*c**
2*d/e**4 - 10*A*c**3*d**3/e**6 + 3*B*a**2*c/e**3 + 18*B*a*c**2*d**2/e**5 +
15*B*c**3*d**4/e**7 - 9*C*a**2*c*d/e**4 - 30*C*a*c**2*d**3/e**6 - 21*C*c**3
*d**5/e**8) + (-A*a**3*e**8 + 9*A*a**2*c*d**2*e**6 + 21*A*a*c**2*d**4*e**4
+ 11*A*c**3*d**6*e**2 - B*a**3*d*e**7 - 15*B*a**2*c*d**3*e**5 - 27*B*a*c**2
*d**5*e**3 - 13*B*c**3*d**7*e + 3*C*a**3*d**2*e**6 + 21*C*a**2*c*d**4*e**4
+ 33*C*a*c**2*d**6*e**2 + 15*C*c**3*d**8 + x*(12*A*a**2*c*d*e**7 + 24*A*a*c
**2*d**3*e**5 + 12*A*c**3*d**5*e**3 - 2*B*a**3*e**8 - 18*B*a**2*c*d**2*e**6
 - 30*B*a*c**2*d**4*e**4 - 14*B*c**3*d**6*e**2 + 4*C*a**3*d*e**7 + 24*C*a**
2*c*d**3*e**5 + 36*C*a*c**2*d**5*e**3 + 16*C*c**3*d**7*e))/(2*d**2*e**9 + 4
*d*e**10*x + 2*e**11*x**2) + (a*e**2 + c*d**2)*(3*A*a*c*e**4 + 15*A*c**2*d*
*2*e**2 - 9*B*a*c*d*e**3 - 21*B*c**2*d**3*e + C*a**2*e**4 + 17*C*a*c*d**2*e
**2 + 28*C*c**2*d**4)*log(d + e*x)/e**9
```

## 3.39 $\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

**Rubi [A]** time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+4bcx+3bdx^2)}{(c+dx)^2} \, dx = \frac{(a+bx^2)^2}{c+dx}$$

**Mathematica [B]** time = 0.04, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2(c^2+cdx+d^2x^2) + b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + 4*b*c*x + 3*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

**fricas [B]** time = 0.95, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4 + 2\,abd^4x^2 + b^2c^4 + 2\,abc^2d^2 + a^2d^4 + \left(b^2c^3d + 2\,abcd^3\right)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="fricas"
)
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

**giac [B]** time = 0.17, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

**maple [B]** time = 0.01, size = 76, normalized size = 4.47

$$\frac{\left(b\,d^2x^3 - bcd\,x^2 + 2a\,d^2x + b\,c^2x\right)b}{d^3} - \frac{-a^2d^4 - 2ab\,c^2d^2 - b^2c^4}{(dx+c)\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^3*(b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/d^4/(d*x+c)
```

**maxima [B]** time = 0.46, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + \left(b^2c^2 + 2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(3*b*d*x^2+4*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxima"
)
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

**mupad [B]** time = 0.08, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)+\frac{b^2\,x^3}{d}+\frac{a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4}{d\,\left(x\,d^4+c\,d^3\right)}-\frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(4*b*c*x - a*d + 3*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

**sympy [B]** time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right)+\frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(3*b*d*x**2+4*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

## 3.40 $\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx^2)^2}{c+dx}$$

```
[Out] (b*x^2+a)^2/(d*x+c)
```

**Rubi [A]** time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1590}

$$\frac{(a+bx^2)^2}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^2/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)(-ad+bx(4c+3dx))}{(c+dx)^2} dx = \frac{(a+bx^2)^2}{c+dx}$$

**Mathematica [B]** time = 0.02, size = 62, normalized size = 3.65

$$\frac{a^2d^4 + 2abd^2(c^2+cdx+d^2x^2) + b^2(c^4+c^3dx+d^4x^4)}{d^4(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(-(a*d) + b*x*(4*c + 3*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^2*d^4 + 2*a*b*d^2*(c^2 + c*d*x + d^2*x^2) + b^2*(c^4 + c^3*d*x + d^4*x^4
))/(d^4*(c + d*x))
```

**fricas [B]** time = 0.84, size = 78, normalized size = 4.59

$$\frac{b^2d^4x^4 + 2\,abd^4x^2 + b^2c^4 + 2\,abc^2d^2 + a^2d^4 + \left(b^2c^3d + 2\,abcd^3\right)x}{d^5x + cd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^4*x^4 + 2*a*b*d^4*x^2 + b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4 + (b^2*c^3
*d + 2*a*b*c*d^3)*x)/(d^5*x + c*d^4)
```

**giac [B]** time = 0.15, size = 111, normalized size = 6.53

$$\frac{\left(b^2 - \frac{4\,b^2c}{dx+c} + \frac{6\,b^2c^2}{(dx+c)^2} + \frac{2\,abd^2}{(dx+c)^2}\right)(dx+c)^3}{d^4} + \frac{\frac{b^2c^4d^3}{dx+c} + \frac{2\,abc^2d^5}{dx+c} + \frac{a^2d^7}{dx+c}}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^2 - 4*b^2*c/(d*x + c) + 6*b^2*c^2/(d*x + c)^2 + 2*a*b*d^2/(d*x + c)^2)*(
d*x + c)^3/d^4 + (b^2*c^4*d^3/(d*x + c) + 2*a*b*c^2*d^5/(d*x + c) + a^2*d^7
/(d*x + c))/d^7
```

**maple [B]** time = 0.00, size = 76, normalized size = 4.47

$$\frac{\left(b\,d^2x^3 - bcd\,x^2 + 2a\,d^2x + b\,c^2x\right)b}{d^3} - \frac{-a^2d^4 - 2ab\,c^2d^2 - b^2c^4}{(dx+c)\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x)
```

```
[Out] (b*d^2*x^3-b*c*d*x^2+2*a*d^2*x+b*c^2*x)*b/d^3-(-a^2*d^4-2*a*b*c^2*d^2-b^2*c
^4)/(d*x+c)/d^4
```

**maxima [B]** time = 0.47, size = 82, normalized size = 4.82

$$\frac{b^2c^4 + 2\,abc^2d^2 + a^2d^4}{d^5x + cd^4} + \frac{b^2d^2x^3 - b^2cdx^2 + \left(b^2c^2 + 2\,abd^2\right)x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(d^5*x + c*d^4) + (b^2*d^2*x^3 - b^2*c*
d*x^2 + (b^2*c^2 + 2*a*b*d^2)*x)/d^3
```

**mupad [B]** time = 3.84, size = 85, normalized size = 5.00

$$x\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)+\frac{b^2\,x^3}{d}+\frac{a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4}{d\,\left(x\,d^4+c\,d^3\right)}-\frac{b^2\,c\,x^2}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(4*c + 3*d*x))*(a + b*x^2))/(c + d*x)^2,x)
```

```
[Out] x*((b^2*c^2)/d^3 + (2*a*b)/d) + (b^2*x^3)/d + (a^2*d^4 + b^2*c^4 + 2*a*b*c^
2*d^2)/(d*(c*d^3 + d^4*x)) - (b^2*c*x^2)/d^2
```

**sympy [B]** time = 0.37, size = 73, normalized size = 4.29

$$-\frac{b^2cx^2}{d^2}+\frac{b^2x^3}{d}+x\left(\frac{2ab}{d}+\frac{b^2c^2}{d^3}\right)+\frac{a^2d^4+2abc^2d^2+b^2c^4}{cd^4+d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(-a*d+b*x*(3*d*x+4*c))/(d*x+c)**2,x)
```

```
[Out] -b**2*c*x**2/d**2 + b**2*x**3/d + x*(2*a*b/d + b**2*c**2/d**3) + (a**2*d**4
 + 2*a*b*c**2*d**2 + b**2*c**4)/(c*d**4 + d**5*x)
```

# 3.41 $\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} \, dx$

**Optimal**. Leaf size=17

$$\frac{(a+bx^2)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

**Rubi [A]** time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1590}

$$\frac{(a+bx^2)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+bx^2)^2(-ad+6bcx+5bdx^2)}{(c+dx)^2} \, dx = \frac{(a+bx^2)^3}{c+dx}$$

**Mathematica [B]** time = 0.04, size = 90, normalized size = 5.29

$$\frac{a^3d^6 + 3a^2bd^4(c^2+cdx+d^2x^2) + 3ab^2d^2(c^4+c^3dx+d^4x^4) + b^3(c^6+c^5dx+d^6x^6)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + 6*b*c*x + 5*b*d*x^2))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

**fricas [B]** time = 0.87, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6 + 3\,ab^2d^6x^4 + 3\,a^2bd^6x^2 + b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6 + \left(b^3c^5d + 3\,ab^2c^3d^3 + 3\,a^2bcd^5\right)x}{d^7x + cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="frica
s")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

**giac [B]** time = 0.18, size = 216, normalized size = 12.71

$$\frac{\left(b^3 - \frac{6\,b^3c}{dx+c} + \frac{15\,b^3c^2}{(dx+c)^2} - \frac{20\,b^3c^3}{(dx+c)^3} + \frac{15\,b^3c^4}{(dx+c)^4} + \frac{3\,ab^2d^2}{(dx+c)^2} - \frac{12\,ab^2cd^2}{(dx+c)^3} + \frac{18\,ab^2c^2d^2}{(dx+c)^4} + \frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c} + \frac{3\,ab^2c^4d^7}{dx+c} + \frac{3\,a^2bc^2}{dx+}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="giac"
)
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

**maple [B]** time = 0.01, size = 157, normalized size = 9.24

$$\frac{\left(b^2d^4x^5 - b^2c\,d^3x^4 + 3ab\,d^4x^3 + b^2c^2d^2x^3 - 3abc\,d^3x^2 - b^2c^3d\,x^2 + 3a^2d^4x + 3ab\,c^2d^2x + b^2c^4x\right)b}{d^5} - \frac{-a^3d^6 - 3a^2b}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x)
```

```
[Out] b/d^5*(b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*
x^2-b^2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/d^6/(d*x+c)
```

**maxima [B]** time = 0.45, size = 160, normalized size = 9.41

$$\frac{b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6}{d^7x + cd^6} + \frac{b^3d^4x^5 - b^3cd^3x^4 + \left(b^3c^2d^2 + 3\,ab^2d^4\right)x^3 - \left(b^3c^3d + 3\,ab^2cd^3\right)x^2 + \left(b^3c^4 +\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(5*b*d*x^2+6*b*c*x-a*d)/(d*x+c)^2,x, algorithm="maxim
a")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

**mupad [B]** time = 3.78, size = 252, normalized size = 14.82

$$x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right)-x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d}+\frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right)+x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(9\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(6*b*c*x - a*d + 5*b*d*x^2))/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

**sympy [B]** time = 0.59, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(5*b*d*x**2+6*b*c*x-a*d)/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

# 3.42 $\int \frac{\left(a+bx^2\right)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx$

**Optimal**. Leaf size=17

$$\frac{\left(a+bx^2\right)^3}{c+dx}$$

```
[Out] (b*x^2+a)^3/(d*x+c)
```

**Rubi [A]** time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {1590}

$$\frac{\left(a+bx^2\right)^3}{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a + b*x^2)^3/(c + d*x)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^2\right)^2(-ad+bx(6c+5dx))}{(c+dx)^2} dx = \frac{\left(a+bx^2\right)^3}{c+dx}$$

**Mathematica [B]** time = 0.02, size = 90, normalized size = 5.29

$$\frac{a^3d^6+3a^2bd^4\left(c^2+cdx+d^2x^2\right)+3ab^2d^2\left(c^4+c^3dx+d^4x^4\right)+b^3\left(c^6+c^5dx+d^6x^6\right)}{d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(-(a*d) + b*x*(6*c + 5*d*x)))/(c + d*x)^2,x]
```

```
[Out] (a^3*d^6 + 3*a^2*b*d^4*(c^2 + c*d*x + d^2*x^2) + 3*a*b^2*d^2*(c^4 + c^3*d*x
 + d^4*x^4) + b^3*(c^6 + c^5*d*x + d^6*x^6))/(d^6*(c + d*x))
```

**fricas [B]** time = 0.89, size = 120, normalized size = 7.06

$$\frac{b^3d^6x^6 + 3\,ab^2d^6x^4 + 3\,a^2bd^6x^2 + b^3c^6 + 3\,ab^2c^4d^2 + 3\,a^2bc^2d^4 + a^3d^6 + \left(b^3c^5d + 3\,ab^2c^3d^3 + 3\,a^2bcd^5\right)x}{d^7x + cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="fricas
")
```

```
[Out] (b^3*d^6*x^6 + 3*a*b^2*d^6*x^4 + 3*a^2*b*d^6*x^2 + b^3*c^6 + 3*a*b^2*c^4*d^
2 + 3*a^2*b*c^2*d^4 + a^3*d^6 + (b^3*c^5*d + 3*a*b^2*c^3*d^3 + 3*a^2*b*c*d^
5)*x)/(d^7*x + c*d^6)
```

**giac [B]** time = 0.20, size = 216, normalized size = 12.71

$$\frac{\left(b^3 - \frac{6\,b^3c}{dx+c} + \frac{15\,b^3c^2}{(dx+c)^2} - \frac{20\,b^3c^3}{(dx+c)^3} + \frac{15\,b^3c^4}{(dx+c)^4} + \frac{3\,ab^2d^2}{(dx+c)^2} - \frac{12\,ab^2cd^2}{(dx+c)^3} + \frac{18\,ab^2c^2d^2}{(dx+c)^4} + \frac{3\,a^2bd^4}{(dx+c)^4}\right)(dx+c)^5}{d^6} + \frac{\frac{b^3c^6d^5}{dx+c} + \frac{3\,ab^2c^4d^7}{dx+c} + \frac{3\,a^2bc}{dx+}}{d^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b^3 - 6*b^3*c/(d*x + c) + 15*b^3*c^2/(d*x + c)^2 - 20*b^3*c^3/(d*x + c)^3
+ 15*b^3*c^4/(d*x + c)^4 + 3*a*b^2*d^2/(d*x + c)^2 - 12*a*b^2*c*d^2/(d*x +
c)^3 + 18*a*b^2*c^2*d^2/(d*x + c)^4 + 3*a^2*b*d^4/(d*x + c)^4)*(d*x + c)^5/
d^6 + (b^3*c^6*d^5/(d*x + c) + 3*a*b^2*c^4*d^7/(d*x + c) + 3*a^2*b*c^2*d^9/
(d*x + c) + a^3*d^11/(d*x + c))/d^11
```

**maple [B]** time = 0.01, size = 157, normalized size = 9.24

$$\frac{\left(b^2d^4x^5 - b^2c\,d^3x^4 + 3ab\,d^4x^3 + b^2c^2d^2x^3 - 3abc\,d^3x^2 - b^2c^3d\,x^2 + 3a^2d^4x + 3ab\,c^2d^2x + b^2c^4x\right)b}{d^5} - \frac{-a^3d^6 - 3a^2b}{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x)
```

```
[Out] (b^2*d^4*x^5-b^2*c*d^3*x^4+3*a*b*d^4*x^3+b^2*c^2*d^2*x^3-3*a*b*c*d^3*x^2-b^
2*c^3*d*x^2+3*a^2*d^4*x+3*a*b*c^2*d^2*x+b^2*c^4*x)*b/d^5-(-a^3*d^6-3*a^2*b*
c^2*d^4-3*a*b^2*c^4*d^2-b^3*c^6)/(d*x+c)/d^6
```

**maxima** [B] time = 0.44, size = 160, normalized size = 9.41

$$\frac{b^3c^6+3\,ab^2c^4d^2+3\,a^2bc^2d^4+a^3d^6}{d^7x+cd^6}+\frac{b^3d^4x^5-b^3cd^3x^4+\left(b^3c^2d^2+3\,ab^2d^4\right)x^3-\left(b^3c^3d+3\,ab^2cd^3\right)x^2+\left(b^3c^4+\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)^2,x, algorithm="maxima
")
```

```
[Out] (b^3*c^6 + 3*a*b^2*c^4*d^2 + 3*a^2*b*c^2*d^4 + a^3*d^6)/(d^7*x + c*d^6) + (
b^3*d^4*x^5 - b^3*c*d^3*x^4 + (b^3*c^2*d^2 + 3*a*b^2*d^4)*x^3 - (b^3*c^3*d
+ 3*a*b^2*c*d^3)*x^2 + (b^3*c^4 + 3*a*b^2*c^2*d^2 + 3*a^2*b*d^4)*x)/d^5
```

**mupad** [B] time = 0.05, size = 252, normalized size = 14.82

$$x^3\left(\frac{3\,a\,b^2}{d}+\frac{b^3\,c^2}{d^3}\right)-x\left(\frac{2\,c\left(\frac{4\,b^3\,c^3}{d^4}-\frac{2\,c\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d}+\frac{12\,a\,b^2\,c}{d^2}\right)}{d}+\frac{c^2\left(\frac{9\,a\,b^2}{d}+\frac{3\,b^3\,c^2}{d^3}\right)}{d^2}-\frac{3\,a^2\,b}{d}\right)+x^2\left(\frac{2\,b^3\,c^3}{d^4}-\frac{c\left(9\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*d - b*x*(6*c + 5*d*x))*(a + b*x^2)^2)/(c + d*x)^2,x)
```

```
[Out] x^3*((3*a*b^2)/d + (b^3*c^2)/d^3) - x*((2*c*((4*b^3*c^3)/d^4 - (2*c*((9*a*b
^2)/d + (3*b^3*c^2)/d^3))/d + (12*a*b^2*c)/d^2))/d + (c^2*((9*a*b^2)/d + (3
*b^3*c^2)/d^3))/d^2 - (3*a^2*b)/d) + x^2*((2*b^3*c^3)/d^4 - (c*((9*a*b^2)/d
 + (3*b^3*c^2)/d^3))/d + (6*a*b^2*c)/d^2) + (a^3*d^6 + b^3*c^6 + 3*a*b^2*c^
4*d^2 + 3*a^2*b*c^2*d^4)/(d*(c*d^5 + d^6*x)) + (b^3*x^5)/d - (b^3*c*x^4)/d^
2
```

**sympy** [B] time = 0.61, size = 153, normalized size = 9.00

$$-\frac{b^3cx^4}{d^2}+\frac{b^3x^5}{d}+x^3\left(\frac{3ab^2}{d}+\frac{b^3c^2}{d^3}\right)+x^2\left(-\frac{3ab^2c}{d^2}-\frac{b^3c^3}{d^4}\right)+x\left(\frac{3a^2b}{d}+\frac{3ab^2c^2}{d^3}+\frac{b^3c^4}{d^5}\right)+\frac{a^3d^6+3a^2bc^2d^4+3ab^2c^4d}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(-a*d+b*x*(5*d*x+6*c))/(d*x+c)**2,x)
```

```
[Out] -b**3*c*x**4/d**2 + b**3*x**5/d + x**3*(3*a*b**2/d + b**3*c**2/d**3) + x**2
*(-3*a*b**2*c/d**2 - b**3*c**3/d**4) + x*(3*a**2*b/d + 3*a*b**2*c**2/d**3 +
 b**3*c**4/d**5) + (a**3*d**6 + 3*a**2*b*c**2*d**4 + 3*a*b**2*c**4*d**2 + b
**3*c**6)/(c*d**6 + d**7*x)
```

## 3.43 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$

**Optimal**. Leaf size=240

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(cd^2-3ae^2\right)+a\left(ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+\right.}{2c^3}$$

```
[Out] -(a*e^2*(B*e+3*C*d)-c*d*(C*d^2+3*e*(A*e+B*d)))*x/c^2-1/2*e*(a*C*e^2-c*(3*C*
d^2+e*(A*e+3*B*d)))*x^2/c^2+1/3*e^2*(B*e+3*C*d)*x^3/c+1/4*C*e^3*x^4/c+1/2*(
B*c*d*(-3*a*e^2+c*d^2)+(A*c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/c^3+(A*c*d
*(-3*a*e^2+c*d^2)+a*(a*e^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)
/a^(1/2))/c^(5/2)/a^(1/2)
```

**Rubi [A]** time = 0.47, antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{ex^2\left(-aCe^2+ce(Ae+3Bd)+3cCd^2\right)}{2c^2}+\frac{\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)+Bcd\left(cd^2-3ae^2\right)\right)}{2c^3}+\frac{x\left(-ae^2(Be+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^3 + 3*c*d*e*(B*d + A*e) - a*e^2*(3*C*d + B*e))*x)/c^2 + (e*(3*c*C*d
^2 - a*C*e^2 + c*e*(3*B*d + A*e))*x^2)/(2*c^2) + (e^2*(3*C*d + B*e)*x^3)/(3
*c) + (C*e^3*x^4)/(4*c) + ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e
) - c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) +
((B*c*d*(c*d^2 - 3*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2]
)/(2*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx = \int\left(\frac{cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+A\right.}{c^2}\right.$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+A\right.}{2c^2}$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+A\right.}{2c^2}$$

$$=\frac{\left(cCd^3+3cde(Bd+Ae)-ae^2(3Cd+Be)\right)x}{c^2}+\frac{e\left(3cCd^2-aCe^2+ce(3Bd+A\right.}{2c^2}$$

**Mathematica [A]** time = 0.24, size = 223, normalized size = 0.93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(cd^2-3ae^2\right)+a\left(ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{6\log\left(a+cx^2\right)\left(e(Ac-aC)\left(3cd^2-ae^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (c*x*(-6*a*e^2*(6*C*d + 2*
B*e + C*e*x) + 3*c*C*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + 2*c*e*(3
*A*e*(6*d + e*x) + B*(18*d^2 + 9*d*e*x + 2*e^2*x^2))) + 6*(B*c*d*(c*d^2 - 3
*a*e^2) + (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(12*c^3)
```

**fricas [A]** time = 0.90, size = 592, normalized size = 2.47

$$\left[\frac{3\,Cac^2e^3x^4+4\left(3\,Cac^2de^2+Bac^2e^3\right)x^3+6\left(3\,Cac^2d^2e+3\,Bac^2de^2-\left(Ca^2c-Aac^2\right)e^3\right)x^2+6\left(3\,Bacd^2e-Ba^2e^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*e^3)*x^3 + 6*(3*C*a
*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*x^2 + 6*(3*B*a*c*d^
2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2)*sqrt(-a*c)
*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 12*(C*a*c^2*d^3 + 3*B*a*c^
2*d^2*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3
*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c
*x^2 + a))/(a*c^3), 1/12*(3*C*a*c^2*e^3*x^4 + 4*(3*C*a*c^2*d*e^2 + B*a*c^2*
e^3)*x^3 + 6*(3*C*a*c^2*d^2*e + 3*B*a*c^2*d*e^2 - (C*a^2*c - A*a*c^2)*e^3)*
x^2 - 12*(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*
c)*d*e^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 12*(C*a*c^2*d^3 + 3*B*a*c^2*d^2
*e - B*a^2*c*e^3 - 3*(C*a^2*c - A*a*c^2)*d*e^2)*x + 6*(B*a*c^2*d^3 - 3*B*a^
2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3 - A*a^2*c)*e^3)*log(c*x^2
+ a))/(a*c^3)]
```

**giac [A]** time = 0.17, size = 279, normalized size = 1.16

$$-\frac{\left(Cacd^3-Ac^2d^3+3\,Bacd^2e-3\,Ca^2de^2+3\,Aacde^2-Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2}+\frac{\left(Bc^2d^3-3\,Cacd^2e+3\,Ac^2d^2e-3\,B\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^3 - A*c^2*d^3 + 3*B*a*c*d^2*e - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 - B
*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/2*(B*c^2*d^3 - 3*C*a*c*
d^2*e + 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 + C*a^2*e^3 - A*a*c*e^3)*log(c*x^2 +
a)/c^3 + 1/12*(3*C*c^3*x^4*e^3 + 12*C*c^3*d*x^3*e^2 + 18*C*c^3*d^2*x^2*e +
12*C*c^3*d^3*x + 4*B*c^3*x^3*e^3 + 18*B*c^3*d*x^2*e^2 + 36*B*c^3*d^2*x*e -
6*C*a*c^2*x^2*e^3 + 6*A*c^3*x^2*e^3 - 36*C*a*c^2*d*x*e^2 + 36*A*c^3*d*x*e^2
 - 12*B*a*c^2*x*e^3)/c^4
```

**maple [A]** time = 0.01, size = 399, normalized size = 1.66

$$\frac{C\,e^3x^4}{4c}+\frac{B\,e^3x^3}{3c}+\frac{Cd\,e^2x^3}{c}-\frac{3Aad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}+\frac{A\,e^3x^2}{2c}+\frac{A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}+\frac{B\,a^2e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2}-\frac{3Ba\,d^2e\,\mathrm{arc}}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/4*C*e^3*x^4/c+1/3/c*B*x^3*e^3+1/c*C*x^3*d*e^2+1/2/c*A*x^2*e^3+3/2/c*B*x^2
*d*e^2-1/2/c^2*C*x^2*a*e^3+3/2/c*C*x^2*d^2*e+3/c*A*x*d*e^2-1/c^2*B*x*a*e^3+
3/c*B*x*d^2*e-3/c^2*C*x*a*d*e^2+1/c*C*x*d^3-1/2/c^2*ln(c*x^2+a)*A*a*e^3+3/2
/c*ln(c*x^2+a)*A*d^2*e-3/2/c^2*ln(c*x^2+a)*B*a*d*e^2+1/2/c*ln(c*x^2+a)*B*d^
3+1/2/c^3*ln(c*x^2+a)*C*a^2*e^3-3/2/c^2*ln(c*x^2+a)*C*a*d^2*e-3/c/(a*c)^(1/
2)*arctan(x*c/(a*c)^(1/2))*A*a*d*e^2+1/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
A*d^3+1/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*B*a^2*e^3-3/c/(a*c)^(1/2)*a
rctan(x*c/(a*c)^(1/2))*B*a*d^2*e+3/c^2/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*
C*a^2*d*e^2-1/c/(a*c)^(1/2)*arctan(x*c/(a*c)^(1/2))*C*a*d^3
```

**maxima [A]** time = 0.98, size = 244, normalized size = 1.02

$$-\frac{\left(3\,Bacd^2e - Ba^2e^3 + \left(Cac - Ac^2\right)d^3 - 3\left(Ca^2 - Aac\right)de^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{3\,Cce^3x^4 + 4\left(3\,Ccde^2 + Bce^3\right)x^3 + 6\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(3*B*a*c*d^2*e - B*a^2*e^3 + (C*a*c - A*c^2)*d^3 - 3*(C*a^2 - A*a*c)*d*e^2
)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/12*(3*C*c*e^3*x^4 + 4*(3*C*c*d*
e^2 + B*c*e^3)*x^3 + 6*(3*C*c*d^2*e + 3*B*c*d*e^2 - (C*a - A*c)*e^3)*x^2 +
12*(C*c*d^3 + 3*B*c*d^2*e - B*a*e^3 - 3*(C*a - A*c)*d*e^2)*x)/c^2 + 1/2*(B*
c^2*d^3 - 3*B*a*c*d*e^2 - 3*(C*a*c - A*c^2)*d^2*e + (C*a^2 - A*a*c)*e^3)*lo
g(c*x^2 + a)/c^3
```

**mupad [B]** time = 3.99, size = 277, normalized size = 1.15

$$x^2\left(\frac{3\,C\,d^2\,e + 3\,B\,d\,e^2 + A\,e^3}{2\,c} - \frac{C\,a\,e^3}{2\,c^2}\right) + x\left(\frac{C\,d^3 + 3\,B\,d^2\,e + 3\,A\,d\,e^2}{c} - \frac{a\left(B\,e^3 + 3\,C\,d\,e^2\right)}{c^2}\right) + \frac{x^3\left(B\,e^3 + 3\,C\,d\,e^2\right.}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x^2*((A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(2*c) - (C*a*e^3)/(2*c^2)) + x*((C*d^3
 + 3*A*d*e^2 + 3*B*d^2*e)/c - (a*(B*e^3 + 3*C*d*e^2))/c^2) + (x^3*(B*e^3 +
3*C*d*e^2))/(3*c) + (C*e^3*x^4)/(4*c) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d
^3 + B*a^2*e^3 - C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e)
)/(a^(1/2)*c^(5/2)) + (log(a + c*x^2)*(4*B*a*c^5*d^3 - 4*A*a^2*c^4*e^3 + 4*
```

```
C*a^3*c^3*e^3 - 12*B*a^2*c^4*d*e^2 - 12*C*a^2*c^4*d^2*e + 12*A*a*c^5*d^2*e)
)/(8*a*c^6)
```

**sympy [B]** time = 5.46, size = 1008, normalized size = 4.20

$$\frac{Ce^3x^4}{4c}+x^3\left(\frac{Be^3}{3c}+\frac{Cde^2}{c}\right)+x^2\left(\frac{Ae^3}{2c}+\frac{3Bde^2}{2c}-\frac{Cae^3}{2c^2}+\frac{3Cd^2e}{2c}\right)+x\left(\frac{3Ade^2}{c}-\frac{Bae^3}{c^2}+\frac{3Bd^2e}{c}-\frac{3Cade^2}{c^2}+\frac{Cd^3}{c}\right)+\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**3*x**4/(4*c) + x**3*(B*e**3/(3*c) + C*d*e**2/c) + x**2*(A*e**3/(2*c) +
 3*B*d*e**2/(2*c) - C*a*e**3/(2*c**2) + 3*C*d**2*e/(2*c)) + x*(3*A*d*e**2/c
 - B*a*e**3/c**2 + 3*B*d**2*e/c - 3*C*a*d*e**2/c**2 + C*d**3/c) + ((-A*a*c*
e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a
*c*d**2*e)/(2*c**3) - sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2
*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6))*log(x +
(A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*
a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c**2*d**2*e -
3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2*c**3) - sqr
t(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a*c*d**2*e +
3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2 + A*c**3*d**
3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))
 + ((-A*a*c*e**3 + 3*A*c**2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*
e**3 - 3*C*a*c*d**2*e)/(2*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d
**3 + B*a**2*e**3 - 3*B*a*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**
6))*log(x + (A*a**2*c*e**3 - 3*A*a*c**2*d**2*e + 3*B*a**2*c*d*e**2 - B*a*c*
*2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*e + 2*a*c**3*((-A*a*c*e**3 + 3*A*c*
*2*d**2*e - 3*B*a*c*d*e**2 + B*c**2*d**3 + C*a**2*e**3 - 3*C*a*c*d**2*e)/(2
*c**3) + sqrt(-a*c**7)*(-3*A*a*c*d*e**2 + A*c**2*d**3 + B*a**2*e**3 - 3*B*a
*c*d**2*e + 3*C*a**2*d*e**2 - C*a*c*d**3)/(2*a*c**6)))/(-3*A*a*c**2*d*e**2
+ A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e + 3*C*a**2*c*d*e**2 - C*a
*c**2*d**3))
```

# 3.44 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{a+cx^2}\,dx$

**Optimal**. Leaf size=168

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(2Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}-\frac{x\,(a}{}$$

```
[Out] -(a*C*e^2-c*(C*d^2+e*(A*e+2*B*d)))*x/c^2+1/2*e*(B*e+2*C*d)*x^2/c+1/3*C*e^2*
x^3/c+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2-2*C*a*d*e)*ln(c*x^2+a)/c^2+(A*c*(-a*e^
2+c*d^2)+a*(a*C*e^2-c*d*(2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/c^(5/2)/a^(
1/2)
```

**Rubi** [A] time = 0.26, antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2-2aCde+2Acde+Bcd^2\right)}{2c^2}+\frac{x\left(-aCe^2+ce(Ae+2Bd)+cCd^2\right)}{c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((c*C*d^2 - a*C*e^2 + c*e*(2*B*d + A*e))*x)/c^2 + (e*(2*C*d + B*e)*x^2)/(2*
c) + (C*e^2*x^3)/(3*c) + ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*
B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + ((B*c*d^2 + 2*A*c*d
*e - 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{a+cx^2}dx = \int \left(\frac{cCd^2-aCe^2+ce(2Bd+Ae)}{c^2}+\frac{e(2Cd+Be)x}{c}+\frac{Ce^2x^2}{c}+\frac{Ac\left(cd^2-ae^2\right)+\right.$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}+\frac{\int \frac{Ac\left(cd^2-ae^2\right)+a(a}{}}{}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}+\frac{\left(Bcd^2+2Acde\right.}{}$$

$$= \frac{\left(cCd^2-aCe^2+ce(2Bd+Ae)\right)x}{c^2}+\frac{e(2Cd+Be)x^2}{2c}+\frac{Ce^2x^3}{3c}+\frac{\left(Ac\left(cd^2-ae^2\right)\right.}{}$$

**Mathematica [A]** time = 0.17, size = 155, normalized size = 0.92

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(2Be+Cd)\right)\right)}{\sqrt{a}\,c^{5/2}}+\frac{x\left(-6aCe^2+3ce(2Ae+4Bd+Bex)+2cC\left(3d^2+3dex\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)
/Sqrt[a]])/(Sqrt[a]*c^(5/2)) + (x*(-6*a*C*e^2 + 3*c*e*(4*B*d + 2*A*e + B*e*
x) + 2*c*C*(3*d^2 + 3*d*e*x + e^2*x^2)) + 3*(B*c*d^2 + 2*A*c*d*e - 2*a*C*d*
e - a*B*e^2)*Log[a + c*x^2])/(6*c^2)
```

**fricas [A]** time = 0.79, size = 404, normalized size = 2.40

$$\left[\frac{2\,Cac^2e^2x^3+3\left(2\,Cac^2de+Bac^2e^2\right)x^2-3\left(2\,Bacde+\left(Cac-Ac^2\right)d^2-\left(Ca^2-Aac\right)e^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2+2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{6\,ac^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a*c^2*e^2)*x^2 - 3*(2*B*a*c*
d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*sqrt(-a*c)*log((c*x^2 + 2*
sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*c
- A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*d*
e)*log(c*x^2 + a))/(a*c^3), 1/6*(2*C*a*c^2*e^2*x^3 + 3*(2*C*a*c^2*d*e + B*a
*c^2*e^2)*x^2 - 6*(2*B*a*c*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)
*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 6*(C*a*c^2*d^2 + 2*B*a*c^2*d*e - (C*a^2*
c - A*a*c^2)*e^2)*x + 3*(B*a*c^2*d^2 - B*a^2*c*e^2 - 2*(C*a^2*c - A*a*c^2)*
d*e)*log(c*x^2 + a))/(a*c^3)]
```

**giac [A]** time = 0.16, size = 176, normalized size = 1.05

$$\frac{\left(Bcd^2 - 2\,Cade + 2\,Acde - Bae^2\right)\log\left(cx^2 + a\right)}{2\,c^2} - \frac{\left(Cacd^2 - Ac^2d^2 + 2\,Bacde - Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d^2 - 2*C*a*d*e + 2*A*c*d*e - B*a*e^2)*log(c*x^2 + a)/c^2 - (C*a*c
*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c
))/(sqrt(a*c)*c^2) + 1/6*(2*C*c^2*x^3*e^2 + 6*C*c^2*d*x^2*e + 6*C*c^2*d^2*x
 + 3*B*c^2*x^2*e^2 + 12*B*c^2*d*x*e - 6*C*a*c*x*e^2 + 6*A*c^2*x*e^2)/c^3
```

**maple [A]** time = 0.01, size = 256, normalized size = 1.52

$$\frac{C\,e^2x^3}{3c} - \frac{Aa\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c} + \frac{A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{2Bade\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c} + \frac{B\,e^2x^2}{2c} + \frac{C\,a^2e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c^2} - \frac{Ca\,d^2\,\mathrm{ar}\cdots}{\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/3*C*e^2*x^3/c+1/2/c*B*x^2*e^2+1/c*C*x^2*d*e+1/c*A*e^2*x+2/c*B*d*e*x-1/c^2
*a*C*e^2*x+1/c*C*d^2*x+1/c*ln(c*x^2+a)*A*d*e-1/2/c^2*ln(c*x^2+a)*B*a*e^2+1/
2/c*ln(c*x^2+a)*B*d^2-1/c^2*ln(c*x^2+a)*C*a*d*e-1/c/(a*c)^(1/2)*arctan(1/(a
*c)^(1/2)*c*x)*A*a*e^2+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2-2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*d*e+1/c^2/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x)*a^2*C*e^2-1/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*d^2
```

**maxima [A]** time = 0.99, size = 161, normalized size = 0.96

$$\frac{\left(Bcd^2 - Bae^2 - 2\,(Ca - Ac)de\right)\log\left(cx^2 + a\right)}{2\,c^2} - \frac{\left(2\,Bacde + \left(Cac - Ac^2\right)d^2 - \left(Ca^2 - Aac\right)e^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c^2} + \frac{2\,Cc\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 - 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/c^2 - (2*B*a*c*d
*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*c^2) + 1/6*(2*C*c*e^2*x^3 + 3*(2*C*c*d*e + B*c*e^2)*x^2 + 6*(C*c*d^2
+ 2*B*c*d*e - (C*a - A*c)*e^2)*x)/c^2
```

**mupad [B]** time = 3.90, size = 181, normalized size = 1.08

$$x\left(\frac{C\,d^2+2\,B\,d\,e+A\,e^2}{c}-\frac{C\,a\,e^2}{c^2}\right)+\frac{x^2\,\left(B\,e^2+2\,C\,d\,e\right)}{2\,c}+\frac{C\,e^2\,x^3}{3\,c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\right.}{\sqrt{a}\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] x*((A*e^2 + C*d^2 + 2*B*d*e)/c - (C*a*e^2)/c^2) + (x^2*(B*e^2 + 2*C*d*e))/(
2*c) + (C*e^2*x^3)/(3*c) - (atan((c^(1/2)*x)/a^(1/2))*(A*a*c*e^2 - C*a^2*e^
2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(a^(1/2)*c^(5/2)) + (log(a + c*x^
2)*(4*B*a*c^4*d^2 - 4*B*a^2*c^3*e^2 + 8*A*a*c^4*d*e - 8*C*a^2*c^3*d*e))/(8*
a*c^5)
```

**sympy [B]** time = 3.15, size = 638, normalized size = 3.80

$$\frac{Ce^2x^3}{3c}+x^2\left(\frac{Be^2}{2c}+\frac{Cde}{c}\right)+x\left(\frac{Ae^2}{c}+\frac{2Bde}{c}-\frac{Cae^2}{c^2}+\frac{Cd^2}{c}\right)+\left(-\frac{-2Acde+Bae^2-Bcd^2+2Cade}{2c^2}-\frac{\sqrt{-ac^5}\,\left(-Aace^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e**2*x**3/(3*c) + x**2*(B*e**2/(2*c) + C*d*e/c) + x*(A*e**2/c + 2*B*d*e/c
 - C*a*e**2/c**2 + C*d**2/c) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2*C*a*
d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*
a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*d**2 + 2
*C*a*d*e)/(2*c**2) - sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e
 + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**2 - 2*B*
a*c*d*e + C*a**2*e**2 - C*a*c*d**2)) + (-(-2*A*c*d*e + B*a*e**2 - B*c*d**2
+ 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*
d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5))*log(x + (-2*A*a*c*d*e + B*a**2*
```

```
e**2 - B*a*c*d**2 + 2*C*a**2*d*e + 2*a*c**2*(-(-2*A*c*d*e + B*a*e**2 - B*c*
d**2 + 2*C*a*d*e)/(2*c**2) + sqrt(-a*c**5)*(-A*a*c*e**2 + A*c**2*d**2 - 2*B
*a*c*d*e + C*a**2*e**2 - C*a*c*d**2)/(2*a*c**5)))/(-A*a*c*e**2 + A*c**2*d**
2 - 2*B*a*c*d*e + C*a**2*e**2 - C*a*c*d**2))
```

## 3.45 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx$

**Optimal**. Leaf size=93

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}\,c^{3/2}}+\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

```
[Out] (B*e+C*d)*x/c+1/2*C*e*x^2/c+1/2*(A*c*e+B*c*d-C*a*e)*ln(c*x^2+a)/c^2+(A*c*d-
a*(B*e+C*d))*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)}{2c^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(Acd-a(Be+Cd))}{\sqrt{a}\,c^{3/2}}+\frac{x(Be+Cd)}{c}+\frac{Cex^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] ((C*d + B*e)*x)/c + (C*e*x^2)/(2*c) + ((A*c*d - a*(C*d + B*e))*ArcTan[(Sqrt
[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + ((B*c*d + A*c*e - a*C*e)*Log[a + c*x^2
])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{a+cx^2}\,dx &= \int \left(\frac{Cd+Be}{c}+\frac{Cex}{c}+\frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{c\left(a+cx^2\right)}\right)dx\\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{\int \frac{Acd-a(Cd+Be)+(Bcd+Ace-aCe)x}{a+cx^2}\,dx}{c}\\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Bcd+Ace-aCe)\int \frac{x}{a+cx^2}\,dx}{c}+\frac{(Acd-a(Cd+Be))\int}{c}\\
&= \frac{(Cd+Be)x}{c}+\frac{Cex^2}{2c}+\frac{(Acd-a(Cd+Be))\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{(Bcd+Ace-aCe)}{2c^2}\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 86, normalized size = 0.92

$$\frac{\log\left(a+cx^2\right)(-aCe+Ace+Bcd)-\frac{2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd-Acd)}{\sqrt{a}}+cx(2Be+2Cd+Cex)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x]
```

```
[Out] (c*x*(2*C*d + 2*B*e + C*e*x) - (2*Sqrt[c]*(-(A*c*d) + a*C*d + a*B*e)*ArcTan
[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] + (B*c*d + A*c*e - a*C*e)*Log[a + c*x^2])/(2
*c^2)
```

**fricas [A]** time = 0.58, size = 206, normalized size = 2.22

$$\left[\frac{Cacex^2-(Bae+(Ca-Ac)d)\sqrt{-ac}\,\log\left(\frac{cx^2+2\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\,(Cacd+Bace)x+\left(Bacd-\left(Ca^2-Aac\right)e\right)\log\left(cx^2\right.}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(C*a*c*e*x^2 - (B*a*e + (C*a - A*c)*d)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(
-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d - (C*a^2 - A
*a*c)*e)*log(c*x^2 + a))/(a*c^2), 1/2*(C*a*c*e*x^2 - 2*(B*a*e + (C*a - A*c)
*d)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 2*(C*a*c*d + B*a*c*e)*x + (B*a*c*d -
(C*a^2 - A*a*c)*e)*log(c*x^2 + a))/(a*c^2)]
```

**giac** [A] time = 0.16, size = 91, normalized size = 0.98

$$-\frac{(Cad - Acd + Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c} + \frac{(Bcd - Cae + Ace)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{Ccx^2e + 2\,Ccdx + 2\,Bcxe}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*d - A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(B*c*d -
 C*a*e + A*c*e)*log(c*x^2 + a)/c^2 + 1/2*(C*c*x^2*e + 2*C*c*d*x + 2*B*c*x*e
)/c^2
```

**maple** [A] time = 0.01, size = 133, normalized size = 1.43

$$\frac{Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{Bae\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c} - \frac{Cad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c} + \frac{Ce\,x^2}{2c} + \frac{Ae\ln\left(c\,x^2 + a\right)}{2c} + \frac{Bd\ln\left(c\,x^2 + a\right)}{2c} + \frac{Bex}{c} - \frac{Cae\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] 1/2*C*e*x^2/c+1/c*B*e*x+1/c*C*d*x+1/2/c*ln(c*x^2+a)*A*e+1/2/c*ln(c*x^2+a)*B
*d-1/2/c^2*ln(c*x^2+a)*a*C*e+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d-1/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*e-1/c/(a*c)^(1/2)*arctan(1/(a*c
)^(1/2)*c*x)*C*a*d
```

**maxima** [A] time = 0.97, size = 86, normalized size = 0.92

$$-\frac{(Bae + (Ca - Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c} + \frac{Cex^2 + 2\,(Cd + Be)x}{2\,c} + \frac{(Bcd - (Ca - Ac)e)\log\left(cx^2 + a\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -(B*a*e + (C*a - A*c)*d)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*c) + 1/2*(C*e*x^2
 + 2*(C*d + B*e)*x)/c + 1/2*(B*c*d - (C*a - A*c)*e)*log(c*x^2 + a)/c^2
```

**mupad [B]** time = 3.78, size = 97, normalized size = 1.04

$$\frac{x\,\left(B\,e+C\,d\right)}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(B\,a\,e-A\,c\,d+C\,a\,d\right)}{\sqrt{a}\,c^{3/2}}+\frac{C\,e\,x^2}{2\,c}+\frac{\ln\left(c\,x^2+a\right)\,\left(4\,A\,a\,c^3\,e+4\,B\,a\,c^3\,d-4\,C\,a^2\,c^2\,e\right)}{8\,a\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2),x)
```

```
[Out] (x*(B*e + C*d))/c - (atan((c^(1/2)*x)/a^(1/2))*(B*a*e - A*c*d + C*a*d))/(a^
(1/2)*c^(3/2)) + (C*e*x^2)/(2*c) + (log(a + c*x^2)*(4*A*a*c^3*e + 4*B*a*c^3
*d - 4*C*a^2*c^2*e))/(8*a*c^4)
```

**sympy [B]** time = 1.66, size = 337, normalized size = 3.62

$$\frac{Cex^2}{2c}+x\left(\frac{Be}{c}+\frac{Cd}{c}\right)+\left(-\frac{-Ace-Bcd+Cae}{2c^2}-\frac{\sqrt{-ac^5}\,(-Acd+Bae+Cad)}{2ac^4}\right)\log\left(x+\frac{Aace+Bacd-Ca^2e-2ac}{-Ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*e*x**2/(2*c) + x*(B*e/c + C*d/c) + (-(-A*c*e - B*c*d + C*a*e)/(2*c**2) -
sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c
*d - C*a**2*e - 2*a*c**2*(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) - sqrt(-a*c**5
)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)))/(-A*c**2*d + B*a*c*e + C*a*c*d)) +
(-(-A*c*e - B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d
)/(2*a*c**4))*log(x + (A*a*c*e + B*a*c*d - C*a**2*e - 2*a*c**2*(-(-A*c*e -
B*c*d + C*a*e)/(2*c**2) + sqrt(-a*c**5)*(-A*c*d + B*a*e + C*a*d)/(2*a*c**4)
))/(-A*c**2*d + B*a*c*e + C*a*c*d))
```

## 3.46 $\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx$

**Optimal**. Leaf size=55

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+(A*c-C*a)*arctan(x*c^(1/2)/a^(1/2))/c^(3/2)/a^(1/
2)
```

**Rubi** [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1810, 635, 205, 260}

$$\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c + ((A*c - a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) + (B*
Log[a + c*x^2])/(2*c)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{a+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC+Bcx}{c\left(a+cx^2\right)}\right)dx\\ &= \frac{Cx}{c}+\frac{\int \frac{Ac-aC+Bcx}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+B\int \frac{x}{a+cx^2}\,dx+\frac{(Ac-aC)\int \frac{1}{a+cx^2}\,dx}{c}\\ &= \frac{Cx}{c}+\frac{(Ac-aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 56, normalized size = 1.02

$$-\frac{(aC-Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{B\log\left(a+cx^2\right)}{2c}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2),x]
```

```
[Out] (C*x)/c - ((-(A*c) + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*c^(3/2)) +
(B*Log[a + c*x^2])/(2*c)
```

**fricas** [A] time = 0.80, size = 125, normalized size = 2.27

$$\left[\frac{2\,Cacx+Bac\log\left(cx^2+a\right)+(Ca-Ac)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)}{2\,ac^2},\ \frac{2\,Cacx+Bac\log\left(cx^2+a\right)-2\,(Ca-Ac)\sqrt{a}}{2\,ac^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*C*a*c*x + B*a*c*log(c*x^2 + a) + (C*a - A*c)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)))/(a*c^2), 1/2*(2*C*a*c*x + B*a*c*log(c*x
^2 + a) - 2*(C*a - A*c)*sqrt(a*c)*arctan(sqrt(a*c)*x/a))/(a*c^2)]
```

**giac** [A] time = 0.16, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B\log\left(cx^2 + a\right)}{2\,c} - \frac{(Ca - Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

**maple** [A] time = 0.00, size = 59, normalized size = 1.07

$$\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}} - \frac{Ca\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c} + \frac{B\ln\left(c\,x^2 + a\right)}{2c} + \frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a),x)
```

```
[Out] C*x/c+1/2*B*ln(c*x^2+a)/c+1/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A-1/c/(a*
c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*a*C
```

**maxima** [A] time = 0.97, size = 48, normalized size = 0.87

$$\frac{Cx}{c} + \frac{B\log\left(cx^2 + a\right)}{2\,c} - \frac{(Ca - Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + 1/2*B*log(c*x^2 + a)/c - (C*a - A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*
c)*c)
```

**mupad** [B] time = 3.73, size = 56, normalized size = 1.02

$$\frac{B\,\ln\left(c\,x^2 + a\right)}{2\,c} + \frac{C\,x}{c} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2),x)
```

```
[Out] (B*log(a + c*x^2))/(2*c) + (C*x)/c + (A*atan((c^(1/2)*x)/a^(1/2)))/(a^(1/2)
*c^(1/2)) - (C*a^(1/2)*atan((c^(1/2)*x)/a^(1/2)))/c^(3/2)
```

**sympy [B]** time = 0.49, size = 156, normalized size = 2.84

$$\frac{Cx}{c}+\left(\frac{B}{2c}-\frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)\log\left(x+\frac{Ba-2ac\left(\frac{B}{2c}-\frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)}{-Ac+Ca}\right)+\left(\frac{B}{2c}+\frac{\sqrt{-ac^3}\,(-Ac+Ca)}{2ac^3}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a),x)
```

```
[Out] C*x/c + (B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*
a*c*(B/(2*c) - sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a)) + (B/(
2*c) + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3))*log(x + (B*a - 2*a*c*(B/(2*c)
 + sqrt(-a*c**3)*(-A*c + C*a)/(2*a*c**3)))/(-A*c + C*a))
```

# 3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)(a+cx^2)}\, dx$

**Optimal**. Leaf size=133

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\,\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] (A*e^2-B*d*e+C*d^2)*ln(e*x+d)/e/(a*e^2+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e)*ln(c
*x^2+a)/c/(a*e^2+c*d^2)+(A*c*d+B*a*e-C*a*d)*arctan(x*c^(1/2)/a^(1/2))/(a*e^
2+c*d^2)/a^(1/2)/c^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)(aCe-Ace+Bcd)}{2c\left(ae^2+cd^2\right)}+\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{e\left(ae^2+cd^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+Acd)}{\sqrt{a}\,\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] ((A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]*(c*d
^2 + a*e^2)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(e*(c*d^2 + a*e^2)) +
 ((B*c*d - A*c*e + a*C*e)*Log[a + c*x^2])/(2*c*(c*d^2 + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)}dx = \int \left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)}+\frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{\int \frac{Acd-aCd+aBe+(Bcd-Ace+aCe)x}{a+cx^2}dx}{cd^2+ae^2}$$

$$= \frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{(Acd-aCd+aBe)\int \frac{1}{a+cx^2}dx}{cd^2+ae^2}+\frac{(Bcd-Ace+aCe}{cd^2+a}$$

$$= \frac{(Acd-aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{e\left(cd^2+ae^2\right)}+\frac{(Bcd-Ace+a}{2c\left(cd\right.}$$

**Mathematica [A]** time = 0.10, size = 120, normalized size = 0.90

$$\frac{\sqrt{a}\left(e\log\left(a+cx^2\right)(aCe-Ace+Bcd)+2c\log(d+ex)\left(Ae^2-Bde+Cd^2\right)\right)+2\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe-aCd+}{2\sqrt{a}\,ce\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)),x]
```

```
[Out] (2*Sqrt[c]*e*(A*c*d - a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[a]*
(2*c*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x] + e*(B*c*d - A*c*e + a*C*e)*Log[a
 + c*x^2]))/(2*Sqrt[a]*c*e*(c*d^2 + a*e^2))
```

**fricas [A]** time = 14.21, size = 262, normalized size = 1.97

$$\left[-\frac{\left(Bae^2-(Ca-Ac)de\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)-\left(Bacde+\left(Ca^2-Aac\right)e^2\right)\log\left(cx^2+a\right)-2\left(Cacd^2-Bacde\right.}{2\left(ac^2d^2e+a^2ce^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((B*a*e^2 - (C*a - A*c)*d*e)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x -
 a)/(c*x^2 + a)) - (B*a*c*d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) - 2*(C*
a*c*d^2 - B*a*c*d*e + A*a*c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3), 1
/2*(2*(B*a*e^2 - (C*a - A*c)*d*e)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (B*a*c*
d*e + (C*a^2 - A*a*c)*e^2)*log(c*x^2 + a) + 2*(C*a*c*d^2 - B*a*c*d*e + A*a*
c*e^2)*log(e*x + d))/(a*c^2*d^2*e + a^2*c*e^3)]
```

**giac [A]** time = 0.16, size = 125, normalized size = 0.94

$$\frac{(Bcd + Cae - Ace)\log\left(cx^2 + a\right)}{2\left(c^2d^2 + ace^2\right)} + \frac{\left(Cd^2 - Bde + Ae^2\right)\log\left(|xe + d|\right)}{cd^2e + ae^3} - \frac{(Cad - Acd - Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2 + ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d + C*a*e - A*c*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(abs(x*e + d))/(c*d^2*e + a*e^3) - (C*a*d - A*c*d - B*a*e)
*arctan(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

**maple [A]** time = 0.01, size = 247, normalized size = 1.86

$$\frac{Acd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2 + c\,d^2\right)\sqrt{ac}} + \frac{Bae\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2 + c\,d^2\right)\sqrt{ac}} - \frac{Cad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2 + c\,d^2\right)\sqrt{ac}} - \frac{Ae\ln\left(c\,x^2 + a\right)}{2\left(a\,e^2 + c\,d^2\right)} + \frac{Ae\ln\left(ex + d\right)}{a\,e^2 + c\,d^2} + \frac{Bd\ln\left(c\,x^2 + a\right)}{2a\,e^2 + 2c\,d^2} - \frac{Bc}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x)
```

```
[Out] 1/(a*e^2+c*d^2)*e*ln(e*x+d)*A-1/(a*e^2+c*d^2)*ln(e*x+d)*B*d+1/(a*e^2+c*d^2)
/e*ln(e*x+d)*C*d^2-1/2/(a*e^2+c*d^2)*ln(c*x^2+a)*A*e+1/2/(a*e^2+c*d^2)*ln(c
*x^2+a)*B*d+1/2/(a*e^2+c*d^2)/c*ln(c*x^2+a)*a*C*e+1/(a*e^2+c*d^2)/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*d+1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(1/(a*
c)^(1/2)*c*x)*B*a*e-1/(a*e^2+c*d^2)/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
*a*d
```

**maxima [A]** time = 0.97, size = 123, normalized size = 0.92

$$\frac{(Bcd + (Ca - Ac)e)\log\left(cx^2 + a\right)}{2\left(c^2d^2 + ace^2\right)} + \frac{\left(Cd^2 - Bde + Ae^2\right)\log\left(ex + d\right)}{cd^2e + ae^3} + \frac{(Bae - (Ca - Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(cd^2 + ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d + (C*a - A*c)*e)*log(c*x^2 + a)/(c^2*d^2 + a*c*e^2) + (C*d^2 - B
*d*e + A*e^2)*log(e*x + d)/(c*d^2*e + a*e^3) + (B*a*e - (C*a - A*c)*d)*arct
an(c*x/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))
```

**mupad [B]** time = 6.49, size = 840, normalized size = 6.32

$$\frac{\ln(d+e\,x)\,\left(C\,d^2-B\,d\,e+A\,e^2\right)}{c\,d^2\,e+a\,e^3}-\frac{\ln\left(x\,\left(c\,e\,B^2-c\,d\,B\,C+a\,e\,C^2-A\,c\,e\,C\right)+C^2\,a\,d+\frac{\left(c^2\left(\frac{A\,a\,e}{2}-\frac{B\,a\,d}{2}\right)-c\left(\frac{C\,a^2\,e}{2}-\frac{A\,d\,\sqrt{\cdot}}{2}\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)),x)
```

```
[Out] (log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 + c*d^2*e) - (log(x*(C^2*a*e
+ B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/2 - (B*a*d)/2) -
c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e*(-a*c^3)^(1/2))/2 - (C*a*
d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*a*c^2*d*e^2)*(c^2
*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*
e*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2)
- x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e) + B*a*c*e^2 - A*c
^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e - A*C*c*d)*(c^2*
((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 - (A*d*(-a*c^3)^(1/2))/2) + (B*a*e
*(-a*c^3)^(1/2))/2 - (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2 + a^2*c^2*e^2) -
 (log(x*(C^2*a*e + B^2*c*e - A*C*c*e - B*C*c*d) + C^2*a*d + ((c^2*((A*a*e)/
2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(1/2))/2) - (B*a*e*(-a*c^3)
^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2)*(((x*(6*a*c^2*e^3 - 2*c^3*d^2*e) + 8*
a*c^2*d*e^2)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^
(1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^
2 + a^2*c^2*e^2) - x*(3*A*c^2*e^2 + 2*C*c^2*d^2 - 5*C*a*c*e^2 - B*c^2*d*e)
+ B*a*c*e^2 - A*c^2*d*e + 5*C*a*c*d*e))/(a*c^3*d^2 + a^2*c^2*e^2) + A*B*c*e
 - A*C*c*d)*(c^2*((A*a*e)/2 - (B*a*d)/2) - c*((C*a^2*e)/2 + (A*d*(-a*c^3)^(
1/2))/2) - (B*a*e*(-a*c^3)^(1/2))/2 + (C*a*d*(-a*c^3)^(1/2))/2))/(a*c^3*d^2
 + a^2*c^2*e^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a),x)
```

```
[Out] Timed out
```

# 3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)}\,dx$

**Optimal**. Leaf size=214

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde+\right.}{\left(ae^2+cd^2\right)^2}$$

```
[Out] (-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)-(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*
C*a*d*e)*ln(e*x+d)/(a*e^2+c*d^2)^2+1/2*(-2*A*c*d*e-B*a*e^2+B*c*d^2+2*C*a*d*
e)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C
*d)))*arctan(x*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

**Rubi [A]** time = 0.36, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{Ae^2-Bde+Cd^2}{e(d+ex)\left(ae^2+cd^2\right)}-\frac{\log(d+ex)\left(-aBe^2+2aCde-2Acde+\right.}{\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] -((C*d^2 - B*d*e + A*e^2)/(e*(c*d^2 + a*e^2)*(d + e*x))) + ((A*c*(c*d^2 - a
*e^2) + a*(a*C*e^2 - c*d*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^
2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 + ((B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*
B*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)}\,dx = \int\left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^2}+\frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{Ac\left(cd^2-ae^2\right)+a\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}+\frac{\int\frac{Ac\left(cd^2-ae^2\right)}{}}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}-\frac{\left(Bcd^2-2Acde+2aCde-aBe^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2}+\frac{\left(c\left(Bcd^2-2\right.\right.}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{e\left(cd^2+ae^2\right)(d+ex)}+\frac{\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2}-$$

**Mathematica** [**A**]     time = 0.31, size = 188, normalized size = 0.88

$$\frac{\log\left(a+cx^2\right)\left(-aBe^2+2aCde-2Acde+Bcd^2\right)-\frac{2\left(ae^2+cd^2\right)\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)}+\log(d+ex)\left(2aBe^2-4aCde+4Acde\right.}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)) + (2*(A*c*(c
*d^2 - a*e^2) + a*(a*C*e^2 + c*d*(-(C*d) + 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt
[a]])/(Sqrt[a]*Sqrt[c]) + (-2*B*c*d^2 + 4*A*c*d*e - 4*a*C*d*e + 2*a*B*e^2)*
Log[d + e*x] + (B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2)*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

**fricas [B]** time = 69.91, size = 904, normalized size = 4.22

$$\left[-\frac{2\,Cac^2d^4 - 2\,Bac^2d^3e - 2\,Ba^2cde^3 + 2\,Aa^2ce^4 + 2\left(Ca^2c + Aac^2\right)d^2e^2 - \left(2\,Bacd^2e^2 - \left(Cac - Ac^2\right)d^3e + \left(Ca^2 \right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3 + 2*A*a^2*c*e^4 +
2*(C*a^2*c + A*a*c^2)*d^2*e^2 - (2*B*a*c*d^2*e^2 - (C*a*c - A*c^2)*d^3*e +
(C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2)*d^2*e^2 + (C*a^2 -
 A*a*c)*e^4)*x)*sqrt(-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) -
(B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d
^2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(
B*a*c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^
2*e^2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*
d^5*e + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^
4 + a^3*c*e^6)*x), -1/2*(2*C*a*c^2*d^4 - 2*B*a*c^2*d^3*e - 2*B*a^2*c*d*e^3
+ 2*A*a^2*c*e^4 + 2*(C*a^2*c + A*a*c^2)*d^2*e^2 - 2*(2*B*a*c*d^2*e^2 - (C*a
*c - A*c^2)*d^3*e + (C*a^2 - A*a*c)*d*e^3 + (2*B*a*c*d*e^3 - (C*a*c - A*c^2
)*d^2*e^2 + (C*a^2 - A*a*c)*e^4)*x)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (B*a*
c^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^
2 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(c*x^2 + a) + 2*(B*a*c
^2*d^3*e - B*a^2*c*d*e^3 + 2*(C*a^2*c - A*a*c^2)*d^2*e^2 + (B*a*c^2*d^2*e^2
 - B*a^2*c*e^4 + 2*(C*a^2*c - A*a*c^2)*d*e^3)*x)*log(e*x + d))/(a*c^3*d^5*e
 + 2*a^2*c^2*d^3*e^3 + a^3*c*d*e^5 + (a*c^3*d^4*e^2 + 2*a^2*c^2*d^2*e^4 + a
^3*c*e^6)*x)]
```

**giac [A]** time = 0.17, size = 270, normalized size = 1.26

$$-\frac{\left(Cacd^2e^2 - Ac^2d^2e^2 - 2\,Bacde^3 - Ca^2e^4 + Aace^4\right)\arctan\left(\frac{\left(cd - \frac{cd^2}{xe+d} - \frac{ae^2}{xe+d}\right)e^{(-1)}}{\sqrt{ac}}\right)e^{(-2)}}{\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)\sqrt{ac}} + \frac{\left(Bcd^2 + 2\,Cade - 2\,Acde - \right.}{2\left(c^2d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a*c*d^2*e^2 - A*c^2*d^2*e^2 - 2*B*a*c*d*e^3 - C*a^2*e^4 + A*a*c*e^4)*ar
ctan((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((c
^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/2*(B*c*d^2 + 2*C*a*d*e - 2
*A*c*d*e - B*a*e^2)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*
```

```
e + d)^2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - (C*d^2*e/(x*e + d) - B*d*e^
2/(x*e + d) + A*e^3/(x*e + d))/(c*d^2*e^2 + a*e^4)
```

**maple [B]** time = 0.01, size = 462, normalized size = 2.16

$$-\frac{Aac\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{A\,c^2d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{2Bacde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}+\frac{C\,a^2e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}-\frac{Cac\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^2\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x)
```

```
[Out] -1/(a*e^2+c*d^2)*e/(e*x+d)*A+1/(a*e^2+c*d^2)/(e*x+d)*B*d-1/(a*e^2+c*d^2)/e/
(e*x+d)*C*d^2+2/(a*e^2+c*d^2)^2*ln(e*x+d)*A*c*d*e+1/(a*e^2+c*d^2)^2*ln(e*x+
d)*B*a*e^2-1/(a*e^2+c*d^2)^2*ln(e*x+d)*B*c*d^2-2/(a*e^2+c*d^2)^2*ln(e*x+d)*
C*a*d*e-1/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*A*d*e-1/2/(a*e^2+c*d^2)^2*ln(c*x^2+
a)*e^2*B*a+1/2/(a*e^2+c*d^2)^2*c*ln(c*x^2+a)*d^2*B+1/(a*e^2+c*d^2)^2*ln(c*x
^2+a)*C*a*d*e-1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*a*c
*e^2+1/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2+2/(a
*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*c*d*e+1/(a*e^2+c*d^
2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*a^2*C*e^2-1/(a*e^2+c*d^2)^2/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*c*d^2
```

**maxima [A]** time = 1.04, size = 255, normalized size = 1.19

$$\frac{\left(Bcd^2-Bae^2+2\,(Ca-Ac)de\right)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{\left(Bcd^2-Bae^2+2\,(Ca-Ac)de\right)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}+\frac{\left(2\,Bacde-\left(Cac-Ac\right.\right.}{\left(c^2d^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c
*d^2*e^2 + a^2*e^4) - (B*c*d^2 - B*a*e^2 + 2*(C*a - A*c)*d*e)*log(e*x + d)/
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + (2*B*a*c*d*e - (C*a*c - A*c^2)*d^2 +
(C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*sqrt(a*c)) - (C*d^2 - B*d*e + A*e^2)/(c*d^3*e + a*d*e^3 + (c*d^2*e^2 +
 a*e^4)*x)
```

**mupad [B]** time = 6.77, size = 1199, normalized size = 5.60

$$\ln\left(C\,c\,d^4\,(-a\,c)^{3/2}-A\,a\,e^4\,(-a\,c)^{3/2}+3\,B\,a\,c^3\,d^4+3\,B\,a^3\,c\,e^4+A\,c^4\,d^4\,x+A\,c^3\,d^4\,\sqrt{-a\,c}-C\,a^3\,e^4\,\sqrt{-a\,c}-C\,a\,c^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^2),x)
```

```
[Out] (log(C*c*d^4*(-a*c)^(3/2) - A*a*e^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*
c*e^4 + A*c^4*d^4*x + A*c^3*d^4*(-a*c)^(1/2) - C*a^3*e^4*(-a*c)^(1/2) - C*a
*c^3*d^4*x - C*a^3*c*e^4*x + 14*A*c*d^2*e^2*(-a*c)^(3/2) - 14*C*a*d^2*e^2*(
-a*c)^(3/2) - 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*
d^3*e + A*a^2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 + 8*B*a*d*e^3*(-a*c)^(3/2) -
 8*B*c*d^3*e*(-a*c)^(3/2) + 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*
C*a^3*c*d*e^3 + 14*C*a^2*c^2*d^2*e^2*x + 8*A*c*d*e^3*x*(-a*c)^(3/2) - 8*C*a
*d*e^3*x*(-a*c)^(3/2) + 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x + 8*
A*c^3*d^3*e*x*(-a*c)^(1/2) - 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2
*e^2*x - 8*B*a^2*c^2*d*e^3*x)*(c^2*(a*((B*d^2)/2 - A*d*e) + (A*d^2*(-a*c)^(
1/2))/2) - c*(a^2*((B*e^2)/2 - C*d*e) + a*((A*e^2*(-a*c)^(1/2))/2 + (C*d^2*
(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (log(d + e*x)*(c*(B*d^2 - 2*A*d*e)
 - a*(B*e^2 - 2*C*d*e)))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2) - (log(A*a*e^4
*(-a*c)^(3/2) - C*c*d^4*(-a*c)^(3/2) + 3*B*a*c^3*d^4 + 3*B*a^3*c*e^4 + A*c^
4*d^4*x - A*c^3*d^4*(-a*c)^(1/2) + C*a^3*e^4*(-a*c)^(1/2) - C*a*c^3*d^4*x -
 C*a^3*c*e^4*x - 14*A*c*d^2*e^2*(-a*c)^(3/2) + 14*C*a*d^2*e^2*(-a*c)^(3/2)
+ 3*B*c^3*d^4*x*(-a*c)^(1/2) + 8*A*a^2*c^2*d*e^3 + 8*C*a^2*c^2*d^3*e + A*a^
2*c^2*e^4*x - 10*B*a^2*c^2*d^2*e^2 - 8*B*a*d*e^3*(-a*c)^(3/2) + 8*B*c*d^3*e
*(-a*c)^(3/2) - 3*B*a*e^4*x*(-a*c)^(3/2) - 8*A*a*c^3*d^3*e - 8*C*a^3*c*d*e^
3 + 14*C*a^2*c^2*d^2*e^2*x - 8*A*c*d*e^3*x*(-a*c)^(3/2) + 8*C*a*d*e^3*x*(-a
*c)^(3/2) - 8*C*c*d^3*e*x*(-a*c)^(3/2) + 8*B*a*c^3*d^3*e*x - 8*A*c^3*d^3*e*
x*(-a*c)^(1/2) + 10*B*c*d^2*e^2*x*(-a*c)^(3/2) - 14*A*a*c^3*d^2*e^2*x - 8*B
*a^2*c^2*d*e^3*x)*(c*(a^2*((B*e^2)/2 - C*d*e) - a*((A*e^2*(-a*c)^(1/2))/2 +
 (C*d^2*(-a*c)^(1/2))/2 - B*d*e*(-a*c)^(1/2))) - c^2*(a*((B*d^2)/2 - A*d*e)
 - (A*d^2*(-a*c)^(1/2))/2) + (C*a^2*e^2*(-a*c)^(1/2))/2))/(a*c^3*d^4 + a^3*
c*e^4 + 2*a^2*c^2*d^2*e^2) - (A*e^2 + C*d^2 - B*d*e)/(e*(a*e^2 + c*d^2)*(d
+ e*x))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a),x)
```

```
[Out] Timed out
```

# 3.49 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)}\,dx$

**Optimal**. Leaf size=305

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+}{(d+ex)\left(ae^2+cd^2\right)^2}$$

```
[Out] 1/2*(-A*e^2+B*d*e-C*d^2)/e/(a*e^2+c*d^2)/(e*x+d)^2+(-2*A*c*d*e-B*a*e^2+B*c*
d^2+2*C*a*d*e)/(a*e^2+c*d^2)^2/(e*x+d)-(B*c*d*(-3*a*e^2+c*d^2)-(A*c-C*a)*e*
(-a*e^2+3*c*d^2))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*(B*c*d*(-3*a*e^2+c*d^2)-(A*
c-C*a)*e*(-a*e^2+3*c*d^2))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+(A*c*d*(-3*a*e^2+c*d
^2)-a*(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*c^
(1/2)/(a*e^2+c*d^2)^3/a^(1/2)
```

**Rubi [A]** time = 0.65, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1629, 635, 205, 260}

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)}{2\left(ae^2+cd^2\right)^3}-\frac{Ae^2-Bde+Cd^2}{2e(d+ex)^2\left(ae^2+cd^2\right)}+\frac{-aBe^2+2aCde-2Acde+}{(d+ex)\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] -(C*d^2 - B*d*e + A*e^2)/(2*e*(c*d^2 + a*e^2)*(d + e*x)^2) + (B*c*d^2 - 2*A
*c*d*e + 2*a*C*d*e - a*B*e^2)/((c*d^2 + a*e^2)^2*(d + e*x)) + (Sqrt[c]*(A*c
*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)^3) - ((B*c*d*(c*d^2 - 3*a*
e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x])/(c*d^2 + a*e^2)^3 + (
(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])
/(2*(c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)}\,dx = \int\left(\frac{Cd^2-Bde+Ae^2}{\left(cd^2+ae^2\right)(d+ex)^3}+\frac{e\left(-Bcd^2+2Acde-2aCde+aBe^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)^2}+\frac{e\left(-Bcd\left(cd^2-3a\right.\right.}{\left(cd\right.}\right.$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{\left(Bcd\left(cd^2-3ae^2\right)-(\right.}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}-\frac{\left(Bcd\left(cd^2-3ae^2\right)-(\right.}{}$$

$$= -\frac{Cd^2-Bde+Ae^2}{2e\left(cd^2+ae^2\right)(d+ex)^2}+\frac{Bcd^2-2Acde+2aCde-aBe^2}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{\sqrt{c}\left(Acd\left(cd^2-3ae^2\right)\right.}{}$$

**Mathematica [A]** time = 0.31, size = 277, normalized size = 0.91

$$\frac{\log\left(a+cx^2\right)\left(Bcd\left(cd^2-3ae^2\right)-e(Ac-aC)\left(3cd^2-ae^2\right)\right)-\frac{\left(ae^2+cd^2\right)^2\left(e(Ae-Bd)+Cd^2\right)}{e(d+ex)^2}+\frac{2\left(ae^2+cd^2\right)\left(-aBe^2+2aCde-2Acde\right.}{d+ex}}{2\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)),x]
```

```
[Out] (-(((c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(e*(d + e*x)^2)) + (2*(c*
d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2))/(d + e*x) + (2*Sq
rt[c]*(A*c*d*(c*d^2 - 3*a*e^2) + a*(a*e^2*(3*C*d - B*e) + c*d^2*(-(C*d) + 3
```

```
*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[a] - 2*(B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[d + e*x] + (B*c*d*(c*d^2 - 3*a*e^2) -
 (A*c - a*C)*e*(3*c*d^2 - a*e^2))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.20, size = 489, normalized size = 1.60

$$\frac{\left(Bc^2d^3 + 3\,Cacd^2e - 3\,Ac^2d^2e - 3\,Bacde^2 - Ca^2e^3 + Aace^3\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} - \frac{\left(Bc^2d^3e + 3\,Cacd^2e^2 - 3\,Ac^2d^2e^2 - 3\,\right.}{c^3d^6e + 3\,ac^2d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d^3 + 3*C*a*c*d^2*e - 3*A*c^2*d^2*e - 3*B*a*c*d*e^2 - C*a^2*e^3
+ A*a*c*e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 +
a^3*e^6) - (B*c^2*d^3*e + 3*C*a*c*d^2*e^2 - 3*A*c^2*d^2*e^2 - 3*B*a*c*d*e^3
 - C*a^2*e^4 + A*a*c*e^4)*log(abs(x*e + d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 +
3*a^2*c*d^2*e^5 + a^3*e^7) - (C*a*c^2*d^3 - A*c^3*d^3 - 3*B*a*c^2*d^2*e - 3
*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 + B*a^2*c*e^3)*arctan(c*x/sqrt(a*c))/((c^3
*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c^2
*d^6 - 3*B*c^2*d^5*e - 2*C*a*c*d^4*e^2 + 5*A*c^2*d^4*e^2 - 2*B*a*c*d^3*e^3
- 3*C*a^2*d^2*e^4 + 6*A*a*c*d^2*e^4 + B*a^2*d*e^5 + A*a^2*e^6 - 2*(B*c^2*d^
4*e^2 + 2*C*a*c*d^3*e^3 - 2*A*c^2*d^3*e^3 + 2*C*a^2*d*e^5 - 2*A*a*c*d*e^5 -
 B*a^2*e^6)*x)*e^(-1)/((c*d^2 + a*e^2)^3*(x*e + d)^2)
```

**maple [B]** time = 0.02, size = 754, normalized size = 2.47

$$-\frac{3Aa\,c^2d\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{A\,c^3d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}-\frac{B\,a^2c\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{3Ba\,c^2d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\left(a\,e^2+c\,d^2\right)^3\sqrt{ac}}+\frac{3C\,a^2cd\,e^2\,\mathrm{arc}}{\left(a\,e^2+c\,d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x)
```

```
[Out] -1/2/(a*e^2+c*d^2)*e/(e*x+d)^2*A+1/2/(a*e^2+c*d^2)/(e*x+d)^2*B*d+2/(a*e^2+c
*d^2)^2/(e*x+d)*C*a*d*e+1/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*A*e^3*a-3/2*c^2/(
a*e^2+c*d^2)^3*ln(c*x^2+a)*A*d^2*e+c^3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*A*d^3-1/(a*e^2+c*d^2)^3*ln(e*x+d)*A*c*e^3*a+3/(a*e^2+c*d^
2)^3*ln(e*x+d)*A*c^2*d^2*e-2/(a*e^2+c*d^2)^2/(e*x+d)*A*c*d*e-3/2*c/(a*e^2+c
*d^2)^3*ln(c*x^2+a)*B*d*e^2*a+3/2*c/(a*e^2+c*d^2)^3*ln(c*x^2+a)*C*a*d^2*e-c
/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a^2*e^3-c^2/(a*e^2
+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a*d^3+3/(a*e^2+c*d^2)^3*l
n(e*x+d)*B*c*d*e^2*a-3/(a*e^2+c*d^2)^3*ln(e*x+d)*C*a*c*d^2*e-3*c^2/(a*e^2+c
*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*a*d*e^2+3*c^2/(a*e^2+c*d^2)
^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*a*d^2*e+3*c/(a*e^2+c*d^2)^3/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*a^2*d*e^2+1/2*c^2/(a*e^2+c*d^2)^3*ln(c*
x^2+a)*d^3*B-1/(a*e^2+c*d^2)^3*ln(e*x+d)*d^3*c^2*B+1/(a*e^2+c*d^2)^3*ln(e*x
+d)*C*a^2*e^3-1/2/(a*e^2+c*d^2)/e/(e*x+d)^2*C*d^2-1/(a*e^2+c*d^2)^2/(e*x+d)
*B*a*e^2+1/(a*e^2+c*d^2)^2/(e*x+d)*B*c*d^2-1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*
C*a^2*e^3
```

**maxima** [**A**]   time = 1.05, size = 495, normalized size = 1.62

$$\frac{\left(Bc^2d^3 - 3\,Bacde^2 + 3\left(Cac - Ac^2\right)d^2e - \left(Ca^2 - Aac\right)e^3\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} - \frac{\left(Bc^2d^3 - 3\,Bacde^2 + 3\left(Cac - Ac^2\right)d^2e \cdots\right.}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*
e^3)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6)
 - (B*c^2*d^3 - 3*B*a*c*d*e^2 + 3*(C*a*c - A*c^2)*d^2*e - (C*a^2 - A*a*c)*e
^3)*log(e*x + d)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) +
(3*B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 - A*c^3)*d^3 + 3*(C*a^2*c - A*a*c
^2)*d*e^2)*arctan(c*x/sqrt(a*c))/((c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*
e^4 + a^3*e^6)*sqrt(a*c)) - 1/2*(C*c*d^4 - 3*B*c*d^3*e + B*a*d*e^3 + A*a*e^
4 - (3*C*a - 5*A*c)*d^2*e^2 - 2*(B*c*d^2*e^2 - B*a*e^4 + 2*(C*a - A*c)*d*e^
3)*x)/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^2*e^5 + (c^2*d^4*e^3 + 2*a*c*d^2*e
^5 + a^2*e^7)*x^2 + 2*(c^2*d^5*e^2 + 2*a*c*d^3*e^4 + a^2*d*e^6)*x)
```

**mupad** [**B**]   time = 9.19, size = 2980, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)*(d + e*x)^3),x)
```

```
[Out] (log(d + e*x)*(e^3*(C*a^2 - A*a*c) - B*c^2*d^3 + d^2*e*(3*A*c^2 - 3*C*a*c)
+ 3*B*a*c*d*e^2))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4) -
 (log(9*A^2*a^5*e^10*(-a*c)^(5/2) + A^2*c^5*d^10*(-a*c)^(5/2) - B^2*a^7*e^1
0*(-a*c)^(3/2) - 9*B^2*c^3*d^10*(-a*c)^(7/2) + 9*C^2*a^9*e^10*(-a*c)^(1/2)
+ C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x - 6*A^2*a*d^4*e^6*(-a*c)^(9/
2) - 6*B^2*a*d^6*e^4*(-a*c)^(9/2) + 106*A^2*c*d^6*e^4*(-a*c)^(9/2) + 77*C^2
*a*d^8*e^2*(-a*c)^(9/2) - 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*
x + 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*
a^4*c^6*d^10*x + 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) - 106*B^2*a^3*d^4*e^6*(-a*
c)^(7/2) + 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) - 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2
) - 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) - 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) + 27*
C^2*a^7*d^2*e^8*(-a*c)^(3/2) + 18*A*C*a^7*e^10*(-a*c)^(3/2) + 2*A*C*c^3*d^1
0*(-a*c)^(7/2) + 224*A*B*a*d^5*e^5*(-a*c)^(9/2) - 48*A*B*a^5*d*e^9*(-a*c)^(
5/2) - 212*A*C*a*d^6*e^4*(-a*c)^(9/2) + 64*A*B*c*d^7*e^3*(-a*c)^(9/2) + 48*
A*B*c^3*d^9*e*(-a*c)^(7/2) - 64*B*C*a*d^7*e^3*(-a*c)^(9/2) - 48*B*C*a^7*d*e
^9*(-a*c)^(3/2) - 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x
 + 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2
*e^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c
^4*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^
2*a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x -
2*A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x - 64*A*B*a^3*d^3*e^7*(-a*c)^(7
/2) - 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) + 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) + 2
24*B*C*a^3*d^5*e^5*(-a*c)^(7/2) - 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) + 48*B*C*
c*d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48
*B*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x +
224*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*
e^2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c
^3*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*
C*a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 - (3*C*a^2*d*(-a*c)^(1/2))/2 + (
3*A*a*c*d*(-a*c)^(1/2))/2) + e^3*((C*a^3)/2 - (A*a^2*c)/2 + (B*a^2*(-a*c)^(
1/2))/2) - e*((3*C*a^2*c*d^2)/2 - (3*A*a*c^2*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(
1/2))/2) - (B*a*c^2*d^3)/2 - (A*c^2*d^3*(-a*c)^(1/2))/2 + (C*a*c*d^3*(-a*c)
^(1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - (
log(B^2*a^7*e^10*(-a*c)^(3/2) - A^2*c^5*d^10*(-a*c)^(5/2) - 9*A^2*a^5*e^10*
(-a*c)^(5/2) + 9*B^2*c^3*d^10*(-a*c)^(7/2) - 9*C^2*a^9*e^10*(-a*c)^(1/2) -
C^2*c*d^10*(-a*c)^(9/2) + 9*C^2*a^9*c*e^10*x + 6*A^2*a*d^4*e^6*(-a*c)^(9/2)
 + 6*B^2*a*d^6*e^4*(-a*c)^(9/2) - 106*A^2*c*d^6*e^4*(-a*c)^(9/2) - 77*C^2*a
*d^8*e^2*(-a*c)^(9/2) + 27*B^2*c*d^8*e^2*(-a*c)^(9/2) + A^2*a^2*c^8*d^10*x
+ 9*A^2*a^7*c^3*e^10*x + 9*B^2*a^3*c^7*d^10*x + B^2*a^8*c^2*e^10*x + C^2*a^
4*c^6*d^10*x - 27*A^2*a^3*d^2*e^8*(-a*c)^(7/2) + 106*B^2*a^3*d^4*e^6*(-a*c)
^(7/2) - 77*B^2*a^5*d^2*e^8*(-a*c)^(5/2) + 77*A^2*c^3*d^8*e^2*(-a*c)^(7/2)
+ 106*C^2*a^3*d^6*e^4*(-a*c)^(7/2) + 6*C^2*a^5*d^4*e^6*(-a*c)^(5/2) - 27*C^
2*a^7*d^2*e^8*(-a*c)^(3/2) - 18*A*C*a^7*e^10*(-a*c)^(3/2) - 2*A*C*c^3*d^10*
(-a*c)^(7/2) - 224*A*B*a*d^5*e^5*(-a*c)^(9/2) + 48*A*B*a^5*d*e^9*(-a*c)^(5/
2) + 212*A*C*a*d^6*e^4*(-a*c)^(9/2) - 64*A*B*c*d^7*e^3*(-a*c)^(9/2) - 48*A*
```

```
B*c^3*d^9*e*(-a*c)^(7/2) + 64*B*C*a*d^7*e^3*(-a*c)^(9/2) + 48*B*C*a^7*d*e^9
*(-a*c)^(3/2) + 154*A*C*c*d^8*e^2*(-a*c)^(9/2) + 77*A^2*a^3*c^7*d^8*e^2*x +
 106*A^2*a^4*c^6*d^6*e^4*x - 6*A^2*a^5*c^5*d^4*e^6*x - 27*A^2*a^6*c^4*d^2*e
^8*x - 27*B^2*a^4*c^6*d^8*e^2*x - 6*B^2*a^5*c^5*d^6*e^4*x + 106*B^2*a^6*c^4
*d^4*e^6*x + 77*B^2*a^7*c^3*d^2*e^8*x + 77*C^2*a^5*c^5*d^8*e^2*x + 106*C^2*
a^6*c^4*d^6*e^4*x - 6*C^2*a^7*c^3*d^4*e^6*x - 27*C^2*a^8*c^2*d^2*e^8*x - 2*
A*C*a^3*c^7*d^10*x - 18*A*C*a^8*c^2*e^10*x + 64*A*B*a^3*d^3*e^7*(-a*c)^(7/2
) + 12*A*C*a^3*d^4*e^6*(-a*c)^(7/2) - 54*A*C*a^5*d^2*e^8*(-a*c)^(5/2) - 224
*B*C*a^3*d^5*e^5*(-a*c)^(7/2) + 64*B*C*a^5*d^3*e^7*(-a*c)^(5/2) - 48*B*C*c*
d^9*e*(-a*c)^(9/2) - 48*A*B*a^3*c^7*d^9*e*x - 48*A*B*a^7*c^3*d*e^9*x + 48*B
*C*a^4*c^6*d^9*e*x + 48*B*C*a^8*c^2*d*e^9*x + 64*A*B*a^4*c^6*d^7*e^3*x + 22
4*A*B*a^5*c^5*d^5*e^5*x + 64*A*B*a^6*c^4*d^3*e^7*x - 154*A*C*a^4*c^6*d^8*e^
2*x - 212*A*C*a^5*c^5*d^6*e^4*x + 12*A*C*a^6*c^4*d^4*e^6*x + 54*A*C*a^7*c^3
*d^2*e^8*x - 64*B*C*a^5*c^5*d^7*e^3*x - 224*B*C*a^6*c^4*d^5*e^5*x - 64*B*C*
a^7*c^3*d^3*e^7*x)*(e^2*((3*B*a^2*c*d)/2 + (3*C*a^2*d*(-a*c)^(1/2))/2 - (3*
A*a*c*d*(-a*c)^(1/2))/2) - e^3*((A*a^2*c)/2 - (C*a^3)/2 + (B*a^2*(-a*c)^(1/
2))/2) + e*((3*A*a*c^2*d^2)/2 - (3*C*a^2*c*d^2)/2 + (3*B*a*c*d^2*(-a*c)^(1/
2))/2) - (B*a*c^2*d^3)/2 + (A*c^2*d^3*(-a*c)^(1/2))/2 - (C*a*c*d^3*(-a*c)^(
1/2))/2))/(a^4*e^6 + a*c^3*d^6 + 3*a^3*c*d^2*e^4 + 3*a^2*c^2*d^4*e^2) - ((A
*a*e^4 + C*c*d^4 + B*a*d*e^3 - 3*B*c*d^3*e + 5*A*c*d^2*e^2 - 3*C*a*d^2*e^2)
/(2*e*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(B*a*e^3 + 2*A*c*d*e^2 - 2*
C*a*d*e^2 - B*c*d^2*e))/(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2
 + 2*d*e*x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a),x)
```

```
[Out] Timed out
```

# 3.50 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=216

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)\right.\right.}{2c^3}$$

```
[Out] -3/2*e^2*(A*c*d-a*(B*e+3*C*d))*x/a/c^2-1/2*(A*c-2*C*a)*e^3*x^2/a/c^2-1/2*(a
*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)+1/2*(A*c*d*(3*a*e^2+c*d^2)-a*(3*a*e
^2*(B*e+3*C*d)-c*d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(5/2
)-1/2*e*(2*a*C*e^2-c*(3*C*d^2+e*(A*e+3*B*d)))*ln(c*x^2+a)/c^3
```

**Rubi [A]** time = 0.50, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)-a\left(3ae^2(Be+3Cd)-cd^2(3Be+Cd)\right)\right)}{2a^{3/2}c^{5/2}}-\frac{e\log\left(a+cx^2\right)\left(2aCe^2-c\left(e(Ae+3Bd)\right.\right.}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (-3*e^2*(A*c*d - a*(3*C*d + B*e))*x)/(2*a*c^2) - ((A*c - 2*a*C)*e^3*x^2)/(2
*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(2*a*c*(a + c*x^2)) + ((A*c*d
*(c*d^2 + 3*a*e^2) - a*(3*a*e^2*(3*C*d + B*e) - c*d^2*(C*d + 3*B*e)))*ArcTa
n[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) - (e*(2*a*C*e^2 - c*(3*C*d^2 +
e*(3*B*d + A*e)))*Log[a + c*x^2])/(2*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)^2(-Acd-aCd-3aBe+2(Ac-2aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)} - \frac{\int \left(\frac{3e^2(Acd-3aCd-aBe)}{c} + \frac{2(Ac-2aC)e^3x}{c} - \frac{Acd\left(cd^2+3\right.}{}\right.}{}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)}$$

$$= -\frac{3e^2(Acd-a(3Cd+Be))x}{2ac^2} - \frac{(Ac-2aC)e^3x^2}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^3}{2ac\left(a+cx^2\right)}$$

**Mathematica [A]** time = 0.20, size = 233, normalized size = 1.08

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2(3Be+Cd)-3ae^2(Be+3Cd)\right)\right)}{a^{3/2}}+\frac{-a^3Ce^3+a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+Cd^2\right.}{a\left(a+cx^2\right)}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*(3*C*d + B*e)*x + c*C*e^3*x^2 + (-(a^3*C*e^3) + A*c^3*d^3*x - a*c^
2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) + a^2*c*e*(3*C*d*(d + e*x
) + e*(3*B*d + A*e + B*e*x)))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c*d^2 + 3*
a*e^2) + a*(-3*a*e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*
x)/Sqrt[a]])/a^(3/2) + e*(3*c*C*d^2 - 2*a*C*e^2 + c*e*(3*B*d + A*e))*Log[a
+ c*x^2])/(2*c^3)
```

**fricas [B]** time = 0.92, size = 931, normalized size = 4.31

$$\left[\frac{2\,Ca^2c^2e^3x^4+2\,Ca^3ce^3x^2-2\,Ba^2c^2d^3+6\,Ba^3cde^2+6\left(Ca^3c-Aa^2c^2\right)d^2e-2\left(Ca^4-Aa^3c\right)e^3+4\left(3\,Ca^2c^2de^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*C*a^2*c^2*e^3*x^4 + 2*C*a^3*c*e^3*x^2 - 2*B*a^2*c^2*d^3 + 6*B*a^3*c
*d*e^2 + 6*(C*a^3*c - A*a^2*c^2)*d^2*e - 2*(C*a^4 - A*a^3*c)*e^3 + 4*(3*C*a
^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*c*d^2*e - 3*B*a^3*e^3 + (C*a^2
*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e^2 + (3*B*a*c^2*d^2*e - 3*B*a^
2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*c - A*a*c^2)*d*e^2)*x^2)*sqrt(
-a*c)*log((c*x^2 + 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(3*B*a^2*c^2*d^2*e
- 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*d*e
^2)*x + 2*(3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (3
*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*lo
g(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(C*a^2*c^2*e^3*x^4 + C*a^3*c*e^3
*x^2 - B*a^2*c^2*d^3 + 3*B*a^3*c*d*e^2 + 3*(C*a^3*c - A*a^2*c^2)*d^2*e - (C
*a^4 - A*a^3*c)*e^3 + 2*(3*C*a^2*c^2*d*e^2 + B*a^2*c^2*e^3)*x^3 + (3*B*a^2*
c*d^2*e - 3*B*a^3*e^3 + (C*a^2*c + A*a*c^2)*d^3 - 3*(3*C*a^3 - A*a^2*c)*d*e
^2 + (3*B*a*c^2*d^2*e - 3*B*a^2*c*e^3 + (C*a*c^2 + A*c^3)*d^3 - 3*(3*C*a^2*
c - A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) - (3*B*a^2*c^2*d^2
*e - 3*B*a^3*c*e^3 + (C*a^2*c^2 - A*a*c^3)*d^3 - 3*(3*C*a^3*c - A*a^2*c^2)*
d*e^2)*x + (3*C*a^3*c*d^2*e + 3*B*a^3*c*d*e^2 - (2*C*a^4 - A*a^3*c)*e^3 + (
3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)*e^3)*x^2)*l
og(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

**giac [A]** time = 0.17, size = 289, normalized size = 1.34

$$\frac{\left(3\,Ccd^2e+3\,Bcde^2-2\,Cae^3+Ace^3\right)\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+Ac^2d^3+3\,Bacd^2e-9\,Ca^2de^2+3\,Aacde^2-3\,Ba^2\right.}{2\,\sqrt{ac}\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - 2*C*a*e^3 + A*c*e^3)*log(c*x^2 + a)/c^3 +
1/2*(C*a*c*d^3 + A*c^2*d^3 + 3*B*a*c*d^2*e - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2
- 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2) + 1/2*(C*c^2*x^2*e^3
 + 6*C*c^2*d*x*e^2 + 2*B*c^2*x*e^3)/c^4 - 1/2*(B*a*c^2*d^3 - 3*C*a^2*c*d^2*
e + 3*A*a*c^2*d^2*e - 3*B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*
d^3 - A*c^3*d^3 + 3*B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - B*a
^2*c*e^3)*x)/((c*x^2 + a)*a*c^3)
```

**maple [B]** time = 0.02, size = 484, normalized size = 2.24

$$\frac{A\,d^3x}{2\left(c\,x^2+a\right)a}+\frac{A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}-\frac{3Ad\,e^2x}{2\left(c\,x^2+a\right)c}+\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Ba\,e^3x}{2\left(c\,x^2+a\right)c^2}-\frac{3Ba\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] -1/2/c/(c*x^2+a)*B*d^3+1/2/c^2*ln(c*x^2+a)*A*e^3+1/2*e^3/c^2*C*x^2+e^3/c^2*
B*x+3/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2-3/2/c^2*a/(a*c)^(1/
2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^3+3/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c
*x)*B*d^2*e-3/2/c/(c*x^2+a)*A*x*d*e^2+1/2/c^2/(c*x^2+a)*B*x*a*e^3-3/2/c/(c*
x^2+a)*B*x*d^2*e+3/2/c^2/(c*x^2+a)*B*a*d*e^2+3/2/c^2/(c*x^2+a)*C*a*d^2*e+1/
2/(c*x^2+a)/a*x*A*d^3+3*e^2/c^2*C*d*x+3/2/c^2/(c*x^2+a)*C*x*a*d*e^2-9/2/c^2
*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+3/2/c^2*ln(c*x^2+a)*B*d*e^
2-1/c^3*a*ln(c*x^2+a)*C*e^3+3/2/c^2*ln(c*x^2+a)*C*d^2*e+1/2/a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*A*d^3+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*
C*d^3-1/2/c/(c*x^2+a)*C*x*d^3+1/2/c^2/(c*x^2+a)*A*a*e^3-3/2/c/(c*x^2+a)*A*d
^2*e-1/2/c^3/(c*x^2+a)*C*a^2*e^3
```

**maxima [A]** time = 0.98, size = 287, normalized size = 1.33

$$-\frac{Bac^2d^3-3\,Ba^2cde^2-3\left(Ca^2c-Aac^2\right)d^2e+\left(Ca^3-Aa^2c\right)e^3+\left(3\,Bac^2d^2e-Ba^2ce^3+\left(Cac^2-Ac^3\right)d^3-3\left(Ca^2\right.\right.}{2\left(ac^4x^2+a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a*c^2*d^3 - 3*B*a^2*c*d*e^2 - 3*(C*a^2*c - A*a*c^2)*d^2*e + (C*a^3
- A*a^2*c)*e^3 + (3*B*a*c^2*d^2*e - B*a^2*c*e^3 + (C*a*c^2 - A*c^3)*d^3 - 3
*(C*a^2*c - A*a*c^2)*d*e^2)*x)/(a*c^4*x^2 + a^2*c^3) + 1/2*(C*e^3*x^2 + 2*(
3*C*d*e^2 + B*e^3)*x)/c^2 + 1/2*(3*C*c*d^2*e + 3*B*c*d*e^2 - (2*C*a - A*c)*
e^3)*log(c*x^2 + a)/c^3 + 1/2*(3*B*a*c*d^2*e - 3*B*a^2*e^3 + (C*a*c + A*c^2
)*d^3 - 3*(3*C*a^2 - A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

**mupad [B]** time = 4.01, size = 303, normalized size = 1.40

$$\frac{x\,\left(B\,e^3+3\,C\,d\,e^2\right)}{c^2}-\frac{\frac{C\,a^2\,e^3-3\,C\,a\,c\,d^2\,e-3\,B\,a\,c\,d\,e^2-A\,a\,c\,e^3+B\,c^2\,d^3+3\,A\,c^2\,d^2\,e}{2\,c}-\frac{x\,\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3-C\,a\,c\,d^3-3\,B\,a\,c\,d^2\,e-3\,A\,a\,c\,d\,e^2+A\,c^2\right.}{2\,a}}{c^3\,x^2+a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (x*(B*e^3 + 3*C*d*e^2))/c^2 - ((B*c^2*d^3 + C*a^2*e^3 - A*a*c*e^3 + 3*A*c^2
*d^2*e - 3*B*a*c*d*e^2 - 3*C*a*c*d^2*e)/(2*c) - (x*(A*c^2*d^3 + B*a^2*e^3 -
 C*a*c*d^3 + 3*C*a^2*d*e^2 - 3*A*a*c*d*e^2 - 3*B*a*c*d^2*e))/(2*a))/(a*c^2
+ c^3*x^2) + (C*e^3*x^2)/(2*c^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^3 -
3*B*a^2*e^3 + C*a*c*d^3 - 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(
2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*A*a^3*c^4*e^3 - 32*C*a^4*c^3*e^3 +
 48*B*a^3*c^4*d*e^2 + 48*C*a^3*c^4*d^2*e))/(32*a^3*c^6)
```

**sympy [B]** time = 34.46, size = 952, normalized size = 4.41

$$\frac{Ce^3x^2}{2c^2}+x\left(\frac{Be^3}{c^2}+\frac{3Cde^2}{c^2}\right)+\left(-\frac{e\left(-Ace^2-3Bcde+2Cae^2-3Ccd^2\right)}{2c^3}-\frac{\sqrt{-a^3c^7}\left(-3Aacde^2-Ac^2d^3+3Ba^2e^3-3B\right.}{4a^3c^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**3*x**2/(2*c**2) + x*(B*e**3/c**2 + 3*C*d*e**2/c**2) + (-e*(-A*c*e**2 -
 3*B*c*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c
*d*e**2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 -
C*a*c*d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4
*C*a**3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e +
 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) - sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*
c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/
(4*a**3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a
*c**2*d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (-e*(-A*c*e**2 - 3*B*c
```

```
*d*e + 2*C*a*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**
2 - A*c**2*d**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*
d**3)/(4*a**3*c**6))*log(x + (2*A*a**2*c*e**3 + 6*B*a**2*c*d*e**2 - 4*C*a**
3*e**3 + 6*C*a**2*c*d**2*e - 4*a**2*c**3*(-e*(-A*c*e**2 - 3*B*c*d*e + 2*C*a
*e**2 - 3*C*c*d**2)/(2*c**3) + sqrt(-a**3*c**7)*(-3*A*a*c*d*e**2 - A*c**2*d
**3 + 3*B*a**2*e**3 - 3*B*a*c*d**2*e + 9*C*a**2*d*e**2 - C*a*c*d**3)/(4*a**
3*c**6)))/(-3*A*a*c**2*d*e**2 - A*c**3*d**3 + 3*B*a**2*c*e**3 - 3*B*a*c**2*
d**2*e + 9*C*a**2*c*d*e**2 - C*a*c**2*d**3)) + (A*a**2*c*e**3 - 3*A*a*c**2*
d**2*e + 3*B*a**2*c*d*e**2 - B*a*c**2*d**3 - C*a**3*e**3 + 3*C*a**2*c*d**2*
e + x*(-3*A*a*c**2*d*e**2 + A*c**3*d**3 + B*a**2*c*e**3 - 3*B*a*c**2*d**2*e
 + 3*C*a**2*c*d*e**2 - C*a*c**2*d**3))/(2*a**2*c**3 + 2*a*c**4*x**2)
```

## 3.51 $\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+\right.}{}$$

```
[Out] -1/2*(A*c-3*C*a)*e^2*x/a/c^2-1/2*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)+
1/2*(a*(A*c-3*C*a)*e^2+c*d*(A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))
/a^(3/2)/c^(5/2)+1/2*e*(B*e+2*C*d)*ln(c*x^2+a)/c^2
```

**Rubi [A]** time = 0.25, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 774, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+Acd)+ae^2(Ac-3aC)\right)}{2a^{3/2}c^{5/2}}-\frac{(d+ex)^2(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}-\frac{e^2x(Ac-3aC)}{2ac^2}+\frac{e\log\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((A*c - 3*a*C)*e^2*x)/(2*a*c^2) - ((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(2*a
*c*(a + c*x^2)) + ((a*(A*c - 3*a*C)*e^2 + c*d*(A*c*d + a*C*d + 2*a*B*e))*Ar
cTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(5/2)) + (e*(2*C*d + B*e)*Log[a + c
*x^2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{(d+ex)(-Acd-aCd-2aBe+(Ac-3aC)ex)}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} - \frac{\int \frac{-a(Ac-3aC)e^2+cd(-Acd-aCd-2}{}}{}$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{(e(2Cd+Be))\int \frac{x}{a+cx^2}\,dx}{c} +$$

$$= -\frac{(Ac-3aC)e^2x}{2ac^2} - \frac{(aB-(Ac-aC)x)(d+ex)^2}{2ac\left(a+cx^2\right)} + \frac{\left(a(Ac-3aC)e^2+cd(Acd\right.}{2a^3}$$

**Mathematica [A]** time = 0.14, size = 175, normalized size = 1.20

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+cd^2\right)+a\left(cd(2Be+Cd)-3aCe^2\right)\right)}{a^{3/2}} + \frac{\sqrt{c}\left(a^2e(Be+2Cd+Cex)-ac\left(Ae(2d+ex)+Bd(d+2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)} + \sqrt{c}\,e\log\left(a+cx\right.}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] (2*Sqrt[c]*C*e^2*x + (Sqrt[c]*(A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*x) -
a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x))))/(a*(a + c*x^2)) + ((A*c
*(c*d^2 + a*e^2) + a*(-3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/a^(3/2) + Sqrt[c]*e*(2*C*d + B*e)*Log[a + c*x^2])/(2*c^(5/2))
```

**fricas [B]** time = 0.97, size = 631, normalized size = 4.32

$$\left[\frac{4\,Ca^2c^2e^2x^3 - 2\,Ba^2c^2d^2 + 2\,Ba^3ce^2 + 4\left(Ca^3c - Aa^2c^2\right)de - \left(2\,Ba^2cde + \left(Ca^2c + Aac^2\right)d^2 - \left(3\,Ca^3 - Aa^2c\right)e^2 + \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*C*a^2*c^2*e^2*x^3 - 2*B*a^2*c^2*d^2 + 2*B*a^3*c*e^2 + 4*(C*a^3*c -
A*a^2*c^2)*d*e - (2*B*a^2*c*d*e + (C*a^2*c + A*a*c^2)*d^2 - (3*C*a^3 - A*a^
2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e
^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(2*B*
a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2)*x +
2*(2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)*log
(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3), 1/2*(2*C*a^2*c^2*e^2*x^3 - B*a^2*c^2*
d^2 + B*a^3*c*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d*e + (2*B*a^2*c*d*e + (C*a^2*c
 + A*a*c^2)*d^2 - (3*C*a^3 - A*a^2*c)*e^2 + (2*B*a*c^2*d*e + (C*a*c^2 + A*c
^3)*d^2 - (3*C*a^2*c - A*a*c^2)*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) -
 (2*B*a^2*c^2*d*e + (C*a^2*c^2 - A*a*c^3)*d^2 - (3*C*a^3*c - A*a^2*c^2)*e^2
)*x + (2*C*a^3*c*d*e + B*a^3*c*e^2 + (2*C*a^2*c^2*d*e + B*a^2*c^2*e^2)*x^2)
*log(c*x^2 + a))/(a^2*c^4*x^2 + a^3*c^3)]
```

**giac [A]** time = 0.17, size = 184, normalized size = 1.26

$$\frac{Cxe^2}{c^2} + \frac{\left(2\,Cde + Be^2\right)\log\left(cx^2 + a\right)}{2\,c^2} + \frac{\left(Cacd^2 + Ac^2d^2 + 2\,Bacde - 3\,Ca^2e^2 + Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac^2} - \frac{Bacd^2 - 2\,Ca^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] C*x*e^2/c^2 + 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(C*a*c*d^2 + A
*c^2*d^2 + 2*B*a*c*d*e - 3*C*a^2*e^2 + A*a*c*e^2)*arctan(c*x/sqrt(a*c))/(sq
rt(a*c)*a*c^2) - 1/2*(B*a*c*d^2 - 2*C*a^2*d*e + 2*A*a*c*d*e - B*a^2*e^2 + (
```

```
C*a*c*d^2 - A*c^2*d^2 + 2*B*a*c*d*e - C*a^2*e^2 + A*a*c*e^2)*x)/((c*x^2 + a
)*a*c^2)
```

**maple [B]** time = 0.01, size = 323, normalized size = 2.21

$$\frac{A\,d^2x}{2\left(c\,x^2+a\right)a}+\frac{A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}-\frac{A\,e^2x}{2\left(c\,x^2+a\right)c}+\frac{A\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}-\frac{Bdex}{\left(c\,x^2+a\right)c}+\frac{Bde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}\;c}+\frac{Ca\,e}{2\left(c\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] C*e^2/c^2*x-1/2/c/(c*x^2+a)*A*e^2*x+1/2/(c*x^2+a)/a*x*A*d^2-1/c/(c*x^2+a)*B
*d*e*x+1/2/c^2/(c*x^2+a)*a*C*e^2*x-1/2/c/(c*x^2+a)*C*d^2*x-1/c/(c*x^2+a)*A*
d*e+1/2/c^2/(c*x^2+a)*B*a*e^2-1/2/c/(c*x^2+a)*B*d^2+1/c^2/(c*x^2+a)*C*a*d*e
+1/2/c^2*ln(c*x^2+a)*B*e^2+1/c^2*ln(c*x^2+a)*C*d*e+1/2/c/(a*c)^(1/2)*arctan
(1/(a*c)^(1/2)*c*x)*A*e^2+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2
+1/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d*e-3/2/c^2*a/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*C*e^2+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
^2
```

**maxima [A]** time = 0.96, size = 188, normalized size = 1.29

$$\frac{Ce^2x}{c^2}-\frac{Bacd^2-Ba^2e^2-2\left(Ca^2-Aac\right)de+\left(2\,Bacde+\left(Cac-Ac^2\right)d^2-\left(Ca^2-Aac\right)e^2\right)x}{2\left(ac^3x^2+a^2c^2\right)}+\frac{\left(2\,Cde+Be^2\right)\log\left(c\right.}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*e^2*x/c^2 - 1/2*(B*a*c*d^2 - B*a^2*e^2 - 2*(C*a^2 - A*a*c)*d*e + (2*B*a*c
*d*e + (C*a*c - A*c^2)*d^2 - (C*a^2 - A*a*c)*e^2)*x)/(a*c^3*x^2 + a^2*c^2)
+ 1/2*(2*C*d*e + B*e^2)*log(c*x^2 + a)/c^2 + 1/2*(2*B*a*c*d*e + (C*a*c + A*
c^2)*d^2 - (3*C*a^2 - A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c^2)
```

**mupad [B]** time = 0.23, size = 195, normalized size = 1.34

$$\frac{C\,e^2\,x}{c^2}-\frac{\frac{x\left(-C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2-A\,c^2\,d^2\right)}{2\,a}-\frac{B\,a\,e^2}{2}+\frac{B\,c\,d^2}{2}+A\,c\,d\,e-C\,a\,d\,e}{c^3\,x^2+a\,c^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(-3\,C\,a^2\,e^2+C\,a\,c\right.}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e^2*x)/c^2 - ((x*(A*a*c*e^2 - C*a^2*e^2 - A*c^2*d^2 + C*a*c*d^2 + 2*B*a*
c*d*e))/(2*a) - (B*a*e^2)/2 + (B*c*d^2)/2 + A*c*d*e - C*a*d*e)/(a*c^2 + c^3
*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(A*c^2*d^2 - 3*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 2*B*a*c*d*e))/(2*a^(3/2)*c^(5/2)) + (log(a + c*x^2)*(16*B*a^3*c^
3*e^2 + 32*C*a^3*c^3*d*e))/(32*a^3*c^5)
```

**sympy [B]** time = 18.40, size = 593, normalized size = 4.06

$$\frac{Ce^2x}{c^2}+\left(\frac{e(Be+2Cd)}{2c^2}-\frac{\sqrt{-a^3c^5}\left(-Aace^2-Ac^2d^2-2Bacde+3Ca^2e^2-Cacd^2\right)}{4a^3c^5}\right)\log\left(x+\frac{2Ba^2e^2+4Ca^2de-4a^2}{-Aace}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] C*e**2*x/c**2 + (e*(B*e + 2*C*d)/(2*c**2) - sqrt(-a**3*c**5)*(-A*a*c*e**2 -
 A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5))*log
(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*C*d)/(2*c**2)
- sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2
 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3
*C*a**2*e**2 - C*a*c*d**2)) + (e*(B*e + 2*C*d)/(2*c**2) + sqrt(-a**3*c**5)*
(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)/(4*a
**3*c**5))*log(x + (2*B*a**2*e**2 + 4*C*a**2*d*e - 4*a**2*c**2*(e*(B*e + 2*
C*d)/(2*c**2) + sqrt(-a**3*c**5)*(-A*a*c*e**2 - A*c**2*d**2 - 2*B*a*c*d*e +
 3*C*a**2*e**2 - C*a*c*d**2)/(4*a**3*c**5)))/(-A*a*c*e**2 - A*c**2*d**2 - 2
*B*a*c*d*e + 3*C*a**2*e**2 - C*a*c*d**2)) + (-2*A*a*c*d*e + B*a**2*e**2 - B
*a*c*d**2 + 2*C*a**2*d*e + x*(-A*a*c*e**2 + A*c**2*d**2 - 2*B*a*c*d*e + C*a
**2*e**2 - C*a*c*d**2))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

# 3.52 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2} \, dx$

**Optimal**. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

```
[Out] -1/2*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)+1/2*(A*c*d+B*a*e+C*a*d)*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/c^(3/2)+1/2*C*e*ln(c*x^2+a)/c^2
```

**Rubi [A]** time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1645, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{2a^{3/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{2ac\left(a+cx^2\right)}+\frac{Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(2*a*c*(a + c*x^2)) + ((A*c*d + a*C*d +
a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2)) + (C*e*Log[a + c*x^
2])/(2*c^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^2}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} - \frac{\int \frac{-Acd-a(Cd+Be)-2aCex}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} + \frac{(Ce)\int \frac{x}{a+cx^2}\,dx}{c} + \frac{(Acd+aCd+aBe)\int \frac{1}{a+cx^2}\,dx}{2ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{2ac\left(a+cx^2\right)} + \frac{(Acd+aCd+aBe)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}} + \frac{Ce\log\left(a+c\right.}{2c^2}$$

**Mathematica [A]** time = 0.09, size = 102, normalized size = 1.05

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+Acd)}{a^{3/2}} + \frac{a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx}{a\left(a+cx^2\right)} + Ce\log\left(a+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x]
```

```
[Out] ((a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x)))/(a*(a + c*x^2)) +
(Sqrt[c]*(A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + C*e
*Log[a + c*x^2])/(2*c^2)
```

**fricas [A]** time = 0.97, size = 337, normalized size = 3.47

$$\left[-\frac{2Ba^2cd+\left(Ba^2e+\left(Bace+\left(Cac+Ac^2\right)d\right)x^2+\left(Ca^2+Aac\right)d\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\sqrt{-ac}\,x-a}{cx^2+a}\right)-2\left(Ca^3-Aa^2c\right)e+2}{4\left(a^2c^3x^2+a^3c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c*d + (B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2
+ A*a*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(C
*a^3 - A*a^2*c)*e + 2*(B*a^2*c*e + (C*a^2*c - A*a*c^2)*d)*x - 2*(C*a^2*c*e*
x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2), -1/2*(B*a^2*c*d - (
B*a^2*e + (B*a*c*e + (C*a*c + A*c^2)*d)*x^2 + (C*a^2 + A*a*c)*d)*sqrt(a*c)*
arctan(sqrt(a*c)*x/a) - (C*a^3 - A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - A*a*c
^2)*d)*x - (C*a^2*c*e*x^2 + C*a^3*e)*log(c*x^2 + a))/(a^2*c^3*x^2 + a^3*c^2
)]
```

**giac [A]** time = 0.22, size = 112, normalized size = 1.15

$$\frac{Ce\log\left(cx^2+a\right)}{2\,c^2}+\frac{(Cad+Acd+Bae)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}-\frac{(Cad-Acd+Bae)x+\frac{Bacd-Ca^2e+Aace}{c}}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 + 1/2*(C*a*d + A*c*d + B*a*e)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a*c) - 1/2*((C*a*d - A*c*d + B*a*e)*x + (B*a*c*d - C*a^2*e +
 A*a*c*e)/c)/((c*x^2 + a)*a*c)
```

**maple [A]** time = 0.01, size = 134, normalized size = 1.38

$$\frac{Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;a}+\frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\;c}+\frac{Ce\ln\left(c\,x^2+a\right)}{2c^2}+\frac{\frac{(Acd-Bae-Cad)x}{2ac}-\frac{Ace+Bcd-aCe}{2c^2}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c*d-B*a*e-C*a*d)/a/c*x-1/2*(A*c*e+B*c*d-C*a*e)/c^2)/(c*x^2+a)+1/2*C
*e*ln(c*x^2+a)/c^2+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d+1/2/c/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*C*d
```

**maxima [A]** time = 0.97, size = 113, normalized size = 1.16

$$\frac{Ce\log\left(cx^2+a\right)}{2\,c^2}-\frac{Bacd-\left(Ca^2-Aac\right)e+\left(Bace+\left(Cac-Ac^2\right)d\right)x}{2\left(ac^3x^2+a^2c^2\right)}+\frac{(Bae+(Ca+Ac)d)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*C*e*log(c*x^2 + a)/c^2 - 1/2*(B*a*c*d - (C*a^2 - A*a*c)*e + (B*a*c*e +
(C*a*c - A*c^2)*d)*x)/(a*c^3*x^2 + a^2*c^2) + 1/2*(B*a*e + (C*a + A*c)*d)*a
rctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c)
```

**mupad [B]** time = 0.14, size = 191, normalized size = 1.97

$$\frac{C\,e\,\ln\left(c\,x^2+a\right)}{2\,c^2}-\frac{B\,d}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{B\,e\,x}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{C\,d\,x}{2\,\left(c^2\,x^2+a\,c\right)}-\frac{A\,e}{2\,\left(c^2\,x^2+a\,c\right)}+\frac{C\,a\,e}{2\,\left(c^3\,x^2+a\,c^2\right)}+\frac{A\,d\,x}{2\,\left(a^2+c\,a\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^2,x)
```

```
[Out] (C*e*log(a + c*x^2))/(2*c^2) - (B*d)/(2*(a*c + c^2*x^2)) - (B*e*x)/(2*(a*c
+ c^2*x^2)) - (C*d*x)/(2*(a*c + c^2*x^2)) - (A*e)/(2*(a*c + c^2*x^2)) + (C*
a*e)/(2*(a*c^2 + c^3*x^2)) + (A*d*x)/(2*(a^2 + a*c*x^2)) + (A*d*atan((c^(1/
2)*x)/a^(1/2)))/(2*a^(3/2)*c^(1/2)) + (B*e*atan((c^(1/2)*x)/a^(1/2)))/(2*a^
(1/2)*c^(3/2)) + (C*d*atan((c^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*c^(3/2))
```

**sympy [B]** time = 6.41, size = 318, normalized size = 3.28

$$\left(\frac{Ce}{2c^2}-\frac{\sqrt{-a^3c^5}\,\left(Acd+Bae+Cad\right)}{4a^3c^4}\right)\log\left(x+\frac{-2Ca^2e+4a^2c^2\left(\frac{Ce}{2c^2}-\frac{\sqrt{-a^3c^5}\,(Acd+Bae+Cad)}{4a^3c^4}\right)}{Ac^2d+Bace+Cacd}\right)+\left(\frac{Ce}{2c^2}+\frac{\sqrt{-a^3c^5}\,\left(Ac\right.}{4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] (C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log
(x + (-2*C*a**2*e + 4*a**2*c**2*(C*e/(2*c**2) - sqrt(-a**3*c**5)*(A*c*d + B
*a*e + C*a*d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (C*e/(2*c**
2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*d)/(4*a**3*c**4))*log(x + (-2*C*
a**2*e + 4*a**2*c**2*(C*e/(2*c**2) + sqrt(-a**3*c**5)*(A*c*d + B*a*e + C*a*
d)/(4*a**3*c**4)))/(A*c**2*d + B*a*c*e + C*a*c*d)) + (-A*a*c*e - B*a*c*d +
C*a**2*e + x*(A*c**2*d - B*a*c*e - C*a*c*d))/(2*a**2*c**2 + 2*a*c**3*x**2)
```

# 3.53 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2} dx$

**Optimal**. Leaf size=69

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

```
[Out] 1/2*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)+1/2*(A*c+C*a)*arctan(x*c^(1/2)/a^(1/2)
)/a^(3/2)/c^(3/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1814, 12, 205}

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}-\frac{aB-x(Ac-aC)}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^2}\,dx &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}-\frac{\int \frac{-A-\frac{aC}{c}}{a+cx^2}\,dx}{2a}\\ &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\int \frac{1}{a+cx^2}\,dx}{2ac}\\ &= -\frac{aB-(Ac-aC)x}{2ac\left(a+cx^2\right)}+\frac{(Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 68, normalized size = 0.99

$$\frac{(aC+Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2a^{3/2}c^{3/2}}+\frac{-aB-aCx+Acx}{2ac\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^2,x]
```

```
[Out] (-(a*B) + A*c*x - a*C*x)/(2*a*c*(a + c*x^2)) + ((A*c + a*C)*ArcTan[(Sqrt[c]
*x)/Sqrt[a]])/(2*a^(3/2)*c^(3/2))
```

**fricas [A]** time = 0.81, size = 195, normalized size = 2.83

$$\left[-\frac{2\,Ba^2c+\left(Ca^2+Aac+\left(Cac+Ac^2\right)x^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2\,\sqrt{-ac}\,x-a}{cx^2+a}\right)+2\left(Ca^2c-Aac^2\right)x}{4\left(a^2c^3x^2+a^3c^2\right)},-\frac{Ba^2c-\left(Ca^2+Aac+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c + (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(-a*c)*log((c*
x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^2*c - A*a*c^2)*x)/(a^2*c^3*
x^2 + a^3*c^2), -1/2*(B*a^2*c - (C*a^2 + A*a*c + (C*a*c + A*c^2)*x^2)*sqrt(
a*c)*arctan(sqrt(a*c)*x/a) + (C*a^2*c - A*a*c^2)*x)/(a^2*c^3*x^2 + a^3*c^2)
]
```

**giac** [A] time = 0.18, size = 60, normalized size = 0.87

$$\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,ac}-\frac{Cax-Acx+Ba}{2\left(cx^2+a\right)ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a + A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a*c) - 1/2*(C*a*x - A*c*x
+ B*a)/((c*x^2 + a)*a*c)
```

**maple** [A] time = 0.01, size = 76, normalized size = 1.10

$$\frac{A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,a}+\frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,c}+\frac{-\frac{B}{2c}+\frac{(Ac-aC)x}{2ac}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^2,x)
```

```
[Out] (1/2*(A*c-C*a)/a/c*x-1/2*B/c)/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*A+1/2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
```

**maxima** [A] time = 0.96, size = 62, normalized size = 0.90

$$-\frac{Ba+(Ca-Ac)x}{2\left(ac^2x^2+a^2c\right)}+\frac{(Ca+Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(B*a + (C*a - A*c)*x)/(a*c^2*x^2 + a^2*c) + 1/2*(C*a + A*c)*arctan(c*x
/sqrt(a*c))/(sqrt(a*c)*a*c)
```

**mupad** [B] time = 0.10, size = 60, normalized size = 0.87

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(A\,c+C\,a)}{2\,a^{3/2}\,c^{3/2}}-\frac{\frac{B}{2\,c}-\frac{x\,(A\,c-C\,a)}{2\,a\,c}}{c\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^2,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(A*c + C*a))/(2*a^(3/2)*c^(3/2)) - (B/(2*c) - (x
*(A*c - C*a))/(2*a*c))/(a + c*x^2)
```

**sympy [A]** time = 0.65, size = 116, normalized size = 1.68

$$-\frac{\sqrt{-\frac{1}{a^3c^3}}\left(Ac+Ca\right)\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3c^3}}\left(Ac+Ca\right)\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}}+x\right)}{4}+\frac{-Ba+x\left(Ac-Ca\right)}{2a^2c+2ac^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
 sqrt(-1/(a**3*c**3))*(A*c + C*a)*log(a**2*c*sqrt(-1/(a**3*c**3)) + x)/4 +
(-B*a + x*(A*c - C*a))/(2*a**2*c + 2*a*c**2*x**2)
```

## 3.54 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2} \, dx$

**Optimal**. Leaf size=226

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(\right.}{}$$

```
[Out] 1/2*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)+e*(A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(a*e^2+c*d^2)^2-1/2*e*(A*e^2-B*d*e+C
*d^2)*ln(c*x^2+a)/(a*e^2+c*d^2)^2+1/2*(a*(-B*e+C*d)*(-a*e^2+c*d^2)+A*c*d*(3
*a*e^2+c*d^2))*arctan(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^2/c^(1/2)
```

**Rubi [A]** time = 0.43, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{2a^{3/2}\sqrt{c}\left(ae^2+cd^2\right)^2}-\frac{a(aCe-Ace+Bcd)-cx(aBe-aCd+Acd)}{2ac\left(a+cx^2\right)\left(ae^2+cd^2\right)}-\frac{e\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(2*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 + 3*a*
e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2 + a*e^2)^2) +
(e*(C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/(c*d^2 + a*e^2)^2 - (e*(C*d^2 - B*
d*e + A*e^2)*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^2}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(cd^2+2ae^2\right)\right)}{cd^2+ae^2}-\frac{ce(Acd-aCd+aBe)x}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}}{2ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} - \frac{\int\left(-\frac{2ace^2\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{c\left(-a(Cd-Be)\left(cd\right.\right.}{}\right.}{}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - $$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{e\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{\left(cd^2+ae^2\right)^2} - $$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{2ac\left(cd^2+ae^2\right)\left(a+cx^2\right)} + \frac{\left(a(Cd-Be)\left(cd^2-ae^2\right)+Acd\left(cd^2\right.\right.}{2a^{3/2}\sqrt{c}\left(cd^2+ae\right.}$$

**Mathematica [A]** time = 0.22, size = 195, normalized size = 0.86

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+cd^2\right)+a\left(cd^2-ae^2\right)(Cd-Be)\right)}{a^{3/2}\sqrt{c}}+\frac{\left(ae^2+cd^2\right)\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)}-e\log\left(a+cx^2\right)\left(e(Ae-Bd)+\right.}{2\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x + B*e
*x)))/(a*c*(a + c*x^2)) + ((a*(C*d - B*e)*(c*d^2 - a*e^2) + A*c*d*(c*d^2 +
3*a*e^2))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(C*d^2 + e*(
-(B*d) + A*e))*Log[d + e*x] - e*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/
(2*(c*d^2 + a*e^2)^2)
```

**fricas [B]** time = 64.28, size = 1024, normalized size = 4.53

$$\left[-\frac{2\,Ba^2c^2d^3+2\,Ba^3cde^2+2\left(Ca^3c-Aa^2c^2\right)d^2e+2\left(Ca^4-Aa^3c\right)e^3-\left(Ba^2cd^2e-Ba^3e^3-\left(Ca^2c+Aac^2\right)d^3+\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*c^2*d^3 + 2*B*a^3*c*d*e^2 + 2*(C*a^3*c - A*a^2*c^2)*d^2*e +
2*(C*a^4 - A*a^3*c)*e^3 - (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*c + A*a*c^2)*
d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3 - (C*a*c^2 +
 A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*s
qrt(-a*c)*x - a)/(c*x^2 + a)) - 2*(B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c
^2 - A*a*c^3)*d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + 2*(C*a^3*c*d^2*e - B*a
^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e
^3)*x^2)*log(c*x^2 + a) - 4*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 +
(C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3
*c^3*d^4 + 2*a^4*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2
 + a^4*c^2*e^4)*x^2), -1/2*(B*a^2*c^2*d^3 + B*a^3*c*d*e^2 + (C*a^3*c - A*a^
2*c^2)*d^2*e + (C*a^4 - A*a^3*c)*e^3 + (B*a^2*c*d^2*e - B*a^3*e^3 - (C*a^2*
c + A*a*c^2)*d^3 + (C*a^3 - 3*A*a^2*c)*d*e^2 + (B*a*c^2*d^2*e - B*a^2*c*e^3
 - (C*a*c^2 + A*c^3)*d^3 + (C*a^2*c - 3*A*a*c^2)*d*e^2)*x^2)*sqrt(a*c)*arct
an(sqrt(a*c)*x/a) - (B*a^2*c^2*d^2*e + B*a^3*c*e^3 - (C*a^2*c^2 - A*a*c^3)*
d^3 - (C*a^3*c - A*a^2*c^2)*d*e^2)*x + (C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a
^3*c*e^3 + (C*a^2*c^2*d^2*e - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(c*x
^2 + a) - 2*(C*a^3*c*d^2*e - B*a^3*c*d*e^2 + A*a^3*c*e^3 + (C*a^2*c^2*d^2*e
```

```
 - B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2)*log(e*x + d))/(a^3*c^3*d^4 + 2*a^4
*c^2*d^2*e^2 + a^5*c*e^4 + (a^2*c^4*d^4 + 2*a^3*c^3*d^2*e^2 + a^4*c^2*e^4)*
x^2)]
```

**giac [A]** time = 0.19, size = 350, normalized size = 1.55

$$-\frac{\left(Cd^2e - Bde^2 + Ae^3\right)\log\left(cx^2 + a\right)}{2\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} + \frac{\left(Cd^2e^2 - Bde^3 + Ae^4\right)\log\left(|xe + d|\right)}{c^2d^4e + 2\,acd^2e^3 + a^2e^5} + \frac{\left(Cacd^3 + Ac^2d^3 - Bacd^2e - Ca^2de^2 +\right.}{2\left(ac^2d^4 + 2\,a^2cd^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e^2 - B*d*e^3 + A*e^4)*log(abs(x*e + d))/(c^2*d^4*e + 2*a
*c*d^2*e^3 + a^2*e^5) + 1/2*(C*a*c*d^3 + A*c^2*d^3 - B*a*c*d^2*e - C*a^2*d*
e^2 + 3*A*a*c*d*e^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/2*(B*a*c^2*d^3 + C*a^2*c*d^2*e - A*a*c^
2*d^2*e + B*a^2*c*d*e^2 + C*a^3*e^3 - A*a^2*c*e^3 + (C*a*c^2*d^3 - A*c^3*d^
3 - B*a*c^2*d^2*e + C*a^2*c*d*e^2 - A*a*c^2*d*e^2 - B*a^2*c*e^3)*x)/((c*d^2
 + a*e^2)^2*(c*x^2 + a)*a*c)
```

**maple [B]** time = 0.02, size = 742, normalized size = 3.28

$$\frac{A\,c^2d^3x}{2\left(a\,e^2 + c\,d^2\right)^2\left(c\,x^2 + a\right)a} + \frac{A\,c^2d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(a\,e^2 + c\,d^2\right)^2\sqrt{ac}\;a} + \frac{Acd\,e^2x}{2\left(a\,e^2 + c\,d^2\right)^2\left(c\,x^2 + a\right)} + \frac{3Acd\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\left(a\,e^2 + c\,d^2\right)^2\sqrt{ac}} + \frac{B}{2\left(a\,e^2 + c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x)
```

```
[Out] e^3/(a*e^2+c*d^2)^2*ln(e*x+d)*A-e^2/(a*e^2+c*d^2)^2*ln(e*x+d)*B*d+e/(a*e^2+
c*d^2)^2*ln(e*x+d)*C*d^2+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*A*x*c*d*e^2+1/2/(a*e
^2+c*d^2)^2/(c*x^2+a)/a*x*A*c^2*d^3+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*B*x*a*e^3
+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*B*x*c*d^2*e-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*
x*a*d*e^2-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*x*c*d^3+1/2/(a*e^2+c*d^2)^2/(c*x^
2+a)*A*e^3*a+1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*c*A*d^2*e-1/2/(a*e^2+c*d^2)^2/(c
*x^2+a)*B*d*e^2*a-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*c*d^3*B-1/2/(a*e^2+c*d^2)^2
/(c*x^2+a)/c*C*a^2*e^3-1/2/(a*e^2+c*d^2)^2/(c*x^2+a)*C*a*d^2*e-1/2/(a*e^2+c
*d^2)^2*ln(c*x^2+a)*A*e^3+1/2/(a*e^2+c*d^2)^2*ln(c*x^2+a)*B*d*e^2-1/2/(a*e^
2+c*d^2)^2*ln(c*x^2+a)*C*d^2*e+3/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*
c)^(1/2)*c*x)*A*c*d*e^2+1/2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(1/(a*c)^(1
/2)*c*x)*A*c^2*d^3+1/2/(a*e^2+c*d^2)^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c
*x)*B*e^3-1/2/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d^2
```

```
*e-1/2/(a*e^2+c*d^2)^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+1/2/
(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^3
```

**maxima [A]** time = 1.00, size = 293, normalized size = 1.30

$$-\frac{\left(Cd^2e-Bde^2+Ae^3\right)\log\left(cx^2+a\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(Cd^2e-Bde^2+Ae^3\right)\log\left(ex+d\right)}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(Bacd^2e-Ba^2e^3-\left(Cac+Ac^2\right)d^3+\left(\right.\right.}{2\left(ac^2d^4+2\,a^2cd^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e - B*d*e^2 + A*e^3)*log(c*x^2 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4) + (C*d^2*e - B*d*e^2 + A*e^3)*log(e*x + d)/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4) - 1/2*(B*a*c*d^2*e - B*a^2*e^3 - (C*a*c + A*c^2)*d^3 + (C*a^2
- 3*A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4)*sqrt(a*c)) - 1/2*(B*a*c*d + (C*a^2 - A*a*c)*e - (B*a*c*e - (C*a*c - A
*c^2)*d)*x)/(a^2*c^2*d^2 + a^3*c*e^2 + (a*c^3*d^2 + a^2*c^2*e^2)*x^2)
```

**mupad [B]** time = 7.68, size = 1493, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)),x)
```

```
[Out] (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2) + 6*A*a^4*c*e^5 -
B*a^4*c*e^5*x - 2*A*a^2*c^3*d^4*e - 8*C*a^3*c^2*d^4*e + 8*C*a^4*c*d^2*e^3 +
 C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e^4*(-a^3*c)^(1/2)
- 12*A*a^3*c^2*d^2*e^3 + 8*B*a^3*c^2*d^3*e^2 - 8*B*a^4*c*d*e^4 + A*a*c^4*d^
5*x + 2*A*a^2*c^3*d^3*e^2*x + 14*B*a^3*c^2*d^2*e^3*x - 14*C*a^3*c^2*d^3*e^2
*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^3*(-a^3*c)^(1/2) -
 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) + C*a^4*c*d*e^4*x - 15*A*a^3*c^2*d*e^4*x
 - B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) - B*a*c^2*d^4*e*(-a^
3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4*e*x*(-a^3*c)^(1/2
) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*(-a^3*c)^(1/2) + 1
2*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*(-a^3*c)^(1/2) - 8
*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(a^2*((B*e^3*(-a^3*c)^(1/2))/4 - (C*d*e^
2*(-a^3*c)^(1/2))/4) - c*(a^3*((A*e^3)/2 - (B*d*e^2)/2 + (C*d^2*e)/2) - a*(
(C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*c)^(1/2))/4 - (B*d^2*e*(-a^3*c)
^(1/2))/4)) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 + a^3*c^3*d^4 + 2*a
^4*c^2*d^2*e^2) - (log(A*c^3*d^5*(-a^3*c)^(1/2) - B*a^3*e^5*(-a^3*c)^(1/2)
- 6*A*a^4*c*e^5 + B*a^4*c*e^5*x + 2*A*a^2*c^3*d^4*e + 8*C*a^3*c^2*d^4*e - 8
*C*a^4*c*d^2*e^3 - C*a^2*c^3*d^5*x + C*a*c^2*d^5*(-a^3*c)^(1/2) + C*a^3*d*e
```

```
^4*(-a^3*c)^(1/2) + 12*A*a^3*c^2*d^2*e^3 - 8*B*a^3*c^2*d^3*e^2 + 8*B*a^4*c*
d*e^4 - A*a*c^4*d^5*x - 2*A*a^2*c^3*d^3*e^2*x - 14*B*a^3*c^2*d^2*e^3*x + 14
*C*a^3*c^2*d^3*e^2*x + 2*A*a*c^2*d^3*e^2*(-a^3*c)^(1/2) + 14*B*a^2*c*d^2*e^
3*(-a^3*c)^(1/2) - 14*C*a^2*c*d^3*e^2*(-a^3*c)^(1/2) - C*a^4*c*d*e^4*x + 15
*A*a^3*c^2*d*e^4*x + B*a^2*c^3*d^4*e*x - 15*A*a^2*c*d*e^4*(-a^3*c)^(1/2) -
B*a*c^2*d^4*e*(-a^3*c)^(1/2) - 6*A*a^2*c*e^5*x*(-a^3*c)^(1/2) + 2*A*c^3*d^4
*e*x*(-a^3*c)^(1/2) + 8*B*a^2*c*d*e^4*x*(-a^3*c)^(1/2) + 8*C*a*c^2*d^4*e*x*
(-a^3*c)^(1/2) + 12*A*a*c^2*d^2*e^3*x*(-a^3*c)^(1/2) - 8*B*a*c^2*d^3*e^2*x*
(-a^3*c)^(1/2) - 8*C*a^2*c*d^2*e^3*x*(-a^3*c)^(1/2))*(c*(a^3*((A*e^3)/2 - (
B*d*e^2)/2 + (C*d^2*e)/2) + a*((C*d^3*(-a^3*c)^(1/2))/4 + (3*A*d*e^2*(-a^3*
c)^(1/2))/4 - (B*d^2*e*(-a^3*c)^(1/2))/4)) + a^2*((B*e^3*(-a^3*c)^(1/2))/4
- (C*d*e^2*(-a^3*c)^(1/2))/4) + (A*c^2*d^3*(-a^3*c)^(1/2))/4))/(a^5*c*e^4 +
 a^3*c^3*d^4 + 2*a^4*c^2*d^2*e^2) - ((B*c*d - A*c*e + C*a*e)/(2*c*(a*e^2 +
c*d^2)) - (x*(A*c*d + B*a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a + c*x^2) +
(e*log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

# 3.55 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=374

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\,\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2aC\right.}{}$$

```
[Out] -e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)+1/2*(-a*(-2*A*c*d*e-B*a*e^2+
B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a/(
a*e^2+c*d^2)^2/(c*x^2+a)-e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^3+1/2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2-e*(-4*
A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^3+1/2*(A*c*(-3*a^2*e^4+6*a*c*d^2*e^2
+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*e+C*d)-6*a*c*d*e^2*(-B*e+C*d)))*arctan
(x*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2)^3/c^(1/2)
```

**Rubi [A]** time = 0.95, antiderivative size = 371, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4-6acde^2(Cd-Be)+c^2d^3(Cd-2Be)\right)\right)}{2a^{3/2}\sqrt{c}\,\left(ae^2+cd^2\right)^3}-\frac{a\left(-aBe^2+2aC\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] -((e*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^2*(d + e*x))) - (a*(B*c*d^2
- 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 - c*
d*(C*d - 2*B*e)))*x)/(2*a*(c*d^2 + a*e^2)^2*(a + c*x^2)) + ((A*c*(c^2*d^4 +
 6*a*c*d^2*e^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) - 6*a*c*
d*e^2*(C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[c]*(c*d^2
+ a*e^2)^3) + (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e))*
Log[d + e*x])/(c*d^2 + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a
*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)\cdots}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)\cdots}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\cdots\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\cdots\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(\cdots\right.\right.}{2a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}$$

**Mathematica** [A] time = 0.41, size = 320, normalized size = 0.86

$$\frac{\frac{\left(ae^2+cd^2\right)\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)+a\left(a^2Ce^4+6acde^2(Be-Cd)+c\cdots\right.\right.}{a^{3/2}\sqrt{c}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^2),x]
```

```
[Out] ((-2*e*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + ((c*d^2 + a*
e^2)*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d*(d -
2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)) + ((A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*d
) + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[c]) + 2*e*(2*c*C*d^3
+ c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - e*(2*c*C*d^
3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(2*(c*d^
2 + a*e^2)^3)
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.19, size = 608, normalized size = 1.63

$$\frac{\left(Cac^2d^4e^2 + Ac^3d^4e^2 - 2\,Bac^2d^3e^3 - 6\,Ca^2cd^2e^4 + 6\,Aac^2d^2e^4 + 6\,Ba^2cde^5 + Ca^3e^6 - 3\,Aa^2ce^6\right)\arctan\left(\frac{\left(cd-\frac{cd^2}{xe+d}-\cdots\right.}{\sqrt{a\cdots}}\right.}{2\left(ac^3d^6 + 3\,a^2c^2d^4e^2 + 3\,a^3cd^2e^4 + a^4e^6\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*a*c^2*d^4*e^2 + A*c^3*d^4*e^2 - 2*B*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^
4 + 6*A*a*c^2*d^2*e^4 + 6*B*a^2*c*d*e^5 + C*a^3*e^6 - 3*A*a^2*c*e^6)*arctan
((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a*c^3
*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(2*C
*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + 4*A*c*d*e^3 + B*a*e^4)*log(c - 2*c
*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 + 3*a*c^2*d^
4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - (C*d^2*e^5/(x*e + d) - B*d*e^6/(x*e +
d) + A*e^7/(x*e + d))/(c^2*d^4*e^4 + 2*a*c*d^2*e^6 + a^2*e^8) - 1/2*((C*a*c
^2*d^3*e - A*c^3*d^3*e - 3*B*a*c^2*d^2*e^2 - 3*C*a^2*c*d*e^3 + 3*A*a*c^2*d*
e^3 + B*a^2*c*e^4)/(c*d^2 + a*e^2) - (C*a*c^2*d^4*e^2 - A*c^3*d^4*e^2 - 4*B
*a*c^2*d^3*e^3 - 6*C*a^2*c*d^2*e^4 + 6*A*a*c^2*d^2*e^4 + 4*B*a^2*c*d*e^5 +
C*a^3*e^6 - A*a^2*c*e^6)*e^(-1)/((c*d^2 + a*e^2)*(x*e + d)))/((c*d^2 + a*e^
2)^2*a*(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

**maple [B]** time = 0.02, size = 1036, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x)
```

```
[Out] -2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*d*A*e^3+3/2/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*
e^2*d^2*B-1/(a*e^2+c*d^2)^3*c*ln(c*x^2+a)*C*d^3*e+1/2/(a*e^2+c*d^2)^3/(a*c)
^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^4+1/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*
C*d*e^3+1/2/(a*e^2+c*d^2)^3*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^4
+2*e/(a*e^2+c*d^2)^3*ln(e*x+d)*C*c*d^3+4*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*A*c*
d-3*e^2/(a*e^2+c*d^2)^3*ln(e*x+d)*B*c*d^2-2*e^3/(a*e^2+c*d^2)^3*ln(e*x+d)*C
*a*d+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*a^2*C*e^4*x-1/2/(a*e^2+c*d^2)^3/(c*x^2+a
```

```
)*C*c^2*d^4*x+1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*c^2*d^3*e-1/(a*e^2+c*d^2)^3/(c*
x^2+a)*C*a^2*d*e^3-e^3/(a*e^2+c*d^2)^2/(e*x+d)*A-1/(a*e^2+c*d^2)^3/(c*x^2+a
)*C*a*c*d^3*e-3/2/(a*e^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A
*c*e^4+1/2/(a*e^2+c*d^2)^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^3*d^
4+3/(a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2*e^2-1/(
a*e^2+c*d^2)^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^3*e-1/2/(a*e^2
+c*d^2)^3/(c*x^2+a)*A*a*c*e^4*x+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)/a*x*A*c^3*d^4
+1/(a*e^2+c*d^2)^3/(c*x^2+a)*B*c^2*d^3*e*x+1/(a*e^2+c*d^2)^3/(c*x^2+a)*A*a*
c*d*e^3+1/(a*e^2+c*d^2)^3/(c*x^2+a)*d*a*c*B*e^3*x-3/(a*e^2+c*d^2)^3*a/(a*c)
^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^2*e^2+3/(a*e^2+c*d^2)^3*a/(a*c)^(1/2
)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d*e^3-1/2/(a*e^2+c*d^2)^3*a*ln(c*x^2+a)*e^4
*B+e^4/(a*e^2+c*d^2)^3*ln(e*x+d)*B*a+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*B*a^2*e^
4-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)*B*c^2*d^4+e^2/(a*e^2+c*d^2)^2/(e*x+d)*B*d-e
/(a*e^2+c*d^2)^2/(e*x+d)*C*d^2
```

**maxima [A]** time = 1.04, size = 604, normalized size = 1.61

$$-\frac{\left(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3\right)\log\left(cx^2+a\right)}{2\left(c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e^6\right)}+\frac{\left(2\,Ccd^3e-3\,Bcd^2e^2+Bae^4-2\,(Ca-2\,Ac)de^3\right.}{c^3d^6+3\,ac^2d^4e^2+3\,a^2cd^2e^4+a^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(c*
x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (2*C*c*d
^3*e - 3*B*c*d^2*e^2 + B*a*e^4 - 2*(C*a - 2*A*c)*d*e^3)*log(e*x + d)/(c^3*d
^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/2*(2*B*a*c^2*d^3*e -
6*B*a^2*c*d*e^3 - (C*a*c^2 + A*c^3)*d^4 + 6*(C*a^2*c - A*a*c^2)*d^2*e^2 - (
C*a^3 - 3*A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/((a*c^3*d^6 + 3*a^2*c^2*d^4*e
^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*sqrt(a*c)) - 1/2*(B*a*c*d^3 - 3*B*a^2*d*e^2
 + 2*A*a^2*e^3 + 2*(2*C*a^2 - A*a*c)*d^2*e - (4*B*a*c*d*e^2 - (3*C*a*c - A*
c^2)*d^2*e + (C*a^2 - 3*A*a*c)*e^3)*x^2 - (B*a*c*d^2*e + B*a^2*e^3 - (C*a*c
 - A*c^2)*d^3 - (C*a^2 - A*a*c)*d*e^2)*x)/(a^2*c^2*d^5 + 2*a^3*c*d^3*e^2 +
a^4*d*e^4 + (a*c^3*d^4*e + 2*a^2*c^2*d^2*e^3 + a^3*c*e^5)*x^3 + (a*c^3*d^5
+ 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*x^2 + (a^2*c^2*d^4*e + 2*a^3*c*d^2*e^3 +
 a^4*e^5)*x)
```

**mupad [B]** time = 9.91, size = 2094, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^2),x)
```

```
[Out] ((x^2*(C*a^2*e^3 - 3*A*a*c*e^3 + A*c^2*d^2*e + 4*B*a*c*d*e^2 - 3*C*a*c*d^2*
e))/(2*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (2*A*a*e^3 + B*c*d^3 - 3*B*
a*d*e^2 - 2*A*c*d^2*e + 4*C*a*d^2*e)/(2*(a*e^2 + c*d^2)^2) + (x*(A*c*d + B*
a*e - C*a*d))/(2*a*(a*e^2 + c*d^2)))/(a*d + a*e*x + c*d*x^2 + c*e*x^3) - (l
og(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a^3*c)^(
1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) + 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^3*c)^(3
/2) - 6*B*e^6*x*(-a^3*c)^(3/2) - C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*c)^(3/2
) - 2*A*a^2*c^4*d^5*e + 30*A*a^4*c^2*d*e^5 - 14*C*a^3*c^3*d^5*e + 3*A*a^4*c
^2*e^6*x + C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) - 36*A*a^3*c^3*d^3*
e^3 + 22*B*a^3*c^3*d^4*e^2 - 36*B*a^4*c^2*d^2*e^4 + 36*C*a^4*c^2*d^3*e^3 -
14*C*a^5*c*d*e^5 + A*a*c^5*d^6*x + 5*A*a^2*c^4*d^4*e^2*x - 57*A*a^3*c^3*d^2
*e^4*x + 44*B*a^3*c^3*d^3*e^3*x - 31*C*a^3*c^3*d^4*e^2*x + 31*C*a^4*c^2*d^2
*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a^3*c)^(
1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-a^3*c)^
(1/2) - 2*B*a^2*c^4*d^5*e*x - 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e*(-a^3*
c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(-a^3*c)
^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*(-a^3*c
)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x*(-a^3*
c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^3*c)^(1
/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2) + a^3
*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c)^(1/2)
)/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2) - a^4*
((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(-a^3*c)
^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2*d^2*e^
4) + (log(3*A*e^6*(-a^3*c)^(3/2) - A*c^4*d^6*(-a^3*c)^(1/2) + C*a^4*e^6*(-a
^3*c)^(1/2) + 31*C*d^2*e^4*(-a^3*c)^(3/2) - 6*B*a^5*c*e^6 - 18*B*d*e^5*(-a^
3*c)^(3/2) - 6*B*e^6*x*(-a^3*c)^(3/2) + C*a^5*c*e^6*x + 14*C*d*e^5*x*(-a^3*
c)^(3/2) + 2*A*a^2*c^4*d^5*e - 30*A*a^4*c^2*d*e^5 + 14*C*a^3*c^3*d^5*e - 3*
A*a^4*c^2*e^6*x - C*a^2*c^4*d^6*x - C*a*c^3*d^6*(-a^3*c)^(1/2) + 36*A*a^3*c
^3*d^3*e^3 - 22*B*a^3*c^3*d^4*e^2 + 36*B*a^4*c^2*d^2*e^4 - 36*C*a^4*c^2*d^3
*e^3 + 14*C*a^5*c*d*e^5 - A*a*c^5*d^6*x - 5*A*a^2*c^4*d^4*e^2*x + 57*A*a^3*
c^3*d^2*e^4*x - 44*B*a^3*c^3*d^3*e^3*x + 31*C*a^3*c^3*d^4*e^2*x - 31*C*a^4*
c^2*d^2*e^4*x - 5*A*a*c^3*d^4*e^2*(-a^3*c)^(1/2) + 57*A*a^2*c^2*d^2*e^4*(-a
^3*c)^(1/2) - 44*B*a^2*c^2*d^3*e^3*(-a^3*c)^(1/2) + 31*C*a^2*c^2*d^4*e^2*(-
a^3*c)^(1/2) + 2*B*a^2*c^4*d^5*e*x + 18*B*a^4*c^2*d*e^5*x + 2*B*a*c^3*d^5*e
*(-a^3*c)^(1/2) - 2*A*c^4*d^5*e*x*(-a^3*c)^(1/2) - 36*B*a^2*c^2*d^2*e^4*x*(
-a^3*c)^(1/2) + 36*C*a^2*c^2*d^3*e^3*x*(-a^3*c)^(1/2) - 14*C*a*c^3*d^5*e*x*
(-a^3*c)^(1/2) - 36*A*a*c^3*d^3*e^3*x*(-a^3*c)^(1/2) + 30*A*a^2*c^2*d*e^5*x
*(-a^3*c)^(1/2) + 22*B*a*c^3*d^4*e^2*x*(-a^3*c)^(1/2))*(c^2*(a*((C*d^4*(-a^
3*c)^(1/2))/4 + (3*A*d^2*e^2*(-a^3*c)^(1/2))/2 - (B*d^3*e*(-a^3*c)^(1/2))/2
) - a^3*(2*A*d*e^3 - (3*B*d^2*e^2)/2 + C*d^3*e)) - c*(a^2*((3*A*e^4*(-a^3*c
)^(1/2))/4 + (3*C*d^2*e^2*(-a^3*c)^(1/2))/2 - (3*B*d*e^3*(-a^3*c)^(1/2))/2)
 + a^4*((B*e^4)/2 - C*d*e^3)) + (A*c^3*d^4*(-a^3*c)^(1/2))/4 + (C*a^3*e^4*(
-a^3*c)^(1/2))/4))/(a^6*c*e^6 + a^3*c^4*d^6 + 3*a^4*c^3*d^4*e^2 + 3*a^5*c^2
*d^2*e^4) + (log(d + e*x)*(a*(B*e^4 - 2*C*d*e^3) + c*(4*A*d*e^3 - 3*B*d^2*e
```

```
^2 + 2*C*d^3*e)))/(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**2,x)
[Out] Timed out
```

## 3.56 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2} dx$

**Optimal**. Leaf size=524

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2d^2\left(3Cd^2-2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2\right.}{}$$

```
[Out] -1/2*e*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^2/(e*x+d)^2+e*(a*e^2*(-B*e+2*C*d)-
c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))/(a*e^2+c*d^2)^3/(e*x+d)+1/2*(-a*(B*c*d*(-3*
a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*(c*d
^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)+e*(a^2*
C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d
)))*ln(e*x+d)/(a*e^2+c*d^2)^4-1/2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e
+3*B*d))-2*a*c*e^2*(4*C*d^2-e*(-A*e+3*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/
2*(A*c*d*(-15*a^2*e^4+10*a*c*d^2*e^2+c^2*d^4)-a*(2*a*c*d^2*e^2*(-9*B*e+7*C*
d)-c^2*d^4*(-3*B*e+C*d)-3*a^2*e^4*(-B*e+3*C*d)))*arctan(x*c^(1/2)/a^(1/2))*
c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^4
```

**Rubi [A]** time = 1.55, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$-\frac{e\log\left(a+cx^2\right)\left(a^2Ce^4-2ace^2\left(4Cd^2-e(3Bd-Ae)\right)+c^2\left(3Cd^4-2d^2e(3Bd-5Ae)\right)\right)}{2\left(ae^2+cd^2\right)^4}+\frac{e\log(d+ex)\left(a^2Ce^4-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] -(e*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^2*(d + e*x)^2) - (e*(2*c*C*
d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^3*(d +
 e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2)) - c
*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e)))*
x)/(2*a*(c*d^2 + a*e^2)^3*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*
d^2*e^2 - 15*a^2*e^4) - a*(2*a*c*d^2*e^2*(7*C*d - 9*B*e) - c^2*d^4*(C*d - 3
*B*e) - 3*a^2*e^4*(3*C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(2*a^(3/2)*(
c*d^2 + a*e^2)^4) + (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e))
 - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^4 -
 (e*(a^2*C*e^4 + c^2*(3*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d
^2 - e*(3*B*d - A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^2}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{2a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd\left(\right.\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd\left(\right.\right.}{}$$

$$= -\frac{e\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^2(d+ex)^2} - \frac{e\left(2cCd^3-cde(3Bd-4Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd\left(\right.\right.}{}$$

**Mathematica** [A] time = 0.63, size = 466, normalized size = 0.89

$$\frac{-\log\left(a+cx^2\right)\left(a^2Ce^5-2ace^3\left(e(Ae-3Bd)+4Cd^2\right)+c^2d^2e\left(2e(5Ae-3Bd)+3Cd^2\right)\right)+2\log(d+ex)\left(a^2Ce^5-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^2),x]
```

```
[Out] (-((e*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2) - (2*e*(c*
d^2 + a*e^2)*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + a*e^2*(-2*C*d + B*e)))/(
d + e*x) + ((c*d^2 + a*e^2)*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d^2*x + B
*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(-3*B*d +
 A*e + B*e*x))))/(a*(a + c*x^2)) + (Sqrt[c]*(A*c*d*(c^2*d^4 + 10*a*c*d^2*e^
2 - 15*a^2*e^4) + a*(-2*a*c*d^2*e^2*(7*C*d - 9*B*e) + c^2*d^4*(C*d - 3*B*e)
 - 3*a^2*e^4*(-3*C*d + B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(3/2) + 2*(a^2
*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*
(-3*B*d + 5*A*e)))*Log[d + e*x] - (a^2*C*e^5 - 2*a*c*e^3*(4*C*d^2 + e*(-3*B
*d + A*e)) + c^2*d^2*e*(3*C*d^2 + 2*e*(-3*B*d + 5*A*e)))*Log[a + c*x^2])/(2
*(c*d^2 + a*e^2)^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.19, size = 957, normalized size = 1.83

$$-\frac{\left(3\,Cc^{2}d^{4}e-6\,Bc^{2}d^{3}e^{2}-8\,Cacd^{2}e^{3}+10\,Ac^{2}d^{2}e^{3}+6\,Bacde^{4}+Ca^{2}e^{5}-2\,Aace^{5}\right)\log\left(cx^{2}+a\right)}{2\left(c^{4}d^{8}+4\,ac^{3}d^{6}e^{2}+6\,a^{2}c^{2}d^{4}e^{4}+4\,a^{3}cd^{2}e^{6}+a^{4}e^{8}\right)}+\frac{\left(3\,Cc^{2}d^{4}e^{2}-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 - 8*C*a*c*d^2*e^3 + 10*A*c^2*d^2*e^3
+ 6*B*a*c*d*e^4 + C*a^2*e^5 - 2*A*a*c*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^
3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e
^2 - 6*B*c^2*d^3*e^3 - 8*C*a*c*d^2*e^4 + 10*A*c^2*d^2*e^4 + 6*B*a*c*d*e^5 +
 C*a^2*e^6 - 2*A*a*c*e^6)*log(abs(x*e + d))/(c^4*d^8*e + 4*a*c^3*d^6*e^3 +
6*a^2*c^2*d^4*e^5 + 4*a^3*c*d^2*e^7 + a^4*e^9) + 1/2*(C*a*c^3*d^5 + A*c^4*d
^5 - 3*B*a*c^3*d^4*e - 14*C*a^2*c^2*d^3*e^2 + 10*A*a*c^3*d^3*e^2 + 18*B*a^2
*c^2*d^2*e^3 + 9*C*a^3*c*d*e^4 - 15*A*a^2*c^2*d*e^4 - 3*B*a^3*c*e^5)*arctan
(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4
*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) - 1/2*(B*a*c^3*d^7 + 8*C*a^2*c^2*d^6*e - 3
*A*a*c^3*d^6*e - 9*B*a^2*c^2*d^5*e^2 + 4*C*a^3*c*d^4*e^3 + 7*A*a^2*c^2*d^4*
e^3 - 9*B*a^3*c*d^3*e^4 - 4*C*a^4*d^2*e^5 + 11*A*a^3*c*d^2*e^5 + B*a^4*d*e^
6 + A*a^4*e^7 + (5*C*a*c^3*d^5*e^2 - A*c^4*d^5*e^2 - 9*B*a*c^3*d^4*e^3 - 2*
C*a^2*c^2*d^3*e^4 + 10*A*a*c^3*d^3*e^4 - 6*B*a^2*c^2*d^2*e^5 - 7*C*a^3*c*d*
e^6 + 11*A*a^2*c^2*d*e^6 + 3*B*a^3*c*e^7)*x^3 + (7*C*a*c^3*d^6*e - 2*A*c^4*
d^6*e - 12*B*a*c^3*d^5*e^2 + C*a^2*c^2*d^4*e^3 + 10*A*a*c^3*d^4*e^3 - 12*B*
a^2*c^2*d^3*e^4 - 7*C*a^3*c*d^2*e^5 + 14*A*a^2*c^2*d^2*e^5 - C*a^4*e^7 + 2*
A*a^3*c*e^7)*x^2 + (C*a*c^3*d^7 - A*c^4*d^7 - B*a*c^3*d^6*e + 8*C*a^2*c^2*d
^5*e^2 - 4*A*a*c^3*d^5*e^2 - 12*B*a^2*c^2*d^4*e^3 + C*a^3*c*d^3*e^4 + 7*A*a
^2*c^2*d^3*e^4 - 9*B*a^3*c*d^2*e^5 - 6*C*a^4*d*e^6 + 10*A*a^3*c*d*e^6 + 2*B
*a^4*e^7)*x)/((c*d^2 + a*e^2)^4*(c*x^2 + a)*(x*e + d)^2*a)
```

**maple [B]** time = 0.03, size = 1588, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x)
```

```
[Out] -1/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*A*e^5*a^2-1/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a
)*C*x*d^5+3/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*A*d^4*e-5/(a*e^2+c*d^2)^4*c^2*l
n(c*x^2+a)*A*d^2*e^3+3/(a*e^2+c*d^2)^4*c^2*ln(c*x^2+a)*d^3*e^2*B-3/2/(a*e^2
+c*d^2)^4*c^2*ln(c*x^2+a)*C*d^4*e+1/2/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arcta
n(1/(a*c)^(1/2)*c*x)*C*d^5+1/(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*A*e^5+2*e^3/(a
*e^2+c*d^2)^3/(e*x+d)*C*a*d-2*e/(a*e^2+c*d^2)^3/(e*x+d)*C*c*d^3-2*e^5/(a*e^
2+c*d^2)^4*ln(e*x+d)*A*a*c+10*e^3/(a*e^2+c*d^2)^4*ln(e*x+d)*A*c^2*d^2-6*e^2
/(a*e^2+c*d^2)^4*ln(e*x+d)*B*c^2*d^3+3*e/(a*e^2+c*d^2)^4*ln(e*x+d)*C*c^2*d^
4-4*e^3/(a*e^2+c*d^2)^3/(e*x+d)*A*c*d+3*e^2/(a*e^2+c*d^2)^3/(e*x+d)*B*c*d^2
-1/2*e^3/(a*e^2+c*d^2)^2/(e*x+d)^2*A+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*C*x*a*
d^3*e^2+9/2/(a*e^2+c*d^2)^4*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
*e^4-15/2/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e
^4+9/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e^3-
7/(a*e^2+c*d^2)^4*c^2*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^3*e^2+3/2
/(a*e^2+c*d^2)^4*c/(c*x^2+a)*C*x*a^2*d*e^4-3/2/(a*e^2+c*d^2)^4*c^2/(c*x^2+a
)*A*x*a*d*e^4+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*B*x*a*d^2*e^3-1/(a*e^2+c*d^2)
^4*c^3/(c*x^2+a)*A*x*d^3*e^2+1/2/(a*e^2+c*d^2)^4*c^4/(c*x^2+a)/a*x*A*d^5+3/
2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*B*x*d^4*e+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*A
*d^2*e^3*a+1/(a*e^2+c*d^2)^4*c^2/(c*x^2+a)*d^3*e^2*B*a-3/2/(a*e^2+c*d^2)^4*
c^2/(c*x^2+a)*C*a*d^4*e+6*e^4/(a*e^2+c*d^2)^4*ln(e*x+d)*B*a*c*d-8*e^3/(a*e^
2+c*d^2)^4*ln(e*x+d)*C*a*c*d^2-1/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*B*x*a^2*e^5+
3/2/(a*e^2+c*d^2)^4*c/(c*x^2+a)*d*e^4*B*a^2-1/(a*e^2+c*d^2)^4*c/(c*x^2+a)*C
*a^2*d^2*e^3+1/2/(a*e^2+c*d^2)^4*c^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x
)*A*d^5+5/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^3*e
^2-3/2/(a*e^2+c*d^2)^4*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^4*e-3/
(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*d*e^4*B+4/(a*e^2+c*d^2)^4*c*a*ln(c*x^2+a)*C
*d^2*e^3-3/2/(a*e^2+c*d^2)^4*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*
e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)*C*a^3*e^5-e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*a
+1/2*e^2/(a*e^2+c*d^2)^2/(e*x+d)^2*B*d-1/2*e/(a*e^2+c*d^2)^2/(e*x+d)^2*C*d^
2+e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*a^2*C-1/2/(a*e^2+c*d^2)^4*c^3/(c*x^2+a)*d^5
*B-1/2/(a*e^2+c*d^2)^4*a^2*ln(c*x^2+a)*C*e^5
```

**maxima [B]** time = 1.22, size = 1030, normalized size = 1.97

$$-\frac{\left(3\,Cc^2d^4e-6\,Bc^2d^3e^2+6\,Bacde^4-2\left(4\,Cac-5\,Ac^2\right)d^2e^3+\left(Ca^2-2\,Aac\right)e^5\right)\log\left(cx^2+a\right)}{2\left(c^4d^8+4\,ac^3d^6e^2+6\,a^2c^2d^4e^4+4\,a^3cd^2e^6+a^4e^8\right)}+\frac{\left(3\,Cc^2d^4e-6\,Bc^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*C*c^2*d^4*e - 6*B*c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^
2)*d^2*e^3 + (C*a^2 - 2*A*a*c)*e^5)*log(c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e
```

```
^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8) + (3*C*c^2*d^4*e - 6*B*
c^2*d^3*e^2 + 6*B*a*c*d*e^4 - 2*(4*C*a*c - 5*A*c^2)*d^2*e^3 + (C*a^2 - 2*A*
a*c)*e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a
^3*c*d^2*e^6 + a^4*e^8) - 1/2*(3*B*a*c^3*d^4*e - 18*B*a^2*c^2*d^2*e^3 + 3*B
*a^3*c*e^5 - (C*a*c^3 + A*c^4)*d^5 + 2*(7*C*a^2*c^2 - 5*A*a*c^3)*d^3*e^2 -
3*(3*C*a^3*c - 5*A*a^2*c^2)*d*e^4)*arctan(c*x/sqrt(a*c))/((a*c^4*d^8 + 4*a^
2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(a*c)) -
 1/2*(B*a*c^2*d^5 - 10*B*a^2*c*d^3*e^2 + B*a^3*d*e^4 + A*a^3*e^5 + (8*C*a^2
*c - 3*A*a*c^2)*d^4*e - 2*(2*C*a^3 - 5*A*a^2*c)*d^2*e^3 - (9*B*a*c^2*d^2*e^
3 - 3*B*a^2*c*e^5 - (5*C*a*c^2 - A*c^3)*d^3*e^2 + (7*C*a^2*c - 11*A*a*c^2)*
d*e^4)*x^3 - (12*B*a*c^2*d^3*e^2 - (7*C*a*c^2 - 2*A*c^3)*d^4*e + 6*(C*a^2*c
 - 2*A*a*c^2)*d^2*e^3 + (C*a^3 - 2*A*a^2*c)*e^5)*x^2 - (B*a*c^2*d^4*e + 11*
B*a^2*c*d^2*e^3 - 2*B*a^3*e^5 - (C*a*c^2 - A*c^3)*d^5 - (7*C*a^2*c - 3*A*a*
c^2)*d^3*e^2 + 2*(3*C*a^3 - 5*A*a^2*c)*d*e^4)*x)/(a^2*c^3*d^8 + 3*a^3*c^2*d
^6*e^2 + 3*a^4*c*d^4*e^4 + a^5*d^2*e^6 + (a*c^4*d^6*e^2 + 3*a^2*c^3*d^4*e^4
 + 3*a^3*c^2*d^2*e^6 + a^4*c*e^8)*x^4 + 2*(a*c^4*d^7*e + 3*a^2*c^3*d^5*e^3
+ 3*a^3*c^2*d^3*e^5 + a^4*c*d*e^7)*x^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6
*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*x^2 + 2*(a^2*c^3*d^7*e + 3*a^
3*c^2*d^5*e^3 + 3*a^4*c*d^3*e^5 + a^5*d*e^7)*x)
```

**mupad [B]** time = 14.48, size = 2828, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^2*(d + e*x)^3),x)

[Out] (log(C*c^2*d^7*(-a^3*c)^(3/2) - 3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 +
12*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e +
 72*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x + 39*A*a^2*d*e^6*(-
a^3*c)^(3/2) + 21*C*a^6*d*e^6*(-a^3*c)^(1/2) - 3*B*c^2*d^6*e*(-a^3*c)^(3/2)
 + 12*A*a^2*e^7*x*(-a^3*c)^(3/2) + 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*
c^3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*
d^3*e^4 - 94*C*a^6*c^2*d^4*e^3 - A*a^2*c^4*d^7*(-a^3*c)^(1/2) - 93*B*a^2*d^
2*e^5*(-a^3*c)^(3/2) + 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) + 119*C*a^2*d^3*e^4*(
-a^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^
3*e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^
5*e^2*x - 119*C*a^6*c^2*d^3*e^4*x + 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) + 72*
C*a^2*d^2*e^5*x*(-a^3*c)^(3/2) - 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a
^7*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x - 145*A*a*c*d^3*e
^4*(-a^3*c)^(3/2) + 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) - 51*C*a*c*d^5*e^2*(-a^
3*c)^(3/2) - 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) + 20*C*c^2*d^6*e*x*(-a^3*c)^(3
/2) - 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) + 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/
2) - 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) - 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2)
)*(e^2*(3*B*a^3*c^2*d^3 + (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 - (7*C*a^2*c*d^3
```

*(-a^3*c)^(1/2))/2) + e^3*(4*C*a^4*c*d^2 - 5*A*a^3*c^2*d^2 + (9*B*a^2*c*d^2
*(-a^3*c)^(1/2))/2) - e^4*(3*B*a^4*c*d - (9*C*a^3*d*(-a^3*c)^(1/2))/4 + (15
*A*a^2*c*d*(-a^3*c)^(1/2))/4) - e*((3*C*a^3*c^2*d^4)/2 + (3*B*a*c^2*d^4*(-a
^3*c)^(1/2))/4) - e^5*((C*a^5)/2 + (3*B*a^3*(-a^3*c)^(1/2))/4 - A*a^4*c) +
(A*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 +
a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - (l
og(3*B*a^6*e^7*(-a^3*c)^(1/2) - 6*C*a^8*e^7 - C*c^2*d^7*(-a^3*c)^(3/2) + 12
*A*a^7*c*e^7 - 3*B*a^7*c*e^7*x + 2*A*a^4*c^4*d^6*e + 20*C*a^5*c^3*d^6*e + 7
2*C*a^7*c*d^2*e^5 - A*a^3*c^5*d^7*x - C*a^4*c^4*d^7*x - 39*A*a^2*d*e^6*(-a^
3*c)^(3/2) - 21*C*a^6*d*e^6*(-a^3*c)^(1/2) + 3*B*c^2*d^6*e*(-a^3*c)^(3/2) -
12*A*a^2*e^7*x*(-a^3*c)^(3/2) - 6*C*a^6*e^7*x*(-a^3*c)^(1/2) + 80*A*a^5*c^
3*d^4*e^3 - 102*A*a^6*c^2*d^2*e^5 - 42*B*a^5*c^3*d^5*e^2 + 108*B*a^6*c^2*d^
3*e^4 - 94*C*a^6*c^2*d^4*e^3 + A*a^2*c^4*d^7*(-a^3*c)^(1/2) + 93*B*a^2*d^2*
e^5*(-a^3*c)^(3/2) - 9*A*c^2*d^5*e^2*(-a^3*c)^(3/2) - 119*C*a^2*d^3*e^4*(-a
^3*c)^(3/2) - 42*B*a^7*c*d*e^6 - 9*A*a^4*c^4*d^5*e^2*x + 145*A*a^5*c^3*d^3*
e^4*x - 93*B*a^5*c^3*d^4*e^3*x + 93*B*a^6*c^2*d^2*e^5*x + 51*C*a^5*c^3*d^5*
e^2*x - 119*C*a^6*c^2*d^3*e^4*x - 80*A*c^2*d^4*e^3*x*(-a^3*c)^(3/2) - 72*C*
a^2*d^2*e^5*x*(-a^3*c)^(3/2) + 42*B*c^2*d^5*e^2*x*(-a^3*c)^(3/2) + 21*C*a^7
*c*d*e^6*x - 39*A*a^6*c^2*d*e^6*x + 3*B*a^4*c^4*d^6*e*x + 145*A*a*c*d^3*e^4
*(-a^3*c)^(3/2) - 93*B*a*c*d^4*e^3*(-a^3*c)^(3/2) + 51*C*a*c*d^5*e^2*(-a^3*
c)^(3/2) + 42*B*a^2*d*e^6*x*(-a^3*c)^(3/2) - 20*C*c^2*d^6*e*x*(-a^3*c)^(3/2
) + 102*A*a*c*d^2*e^5*x*(-a^3*c)^(3/2) - 108*B*a*c*d^3*e^4*x*(-a^3*c)^(3/2)
+ 94*C*a*c*d^4*e^3*x*(-a^3*c)^(3/2) + 2*A*a^2*c^4*d^6*e*x*(-a^3*c)^(1/2))*
(e^3*(5*A*a^3*c^2*d^2 - 4*C*a^4*c*d^2 + (9*B*a^2*c*d^2*(-a^3*c)^(1/2))/2) -
e^2*(3*B*a^3*c^2*d^3 - (5*A*a*c^2*d^3*(-a^3*c)^(1/2))/2 + (7*C*a^2*c*d^3*(
-a^3*c)^(1/2))/2) + e^4*(3*B*a^4*c*d + (9*C*a^3*d*(-a^3*c)^(1/2))/4 - (15*A
*a^2*c*d*(-a^3*c)^(1/2))/4) + e*((3*C*a^3*c^2*d^4)/2 - (3*B*a*c^2*d^4*(-a^3
*c)^(1/2))/4) - e^5*((3*B*a^3*(-a^3*c)^(1/2))/4 - (C*a^5)/2 + A*a^4*c) + (A
*c^3*d^5*(-a^3*c)^(1/2))/4 + (C*a*c^2*d^5*(-a^3*c)^(1/2))/4))/(a^7*e^8 + a^
3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4) - ((A*
a^2*e^5 + B*c^2*d^5 + B*a^2*d*e^4 - 3*A*c^2*d^4*e - 4*C*a^2*d^2*e^3 + 8*C*a
*c*d^4*e + 10*A*a*c*d^2*e^3 - 10*B*a*c*d^3*e^2)/(2*(a*e^2 + c*d^2)*(a^2*e^4
+ c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*B*a^2*c*e^5 - A*c^3*d^3*e^2 - 9*B*a*
c^2*d^2*e^3 + 5*C*a*c^2*d^3*e^2 + 11*A*a*c^2*d*e^4 - 7*C*a^2*c*d*e^4))/(2*a
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (x*(A*c^3*d^5 -
2*B*a^3*e^5 - C*a*c^2*d^5 + 6*C*a^3*d*e^4 + 3*A*a*c^2*d^3*e^2 + 11*B*a^2*c
*d^2*e^3 - 7*C*a^2*c*d^3*e^2 - 10*A*a^2*c*d*e^4 + B*a*c^2*d^4*e))/(2*a*(a*e
^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) - (x^2*(C*a^3*e^5 - 2*A*a^
2*c*e^5 + 2*A*c^3*d^4*e - 12*A*a*c^2*d^2*e^3 + 12*B*a*c^2*d^3*e^2 + 6*C*a^2
*c*d^2*e^3 - 7*C*a*c^2*d^4*e))/(2*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))/(a*d^2 + x^2*(a*e^2 + c*d^2) + c*e^2*x^4 + 2*a*d*e*x + 2*c*d
*e*x^3) + (log(d + e*x)*(c^2*(10*A*d^2*e^3 - 6*B*d^3*e^2 + 3*C*d^4*e) - c*(
2*A*a*e^5 - 6*B*a*d*e^4 + 8*C*a*d^2*e^3) + C*a^2*e^5))/(a^4*e^8 + c^4*d^8 +
4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

## 3.57 $\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=209

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+3Acd)+3ae^2(aBe+3aCd+Acd)\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac\right.\right.}{8a^2c^2\left(a+cx\right.}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*e*(3*A*c*d+
3*B*a*e+5*C*a*d)-(3*A*c^2*d^2-a*(4*a*C*e^2-c*d*(3*B*e+C*d)))*x)/a^2/c^2/(c*
x^2+a)+1/8*(3*A*c*d*(a*e^2+c*d^2)+a*(3*a*e^2*(B*e+3*C*d)+c*d^2*(3*B*e+C*d))
)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)+1/2*C*e^3*ln(c*x^2+a)/c^3
```

**Rubi [A]** time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1645, 819, 635, 205, 260}

$$-\frac{(d+ex)\left(ae(3aBe+5aCd+3Acd)-x\left(3Ac^2d^2-a\left(4aCe^2-cd(3Be+Cd)\right)\right)\right)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd^2(3aBe+aCd+\right.}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(4*a*c*(a + c*x^2)^2) - ((d + e*x)*(a*
e*(3*A*c*d + 5*a*C*d + 3*a*B*e) - (3*A*c^2*d^2 - a*(4*a*C*e^2 - c*d*(C*d +
3*B*e)))*x))/(8*a^2*c^2*(a + c*x^2)) + ((3*a*e^2*(A*c*d + 3*a*C*d + a*B*e)
+ c*d^2*(3*A*c*d + a*C*d + 3*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2
)*c^(5/2)) + (C*e^3*Log[a + c*x^2])/(2*c^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3 \left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3} dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)^2(-3Acd-aCd-3aBe-4aCex)}{\left(a+cx^2\right)^2} dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{4ac\left(a+cx^2\right)^2} - \frac{(d+ex)\left(ae(3Acd+5aCd+3aBe)-\left(3Ac^2d^2\right.\right.}{8a^2c^2\left(a+cx^2\right)}$$

**Mathematica [A]** time = 0.25, size = 281, normalized size = 1.34

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(3Acd\left(ae^2+cd^2\right)+a\left(3ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{a^{5/2}} + \frac{-2a^3Ce^3+2a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))-2ac^2d\left(3Ae(d+ex)+Bd(d+3ex)+\right.}{a\left(a+cx^2\right)^2}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((-2*a^3*C*e^3 + 2*A*c^3*d^3*x - 2*a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d
*(d + 3*e*x)) + 2*a^2*c*e*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x)))/(a*(
a + c*x^2)^2) + (8*a^3*C*e^3 + 3*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d +
A*e))*x - a^2*c*e*(3*C*d*(4*d + 5*e*x) + e*(12*B*d + 4*A*e + 5*B*e*x)))/(a^
2*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c*d^2 + a*e^2) + a*(3*a*e^2*(3*C*d + B*
e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 4*C*e^3*L
og[a + c*x^2])/(8*c^3)
```

**fricas [B]** time = 1.43, size = 1138, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 + 12*(C*a^4*c + A*a^3*c^2)*d^2*e
 - 4*(3*C*a^5 - A*a^4*c)*e^3 - 2*(3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*
```

```
a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 8*(3*C*
a^3*c^2*d^2*e + 3*B*a^3*c^2*d*e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 + (3*B
*a^3*c*d^2*e + 3*B*a^4*e^3 + (3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3
+ 3*A*c^4)*d^3 + 3*(3*C*a^2*c^2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*
c^2)*d^3 + 3*(3*C*a^4 + A*a^3*c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e
^3 + (C*a^2*c^2 + 3*A*a*c^3)*d^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sq
rt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(3*B*a^3*c^2*d^2
*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3*C*a^4*c + A*a^3*c
^2)*d*e^2)*x - 8*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 + C*a^5*e^3)*log(c*
x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^3 +
6*B*a^4*c*d*e^2 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e - 2*(3*C*a^5 - A*a^4*c)*e^3
 - (3*B*a^2*c^3*d^2*e - 5*B*a^3*c^2*e^3 + (C*a^2*c^3 + 3*A*a*c^4)*d^3 - 3*(
5*C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 4*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d*
e^2 - (2*C*a^4*c - A*a^3*c^2)*e^3)*x^2 - (3*B*a^3*c*d^2*e + 3*B*a^4*e^3 + (
3*B*a*c^3*d^2*e + 3*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 + 3*(3*C*a^2*c^
2 + A*a*c^3)*d*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^3 + 3*(3*C*a^4 + A*a^3*
c)*d*e^2 + 2*(3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 + 3*A*a*c^3)*d
^3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a)
+ (3*B*a^3*c^2*d^2*e + 3*B*a^4*c*e^3 + (C*a^3*c^2 - 5*A*a^2*c^3)*d^3 + 3*(3
*C*a^4*c + A*a^3*c^2)*d*e^2)*x - 4*(C*a^3*c^2*e^3*x^4 + 2*C*a^4*c*e^3*x^2 +
 C*a^5*e^3)*log(c*x^2 + a))/(a^3*c^5*x^4 + 2*a^4*c^4*x^2 + a^5*c^3)]
```

**giac [A]** time = 0.27, size = 348, normalized size = 1.67

$$\frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3}+\frac{\left(Cacd^3+3\,Ac^2d^3+3\,Bacd^2e+9\,Ca^2de^2+3\,Aacde^2+3\,Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}+\frac{\left(Cac^2d^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 + 1/8*(C*a*c*d^3 + 3*A*c^2*d^3 + 3*B*a*c*d^2*e
 + 9*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt
(a*c)*a^2*c^2) + 1/8*((C*a*c^2*d^3 + 3*A*c^3*d^3 + 3*B*a*c^2*d^2*e - 15*C*a
^2*c*d*e^2 + 3*A*a*c^2*d*e^2 - 5*B*a^2*c*e^3)*x^3 - 4*(3*C*a^2*c*d^2*e + 3*
B*a^2*c*d*e^2 - 2*C*a^3*e^3 + A*a^2*c*e^3)*x^2 - (C*a^2*c*d^3 - 5*A*a*c^2*d
^3 + 3*B*a^2*c*d^2*e + 9*C*a^3*d*e^2 + 3*A*a^2*c*d*e^2 + 3*B*a^3*e^3)*x - 2
*(B*a^2*c^2*d^3 + 3*C*a^3*c*d^2*e + 3*A*a^2*c^2*d^2*e + 3*B*a^3*c*d*e^2 - 3
*C*a^4*e^3 + A*a^3*c*e^3)/c)/((c*x^2 + a)^2*a^2*c^2)
```

**maple [B]** time = 0.01, size = 402, normalized size = 1.92

$$\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{3A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2}+\frac{3B\,d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{3B\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;c^2}+\frac{C\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{9C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*a*c*d*e^2+3*A*c^2*d^3-5*B*a^2*e^3+3*B*a*c*d^2*e-15*C*a^2*d*e^2+C*
a*c*d^3)/a^2/c*x^3-1/2*e*(A*c*e^2+3*B*c*d*e-2*C*a*e^2+3*C*c*d^2)/c^2*x^2-1/
8*(3*A*a*c*d*e^2-5*A*c^2*d^3+3*B*a^2*e^3+3*B*a*c*d^2*e+9*C*a^2*d*e^2+C*a*c*
d^3)/a/c^2*x-1/4*(A*a*c*e^3+3*A*c^2*d^2*e+3*B*a*c*d*e^2+B*c^2*d^3-3*C*a^2*e
^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^2+1/2*C*e^3*ln(c*x^2+a)/c^3+3/8/a/c/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2+3/8/a^2/(a*c)^(1/2)*arctan(1/(a*c)^
(1/2)*c*x)*A*d^3+3/8/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^3+3/8/a/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e+9/8/c^2/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*C*d*e^2+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*
d^3
```

**maxima [A]** time = 1.00, size = 379, normalized size = 1.81

$$\frac{Ce^3\log\left(cx^2+a\right)}{2\,c^3}-\frac{2\,Ba^2c^2d^3+6\,Ba^3cde^2+6\left(Ca^3c+Aa^2c^2\right)d^2e-2\left(3\,Ca^4-Aa^3c\right)e^3-\left(3\,Bac^3d^2e-5\,Ba^2c^2e^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*C*e^3*log(c*x^2 + a)/c^3 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 + 6*(
C*a^3*c + A*a^2*c^2)*d^2*e - 2*(3*C*a^4 - A*a^3*c)*e^3 - (3*B*a*c^3*d^2*e -
 5*B*a^2*c^2*e^3 + (C*a*c^3 + 3*A*c^4)*d^3 - 3*(5*C*a^2*c^2 - A*a*c^3)*d*e^
2)*x^3 + 4*(3*C*a^2*c^2*d^2*e + 3*B*a^2*c^2*d*e^2 - (2*C*a^3*c - A*a^2*c^2)
*e^3)*x^2 + (3*B*a^2*c^2*d^2*e + 3*B*a^3*c*e^3 + (C*a^2*c^2 - 5*A*a*c^3)*d^
3 + 3*(3*C*a^3*c + A*a^2*c^2)*d*e^2)*x)/(a^2*c^5*x^4 + 2*a^3*c^4*x^2 + a^4*
c^3) + 1/8*(3*B*a*c*d^2*e + 3*B*a^2*e^3 + (C*a*c + 3*A*c^2)*d^3 + 3*(3*C*a^
2 + A*a*c)*d*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2)
```

**mupad [B]** time = 1.77, size = 920, normalized size = 4.40

$$\frac{5\,A\,d^3\,x}{8\,\left(a^3+2\,a^2\,c\,x^2+a\,c^2\,x^4\right)}-\frac{B\,d^3}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+\frac{3\,C\,a^2\,e^3}{4\,\left(a^2\,c^3+2\,a\,c^4\,x^2+c^5\,x^4\right)}-\frac{3\,A\,d^2\,e}{4\,\left(a^2\,c+2\,a\,c^2\,x^2+c^3\,x^4\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (5*A*d^3*x)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (B*d^3)/(4*(a^2*c + c^3*x
^4 + 2*a*c^2*x^2)) + (3*C*a^2*e^3)/(4*(a^2*c^3 + c^5*x^4 + 2*a*c^4*x^2)) -
(3*A*d^2*e)/(4*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*d^3*x^3)/(8*(a^3 + 2*a
```

```
^2*c*x^2 + a*c^2*x^4)) - (C*d^3*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (A
*a*e^3)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) - (A*e^3*x^2)/(2*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) - (5*B*e^3*x^3)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) +
(C*e^3*log(a + c*x^2))/(2*c^3) - (3*B*a*d*e^2)/(4*(a^2*c^2 + c^4*x^4 + 2*a*
c^3*x^2)) - (3*C*a*d^2*e)/(4*(a^2*c^2 + c^4*x^4 + 2*a*c^3*x^2)) + (3*A*c*d^
3*x^3)/(8*(a^4 + 2*a^3*c*x^2 + a^2*c^2*x^4)) - (3*B*a*e^3*x)/(8*(a^2*c^2 +
c^4*x^4 + 2*a*c^3*x^2)) - (3*B*d*e^2*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2
)) - (3*C*d^2*e*x^2)/(2*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (15*C*d*e^2*x^3)
/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) + (C*a*e^3*x^2)/(a^2*c^2 + c^4*x^4 + 2
*a*c^3*x^2) + (3*A*d^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*c^(1/2)) + (3*
B*e^3*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) + (C*d^3*atan((c^(1/2)
*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (3*A*d*e^2*x^3)/(8*(a^3 + 2*a^2*c*x^2 +
 a*c^2*x^4)) + (3*B*d^2*e*x^3)/(8*(a^3 + 2*a^2*c*x^2 + a*c^2*x^4)) - (3*A*d
*e^2*x)/(8*(a^2*c + c^3*x^4 + 2*a*c^2*x^2)) - (3*B*d^2*e*x)/(8*(a^2*c + c^3
*x^4 + 2*a*c^2*x^2)) + (3*A*d*e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(
3/2)) + (3*B*d^2*e*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(3/2)*c^(3/2)) + (9*C*d*
e^2*atan((c^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*c^(5/2)) - (9*C*a*d*e^2*x)/(8*(a^
2*c^2 + c^4*x^4 + 2*a*c^3*x^2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

## 3.58 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^3}\,dx$

**Optimal**. Leaf size=156

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd)+ae^2(3aC+Ac)\right)}{8a^{5/2}c^{5/2}}-\frac{(d+ex)(ae(3aC+Ac)-cx(2aBe+aCd+3Acd))}{8a^2c^2\left(a+cx^2\right)}-\frac{(d\cdots}{}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^2-1/8*(e*x+d)*(a*(A*c+3*C*a)
*e-c*(3*A*c*d+2*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(a*(A*c+3*C*a)*e^2+c*
d*(3*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(5/2)
```

**Rubi [A]** time = 0.23, antiderivative size = 175, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1645, 778, 205}

$$-\frac{x\left(ae^2(3aC+Ac)-cd(2aBe+aCd+3Acd)\right)+2ae(aBe+2aCd+2Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+3Acd\cdots\right.}{8a^{5/2}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(4*a*c*(a + c*x^2)^2) - (2*a*e*(2*A*c*
d + 2*a*C*d + a*B*e) + (a*(A*c + 3*a*C)*e^2 - c*d*(3*A*c*d + a*C*d + 2*a*B*
e))*x)/(8*a^2*c^2*(a + c*x^2)) + ((a*(A*c + 3*a*C)*e^2 + c*d*(3*A*c*d + a*C
*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{\int \frac{(d+ex)(-3Acd-aCd-2aBe-(Ac+3aC)ex)}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^2 \cdots\right.}{8a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{4ac\left(a+cx^2\right)^2} - \frac{2ae(2Acd+2aCd+aBe)+\left(a(Ac+3aC)e^2 \cdots\right.}{8a^2c^2\left(a+cx^2\right)}$$

**Mathematica [A]** time = 0.14, size = 211, normalized size = 1.35

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+3cd^2\right)+a\left(3aCe^2+cd(2Be+Cd)\right)\right)}{8a^{5/2}c^{5/2}}+\frac{a^2(-e)(4Be+8Cd+5Cex)+acx\left(e(Ae+2Bd)+\cdots\right.}{8a^2c^2\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(8*C*d + 4*B*e + 5
*C*e*x))/(8*a^2*c^2*(a + c*x^2)) + (A*c^2*d^2*x + a^2*e*(2*C*d + B*e + C*e*
x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(4*a*c^2*(a + c*x^2
)^2) + ((A*c*(3*c*d^2 + a*e^2) + a*(3*a*C*e^2 + c*d*(C*d + 2*B*e)))*ArcTan[
(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(5/2))
```

**fricas [B]** time = 1.22, size = 806, normalized size = 5.17

$$\left[-\frac{4\,Ba^3c^2d^2+4\,Ba^4ce^2-2\left(2\,Ba^2c^3de+\left(Ca^2c^3+3\,Aac^4\right)d^2-\left(5\,Ca^3c^2-Aa^2c^3\right)e^2\right)x^3+8\left(Ca^4c+Aa^3c^2\right)de+8}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c^2*d^2 + 4*B*a^4*c*e^2 - 2*(2*B*a^2*c^3*d*e + (C*a^2*c^3 +
 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 8*(C*a^4*c + A*a^3*c
^2)*d*e + 8*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 + (2*B*a^3*c*d*e + (2*B*a
*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*c^3)*e^2)*x^4 + (C*
a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(2*B*a^2*c^2*d*e + (
C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2)*x^2)*sqrt(-a*c)*l
og((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(2*B*a^3*c^2*d*e + (C*a^3*
c^2 - 5*A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a
^4*c^4*x^2 + a^5*c^3), -1/8*(2*B*a^3*c^2*d^2 + 2*B*a^4*c*e^2 - (2*B*a^2*c^3
*d*e + (C*a^2*c^3 + 3*A*a*c^4)*d^2 - (5*C*a^3*c^2 - A*a^2*c^3)*e^2)*x^3 + 4
*(C*a^4*c + A*a^3*c^2)*d*e + 4*(2*C*a^3*c^2*d*e + B*a^3*c^2*e^2)*x^2 - (2*B
*a^3*c*d*e + (2*B*a*c^3*d*e + (C*a*c^3 + 3*A*c^4)*d^2 + (3*C*a^2*c^2 + A*a*
c^3)*e^2)*x^4 + (C*a^3*c + 3*A*a^2*c^2)*d^2 + (3*C*a^4 + A*a^3*c)*e^2 + 2*(
2*B*a^2*c^2*d*e + (C*a^2*c^2 + 3*A*a*c^3)*d^2 + (3*C*a^3*c + A*a^2*c^2)*e^2
)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (2*B*a^3*c^2*d*e + (C*a^3*c^2 - 5*
A*a^2*c^3)*d^2 + (3*C*a^4*c + A*a^3*c^2)*e^2)*x)/(a^3*c^5*x^4 + 2*a^4*c^4*x
^2 + a^5*c^3)]
```

**giac [A]** time = 0.16, size = 254, normalized size = 1.63

$$\frac{\left(Cacd^2+3\,Ac^2d^2+2\,Bacde+3\,Ca^2e^2+Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c^2}+\frac{Cac^2d^2x^3+3\,Ac^3d^2x^3+2\,Bac^2dx^3e-5\,Ca^2cx^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c*d^2 + 3*A*c^2*d^2 + 2*B*a*c*d*e + 3*C*a^2*e^2 + A*a*c*e^2)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c^2) + 1/8*(C*a*c^2*d^2*x^3 + 3*A*c^3*d^2*x
^3 + 2*B*a*c^2*d*x^3*e - 5*C*a^2*c*x^3*e^2 + A*a*c^2*x^3*e^2 - 8*C*a^2*c*d*
x^2*e - C*a^2*c*d^2*x + 5*A*a*c^2*d^2*x - 4*B*a^2*c*x^2*e^2 - 2*B*a^2*c*d*x
*e - 2*B*a^2*c*d^2 - 3*C*a^3*x*e^2 - A*a^2*c*x*e^2 - 4*C*a^3*d*e - 4*A*a^2*
c*d*e - 2*B*a^3*e^2)/((c*x^2 + a)^2*a^2*c^2)
```

**maple [A]** time = 0.01, size = 283, normalized size = 1.81

$$\frac{A e^{2} \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\ ac}+\frac{3A d^{2} \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\ a^{2}}+\frac{Bde \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\ ac}+\frac{C d^{2} \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\ ac}+\frac{3C e^{2} \arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\ c^{2}}+\frac{-\frac{(Be+2C}{2c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(A*a*c*e^2+3*A*c^2*d^2+2*B*a*c*d*e-5*C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/
2*e*(B*e+2*C*d)*x^2/c-1/8*(A*a*c*e^2-5*A*c^2*d^2+2*B*a*c*d*e+3*C*a^2*e^2+C*
a*c*d^2)/a/c^2*x-1/4*(2*A*c*d*e+B*a*e^2+B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^2
+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^2+3/8/a^2/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x)*A*d^2+1/4/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)
*B*d*e+3/8/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^2+1/8/a/c/(a*c)^(1
/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^2
```

**maxima [A]** time = 0.99, size = 253, normalized size = 1.62

$$-\frac{2\,Ba^{2}cd^{2}+2\,Ba^{3}e^{2}-\left(2\,Bac^{2}de+\left(Cac^{2}+3\,Ac^{3}\right)d^{2}-\left(5\,Ca^{2}c-Aac^{2}\right)e^{2}\right)x^{3}+4\left(Ca^{3}+Aa^{2}c\right)de+4\left(2\,Ca^{2}cd\right.}{8\left(a^{2}c^{4}x^{4}+2\,a^{3}c^{3}x^{2}+a^{4}c^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*B*a^2*c*d^2 + 2*B*a^3*e^2 - (2*B*a*c^2*d*e + (C*a*c^2 + 3*A*c^3)*d^
2 - (5*C*a^2*c - A*a*c^2)*e^2)*x^3 + 4*(C*a^3 + A*a^2*c)*d*e + 4*(2*C*a^2*c
*d*e + B*a^2*c*e^2)*x^2 + (2*B*a^2*c*d*e + (C*a^2*c - 5*A*a*c^2)*d^2 + (3*C
*a^3 + A*a^2*c)*e^2)*x)/(a^2*c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(2*B*
a*c*d*e + (C*a*c + 3*A*c^2)*d^2 + (3*C*a^2 + A*a*c)*e^2)*arctan(c*x/sqrt(a*
c))/(sqrt(a*c)*a^2*c^2)
```

**mupad [B]** time = 3.96, size = 230, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(3\,C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+3\,A\,c^2\,d^2\right)}{8\,a^{5/2}\,c^{5/2}}-\frac{\frac{B\,a\,e^2+B\,c\,d^2+2\,A\,c\,d\,e+2\,C\,a\,d\,e}{4\,c^2}+\frac{x^2\left(B\,e^2+2\,C\,d\,e\right)}{2\,c}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c^2*d^2 + 3*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d
^2 + 2*B*a*c*d*e))/(8*a^(5/2)*c^(5/2)) - ((B*a*e^2 + B*c*d^2 + 2*A*c*d*e +
```

```
2*C*a*d*e)/(4*c^2) + (x^2*(B*e^2 + 2*C*d*e))/(2*c) + (x*(3*C*a^2*e^2 - 5*A*
c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a*c^2) - (x^3*(3*A*c^2*d
^2 - 5*C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(8*a^2*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2)
```

**sympy [B]** time = 141.18, size = 391, normalized size = 2.51

$$-\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2+2Bacde+3Ca^2e^2+Cacd^2\right)\log\left(-a^3c^2\sqrt{-\frac{1}{a^5c^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^5}}\left(Aace^2+3Ac^2d^2+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*
e**2 + C*a*c*d**2)*log(-a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + sqrt(-1/(a
**5*c**5))*(A*a*c*e**2 + 3*A*c**2*d**2 + 2*B*a*c*d*e + 3*C*a**2*e**2 + C*a*
c*d**2)*log(a**3*c**2*sqrt(-1/(a**5*c**5)) + x)/16 + (-4*A*a**2*c*d*e - 2*B
*a**3*e**2 - 2*B*a**2*c*d**2 - 4*C*a**3*d*e + x**3*(A*a*c**2*e**2 + 3*A*c**
3*d**2 + 2*B*a*c**2*d*e - 5*C*a**2*c*e**2 + C*a*c**2*d**2) + x**2*(-4*B*a**
2*c*e**2 - 8*C*a**2*c*d*e) + x*(-A*a**2*c*e**2 + 5*A*a*c**2*d**2 - 2*B*a**2
*c*d*e - 3*C*a**3*e**2 - C*a**2*c*d**2))/(8*a**4*c**2 + 16*a**3*c**3*x**2 +
 8*a**2*c**4*x**4)
```

## 3.59 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=130

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

```
[Out] -1/4*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^2+1/8*(-2*a*(A*c+C*a)*e+c*(3*A
*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)+1/8*(3*A*c*d+B*a*e+C*a*d)*arctan(x*c
^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1645, 639, 205}

$$-\frac{2ae(aC+Ac)-cx(aBe+aCd+3Acd)}{8a^2c^2\left(a+cx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^{3/2}}-\frac{(d+ex)(aB-x(Ac-aC))}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(4*a*c*(a + c*x^2)^2) - (2*a*(A*c + a*C)
*e - c*(3*A*c*d + a*C*d + a*B*e)*x)/(8*a^2*c^2*(a + c*x^2)) + ((3*A*c*d + a
*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^3}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3Acd-a(Cd+Be)-2(Ac+aC)ex}{\left(a+cx^2\right)^2}\,dx}{4ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}+\frac{(3Acd}{}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{4ac\left(a+cx^2\right)^2}-\frac{2a(Ac+aC)e-c(3Acd+aCd+aBe)x}{8a^2c^2\left(a+cx^2\right)}+\frac{(3Acd}{}$$

**Mathematica [A]** time = 0.10, size = 137, normalized size = 1.05

$$\frac{\frac{\sqrt{a}\left(-4a^2Ce+acx(Be+Cd)+3Ac^2dx\right)}{a+cx^2}+\frac{2a^{3/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^2}+\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+3Acd)}{8a^{5/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-4*a^2*C*e + 3*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2) + (2*
a^(3/2)*(a^2*C*e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2
)^2 + Sqrt[c]*(3*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(
5/2)*c^2)
```

**fricas [A]** time = 1.20, size = 470, normalized size = 3.62

$$\left[-\frac{8\,Ca^3cex^2+4\,Ba^3cd-2\left(Ba^2c^2e+\left(Ca^2c^2+3\,Aac^3\right)d\right)x^3+\left(Ba^3e+\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^4+2\left(Ba^2ce+\right.\right.}{16\left(a^3c^4x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^3*c*e*x^2 + 4*B*a^3*c*d - 2*(B*a^2*c^2*e + (C*a^2*c^2 + 3*A*a
*c^3)*d)*x^3 + (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*a^
2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(-a*c)*lo
g((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 4*(C*a^4 + A*a^3*c)*e + 2*(B*
a^3*c*e + (C*a^3*c - 5*A*a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*
c^2), -1/8*(4*C*a^3*c*e*x^2 + 2*B*a^3*c*d - (B*a^2*c^2*e + (C*a^2*c^2 + 3*A
*a*c^3)*d)*x^3 - (B*a^3*e + (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x^4 + 2*(B*
a^2*c*e + (C*a^2*c + 3*A*a*c^2)*d)*x^2 + (C*a^3 + 3*A*a^2*c)*d)*sqrt(a*c)*a
rctan(sqrt(a*c)*x/a) + 2*(C*a^4 + A*a^3*c)*e + (B*a^3*c*e + (C*a^3*c - 5*A*
a^2*c^2)*d)*x)/(a^3*c^4*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

**giac [A]** time = 0.19, size = 152, normalized size = 1.17

$$\frac{\left(Cad+3\,Acd+Bae\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^{2}c}+\frac{Cac^{2}dx^{3}+3\,Ac^{3}dx^{3}+Bac^{2}x^{3}e-4\,Ca^{2}cx^{2}e-Ca^{2}cdx+5\,Aac^{2}dx-Ba^{2}cx}{8\left(cx^{2}+a\right)^{2}a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*d + 3*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8
*(C*a*c^2*d*x^3 + 3*A*c^3*d*x^3 + B*a*c^2*x^3*e - 4*C*a^2*c*x^2*e - C*a^2*c
*d*x + 5*A*a*c^2*d*x - B*a^2*c*x*e - 2*B*a^2*c*d - 2*C*a^3*e - 2*A*a^2*c*e)
/((c*x^2 + a)^2*a^2*c^2)
```

**maple [A]** time = 0.01, size = 157, normalized size = 1.21

$$\frac{3Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^{2}}+\frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac}+\frac{-\frac{Ce\,x^{2}}{2c}+\frac{(3Acd+Bae+Cad)x^{3}}{8a^{2}}+\frac{(5Acd-Bae-Cad)x}{8ac}-\frac{Ace+Bcd+a}{4c^{2}}}{\left(c\,x^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c*d+B*a*e+C*a*d)/a^2*x^3-1/2*C/c*e*x^2+1/8*(5*A*c*d-B*a*e-C*a*d)/
a/c*x-1/4*(A*c*e+B*c*d+C*a*e)/c^2)/(c*x^2+a)^2+3/8/a^2/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*A*d+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/8
/a/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d
```

**maxima [A]** time = 0.98, size = 160, normalized size = 1.23

$$-\frac{4\,Ca^2cex^2+2\,Ba^2cd-\left(Bac^2e+\left(Cac^2+3\,Ac^3\right)d\right)x^3+2\left(Ca^3+Aa^2c\right)e+\left(Ba^2ce+\left(Ca^2c-5\,Aac^2\right)d\right)x}{8\left(a^2c^4x^4+2\,a^3c^3x^2+a^4c^2\right)}+\frac{(Bae\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a^2*c*e*x^2 + 2*B*a^2*c*d - (B*a*c^2*e + (C*a*c^2 + 3*A*c^3)*d)*x
^3 + 2*(C*a^3 + A*a^2*c)*e + (B*a^2*c*e + (C*a^2*c - 5*A*a*c^2)*d)*x)/(a^2*
c^4*x^4 + 2*a^3*c^3*x^2 + a^4*c^2) + 1/8*(B*a*e + (C*a + 3*A*c)*d)*arctan(c
*x/sqrt(a*c))/(sqrt(a*c)*a^2*c)
```

**mupad [B]** time = 0.15, size = 128, normalized size = 0.98

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^{5/2}\,c^{3/2}}-\frac{\frac{A\,c\,e+B\,c\,d+C\,a\,e}{4\,c^2}-\frac{x^3\,\left(3\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{8\,a^2}+\frac{C\,e\,x^2}{2\,c}+\frac{x\,\left(B\,a\,e-5\,A\,c\,d+C\,a\,d\right)}{8\,a\,c}}{a^2+2\,a\,c\,x^2+c^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^3,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(3*A*c*d + B*a*e + C*a*d))/(8*a^(5/2)*c^(3/2)) -
 ((A*c*e + B*c*d + C*a*e)/(4*c^2) - (x^3*(3*A*c*d + B*a*e + C*a*d))/(8*a^2)
 + (C*e*x^2)/(2*c) + (x*(B*a*e - 5*A*c*d + C*a*d))/(8*a*c))/(a^2 + c^2*x^4
+ 2*a*c*x^2)
```

**sympy [A]** time = 32.42, size = 240, normalized size = 1.85

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Acd+Bae+Cad)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}}+x\right)}{16}+\frac{-2A\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(-a**3*c*sqrt(-1/(a**5*c
**3)) + x)/16 + sqrt(-1/(a**5*c**3))*(3*A*c*d + B*a*e + C*a*d)*log(a**3*c*s
qrt(-1/(a**5*c**3)) + x)/16 + (-2*A*a**2*c*e - 2*B*a**2*c*d - 2*C*a**3*e -
4*C*a**2*c*e*x**2 + x**3*(3*A*c**3*d + B*a*c**2*e + C*a*c**2*d) + x*(5*A*a*
c**2*d - B*a**2*c*e - C*a**2*c*d))/(8*a**4*c**2 + 16*a**3*c**3*x**2 + 8*a**
2*c**4*x**4)
```

# 3.60 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3} dx$

**Optimal**. Leaf size=98

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

```
[Out] 1/4*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^2+1/8*(3*A*c+C*a)*x/a^2/c/(c*x^2+a)+1/
8*(3*A*c+C*a)*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/c^(3/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{x(aC+3Ac)}{8a^2c\left(a+cx^2\right)}-\frac{aB-x(Ac-aC)}{4ac\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(4*a*c*(a + c*x^2)^2) + ((3*A*c + a*C)*x)/(8*a^2*c*(
a + c*x^2)) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^3}\,dx &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{c}}{\left(a+cx^2\right)^2}\,dx}{4a}\\ &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{4ac}\\ &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\int \frac{1}{a+cx^2}\,dx}{8a^2c}\\ &= -\frac{aB-(Ac-aC)x}{4ac\left(a+cx^2\right)^2}+\frac{(3Ac+aC)x}{8a^2c\left(a+cx^2\right)}+\frac{(3Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 90, normalized size = 0.92

$$\frac{(aC+3Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8a^{5/2}c^{3/2}}+\frac{-a^2(2B+Cx)+acx\left(5A+Cx^2\right)+3Ac^2x^3}{8a^2c\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^3,x]
```

```
[Out] (3*A*c^2*x^3 - a^2*(2*B + C*x) + a*c*x*(5*A + C*x^2))/(8*a^2*c*(a + c*x^2)^
2) + ((3*A*c + a*C)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*c^(3/2))
```

**fricas** [A] time = 1.26, size = 314, normalized size = 3.20

$$\left[-\frac{4\,Ba^3c - 2\left(Ca^2c^2 + 3\,Aac^3\right)x^3 + \left(\left(Cac^2 + 3\,Ac^3\right)x^4 + Ca^3 + 3\,Aa^2c + 2\left(Ca^2c + 3\,Aac^2\right)x^2\right)\sqrt{-ac}\,\log\left(\frac{cx^2-2}{c}\right.}{16\left(a^3c^4x^4 + 2\,a^4c^3x^2 + a^5c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*B*a^3*c - 2*(C*a^2*c^2 + 3*A*a*c^3)*x^3 + ((C*a*c^2 + 3*A*c^3)*x^
4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)*x^2)*sqrt(-a*c)*log((c*x^2
- 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 2*(C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4*
x^4 + 2*a^4*c^3*x^2 + a^5*c^2), -1/8*(2*B*a^3*c - (C*a^2*c^2 + 3*A*a*c^3)*x
^3 - ((C*a*c^2 + 3*A*c^3)*x^4 + C*a^3 + 3*A*a^2*c + 2*(C*a^2*c + 3*A*a*c^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + (C*a^3*c - 5*A*a^2*c^2)*x)/(a^3*c^4
*x^4 + 2*a^4*c^3*x^2 + a^5*c^2)]
```

**giac** [A] time = 0.16, size = 84, normalized size = 0.86

$$\frac{(Ca + 3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c} + \frac{Cacx^3 + 3\,Ac^2x^3 - Ca^2x + 5\,Aacx - 2\,Ba^2}{8\left(cx^2 + a\right)^2a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2*c) + 1/8*(C*a*c*x^3
+ 3*A*c^2*x^3 - C*a^2*x + 5*A*a*c*x - 2*B*a^2)/((c*x^2 + a)^2*a^2*c)
```

**maple** [A] time = 0.01, size = 96, normalized size = 0.98

$$\frac{3A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2} + \frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;ac} + \frac{\frac{(3Ac+aC)x^3}{8a^2} - \frac{B}{4c} + \frac{(5Ac-aC)x}{8ac}}{\left(c\,x^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^3,x)
```

```
[Out] (1/8*(3*A*c+C*a)/a^2*x^3+1/8*(5*A*c-C*a)/a/c*x-1/4*B/c)/(c*x^2+a)^2+3/8/a^2
/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A+1/8/a/c/(a*c)^(1/2)*arctan(1/(a*c)
^(1/2)*c*x)*C
```

**maxima [A]** time = 0.97, size = 98, normalized size = 1.00

$$\frac{(Cac + 3\,Ac^2)x^3 - 2\,Ba^2 - (Ca^2 - 5\,Aac)x}{8\,(a^2c^3x^4 + 2\,a^3c^2x^2 + a^4c)} + \frac{(Ca + 3\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((C*a*c + 3*A*c^2)*x^3 - 2*B*a^2 - (C*a^2 - 5*A*a*c)*x)/(a^2*c^3*x^4 +
2*a^3*c^2*x^2 + a^4*c) + 1/8*(C*a + 3*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)
*a^2*c)
```

**mupad [B]** time = 3.84, size = 88, normalized size = 0.90

$$\frac{\frac{x^3\,(3\,A\,c+C\,a)}{8\,a^2} - \frac{B}{4\,c} + \frac{x\,(5\,A\,c-C\,a)}{8\,a\,c}}{a^2 + 2\,a\,c\,x^2 + c^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(3\,A\,c + C\,a)}{8\,a^{5/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^3,x)
```

```
[Out] ((x^3*(3*A*c + C*a))/(8*a^2) - B/(4*c) + (x*(5*A*c - C*a))/(8*a*c))/(a^2 +
c^2*x^4 + 2*a*c*x^2) + (atan((c^(1/2)*x)/a^(1/2))*(3*A*c + C*a))/(8*a^(5/2)
*c^(3/2))
```

**sympy [A]** time = 1.23, size = 156, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac + Ca)\log\left(-a^3c\sqrt{-\frac{1}{a^5c^3}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5c^3}}\,(3Ac + Ca)\log\left(a^3c\sqrt{-\frac{1}{a^5c^3}} + x\right)}{16} + \frac{-2Ba^2 + x^3\left(3Ac^2 + C\right.}{8a^4c + 16a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(-a**3*c*sqrt(-1/(a**5*c**3)) + x)/1
6 + sqrt(-1/(a**5*c**3))*(3*A*c + C*a)*log(a**3*c*sqrt(-1/(a**5*c**3)) + x)
/16 + (-2*B*a**2 + x**3*(3*A*c**2 + C*a*c) + x*(5*A*a*c - C*a**2))/(8*a**4*
c + 16*a**3*c**2*x**2 + 8*a**2*c**3*x**4)
```

# 3.61 $\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=353

$$\frac{4a^2e\left(Ae^2-Bde+Cd^2\right)+x\left(Acd\left(7ae^2+3cd^2\right)+a\left(cd^2-3ae^2\right)(Cd-Be)\right)}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd\right.\right.}{}$$

```
[Out] 1/4*(-a*(-A*c*e+B*c*d+C*a*e)+c*(A*c*d+B*a*e-C*a*d)*x)/a/c/(a*e^2+c*d^2)/(c*
x^2+a)^2+1/8*(4*a^2*e*(A*e^2-B*d*e+C*d^2)+(a*(-B*e+C*d)*(-3*a*e^2+c*d^2)+A*
c*d*(7*a*e^2+3*c*d^2))*x)/a^2/(a*e^2+c*d^2)^2/(c*x^2+a)+e^3*(A*e^2-B*d*e+C*
d^2)*ln(e*x+d)/(a*e^2+c*d^2)^3-1/2*e^3*(A*e^2-B*d*e+C*d^2)*ln(c*x^2+a)/(a*e
^2+c*d^2)^3+1/8*(a*(-B*e+C*d)*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+A*c*d*(15*
a^2*e^4+10*a*c*d^2*e^2+3*c^2*d^4))*arctan(x*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2
+c*d^2)^3/c^(1/2)
```

**Rubi [A]** time = 0.73, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1647, 823, 801, 635, 205, 260}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(15a^2e^4+10acd^2e^2+3c^2d^4\right)+a\left(-3a^2e^4+6acd^2e^2+c^2d^4\right)(Cd-Be)\right)}{8a^{5/2}\sqrt{c}\left(ae^2+cd^2\right)^3}+\frac{4a^2e\left(Ae^2-Bde+C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] -(a*(B*c*d - A*c*e + a*C*e) - c*(A*c*d - a*C*d + a*B*e)*x)/(4*a*c*(c*d^2 +
a*e^2)*(a + c*x^2)^2) + (4*a^2*e*(C*d^2 - B*d*e + A*e^2) + (a*(C*d - B*e)*(
c*d^2 - 3*a*e^2) + A*c*d*(3*c*d^2 + 7*a*e^2))*x)/(8*a^2*(c*d^2 + a*e^2)^2*(
a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2 - 3*a^2*e^4) + A*c*d
*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8
*a^(5/2)*Sqrt[c]*(c*d^2 + a*e^2)^3) + (e^3*(C*d^2 - B*d*e + A*e^2)*Log[d +
e*x])/(c*d^2 + a*e^2)^3 - (e^3*(C*d^2 - B*d*e + A*e^2)*Log[a + c*x^2])/(2*(
c*d^2 + a*e^2)^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)\left(a+cx^2\right)^3}\,dx = -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{\int \frac{-\frac{c\left(ad(Cd-Be)+A\left(3cd^2+4ae^2\right)\right)}{cd^2+ae^2}-\frac{3ce(Acd-aCd+aB}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)^2}}{4ac}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-\right.}{8a^2\left(cd^2+\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-\right.}{8a^2\left(cd^2+\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-\right.}{8a^2\left(cd^2+\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-\right.}{8a^2\left(cd^2+\right.}$$

$$= -\frac{a(Bcd-Ace+aCe)-c(Acd-aCd+aBe)x}{4ac\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{4a^2e\left(Cd^2-Bde+Ae^2\right)+\left(a(Cd-\right.}{8a^2\left(cd^2+\right.}$$

**Mathematica [A]** time = 0.42, size = 321, normalized size = 0.91

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2(-C)e+ac(Ae-Bd+Bex-Cdx)+Ac^2dx\right)}{ac\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^2e(e(4Ae-4Bd+3Bex)+Cd(4d-3ex))+acdx\left(e(7Ae-Bd)+Cd^2\right)+3Ac^2d^3x\right)}{a^2\left(a+cx^2\right)}+\frac{\tan}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)*(a + c*x^2)^3),x]
```

```
[Out] ((2*(c*d^2 + a*e^2)^2*(-(a^2*C*e) + A*c^2*d*x + a*c*(-(B*d) + A*e - C*d*x +
 B*e*x)))/(a*c*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*A*c^2*d^3*x + a*c*d*(C*
d^2 + e*(-(B*d) + 7*A*e))*x + a^2*e*(C*d*(4*d - 3*e*x) + e*(-4*B*d + 4*A*e
+ 3*B*e*x))))/(a^2*(a + c*x^2)) + ((a*(C*d - B*e)*(c^2*d^4 + 6*a*c*d^2*e^2
- 3*a^2*e^4) + A*c*d*(3*c^2*d^4 + 10*a*c*d^2*e^2 + 15*a^2*e^4))*ArcTan[(Sqr
t[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[
```

```
d + e*x] - 4*e^3*(C*d^2 + e*(-(B*d) + A*e))*Log[a + c*x^2])/(8*(c*d^2 + a*e
^2)^3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.18, size = 715, normalized size = 2.03

$$-\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)} + \frac{\left(Cd^2e^4 - Bde^5 + Ae^6\right)\log\left(|xe + d|\right)}{c^3d^6e + 3\,ac^2d^4e^3 + 3\,a^2cd^2e^5 + a^3e^7} + \frac{\left(Cac^2d^5 + 3\,Ac^3d^5 - Bac^2d^4e + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^4 - B*d*e^5 + A*e^6)*log(abs(x*e
+ d))/(c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7) + 1/8*(C*a*
c^2*d^5 + 3*A*c^3*d^5 - B*a*c^2*d^4*e + 6*C*a^2*c*d^3*e^2 + 10*A*a*c^2*d^3*
e^2 - 6*B*a^2*c*d^2*e^3 - 3*C*a^3*d*e^4 + 15*A*a^2*c*d*e^4 + 3*B*a^3*e^5)*a
rctan(c*x/sqrt(a*c))/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 +
a^5*e^6)*sqrt(a*c)) - 1/8*(2*B*a^2*c^3*d^5 - 2*C*a^3*c^2*d^4*e - 2*A*a^2*c^
3*d^4*e + 8*B*a^3*c^2*d^3*e^2 - 8*A*a^3*c^2*d^2*e^3 + 6*B*a^4*c*d*e^4 + 2*C
*a^5*e^5 - 6*A*a^4*c*e^5 - (C*a*c^4*d^5 + 3*A*c^5*d^5 - B*a*c^4*d^4*e - 2*C
*a^2*c^3*d^3*e^2 + 10*A*a*c^4*d^3*e^2 + 2*B*a^2*c^3*d^2*e^3 - 3*C*a^3*c^2*d
*e^4 + 7*A*a^2*c^3*d*e^4 + 3*B*a^3*c^2*e^5)*x^3 - 4*(C*a^2*c^3*d^4*e - B*a^
2*c^3*d^3*e^2 + C*a^3*c^2*d^2*e^3 + A*a^2*c^3*d^2*e^3 - B*a^3*c^2*d*e^4 + A
*a^3*c^2*e^5)*x^2 + (C*a^2*c^3*d^5 - 5*A*a*c^4*d^5 - B*a^2*c^3*d^4*e + 6*C*
a^3*c^2*d^3*e^2 - 14*A*a^2*c^3*d^3*e^2 - 6*B*a^3*c^2*d^2*e^3 + 5*C*a^4*c*d*
e^4 - 9*A*a^3*c^2*d*e^4 - 5*B*a^4*c*e^5)*x)/((c*d^2 + a*e^2)^3*(c*x^2 + a)^
2*a^2*c)
```

**maple [B]** time = 0.02, size = 1598, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x)
```

```
[Out] 5/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*a^2*e^5-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2
*C*x*c^2*d^5-3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*d*e^4*B*a^2+3/8/(a*e^2+c*d^2)^
3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^5+1/4/(a*e^2+c*d^2)^3/(c*x^2+
a)^2*c^2*A*d^4*e-1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2/c*C*a^3*e^5+e^5/(a*e^2+c*d
^2)^3*ln(e*x+d)*A-1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*A*e^5+5/4/(a*e^2+c*d^2)^3
/(c*x^2+a)^2*c^3/a*x^3*A*d^3*e^2-1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*
B*d^4*e+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^2*a*c*d^2*e^3+5/4/(a*e^2+c*d^2)
^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^3*e^2-1/8/(a*e^2+c*d^2)^
3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^4*e+9/8/(a*e^2+c*d^2)^3/(
c*x^2+a)^2*A*x*a*c*d*e^4+3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*a*c*d^2*e^3-3/
4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x*a*c*d^3*e^2-3/8/(a*e^2+c*d^2)^3/(c*x^2+a)
^2*C*x^3*a*c*d*e^4-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^2*a*c*d*e^4-3/8/(a*e
^2+c*d^2)^3*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^4+1/8/(a*e^2+c*d^
2)^3/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^5+3/4/(a*e^2+c*d^2)^3/
(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c*d^3*e^2+15/8/(a*e^2+c*d^2)^3/(a*c
)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*d*e^4-3/4/(a*e^2+c*d^2)^3/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x)*B*c*d^2*e^3-1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^
2*c^2*d^3*e^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*C*x^2*c^2*d^4*e+7/8/(a*e^2+c*
d^2)^3/(c*x^2+a)^2*A*x^3*c^2*d*e^4+3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*a*
c*e^5+1/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x^3*c^2*d^2*e^3-1/4/(a*e^2+c*d^2)^3
/(c*x^2+a)^2*C*x^3*c^2*d^3*e^2+1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c^3/a*x^3*C*
d^5+5/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2/a*x*A*c^3*d^5+1/(a*e^2+c*d^2)^3/(c*x^2+
a)^2*c*A*d^2*e^3*a+1/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*B*x*c^2*d^4*e-5/8/(a*e^2
+c*d^2)^3/(c*x^2+a)^2*C*x*a^2*d*e^4+7/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x*c^2
*d^3*e^2+1/2/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*x^2*a*c*e^5+1/2/(a*e^2+c*d^2)^3/
(c*x^2+a)^2*A*x^2*c^2*d^2*e^3-1/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*d^3*e^2*B*a+1
/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*c*C*a*d^4*e+3/8/(a*e^2+c*d^2)^3/(c*x^2+a)^2*
c^4/a^2*x^3*A*d^5+3/8/(a*e^2+c*d^2)^3/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*
c*x)*A*c^3*d^5+3/4/(a*e^2+c*d^2)^3/(c*x^2+a)^2*A*e^5*a^2-1/4/(a*e^2+c*d^2)^
3/(c*x^2+a)^2*c^2*d^5*B+1/2/(a*e^2+c*d^2)^3*ln(c*x^2+a)*d*e^4*B-1/2/(a*e^2+
c*d^2)^3*ln(c*x^2+a)*C*d^2*e^3-e^4/(a*e^2+c*d^2)^3*ln(e*x+d)*B*d+e^3/(a*e^2
+c*d^2)^3*ln(e*x+d)*C*d^2
```

**maxima [A]** time = 1.10, size = 655, normalized size = 1.86

$$-\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(cx^2 + a\right)}{2\left(c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6\right)}+\frac{\left(Cd^2e^3 - Bde^4 + Ae^5\right)\log\left(ex + d\right)}{c^3d^6 + 3\,ac^2d^4e^2 + 3\,a^2cd^2e^4 + a^3e^6}-\frac{\left(Bac^2d^4e + 6\,Ba^2cd^2e^3 - 3\,Ba^3e^5 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(C*d^2*e^3 - B*d*e^4 + A*e^5)*log(c*x^2 + a)/(c^3*d^6 + 3*a*c^2*d^4*e^
2 + 3*a^2*c*d^2*e^4 + a^3*e^6) + (C*d^2*e^3 - B*d*e^4 + A*e^5)*log(e*x + d)
/(c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + a^3*e^6) - 1/8*(B*a*c^2*d^4
```

```
*e + 6*B*a^2*c*d^2*e^3 - 3*B*a^3*e^5 - (C*a*c^2 + 3*A*c^3)*d^5 - 2*(3*C*a^2
*c + 5*A*a*c^2)*d^3*e^2 + 3*(C*a^3 - 5*A*a^2*c)*d*e^4)*arctan(c*x/sqrt(a*c)
)/((a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^2*d^3 + 6*B*a^3*c*d*e^2 - 2*(C*a^3*c + A*a^2*c^2)*d^2*e +
 2*(C*a^4 - 3*A*a^3*c)*e^3 + (B*a*c^3*d^2*e - 3*B*a^2*c^2*e^3 - (C*a*c^3 +
3*A*c^4)*d^3 + (3*C*a^2*c^2 - 7*A*a*c^3)*d*e^2)*x^3 - 4*(C*a^2*c^2*d^2*e -
B*a^2*c^2*d*e^2 + A*a^2*c^2*e^3)*x^2 - (B*a^2*c^2*d^2*e + 5*B*a^3*c*e^3 - (
C*a^2*c^2 - 5*A*a*c^3)*d^3 - (5*C*a^3*c - 9*A*a^2*c^2)*d*e^2)*x)/(a^4*c^3*d
^4 + 2*a^5*c^2*d^2*e^2 + a^6*c*e^4 + (a^2*c^5*d^4 + 2*a^3*c^4*d^2*e^2 + a^4
*c^3*e^4)*x^4 + 2*(a^3*c^4*d^4 + 2*a^4*c^3*d^2*e^2 + a^5*c^2*e^4)*x^2)
```

**mupad [B]** time = 9.90, size = 2392, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)),x)
```

```
[Out] ((x^2*(A*c*e^3 - B*c*d*e^2 + C*c*d^2*e))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*
e^2)) - (B*c^2*d^3 + C*a^2*e^3 - 3*A*a*c*e^3 - A*c^2*d^2*e + 3*B*a*c*d*e^2
- C*a*c*d^2*e)/(4*c*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3
+ 5*B*a^2*e^3 - C*a*c*d^3 - 5*C*a^2*d*e^2 + 9*A*a*c*d*e^2 + B*a*c*d^2*e))/(
8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 3*B*a^2*c*e^
3 + C*a*c^2*d^3 + 7*A*a*c^2*d*e^2 - B*a*c^2*d^2*e - 3*C*a^2*c*d*e^2))/(8*a^
2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2 + c^2*x^4 + 2*a*c*x^2) - (log(
3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5*c)^(1/2) - 24*A*a^6*c*e^7 +
3*B*a^6*c*e^7*x + 6*A*a^3*c^4*d^6*e + 2*C*a^4*c^3*d^6*e - 30*C*a^6*c*d^2*e^
5 - 3*A*a^2*c^5*d^7*x - C*a^3*c^4*d^7*x + C*a*c^3*d^7*(-a^5*c)^(1/2) + 3*C*
a^4*d*e^6*(-a^5*c)^(1/2) + 20*A*a^4*c^3*d^4*e^3 + 54*A*a^5*c^2*d^2*e^5 - 2*
B*a^4*c^3*d^5*e^2 - 36*B*a^5*c^2*d^3*e^4 + 36*C*a^5*c^2*d^4*e^3 + 30*B*a^6*
c*d*e^6 - 7*A*a^3*c^4*d^5*e^2*x - 5*A*a^4*c^3*d^3*e^4*x + 5*B*a^4*c^3*d^4*e
^3*x - 57*B*a^5*c^2*d^2*e^5*x - 5*C*a^4*c^3*d^5*e^2*x + 57*C*a^5*c^2*d^3*e^
4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*a^3*c*d^2*e^5*(-a^5*c)^(1/2)
- 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) - 3*C*a^6*c*d*e^6*x + 5*A*a^2*c^2*d^3*e
^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*c)^(1/2) + 5*C*a^2*c^2*d^5*e^
2*(-a^5*c)^(1/2) + 63*A*a^5*c^2*d*e^6*x + B*a^3*c^4*d^6*e*x - 63*A*a^3*c*d*
e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1/2) - 24*A*a^3*c*e^7*x*(-a^5*
c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54*A*a^2*c^2*d^2*e^5*x*(-a^5*c)
^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2) + 36*C*a^2*c^2*d^4*e^3*x*(-a
^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2) + 2*C*a*c^3*d^6*e*x*(-a^5*c
)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2) - 2*B*a*c^3*d^5*e^2*x*(-a^5*c
)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))*(c*(a^2*((3*C*d^3*e^2*(-a^5*
c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 + (15*A*d*e^4*(-a^5*c)^(1/2))/
16) + a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^4)/2)) + a^3*((3*B*e^5*(-a^5*
c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16) + a*c^2*((C*d^5*(-a^5*c)^(1/2
```

```
))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^4*e*(-a^5*c)^(1/2))/16) + (3*
A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^5*c^4*d^6 + 3*a^6*c^3*d^4*e^2
 + 3*a^7*c^2*d^2*e^4) + (log(3*A*c^4*d^7*(-a^5*c)^(1/2) - 3*B*a^4*e^7*(-a^5
*c)^(1/2) + 24*A*a^6*c*e^7 - 3*B*a^6*c*e^7*x - 6*A*a^3*c^4*d^6*e - 2*C*a^4*
c^3*d^6*e + 30*C*a^6*c*d^2*e^5 + 3*A*a^2*c^5*d^7*x + C*a^3*c^4*d^7*x + C*a*
c^3*d^7*(-a^5*c)^(1/2) + 3*C*a^4*d*e^6*(-a^5*c)^(1/2) - 20*A*a^4*c^3*d^4*e^
3 - 54*A*a^5*c^2*d^2*e^5 + 2*B*a^4*c^3*d^5*e^2 + 36*B*a^5*c^2*d^3*e^4 - 36*
C*a^5*c^2*d^4*e^3 - 30*B*a^6*c*d*e^6 + 7*A*a^3*c^4*d^5*e^2*x + 5*A*a^4*c^3*
d^3*e^4*x - 5*B*a^4*c^3*d^4*e^3*x + 57*B*a^5*c^2*d^2*e^5*x + 5*C*a^4*c^3*d^
5*e^2*x - 57*C*a^5*c^2*d^3*e^4*x + 7*A*a*c^3*d^5*e^2*(-a^5*c)^(1/2) + 57*B*
a^3*c*d^2*e^5*(-a^5*c)^(1/2) - 57*C*a^3*c*d^3*e^4*(-a^5*c)^(1/2) + 3*C*a^6*
c*d*e^6*x + 5*A*a^2*c^2*d^3*e^4*(-a^5*c)^(1/2) - 5*B*a^2*c^2*d^4*e^3*(-a^5*
c)^(1/2) + 5*C*a^2*c^2*d^5*e^2*(-a^5*c)^(1/2) - 63*A*a^5*c^2*d*e^6*x - B*a^
3*c^4*d^6*e*x - 63*A*a^3*c*d*e^6*(-a^5*c)^(1/2) - B*a*c^3*d^6*e*(-a^5*c)^(1
/2) - 24*A*a^3*c*e^7*x*(-a^5*c)^(1/2) + 6*A*c^4*d^6*e*x*(-a^5*c)^(1/2) + 54
*A*a^2*c^2*d^2*e^5*x*(-a^5*c)^(1/2) - 36*B*a^2*c^2*d^3*e^4*x*(-a^5*c)^(1/2)
 + 36*C*a^2*c^2*d^4*e^3*x*(-a^5*c)^(1/2) + 30*B*a^3*c*d*e^6*x*(-a^5*c)^(1/2
) + 2*C*a*c^3*d^6*e*x*(-a^5*c)^(1/2) + 20*A*a*c^3*d^4*e^3*x*(-a^5*c)^(1/2)
- 2*B*a*c^3*d^5*e^2*x*(-a^5*c)^(1/2) - 30*C*a^3*c*d^2*e^5*x*(-a^5*c)^(1/2))
*(c*(a^2*((3*C*d^3*e^2*(-a^5*c)^(1/2))/8 - (3*B*d^2*e^3*(-a^5*c)^(1/2))/8 +
 (15*A*d*e^4*(-a^5*c)^(1/2))/16) - a^5*((A*e^5)/2 + (C*d^2*e^3)/2 - (B*d*e^
4)/2)) + a^3*((3*B*e^5*(-a^5*c)^(1/2))/16 - (3*C*d*e^4*(-a^5*c)^(1/2))/16)
+ a*c^2*((C*d^5*(-a^5*c)^(1/2))/16 + (5*A*d^3*e^2*(-a^5*c)^(1/2))/8 - (B*d^
4*e*(-a^5*c)^(1/2))/16) + (3*A*c^3*d^5*(-a^5*c)^(1/2))/16))/(a^8*c*e^6 + a^
5*c^4*d^6 + 3*a^6*c^3*d^4*e^2 + 3*a^7*c^2*d^2*e^4) + (e^3*log(d + e*x)*(A*e
^2 + C*d^2 - B*d*e))/(a*e^2 + c*d^2)^3
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

## 3.62 $\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3} dx$

**Optimal**. Leaf size=571

$$-\frac{4a^2e\left(ae^2(2Cd-Be)-cd\left(2Cd^2-e(3Bd-4Ae)\right)\right)-x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2(6\cdots\right.\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

```
[Out] -e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)+1/4*(-a*(-2*A*c*d*e-B*a*e^
2+B*c*d^2+2*C*a*d*e)+(A*c*(-a*e^2+c*d^2)+a*(a*C*e^2-c*d*(-2*B*e+C*d)))*x)/a
/(a*e^2+c*d^2)^2/(c*x^2+a)^2+1/8*(-4*a^2*e*(a*e^2*(-B*e+2*C*d)-c*d*(2*C*d^2
-e*(-4*A*e+3*B*d)))+(A*c*(-7*a^2*e^4+12*a*c*d^2*e^2+3*c^2*d^4)+a*(3*a^2*C*e
^4-2*a*c*d*e^2*(-7*B*e+6*C*d)+c^2*d^3*(-2*B*e+C*d)))*x)/a^2/(a*e^2+c*d^2)^3
/(c*x^2+a)-e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))*ln(e*x+d
)/(a*e^2+c*d^2)^4+1/2*e^3*(a*e^2*(-B*e+2*C*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)
))*ln(c*x^2+a)/(a*e^2+c*d^2)^4+1/8*(3*A*c*(-5*a^3*e^6+15*a^2*c*d^2*e^4+5*a*
c^2*d^4*e^2+c^3*d^6)+a*(3*a^3*C*e^6+a*c^2*d^3*e^2*(-20*B*e+13*C*d)-3*a^2*c*
d*e^4*(-10*B*e+11*C*d)+c^3*d^5*(-2*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(
5/2)/(a*e^2+c*d^2)^4/c^(1/2)
```

**Rubi [A]** time = 1.92, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{x\left(Ac\left(-7a^2e^4+12acd^2e^2+3c^2d^4\right)+a\left(3a^2Ce^4-2acde^2(6Cd-7Be)+c^2d^3(Cd-2Be)\right)\right)+4a^2e\left(-ae^2(2Cd-Be\cdots\right.}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] -((e^3*(C*d^2 - B*d*e + A*e^2))/((c*d^2 + a*e^2)^3*(d + e*x))) - (a*(B*c*d^
2 - 2*A*c*d*e + 2*a*C*d*e - a*B*e^2) - (A*c*(c*d^2 - a*e^2) + a*(a*C*e^2 -
c*d*(C*d - 2*B*e)))*x)/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^2)^2) + (4*a^2*e*(2*
c*C*d^3 - c*d*e*(3*B*d - 4*A*e) - a*e^2*(2*C*d - B*e)) + (A*c*(3*c^2*d^4 +
12*a*c*d^2*e^2 - 7*a^2*e^4) + a*(3*a^2*C*e^4 - 2*a*c*d*e^2*(6*C*d - 7*B*e)
+ c^2*d^3*(C*d - 2*B*e)))*x)/(8*a^2*(c*d^2 + a*e^2)^3*(a + c*x^2)) + ((3*A*
c*(c^3*d^6 + 5*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e
^6 + a*c^2*d^3*e^2*(13*C*d - 20*B*e) - 3*a^2*c*d*e^4*(11*C*d - 10*B*e) + c^
3*d^5*(C*d - 2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c]*(c*d^
2 + a*e^2)^4) + (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*
```

```
e))*Log[d + e*x])/(c*d^2 + a*e^2)^4 - (e^3*(4*c*C*d^3 - c*d*e*(5*B*d - 6*A*
e) - a*e^2*(2*C*d - B*e))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^4)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(aCe^2-cd(Cd-2Be)\right)\right)x}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{\left(cd^2+ae^2\right)^3(d+ex)} - \frac{a\left(Bcd^2-2Acde+2aCde-aBe^2\right)-\left(Ac\left(cd^2-ae^2\right)+a\left(a\right.\right.}{4a\left(cd^2+ae^2\right)^2\left(a+cx^2\right)^2}$$

**Mathematica [A]** time = 0.76, size = 498, normalized size = 0.87

$$\frac{\frac{2\left(ae^2+cd^2\right)^2\left(a^2e(Be-2Cd+Cex)-ac\left(Ae(ex-2d)+Bd(d-2ex)+Cd^2x\right)+Ac^2d^2x\right)}{a\left(a+cx^2\right)^2}+\frac{\left(ae^2+cd^2\right)\left(a^3e^3(4Be-8Cd+3Cex)+a^2ce\left(e(Ae(16d-7ex)-2Bd(6d-7ex)\right.\right.}{a^2\left(a+cx^2\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(a + c*x^2)^3),x]
```

```
[Out] ((-8*e^3*(c*d^2 + a*e^2)*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x) + (2*(c*d^2
+ a*e^2)^2*(A*c^2*d^2*x + a^2*e*(-2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + B*d
*(d - 2*e*x) + A*e*(-2*d + e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(3*
A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 2*e*(-(B*d) + 6*A*e))*x + a^3*e^3*(-8*C*d
+ 4*B*e + 3*C*e*x) + a^2*c*e*(4*C*d^2*(2*d - 3*e*x) + e*(-2*B*d*(6*d - 7*e*
```

```
x) + A*e*(16*d - 7*e*x)))))/(a^2*(a + c*x^2)) + ((3*A*c*(c^3*d^6 + 5*a*c^2*
d^4*e^2 + 15*a^2*c*d^2*e^4 - 5*a^3*e^6) + a*(3*a^3*C*e^6 + a*c^2*d^3*e^2*(1
3*C*d - 20*B*e) + c^3*d^5*(C*d - 2*B*e) + 3*a^2*c*d*e^4*(-11*C*d + 10*B*e))
)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[c]) + 8*e^3*(4*c*C*d^3 + c*d*e
*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[d + e*x] - 4*e^3*(4*c*C*d^3 +
 c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e))*Log[a + c*x^2])/(8*(c*d^2 +
 a*e^2)^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.24, size = 1107, normalized size = 1.94

$$\frac{\left(Cac^3d^6e^2 + 3\,Ac^4d^6e^2 - 2\,Bac^3d^5e^3 + 13\,Ca^2c^2d^4e^4 + 15\,Aac^3d^4e^4 - 20\,Ba^2c^2d^3e^5 - 33\,Ca^3cd^2e^6 + 45\,Aa^2c^2d^2\cdots\right.}{8\left(a^2c^4d^8 + 4\,a^3c^3d^6e^2 + 6\,a^4c^2d^4e^4 + 4\,a^5cd^2e^6 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a*c^3*d^6*e^2 + 3*A*c^4*d^6*e^2 - 2*B*a*c^3*d^5*e^3 + 13*C*a^2*c^2*d
^4*e^4 + 15*A*a*c^3*d^4*e^4 - 20*B*a^2*c^2*d^3*e^5 - 33*C*a^3*c*d^2*e^6 + 4
5*A*a^2*c^2*d^2*e^6 + 30*B*a^3*c*d*e^7 + 3*C*a^4*e^8 - 15*A*a^3*c*e^8)*arct
an((c*d - c*d^2/(x*e + d) - a*e^2/(x*e + d))*e^(-1)/sqrt(a*c))*e^(-2)/((a^2
*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^
8)*sqrt(a*c)) - 1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 - 2*C*a*d*e^5 + 6*A*c*d*
e^5 + B*a*e^6)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d
)^2)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4
*e^8) - (C*d^2*e^9/(x*e + d) - B*d*e^10/(x*e + d) + A*e^11/(x*e + d))/(c^3*
d^6*e^6 + 3*a*c^2*d^4*e^8 + 3*a^2*c*d^2*e^10 + a^3*e^12) + 1/8*(C*a*c^4*d^5
*e + 3*A*c^5*d^5*e - 2*B*a*c^4*d^4*e^2 - 22*C*a^2*c^3*d^3*e^3 + 14*A*a*c^4*
d^3*e^3 + 32*B*a^2*c^3*d^2*e^4 + 17*C*a^3*c^2*d*e^5 - 29*A*a^2*c^3*d*e^5 -
6*B*a^3*c^2*e^6 - (3*C*a*c^4*d^6*e^2 + 9*A*c^5*d^6*e^2 - 6*B*a*c^4*d^5*e^3
- 77*C*a^2*c^3*d^4*e^4 + 41*A*a*c^4*d^4*e^4 + 116*B*a^2*c^3*d^3*e^5 + 77*C*
a^3*c^2*d^2*e^6 - 121*A*a^2*c^3*d^2*e^6 - 38*B*a^3*c^2*d*e^7 - 3*C*a^4*c*e^
8 + 7*A*a^3*c^2*e^8)*e^(-1)/(x*e + d) + (3*C*a*c^4*d^7*e^3 + 9*A*c^5*d^7*e^
```

```
3 - 6*B*a*c^4*d^6*e^4 - 89*C*a^2*c^3*d^5*e^5 + 45*A*a*c^4*d^5*e^5 + 140*B*a
^2*c^3*d^4*e^6 + 85*C*a^3*c^2*d^3*e^7 - 145*A*a^2*c^3*d^3*e^7 - 22*B*a^3*c^
2*d^2*e^8 + 17*C*a^4*c*d*e^9 - 21*A*a^3*c^2*d*e^9 - 8*B*a^4*c*e^10)*e^(-2)/
(x*e + d)^2 - (C*a*c^4*d^8*e^4 + 3*A*c^5*d^8*e^4 - 2*B*a*c^4*d^7*e^5 - 34*C
*a^2*c^3*d^6*e^6 + 18*A*a*c^4*d^6*e^6 + 58*B*a^2*c^3*d^5*e^7 + 20*C*a^3*c^2
*d^4*e^8 - 60*A*a^2*c^3*d^4*e^8 + 26*B*a^3*c^2*d^3*e^9 + 50*C*a^4*c*d^2*e^1
0 - 66*A*a^3*c^2*d^2*e^10 - 34*B*a^4*c*d*e^11 - 5*C*a^5*e^12 + 9*A*a^4*c*e^
12)*e^(-3)/(x*e + d)^3)/((c*d^2 + a*e^2)^4*a^2*(c - 2*c*d/(x*e + d) + c*d^2
/(x*e + d)^2 + a*e^2/(x*e + d)^2)^2)
```

**maple [B]** time = 0.03, size = 2159, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x)
```

```
[Out] -3/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*d*A*e^5+1/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*C*
d*e^5+3/8/(a*e^2+c*d^2)^4*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^6-3
/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^3*d*e^5+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*
A*c^3*d^5*e+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*a^3*C*e^6*x-1/8/(a*e^2+c*d^2)^4
/(c*x^2+a)^2*C*c^3*d^6*x+5/2/(a*e^2+c*d^2)^4*c*ln(c*x^2+a)*d^2*e^4*B-2/(a*e
^2+c*d^2)^4*c*ln(c*x^2+a)*C*d^3*e^3+6*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*A*c*d-5
*e^4/(a*e^2+c*d^2)^4*ln(e*x+d)*B*c*d^2-2*e^5/(a*e^2+c*d^2)^4*ln(e*x+d)*C*a*
d+4*e^3/(a*e^2+c*d^2)^4*ln(e*x+d)*C*c*d^3-e^5/(a*e^2+c*d^2)^3/(e*x+d)*A-7/8
/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^2*e^4*x-13/8/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*C*a*c^2*d^4*e^2*x+7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^3*a*c^2*d*e^5-9/
8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*a*c^2*d^2*e^4+2/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*A*x^2*a*c^2*d*e^5-1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*a*c^2*d^2*e^4-1/
(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*a^2*c*d*e^5+15/8/(a*e^2+c*d^2)^4/a/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^3*d^4*e^2+15/4/(a*e^2+c*d^2)^4*a/(a*c)^
(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c*d*e^5-1/4/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)
*arctan(1/(a*c)^(1/2)*c*x)*B*c^3*d^5*e-33/8/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*C*c*d^2*e^4+15/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a
*x^3*A*d^4*e^2-1/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*B*d^5*e+3/8/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^2*e^4*x+9/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*d*a
^2*c*B*e^5*x+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^3*e^3*x+e^6/(a*e^2+c
*d^2)^4*ln(e*x+d)*B*a+e^4/(a*e^2+c*d^2)^3/(e*x+d)*B*d-e^3/(a*e^2+c*d^2)^3/(
e*x+d)*C*d^2+3/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a^3*e^6-1/4/(a*e^2+c*d^2)^4/
(c*x^2+a)^2*B*c^3*d^6-1/2/(a*e^2+c*d^2)^4*a*ln(c*x^2+a)*e^6*B+45/8/(a*e^2+c
*d^2)^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^2*d^2*e^4-5/2/(a*e^2+c*d^
2)^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*c^2*d^3*e^3+13/8/(a*e^2+c*d^2)
^4/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^2*d^4*e^2+3/2/(a*e^2+c*d^2)^4/
(c*x^2+a)^2*B*x^3*c^3*d^3*e^3-11/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^3*c^3*d^
4*e^2+2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^2*c^3*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(
```

```
c*x^2+a)^2*B*x^2*a^2*c*e^6-3/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*x^2*c^3*d^4*e^
2+1/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^2*c^3*d^5*e-9/8/(a*e^2+c*d^2)^4/(c*x^2+
a)^2*A*a^2*c*e^6*x+17/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*c^3*d^4*e^2*x+1/4/(a*
e^2+c*d^2)^4/(c*x^2+a)^2*B*c^3*d^5*e*x+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*x^
3*a^2*c*e^6-7/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*x^3*a*c^2*e^6+5/8/(a*e^2+c*d^
2)^4/(c*x^2+a)^2*A*x^3*c^3*d^2*e^4+5/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a^2*c*
d*e^5+3/(a*e^2+c*d^2)^4/(c*x^2+a)^2*A*a*c^2*d^3*e^3-3/4/(a*e^2+c*d^2)^4/(c*
x^2+a)^2*B*a^2*c*d^2*e^4-7/4/(a*e^2+c*d^2)^4/(c*x^2+a)^2*B*a*c^2*d^4*e^2-1/
(a*e^2+c*d^2)^4/(c*x^2+a)^2*C*a^2*c*d^3*e^3+1/2/(a*e^2+c*d^2)^4/(c*x^2+a)^2
*C*a*c^2*d^5*e+3/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2*c^5/a^2*x^3*A*d^6+1/8/(a*e^2
+c*d^2)^4/(c*x^2+a)^2*c^4/a*x^3*C*d^6+5/8/(a*e^2+c*d^2)^4/(c*x^2+a)^2/a*x*A
*c^4*d^6-15/8/(a*e^2+c*d^2)^4*a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c*e
^6+3/8/(a*e^2+c*d^2)^4/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*c^4*d^6+
1/8/(a*e^2+c*d^2)^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*c^3*d^6
```

**maxima [B]** time = 1.24, size = 1196, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^2/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*e^5)*log(
c*x^2 + a)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6
 + a^4*e^8) + (4*C*c*d^3*e^3 - 5*B*c*d^2*e^4 + B*a*e^6 - 2*(C*a - 3*A*c)*d*
e^5)*log(e*x + d)/(c^4*d^8 + 4*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 + 4*a^3*c*
d^2*e^6 + a^4*e^8) - 1/8*(2*B*a*c^3*d^5*e + 20*B*a^2*c^2*d^3*e^3 - 30*B*a^3
*c*d*e^5 - (C*a*c^3 + 3*A*c^4)*d^6 - (13*C*a^2*c^2 + 15*A*a*c^3)*d^4*e^2 +
3*(11*C*a^3*c - 15*A*a^2*c^2)*d^2*e^4 - 3*(C*a^4 - 5*A*a^3*c)*e^6)*arctan(c
*x/sqrt(a*c))/((a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5
*c*d^2*e^6 + a^6*e^8)*sqrt(a*c)) - 1/8*(2*B*a^2*c^2*d^5 + 12*B*a^3*c*d^3*e^
2 - 14*B*a^4*d*e^4 + 8*A*a^4*e^5 - 4*(C*a^3*c + A*a^2*c^2)*d^4*e + 20*(C*a^
4 - A*a^3*c)*d^2*e^3 + (2*B*a*c^3*d^3*e^2 - 22*B*a^2*c^2*d*e^4 - (C*a*c^3 +
 3*A*c^4)*d^4*e + 4*(5*C*a^2*c^2 - 3*A*a*c^3)*d^2*e^3 - 3*(C*a^3*c - 5*A*a^
2*c^2)*e^5)*x^4 + (2*B*a*c^3*d^4*e - 2*B*a^2*c^2*d^2*e^3 - 4*B*a^3*c*e^5 -
(C*a*c^3 + 3*A*c^4)*d^5 + 4*(C*a^2*c^2 - 3*A*a*c^3)*d^3*e^2 + (5*C*a^3*c -
9*A*a^2*c^2)*d*e^4)*x^3 + (10*B*a^2*c^2*d^3*e^2 - 38*B*a^3*c*d*e^4 - (7*C*a
^2*c^2 + 5*A*a*c^3)*d^4*e + 4*(9*C*a^3*c - 7*A*a^2*c^2)*d^2*e^3 - 5*(C*a^4
- 5*A*a^3*c)*e^5)*x^2 - (6*B*a^3*c*d^2*e^3 + 6*B*a^4*e^5 - (C*a^2*c^2 - 5*A
*a*c^3)*d^5 - 8*(C*a^3*c - 2*A*a^2*c^2)*d^3*e^2 - (7*C*a^4 - 11*A*a^3*c)*d*
e^4)*x)/(a^4*c^3*d^7 + 3*a^5*c^2*d^5*e^2 + 3*a^6*c*d^3*e^4 + a^7*d*e^6 + (a
^2*c^5*d^6*e + 3*a^3*c^4*d^4*e^3 + 3*a^4*c^3*d^2*e^5 + a^5*c^2*e^7)*x^5 + (
a^2*c^5*d^7 + 3*a^3*c^4*d^5*e^2 + 3*a^4*c^3*d^3*e^4 + a^5*c^2*d*e^6)*x^4 +
2*(a^3*c^4*d^6*e + 3*a^4*c^3*d^4*e^3 + 3*a^5*c^2*d^2*e^5 + a^6*c*e^7)*x^3 +
```

```
 2*(a^3*c^4*d^7 + 3*a^4*c^3*d^5*e^2 + 3*a^5*c^2*d^3*e^4 + a^6*c*d*e^6)*x^2
+ (a^4*c^3*d^6*e + 3*a^5*c^2*d^4*e^3 + 3*a^6*c*d^2*e^5 + a^7*e^7)*x)
```

**mupad [B]** time = 6.66, size = 6848, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^2),x)
```

```
[Out] symsum(log(root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 1
4336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*
e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*
c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*
d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*
C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*
z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c
^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 8
40*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*
e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c
^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*
C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*
e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c
^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z +
1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*
z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z
 + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A
^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c
^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*
C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^
2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*
B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405
*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^
4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*
c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A
*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*
c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c
^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*
d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9
*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k)*((120*A*a^8*c^2*e^13 - 24*C*a^9
*c*e^13 + 24*A*a^2*c^8*d^12*e - 112*B*a^8*c^2*d*e^12 + 8*C*a^3*c^7*d^12*e +
 144*A*a^3*c^7*d^10*e^3 + 456*A*a^4*c^6*d^8*e^5 + 864*A*a^5*c^5*d^6*e^7 + 9
36*A*a^6*c^4*d^4*e^9 + 528*A*a^7*c^3*d^2*e^11 - 16*B*a^3*c^7*d^11*e^2 - 176
*B*a^4*c^6*d^9*e^4 - 544*B*a^5*c^5*d^7*e^6 - 736*B*a^6*c^4*d^5*e^8 - 464*B*
a^7*c^3*d^3*e^10 + 112*C*a^4*c^6*d^10*e^3 + 344*C*a^5*c^5*d^8*e^5 + 416*C*a
```

^6*c^4*d^6*e^7 + 184*C*a^7*c^3*d^4*e^9 - 16*C*a^8*c^2*d^2*e^11)/(64*(a^10*e
^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8
*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e^8)) + root(17920*a^9*c^5*d^8*e
^8*z^3 + 14336*a^10*c^4*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^
11*c^3*d^4*e^12*z^3 + 7168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^
3 + 2048*a^6*c^8*d^14*e^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3
+ 948*B*C*a^7*c*d*e^11*z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7
*z - 7476*B*C*a^6*c^2*d^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^
5*d^9*e^3*z - 12486*A*C*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 2
82*A*C*a^3*c^5*d^8*e^4*z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10
*e^2*z + 14820*A*B*a^5*c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*
a^3*c^5*d^7*e^5*z - 180*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 32
04*A*B*a^6*c^2*d*e^11*z + 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6
*e^6*z + 103*C^2*a^4*c^4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a
^5*c^3*d^4*e^8*z + 2820*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z
+ 80*B^2*a^3*c^5*d^8*e^4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^
2*e^10*z + 1575*A^2*a^4*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^
2*a^2*c^6*d^8*e^4*z - 90*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^
7*c*d^2*e^10*z + 90*A^2*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6
*c^2*e^12*z - 192*B^2*a^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z +
78*A*B*C*a*c^4*d^6*e^4 + 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*
e^8 - 129*B*C^2*a^4*c*d^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d
^5*e^5 - 24*A*C^2*a*c^4*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c
^2*d*e^9 - 60*A*B^2*a*c^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c
*d*e^9 - 624*B^2*C*a^3*c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a
^2*c^3*d^5*e^5 + 21*B*C^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 39
6*A*C^2*a^3*c^2*d^3*e^7 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^
2*e^8 - 672*A*B^2*a^2*c^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e
^7 - 1350*A^3*a^2*c^3*d*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3
+ 45*A^2*B*c^5*d^6*e^4 - 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 -
4*C^3*a^2*c^3*d^7*e^3 + 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 +
18*C^3*a^5*d*e^9 - 64*B^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^
5, z, k)*((512*a^11*c^2*d*e^14 + 512*a^5*c^8*d^13*e^2 + 3072*a^6*c^7*d^11*e
^4 + 7680*a^7*c^6*d^9*e^6 + 10240*a^8*c^5*d^7*e^8 + 7680*a^9*c^4*d^5*e^10 +
3072*a^10*c^3*d^3*e^12)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8)) + (x*(384*a^11*c^2*e^15 - 128*a^4*c^9*d^14*e - 384*a^5*c^8*d^12*e
^3 + 384*a^6*c^7*d^10*e^5 + 3200*a^7*c^6*d^8*e^7 + 5760*a^8*c^5*d^6*e^9 + 4
992*a^9*c^4*d^4*e^11 + 2176*a^10*c^3*d^2*e^13))/(64*(a^10*e^12 + a^4*c^6*d^
12 + 6*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^
3*d^6*e^6 + 15*a^8*c^2*d^4*e^8))) + (x*(192*B*a^8*c^2*e^13 + 912*A*a^7*c^3*
d*e^12 - 336*C*a^8*c^2*d*e^12 + 48*A*a^2*c^8*d^11*e^2 + 336*A*a^3*c^7*d^9*e
^4 + 1632*A*a^4*c^6*d^7*e^6 + 3360*A*a^5*c^5*d^5*e^8 + 2928*A*a^6*c^4*d^3*e
^10 - 32*B*a^3*c^7*d^10*e^3 - 704*B*a^4*c^6*d^8*e^5 - 1728*B*a^5*c^5*d^6*e^
7 - 1280*B*a^6*c^4*d^4*e^9 - 32*B*a^7*c^3*d^2*e^11 + 16*C*a^3*c^7*d^11*e^2

+ 496*C*a^4*c^6*d^9*e^4 + 1056*C*a^5*c^5*d^7*e^6 + 352*C*a^6*c^4*d^5*e^8 -
560*C*a^7*c^3*d^3*e^10))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 +
6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*
d^4*e^8))) + (9*A^2*c^7*d^9*e^2 + 198*A^2*a^2*c^5*d^5*e^6 + 216*A^2*a^3*c^4
*d^3*e^8 + 4*B^2*a^2*c^5*d^7*e^4 - 8*B^2*a^3*c^4*d^5*e^6 - 412*B^2*a^4*c^3*
d^3*e^8 + C^2*a^2*c^5*d^9*e^2 - 8*C^2*a^3*c^4*d^7*e^4 - 250*C^2*a^4*c^3*d^5
*e^6 + 296*C^2*a^5*c^2*d^3*e^8 - 120*A*B*a^5*c^2*e^11 - 39*C^2*a^6*c*d*e^10
+ 72*A^2*a*c^6*d^7*e^4 - 495*A^2*a^4*c^3*d*e^10 + 176*B^2*a^5*c^2*d*e^10 +
24*B*C*a^6*c*e^11 - 12*A*B*a*c^6*d^8*e^3 + 6*A*C*a*c^6*d^9*e^2 + 294*A*C*a
^5*c^2*d*e^10 - 36*A*B*a^2*c^5*d^6*e^5 + 36*A*B*a^3*c^4*d^4*e^7 + 1092*A*B*
a^4*c^3*d^2*e^9 - 108*A*C*a^3*c^4*d^5*e^6 - 960*A*C*a^4*c^3*d^3*e^8 - 4*B*C
*a^2*c^5*d^8*e^3 + 20*B*C*a^3*c^4*d^6*e^5 + 652*B*C*a^4*c^3*d^4*e^7 - 500*B
*C*a^5*c^2*d^2*e^9)/(64*(a^10*e^12 + a^4*c^6*d^12 + 6*a^9*c*d^2*e^10 + 6*a^
5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*e^6 + 15*a^8*c^2*d^4*e
^8)) + (x*(225*A^2*a^4*c^3*e^11 + 9*A^2*c^7*d^8*e^3 + 9*C^2*a^6*c*e^11 + 54
*A^2*a^2*c^5*d^4*e^7 - 360*A^2*a^3*c^4*d^2*e^9 + 4*B^2*a^2*c^5*d^6*e^5 - 88
*B^2*a^3*c^4*d^4*e^7 + 484*B^2*a^4*c^3*d^2*e^9 + C^2*a^2*c^5*d^8*e^3 - 40*C
^2*a^3*c^4*d^6*e^5 + 406*C^2*a^4*c^3*d^4*e^7 - 120*C^2*a^5*c^2*d^2*e^9 - 90
*A*C*a^5*c^2*e^11 + 72*A^2*a*c^6*d^6*e^5 - 12*A*B*a*c^6*d^7*e^4 - 660*A*B*a
^4*c^3*d*e^10 + 6*A*C*a*c^6*d^8*e^3 + 132*B*C*a^5*c^2*d*e^10 + 84*A*B*a^2*c
^5*d^5*e^6 + 588*A*B*a^3*c^4*d^3*e^8 - 96*A*C*a^2*c^5*d^6*e^5 - 492*A*C*a^3
*c^4*d^4*e^7 + 672*A*C*a^4*c^3*d^2*e^9 - 4*B*C*a^2*c^5*d^7*e^4 + 124*B*C*a^
3*c^4*d^5*e^6 - 892*B*C*a^4*c^3*d^3*e^8))/(64*(a^10*e^12 + a^4*c^6*d^12 + 6
*a^9*c*d^2*e^10 + 6*a^5*c^5*d^10*e^2 + 15*a^6*c^4*d^8*e^4 + 20*a^7*c^3*d^6*
e^6 + 15*a^8*c^2*d^4*e^8)))*root(17920*a^9*c^5*d^8*e^8*z^3 + 14336*a^10*c^4
*d^6*e^10*z^3 + 14336*a^8*c^6*d^10*e^6*z^3 + 7168*a^11*c^3*d^4*e^12*z^3 + 7
168*a^7*c^7*d^12*e^4*z^3 + 2048*a^12*c^2*d^2*e^14*z^3 + 2048*a^6*c^8*d^14*e
^2*z^3 + 256*a^5*c^9*d^16*z^3 + 256*a^13*c*e^16*z^3 + 948*B*C*a^7*c*d*e^11*
z - 12*A*B*a*c^7*d^11*e*z + 9768*B*C*a^5*c^3*d^5*e^7*z - 7476*B*C*a^6*c^2*d
^3*e^9*z - 328*B*C*a^4*c^4*d^7*e^5*z - 92*B*C*a^3*c^5*d^9*e^3*z - 12486*A*C
*a^5*c^3*d^4*e^8*z + 5868*A*C*a^6*c^2*d^2*e^10*z + 282*A*C*a^3*c^5*d^8*e^4*
z + 168*A*C*a^4*c^4*d^6*e^6*z + 108*A*C*a^2*c^6*d^10*e^2*z + 14820*A*B*a^5*
c^3*d^3*e^9*z - 840*A*B*a^4*c^4*d^5*e^7*z - 600*A*B*a^3*c^5*d^7*e^5*z - 180
*A*B*a^2*c^6*d^9*e^3*z - 4*B*C*a^2*c^6*d^11*e*z - 3204*A*B*a^6*c^2*d*e^11*z
+ 4239*C^2*a^6*c^2*d^4*e^8*z - 3924*C^2*a^5*c^3*d^6*e^6*z + 103*C^2*a^4*c^
4*d^8*e^4*z + 26*C^2*a^3*c^5*d^10*e^2*z - 6000*B^2*a^5*c^3*d^4*e^8*z + 2820
*B^2*a^6*c^2*d^2*e^10*z + 280*B^2*a^4*c^4*d^6*e^6*z + 80*B^2*a^3*c^5*d^8*e^
4*z + 4*B^2*a^2*c^6*d^10*e^2*z - 8262*A^2*a^5*c^3*d^2*e^10*z + 1575*A^2*a^4
*c^4*d^4*e^8*z + 1260*A^2*a^3*c^5*d^6*e^6*z + 495*A^2*a^2*c^6*d^8*e^4*z - 9
0*A*C*a^7*c*e^12*z + 6*A*C*a*c^7*d^12*z - 966*C^2*a^7*c*d^2*e^10*z + 90*A^2
*a*c^7*d^10*e^2*z + C^2*a^2*c^6*d^12*z + 225*A^2*a^6*c^2*e^12*z - 192*B^2*a
^7*c*e^12*z + 9*A^2*c^8*d^12*z + 9*C^2*a^8*e^12*z + 78*A*B*C*a*c^4*d^6*e^4
+ 942*A*B*C*a^2*c^3*d^4*e^6 - 342*A*B*C*a^3*c^2*d^2*e^8 - 129*B*C^2*a^4*c*d
^2*e^8 + 990*A^2*C*a^3*c^2*d*e^9 - 234*A^2*C*a*c^4*d^5*e^5 - 24*A*C^2*a*c^4
*d^7*e^3 + 333*A^2*B*a*c^4*d^4*e^6 - 252*A*B^2*a^3*c^2*d*e^9 - 60*A*B^2*a*c

```
^4*d^5*e^5 + 204*B^2*C*a^4*c*d*e^9 - 234*A*C^2*a^4*c*d*e^9 - 624*B^2*C*a^3*
c^2*d^3*e^7 + 405*B*C^2*a^3*c^2*d^4*e^6 - 36*B^2*C*a^2*c^3*d^5*e^5 + 21*B*C
^2*a^2*c^3*d^6*e^4 - 1296*A^2*C*a^2*c^3*d^3*e^7 + 396*A*C^2*a^3*c^2*d^3*e^7
 - 330*A*C^2*a^2*c^3*d^5*e^5 + 1863*A^2*B*a^2*c^3*d^2*e^8 - 672*A*B^2*a^2*c
^3*d^3*e^7 + 90*A*B*C*a^4*c*e^10 + 8*C^3*a^4*c*d^3*e^7 - 1350*A^3*a^2*c^3*d
*e^9 - 324*A^3*a*c^4*d^3*e^7 - 36*A^2*C*c^5*d^7*e^3 + 45*A^2*B*c^5*d^6*e^4
- 225*A^2*B*a^3*c^2*e^10 - 86*C^3*a^3*c^2*d^5*e^5 - 4*C^3*a^2*c^3*d^7*e^3 +
 316*B^3*a^3*c^2*d^2*e^8 + 20*B^3*a^2*c^3*d^4*e^6 + 18*C^3*a^5*d*e^9 - 64*B
^3*a^4*c*e^10 - 9*B*C^2*a^5*e^10 - 54*A^3*c^5*d^5*e^5, z, k), k, 1, 3) + ((
x^4*(3*C*a^3*c*e^5 + 3*A*c^4*d^4*e - 15*A*a^2*c^2*e^5 + 12*A*a*c^3*d^2*e^3
- 2*B*a*c^3*d^3*e^2 + 22*B*a^2*c^2*d*e^4 - 20*C*a^2*c^2*d^2*e^3 + C*a*c^3*d
^4*e))/(8*a^2*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (4
*A*a^2*e^5 + B*c^2*d^5 - 7*B*a^2*d*e^4 - 2*A*c^2*d^4*e + 10*C*a^2*d^2*e^3 -
 2*C*a*c*d^4*e - 10*A*a*c*d^2*e^3 + 6*B*a*c*d^3*e^2)/(4*(a*e^2 + c*d^2)*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x^3*(3*A*c^3*d^3 + 4*B*a^2*c*e^3 + C*a
*c^2*d^3 + 9*A*a*c^2*d*e^2 - 2*B*a*c^2*d^2*e - 5*C*a^2*c*d*e^2))/(8*a^2*(a^
2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (x*(5*A*c^2*d^3 + 6*B*a^2*e^3 - C*a*c*d
^3 - 7*C*a^2*d*e^2 + 11*A*a*c*d*e^2))/(8*a*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e
^2)) + (x^2*(5*C*a^3*e^5 - 25*A*a^2*c*e^5 + 5*A*c^3*d^4*e + 28*A*a*c^2*d^2*
e^3 - 10*B*a*c^2*d^3*e^2 - 36*C*a^2*c*d^2*e^3 + 38*B*a^2*c*d*e^4 + 7*C*a*c^
2*d^4*e))/(8*a*(a*e^2 + c*d^2)*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a^2*d
 + c^2*d*x^4 + c^2*e*x^5 + a^2*e*x + 2*a*c*d*x^2 + 2*a*c*e*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**2/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

# 3.63 $\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=753

$$-\frac{e^3\log\left(a+cx^2\right)\left(a^2Ce^4-ace^2\left(3Ae^2-9Bde+13Cd^2\right)+c^2d^2\left(10Cd^2-3e(5Bd-7Ae)\right)\right)}{2\left(ae^2+cd^2\right)^5}+\frac{e^3\log(d+ex)\left(a^2Ce^4\right.}{}$$

```
[Out] -1/2*e^3*(A*e^2-B*d*e+C*d^2)/(a*e^2+c*d^2)^3/(e*x+d)^2+e^3*(a*e^2*(-B*e+2*C
*d)-c*d*(4*C*d^2-e*(-6*A*e+5*B*d)))/(a*e^2+c*d^2)^4/(e*x+d)+1/4*(-a*(B*c*d*
(-3*a*e^2+c*d^2)-(A*c-C*a)*e*(-a*e^2+3*c*d^2))+c*(A*c*d*(-3*a*e^2+c*d^2)-a*
(c*d^2*(-3*B*e+C*d)-a*e^2*(-B*e+3*C*d)))*x)/a/(a*e^2+c*d^2)^3/(c*x^2+a)^2+1
/8*(4*a^2*e*(a^2*C*e^4+c^2*d^2*(3*C*d^2-2*e*(-5*A*e+3*B*d))-2*a*c*e^2*(4*C*
d^2-e*(-A*e+3*B*d)))+c*(3*A*c*d*(-11*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)-a*(2*a*
c*d^2*e^2*(-19*B*e+13*C*d)-c^2*d^4*(-3*B*e+C*d)-7*a^2*e^4*(-B*e+3*C*d)))*x)
/a^2/(a*e^2+c*d^2)^4/(c*x^2+a)+e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C
*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7*A*e+5*B*d)))*ln(e*x+d)/(a*e^2+c*d^2)^5-1/2*
e^3*(a^2*C*e^4-a*c*e^2*(3*A*e^2-9*B*d*e+13*C*d^2)+c^2*d^2*(10*C*d^2-3*e*(-7
*A*e+5*B*d)))*ln(c*x^2+a)/(a*e^2+c*d^2)^5+1/8*(3*A*c*d*(-35*a^3*e^6+35*a^2*
c*d^2*e^4+7*a*c^2*d^4*e^2+c^3*d^6)+a*(a*c^2*d^4*e^2*(-45*B*e+23*C*d)-5*a^2*
c*d^2*e^4*(-27*B*e+25*C*d)+c^3*d^6*(-3*B*e+C*d)+15*a^3*e^6*(-B*e+3*C*d)))*a
rctan(x*c^(1/2)/a^(1/2))*c^(1/2)/a^(5/2)/(a*e^2+c*d^2)^5
```

**Rubi [A]** time = 3.14, antiderivative size = 753, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1647, 1629, 635, 205, 260}

$$\frac{cx\left(3Acd\left(-11a^2e^4+6acd^2e^2+c^2d^4\right)-a\left(-7a^2e^4(3Cd-Be)+2acd^2e^2(13Cd-19Be)-c^2d^4(Cd-3Be)\right)\right)+4a^2e}{8a^2\left(a+cx^2\right)\left(ae^2+cd^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] -(e^3*(C*d^2 - B*d*e + A*e^2))/(2*(c*d^2 + a*e^2)^3*(d + e*x)^2) - (e^3*(4*
c*C*d^3 - c*d*e*(5*B*d - 6*A*e) - a*e^2*(2*C*d - B*e)))/((c*d^2 + a*e^2)^4*
(d + e*x)) - (a*(B*c*d*(c*d^2 - 3*a*e^2) - (A*c - a*C)*e*(3*c*d^2 - a*e^2))
 - c*(A*c*d*(c*d^2 - 3*a*e^2) - a*(c*d^2*(C*d - 3*B*e) - a*e^2*(3*C*d - B*e
)))*x)/(4*a*(c*d^2 + a*e^2)^3*(a + c*x^2)^2) + (4*a^2*e*(a^2*C*e^4 + c^2*(3
*C*d^4 - 2*d^2*e*(3*B*d - 5*A*e)) - 2*a*c*e^2*(4*C*d^2 - e*(3*B*d - A*e)))
+ c*(3*A*c*d*(c^2*d^4 + 6*a*c*d^2*e^2 - 11*a^2*e^4) - a*(2*a*c*d^2*e^2*(13*
C*d - 19*B*e) - c^2*d^4*(C*d - 3*B*e) - 7*a^2*e^4*(3*C*d - B*e)))*x)/(8*a^2
```

```
*(c*d^2 + a*e^2)^4*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*
e^2 + 35*a^2*c*d^2*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) -
 5*a^2*c*d^2*e^4*(25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) + 15*a^3*e^6*(3*
C*d - B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(8*a^(5/2)*(c*d^2 + a*e^2)^5) + (
e^3*(a^2*C*e^4 - a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3
*d^2*e*(5*B*d - 7*A*e)))*Log[d + e*x])/(c*d^2 + a*e^2)^5 - (e^3*(a^2*C*e^4
- a*c*e^2*(13*C*d^2 - 9*B*d*e + 3*A*e^2) + c^2*(10*C*d^4 - 3*d^2*e*(5*B*d -
 7*A*e)))*Log[a + c*x^2])/(2*(c*d^2 + a*e^2)^5)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3\left(a+cx^2\right)^3}\,dx = -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{a\left(Bcd\left(cd^2-3ae^2\right)-(Ac-aC)e\left(3cd^2-ae^2\right)\right)-c\left(Acd\left(cd^2-3ae^2\right)-a\left(cd^2(Cd-\right.\right.}{4a\left(cd^2+ae^2\right)^3\left(a+cx^2\right)^2}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

$$= -\frac{e^3\left(Cd^2-Bde+Ae^2\right)}{2\left(cd^2+ae^2\right)^3(d+ex)^2}-\frac{e^3\left(4cCd^3-cde(5Bd-6Ae)-ae^2(2Cd-Be)\right)}{\left(cd^2+ae^2\right)^4(d+ex)}-\frac{a\left(Bcd\right.}{}$$

**Mathematica [A]** time = 1.08, size = 672, normalized size = 0.89

$$\frac{-4\log\left(a+cx^2\right)\left(a^2Ce^7+ace^5\left(-3Ae^2+9Bde-13Cd^2\right)+c^2d^2e^3\left(3e(7Ae-5Bd)+10Cd^2\right)\right)+8\log(d+ex)\left(a^2C\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(a + c*x^2)^3),x]
```

```
[Out] ((-4*e^3*(c*d^2 + a*e^2)^2*(C*d^2 + e*(-(B*d) + A*e)))/(d + e*x)^2 - (8*e^3
*(c*d^2 + a*e^2)*(4*c*C*d^3 + c*d*e*(-5*B*d + 6*A*e) + a*e^2*(-2*C*d + B*e)
))/(d + e*x) + (2*(c*d^2 + a*e^2)^2*(a^3*C*e^3 + A*c^3*d^3*x - a*c^2*d*(C*d
^2*x + B*d*(d - 3*e*x) + 3*A*e*(-d + e*x)) - a^2*c*e*(3*C*d*(d - e*x) + e*(
-3*B*d + A*e + B*e*x))))/(a*(a + c*x^2)^2) + ((c*d^2 + a*e^2)*(4*a^4*C*e^5
```

```
+ 3*A*c^4*d^5*x + a*c^3*d^3*(C*d^2 + 3*e*(-(B*d) + 6*A*e))*x + a^3*c*e^3*(C
*d*(-32*d + 21*e*x) + e*(24*B*d - 8*A*e - 7*B*e*x)) + a^2*c^2*d*e*(2*C*d^2*
(6*d - 13*e*x) + e*(-24*B*d^2 + 40*A*d*e + 38*B*d*e*x - 33*A*e^2*x))))/(a^2
*(a + c*x^2)) + (Sqrt[c]*(3*A*c*d*(c^3*d^6 + 7*a*c^2*d^4*e^2 + 35*a^2*c*d^2
*e^4 - 35*a^3*e^6) + a*(a*c^2*d^4*e^2*(23*C*d - 45*B*e) - 5*a^2*c*d^2*e^4*(
25*C*d - 27*B*e) + c^3*d^6*(C*d - 3*B*e) - 15*a^3*e^6*(-3*C*d + B*e)))*ArcT
an[(Sqrt[c]*x)/Sqrt[a]])/a^(5/2) + 8*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*
d*e - 3*A*e^2) + c^2*d^2*e^3*(10*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[d + e*x
] - 4*(a^2*C*e^7 + a*c*e^5*(-13*C*d^2 + 9*B*d*e - 3*A*e^2) + c^2*d^2*e^3*(1
0*C*d^2 + 3*e*(-5*B*d + 7*A*e)))*Log[a + c*x^2])/(8*(c*d^2 + a*e^2)^5)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.20, size = 1532, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 - 13*C*a*c*d^2*e^5 + 21*A*c^2*d^2
*e^5 + 9*B*a*c*d*e^6 + C*a^2*e^7 - 3*A*a*c*e^7)*log(c*x^2 + a)/(c^5*d^10 +
5*a*c^4*d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8
 + a^5*e^10) + (10*C*c^2*d^4*e^4 - 15*B*c^2*d^3*e^5 - 13*C*a*c*d^2*e^6 + 21
*A*c^2*d^2*e^6 + 9*B*a*c*d*e^7 + C*a^2*e^8 - 3*A*a*c*e^8)*log(abs(x*e + d))
/(c^5*d^10*e + 5*a*c^4*d^8*e^3 + 10*a^2*c^3*d^6*e^5 + 10*a^3*c^2*d^4*e^7 +
5*a^4*c*d^2*e^9 + a^5*e^11) + 1/8*(C*a*c^4*d^7 + 3*A*c^5*d^7 - 3*B*a*c^4*d^
6*e + 23*C*a^2*c^3*d^5*e^2 + 21*A*a*c^4*d^5*e^2 - 45*B*a^2*c^3*d^4*e^3 - 12
5*C*a^3*c^2*d^3*e^4 + 105*A*a^2*c^3*d^3*e^4 + 135*B*a^3*c^2*d^2*e^5 + 45*C*
a^4*c*d*e^6 - 105*A*a^3*c^2*d*e^6 - 15*B*a^4*c*e^7)*arctan(c*x/sqrt(a*c))/(
(a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6
 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c)) + 1/8*(C*a*c^4*d^5*x^5*e^2 + 3*A*
c^5*d^5*x^5*e^2 + 2*C*a*c^4*d^6*x^4*e + 6*A*c^5*d^6*x^4*e + C*a*c^4*d^7*x^3
 + 3*A*c^5*d^7*x^3 - 3*B*a*c^4*d^4*x^5*e^3 - 6*B*a*c^4*d^5*x^4*e^2 - 3*B*a*
c^4*d^6*x^3*e - 58*C*a^2*c^3*d^3*x^5*e^4 + 18*A*a*c^4*d^3*x^5*e^4 - 76*C*a^
2*c^3*d^4*x^4*e^3 + 36*A*a*c^4*d^4*x^4*e^3 - 3*C*a^2*c^3*d^5*x^3*e^2 + 23*A
*a*c^4*d^5*x^3*e^2 + 10*C*a^2*c^3*d^6*x^2*e + 10*A*a*c^4*d^6*x^2*e - C*a^2*
c^3*d^7*x + 5*A*a*c^4*d^7*x + 78*B*a^2*c^3*d^2*x^5*e^5 + 96*B*a^2*c^3*d^3*x
```

```
^4*e^4 - 7*B*a^2*c^3*d^4*x^3*e^3 - 20*B*a^2*c^3*d^5*x^2*e^2 - B*a^2*c^3*d^6
*x*e - 2*B*a^2*c^3*d^7 + 37*C*a^3*c^2*d*x^5*e^6 - 81*A*a^2*c^3*d*x^5*e^6 +
22*C*a^3*c^2*d^2*x^4*e^5 - 78*A*a^2*c^3*d^2*x^4*e^5 - 129*C*a^3*c^2*d^3*x^3
*e^4 + 61*A*a^2*c^3*d^3*x^3*e^4 - 142*C*a^3*c^2*d^4*x^2*e^3 + 74*A*a^2*c^3*
d^4*x^2*e^3 - 10*C*a^3*c^2*d^5*x*e^2 + 26*A*a^2*c^3*d^5*x*e^2 + 6*C*a^3*c^2
*d^6*e + 6*A*a^2*c^3*d^6*e - 15*B*a^3*c^2*x^5*e^7 + 6*B*a^3*c^2*d*x^4*e^6 +
 163*B*a^3*c^2*d^2*x^3*e^5 + 176*B*a^3*c^2*d^3*x^2*e^4 + 2*B*a^3*c^2*d^4*x*
e^3 - 20*B*a^3*c^2*d^5*e^2 + 4*C*a^4*c*x^4*e^7 - 12*A*a^3*c^2*x^4*e^7 + 67*
C*a^4*c*d*x^3*e^6 - 151*A*a^3*c^2*d*x^3*e^6 + 46*C*a^4*c*d^2*x^2*e^5 - 146*
A*a^3*c^2*d^2*x^2*e^5 - 77*C*a^4*c*d^3*x*e^4 + 49*A*a^3*c^2*d^3*x*e^4 - 72*
C*a^4*c*d^4*e^3 + 44*A*a^3*c^2*d^4*e^3 - 25*B*a^4*c*x^3*e^7 + 4*B*a^4*c*d*x
^2*e^6 + 91*B*a^4*c*d^2*x*e^5 + 74*B*a^4*c*d^3*e^4 + 6*C*a^5*x^2*e^7 - 18*A
*a^4*c*x^2*e^7 + 28*C*a^5*d*x*e^6 - 68*A*a^4*c*d*x*e^6 + 18*C*a^5*d^2*e^5 -
 62*A*a^4*c*d^2*e^5 - 8*B*a^5*x*e^7 - 4*B*a^5*d*e^6 - 4*A*a^5*e^7)/((a^2*c^
4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*
(c*x^3*e + c*d*x^2 + a*x*e + a*d)^2)
```

**maple [B]** time = 0.04, size = 2737, normalized size = 3.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x)
```

```
[Out] 15/2/(a*e^2+c*d^2)^5*c^2*ln(c*x^2+a)*d^3*e^4*B-5/(a*e^2+c*d^2)^5*c^2*ln(c*x
^2+a)*C*d^4*e^3+3/2/(a*e^2+c*d^2)^5*c*a*ln(c*x^2+a)*A*e^7+2*e^5/(a*e^2+c*d^
2)^4/(e*x+d)*C*a*d-4*e^3/(a*e^2+c*d^2)^4/(e*x+d)*C*c*d^3-3*e^7/(a*e^2+c*d^2
)^5*ln(e*x+d)*A*a*c+21*e^5/(a*e^2+c*d^2)^5*ln(e*x+d)*A*c^2*d^2-15*e^4/(a*e^
2+c*d^2)^5*ln(e*x+d)*B*c^2*d^3+10*e^3/(a*e^2+c*d^2)^5*ln(e*x+d)*C*c^2*d^4-6
*e^5/(a*e^2+c*d^2)^4/(e*x+d)*A*c*d+5*e^4/(a*e^2+c*d^2)^4/(e*x+d)*B*c*d^2-5/
4/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*A*e^7*a^3+3/4/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)
^2*A*d^6*e-1/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x*d^7-21/2/(a*e^2+c*d^2)^5
*c^2*ln(c*x^2+a)*A*d^2*e^5-1/2*e^5/(a*e^2+c*d^2)^3/(e*x+d)^2*A-7/2/(a*e^2+c
*d^2)^5*c^2/(c*x^2+a)^2*C*x^2*a^2*d^2*e^5-5/2/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)
^2*C*x^2*a*d^4*e^3+21/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*A*d^5*e^2-3/8
/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*B*d^6*e+31/8/(a*e^2+c*d^2)^5*c^3/(c*
x^2+a)^2*B*x^3*a*d^2*e^5-39/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*x*a^2*d*e^6
-25/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x*a*d^3*e^4+33/8/(a*e^2+c*d^2)^5*c^
2/(c*x^2+a)^2*B*x*a^2*d^2*e^5+45/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*B*x*a*d^
4*e^3+5/8/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*x*a^2*d^3*e^4-23/8/(a*e^2+c*d^2
)^5*c^3/(c*x^2+a)^2*C*x*a*d^5*e^2-105/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*a
rctan(1/(a*c)^(1/2)*c*x)*A*d*e^6+21/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*A*d^5*e^2+135/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*ar
ctan(1/(a*c)^(1/2)*c*x)*B*d^2*e^5-3/8/(a*e^2+c*d^2)^5*c^4/a/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*B*d^6*e-125/8/(a*e^2+c*d^2)^5*c^2*a/(a*c)^(1/2)*arct
```

```
an(1/(a*c)^(1/2)*c*x)*C*d^3*e^4+45/8/(a*e^2+c*d^2)^5*c*a^2/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*C*d*e^6+27/8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*x*a^3*d*
e^6-33/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x^3*a*d*e^6+21/8/(a*e^2+c*d^2)^5
*c^2/(c*x^2+a)^2*C*x^3*a^2*d*e^6-5/8/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*C*x^3*
a*d^3*e^4+4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*x^2*a*d^2*e^5+3/(a*e^2+c*d^2)
^5*c^2/(c*x^2+a)^2*B*x^2*a^2*d*e^6-1/2/(a*e^2+c*d^2)^5*a^2*ln(c*x^2+a)*C*e^
7-e^6/(a*e^2+c*d^2)^4/(e*x+d)*B*a+1/2*e^4/(a*e^2+c*d^2)^3/(e*x+d)^2*B*d-1/2
*e^3/(a*e^2+c*d^2)^3/(e*x+d)^2*C*d^2+e^7/(a*e^2+c*d^2)^5*ln(e*x+d)*a^2*C+3/
4/(a*e^2+c*d^2)^5/(c*x^2+a)^2*C*a^4*e^7-1/4/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2
*d^7*B-15/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*A*x^3*d^3*e^4+35/8/(a*e^2+c*d^2
)^5*c^4/(c*x^2+a)^2*B*x^3*d^4*e^3-25/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^
3*d^5*e^2-1/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*x^2*a^2*e^7+5/(a*e^2+c*d^2)^5
*c^4/(c*x^2+a)^2*A*x^2*d^4*e^3-3/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B*x^2*d^5*
e^2+3/2/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*C*x^2*d^6*e-7/8/(a*e^2+c*d^2)^5*c^2
/(c*x^2+a)^2*B*x^3*a^2*e^7+3/8/(a*e^2+c*d^2)^5*c^6/(c*x^2+a)^2/a^2*x^3*A*d^
7+1/8/(a*e^2+c*d^2)^5*c^5/(c*x^2+a)^2/a*x^3*C*d^7+5/8/(a*e^2+c*d^2)^5*c^5/(
c*x^2+a)^2/a*x*A*d^7+17/4/(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*A*d^2*e^5*a^2+25/
4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*A*d^4*e^3*a+5/4/(a*e^2+c*d^2)^5*c^2/(c*x^
2+a)^2*d^3*e^4*B*a^2-11/4/(a*e^2+c*d^2)^5*c^3/(c*x^2+a)^2*d^5*e^2*B*a-15/4/
(a*e^2+c*d^2)^5*c^2/(c*x^2+a)^2*C*a^2*d^4*e^3+3/4/(a*e^2+c*d^2)^5*c^3/(c*x^
2+a)^2*C*a*d^6*e+9*e^6/(a*e^2+c*d^2)^5*ln(e*x+d)*B*a*c*d-13*e^5/(a*e^2+c*d^
2)^5*ln(e*x+d)*C*a*c*d^2+1/2/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*x^2*a^3*e^7-9/
8/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*B*x*a^3*e^7+15/4/(a*e^2+c*d^2)^5*c/(c*x^2+a
)^2*d*e^6*B*a^3-15/4/(a*e^2+c*d^2)^5*c/(c*x^2+a)^2*C*a^3*d^2*e^5+1/8/(a*e^2
+c*d^2)^5*c^4/a/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^7-9/2/(a*e^2+c*d^
2)^5*c*a*ln(c*x^2+a)*d*e^6*B+13/2/(a*e^2+c*d^2)^5*c*a*ln(c*x^2+a)*C*d^2*e^5
-15/8/(a*e^2+c*d^2)^5*c*a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e^7+105
/8/(a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^3*e^4-45/8
/(a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^4*e^3+23/8/(
a*e^2+c*d^2)^5*c^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^5*e^2+3/8/(a*e
^2+c*d^2)^5*c^5/a^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^7+19/8/(a*e^2
+c*d^2)^5*c^4/(c*x^2+a)^2*A*x*d^5*e^2+3/8/(a*e^2+c*d^2)^5*c^4/(c*x^2+a)^2*B
*x*d^6*e
```

**maxima [B]** time = 1.27, size = 1835, normalized size = 2.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^3/(c*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 21*
A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(c*x^2 + a)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) + (10*C*c^2*d^4*e^3 - 15*B*c^2*d^3*e^4 + 9*B*a*c*d*e^6 - (13*C*a*c - 2
```

```
1*A*c^2)*d^2*e^5 + (C*a^2 - 3*A*a*c)*e^7)*log(e*x + d)/(c^5*d^10 + 5*a*c^4*
d^8*e^2 + 10*a^2*c^3*d^6*e^4 + 10*a^3*c^2*d^4*e^6 + 5*a^4*c*d^2*e^8 + a^5*e
^10) - 1/8*(3*B*a*c^4*d^6*e + 45*B*a^2*c^3*d^4*e^3 - 135*B*a^3*c^2*d^2*e^5
+ 15*B*a^4*c*e^7 - (C*a*c^4 + 3*A*c^5)*d^7 - (23*C*a^2*c^3 + 21*A*a*c^4)*d^
5*e^2 + 5*(25*C*a^3*c^2 - 21*A*a^2*c^3)*d^3*e^4 - 15*(3*C*a^4*c - 7*A*a^3*c
^2)*d*e^6)*arctan(c*x/sqrt(a*c))/((a^2*c^5*d^10 + 5*a^3*c^4*d^8*e^2 + 10*a^
4*c^3*d^6*e^4 + 10*a^5*c^2*d^4*e^6 + 5*a^6*c*d^2*e^8 + a^7*e^10)*sqrt(a*c))
 - 1/8*(2*B*a^2*c^3*d^7 + 20*B*a^3*c^2*d^5*e^2 - 74*B*a^4*c*d^3*e^4 + 4*B*a
^5*d*e^6 + 4*A*a^5*e^7 - 6*(C*a^3*c^2 + A*a^2*c^3)*d^6*e + 4*(18*C*a^4*c -
11*A*a^3*c^2)*d^4*e^3 - 2*(9*C*a^5 - 31*A*a^4*c)*d^2*e^5 + (3*B*a*c^4*d^4*e
^3 - 78*B*a^2*c^3*d^2*e^5 + 15*B*a^3*c^2*e^7 - (C*a*c^4 + 3*A*c^5)*d^5*e^2
+ 2*(29*C*a^2*c^3 - 9*A*a*c^4)*d^3*e^4 - (37*C*a^3*c^2 - 81*A*a^2*c^3)*d*e^
6)*x^5 + 2*(3*B*a*c^4*d^5*e^2 - 48*B*a^2*c^3*d^3*e^4 - 3*B*a^3*c^2*d*e^6 -
(C*a*c^4 + 3*A*c^5)*d^6*e + 2*(19*C*a^2*c^3 - 9*A*a*c^4)*d^4*e^3 - (11*C*a^
3*c^2 - 39*A*a^2*c^3)*d^2*e^5 - 2*(C*a^4*c - 3*A*a^3*c^2)*e^7)*x^4 + (3*B*a
*c^4*d^6*e + 7*B*a^2*c^3*d^4*e^3 - 163*B*a^3*c^2*d^2*e^5 + 25*B*a^4*c*e^7 -
 (C*a*c^4 + 3*A*c^5)*d^7 + (3*C*a^2*c^3 - 23*A*a*c^4)*d^5*e^2 + (129*C*a^3*
c^2 - 61*A*a^2*c^3)*d^3*e^4 - (67*C*a^4*c - 151*A*a^3*c^2)*d*e^6)*x^3 + 2*(
10*B*a^2*c^3*d^5*e^2 - 88*B*a^3*c^2*d^3*e^4 - 2*B*a^4*c*d*e^6 - 5*(C*a^2*c^
3 + A*a*c^4)*d^6*e + (71*C*a^3*c^2 - 37*A*a^2*c^3)*d^4*e^3 - (23*C*a^4*c -
73*A*a^3*c^2)*d^2*e^5 - 3*(C*a^5 - 3*A*a^4*c)*e^7)*x^2 + (B*a^2*c^3*d^6*e -
 2*B*a^3*c^2*d^4*e^3 - 91*B*a^4*c*d^2*e^5 + 8*B*a^5*e^7 + (C*a^2*c^3 - 5*A*
a*c^4)*d^7 + 2*(5*C*a^3*c^2 - 13*A*a^2*c^3)*d^5*e^2 + 7*(11*C*a^4*c - 7*A*a
^3*c^2)*d^3*e^4 - 4*(7*C*a^5 - 17*A*a^4*c)*d*e^6)*x)/(a^4*c^4*d^10 + 4*a^5*
c^3*d^8*e^2 + 6*a^6*c^2*d^6*e^4 + 4*a^7*c*d^4*e^6 + a^8*d^2*e^8 + (a^2*c^6*
d^8*e^2 + 4*a^3*c^5*d^6*e^4 + 6*a^4*c^4*d^4*e^6 + 4*a^5*c^3*d^2*e^8 + a^6*c
^2*e^10)*x^6 + 2*(a^2*c^6*d^9*e + 4*a^3*c^5*d^7*e^3 + 6*a^4*c^4*d^5*e^5 + 4
*a^5*c^3*d^3*e^7 + a^6*c^2*d*e^9)*x^5 + (a^2*c^6*d^10 + 6*a^3*c^5*d^8*e^2 +
 14*a^4*c^4*d^6*e^4 + 16*a^5*c^3*d^4*e^6 + 9*a^6*c^2*d^2*e^8 + 2*a^7*c*e^10
)*x^4 + 4*(a^3*c^5*d^9*e + 4*a^4*c^4*d^7*e^3 + 6*a^5*c^3*d^5*e^5 + 4*a^6*c^
2*d^3*e^7 + a^7*c*d*e^9)*x^3 + (2*a^3*c^5*d^10 + 9*a^4*c^4*d^8*e^2 + 16*a^5
*c^3*d^6*e^4 + 14*a^6*c^2*d^4*e^6 + 6*a^7*c*d^2*e^8 + a^8*e^10)*x^2 + 2*(a^
4*c^4*d^9*e + 4*a^5*c^3*d^7*e^3 + 6*a^6*c^2*d^5*e^5 + 4*a^7*c*d^3*e^7 + a^8
*d*e^9)*x)
```

**mupad [B]** time = 7.24, size = 8774, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a + c*x^2)^3*(d + e*x)^3),x)
```

```
[Out] ((x^5*(3*A*c^5*d^5*e^2 - 15*B*a^3*c^2*e^7 + 18*A*a*c^4*d^3*e^4 - 81*A*a^2*c
^3*d*e^6 - 3*B*a*c^4*d^4*e^3 + C*a*c^4*d^5*e^2 + 37*C*a^3*c^2*d*e^6 + 78*B*
a^2*c^3*d^2*e^5 - 58*C*a^2*c^3*d^3*e^4))/(8*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^
```

3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) - (2*A*a^3*e^7 + B*c^3*d^
7 + 2*B*a^3*d*e^6 - 3*A*c^3*d^6*e - 9*C*a^3*d^2*e^5 - 22*A*a*c^2*d^4*e^3 +
31*A*a^2*c*d^2*e^5 + 10*B*a*c^2*d^5*e^2 - 37*B*a^2*c*d^3*e^4 + 36*C*a^2*c*d
^4*e^3 - 3*C*a*c^2*d^6*e)/(4*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c
*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x*(5*A*c^4*d^7 - 8*B*a^4*e^7 - C*a*c^3*d^
7 + 28*C*a^4*d*e^6 + 26*A*a*c^3*d^5*e^2 + 91*B*a^3*c*d^2*e^5 - 77*C*a^3*c*d
^3*e^4 + 49*A*a^2*c^2*d^3*e^4 + 2*B*a^2*c^2*d^4*e^3 - 10*C*a^2*c^2*d^5*e^2
- 68*A*a^3*c*d*e^6 - B*a*c^3*d^6*e))/(8*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^2*(3*C*a^4*e^7 - 9*A*a^3*c
*e^7 + 5*A*c^4*d^6*e + 37*A*a*c^3*d^4*e^3 - 10*B*a*c^3*d^5*e^2 + 23*C*a^3*c
*d^2*e^5 - 73*A*a^2*c^2*d^2*e^5 + 88*B*a^2*c^2*d^3*e^4 - 71*C*a^2*c^2*d^4*e
^3 + 2*B*a^3*c*d*e^6 + 5*C*a*c^3*d^6*e))/(4*a*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)) + (x^3*(3*A*c^5*d^7 - 25*B*
a^4*c*e^7 + C*a*c^4*d^7 + 23*A*a*c^4*d^5*e^2 - 151*A*a^3*c^2*d*e^6 + 61*A*a
^2*c^3*d^3*e^4 - 7*B*a^2*c^3*d^4*e^3 + 163*B*a^3*c^2*d^2*e^5 - 3*C*a^2*c^3*
d^5*e^2 - 129*C*a^3*c^2*d^3*e^4 - 3*B*a*c^4*d^6*e + 67*C*a^4*c*d*e^6))/(8*a
^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e
^4)) + (x^4*(2*C*a^4*c*e^7 + 3*A*c^5*d^6*e - 6*A*a^3*c^2*e^7 + 18*A*a*c^4*d
^4*e^3 - 3*B*a*c^4*d^5*e^2 + 3*B*a^3*c^2*d*e^6 - 39*A*a^2*c^3*d^2*e^5 + 48*
B*a^2*c^3*d^3*e^4 - 38*C*a^2*c^3*d^4*e^3 + 11*C*a^3*c^2*d^2*e^5 + C*a*c^4*d
^6*e))/(4*a^2*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^
2*c^2*d^4*e^4)))/(x^2*(a^2*e^2 + 2*a*c*d^2) + x^4*(c^2*d^2 + 2*a*c*e^2) + a
^2*d^2 + c^2*e^2*x^6 + 2*a^2*d*e*x + 2*c^2*d*e*x^5 + 4*a*c*d*e*x^3) + symsu
m(log(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*
a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^1
4*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^
7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 25
6*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*
B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3
*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*
A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d
^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*
C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^
3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*
B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z
+ 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^
5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z +
71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*
c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z +
9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^
3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 10
71*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 70
17*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1
728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*
a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B

*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 -
60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*
e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3
*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B
*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4
+ 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^
4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 1
4283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^
3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^
5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c
^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e
^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^
8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*
c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11
, z, k)*(root(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 537
60*a^11*c^4*d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*
e^14*z^3 + 30720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520
*a^7*c^8*d^16*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 +
256*a^15*e^20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 1479
30*B*C*a^6*c^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*
d^3*e^11*z - 1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 1520
52*A*C*a^6*c^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^
2*d^2*e^12*z + 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180
*A*C*a^2*c^7*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3
*d^3*e^11*z - 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396
*A*B*a^2*c^7*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13
*z + 67615*C^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2
*a^5*c^4*d^8*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z
+ 71415*B^2*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a
^7*c^2*d^2*e^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z
+ 9*B^2*a^2*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6
*c^3*d^2*e^12*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z +
1071*A^2*a^2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z +
7017*C^2*a^8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z
- 1728*A^2*a^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C
^2*a^9*e^14*z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*
A*B*C*a^3*c^3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^
5 - 60*A*C^2*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d
^5*e^6 - 270*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*
c^3*d^4*e^7 + 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 197
7*B*C^2*a^4*c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*
e^4 + 15921*A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2
*a^4*c^2*d^2*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5
+ 14283*A^2*B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3
*c^3*d^2*e^9 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a

*c^5*d^4*e^7 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c^2*e^11 + 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e^11 - 397*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^8*e^3 + 3294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*c^4*d^2*e^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11, z, k)*((512*a^13*c^2*d*e^18 + 512*a^5*c^10*d^17*e^2 + 4096*a^6*c^9*d^15*e^4 + 14336*a^7*c^8*d^13*e^6 + 28672*a^8*c^7*d^11*e^8 + 35840*a^9*c^6*d^9*e^10 + 28672*a^10*c^5*d^7*e^12 + 14336*a^11*c^4*d^5*e^14 + 4096*a^12*c^3*d^3*e^16)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(384*a^13*c^2*e^19 - 128*a^4*c^11*d^18*e - 640*a^5*c^10*d^16*e^3 - 512*a^6*c^9*d^14*e^5 + 3584*a^7*c^8*d^12*e^7 + 12544*a^8*c^7*d^10*e^9 + 19712*a^9*c^6*d^8*e^11 + 17920*a^10*c^5*d^6*e^13 + 9728*a^11*c^4*d^4*e^15 + 2944*a^12*c^3*d^2*e^17))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12))) + (120*B*a^10*c^2*e^16 + 24*A*a^2*c^10*d^15*e + 456*A*a^9*c^3*d*e^15 + 8*C*a^3*c^9*d^15*e - 232*C*a^10*c^2*d*e^15 + 216*A*a^3*c^9*d^13*e^3 + 1176*A*a^4*c^8*d^11*e^5 + 3480*A*a^5*c^7*d^9*e^7 + 5640*A*a^6*c^6*d^7*e^9 + 5064*A*a^7*c^5*d^5*e^11 + 2376*A*a^8*c^4*d^3*e^13 - 24*B*a^3*c^9*d^14*e^2 - 408*B*a^4*c^8*d^12*e^4 - 1560*B*a^5*c^7*d^10*e^6 - 2520*B*a^6*c^6*d^8*e^8 - 1800*B*a^7*c^5*d^6*e^10 - 264*B*a^8*c^4*d^4*e^12 + 312*B*a^9*c^3*d^2*e^14 + 200*C*a^4*c^8*d^13*e^3 + 648*C*a^5*c^7*d^11*e^5 + 520*C*a^6*c^6*d^9*e^7 - 680*C*a^7*c^5*d^7*e^9 - 1512*C*a^8*c^4*d^5*e^11 - 1000*C*a^9*c^3*d^3*e^13)/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)) + (x*(192*C*a^10*c^2*e^16 - 576*A*a^9*c^3*e^16 + 1488*B*a^9*c^3*d*e^15 + 48*A*a^2*c^10*d^14*e^2 + 480*A*a^3*c^9*d^12*e^4 + 4176*A*a^4*c^8*d^10*e^6 + 12288*A*a^5*c^7*d^8*e^8 + 15312*A*a^6*c^6*d^6*e^10 + 7776*A*a^7*c^5*d^4*e^12 + 432*A*a^8*c^4*d^2*e^14 - 48*B*a^3*c^9*d^13*e^3 - 1824*B*a^4*c^8*d^11*e^5 - 5328*B*a^5*c^7*d^9*e^7 - 4032*B*a^6*c^6*d^7*e^9 + 2352*B*a^7*c^5*d^5*e^11 + 4320*B*a^8*c^4*d^3*e^13 + 16*C*a^3*c^9*d^14*e^2 + 1056*C*a^4*c^8*d^12*e^4 + 2160*C*a^5*c^7*d^10*e^6 - 1408*C*a^6*c^6*d^8*e^8 - 6672*C*a^7*c^5*d^6*e^10 - 5472*C*a^8*c^4*d^4*e^12 - 1136*C*a^9*c^3*d^2*e^14))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12))) + (9*A^2*c^9*d^11*e^2 + 342*A^2*a^2*c^7*d^7*e^6 + 36*A^2*a^3*c^6*d^5*e^8 - 7479*A^2*a^4*c^5*d^3*e^10 + 9*B^2*a^2*c^7*d^9*e^4 - 108*B^2*a^3*c^6*d^7*e^6 - 3402*B^2*a^4*c^5*d^5*e^8 + 5076*B^2*a^5*c^4*d^3*e^10 + C^2*a^2*c^7*d^11*e^2 - 36*C^2*a^3*c^6*d^9*e^4 - 1306*C^2*a^4*c^5*d^7*e^6 + 4708*C^2*a^5*c^4*d^5*e^8 - 2943*C^2*a^6*c^3*d^3*e^10 + 360*A*B*a^6*c^3*e^13 - 120*B*C*a^7*c^2*e^13 + 108*A^2*a*c^8*d^9*e^4 + 1944*A^2*a^5*c^4*d*e^12 - 855*B^2*a^6*c^3*d*e^12 + 296*C^2*a^7*c^2*d*e^12 - 18*A*B*a*c^8*d^10*e^3 + 6*A*C*a*c^8*d^11*e^2 - 1536*A*C*a^6*c^3*d*e^12 + 756*A*B*a^3*c^6*d^6*e^7 + 11016*A*B*a^4*c^5*d^4*e^9

- 7794*A*B*a^5*c^4*d^2*e^11 - 72*A*C*a^2*c^7*d^9*e^4 - 732*A*C*a^3*c^6*d^7*
e^6 - 7368*A*C*a^4*c^5*d^5*e^8 + 10182*A*C*a^5*c^4*d^3*e^10 - 6*B*C*a^2*c^7
*d^10*e^3 + 144*B*C*a^3*c^6*d^8*e^5 + 4284*B*C*a^4*c^5*d^6*e^7 - 10440*B*C*
a^5*c^4*d^4*e^9 + 3738*B*C*a^6*c^3*d^2*e^11)/(64*(a^12*e^16 + a^4*c^8*d^16
+ 8*a^11*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5
*d^10*e^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12
)) + (x*(225*B^2*a^6*c^3*e^13 + 9*A^2*c^9*d^10*e^3 - 162*A^2*a^2*c^7*d^6*e^
7 - 2916*A^2*a^3*c^6*d^4*e^9 + 6561*A^2*a^4*c^5*d^2*e^11 + 9*B^2*a^2*c^7*d^
8*e^5 - 468*B^2*a^3*c^6*d^6*e^7 + 6174*B^2*a^4*c^5*d^4*e^9 - 2340*B^2*a^5*c
^4*d^2*e^11 + C^2*a^2*c^7*d^10*e^3 - 116*C^2*a^3*c^6*d^8*e^5 + 3438*C^2*a^4
*c^5*d^6*e^7 - 4292*C^2*a^5*c^4*d^4*e^9 + 1369*C^2*a^6*c^3*d^2*e^11 + 108*A
^2*a*c^8*d^8*e^5 - 18*A*B*a*c^8*d^9*e^4 + 2430*A*B*a^5*c^4*d*e^12 + 6*A*C*a
*c^8*d^10*e^3 - 1110*B*C*a^6*c^3*d*e^12 + 360*A*B*a^2*c^7*d^7*e^6 + 3204*A*
B*a^3*c^6*d^5*e^8 - 13176*A*B*a^4*c^5*d^3*e^10 - 312*A*C*a^2*c^7*d^8*e^5 -
2028*A*C*a^3*c^6*d^6*e^7 + 10728*A*C*a^4*c^5*d^4*e^9 - 5994*A*C*a^5*c^4*d^2
*e^11 - 6*B*C*a^2*c^7*d^9*e^4 + 504*B*C*a^3*c^6*d^7*e^6 - 9300*B*C*a^4*c^5*
d^5*e^8 + 7512*B*C*a^5*c^4*d^3*e^10))/(64*(a^12*e^16 + a^4*c^8*d^16 + 8*a^1
1*c*d^2*e^14 + 8*a^5*c^7*d^14*e^2 + 28*a^6*c^6*d^12*e^4 + 56*a^7*c^5*d^10*e
^6 + 70*a^8*c^4*d^8*e^8 + 56*a^9*c^3*d^6*e^10 + 28*a^10*c^2*d^4*e^12)))*roo
t(2560*a^14*c*d^2*e^18*z^3 + 64512*a^10*c^5*d^10*e^10*z^3 + 53760*a^11*c^4*
d^8*e^12*z^3 + 53760*a^9*c^6*d^12*e^8*z^3 + 30720*a^12*c^3*d^6*e^14*z^3 + 3
0720*a^8*c^7*d^14*e^6*z^3 + 11520*a^13*c^2*d^4*e^16*z^3 + 11520*a^7*c^8*d^1
6*e^4*z^3 + 2560*a^6*c^9*d^18*e^2*z^3 + 256*a^5*c^10*d^20*z^3 + 256*a^15*e^
20*z^3 - 4806*B*C*a^8*c*d*e^13*z - 18*A*B*a*c^8*d^13*e*z - 147930*B*C*a^6*c
^3*d^5*e^9*z + 74760*B*C*a^5*c^4*d^7*e^7*z + 66588*B*C*a^7*c^2*d^3*e^11*z -
1050*B*C*a^4*c^5*d^9*e^5*z - 228*B*C*a^3*c^6*d^11*e^3*z + 152052*A*C*a^6*c
^3*d^4*e^10*z - 109830*A*C*a^5*c^4*d^6*e^8*z - 32490*A*C*a^7*c^2*d^2*e^12*z
+ 426*A*C*a^3*c^6*d^10*e^4*z - 360*A*C*a^4*c^5*d^8*e^6*z + 180*A*C*a^2*c^7
*d^12*e^2*z + 158130*A*B*a^5*c^4*d^5*e^9*z - 121356*A*B*a^6*c^3*d^3*e^11*z
- 3240*A*B*a^4*c^5*d^7*e^7*z - 1710*A*B*a^3*c^6*d^9*e^5*z - 396*A*B*a^2*c^7
*d^11*e^3*z - 6*B*C*a^2*c^7*d^13*e*z + 13518*A*B*a^7*c^2*d*e^13*z + 67615*C
^2*a^6*c^3*d^6*e^8*z - 47538*C^2*a^7*c^2*d^4*e^10*z - 24860*C^2*a^5*c^4*d^8
*e^6*z + 279*C^2*a^4*c^5*d^10*e^4*z + 46*C^2*a^3*c^6*d^12*e^2*z + 71415*B^2
*a^6*c^3*d^4*e^10*z - 55260*B^2*a^5*c^4*d^6*e^8*z - 19602*B^2*a^7*c^2*d^2*e
^12*z + 1215*B^2*a^4*c^5*d^8*e^6*z + 270*B^2*a^3*c^6*d^10*e^4*z + 9*B^2*a^2
*c^7*d^12*e^2*z - 106722*A^2*a^5*c^4*d^4*e^10*z + 35217*A^2*a^6*c^3*d^2*e^1
2*z + 6615*A^2*a^4*c^5*d^6*e^8*z + 3780*A^2*a^3*c^6*d^8*e^6*z + 1071*A^2*a^
2*c^7*d^10*e^4*z + 1152*A*C*a^8*c*e^14*z + 6*A*C*a*c^8*d^14*z + 7017*C^2*a^
8*c*d^2*e^12*z + 126*A^2*a*c^8*d^12*e^2*z + C^2*a^2*c^7*d^14*z - 1728*A^2*a
^7*c^2*e^14*z + 225*B^2*a^8*c*e^14*z + 9*A^2*c^9*d^14*z - 192*C^2*a^9*e^14*
z + 3168*A*B*C*a^4*c^2*d*e^10 + 270*A*B*C*a*c^5*d^7*e^4 - 6930*A*B*C*a^3*c^
3*d^3*e^8 + 5148*A*B*C*a^2*c^4*d^5*e^6 - 819*A^2*C*a*c^5*d^6*e^5 - 60*A*C^2
*a*c^5*d^8*e^3 - 6102*A^2*B*a^3*c^3*d*e^10 + 1512*A^2*B*a*c^5*d^5*e^6 - 270
*A*B^2*a*c^5*d^6*e^5 - 378*B*C^2*a^5*c*d*e^10 - 5049*B^2*C*a^3*c^3*d^4*e^7
+ 4698*B^2*C*a^4*c^2*d^2*e^9 + 2508*B*C^2*a^3*c^3*d^5*e^6 - 1977*B*C^2*a^4*

```
c^2*d^3*e^8 - 180*B^2*C*a^2*c^4*d^6*e^5 + 75*B*C^2*a^2*c^4*d^7*e^4 + 15921*
A^2*C*a^3*c^3*d^2*e^9 - 7848*A^2*C*a^2*c^4*d^4*e^7 - 6363*A*C^2*a^4*c^2*d^2
*e^9 + 4926*A*C^2*a^3*c^3*d^4*e^7 - 1443*A*C^2*a^2*c^4*d^6*e^5 + 14283*A^2*
B*a^2*c^4*d^3*e^8 - 4617*A*B^2*a^2*c^4*d^4*e^7 - 1944*A*B^2*a^3*c^3*d^2*e^9
 + 791*C^3*a^5*c*d^2*e^9 - 2025*B^3*a^4*c^2*d*e^10 - 1674*A^3*a*c^5*d^4*e^7
 - 90*A^2*C*c^6*d^8*e^3 + 135*A^2*B*c^6*d^7*e^4 - 1728*A^2*C*a^4*c^2*e^11 +
 675*A*B^2*a^4*c^2*e^11 - 225*B^2*C*a^5*c*e^11 + 576*A*C^2*a^5*c*e^11 - 397
*C^3*a^3*c^3*d^6*e^5 - 108*C^3*a^4*c^2*d^4*e^7 - 10*C^3*a^2*c^4*d^8*e^3 + 3
294*B^3*a^3*c^3*d^3*e^8 + 135*B^3*a^2*c^4*d^5*e^6 - 11853*A^3*a^2*c^4*d^2*e
^9 - 189*A^3*c^6*d^6*e^5 + 1728*A^3*a^3*c^3*e^11 - 64*C^3*a^6*e^11, z, k),
k, 1, 3)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**3/(c*x**2+a)**3,x)
```

```
[Out] Timed out
```

## 3.64 $\int \frac{(d+ex)^4(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

**Optimal**. Leaf size=234

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)\left(cd(4aBe+aCd+5Acd)\right.}{16a^3c^3\left(a+cx^2\right)}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^4/a/c/(c*x^2+a)^3-1/24*(e*x+d)^3*(a*(A*c+5*C
*a)*e-c*(5*A*c*d+4*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/16*(a*(A*c+5*C*a)*
e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*(-c*d*x+a*e)*(e*x+d)/a^3/c^3/(c*x^2+a)+1/1
6*(a*e^2+c*d^2)*(a*(A*c+5*C*a)*e^2+c*d*(5*A*c*d+4*B*a*e+C*a*d))*arctan(x*c^
(1/2)/a^(1/2))/a^(7/2)/c^(7/2)
```

**Rubi [A]** time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 805, 723, 205}

$$\frac{(ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(4aBe+aCd+5Acd)+ae^2(5aC+Ac)\right)}{16a^{7/2}c^{7/2}}-\frac{(d+ex)(ae-cdx)\left(cd(4aBe+aCd+5Acd)\right.}{16a^3c^3\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^4)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^3*(
a*(A*c + 5*a*C)*e - c*(5*A*c*d + a*C*d + 4*a*B*e)*x))/(24*a^2*c^2*(a + c*x^
2)^2) - ((a*(A*c + 5*a*C)*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*(a*e - c*d
*x)*(d + e*x))/(16*a^3*c^3*(a + c*x^2)) + ((c*d^2 + a*e^2)*(a*(A*c + 5*a*C)
*e^2 + c*d*(5*A*c*d + a*C*d + 4*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^
(7/2)*c^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
```

```
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^4\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^3(-5Acd-aCd-4aBe-(Ac+5aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4a}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4a}{24a^2c^2\left(a+cx^2\right)^2}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^4}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^3(a(Ac+5aC)e-c(5Acd+aCd+4a}{24a^2c^2\left(a+cx^2\right)^2}$$

**Mathematica [A]** time = 0.31, size = 437, normalized size = 1.87

$$\frac{\left(ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Ac\left(ae^2+5cd^2\right)+a\left(5aCe^2+cd(4Be+Cd)\right)\right)}{16a^{7/2}c^{7/2}}+\frac{-a^3e^3(8Be+32Cd+11Cex)+a^2ce^2x\left(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^2*c*e^2*(6*C*d
^2 + e*(4*B*d + A*e))*x - a^3*e^3*(32*C*d + 8*B*e + 11*C*e*x))/(16*a^3*c^3*
(a + c*x^2)) + (A*c^3*d^4*x - a^3*e^3*(4*C*d + B*e + C*e*x) - a*c^2*d^2*(4*
A*d*e + C*d^2*x + 6*A*e^2*x + B*d*(d + 4*e*x)) + a^2*c*e*(2*C*d^2*(2*d + 3*
e*x) + e*(A*e*(4*d + e*x) + 2*B*d*(3*d + 2*e*x))))/(6*a*c^3*(a + c*x^2)^3)
+ (5*A*c^3*d^4*x + a*c^2*d^2*(C*d^2 + 4*B*d*e + 6*A*e^2)*x + a^3*e^3*(48*C*
d + 12*B*e + 13*C*e*x) - a^2*c*e*(6*C*d^2*(4*d + 7*e*x) + e*(4*B*d*(9*d + 7
*e*x) + A*e*(24*d + 7*e*x))))/(24*a^2*c^3*(a + c*x^2)^2) + ((c*d^2 + a*e^2)
*(A*c*(5*c*d^2 + a*e^2) + a*(5*a*C*e^2 + c*d*(C*d + 4*B*e)))*ArcTan[(Sqrt[c
]*x)/Sqrt[a]])/(16*a^(7/2)*c^(7/2))
```

**fricas [B]** time = 1.46, size = 1864, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^3*d^4 + 48*B*a^5*c^2*d^2*e^2 + 16*B*a^6*c*e^4 - 6*(4*B*a
^2*c^5*d^3*e + 4*B*a^3*c^4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c
^4 + A*a^2*c^5)*d^2*e^2 - (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 32*(C*a^5*c
^2 + 2*A*a^4*c^3)*d^3*e + 32*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 48*(4*C*a^4*c^
3*d*e^3 + B*a^4*c^3*e^4)*x^4 - 16*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 +
(C*a^3*c^4 + 5*A*a^2*c^5)*d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^
5*c^2 + A*a^4*c^3)*e^4)*x^3 + 48*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 +
 B*a^5*c^2*e^4 + 2*(2*C*a^5*c^2 + A*a^4*c^3)*d*e^3)*x^2 + 3*(4*B*a^4*c^2*d^
3*e + 4*B*a^5*c*d*e^3 + (4*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5
*A*c^6)*d^4 + 6*(C*a^2*c^4 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e
^4)*x^6 + (C*a^4*c^2 + 5*A*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 +
 (5*C*a^6 + A*a^5*c)*e^4 + 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^
2*c^4 + 5*A*a*c^5)*d^4 + 6*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 +
 A*a^3*c^3)*e^4)*x^4 + 3*(4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^
3 + 5*A*a^2*c^4)*d^4 + 6*(C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a
^4*c^2)*e^4)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a))
+ 6*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4*c^3 - 11*A*a^3*c^4)*d^4
```

```
 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c + A*a^5*c^2)*e^4)*x)/(a^4
*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4), -1/48*(8*B*a^4*c^3*d^4
 + 24*B*a^5*c^2*d^2*e^2 + 8*B*a^6*c*e^4 - 3*(4*B*a^2*c^5*d^3*e + 4*B*a^3*c^
4*d*e^3 + (C*a^2*c^5 + 5*A*a*c^6)*d^4 + 6*(C*a^3*c^4 + A*a^2*c^5)*d^2*e^2 -
 (11*C*a^4*c^3 - A*a^3*c^4)*e^4)*x^5 + 16*(C*a^5*c^2 + 2*A*a^4*c^3)*d^3*e +
 16*(2*C*a^6*c + A*a^5*c^2)*d*e^3 + 24*(4*C*a^4*c^3*d*e^3 + B*a^4*c^3*e^4)*
x^4 - 8*(4*B*a^3*c^4*d^3*e - 4*B*a^4*c^3*d*e^3 + (C*a^3*c^4 + 5*A*a^2*c^5)*
d^4 - 6*(C*a^4*c^3 - A*a^3*c^4)*d^2*e^2 - (5*C*a^5*c^2 + A*a^4*c^3)*e^4)*x^
3 + 24*(2*C*a^4*c^3*d^3*e + 3*B*a^4*c^3*d^2*e^2 + B*a^5*c^2*e^4 + 2*(2*C*a^
5*c^2 + A*a^4*c^3)*d*e^3)*x^2 - 3*(4*B*a^4*c^2*d^3*e + 4*B*a^5*c*d*e^3 + (4
*B*a*c^5*d^3*e + 4*B*a^2*c^4*d*e^3 + (C*a*c^5 + 5*A*c^6)*d^4 + 6*(C*a^2*c^4
 + A*a*c^5)*d^2*e^2 + (5*C*a^3*c^3 + A*a^2*c^4)*e^4)*x^6 + (C*a^4*c^2 + 5*A
*a^3*c^3)*d^4 + 6*(C*a^5*c + A*a^4*c^2)*d^2*e^2 + (5*C*a^6 + A*a^5*c)*e^4 +
 3*(4*B*a^2*c^4*d^3*e + 4*B*a^3*c^3*d*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^4 + 6
*(C*a^3*c^3 + A*a^2*c^4)*d^2*e^2 + (5*C*a^4*c^2 + A*a^3*c^3)*e^4)*x^4 + 3*(
4*B*a^3*c^3*d^3*e + 4*B*a^4*c^2*d*e^3 + (C*a^3*c^3 + 5*A*a^2*c^4)*d^4 + 6*(
C*a^4*c^2 + A*a^3*c^3)*d^2*e^2 + (5*C*a^5*c + A*a^4*c^2)*e^4)*x^2)*sqrt(a*c
)*arctan(sqrt(a*c)*x/a) + 3*(4*B*a^4*c^3*d^3*e + 4*B*a^5*c^2*d*e^3 + (C*a^4
*c^3 - 11*A*a^3*c^4)*d^4 + 6*(C*a^5*c^2 + A*a^4*c^3)*d^2*e^2 + (5*C*a^6*c +
 A*a^5*c^2)*e^4)*x)/(a^4*c^7*x^6 + 3*a^5*c^6*x^4 + 3*a^6*c^5*x^2 + a^7*c^4)
]
```

**giac [B]** time = 0.18, size = 636, normalized size = 2.72

$$\frac{\left(Cac^2d^4 + 5\,Ac^3d^4 + 4\,Bac^2d^3e + 6\,Ca^2cd^2e^2 + 6\,Aac^2d^2e^2 + 4\,Ba^2cde^3 + 5\,Ca^3e^4 + Aa^2ce^4\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^3} + \frac{3\,C}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c^2*d^4 + 5*A*c^3*d^4 + 4*B*a*c^2*d^3*e + 6*C*a^2*c*d^2*e^2 + 6*A
*a*c^2*d^2*e^2 + 4*B*a^2*c*d*e^3 + 5*C*a^3*e^4 + A*a^2*c*e^4)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^3) + 1/48*(3*C*a*c^4*d^4*x^5 + 15*A*c^5*d^4*x^5 +
 12*B*a*c^4*d^3*x^5*e + 18*C*a^2*c^3*d^2*x^5*e^2 + 18*A*a*c^4*d^2*x^5*e^2 +
 8*C*a^2*c^3*d^4*x^3 + 40*A*a*c^4*d^4*x^3 + 12*B*a^2*c^3*d*x^5*e^3 + 32*B*a
^2*c^3*d^3*x^3*e - 33*C*a^3*c^2*x^5*e^4 + 3*A*a^2*c^3*x^5*e^4 - 96*C*a^3*c^
2*d*x^4*e^3 - 48*C*a^3*c^2*d^2*x^3*e^2 + 48*A*a^2*c^3*d^2*x^3*e^2 - 48*C*a^
3*c^2*d^3*x^2*e - 3*C*a^3*c^2*d^4*x + 33*A*a^2*c^3*d^4*x - 24*B*a^3*c^2*x^4
*e^4 - 32*B*a^3*c^2*d*x^3*e^3 - 72*B*a^3*c^2*d^2*x^2*e^2 - 12*B*a^3*c^2*d^3
*x*e - 8*B*a^3*c^2*d^4 - 40*C*a^4*c*x^3*e^4 - 8*A*a^3*c^2*x^3*e^4 - 96*C*a^
4*c*d*x^2*e^3 - 48*A*a^3*c^2*d*x^2*e^3 - 18*C*a^4*c*d^2*x*e^2 - 18*A*a^3*c^
2*d^2*x*e^2 - 16*C*a^4*c*d^3*e - 32*A*a^3*c^2*d^3*e - 24*B*a^4*c*x^2*e^4 -
12*B*a^4*c*d*x*e^3 - 24*B*a^4*c*d^2*e^2 - 15*C*a^5*x*e^4 - 3*A*a^4*c*x*e^4
- 32*C*a^5*d*e^3 - 16*A*a^4*c*d*e^3 - 8*B*a^5*e^4)/((c*x^2 + a)^3*a^3*c^3)
```

**maple [B]** time = 0.01, size = 647, normalized size = 2.76

$$\frac{A\,e^{4}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^{2}}+\frac{3A\,d^{2}e^{2}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^{2}c}+\frac{5A\,d^{4}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^{3}}+\frac{Bd\,e^{3}\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a\,c^{2}}+\frac{B\,d^{3}e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{4\sqrt{ac}\;a^{2}c}+\frac{3C\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a^2*c*e^4+6*A*a*c^2*d^2*e^2+5*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*
d^3*e-11*C*a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a^3/c*x^5-1/2*e^3*(B*e+4*
C*d)/c*x^4-1/6*(A*a^2*c*e^4-6*A*a*c^2*d^2*e^2-5*A*c^3*d^4+4*B*a^2*c*d*e^3-4
*B*a*c^2*d^3*e+5*C*a^3*e^4+6*C*a^2*c*d^2*e^2-C*a*c^2*d^4)/a^2/c^2*x^3-1/2*e
*(2*A*c*d*e^2+B*a*e^3+3*B*c*d^2*e+4*C*a*d*e^2+2*C*c*d^3)/c^2*x^2-1/16*(A*a^
2*c*e^4+6*A*a*c^2*d^2*e^2-11*A*c^3*d^4+4*B*a^2*c*d*e^3+4*B*a*c^2*d^3*e+5*C*
a^3*e^4+6*C*a^2*c*d^2*e^2+C*a*c^2*d^4)/a/c^3*x-1/6*(2*A*a*c*d*e^3+4*A*c^2*d
^3*e+B*a^2*e^4+3*B*a*c*d^2*e^2+B*c^2*d^4+4*C*a^2*d*e^3+2*C*a*c*d^3*e)/c^3)/
(c*x^2+a)^3+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^4+3/8/a^2/
c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d^2*e^2+5/16/a^3/(a*c)^(1/2)*arct
an(1/(a*c)^(1/2)*c*x)*A*d^4+1/4/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)
*B*d*e^3+1/4/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^3*e+5/16/c^3/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^4+3/8/a/c^2/(a*c)^(1/2)*arctan(1/(
a*c)^(1/2)*c*x)*C*d^2*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*
C*d^4
```

**maxima [B]** time = 1.04, size = 599, normalized size = 2.56

$$-\frac{8\,Ba^{3}c^{2}d^{4}+24\,Ba^{4}cd^{2}e^{2}+8\,Ba^{5}e^{4}-3\left(4\,Bac^{4}d^{3}e+4\,Ba^{2}c^{3}de^{3}+\left(Cac^{4}+5\,Ac^{5}\right)d^{4}+6\left(Ca^{2}c^{3}+Aac^{4}\right)d^{2}e^{2}-\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c^2*d^4 + 24*B*a^4*c*d^2*e^2 + 8*B*a^5*e^4 - 3*(4*B*a*c^4*d^
3*e + 4*B*a^2*c^3*d*e^3 + (C*a*c^4 + 5*A*c^5)*d^4 + 6*(C*a^2*c^3 + A*a*c^4)
*d^2*e^2 - (11*C*a^3*c^2 - A*a^2*c^3)*e^4)*x^5 + 16*(C*a^4*c + 2*A*a^3*c^2)
*d^3*e + 16*(2*C*a^5 + A*a^4*c)*d*e^3 + 24*(4*C*a^3*c^2*d*e^3 + B*a^3*c^2*e
^4)*x^4 - 8*(4*B*a^2*c^3*d^3*e - 4*B*a^3*c^2*d*e^3 + (C*a^2*c^3 + 5*A*a*c^4
)*d^4 - 6*(C*a^3*c^2 - A*a^2*c^3)*d^2*e^2 - (5*C*a^4*c + A*a^3*c^2)*e^4)*x^
3 + 24*(2*C*a^3*c^2*d^3*e + 3*B*a^3*c^2*d^2*e^2 + B*a^4*c*e^4 + 2*(2*C*a^4*
c + A*a^3*c^2)*d*e^3)*x^2 + 3*(4*B*a^3*c^2*d^3*e + 4*B*a^4*c*d*e^3 + (C*a^3
*c^2 - 11*A*a^2*c^3)*d^4 + 6*(C*a^4*c + A*a^3*c^2)*d^2*e^2 + (5*C*a^5 + A*a
^4*c)*e^4)*x)/(a^3*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/1
```

```
6*(4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3 + (C*a*c^2 + 5*A*c^3)*d^4 + 6*(C*a^2*c
 + A*a*c^2)*d^2*e^2 + (5*C*a^3 + A*a^2*c)*e^4)*arctan(c*x/sqrt(a*c))/(sqrt(
a*c)*a^3*c^3)
```

**mupad** [**B**] time = 4.38, size = 669, normalized size = 2.86

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x\left(c\,d^2+a\,e^2\right)\left(5\,C\,a^2\,e^2+C\,a\,c\,d^2+4\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{\sqrt{a}\,\left(5\,C\,a^3\,e^4+6\,C\,a^2\,c\,d^2\,e^2+4\,B\,a^2\,c\,d\,e^3+A\,a^2\,c\,e^4+C\,a\,c^2\,d^4+4\,B\,a\,c^2\,d^3\,e+6\,A\,a\,c^2\,d^2\,e^2+5\,A\,c^3\,d^4\right)}\right)\left(c\,d^2+a\,e^2\right)\left(5\,C\,a^2\,e^2+C\,a\right.}{16\,a^{7/2}\,c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^4*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 + C
*a*c*d^2 + 4*B*a*c*d*e))/(a^(1/2)*(5*A*c^3*d^4 + 5*C*a^3*e^4 + A*a^2*c*e^4
+ C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4
*B*a^2*c*d*e^3)))*(a*e^2 + c*d^2)*(5*A*c^2*d^2 + 5*C*a^2*e^2 + A*a*c*e^2 +
C*a*c*d^2 + 4*B*a*c*d*e))/(16*a^(7/2)*c^(7/2)) - ((B*a^2*e^4 + B*c^2*d^4 +
4*A*c^2*d^3*e + 4*C*a^2*d*e^3 + 2*A*a*c*d*e^3 + 2*C*a*c*d^3*e + 3*B*a*c*d^2
*e^2)/(6*c^3) + (x^2*(B*a*e^4 + 2*A*c*d*e^3 + 4*C*a*d*e^3 + 2*C*c*d^3*e + 3
*B*c*d^2*e^2))/(2*c^2) + (x^4*(B*e^4 + 4*C*d*e^3))/(2*c) + (x*(5*C*a^3*e^4
- 11*A*c^3*d^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^2*c*
d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a*c^3) - (x^3*(5*A*c^3*d^
4 - 5*C*a^3*e^4 - A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 - 6*C*a^2*c
*d^2*e^2 + 4*B*a*c^2*d^3*e - 4*B*a^2*c*d*e^3))/(6*a^2*c^2) - (x^5*(5*A*c^3*
d^4 - 11*C*a^3*e^4 + A*a^2*c*e^4 + C*a*c^2*d^4 + 6*A*a*c^2*d^2*e^2 + 6*C*a^
2*c*d^2*e^2 + 4*B*a*c^2*d^3*e + 4*B*a^2*c*d*e^3))/(16*a^3*c))/(a^3 + c^3*x^
6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

## 3.65 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

**Optimal**. Leaf size=254

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(Acd\left(3ae^2+5cd^2\right)+a\left(ae^2(Be+3Cd)+cd^2(3Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}-\frac{(d+ex)\left(ae(3aBe+aCd+5Acd)-x\left(3\right.\right.}{48a^3c^{3}}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^3/a/c/(c*x^2+a)^3-1/24*(e*x+d)^2*(2*a*(A*c+2
*C*a)*e-c*(5*A*c*d+3*B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2-1/48*(e*x+d)*(a*e*
(5*A*c*d+3*B*a*e+C*a*d)-(4*a*(A*c+2*C*a)*e^2+3*c*d*(5*A*c*d+3*B*a*e+C*a*d))
*x)/a^3/c^2/(c*x^2+a)+1/16*(A*c*d*(3*a*e^2+5*c*d^2)+a*(a*e^2*(B*e+3*C*d)+c*
d^2*(3*B*e+C*d)))*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/c^(5/2)
```

**Rubi [A]** time = 0.54, antiderivative size = 288, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 821, 778, 205}

$$-\frac{4ae\left(Ac\left(ae^2+5cd^2\right)+a\left(2aCe^2+cd(3Be+Cd)\right)\right)-cx\left(Acd\left(15cd^2-ae^2\right)+a\left(ae^2(7Cd-3Be)+3cd^2(3Be+C\right.\right.}{48a^3c^3\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^3)/(6*a*c*(a + c*x^2)^3) - ((d + e*x)^2*(
2*a*(A*c + 2*a*C)*e - c*(5*A*c*d + a*C*d + 3*a*B*e)*x))/(24*a^2*c^2*(a + c*
x^2)^2) - (4*a*e*(A*c*(5*c*d^2 + a*e^2) + a*(2*a*C*e^2 + c*d*(C*d + 3*B*e))
) - c*(A*c*d*(15*c*d^2 - a*e^2) + a*(a*e^2*(7*C*d - 3*B*e) + 3*c*d^2*(C*d +
 3*B*e)))*x)/(48*a^3*c^3*(a + c*x^2)) + ((A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*
e^2*(3*C*d + B*e) + c*d^2*(C*d + 3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*
a^(7/2)*c^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
```

```
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)^2(-5Acd-aCd-3aBe-2(Ac+2aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^3}{6ac\left(a+cx^2\right)^3} - \frac{(d+ex)^2(2a(Ac+2aC)e-c(5Acd+aCd+$$

**Mathematica [A]** time = 0.30, size = 350, normalized size = 1.38

$$-\frac{3\sqrt{a}\left(8a^3Ce^3-a^2ce^2x(Be+3Cd)-ac^2dx\left(3e(Ae+Bd)+Cd^2\right)-5Ac^3d^3x\right)}{a+cx^2}-\frac{8a^{5/2}\left(a^3Ce^3-a^2ce(e(Ae+3Bd+Bex)+3Cd(d+ex))+ac^2d\left(3Ae(d+ex)+Bd(d+3ex\right.\right.}{\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((-3*Sqrt[a]*(8*a^3*C*e^3 - 5*A*c^3*d^3*x - a^2*c*e^2*(3*C*d + B*e)*x - a*c
^2*d*(C*d^2 + 3*e*(B*d + A*e))*x))/(a + c*x^2) - (8*a^(5/2)*(a^3*C*e^3 - A*
c^3*d^3*x + a*c^2*d*(C*d^2*x + 3*A*e*(d + e*x) + B*d*(d + 3*e*x)) - a^2*c*e
*(3*C*d*(d + e*x) + e*(3*B*d + A*e + B*e*x))))/(a + c*x^2)^3 + (2*a^(3/2)*(
12*a^3*C*e^3 + 5*A*c^3*d^3*x + a*c^2*d*(C*d^2 + 3*e*(B*d + A*e))*x - a^2*c*
e*(3*C*d*(6*d + 7*e*x) + e*(18*B*d + 6*A*e + 7*B*e*x))))/(a + c*x^2)^2 + 3*
Sqrt[c]*(A*c*d*(5*c*d^2 + 3*a*e^2) + a*(a*e^2*(3*C*d + B*e) + c*d^2*(C*d +
3*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^3)
```

**fricas [B]** time = 0.83, size = 1378, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(48*C*a^4*c^2*e^3*x^4 + 16*B*a^4*c^2*d^3 + 24*B*a^5*c*d*e^2 - 6*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 24*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 8*(2*C*a^6 +
A*a^5*c)*e^3 - 16*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2
*c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 24*(3*C*a^4*c^2*d^2*e +
3*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 + 3*(3*B*a^4*c*d^2*e +
 B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3
*(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 +
(C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4
*c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e +
B*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2
)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(3*B*a^
4*c^2*d^2*e + B*a^5*c*e^3 + (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A
*a^4*c^2)*d*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)
, -1/48*(24*C*a^4*c^2*e^3*x^4 + 8*B*a^4*c^2*d^3 + 12*B*a^5*c*d*e^2 - 3*(3*B
*a^2*c^4*d^2*e + B*a^3*c^3*e^3 + (C*a^2*c^4 + 5*A*a*c^5)*d^3 + 3*(C*a^3*c^3
 + A*a^2*c^4)*d*e^2)*x^5 + 12*(C*a^5*c + 2*A*a^4*c^2)*d^2*e + 4*(2*C*a^6 +
A*a^5*c)*e^3 - 8*(3*B*a^3*c^3*d^2*e - B*a^4*c^2*e^3 + (C*a^3*c^3 + 5*A*a^2*
c^4)*d^3 - 3*(C*a^4*c^2 - A*a^3*c^3)*d*e^2)*x^3 + 12*(3*C*a^4*c^2*d^2*e + 3
```

```
*B*a^4*c^2*d*e^2 + (2*C*a^5*c + A*a^4*c^2)*e^3)*x^2 - 3*(3*B*a^4*c*d^2*e +
B*a^5*e^3 + (3*B*a*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*
(C*a^2*c^3 + A*a*c^4)*d*e^2)*x^6 + 3*(3*B*a^2*c^3*d^2*e + B*a^3*c^2*e^3 + (
C*a^2*c^3 + 5*A*a*c^4)*d^3 + 3*(C*a^3*c^2 + A*a^2*c^3)*d*e^2)*x^4 + (C*a^4*
c + 5*A*a^3*c^2)*d^3 + 3*(C*a^5 + A*a^4*c)*d*e^2 + 3*(3*B*a^3*c^2*d^2*e + B
*a^4*c*e^3 + (C*a^3*c^2 + 5*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)
*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(3*B*a^4*c^2*d^2*e + B*a^5*c*e^3
+ (C*a^4*c^2 - 11*A*a^3*c^3)*d^3 + 3*(C*a^5*c + A*a^4*c^2)*d*e^2)*x)/(a^4*c
^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3)]
```

**giac [B]** time = 0.17, size = 475, normalized size = 1.87

$$\frac{\left(Cacd^3 + 5\,Ac^2d^3 + 3\,Bacd^2e + 3\,Ca^2de^2 + 3\,Aacde^2 + Ba^2e^3\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2} + \frac{3\,Cac^4d^3x^5 + 15\,Ac^5d^3x^5 + 9\,Bac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^3 + 5*A*c^2*d^3 + 3*B*a*c*d^2*e + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
^2 + B*a^2*e^3)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^4
*d^3*x^5 + 15*A*c^5*d^3*x^5 + 9*B*a*c^4*d^2*x^5*e + 9*C*a^2*c^3*d*x^5*e^2 +
 9*A*a*c^4*d*x^5*e^2 + 8*C*a^2*c^3*d^3*x^3 + 40*A*a*c^4*d^3*x^3 + 3*B*a^2*c
^3*x^5*e^3 + 24*B*a^2*c^3*d^2*x^3*e - 24*C*a^3*c^2*x^4*e^3 - 24*C*a^3*c^2*d
*x^3*e^2 + 24*A*a^2*c^3*d*x^3*e^2 - 36*C*a^3*c^2*d^2*x^2*e - 3*C*a^3*c^2*d^
3*x + 33*A*a^2*c^3*d^3*x - 8*B*a^3*c^2*x^3*e^3 - 36*B*a^3*c^2*d*x^2*e^2 - 9
*B*a^3*c^2*d^2*x*e - 8*B*a^3*c^2*d^3 - 24*C*a^4*c*x^2*e^3 - 12*A*a^3*c^2*x^
2*e^3 - 9*C*a^4*c*d*x*e^2 - 9*A*a^3*c^2*d*x*e^2 - 12*C*a^4*c*d^2*e - 24*A*a
^3*c^2*d^2*e - 3*B*a^4*c*x*e^3 - 12*B*a^4*c*d*e^2 - 8*C*a^5*e^3 - 4*A*a^4*c
*e^3)/((c*x^2 + a)^3*a^3*c^3)
```

**maple [A]** time = 0.01, size = 464, normalized size = 1.83

$$\frac{3Ad\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{5A\,d^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^3} + \frac{B\,e^3\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^2} + \frac{3B\,d^2e\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c} + \frac{3Cd\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^2} + \frac{C$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(3*A*a*c*d*e^2+5*A*c^2*d^3+B*a^2*e^3+3*B*a*c*d^2*e+3*C*a^2*d*e^2+C*a*
c*d^3)/a^3*x^5-1/2*C/c*e^3*x^4+1/6*(3*A*a*c*d*e^2+5*A*c^2*d^3-B*a^2*e^3+3*B
*a*c*d^2*e-3*C*a^2*d*e^2+C*a*c*d^3)/a^2/c*x^3-1/4*e*(A*c*e^2+3*B*c*d*e+2*C*
a*e^2+3*C*c*d^2)/c^2*x^2-1/16*(3*A*a*c*d*e^2-11*A*c^2*d^3+B*a^2*e^3+3*B*a*c
```

```
*d^2*e+3*C*a^2*d*e^2+C*a*c*d^3)/a/c^2*x-1/12*(A*a*c*e^3+6*A*c^2*d^2*e+3*B*a
*c*d*e^2+2*B*c^2*d^3+2*C*a^2*e^3+3*C*a*c*d^2*e)/c^3)/(c*x^2+a)^3+3/16/a^2/c
/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d*e^2+5/16/a^3/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*A*d^3+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B
*e^3+3/16/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*d^2*e+3/16/a/c^2/(a
*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d*e^2+1/16/a^2/c/(a*c)^(1/2)*arctan(1
/(a*c)^(1/2)*c*x)*C*d^3
```

**maxima [A]** time = 1.02, size = 457, normalized size = 1.80

$$-\frac{24\,Ca^3c^2e^3x^4+8\,Ba^3c^2d^3+12\,Ba^4cde^2-3\left(3\,Bac^4d^2e+Ba^2c^3e^3+\left(Cac^4+5\,Ac^5\right)d^3+3\left(Ca^2c^3+Aac^4\right)de^2\right)x^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(24*C*a^3*c^2*e^3*x^4 + 8*B*a^3*c^2*d^3 + 12*B*a^4*c*d*e^2 - 3*(3*B*a
*c^4*d^2*e + B*a^2*c^3*e^3 + (C*a*c^4 + 5*A*c^5)*d^3 + 3*(C*a^2*c^3 + A*a*c
^4)*d*e^2)*x^5 + 12*(C*a^4*c + 2*A*a^3*c^2)*d^2*e + 4*(2*C*a^5 + A*a^4*c)*e
^3 - 8*(3*B*a^2*c^3*d^2*e - B*a^3*c^2*e^3 + (C*a^2*c^3 + 5*A*a*c^4)*d^3 - 3
*(C*a^3*c^2 - A*a^2*c^3)*d*e^2)*x^3 + 12*(3*C*a^3*c^2*d^2*e + 3*B*a^3*c^2*d
*e^2 + (2*C*a^4*c + A*a^3*c^2)*e^3)*x^2 + 3*(3*B*a^3*c^2*d^2*e + B*a^4*c*e^
3 + (C*a^3*c^2 - 11*A*a^2*c^3)*d^3 + 3*(C*a^4*c + A*a^3*c^2)*d*e^2)*x)/(a^3
*c^6*x^6 + 3*a^4*c^5*x^4 + 3*a^5*c^4*x^2 + a^6*c^3) + 1/16*(3*B*a*c*d^2*e +
 B*a^2*e^3 + (C*a*c + 5*A*c^2)*d^3 + 3*(C*a^2 + A*a*c)*d*e^2)*arctan(c*x/sq
rt(a*c))/(sqrt(a*c)*a^3*c^2)
```

**mupad [B]** time = 4.07, size = 402, normalized size = 1.58

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(3\,C\,a^2\,d\,e^2+B\,a^2\,e^3+C\,a\,c\,d^3+3\,B\,a\,c\,d^2\,e+3\,A\,a\,c\,d\,e^2+5\,A\,c^2\,d^3\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{2\,C\,a^2\,e^3+3\,C\,a\,c\,d^2\,e+3\,B\,a\,c\,d\,e^2+A\,a\,c}{12\,c^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d
*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(16*a^(7/2)*c^(5/2)) - ((2*B*c^2*d^3
 + 2*C*a^2*e^3 + A*a*c*e^3 + 6*A*c^2*d^2*e + 3*B*a*c*d*e^2 + 3*C*a*c*d^2*e)
/(12*c^3) + (x^2*(A*c*e^3 + 2*C*a*e^3 + 3*B*c*d*e^2 + 3*C*c*d^2*e))/(4*c^2)
 - (x^5*(5*A*c^2*d^3 + B*a^2*e^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e^
2 + 3*B*a*c*d^2*e))/(16*a^3) + (C*e^3*x^4)/(2*c) - (x^3*(5*A*c^2*d^3 - B*a^
2*e^3 + C*a*c*d^3 - 3*C*a^2*d*e^2 + 3*A*a*c*d*e^2 + 3*B*a*c*d^2*e))/(6*a^2*
c) + (x*(B*a^2*e^3 - 11*A*c^2*d^3 + C*a*c*d^3 + 3*C*a^2*d*e^2 + 3*A*a*c*d*e
```

```
^2 + 3*B*a*c*d^2*e))/(16*a*c^2))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4
)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

## 3.66 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(a+cx^2)^4}\,dx$

**Optimal**. Leaf size=225

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^{7/2}c^{5/2}}+\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2(aC\right.}{}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)^2/a/c/(c*x^2+a)^3+1/24*(-2*a*e*(4*A*c*d+B*a*
e+2*C*a*d)-(3*a*(A*c+C*a)*e^2-c*d*(5*A*c*d+2*B*a*e+C*a*d))*x)/a^2/c^2/(c*x^
2+a)^2+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*x/a^3/c^2/(c*x^2+
a)+1/16*(a*(A*c+C*a)*e^2+c*d*(5*A*c*d+2*B*a*e+C*a*d))*arctan(x*c^(1/2)/a^(1
/2))/a^(7/2)/c^(5/2)
```

**Rubi [A]** time = 0.40, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 778, 199, 205}

$$\frac{x\left(cd(2aBe+aCd+5Acd)+ae^2(aC+Ac)\right)}{16a^3c^2\left(a+cx^2\right)}-\frac{x\left(3ae^2(aC+Ac)-cd(2aBe+aCd+5Acd)\right)+2ae(aBe+2aCd+}{24a^2c^2\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x)^2)/(6*a*c*(a + c*x^2)^3) - (2*a*e*(4*A*c*
d + 2*a*C*d + a*B*e) + (3*a*(A*c + a*C)*e^2 - c*d*(5*A*c*d + a*C*d + 2*a*B*
e))*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((a*(A*c + a*C)*e^2 + c*d*(5*A*c*d + a*
C*d + 2*a*B*e))*x)/(16*a^3*c^2*(a + c*x^2)) + ((a*(A*c + a*C)*e^2 + c*d*(5*
A*c*d + a*C*d + 2*a*B*e))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{(d+ex)(-5Acd-aCd-2aBe-3(Ac+aC)ex)}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2\right.}{24a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2\right.}{24a^2c^2\left(a+cx^2\right)}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)^2}{6ac\left(a+cx^2\right)^3} - \frac{2ae(4Acd+2aCd+aBe)+\left(3a(Ac+aC)e^2\right.}{24a^2c^2\left(a+cx^2\right)}$$

**Mathematica [A]** time = 0.16, size = 266, normalized size = 1.18

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^{7/2}c^{5/2}}+\frac{x\left(Ac\left(ae^2+5cd^2\right)+a\left(aCe^2+cd(2Be+Cd)\right)\right)}{16a^3c^2\left(a+cx^2\right)}+\frac{a^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d + 2*B*e)))*x)/(16*a^3*c^2*(
a + c*x^2)) + (5*A*c^2*d^2*x + a*c*(C*d^2 + e*(2*B*d + A*e))*x - a^2*e*(12*
C*d + 6*B*e + 7*C*e*x))/(24*a^2*c^2*(a + c*x^2)^2) + (A*c^2*d^2*x + a^2*e*(
2*C*d + B*e + C*e*x) - a*c*(C*d^2*x + A*e*(2*d + e*x) + B*d*(d + 2*e*x)))/(
6*a*c^2*(a + c*x^2)^3) + ((A*c*(5*c*d^2 + a*e^2) + a*(a*C*e^2 + c*d*(C*d +
2*B*e)))*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(5/2))
```

**fricas [B]** time = 0.91, size = 1062, normalized size = 4.72

$$\left[-\frac{16\,Ba^4c^2d^2 + 8\,Ba^5ce^2 - 6\left(2\,Ba^2c^4de + \left(Ca^2c^4 + 5\,Aac^5\right)d^2 + \left(Ca^3c^3 + Aa^2c^4\right)e^2\right)x^5 - 16\left(2\,Ba^3c^3de + \left(Ca^3c^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c^2*d^2 + 8*B*a^5*c*e^2 - 6*(2*B*a^2*c^4*d*e + (C*a^2*c^4
+ 5*A*a*c^5)*d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 16*(2*B*a^3*c^3*d*e +
 (C*a^3*c^3 + 5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 16*(C*a
^5*c + 2*A*a^4*c^2)*d*e + 24*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 + 3*(2*B
*a^4*c*d*e + (2*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^
4)*e^2)*x^6 + 3*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2
 + A*a^2*c^3)*e^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^
2 + 3*(2*B*a^3*c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c
^2)*e^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*
(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e
^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*a^6*c^4*x^2 + a^7*c^3), -1/48*(8*B*
a^4*c^2*d^2 + 4*B*a^5*c*e^2 - 3*(2*B*a^2*c^4*d*e + (C*a^2*c^4 + 5*A*a*c^5)*
d^2 + (C*a^3*c^3 + A*a^2*c^4)*e^2)*x^5 - 8*(2*B*a^3*c^3*d*e + (C*a^3*c^3 +
5*A*a^2*c^4)*d^2 - (C*a^4*c^2 - A*a^3*c^3)*e^2)*x^3 + 8*(C*a^5*c + 2*A*a^4*
c^2)*d*e + 12*(2*C*a^4*c^2*d*e + B*a^4*c^2*e^2)*x^2 - 3*(2*B*a^4*c*d*e + (2
*B*a*c^4*d*e + (C*a*c^4 + 5*A*c^5)*d^2 + (C*a^2*c^3 + A*a*c^4)*e^2)*x^6 + 3
*(2*B*a^2*c^3*d*e + (C*a^2*c^3 + 5*A*a*c^4)*d^2 + (C*a^3*c^2 + A*a^2*c^3)*e
^2)*x^4 + (C*a^4*c + 5*A*a^3*c^2)*d^2 + (C*a^5 + A*a^4*c)*e^2 + 3*(2*B*a^3*
c^2*d*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d^2 + (C*a^4*c + A*a^3*c^2)*e^2)*x^2)*s
qrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(2*B*a^4*c^2*d*e + (C*a^4*c^2 - 11*A*a^3
*c^3)*d^2 + (C*a^5*c + A*a^4*c^2)*e^2)*x)/(a^4*c^6*x^6 + 3*a^5*c^5*x^4 + 3*
a^6*c^4*x^2 + a^7*c^3)]
```

**giac [A]** time = 0.17, size = 328, normalized size = 1.46

$$\frac{\left(Cacd^2+5\,Ac^2d^2+2\,Bacde+Ca^2e^2+Aace^2\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c^2}+\frac{3\,Cac^3d^2x^5+15\,Ac^4d^2x^5+6\,Bac^3dx^5e+3\,Ca^2c^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*c*d^2 + 5*A*c^2*d^2 + 2*B*a*c*d*e + C*a^2*e^2 + A*a*c*e^2)*arctan
(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2) + 1/48*(3*C*a*c^3*d^2*x^5 + 15*A*c^4*d^
2*x^5 + 6*B*a*c^3*d*x^5*e + 3*C*a^2*c^2*x^5*e^2 + 3*A*a*c^3*x^5*e^2 + 8*C*a
^2*c^2*d^2*x^3 + 40*A*a*c^3*d^2*x^3 + 16*B*a^2*c^2*d*x^3*e - 8*C*a^3*c*x^3*
e^2 + 8*A*a^2*c^2*x^3*e^2 - 24*C*a^3*c*d*x^2*e - 3*C*a^3*c*d^2*x + 33*A*a^2
*c^2*d^2*x - 12*B*a^3*c*x^2*e^2 - 6*B*a^3*c*d*x*e - 8*B*a^3*c*d^2 - 3*C*a^4
*x*e^2 - 3*A*a^3*c*x*e^2 - 8*C*a^4*d*e - 16*A*a^3*c*d*e - 4*B*a^4*e^2)/((c*
x^2 + a)^3*a^3*c^2)
```

**maple [A]** time = 0.01, size = 333, normalized size = 1.48

$$\frac{A\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c}+\frac{5A\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^3}+\frac{Bde\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{ac}\;a^2c}+\frac{C\,e^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a\,c^2}+\frac{C\,d^2\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c}+\frac{\left(Aac\,e^2+5A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)/a^3*x^5+1/6*(
A*a*c*e^2+5*A*c^2*d^2+2*B*a*c*d*e-C*a^2*e^2+C*a*c*d^2)/a^2/c*x^3-1/4*(B*e+2
*C*d)/c*e*x^2-1/16*(A*a*c*e^2-11*A*c^2*d^2+2*B*a*c*d*e+C*a^2*e^2+C*a*c*d^2)
/a/c^2*x-1/12*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)/c^2)/(c*x^2+a)^3+1/16
/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*e^2+5/16/a^3/(a*c)^(1/2)*arc
tan(1/(a*c)^(1/2)*c*x)*A*d^2+1/8/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x
)*B*d*e+1/16/a/c^2/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*e^2+1/16/a^2/c/(
a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C*d^2
```

**maxima [A]** time = 1.01, size = 323, normalized size = 1.44

$$-\frac{8\,Ba^3cd^2+4\,Ba^4e^2-3\left(2\,Bac^3de+\left(Cac^3+5\,Ac^4\right)d^2+\left(Ca^2c^2+Aac^3\right)e^2\right)x^5-8\left(2\,Ba^2c^2de+\left(Ca^2c^2+5\,Aac\right.\right.}{48\left(a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(8*B*a^3*c*d^2 + 4*B*a^4*e^2 - 3*(2*B*a*c^3*d*e + (C*a*c^3 + 5*A*c^4)
*d^2 + (C*a^2*c^2 + A*a*c^3)*e^2)*x^5 - 8*(2*B*a^2*c^2*d*e + (C*a^2*c^2 + 5
*A*a*c^3)*d^2 - (C*a^3*c - A*a^2*c^2)*e^2)*x^3 + 8*(C*a^4 + 2*A*a^3*c)*d*e
+ 12*(2*C*a^3*c*d*e + B*a^3*c*e^2)*x^2 + 3*(2*B*a^3*c*d*e + (C*a^3*c - 11*A
*a^2*c^2)*d^2 + (C*a^4 + A*a^3*c)*e^2)*x)/(a^3*c^5*x^6 + 3*a^4*c^4*x^4 + 3*
a^5*c^3*x^2 + a^6*c^2) + 1/16*(2*B*a*c*d*e + (C*a*c + 5*A*c^2)*d^2 + (C*a^2
 + A*a*c)*e^2)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c^2)
```

**mupad [B]** time = 0.23, size = 287, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\,a\,c\,d\,e+A\,a\,c\,e^2+5\,A\,c^2\,d^2\right)}{16\,a^{7/2}\,c^{5/2}}-\frac{\frac{B\,a\,e^2+2\,B\,c\,d^2+4\,A\,c\,d\,e+2\,C\,a\,d\,e}{12\,c^2}-\frac{x^5\left(C\,a^2\,e^2+C\,a\,c\,d^2+2\,B\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2
 + 2*B*a*c*d*e))/(16*a^(7/2)*c^(5/2)) - ((B*a*e^2 + 2*B*c*d^2 + 4*A*c*d*e +
 2*C*a*d*e)/(12*c^2) - (x^5*(5*A*c^2*d^2 + C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^
2 + 2*B*a*c*d*e))/(16*a^3) + (x^2*(B*e^2 + 2*C*d*e))/(4*c) + (x*(C*a^2*e^2
- 11*A*c^2*d^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(16*a*c^2) - (x^3*(5
*A*c^2*d^2 - C*a^2*e^2 + A*a*c*e^2 + C*a*c*d^2 + 2*B*a*c*d*e))/(6*a^2*c))/(
a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] Timed out
```

## 3.67 $\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4} dx$

**Optimal**. Leaf size=165

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}-\frac{(d+ex)(a}{6ac}$$

```
[Out] -1/6*(a*B-(A*c-C*a)*x)*(e*x+d)/a/c/(c*x^2+a)^3+1/24*(-2*a*(2*A*c+C*a)*e+c*(
5*A*c*d+B*a*e+C*a*d)*x)/a^2/c^2/(c*x^2+a)^2+1/16*(5*A*c*d+B*a*e+C*a*d)*x/a^
3/c/(c*x^2+a)+1/16*(5*A*c*d+B*a*e+C*a*d)*arctan(x*c^(1/2)/a^(1/2))/a^(7/2)/
c^(3/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1645, 639, 199, 205}

$$-\frac{2ae(aC+2Ac)-cx(aBe+aCd+5Acd)}{24a^2c^2\left(a+cx^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+aCd+5Acd)}{16a^{7/2}c^{3/2}}+\frac{x(aBe+aCd+5Acd)}{16a^3c\left(a+cx^2\right)}-\frac{(d+ex)(}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] -((a*B - (A*c - a*C)*x)*(d + e*x))/(6*a*c*(a + c*x^2)^3) - (2*a*(2*A*c + a*
C)*e - c*(5*A*c*d + a*C*d + a*B*e)*x)/(24*a^2*c^2*(a + c*x^2)^2) + ((5*A*c*
d + a*C*d + a*B*e)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c*d + a*C*d + a*B*e)*A
rcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(a+cx^2\right)^4}\,dx = -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{\int \frac{-5Acd-a(Cd+Be)-2(2Ac+aC)ex}{\left(a+cx^2\right)^3}\,dx}{6ac}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Ac}{}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Ac}{1}$$

$$= -\frac{(aB-(Ac-aC)x)(d+ex)}{6ac\left(a+cx^2\right)^3} - \frac{2a(2Ac+aC)e-c(5Acd+aCd+aBe)x}{24a^2c^2\left(a+cx^2\right)^2} + \frac{(5Ac}{1}$$

**Mathematica [A]** time = 0.13, size = 171, normalized size = 1.04

$$\frac{\frac{8a^{5/2}\left(a^2Ce-ac(Ae+B(d+ex)+Cdx)+Ac^2dx\right)}{\left(a+cx^2\right)^3} + \frac{2a^{3/2}\left(-6a^2Ce+acx(Be+Cd)+5Ac^2dx\right)}{\left(a+cx^2\right)^2} + \frac{3\sqrt{a}\,cx(aBe+aCd+5Acd)}{a+cx^2} + 3\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aBe+}{48a^{7/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x]
```

```
[Out] ((2*a^(3/2)*(-6*a^2*C*e + 5*A*c^2*d*x + a*c*(C*d + B*e)*x))/(a + c*x^2)^2 +
 (3*Sqrt[a]*c*(5*A*c*d + a*C*d + a*B*e)*x)/(a + c*x^2) + (8*a^(5/2)*(a^2*C*
e + A*c^2*d*x - a*c*(A*e + C*d*x + B*(d + e*x))))/(a + c*x^2)^3 + 3*Sqrt[c]
*(5*A*c*d + a*C*d + a*B*e)*ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(48*a^(7/2)*c^2)
```

**fricas [B]** time = 1.20, size = 636, normalized size = 3.85

$$\left[-\frac{24\,Ca^{4}cex^{2}+16\,Ba^{4}cd-6\left(Ba^{2}c^{3}e+\left(Ca^{2}c^{3}+5\,Aac^{4}\right)d\right)x^{5}-16\left(Ba^{3}c^{2}e+\left(Ca^{3}c^{2}+5\,Aa^{2}c^{3}\right)d\right)x^{3}+3\left(\left(Bac\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(24*C*a^4*c*e*x^2 + 16*B*a^4*c*d - 6*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A
*a*c^4)*d)*x^5 - 16*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 + 3*((B
*a*c^3*e + (C*a*c^3 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c
^2 + 5*A*a*c^3)*d)*x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C
*a^4 + 5*A*a^3*c)*d)*sqrt(-a*c)*log((c*x^2 - 2*sqrt(-a*c)*x - a)/(c*x^2 + a
)) + 8*(C*a^5 + 2*A*a^4*c)*e + 6*(B*a^4*c*e + (C*a^4*c - 11*A*a^3*c^2)*d)*x
)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(12*C*a^4*
c*e*x^2 + 8*B*a^4*c*d - 3*(B*a^2*c^3*e + (C*a^2*c^3 + 5*A*a*c^4)*d)*x^5 - 8
*(B*a^3*c^2*e + (C*a^3*c^2 + 5*A*a^2*c^3)*d)*x^3 - 3*((B*a*c^3*e + (C*a*c^3
 + 5*A*c^4)*d)*x^6 + B*a^4*e + 3*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*
x^4 + 3*(B*a^3*c*e + (C*a^3*c + 5*A*a^2*c^2)*d)*x^2 + (C*a^4 + 5*A*a^3*c)*d
)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 4*(C*a^5 + 2*A*a^4*c)*e + 3*(B*a^4*c*e
+ (C*a^4*c - 11*A*a^3*c^2)*d)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x
^2 + a^7*c^2)]
```

**giac [A]** time = 0.16, size = 194, normalized size = 1.18

$$\frac{\left(Cad+5\,Acd+Bae\right)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^{3}c}+\frac{3\,Cac^{3}dx^{5}+15\,Ac^{4}dx^{5}+3\,Bac^{3}x^{5}e+8\,Ca^{2}c^{2}dx^{3}+40\,Aac^{3}dx^{3}+8\,Ba^{2}c^{2}x}{48\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a*d + 5*A*c*d + B*a*e)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/
48*(3*C*a*c^3*d*x^5 + 15*A*c^4*d*x^5 + 3*B*a*c^3*x^5*e + 8*C*a^2*c^2*d*x^3
```

```
+ 40*A*a*c^3*d*x^3 + 8*B*a^2*c^2*x^3*e - 12*C*a^3*c*x^2*e - 3*C*a^3*c*d*x +
 33*A*a^2*c^2*d*x - 3*B*a^3*c*x*e - 8*B*a^3*c*d - 4*C*a^4*e - 8*A*a^3*c*e)/
((c*x^2 + a)^3*a^3*c^2)
```

**maple [A]** time = 0.01, size = 182, normalized size = 1.10

$$\frac{5Ad\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^3}+\frac{Be\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^2c}+\frac{Cd\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\,a^2c}+\frac{-\frac{Ce\,x^2}{4c}+\frac{(5Acd+Bae+Cad)c\,x^5}{16a^3}+\frac{(5Acd+Bae+Cad)x^3}{6a^2}+\frac{(11Acd-Ba}{16a}}{\left(c\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c*d+B*a*e+C*a*d)/a^3*c*x^5+1/6/a^2*(5*A*c*d+B*a*e+C*a*d)*x^3-1/4
*C/c*e*x^2+1/16*(11*A*c*d-B*a*e-C*a*d)/a/c*x-1/12*(2*A*c*e+2*B*c*d+C*a*e)/c
^2)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A*d+1/16/a^2
/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*B*e+1/16/a^2/c/(a*c)^(1/2)*arctan(
1/(a*c)^(1/2)*c*x)*C*d
```

**maxima [A]** time = 0.98, size = 208, normalized size = 1.26

$$-\frac{12\,Ca^3cex^2+8\,Ba^3cd-3\left(Bac^3e+\left(Cac^3+5\,Ac^4\right)d\right)x^5-8\left(Ba^2c^2e+\left(Ca^2c^2+5\,Aac^3\right)d\right)x^3+4\left(Ca^4+2\,Aa^3c\right.}{48\left(a^3c^5x^6+3\,a^4c^4x^4+3\,a^5c^3x^2+a^6c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(12*C*a^3*c*e*x^2 + 8*B*a^3*c*d - 3*(B*a*c^3*e + (C*a*c^3 + 5*A*c^4)*
d)*x^5 - 8*(B*a^2*c^2*e + (C*a^2*c^2 + 5*A*a*c^3)*d)*x^3 + 4*(C*a^4 + 2*A*a
^3*c)*e + 3*(B*a^3*c*e + (C*a^3*c - 11*A*a^2*c^2)*d)*x)/(a^3*c^5*x^6 + 3*a^
4*c^4*x^4 + 3*a^5*c^3*x^2 + a^6*c^2) + 1/16*(B*a*e + (C*a + 5*A*c)*d)*arcta
n(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

**mupad [B]** time = 3.94, size = 164, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\left(5\,A\,c\,d+B\,a\,e+C\,a\,d\right)}{16\,a^{7/2}\,c^{3/2}}-\frac{\frac{2\,A\,c\,e+2\,B\,c\,d+C\,a\,e}{12\,c^2}-\frac{x^3\,(5\,A\,c\,d+B\,a\,e+C\,a\,d)}{6\,a^2}+\frac{C\,e\,x^2}{4\,c}+\frac{x\,(B\,a\,e-11\,A\,c\,d+C\,a\,d)}{16\,a\,c}-\frac{c\,x^5\,(5\,A\,c}{}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(A + B*x + C*x^2))/(a + c*x^2)^4,x)
```

```
[Out] (atan((c^(1/2)*x)/a^(1/2))*(5*A*c*d + B*a*e + C*a*d))/(16*a^(7/2)*c^(3/2))
- ((2*A*c*e + 2*B*c*d + C*a*e)/(12*c^2) - (x^3*(5*A*c*d + B*a*e + C*a*d))/(
```

```
6*a^2) + (C*e*x^2)/(4*c) + (x*(B*a*e - 11*A*c*d + C*a*d))/(16*a*c) - (c*x^5
*(5*A*c*d + B*a*e + C*a*d))/(16*a^3))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^
2*x^4)
```

**sympy [A]** time = 139.97, size = 298, normalized size = 1.81

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Acd+Bae+Cad)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(-a**4*c*sqrt(-1/(a**7*c
**3)) + x)/32 + sqrt(-1/(a**7*c**3))*(5*A*c*d + B*a*e + C*a*d)*log(a**4*c*s
qrt(-1/(a**7*c**3)) + x)/32 + (-8*A*a**3*c*e - 8*B*a**3*c*d - 4*C*a**4*e -
12*C*a**3*c*e*x**2 + x**5*(15*A*c**4*d + 3*B*a*c**3*e + 3*C*a*c**3*d) + x**
3*(40*A*a*c**3*d + 8*B*a**2*c**2*e + 8*C*a**2*c**2*d) + x*(33*A*a**2*c**2*d
 - 3*B*a**3*c*e - 3*C*a**3*c*d))/(48*a**6*c**2 + 144*a**5*c**3*x**2 + 144*a
**4*c**4*x**4 + 48*a**3*c**5*x**6)
```

## 3.68 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4} dx$

**Optimal**. Leaf size=126

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

```
[Out] 1/6*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^3+1/24*(5*A*c+C*a)*x/a^2/c/(c*x^2+a)^2
+1/16*(5*A*c+C*a)*x/a^3/c/(c*x^2+a)+1/16*(5*A*c+C*a)*arctan(x*c^(1/2)/a^(1/
2))/a^(7/2)/c^(3/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1814, 12, 199, 205}

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{x(aC+5Ac)}{16a^3c\left(a+cx^2\right)}+\frac{x(aC+5Ac)}{24a^2c\left(a+cx^2\right)^2}-\frac{aB-x(Ac-aC)}{6ac\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(6*a*c*(a + c*x^2)^3) + ((5*A*c + a*C)*x)/(24*a^2*c*
(a + c*x^2)^2) + ((5*A*c + a*C)*x)/(16*a^3*c*(a + c*x^2)) + ((5*A*c + a*C)*
ArcTan[(Sqrt[c]*x)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^4}\,dx &= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}-\frac{\int \frac{-5A-\frac{aC}{c}}{\left(a+cx^2\right)^3}\,dx}{6a}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^3}\,dx}{6ac}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)\int \frac{1}{\left(a+cx^2\right)^2}\,dx}{8a^2c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\int \frac{1}{a+cx^2}\,dx}{16a^3c}\\
&= -\frac{aB-(Ac-aC)x}{6ac\left(a+cx^2\right)^3}+\frac{(5Ac+aC)x}{24a^2c\left(a+cx^2\right)^2}+\frac{(5Ac+aC)x}{16a^3c\left(a+cx^2\right)}+\frac{(5Ac+aC)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 112, normalized size = 0.89

$$\frac{(aC+5Ac)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16a^{7/2}c^{3/2}}+\frac{-a^3(8B+3Cx)+a^2cx\left(33A+8Cx^2\right)+ac^2x^3\left(40A+3Cx^2\right)+15Ac^3x^5}{48a^3c\left(a+cx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^4,x]
```

```
[Out] (15*A*c^3*x^5 - a^3*(8*B + 3*C*x) + a*c^2*x^3*(40*A + 3*C*x^2) + a^2*c*x*(3
3*A + 8*C*x^2))/(48*a^3*c*(a + c*x^2)^3) + ((5*A*c + a*C)*ArcTan[(Sqrt[c]*x
)/Sqrt[a]])/(16*a^(7/2)*c^(3/2))
```

**fricas [A]** time = 0.67, size = 430, normalized size = 3.41

$$\left[-\frac{16\,Ba^4c-6\left(Ca^2c^3+5\,Aac^4\right)x^5-16\left(Ca^3c^2+5\,Aa^2c^3\right)x^3+3\left(\left(Cac^3+5\,Ac^4\right)x^6+Ca^4+5\,Aa^3c+3\left(Ca^2c^2+\right.\right.}{96\left(a^4c^5x^6+3\,a^5c^4x^4+3\,a^6c^3x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*B*a^4*c - 6*(C*a^2*c^3 + 5*A*a*c^4)*x^5 - 16*(C*a^3*c^2 + 5*A*a^
2*c^3)*x^3 + 3*((C*a*c^3 + 5*A*c^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2
+ 5*A*a*c^3)*x^4 + 3*(C*a^3*c + 5*A*a^2*c^2)*x^2)*sqrt(-a*c)*log((c*x^2 - 2
*sqrt(-a*c)*x - a)/(c*x^2 + a)) + 6*(C*a^4*c - 11*A*a^3*c^2)*x)/(a^4*c^5*x^
6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2), -1/48*(8*B*a^4*c - 3*(C*a^2*c
^3 + 5*A*a*c^4)*x^5 - 8*(C*a^3*c^2 + 5*A*a^2*c^3)*x^3 - 3*((C*a*c^3 + 5*A*c
^4)*x^6 + C*a^4 + 5*A*a^3*c + 3*(C*a^2*c^2 + 5*A*a*c^3)*x^4 + 3*(C*a^3*c +
5*A*a^2*c^2)*x^2)*sqrt(a*c)*arctan(sqrt(a*c)*x/a) + 3*(C*a^4*c - 11*A*a^3*c
^2)*x)/(a^4*c^5*x^6 + 3*a^5*c^4*x^4 + 3*a^6*c^3*x^2 + a^7*c^2)]
```

**giac [A]** time = 0.18, size = 109, normalized size = 0.87

$$\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}+\frac{3\,Cac^2x^5+15\,Ac^3x^5+8\,Ca^2cx^3+40\,Aac^2x^3-3\,Ca^3x+33\,Aa^2cx-8\,Ba^3}{48\left(cx^2+a\right)^3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c) + 1/48*(3*C*a*c^
2*x^5 + 15*A*c^3*x^5 + 8*C*a^2*c*x^3 + 40*A*a*c^2*x^3 - 3*C*a^3*x + 33*A*a^
2*c*x - 8*B*a^3)/((c*x^2 + a)^3*a^3*c)
```

**maple [A]** time = 0.01, size = 113, normalized size = 0.90

$$\frac{5A\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^3}+\frac{C\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\sqrt{ac}\;a^2c}+\frac{\frac{(5Ac+aC)c\,x^5}{16a^3}+\frac{(5Ac+aC)x^3}{6a^2}-\frac{B}{6c}+\frac{(11Ac-aC)x}{16ac}}{\left(c\,x^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^4,x)
```

```
[Out] (1/16*(5*A*c+C*a)/a^3*c*x^5+1/6/a^2*(5*A*c+C*a)*x^3+1/16*(11*A*c-C*a)/a/c*x
-1/6*B/c)/(c*x^2+a)^3+5/16/a^3/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*A+1/16
/a^2/c/(a*c)^(1/2)*arctan(1/(a*c)^(1/2)*c*x)*C
```

**maxima [A]** time = 0.98, size = 133, normalized size = 1.06

$$\frac{3\left(Cac^2+5\,Ac^3\right)x^5-8\,Ba^3+8\left(Ca^2c+5\,Aac^2\right)x^3-3\left(Ca^3-11\,Aa^2c\right)x}{48\left(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c\right)}+\frac{(Ca+5\,Ac)\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{16\,\sqrt{ac}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(C*a*c^2 + 5*A*c^3)*x^5 - 8*B*a^3 + 8*(C*a^2*c + 5*A*a*c^2)*x^3 - 3
*(C*a^3 - 11*A*a^2*c)*x)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 + a^6
*c) + 1/16*(C*a + 5*A*c)*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^3*c)
```

**mupad [B]** time = 3.89, size = 116, normalized size = 0.92

$$\frac{\frac{x^3\,(5\,A\,c+C\,a)}{6\,a^2}-\frac{B}{6\,c}+\frac{c\,x^5\,(5\,A\,c+C\,a)}{16\,a^3}+\frac{x\,(11\,A\,c-C\,a)}{16\,a\,c}}{a^3+3\,a^2\,c\,x^2+3\,a\,c^2\,x^4+c^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)\,(5\,A\,c+C\,a)}{16\,a^{7/2}\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^4,x)
```

```
[Out] ((x^3*(5*A*c + C*a))/(6*a^2) - B/(6*c) + (c*x^5*(5*A*c + C*a))/(16*a^3) + (
x*(11*A*c - C*a))/(16*a*c))/(a^3 + c^3*x^6 + 3*a^2*c*x^2 + 3*a*c^2*x^4) + (
atan((c^(1/2)*x)/a^(1/2))*(5*A*c + C*a))/(16*a^(7/2)*c^(3/2))
```

**sympy [A]** time = 2.08, size = 196, normalized size = 1.56

$$-\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Ac+Ca)\log\left(-a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^7c^3}}\,(5Ac+Ca)\log\left(a^4c\sqrt{-\frac{1}{a^7c^3}}+x\right)}{32}+\frac{-8Ba^3+x^5\left(15Ac^3+\right.}{48a^6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**4,x)
```

```
[Out] -sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(-a**4*c*sqrt(-1/(a**7*c**3)) + x)/3
2 + sqrt(-1/(a**7*c**3))*(5*A*c + C*a)*log(a**4*c*sqrt(-1/(a**7*c**3)) + x)
/32 + (-8*B*a**3 + x**5*(15*A*c**3 + 3*C*a*c**2) + x**3*(40*A*a*c**2 + 8*C*
a**2*c) + x*(33*A*a**2*c - 3*C*a**3))/(48*a**6*c + 144*a**5*c**2*x**2 + 144
*a**4*c**3*x**4 + 48*a**3*c**4*x**6)
```

## 3.69 $\int \frac{x^3(1+x+x^2)}{(1+x^2)^2}\,dx$

**Optimal**. Leaf size=43

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-\frac{x^3}{2\left(x^2+1\right)}+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] 3/2*x+1/2*x^2-1/2*x^3/(x^2+1)-3/2*arctan(x)-1/2*ln(x^2+1)
```

**Rubi [A]** time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1804, 801, 635, 203, 260}

$$-\frac{x^3}{2\left(x^2+1\right)}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+\frac{3x}{2}-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] (3*x)/2 + x^2/2 - x^3/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{(-3-2x)x^2}{1+x^2}\,dx\\ &= -\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-3-2x+\frac{3+2x}{1+x^2}\right)dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{3+2x}{1+x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= \frac{3x}{2}+\frac{x^2}{2}-\frac{x^3}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 29, normalized size = 0.67

$$\frac{1}{2}\left(x\left(\frac{1}{x^2+1}+x+2\right)-\log\left(x^2+1\right)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (x*(2 + x + (1 + x^2)^(-1)) - 3*ArcTan[x] - Log[1 + x^2])/2
```

**fricas [A]** time = 0.93, size = 46, normalized size = 1.07

$$\frac{x^4+2x^3+x^2-3\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+3x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + 2*x^3 + x^2 - 3*(x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 3
*x)/(x^2 + 1)
```

**giac** [A] time = 0.15, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2 + x + \frac{x}{2\left(x^2+1\right)} - \frac{3}{2}\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

**maple** [A] time = 0.01, size = 30, normalized size = 0.70

$$\frac{x^2}{2} + x + \frac{x}{2x^2+2} - \frac{3\arctan(x)}{2} - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] 1/2*x^2+x+1/2*x/(x^2+1)-1/2*ln(x^2+1)-3/2*arctan(x)
```

**maxima** [A] time = 0.95, size = 29, normalized size = 0.67

$$\frac{1}{2}x^2 + x + \frac{x}{2\left(x^2+1\right)} - \frac{3}{2}\arctan(x) - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*x/(x^2 + 1) - 3/2*arctan(x) - 1/2*log(x^2 + 1)
```

**mupad** [B] time = 0.04, size = 30, normalized size = 0.70

$$x - \frac{\ln\left(x^2+1\right)}{2} - \frac{3\,\mathrm{atan}(x)}{2} + \frac{x}{2\,\left(x^2+1\right)} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x - log(x^2 + 1)/2 - (3*atan(x))/2 + x/(2*(x^2 + 1)) + x^2/2
```

**sympy [A]** time = 0.13, size = 29, normalized size = 0.67

$$\frac{x^2}{2} + x + \frac{x}{2x^2 + 2} - \frac{\log\left(x^2 + 1\right)}{2} - \frac{3\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2+1)**2,x)
[Out] x**2/2 + x + x/(2*x**2 + 2) - log(x**2 + 1)/2 - 3*atan(x)/2
```

# 3.70 $\int \frac{x^2(1+x+x^2)}{(1+x^2)^2} dx$

**Optimal**. Leaf size=30

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

```
[Out] x-1/2*x^2/(x^2+1)-arctan(x)+1/2*ln(x^2+1)
```

**Rubi** [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1804, 774, 635, 203, 260}

$$-\frac{x^2}{2(x^2+1)}+\frac{1}{2}\log(x^2+1)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] x - x^2/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
```

```
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x^2}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{(-2-2x)x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{2-2x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= x-\frac{x^2}{2\left(1+x^2\right)}-\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 27, normalized size = 0.90

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\log\left(x^2+1\right)+x-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] x + 1/(2*(1 + x^2)) - ArcTan[x] + Log[1 + x^2]/2
```

**fricas [A]** time = 0.97, size = 40, normalized size = 1.33

$$\frac{2\,x^3-2\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)+2\,x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 - 2*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) + 2*x + 1)/(x^2
 + 1)
```

**giac** [A] time = 0.21, size = 23, normalized size = 0.77

$$x + \frac{1}{2\left(x^2 + 1\right)} - \arctan(x) + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

**maple** [A] time = 0.01, size = 24, normalized size = 0.80

$$x - \arctan(x) + \frac{\ln\left(x^2 + 1\right)}{2} + \frac{1}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] x+1/2/(x^2+1)+1/2*ln(x^2+1)-arctan(x)
```

**maxima** [A] time = 0.96, size = 23, normalized size = 0.77

$$x + \frac{1}{2\left(x^2 + 1\right)} - \arctan(x) + \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] x + 1/2/(x^2 + 1) - arctan(x) + 1/2*log(x^2 + 1)
```

**mupad** [B] time = 0.03, size = 23, normalized size = 0.77

$$x + \frac{\ln\left(x^2 + 1\right)}{2} - \mathrm{atan}(x) + \frac{1}{2\,\left(x^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 + 1)^2,x)
[Out] x + log(x^2 + 1)/2 - atan(x) + 1/(2*(x^2 + 1))
```

**sympy [A]** time = 0.12, size = 20, normalized size = 0.67

$$x + \frac{\log\left(x^2+1\right)}{2} - \operatorname{atan}(x) + \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2+1)**2,x)
[Out] x + log(x**2 + 1)/2 - atan(x) + 1/(2*x**2 + 2)
```

# 3.71 $\int \frac{x(1+x+x^2)}{(1+x^2)^2} dx$

**Optimal**. Leaf size=29

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)+1/2*ln(x^2+1)
```

**Rubi [A]** time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1804, 635, 203, 260}

$$-\frac{x}{2(x^2+1)}+\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1804

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(1+x+x^2\right)}{\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-1-2x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -\frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 0.79

$$\frac{1}{2}\left(-\frac{x}{x^2+1}+\log\left(x^2+1\right)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 + x^2)^2,x]
[Out] (-(x/(1 + x^2)) + ArcTan[x] + Log[1 + x^2])/2
```

**fricas [A]** time = 0.96, size = 33, normalized size = 1.14

$$\frac{\left(x^2+1\right)\arctan(x)+\left(x^2+1\right)\log\left(x^2+1\right)-x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*((x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - x)/(x^2 + 1)
```

**giac [A]** time = 0.15, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

**maple [A]** time = 0.00, size = 24, normalized size = 0.83

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{\arctan(x)}{2}+\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2+1)^2,x)
```

```
[Out] -1/2/(x^2+1)*x+1/2*arctan(x)+1/2*ln(x^2+1)
```

**maxima [A]** time = 0.96, size = 23, normalized size = 0.79

$$-\frac{x}{2\left(x^{2}+1\right)}+\frac{1}{2}\arctan(x)+\frac{1}{2}\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + 1/2*log(x^2 + 1)
```

**mupad [B]** time = 0.03, size = 25, normalized size = 0.86

$$\frac{\ln\left(x^{2}+1\right)}{2}+\frac{\mathrm{atan}(x)}{2}-\frac{x}{2\left(x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 + 1)^2,x)
```

```
[Out] log(x^2 + 1)/2 + atan(x)/2 - x/(2*(x^2 + 1))
```

**sympy [A]** time = 0.13, size = 20, normalized size = 0.69

$$-\frac{x}{2x^{2}+2}+\frac{\log\left(x^{2}+1\right)}{2}+\frac{\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + log(x**2 + 1)/2 + atan(x)/2
```

## 3.72 $\int \frac{1+x+x^2}{(1+x^2)^2} dx$

**Optimal**. Leaf size=14

$$\tan^{-1}(x) - \frac{1}{2(x^2+1)}$$

```
[Out] -1/2/(x^2+1)+arctan(x)
```

**Rubi [A]** time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1814, 12, 203}

$$\tan^{-1}(x) - \frac{1}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 + x^2)^2,x]
```

```
[Out] -1/(2*(1 + x^2)) + ArcTan[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+x+x^2}{\left(1+x^2\right)^2}\,dx = -\frac{1}{2\left(1+x^2\right)} - \frac{1}{2}\int -\frac{2}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{2\left(1+x^2\right)} + \tan^{-1}(x)$$

**Mathematica** [**A**] time = 0.01, size = 14, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 + x^2)^2,x]
[Out] -1/2*1/(1 + x^2) + ArcTan[x]
```

**fricas** [**A**] time = 0.87, size = 20, normalized size = 1.43

$$\frac{2\left(x^2+1\right)\arctan(x) - 1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] 1/2*(2*(x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

**giac** [**A**] time = 0.15, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(x^2+1\right)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

**maple** [**A**] time = 0.00, size = 13, normalized size = 0.93

$$\arctan(x) - \frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2+1)^2,x)
[Out] -1/2/(x^2+1)+arctan(x)
```

**maxima** [**A**] time = 0.96, size = 12, normalized size = 0.86

$$-\frac{1}{2\left(x^2+1\right)} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+1)^2,x, algorithm="maxima")
[Out] -1/2/(x^2 + 1) + arctan(x)
```

**mupad** [**B**] time = 3.80, size = 14, normalized size = 1.00

$$\mathrm{atan}(x) - \frac{1}{2\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 + 1)^2,x)
[Out] atan(x) - 1/(2*(x^2 + 1))
```

**sympy** [**A**] time = 0.11, size = 10, normalized size = 0.71

$$\mathrm{atan}\,(x) - \frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+1)**2,x)
[Out] atan(x) - 1/(2*x**2 + 2)
```

# 3.73 $\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx$

**Optimal**. Leaf size=31

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

**Rubi [A]** time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{x}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2 + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x+x^2}{x\left(1+x^2\right)^2}\,dx &= \frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-x}{x\left(1+x^2\right)}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{-1+2x}{1+x^2}\right)dx\\
&= \frac{x}{2\left(1+x^2\right)}+\log(x)-\frac{1}{2}\int \frac{-1+2x}{1+x^2}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}+\log(x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= \frac{x}{2\left(1+x^2\right)}+\frac{1}{2}\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.90

$$\frac{1}{2}\left(\frac{x}{x^2+1}-\log\left(x^2+1\right)+2\log(x)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 + x^2)^2),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x] + 2*Log[x] - Log[1 + x^2])/2
```

**fricas [A]** time = 1.02, size = 41, normalized size = 1.32

$$\frac{\left(x^2+1\right)\arctan(x)-\left(x^2+1\right)\log\left(x^2+1\right)+2\left(x^2+1\right)\log(x)+x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - (x^2 + 1)*log(x^2 + 1) + 2*(x^2 + 1)*log(x) + x)
/(x^2 + 1)
```

**giac** [A] time = 0.15, size = 26, normalized size = 0.84

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

**maple** [A] time = 0.01, size = 26, normalized size = 0.84

$$\frac{x}{2x^2+2}+\frac{\arctan(x)}{2}+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)*x+1/2*arctan(x)+ln(x)-1/2*ln(x^2+1)
```

**maxima** [A] time = 0.96, size = 25, normalized size = 0.81

$$\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

**mupad** [B] time = 0.04, size = 32, normalized size = 1.03

$$\ln(x)+\frac{x}{2\left(x^2+1\right)}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 - 1i/4) - log(x - 1i)*(1/2 + 1i/4) + x/(2*(x^2 +
1))
```

**sympy** [A] time = 0.16, size = 24, normalized size = 0.77

$$\frac{x}{2x^2+2}+\log(x)-\frac{\log\left(x^2+1\right)}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 + atan(x)/2
```

# 3.74 $\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2} \, dx$

**Optimal**. Leaf size=33

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

**Rubi [A]** time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 801, 635, 203, 260}

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
```

```
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{x^2\left(1+x^2\right)^2}\,dx &= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x}{x^2\left(1+x^2\right)}\,dx\\ &= \frac{1}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^2}-\frac{2}{x}+\frac{2(1+x)}{1+x^2}\right)dx\\ &= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1+x}{1+x^2}\,dx\\ &= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}+\log(x)-\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= -\frac{1}{x}+\frac{1}{2\left(1+x^2\right)}-\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 33, normalized size = 1.00

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}+\log(x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 + x^2)^2),x]
[Out] -x^(-1) + 1/(2*(1 + x^2)) - ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

**fricas [A]** time = 0.71, size = 49, normalized size = 1.48

$$-\frac{2\,x^2+2\left(x^3+x\right)\arctan(x)+\left(x^3+x\right)\log\left(x^2+1\right)-2\left(x^3+x\right)\log(x)-x+2}{2\left(x^3+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*(x^3 + x)*arctan(x) + (x^3 + x)*log(x^2 + 1) - 2*(x^3 + x)*
log(x) - x + 2)/(x^3 + x)
```

**giac** [A] time = 0.18, size = 35, normalized size = 1.06

$$-\frac{2\,x^2-x+2}{2\left(x^3+x\right)}-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

**maple** [A] time = 0.01, size = 30, normalized size = 0.91

$$-\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{x}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2+1)^2,x)
```

```
[Out] -1/x+1/2/(x^2+1)-arctan(x)+ln(x)-1/2*ln(x^2+1)
```

**maxima** [A] time = 0.95, size = 34, normalized size = 1.03

$$-\frac{2\,x^2-x+2}{2\left(x^3+x\right)}-\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x^2 - x + 2)/(x^3 + x) - arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

**mupad** [B] time = 3.81, size = 38, normalized size = 1.15

$$\ln(x)-\frac{x^2-\frac{x}{2}+1}{x^3+x}+\ln\left(x-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 + 1)^2),x)
```

```
[Out] log(x) - log(x + 1i)*(1/2 + 1i/2) - log(x - 1i)*(1/2 - 1i/2) - (x^2 - x/2 +
 1)/(x + x^3)
```

**sympy** [A] time = 0.16, size = 31, normalized size = 0.94

$$\log(x) - \frac{\log\left(x^2+1\right)}{2} - \operatorname{atan}(x) + \frac{-2x^2+x-2}{2x^3+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x) + (-2*x**2 + x - 2)/(2*x**3 + 2*x)
```

## 3.75 $\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2} dx$

**Optimal**. Leaf size=45

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

```
[Out] -1/2/x^2-1/x-1/2*x/(x^2+1)-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

**Rubi** [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1805, 1802, 635, 203, 260}

$$-\frac{x}{2\left(x^2+1\right)}-\frac{1}{2x^2}+\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{x}-\log(x)-\frac{3}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - x^(-1) - x/(2*(1 + x^2)) - (3*ArcTan[x])/2 - Log[x] + Log[1 +
x^2]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{x^3\left(1+x^2\right)^2}\,dx &= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2-2x+x^3}{x^3\left(1+x^2\right)}\,dx\\
&= -\frac{x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x^3}-\frac{2}{x^2}+\frac{2}{x}+\frac{3-2x}{1+x^2}\right)dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{1}{2}\int \frac{3-2x}{1+x^2}\,dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\log(x)-\frac{3}{2}\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1}{2x^2}-\frac{1}{x}-\frac{x}{2\left(1+x^2\right)}-\frac{3}{2}\tan^{-1}(x)-\log(x)+\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.87

$$\frac{1}{2}\left(-\frac{x}{x^2+1}-\frac{1}{x^2}+\log\left(x^2+1\right)-\frac{2}{x}-2\log(x)-3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 + x^2)^2),x]
[Out] (-x^(-2) - 2/x - x/(1 + x^2) - 3*ArcTan[x] - 2*Log[x] + Log[1 + x^2])/2
```

**fricas [A]** time = 1.07, size = 61, normalized size = 1.36

$$-\frac{3\,x^3+x^2+3\left(x^4+x^2\right)\arctan(x)-\left(x^4+x^2\right)\log\left(x^2+1\right)+2\left(x^4+x^2\right)\log(x)+2\,x+1}{2\left(x^4+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^3 + x^2 + 3*(x^4 + x^2)*arctan(x) - (x^4 + x^2)*log(x^2 + 1) + 2*
(x^4 + x^2)*log(x) + 2*x + 1)/(x^4 + x^2)
```

**giac [A]** time = 0.16, size = 43, normalized size = 0.96

$$-\frac{3\,x^3+x^2+2\,x+1}{2\left(x^2+1\right)x^2}-\frac{3}{2}\,\arctan(x)+\frac{1}{2}\,\log\left(x^2+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/((x^2 + 1)*x^2) - 3/2*arctan(x) + 1/2*log(x^2
+ 1) - log(abs(x))
```

**maple [A]** time = 0.01, size = 38, normalized size = 0.84

$$-\frac{x}{2\left(x^2+1\right)}-\frac{3\arctan(x)}{2}-\ln(x)+\frac{\ln\left(x^2+1\right)}{2}-\frac{1}{x}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2+1)^2,x)
```

```
[Out] -1/2/x^2-1/x-1/2/(x^2+1)*x-3/2*arctan(x)-ln(x)+1/2*ln(x^2+1)
```

**maxima [A]** time = 0.97, size = 41, normalized size = 0.91

$$-\frac{3\,x^3+x^2+2\,x+1}{2\left(x^4+x^2\right)}-\frac{3}{2}\,\arctan(x)+\frac{1}{2}\,\log\left(x^2+1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^3 + x^2 + 2*x + 1)/(x^4 + x^2) - 3/2*arctan(x) + 1/2*log(x^2 + 1)
 - log(x)
```

**mupad [B]** time = 0.04, size = 47, normalized size = 1.04

$$-\ln(x)-\frac{\frac{3\,x^3}{2}+\frac{x^2}{2}+x+\frac{1}{2}}{x^4+x^2}+\ln\left(x-\mathrm{i}\right)\left(\frac{1}{2}+\frac{3}{4}\mathrm{i}\right)+\ln\left(x+1\mathrm{i}\right)\left(\frac{1}{2}-\frac{3}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 + 1)^2),x)
```

```
[Out] log(x - 1i)*(1/2 + 3i/4) + log(x + 1i)*(1/2 - 3i/4) - log(x) - (x + x^2/2 +
 (3*x^3)/2 + 1/2)/(x^2 + x^4)
```

**sympy** [A] time = 0.18, size = 42, normalized size = 0.93

$$-\log(x) + \frac{\log(x^2+1)}{2} - \frac{3\,\mathrm{atan}(x)}{2} + \frac{-3x^3 - x^2 - 2x - 1}{2x^4 + 2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2+1)**2,x)
```

```
[Out] -log(x) + log(x**2 + 1)/2 - 3*atan(x)/2 + (-3*x**3 - x**2 - 2*x - 1)/(2*x**
4 + 2*x**2)
```

## 3.76 $\int \frac{1+2x+x^2}{\left(1+x^2\right)^2} \, dx$

**Optimal**. Leaf size=12

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

[Out] -1/(x^2+1)+arctan(x)

**Rubi [A]** time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.83, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {27, 723, 203}

$$\tan^{-1}(x) - \frac{(1-x)(x+1)}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2)/(1 + x^2)^2,x]

[Out] -((1 - x)*(1 + x))/(2*(1 + x^2)) + ArcTan[x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x+x^2}{\left(1+x^2\right)^2}\,dx = \int \frac{(1+x)^2}{\left(1+x^2\right)^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{(1-x)(1+x)}{2\left(1+x^2\right)} + \tan^{-1}(x)$$

**Mathematica** [A] time = 0.01, size = 12, normalized size = 1.00

$$\tan^{-1}(x) - \frac{1}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)/(1 + x^2)^2,x]
[Out] -(1 + x^2)^(-1) + ArcTan[x]
```

**fricas** [A] time = 1.04, size = 18, normalized size = 1.50

$$\frac{\left(x^2+1\right)\arctan(x) - 1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="fricas")
[Out] ((x^2 + 1)*arctan(x) - 1)/(x^2 + 1)
```

**giac** [A] time = 0.15, size = 12, normalized size = 1.00

$$-\frac{1}{x^2+1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="giac")
[Out] -1/(x^2 + 1) + arctan(x)
```

**maple** [A] time = 0.00, size = 13, normalized size = 1.08

$$\arctan(x) - \frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+1)/(x^2+1)^2,x)
```

```
[Out] -1/(x^2+1)+arctan(x)
```

**maxima [A]** time = 0.95, size = 12, normalized size = 1.00

$$-\frac{1}{x^2+1}+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/(x^2 + 1) + arctan(x)
```

**mupad [B]** time = 0.03, size = 12, normalized size = 1.00

$$\mathrm{atan}(x)-\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2 + 1)/(x^2 + 1)^2,x)
```

```
[Out] atan(x) - 1/(x^2 + 1)
```

**sympy [A]** time = 0.12, size = 8, normalized size = 0.67

$$\mathrm{atan}\,(x)-\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+1)/(x**2+1)**2,x)
```

```
[Out] atan(x) - 1/(x**2 + 1)
```

# 3.77 $\int \frac{2+12x+3x^2}{(4+x^2)^2} dx$

**Optimal**. Leaf size=27

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

```
[Out] 1/4*(-24-5*x)/(x^2+4)+7/8*arctan(1/2*x)
```

**Rubi [A]** time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1814, 12, 203}

$$\frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)-\frac{5x+24}{4\left(x^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
```

```
[Out] -(24 + 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+12x+3x^2}{(4+x^2)^2}\,dx = -\frac{24+5x}{4(4+x^2)} - \frac{1}{8}\int -\frac{14}{4+x^2}\,dx$$
$$= -\frac{24+5x}{4(4+x^2)} + \frac{7}{4}\int \frac{1}{4+x^2}\,dx$$
$$= -\frac{24+5x}{4(4+x^2)} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

**Mathematica** [**A**] time = 0.01, size = 27, normalized size = 1.00

$$\frac{-5x-24}{4(x^2+4)} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 12*x + 3*x^2)/(4 + x^2)^2,x]
[Out] (-24 - 5*x)/(4*(4 + x^2)) + (7*ArcTan[x/2])/8
```

**fricas** [**A**] time = 0.83, size = 25, normalized size = 0.93

$$\frac{7(x^2+4)\arctan\left(\frac{1}{2}x\right) - 10x - 48}{8(x^2+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="fricas")
[Out] 1/8*(7*(x^2 + 4)*arctan(1/2*x) - 10*x - 48)/(x^2 + 4)
```

**giac** [**A**] time = 0.18, size = 21, normalized size = 0.78

$$-\frac{5x+24}{4(x^2+4)} + \frac{7}{8}\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="giac")
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

**maple** [A] time = 0.01, size = 21, normalized size = 0.78

$$\frac{7\arctan\left(\frac{x}{2}\right)}{8}+\frac{-\frac{5x}{4}-6}{x^{2}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+12*x+2)/(x^2+4)^2,x)
```

```
[Out] (-5/4*x-6)/(x^2+4)+7/8*arctan(1/2*x)
```

**maxima** [A] time = 0.97, size = 21, normalized size = 0.78

$$-\frac{5\,x+24}{4\left(x^{2}+4\right)}+\frac{7}{8}\,\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+12*x+2)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(5*x + 24)/(x^2 + 4) + 7/8*arctan(1/2*x)
```

**mupad** [B] time = 3.83, size = 21, normalized size = 0.78

$$\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}-\frac{\frac{5\,x}{4}+6}{x^{2}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x + 3*x^2 + 2)/(x^2 + 4)^2,x)
```

```
[Out] (7*atan(x/2))/8 - ((5*x)/4 + 6)/(x^2 + 4)
```

**sympy** [A] time = 0.13, size = 20, normalized size = 0.74

$$\frac{-5x-24}{4x^{2}+16}+\frac{7\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+12*x+2)/(x**2+4)**2,x)
```

```
[Out] (-5*x - 24)/(4*x**2 + 16) + 7*atan(x/2)/8
```

# 3.78 $\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=390

$$\frac{x\sqrt{a+cx^2}\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(a^2h^2(eh+3fg)-2acg\left(3\right.\right.}{16c^{5/2}}$$

```
[Out] -1/70*(8*a*f*h^2+c*(3*f*g^2-7*h*(2*d*h+e*g)))*(h*x+g)^2*(c*x^2+a)^(3/2)/c^2
/h-1/42*(-7*e*h+3*f*g)*(h*x+g)^3*(c*x^2+a)^(3/2)/c/h+1/7*f*(h*x+g)^4*(c*x^2
+a)^(3/2)/c/h+1/840*(64*a^2*f*h^4-16*a*c*h^2*(15*f*g^2+7*h*(d*h+3*e*g))-8*c
^2*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-3*c*h*(a*h^2*(35*e*h+41*f*g)+2*c*g*(3*f*g
^2-7*h*(7*d*h+e*g)))*x)*(c*x^2+a)^(3/2)/c^3/h+1/16*a*(8*c^2*d*g^3+a^2*h^2*(
e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2)
)/c^(5/2)+1/16*(8*c^2*d*g^3+a^2*h^2*(e*h+3*f*g)-2*a*c*g*(f*g^2+3*h*(d*h+e*g
)))*x*(c*x^2+a)^(1/2)/c^2
```

**Rubi [A]** time = 0.83, antiderivative size = 387, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{\left(a+cx^2\right)^{3/2}\left(8\left(8a^2fh^4-2ach^2\left(7h(dh+3eg)+15fg^2\right)-c^2\left(3fg^4-7g^2h(12dh+eg)\right)\right)-3chx\left(ah^2(35eh+41fg\right.\right.}{840c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*
x*Sqrt[a + c*x^2])/(16*c^2) - ((3*c*f*g^2 + 8*a*f*h^2 - 7*c*h*(e*g + 2*d*h)
)*(g + h*x)^2*(a + c*x^2)^(3/2))/(70*c^2*h) - ((3*f*g - 7*e*h)*(g + h*x)^3*
(a + c*x^2)^(3/2))/(42*c*h) + (f*(g + h*x)^4*(a + c*x^2)^(3/2))/(7*c*h) + (
(8*(8*a^2*f*h^4 - 2*a*c*h^2*(15*f*g^2 + 7*h*(3*e*g + d*h)) - c^2*(3*f*g^4 -
 7*g^2*h*(e*g + 12*d*h))) - 3*c*h*(6*c*f*g^3 - 14*c*g*h*(e*g + 7*d*h) + a*h
^2*(41*f*g + 35*e*h))*x)*(a + c*x^2)^(3/2))/(840*c^3*h) + (a*(8*c^2*d*g^3 +
 a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c
]*x)/Sqrt[a + c*x^2]])/(16*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left((7cd-4af)h^2-ch(3fg-7eh)x\right.}{7ch^2}$$

$$= -\frac{(3fg-7eh)(g+hx)^3\left(a+cx^2\right)^{3/2}}{42ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+}{}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-7eh}{}$$

$$= -\frac{\left(3cfg^2+8afh^2-7ch(eg+2dh)\right)(g+hx)^2\left(a+cx^2\right)^{3/2}}{70c^2h} - \frac{(3fg-7eh}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(}{}$$

$$= \frac{\left(8c^2dg^3+a^2h^2(3fg+eh)-2acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(}{}$$

**Mathematica [A]** time = 0.46, size = 362, normalized size = 0.93

$$\frac{105a\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(a^2h^2(eh+3fg)-2acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)+\sqrt{a+cx^2}\left(16cx^2\left(-4a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(16*a*(8*a^2*f*h^3 + 35*c^2*g^2*(e*g + 3*d*h) - 14*a*c*h*(
3*f*g^2 + h*(3*e*g + d*h))) + 105*c*(8*c^2*d*g^3 - a^2*h^2*(3*f*g + e*h) +
2*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 16*c*(-4*a^2*f*h^3 + 35*c^2*g^2*(e*g
 + 3*d*h) + 7*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 70*c^2*(a*h^2*(3*f*g
 + e*h) + 6*c*(f*g^3 + 3*g*h*(e*g + d*h)))*x^3 + 48*c^2*h*(a*f*h^2 + 7*c*(3
*f*g^2 + h*(3*e*g + d*h)))*x^4 + 280*c^3*h^2*(3*f*g + e*h)*x^5 + 240*c^3*f*
h^3*x^6) + 105*a*Sqrt[c]*(8*c^2*d*g^3 + a^2*h^2*(3*f*g + e*h) - 2*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(1680*c^3)
```

**fricas [A]** time = 0.97, size = 855, normalized size = 2.19

$$\left[-\frac{105\left(6a^2ceg^2h-a^3eh^3-2\left(4ac^2d-a^2cf\right)g^3+3\left(2a^2cd-a^3f\right)gh^2\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3360*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2*(4*a*c^2*d - a^2*c*f)*g^3 +
3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(
c)*x - a) - 2*(240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2*c*e*g*h^2 + 28
0*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*c^3*e*g*h^2 + (
7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^2*h - 32*(7*a^2
*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c^2*e*h^3 + 3*(6
*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e*g*h^2 + 21*(5*
c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 + 105*(6*a*c^2*e*
g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2*d - a^2*c*f)*g
*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/1680*(105*(6*a^2*c*e*g^2*h - a^3*e*h^3 - 2
*(4*a*c^2*d - a^2*c*f)*g^3 + 3*(2*a^2*c*d - a^3*f)*g*h^2)*sqrt(-c)*arctan(s
qrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h^3*x^6 + 560*a*c^2*e*g^3 - 672*a^2
*c*e*g*h^2 + 280*(3*c^3*f*g*h^2 + c^3*e*h^3)*x^5 + 48*(21*c^3*f*g^2*h + 21*
c^3*e*g*h^2 + (7*c^3*d + a*c^2*f)*h^3)*x^4 + 336*(5*a*c^2*d - 2*a^2*c*f)*g^
2*h - 32*(7*a^2*c*d - 4*a^3*f)*h^3 + 70*(6*c^3*f*g^3 + 18*c^3*e*g^2*h + a*c
^2*e*h^3 + 3*(6*c^3*d + a*c^2*f)*g*h^2)*x^3 + 16*(35*c^3*e*g^3 + 21*a*c^2*e
*g*h^2 + 21*(5*c^3*d + a*c^2*f)*g^2*h + (7*a*c^2*d - 4*a^2*c*f)*h^3)*x^2 +
105*(6*a*c^2*e*g^2*h - a^2*c*e*h^3 + 2*(4*c^3*d + a*c^2*f)*g^3 + 3*(2*a*c^2
*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac [A]** time = 0.23, size = 475, normalized size = 1.22

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,fh^3x+\frac{7\left(3\,c^5fgh^2+c^5h^3e\right)}{c^5}\right)x+\frac{6\left(21\,c^5fg^2h+7\,c^5dh^3+ac^4fh^3+21\,c^5gh^2e\right)}{c^5}\right)x+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*f*h^3*x + 7*(3*c^5*f*g*h^2 + c^5*h^3*e
)/c^5)*x + 6*(21*c^5*f*g^2*h + 7*c^5*d*h^3 + a*c^4*f*h^3 + 21*c^5*g*h^2*e)/
c^5)*x + 35*(6*c^5*f*g^3 + 18*c^5*d*g*h^2 + 3*a*c^4*f*g*h^2 + 18*c^5*g^2*h*
e + a*c^4*h^3*e)/c^5)*x + 8*(105*c^5*d*g^2*h + 21*a*c^4*f*g^2*h + 7*a*c^4*d
*h^3 - 4*a^2*c^3*f*h^3 + 35*c^5*g^3*e + 21*a*c^4*g*h^2*e)/c^5)*x + 105*(8*c
^5*d*g^3 + 2*a*c^4*f*g^3 + 6*a*c^4*d*g*h^2 - 3*a^2*c^3*f*g*h^2 + 6*a*c^4*g^
2*h*e - a^2*c^3*h^3*e)/c^5)*x + 16*(105*a*c^4*d*g^2*h - 42*a^2*c^3*f*g^2*h
- 14*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 35*a*c^4*g^3*e - 42*a^2*c^3*g*h^2*e)
/c^5) - 1/16*(8*a*c^2*d*g^3 - 2*a^2*c*f*g^3 - 6*a^2*c*d*g*h^2 + 3*a^3*f*g*h
^2 - 6*a^2*c*g^2*h*e + a^3*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

**maple [A]** time = 0.02, size = 661, normalized size = 1.69

$$\frac{\left(c\,x^{2}+a\right)^{\frac{3}{2}} f\,h^{3} x^{4}}{7c}+\frac{\left(c\,x^{2}+a\right)^{\frac{3}{2}} e\,h^{3} x^{3}}{6c}+\frac{\left(c\,x^{2}+a\right)^{\frac{3}{2}} f g\,h^{2} x^{3}}{2c}+\frac{a^{3} e\,h^{3} \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{16 c^{\frac{5}{2}}}+\frac{3 a^{3} f g\,h^{2} \ln\left(\sqrt{c}\,x+\right.}{16 c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+a)^(3/2)/c*e*g^3+1/2*d*g^3*x*(c*x^2+a)^(1/2)+3/5*x^2*(c*x^2+a)^(
3/2)/c*e*g*h^2-2/5*a/c^2*(c*x^2+a)^(3/2)*f*g^2*h+3/5*x^2*(c*x^2+a)^(3/2)/c*
f*g^2*h-2/5*a/c^2*(c*x^2+a)^(3/2)*e*g*h^2+3/4*x*(c*x^2+a)^(3/2)/c*d*g*h^2+3
/4*x*(c*x^2+a)^(3/2)/c*e*g^2*h-1/8*a/c*x*(c*x^2+a)^(1/2)*f*g^3-3/8*a^2/c^(3
/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g^2*h-4/35*f*h^3*a/c^2*x^2*(c*x^2+a)^(3
/2)+1/2*x^3*(c*x^2+a)^(3/2)/c*f*g*h^2-1/8*a/c^2*x*(c*x^2+a)^(3/2)*e*h^3+1/1
6*a^2/c^2*x*(c*x^2+a)^(1/2)*e*h^3+3/16*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(
1/2))*f*g*h^2-3/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*g*h^2-3/8*a/c
^2*x*(c*x^2+a)^(3/2)*f*g*h^2-3/8*a/c*x*(c*x^2+a)^(1/2)*e*g^2*h+3/16*a^2/c^2
*x*(c*x^2+a)^(1/2)*f*g*h^2-3/8*a/c*x*(c*x^2+a)^(1/2)*d*g*h^2+1/6*x^3*(c*x^2
+a)^(3/2)/c*e*h^3+1/16*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h^3+1/5*
x^2*(c*x^2+a)^(3/2)/c*d*h^3-2/15*a/c^2*(c*x^2+a)^(3/2)*d*h^3+1/4*x*(c*x^2+a
)^(3/2)/c*f*g^3-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^3+1/7*f*h
^3*x^4*(c*x^2+a)^(3/2)/c+8/105*f*h^3*a^2/c^3*(c*x^2+a)^(3/2)+(c*x^2+a)^(3/2
)/c*d*g^2*h+1/2*d*g^3*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.47, size = 436, normalized size = 1.12

$$\frac{\left(cx^{2}+a\right)^{\frac{3}{2}} f h^{3} x^{4}}{7\,c}-\frac{4\left(cx^{2}+a\right)^{\frac{3}{2}} a f h^{3} x^{2}}{35\,c^{2}}+\frac{1}{2}\sqrt{cx^{2}+a}\,d g^{3} x+\frac{a d g^{3} \operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{c}}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}} e g^{3}}{3\,c}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}} d g^{2} h}{c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(3/2)*f*h^3*x^4/c - 4/35*(c*x^2 + a)^(3/2)*a*f*h^3*x^2/c^2
+ 1/2*sqrt(c*x^2 + a)*d*g^3*x + 1/2*a*d*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c)
+ 1/3*(c*x^2 + a)^(3/2)*e*g^3/c + (c*x^2 + a)^(3/2)*d*g^2*h/c + 8/105*(c*x^
2 + a)^(3/2)*a^2*f*h^3/c^3 + 1/6*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*x^3/
c + 1/5*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(3/2)*x^2/c - 1/8*(3*f*
g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a*x/c^2 + 1/16*(3*f*g*h^2 + e*h^3)*sqrt(c*
x^2 + a)*a^2*x/c^2 + 1/4*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a)^(3/2)*
x/c - 1/8*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a*x/c + 1/16*(3*f
*g*h^2 + e*h^3)*a^3*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/8*(f*g^3 + 3*e*g^2*h
```

```
 + 3*d*g*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/15*(3*f*g^2*h + 3*e*g*
h^2 + d*h^3)*(c*x^2 + a)^(3/2)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^3 \sqrt{cx^2+a}\ (fx^2+ex+d)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 28.37, size = 1088, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*e*h**3*x/(16*c**2*sqrt(1 + c*x**2/a)) - 3*a**(5/2)*f*g*h**2*x/(16
*c**2*sqrt(1 + c*x**2/a)) + 3*a**(3/2)*d*g*h**2*x/(8*c*sqrt(1 + c*x**2/a))
+ 3*a**(3/2)*e*g**2*h*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*e*h**3*x**3/(48
*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g**3*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3
/2)*f*g*h**2*x**3/(16*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*g**3*x*sqrt(1 + c*x
**2/a)/2 + 9*sqrt(a)*d*g*h**2*x**3/(8*sqrt(1 + c*x**2/a)) + 9*sqrt(a)*e*g**
2*h*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*e*h**3*x**5/(24*sqrt(1 + c*x**2
/a)) + 3*sqrt(a)*f*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*g*h**2*x*
*5/(8*sqrt(1 + c*x**2/a)) + a**3*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/
2)) + 3*a**3*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - 3*a**2*d*g*h
**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - 3*a**2*e*g**2*h*asinh(sqrt(c)*x
/sqrt(a))/(8*c**(3/2)) - a**2*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2))
+ a*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 3*d*g**2*h*Piecewise((sqr
t(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + d*h**3*Piecewi
se((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x
**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + e*g**3*Piecewi
se((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 3*e*g*h
**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)
/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*f
*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x
**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) +
 f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g*h**2*x**5/(4*sqrt(a)*sqr
t(1 + c*x**2/a)) + 3*c*e*g**2*h*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*h
```

```
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*g**3*x**5/(4*sqrt(a)*sqrt(1 +
 c*x**2/a)) + c*f*g*h**2*x**7/(2*sqrt(a)*sqrt(1 + c*x**2/a))
```

# 3.79 $\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=280

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+\right.}{16c^{5/2}}$$

```
[Out] -1/10*(-2*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h+1/6*f*(h*x+g)^3*(c*x^2+a)^
(3/2)/c/h-1/120*(16*a*h^2*(e*h+2*f*g)+8*c*g*(f*g^2-2*h*(5*d*h+e*g))-3*h*(5*
(-a*f+2*c*d)*h^2-2*c*g*(-2*e*h+f*g))*x)*(c*x^2+a)^(3/2)/c^2/h+1/16*a*(8*c^2
*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(
1/2))/c^(5/2)+1/16*(8*c^2*d*g^2+a^2*f*h^2-2*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c
*x^2+a)^(1/2)/c^2
```

**Rubi [A]** time = 0.50, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\sqrt{a+cx^2}\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{16c^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(a^2fh^2-2ac\left(h(dh+2eg)+fg^2\right)+\right.}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a + c*x
^2])/(16*c^2) - ((f*g - 2*e*h)*(g + h*x)^2*(a + c*x^2)^(3/2))/(10*c*h) + (f
*(g + h*x)^3*(a + c*x^2)^(3/2))/(6*c*h) - ((8*(c*f*g^3 - 2*c*g*h*(e*g + 5*d
*h) + 2*a*h^2*(2*f*g + e*h)) - 3*h*(5*(2*c*d - a*f)*h^2 - 2*c*g*(f*g - 2*e*
h))*x)*(a + c*x^2)^(3/2))/(120*c^2*h) + (a*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c
*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(5/
2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(3(2cd-af)h^2-3ch(fg-2eh\right.}{6ch^2}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+}{}$$

$$= -\frac{(fg-2eh)(g+hx)^2\left(a+cx^2\right)^{3/2}}{10ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{3/2}}{6ch} - \frac{\left(8\left(cf\right.\right.}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh}{}$$

$$= \frac{\left(8c^2dg^2+a^2fh^2-2ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{16c^2} - \frac{(fg-2eh}{}$$

**Mathematica [A]** time = 0.65, size = 256, normalized size = 0.91

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(a^2(-h)(32eh+64fg+15fhx)+2ac\left(5dh(16g+3hx)+e\left(40g^2+30ghx+8h^2x^2\right)+fx\left(15g^2+\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(-(a^2*h*(64*f*g + 32*e*h + 15*f*h*x)) + 2*a*c*(5
*d*h*(16*g + 3*h*x) + f*x*(15*g^2 + 16*g*h*x + 5*h^2*x^2) + e*(40*g^2 + 30*
g*h*x + 8*h^2*x^2)) + 4*c^2*x*(5*d*(6*g^2 + 8*g*h*x + 3*h^2*x^2) + x*(2*e*(
10*g^2 + 15*g*h*x + 6*h^2*x^2) + f*x*(15*g^2 + 24*g*h*x + 10*h^2*x^2)))) +
(15*Sqrt[a]*(8*c^2*d*g^2 + a^2*f*h^2 - 2*a*c*(f*g^2 + h*(2*e*g + d*h)))*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(5/2))
```

**fricas [A]** time = 1.05, size = 595, normalized size = 2.12

$$\left[-\frac{15\left(4a^2cegh-2\left(4ac^2d-a^2cf\right)g^2+\left(2a^2cd-a^3f\right)h^2\right)\sqrt{c}\log\left(-2cx^2-2\sqrt{cx^2+a}\sqrt{c}x-a\right)-2\left(40c^3fh^2x^5\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(4*a^2*c*e*g*h - 2*(4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3
*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(40*c^
3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2 + 48*(2*c^3*f*g*h + c^3*e*h^2
)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^3*d + a*c^2*f)*h^2)*x^3 + 32*
(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2 + a*c^2*e*h^2 + 2*(5*c^3*d +
a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c^3*d + a*c^2*f)*g^2 + (2*a*c^
2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/240*(15*(4*a^2*c*e*g*h - 2*(
4*a*c^2*d - a^2*c*f)*g^2 + (2*a^2*c*d - a^3*f)*h^2)*sqrt(-c)*arctan(sqrt(-c
)*x/sqrt(c*x^2 + a)) + (40*c^3*f*h^2*x^5 + 80*a*c^2*e*g^2 - 32*a^2*c*e*h^2
+ 48*(2*c^3*f*g*h + c^3*e*h^2)*x^4 + 10*(6*c^3*f*g^2 + 12*c^3*e*g*h + (6*c^
3*d + a*c^2*f)*h^2)*x^3 + 32*(5*a*c^2*d - 2*a^2*c*f)*g*h + 16*(5*c^3*e*g^2
+ a*c^2*e*h^2 + 2*(5*c^3*d + a*c^2*f)*g*h)*x^2 + 15*(4*a*c^2*e*g*h + 2*(4*c
^3*d + a*c^2*f)*g^2 + (2*a*c^2*d - a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac [A]** time = 0.21, size = 321, normalized size = 1.15

$$\frac{1}{240}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,fh^2x+\frac{6\left(2\,c^4fgh+c^4h^2e\right)}{c^4}\right)x+\frac{5\left(6\,c^4fg^2+6\,c^4dh^2+ac^3fh^2+12\,c^4ghe\right)}{c^4}\right)x+\frac{8\left(10\,c^4d\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*f*h^2*x + 6*(2*c^4*f*g*h + c^4*h^2*e)/c^4)
*x + 5*(6*c^4*f*g^2 + 6*c^4*d*h^2 + a*c^3*f*h^2 + 12*c^4*g*h*e)/c^4)*x + 8*
(10*c^4*d*g*h + 2*a*c^3*f*g*h + 5*c^4*g^2*e + a*c^3*h^2*e)/c^4)*x + 15*(8*c
^4*d*g^2 + 2*a*c^3*f*g^2 + 2*a*c^3*d*h^2 - a^2*c^2*f*h^2 + 4*a*c^3*g*h*e)/c
^4)*x + 16*(10*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + 5*a*c^3*g^2*e - 2*a^2*c^2*h^
2*e)/c^4) - 1/16*(8*a*c^2*d*g^2 - 2*a^2*c*f*g^2 - 2*a^2*c*d*h^2 + a^3*f*h^2
 - 4*a^2*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple [A]** time = 0.01, size = 446, normalized size = 1.59

$$\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}f\,h^2x^3}{6c}+\frac{a^3f\,h^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{a^2d\,h^2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^2egh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{4c^{\frac{3}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/6*f*h^2*x^3*(c*x^2+a)^(3/2)/c-1/8*f*h^2*a/c^2*x*(c*x^2+a)^(3/2)+1/16*f*h^
2*a^2/c^2*x*(c*x^2+a)^(1/2)+1/16*f*h^2*a^3/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(
1/2))+1/5*x^2*(c*x^2+a)^(3/2)/c*e*h^2+2/5*x^2*(c*x^2+a)^(3/2)/c*f*g*h-2/15*
```

```
a/c^2*(c*x^2+a)^(3/2)*e*h^2-4/15*a/c^2*(c*x^2+a)^(3/2)*f*g*h+1/4*x*(c*x^2+a
)^(3/2)/c*d*h^2+1/2*x*(c*x^2+a)^(3/2)/c*e*g*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g^2
-1/8*a/c*x*(c*x^2+a)^(1/2)*d*h^2-1/4*a/c*x*(c*x^2+a)^(1/2)*e*g*h-1/8*a/c*x*
(c*x^2+a)^(1/2)*f*g^2-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2-1
/4*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/8*a^2/c^(3/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))*f*g^2+2/3*(c*x^2+a)^(3/2)/c*d*g*h+1/3*(c*x^2+a)^(3/2)
/c*e*g^2+1/2*d*g^2*x*(c*x^2+a)^(1/2)+1/2*d*g^2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^
2+a)^(1/2))
```

**maxima [A]** time = 0.46, size = 305, normalized size = 1.09

$$\frac{\left(cx^2+a\right)^{\frac{3}{2}} fh^2x^3}{6\,c}+\frac{1}{2}\,\sqrt{cx^2+a}\,dg^2x-\frac{\left(cx^2+a\right)^{\frac{3}{2}} afh^2x}{8\,c^2}+\frac{\sqrt{cx^2+a}\,a^2fh^2x}{16\,c^2}+\frac{adg^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}+\frac{a^3fh^2\operatorname{arsinh}\left(\right.}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(3/2)*f*h^2*x^3/c + 1/2*sqrt(c*x^2 + a)*d*g^2*x - 1/8*(c*x^
2 + a)^(3/2)*a*f*h^2*x/c^2 + 1/16*sqrt(c*x^2 + a)*a^2*f*h^2*x/c^2 + 1/2*a*d
*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/16*a^3*f*h^2*arcsinh(c*x/sqrt(a*c))
/c^(5/2) + 1/3*(c*x^2 + a)^(3/2)*e*g^2/c + 2/3*(c*x^2 + a)^(3/2)*d*g*h/c +
1/5*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*x^2/c + 1/4*(f*g^2 + 2*e*g*h + d*h^
2)*(c*x^2 + a)^(3/2)*x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a*
x/c - 1/8*(f*g^2 + 2*e*g*h + d*h^2)*a^2*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
15*(2*f*g*h + e*h^2)*(c*x^2 + a)^(3/2)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 21.00, size = 738, normalized size = 2.64

$$-\frac{a^{\frac{5}{2}}fh^2x}{16c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dh^2x}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}eghx}{4c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fg^2x}{8c\sqrt{1+\frac{cx^2}{a}}}-\frac{a^{\frac{3}{2}}fh^2x^3}{48c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dg^2x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,dh^2x^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}}{4\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] -a**(5/2)*f*h**2*x/(16*c**2*sqrt(1 + c*x**2/a)) + a**(3/2)*d*h**2*x/(8*c*sq
rt(1 + c*x**2/a)) + a**(3/2)*e*g*h*x/(4*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*
g**2*x/(8*c*sqrt(1 + c*x**2/a)) - a**(3/2)*f*h**2*x**3/(48*c*sqrt(1 + c*x**
2/a)) + sqrt(a)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*d*h**2*x**3/(8*sq
rt(1 + c*x**2/a)) + 3*sqrt(a)*e*g*h*x**3/(4*sqrt(1 + c*x**2/a)) + 3*sqrt(a)
*f*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*f*h**2*x**5/(24*sqrt(1 + c*
x**2/a)) + a**3*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(5/2)) - a**2*d*h**2
*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*e*g*h*asinh(sqrt(c)*x/sqrt(a)
)/(4*c**(3/2)) - a**2*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3/2)) + a*d*g*
*2*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + 2*d*g*h*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g**2*Piecewise((sqrt(a)
*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*h**2*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 2*f*g*h*Piecewise(
(-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4
*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h**2*x**5/(4*
sqrt(a)*sqrt(1 + c*x**2/a)) + c*e*g*h*x**5/(2*sqrt(a)*sqrt(1 + c*x**2/a)) +
 c*f*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c*f*h**2*x**7/(6*sqrt(a)*sq
rt(1 + c*x**2/a))
```

## 3.80 $\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=175

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+a)^(3/2)/c/h-1/60*(8*a*f*h^2+4*c*(3*f*g^2-5*h*(d*h+e
*g))+3*c*h*(-5*e*h+3*f*g)*x)*(c*x^2+a)^(3/2)/c^2/h+1/8*a*(-a*e*h-a*f*g+4*c*
d*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(4*c*d*g-a*(e*h+f*g))*x
*(c*x^2+a)^(1/2)/c
```

**Rubi [A]** time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1654, 780, 195, 217, 206}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(4\left(2afh^2+c\left(3fg^2-5h(dh+eg)\right)\right)+3chx(3fg-5eh)\right)}{60c^2h}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+4cdg)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d*g - a*(f*g + e*h))*x*Sqrt[a + c*x^2])/(8*c) + (f*(g + h*x)^2*(a + c
*x^2)^(3/2))/(5*c*h) - ((4*(2*a*f*h^2 + c*(3*f*g^2 - 5*h*(e*g + d*h))) + 3*
c*h*(3*f*g - 5*e*h)*x)*(a + c*x^2)^(3/2))/(60*c^2*h) + (a*(4*c*d*g - a*f*g
- a*e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}+\frac{\int (g+hx)\left((5cd-2af)h^2-ch(3fg-5eh)x\right)}{5ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}-\frac{\left(4\left(2afh^2+c\left(3fg^2-5h(eg+dh)\right)\right)+3ch(3}{60c^2h}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}-\frac{\left(4\left(2afh^2+\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}-\frac{\left(4\left(2afh^2+\right.\right.}{}$$

$$= \frac{(4cdg-a(fg+eh))x\sqrt{a+cx^2}}{8c}+\frac{f(g+hx)^2\left(a+cx^2\right)^{3/2}}{5ch}-\frac{\left(4\left(2afh^2+\right.\right.}{}$$

**Mathematica [A]** time = 0.42, size = 153, normalized size = 0.87

$$\frac{\sqrt{a+cx^2}\left(-16a^2fh-\frac{15\sqrt{a}\sqrt{c}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)(aeh+afg-4cdg)}{\sqrt{\frac{cx^2}{a}+1}}+ac(40dh+5e(8g+3hx)+fx(15g+8hx))+2c^2x(10d(3\ldots\right.}{120c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(-16*a^2*f*h + a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g
 + 8*h*x)) + 2*c^2*x*(10*d*(3*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*
g + 4*h*x))) - (15*Sqrt[a]*Sqrt[c]*(-4*c*d*g + a*f*g + a*e*h)*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(120*c^2)
```

**fricas [A]** time = 1.00, size = 329, normalized size = 1.88

$$\left[\frac{15\left(a^2eh-\left(4\,acd-a^2f\right)g\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(24\,c^2fhx^4+40\,aceg+30\left(c^2fg+c^2eh\right)\ldots\right.}{240\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*
x^2 + a)*sqrt(c)*x - a) + 2*(24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^
2*e*h)*x^3 + 8*(5*c^2*e*g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f
)*h + 15*(a*c*e*h + (4*c^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/120*(15
*(a^2*e*h - (4*a*c*d - a^2*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)
) + (24*c^2*f*h*x^4 + 40*a*c*e*g + 30*(c^2*f*g + c^2*e*h)*x^3 + 8*(5*c^2*e*
g + (5*c^2*d + a*c*f)*h)*x^2 + 8*(5*a*c*d - 2*a^2*f)*h + 15*(a*c*e*h + (4*c
^2*d + a*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

**giac [A]** time = 0.21, size = 180, normalized size = 1.03

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(4\,fhx+\frac{5\left(c^3fg+c^3he\right)}{c^3}\right)x+\frac{4\left(5\,c^3dh+ac^2fh+5\,c^3ge\right)}{c^3}\right)x+\frac{15\left(4\,c^3dg+ac^2fg+ac^2he\right)}{c^3}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h*x + 5*(c^3*f*g + c^3*h*e)/c^3)*x + 4*(5
*c^3*d*h + a*c^2*f*h + 5*c^3*g*e)/c^3)*x + 15*(4*c^3*d*g + a*c^2*f*g + a*c^
```

```
2*h*e)/c^3)*x + 8*(5*a*c^2*d*h - 2*a^2*c*f*h + 5*a*c^2*g*e)/c^3) - 1/8*(4*a
*c*d*g - a^2*f*g - a^2*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple** [A] time = 0.01, size = 230, normalized size = 1.31

$$-\frac{a^{2}eh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^{2}fg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{8c^{\frac{3}{2}}}+\frac{adg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{\sqrt{c\,x^{2}+a}\;aehx}{8c}-\frac{\sqrt{c\,x^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h*f*x^2*(c*x^2+a)^(3/2)/c-2/15*h*f*a/c^2*(c*x^2+a)^(3/2)+1/4*x*(c*x^2+a
)^(3/2)/c*e*h+1/4*x*(c*x^2+a)^(3/2)/c*f*g-1/8*a/c*x*(c*x^2+a)^(1/2)*e*h-1/8
*a/c*x*(c*x^2+a)^(1/2)*f*g-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*
h-1/8*a^2/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/3*(c*x^2+a)^(3/2)/c*d
*h+1/3*(c*x^2+a)^(3/2)/c*e*g+1/2*d*g*x*(c*x^2+a)^(1/2)+1/2*d*g*a/c^(1/2)*ln
(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima** [A] time = 0.45, size = 169, normalized size = 0.97

$$\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}fhx^{2}}{5\,c}+\frac{1}{2}\sqrt{cx^{2}+a}\,dgx+\frac{adg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}eg}{3\,c}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}dh}{3\,c}-\frac{2\left(cx^{2}+a\right)^{\frac{3}{2}}afh}{15\,c^{2}}+\frac{\left(cx^{2}+a\right)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(c*x^2 + a)^(3/2)*f*h*x^2/c + 1/2*sqrt(c*x^2 + a)*d*g*x + 1/2*a*d*g*arc
sinh(c*x/sqrt(a*c))/sqrt(c) + 1/3*(c*x^2 + a)^(3/2)*e*g/c + 1/3*(c*x^2 + a)
^(3/2)*d*h/c - 2/15*(c*x^2 + a)^(3/2)*a*f*h/c^2 + 1/4*(c*x^2 + a)^(3/2)*(f*
g + e*h)*x/c - 1/8*sqrt(c*x^2 + a)*(f*g + e*h)*a*x/c - 1/8*(f*g + e*h)*a^2*
arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(g+h\,x\right)\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 11.88, size = 384, normalized size = 2.19

$$\frac{a^{\frac{3}{2}}ehx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}fgx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,ehx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,fgx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2eh\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}-\frac{a^2fg\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*e*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(3/2)*f*g*x/(8*c*sqrt(1 + c*x*
*2/a)) + sqrt(a)*d*g*x*sqrt(1 + c*x**2/a)/2 + 3*sqrt(a)*e*h*x**3/(8*sqrt(1
+ c*x**2/a)) + 3*sqrt(a)*f*g*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*e*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**2*f*g*asinh(sqrt(c)*x/sqrt(a))/(8*c**(3
/2)) + a*d*g*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + d*h*Piecewise((sqrt(a)*
x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + e*g*Piecewise((sqrt
(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + f*h*Piecewise((
-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*
sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*h*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a)) + c*f*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a))
```

# 3.81 $\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=106

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

```
[Out] 1/3*e*(c*x^2+a)^(3/2)/c+1/4*f*x*(c*x^2+a)^(3/2)/c+1/8*a*(-a*f+4*c*d)*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/8*(-a*f+4*c*d)*x*(c*x^2+a)^(1/2)/c
```

**Rubi [A]** time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a(4cd-af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8c^{3/2}}+\frac{x\sqrt{a+cx^2}(4cd-af)}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((4*c*d - a*f)*x*Sqrt[a + c*x^2])/(8*c) + (e*(a + c*x^2)^(3/2))/(3*c) + (f*
x*(a + c*x^2)^(3/2))/(4*c) + (a*(4*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
c*x^2]])/(8*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{\int (4cd-af+4cex)\sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(4cd-af)\int \sqrt{a+cx^2}\,dx}{4c}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\int \frac{\ldots}{\sqrt{a\ldots}}}{8c}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{(a(4cd-af))\,\mathrm{Subs}\ldots}{\ldots}$$

$$= \frac{(4cd-af)x\sqrt{a+cx^2}}{8c}+\frac{e\left(a+cx^2\right)^{3/2}}{3c}+\frac{fx\left(a+cx^2\right)^{3/2}}{4c}+\frac{a(4cd-af)\tanh^{-}\ldots}{8c^{3/2}}$$

**Mathematica [A]** time = 0.20, size = 98, normalized size = 0.92

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\,(a(8e+3fx)+2cx(6d+x(4e+3fx)))-\frac{3\sqrt{a}\,(af-4cd)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(a*(8*e + 3*f*x) + 2*c*x*(6*d + x*(4*e + 3*f*x)))
 - (3*Sqrt[a]*(-4*c*d + a*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)
/a]))/(24*c^(3/2))
```

**fricas [A]** time = 0.95, size = 190, normalized size = 1.79

$$\left[-\frac{3\left(4\,acd-a^2f\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(6\,c^2fx^3+8\,c^2ex^2+8\,ace+3\left(4\,c^2d+acf\right)x\right)\sqrt{cx^2+}}{48\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(4*a*c*d - a^2*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c
)*x - a) - 2*(6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c*f)*x)*
sqrt(c*x^2 + a))/c^2, -1/24*(3*(4*a*c*d - a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x
/sqrt(c*x^2 + a)) - (6*c^2*f*x^3 + 8*c^2*e*x^2 + 8*a*c*e + 3*(4*c^2*d + a*c
*f)*x)*sqrt(c*x^2 + a))/c^2]
```

**giac [A]** time = 0.18, size = 87, normalized size = 0.82

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(3\,fx+4\,e\right)x+\frac{3\left(4\,c^2d+acf\right)}{c^2}\right)x+\frac{8\,ae}{c}\right)-\frac{\left(4\,acd-a^2f\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*x + 4*e)*x + 3*(4*c^2*d + a*c*f)/c^2)*x + 8*a
*e/c) - 1/8*(4*a*c*d - a^2*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2
)
```

**maple [A]** time = 0.00, size = 111, normalized size = 1.05

$$-\frac{a^2f\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{3}{2}}}+\frac{ad\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\,afx}{8c}+\frac{\sqrt{c\,x^2+a}\,dx}{2}+\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}fx}{4c}+\frac{\left(c\,x^2+\right.}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+a)^(3/2)/c-1/8*f*a/c*x*(c*x^2+a)^(1/2)-1/8*f*a^2/c^(3/2)*ln(
c^(1/2)*x+(c*x^2+a)^(1/2))+1/3*e*(c*x^2+a)^(3/2)/c+1/2*d*x*(c*x^2+a)^(1/2)+
1/2*d*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.45, size = 96, normalized size = 0.91

$$\frac{1}{2}\sqrt{cx^2+a}\,dx+\frac{\left(cx^2+a\right)^{\frac{3}{2}}fx}{4\,c}-\frac{\sqrt{cx^2+a}\,afx}{8\,c}+\frac{ad\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}}-\frac{a^2f\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{3}{2}}}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}e}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*d*x + 1/4*(c*x^2 + a)^(3/2)*f*x/c - 1/8*sqrt(c*x^2 + a)
*a*f*x/c + 1/2*a*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/8*a^2*f*arcsinh(c*x/s
qrt(a*c))/c^(3/2) + 1/3*(c*x^2 + a)^(3/2)*e/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 6.90, size = 170, normalized size = 1.60

$$\frac{a^{\frac{3}{2}}fx}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3\sqrt{a}\,fx^3}{8\sqrt{1+\frac{cx^2}{a}}}-\frac{a^2f\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{3}{2}}}+\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2\sqrt{c}}+e\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } c=0\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c} & \text{otherwise}\end{cases}\right)+\frac{}{4\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
```

```
[Out] a**(3/2)*f*x/(8*c*sqrt(1 + c*x**2/a)) + sqrt(a)*d*x*sqrt(1 + c*x**2/a)/2 +
3*sqrt(a)*f*x**3/(8*sqrt(1 + c*x**2/a)) - a**2*f*asinh(sqrt(c)*x/sqrt(a))/(
8*c**(3/2)) + a*d*asinh(sqrt(c)*x/sqrt(a))/(2*sqrt(c)) + e*Piecewise((sqrt(
a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*f*x**5/(4*sqrt
(a)*sqrt(1 + c*x**2/a))
```

## 3.82 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

**Optimal**. Leaf size=206

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}\,h^4}-\frac{\sqrt{ah^2+cg^2}\,\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)}{h^4}+\cdots$$

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h-1/2*(2*c*d*g*h^2+(-e*h+f*g)*(a*h^2+2*c*g^2))*arct
anh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)-(d*h^2-e*g*h+f*g^2)*arctanh((-c*
g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*h^2+c*g^2)^(1/2)/h^4+1/2*(
2*d*h^2-2*e*g*h+2*f*g^2-h*(-e*h+f*g)*x)*(c*x^2+a)^(1/2)/h^3
```

**Rubi [A]** time = 0.39, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\,\left(2\left(dh^2-egh+fg^2\right)-hx(fg-eh)\right)}{2h^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(\left(ah^2+2cg^2\right)(fg-eh)+2cdgh^2\right)}{2\sqrt{c}\,h^4}-\frac{\sqrt{ah^2+cg^2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((2*(f*g^2 - e*g*h + d*h^2) - h*(f*g - e*h)*x)*Sqrt[a + c*x^2])/(2*h^3) + (
f*(a + c*x^2)^(3/2))/(3*c*h) - ((2*c*d*g*h^2 + (f*g - e*h)*(2*c*g^2 + a*h^2
))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*h^4) - (Sqrt[c*g^2 + a*
h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2])])/h^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(3cdh^2-3ch(fg-eh)x\right)\sqrt{a+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{3ac^2h^2\left(fg^2-\right.}{}}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} + \frac{\left(\left(cg^2+ah^2\right)\right.}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(\left(cg^2+ah^2\right)\right.}{}$$

$$= \frac{\left(2\left(fg^2-egh+dh^2\right)-h(fg-eh)x\right)\sqrt{a+cx^2}}{2h^3} + \frac{f\left(a+cx^2\right)^{3/2}}{3ch} - \frac{\left(2cdgh^2+(f\right.}{}$$

**Mathematica [A]** time = 0.44, size = 224, normalized size = 1.09

$$\frac{\left(h(dh-eg)+fg^2\right)\left(-\sqrt{ah^2+cg^2}\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)-\sqrt{c}\,g\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+h\sqrt{a+cx^2}\right)}{h^4} + \frac{\sqrt{a+cx^2}\left(\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(3/2))/(3*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(2*Sqrt[c]*h^2
*Sqrt[1 + (c*x^2)/a]) + ((f*g^2 + h*(-(e*g) + d*h))*(h*Sqrt[a + c*x^2] - Sq
rt[c]*g*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[c*g^2 + a*h^2]*ArcTanh[
(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])]))/h^4
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.22, size = 278, normalized size = 1.35

$$\frac{1}{6}\sqrt{cx^2+a}\left(\left(\frac{2\,fx}{h}-\frac{3\left(cfgh^8-ch^9e\right)}{ch^{10}}\right)x+\frac{2\left(3\,cfg^2h^7+3\,cdh^9+afh^9-3\,cgh^8e\right)}{ch^{10}}\right)+\frac{2\left(cfg^4+cdg^2h^2+afg^2h\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*x/h - 3*(c*f*g*h^8 - c*h^9*e)/(c*h^10))*x + 2*(3*
c*f*g^2*h^7 + 3*c*d*h^9 + a*f*h^9 - 3*c*g*h^8*e)/(c*h^10)) + 2*(c*f*g^4 + c
*d*g^2*h^2 + a*f*g^2*h^2 + a*d*h^4 - c*g^3*h*e - a*g*h^3*e)*arctan(-((sqrt(
c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 -
 a*h^2)*h^4) + 1/2*(2*c^(3/2)*f*g^3 + 2*c^(3/2)*d*g*h^2 + a*sqrt(c)*f*g*h^2
 - 2*c^(3/2)*g^2*h*e - a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a
)))/(c*h^4)
```

**maple [B]** time = 0.02, size = 1265, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+a)^(3/2)/c/h+1/2/h*e*x*(c*x^2+a)^(1/2)+1/2/h*e*a/c^(1/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))-1/2/h^2*f*g*x*(c*x^2+a)^(1/2)-1/2/h^2*f*g*a/c^(1/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/h*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g
^2)/h^2)^(1/2)*d-1/h^2*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2
)*e*g+1/h^3*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1/h
^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*
h^2+c*g^2)/h^2)^(1/2))*d+1/h^3*c^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+((
x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))*e-1/h^4*c^(1/2)*g^3*ln
((-c*g/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)
^(1/2))*f-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+
g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*a*d+1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^
2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+
g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+
g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2-1/h^3/(
(a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x
+g/h))*c*g^2*d+1/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2+c*g^2)/h^2-2*c*
```

```
g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-2*c*g/h*(x+g/h)+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*e-1/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((2*
(a*h^2+c*g^2)/h^2-2*c*g/h*(x+g/h)+2*((a*h^2+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c-
2*c*g/h*(x+g/h)+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f
```

**maxima [A]** time = 0.61, size = 362, normalized size = 1.76

$$-\frac{\sqrt{cx^2+a}\,fgx}{2\,h^2}+\frac{\sqrt{cx^2+a}\,ex}{2\,h}-\frac{\sqrt{c}\,fg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}+\frac{\sqrt{c}\,eg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}-\frac{\sqrt{c}\,dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^2}-\frac{afg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,\sqrt{c}\,h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*f*g*x/h^2 + 1/2*sqrt(c*x^2 + a)*e*x/h - sqrt(c)*f*g^3*
arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^3 - sqr
t(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 1/2*a*f*g*arcsinh(c*x/sqrt(a*c))/(sqr
t(c)*h^2) + 1/2*a*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h) + sqrt(a + c*g^2/h^2
)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/h^3 - sqrt(a + c*g^2/h^2)*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + sqrt(a + c*g^2/h^2)*d*arcsinh(c*g*x/(sq
rt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h + sqrt(c*x^2 + a)*f
*g^2/h^3 - sqrt(c*x^2 + a)*e*g/h^2 + sqrt(c*x^2 + a)*d/h + 1/3*(c*x^2 + a)^
(3/2)*f/(c*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

# 3.83 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=308

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(afh^2+2c\left(3fg^2-h(2eg-dh)\right)\right)}{2\sqrt{c}\,h^4}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)}{}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/2*(a*f*h^2+2
*c*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^4/c^(1/2)
+(a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*
h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(1/2)-1/2*(2*a*h^2*(-e*
h+2*f*g)+2*c*g*(3*f*g^2-h*(-d*h+2*e*g))-h*(a*f*h^2+c*(3*f*g^2-2*h*(-d*h+e*g
)))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)
```

**Rubi [A]** time = 0.51, antiderivative size = 303, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}-\frac{\sqrt{a+cx^2}\left(2\left(ah^2(2fg-eh)-cgh(2eg-dh)+3cfg^3\right)-hx\left(afh^2-2ch(eg-\right.\right.}{2h^3\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((2*(3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*f*g - e*h)) - h*(3*c*f*g^2
 + a*f*h^2 - 2*c*h*(e*g - d*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2))
 - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(h*(c*g^2 + a*h^2)*(g + h*x)
) + ((6*c*f*g^2 + a*f*h^2 - 2*c*h*(2*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a
 + c*x^2]])/(2*Sqrt[c]*h^4) + ((3*c*f*g^3 - c*g*h*(2*e*g - d*h) + a*h^2*(2*
f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(
h^4*Sqrt[c*g^2 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(2eg-\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+cx^2}\cdots}{g+hx}}{cg^2+ah^2}\cdots$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-d\cdots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-d\cdots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-d\cdots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(3cfg^3-cgh(2eg-dh)+ah^2(2fg-eh)\right)-h\left(3cfg^2+afh^2-2ch(eg-d\cdots\right.\right.}{2h^3\left(cg^2+ah^2\right)}$$

**Mathematica [A]** time = 0.26, size = 264, normalized size = 0.86

$$\frac{\frac{h\sqrt{a+cx^2}\left(2h(-dh+2eg+ehx)+f\left(-6g^2-3ghx+h^2x^2\right)\right)}{g+hx}+\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(afh^2+2ch(dh-2eg)+6cfg^2\right)}{\sqrt{c}}+\frac{2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)(a\cdots}{\sqrt{ah^2+\cdots}}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(2*h*(2*e*g - d*h + e*h*x) + f*(-6*g^2 - 3*g*h*x + h^2*
x^2)))/(g + h*x) - (2*(3*c*f*g^3 + c*g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*
h))*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] + ((6*c*f*g^2 + a*f*h^2 + 2*c*h*(-2*e
*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + (2*(3*c*f*g^3 + c*
g*h*(-2*e*g + d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*
h^2]*Sqrt[a + c*x^2]])/Sqrt[c*g^2 + a*h^2])/(2*h^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 2818, normalized size = 9.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] 1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(3/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(3/2)*f*g^2-1/h*c*g/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h^2*c*g^2/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+
g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/h^3*c*g^3/(a*h^2+c*g^2)*(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+1/h^2*c^(3/2)*g^2/(a*h^2+c*g^2)
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*d-1/h^3*c^(3/2)*g^3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)
+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e+1/h^4*c^(3/2)*g^
4/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2))*f+2/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g-1/h^4/((a*h^2+c*g^2)/h^2
)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2
)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e+
2/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*c*g^3*f-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2
*c+(a*h^2+c*g^2)/h^2)^(3/2)*d-2/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*f*g+1/2*f/h^2*x*(c*x^2+a)^(1/2)+1/h^2*c^(1/2)/(a*h^2+c*g^2)
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*a*f*g^2+1/h^3*c^2*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*l
n((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+
g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/h^4*c^2*g^4/(
a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)
```

```
/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2
)/h^2)^(1/2))/(x+g/h))*e+1/h^5*c^2*g^5/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1
/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-
2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-1/h*c/(a*h
^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e*g+1/h^
2*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*
f*g^2-1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e*g+1/h^3*c*g^3/(a*h^2+c*g^2)/
((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^
2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))
/(x+g/h))*a*f+1/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h
)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(
x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d-1/h^2*c*g^2/(a*h^2+c*g^2)
/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h
^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
)/(x+g/h))*a*e+1/2*f/h^2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+c/(a*h^2+c
*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d+1/h^2*(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+c^(1/2)/(a*h^2+c*g^2)*
ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h
^2)^(1/2))*a*d-1/h^3*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e+2/h^4*c^(1/2)*g^2*ln((-c*g/h+(x
+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*f-
1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*a*e
```

**maxima** [A] time = 0.65, size = 478, normalized size = 1.55

$$-\frac{\sqrt{cx^2+a}\,fg^2}{h^4x+gh^3}+\frac{\sqrt{cx^2+a}\,eg}{h^3x+gh^2}-\frac{\sqrt{cx^2+a}\,d}{h^2x+gh}+\frac{\sqrt{cx^2+a}\,fx}{2\,h^2}+\frac{3\,\sqrt{c}\,fg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^4}-\frac{2\,\sqrt{c}\,eg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{h^3}+\sqrt{c}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(h^4*x + g*h^3) + sqrt(c*x^2 + a)*e*g/(h^3*x + g*h^2
) - sqrt(c*x^2 + a)*d/(h^2*x + g*h) + 1/2*sqrt(c*x^2 + a)*f*x/h^2 + 3*sqrt(
c)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^4 - 2*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/
h^3 + sqrt(c)*d*arcsinh(c*x/sqrt(a*c))/h^2 + 1/2*a*f*arcsinh(c*x/sqrt(a*c))
/(sqrt(c)*h^2) - c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + c*e*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h
^4) - c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^3) - 2*sqrt(a + c*g^2/h^2)*f*g*arcsinh(c*g*x
```

```
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(a + c*
g^2/h^2)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/h^2 - 2*sqrt(c*x^2 + a)*f*g/h^3 + sqrt(c*x^2 + a)*e/h^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

## 3.84 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=296

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+\right.}{2h(g+hx)^2\left(ah^2+cg^2\right.}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(2*a
^2*f*h^4+2*c^2*g^3*(-e*h+3*f*g)+a*c*h^2*(9*f*g^2-h*(-d*h+3*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(3/2)-(-
e*h+3*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4+1/2*(2*(-e*h+3*f*
g)*(a*h^2+c*g^2)+h*(2*a*f*h^2+c*(3*f*g^2-h*(-d*h+e*g)))*x)*(c*x^2+a)^(1/2)/
h^3/(a*h^2+c*g^2)/(h*x+g)
```

**Rubi** [A] time = 0.55, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(9fg^2-h(3eg-dh)\right)+2c^2g^3(3fg-eh)\right)}{2h^4\left(ah^2+cg^2\right)^{3/2}}-\frac{\left(a+cx^2\right)^{3/2}\left(dh^2-egh+\right.}{2h(g+hx)^2\left(ah^2+cg^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*(3*f*g - e*h)*(c*g^2 + a*h^2) + h*(3*c*f*g^2 + 2*a*f*h^2 - c*h*(e*g - d
*h))*x)*Sqrt[a + c*x^2])/(2*h^3*(c*g^2 + a*h^2)*(g + h*x)) - ((f*g^2 - e*g*
h + d*h^2)*(a + c*x^2)^(3/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2) - (Sqrt[c]*
(3*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^4 - ((2*a^2*f*h^4 + 2
*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h
- c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^4*(c*g^2 + a*h^2)^(3/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(eg-\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx}}{(g+hx)^2}}{2\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{(}{}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{(}{}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{(}{}$$

$$= \frac{\left(2(3fg-eh)\left(cg^2+ah^2\right)+h\left(3cfg^2+2afh^2-ch(eg-dh)\right)x\right)\sqrt{a+cx^2}}{2h^3\left(cg^2+ah^2\right)(g+hx)} - \frac{(}{}$$

**Mathematica [A]** time = 0.56, size = 318, normalized size = 1.07

$$\frac{-\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-eh)\right)}{\left(ah^2+cg^2\right)^{3/2}} + \frac{\log(g+hx)\left(2a^2fh^4+ach^2\left(h(dh-3eg)+9fg^2\right)+2c^2g^3(3fg-e\right.}{\left(ah^2+cg^2\right)^{3/2}}}{2h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(2*f + (-(f*g^2) + h*(e*g - d*h))/(g + h*x)^2 + (5*c*f*g
^3 + c*g*h*(-3*e*g + d*h) - 2*a*h^2*(-2*f*g + e*h))/((c*g^2 + a*h^2)*(g + h
*x))) + ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*h) + a*c*h^2*(9*f*g^2 + h*(-3*
e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(3/2) + 2*Sqrt[c]*(-3*f*g + e*h)
*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - ((2*a^2*f*h^4 + 2*c^2*g^3*(3*f*g - e*
h) + a*c*h^2*(9*f*g^2 + h*(-3*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a
*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/(2*h^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.35, size = 923, normalized size = 3.12

$$-\frac{\left(6\,c^2fg^4+9\,acfg^2h^2+acdh^4+2\,a^2fh^4-2\,c^2g^3he-3\,acgh^3e\right)\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^2-ah^2}}\right)}{\left(cg^2h^4+ah^6\right)\sqrt{-cg^2-ah^2}}+\frac{\sqrt{cx^2+a}\,f}{h^3}+\frac{6\left(\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] -(6*c^2*f*g^4 + 9*a*c*f*g^2*h^2 + a*c*d*h^4 + 2*a^2*f*h^4 - 2*c^2*g^3*h*e -
 3*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c*g^2*h^4 + a*h^6)*sqrt(-c*g^2 - a*h^2)) + sqrt(c*x^2 + a
)*f/h^3 + (6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g
^2*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e +
 10*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))^2*c^(5/2)*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*
f*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*
x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 6*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e +
 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 14*(sqrt(c)*x - sqrt
(c*x^2 + a))*a*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^
3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*
x^2 + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5
*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 5*a^2*c^(3/2)*f*g^3*h^2 + a^
2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 3*a^2*c^(3/2)*g^2*h^3*e - 2*a^3
*sqrt(c)*h^5*e)/((c*g^2*h^4 + a*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2
*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2) + (3*sqrt(c)*f*g - sqrt(
c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4
```

**maple [B]** time = 0.02, size = 4432, normalized size = 14.97

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x)
```

```
[Out] 5/2/h^3*c*g^2/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2
)^(1/2)*f+3/2/h^3*c^(3/2)*g^2/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(
-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*e-5/2/h^4*c^(3/2)*g^
3/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c
+(a*h^2+c*g^2)/h^2)^(1/2))*f+1/h*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^
2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1/h*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/
h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e+2
/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(3/2)*f*g-1/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(3/2)*d+1/2*c^(3/2)*g/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c
)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*d+1/2*c
^2*g/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*x*d+1/2/h^2*c^(5/2)*g^3/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d-1/2/h^3*c^(5/2)*g^4/(
a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2))*e+1/2/h^4*c^(5/2)*g^5/(a*h^2+c*g^2)^2*ln((-c*g/h+
(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*
f-1/2/h^2*c^(3/2)/(a*h^2+c*g^2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d+1/2/h^2/(a*h^2+c*g^2)/(x+g/h
)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e*g-1/2/h^3/(a*h
^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*
f*g^2-1/2/h*c^2*g^2/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*d+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2*(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-1/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2*(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f-f/h^5/((a*h^2+c*g^2)/h^2)^(1/
2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2
*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2-3/2/h^2
*c*g/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e
-1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c
*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g
/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*e+1/2/h*c^2*g^2/(a*h^2+c*g^2)^
2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*
h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
))/(x+g/h))*a*d-5/2/h^3*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-
2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)
*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f+3/2/h^2*c*g/(a*h^
2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2
+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^
2)^(1/2))/(x+g/h))*a*e+1/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^
(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f+f/h^3*
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)+3/2/h^4*c^2*g^3/(a*h
^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^
2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h
^2)^(1/2))/(x+g/h))*e-5/2/h^5*c^2*g^4/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/
```

```
2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2
*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-2/h^2*c/(a*
h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f*g-2/h
^2*c^(1/2)/(a*h^2+c*g^2)*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f*g-1/2/h*c/(a*h^2+c*g^2)/((a*h^2+c*g
^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^
2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a
*d-1/2/h^3*c^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h
+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^
2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*g^2*d+1/2/h^2*c^2*g^3/(a*h^2+c*g^2)^
2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/2/h*c^(3/2)*
g^2/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*e+1/2/h^5*c^3*g^6/(a*h^2+c*g^2)^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*f-1/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*
h^2+c*g^2)/h^2)^(3/2)*f+1/2/h^3*c^3*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)
^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)
*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-1/2/h^4
*c^3*g^5/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(
a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-1/2/h*c^2*g^2/(a*h^2+c*g^2)^2*(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1/2/h*c*g^2/(a*h^2+c*g^2
)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e+1/2/h^
2*c^(3/2)*g^3/(a*h^2+c*g^2)^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f-1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*
(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*e-f/h^4*c^(1/2)*g*ln((-c
*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1
/2))-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h
^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))/(x+g/h))*a+1/2/h*c/(a*h^2+c*g^2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2)*d-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+
(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/2)*d
```

**maxima [B]** time = 0.70, size = 927, normalized size = 3.13

$$-\frac{\sqrt{cx^2+a}\,cfg^3}{2\left(cg^2h^4x+ah^6x+cg^3h^3+agh^5\right)}+\frac{\sqrt{cx^2+a}\,ceg^2}{2\left(cg^2h^3x+ah^5x+cg^3h^2+agh^4\right)}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fg^2}{2\left(cg^2h^3x^2+ah^5x^2+2\,cg^3h^2x+2\,agh^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima")

[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5)
```

```
+ 1/2*sqrt(c*x^2 + a)*c*e*g^2/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4)
 - 1/2*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x +
 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^
3 + a*h^5) - 1/2*sqrt(c*x^2 + a)*c*d*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a
*g*h^3) + 1/2*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*
x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^2
 + a*h^4) - 1/2*(c*x^2 + a)^(3/2)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*sqrt(c*x^2 + a)*c*d/(c*g^2*h + a*h^3
) + 2*sqrt(c*x^2 + a)*f*g/(h^4*x + g*h^3) - sqrt(c*x^2 + a)*e/(h^3*x + g*h^
2) - 3*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + sqrt(c)*e*arcsinh(c*x/sqrt(
a*c))/h^3 - 1/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 1/2*c^2*e*g^3*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^6) - 1/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) + 5/2*c*f*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
c*g^2/h^2)*h^5) - 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4) + 1/2*c*d*arcsinh(c*g*x/(
sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2
)*h^3) + sqrt(a + c*g^2/h^2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/h^3 + sqrt(c*x^2 + a)*f/h^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**3,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

# 3.85 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

**Optimal**. Leaf size=314

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{2h^4\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg\right.\right.}{}$$

```
[Out] -1/3*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(2
*c^2*f*g^5+a^2*h^4*(-e*h+4*f*g)+a*c*g*h^2*(-d*h^2+5*f*g^2))*arctanh((-c*g*x
+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^4/(a*h^2+c*g^2)^(5/2)+f*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^4-1/2*(2*c^2*f*g^5+a^2*e*h^5+a*c*g*h
^2*(d*h^2+3*f*g^2)+h*(2*a^2*f*h^4+a*c*g*h^2*(-e*h+6*f*g)+c^2*(-d*g^2*h^2+3*
f*g^4))*x)*(c*x^2+a)^(1/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^2
```

**Rubi [A]** time = 0.51, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 811, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(hx\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3fg^4-dg^2h^2\right)\right)+a^2eh^5+acgh^2\left(dh^2+3fg^2\right)+2c^2fg^5\right)}{2h^3(g+hx)^2\left(ah^2+cg^2\right)^2}+\frac{c\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((2*c^2*f*g^5 + a^2*e*h^5 + a*c*g*h^2*(3*f*g^2 + d*h^2) + h*(2*a^2*f*h^4 +
 a*c*g*h^2*(6*f*g - e*h) + c^2*(3*f*g^4 - d*g^2*h^2))*x)*Sqrt[a + c*x^2])/(
2*h^3*(c*g^2 + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)
^(3/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*ArcTanh[(Sqrt[c]*x)/
Sqrt[a + c*x^2]])/h^4 + (c*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2
*(5*f*g^2 - d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x
^2])])/(2*h^4*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-3f\left(\frac{cg^2}{h}+ah\right)x\right)\sqrt{a+cx^2}}{(g+hx)^3}\,dx}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

$$= -\frac{\left(2c^2fg^5+a^2eh^5+acgh^2\left(3fg^2+dh^2\right)+h\left(2a^2fh^4+acgh^2(6fg-eh)+c^2\left(3\right.\right.\right.}{2h^3\left(cg^2+ah^2\right)^2(g+hx)^2}$$

**Mathematica** [A] time = 0.82, size = 382, normalized size = 1.22

$$\frac{\frac{3c\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{\left(ah^2+cg^2\right)^{5/2}} - \frac{3c\log(g+hx)\left(a^2h^4(4fg-eh)+acgh^2\left(5fg^2-dh^2\right)+2c^2fg^5\right)}{\left(ah^2+cg^2\right)^{5/2}} + \frac{h\sqrt{a+\ldots}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(-2*(f*g^2 + h*(-(e*g) + d*h)) + ((7*c*f*g^3 + c*g*h*(-
4*e*g + d*h) - 3*a*h^2*(-2*f*g + e*h))*(g + h*x))/(c*g^2 + a*h^2) - ((6*a^2
*f*h^4 + c^2*(11*f*g^4 - g^2*h*(2*e*g + d*h)) + a*c*h^2*(20*f*g^2 + h*(-5*e
*g + 2*d*h)))*(g + h*x)^2)/(c*g^2 + a*h^2)^2))/(g + h*x)^3 - (3*c*(2*c^2*f*
g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[g + h*x])/(c
*g^2 + a*h^2)^(5/2) + 6*Sqrt[c]*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c
*(2*c^2*f*g^5 + a^2*h^4*(4*f*g - e*h) + a*c*g*h^2*(5*f*g^2 - d*h^2))*Log[a*
h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/(6
*h^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas")
[Out] Timed out
```

**giac [B]** time = 0.50, size = 1719, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
[Out] -(2*c^3*f*g^5 + 5*a*c^2*f*g^3*h^2 - a*c^2*d*g*h^4 + 4*a^2*c*f*g*h^4 - a^2*c
*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 -
 a*h^2))/((c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8)*sqrt(-c*g^2 - a*h^2)) - s
qrt(c)*f*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^4 - 1/3*(18*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*c^3*f*g^5*h^2 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2
*f*g^3*h^4 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*
x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^
3*g^4*h^3*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 54*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
c^(7/2)*f*g^6*h - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 87*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*f*g^4*h^3 + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c
^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*f*h^7 - 12*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*g^5*h^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2
)*g^3*h^4*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 44*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*c^4*f*g^7 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))^3
*c^4*d*g^5*h^2 + 14*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 + 14*(s
qrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*d*g^3*h^4 - 96*(sqrt(c)*x - sqrt(c*x^2
+ a))^3*a^2*c^2*f*g^3*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*
h^6 - 36*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c*f*g*h^6 - 8*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^4*g^6*h*e - 8*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h
^3*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 78*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a*c^(7/2)*f*g^6*h + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a*c^(7/2)*d*g^4*h^3 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^
4*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 12*(sqrt
(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 12*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a*c^(7/2)*g^5*h^2*e + 30*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)
```

```
*g^3*h^4*e - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*g*h^6*e + 48*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 - 6*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*d*g^3*h^4 + 87*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*f*g^3*h^4
 + 9*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^4*c*f*g*h^6 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*g^4*h^3*
e - 18*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*e + 3*(sqrt(c)*x - sqr
t(c*x^2 + a))*a^4*c*h^7*e - 11*a^3*c^(5/2)*f*g^4*h^3 + a^3*c^(5/2)*d*g^2*h^
5 - 20*a^4*c^(3/2)*f*g^2*h^5 - 2*a^4*c^(3/2)*d*h^7 - 6*a^5*sqrt(c)*f*h^7 +
2*a^3*c^(5/2)*g^3*h^4*e + 5*a^4*c^(3/2)*g*h^6*e)/((c^2*g^4*h^4 + 2*a*c*g^2*
h^6 + a^2*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x
^2 + a))*sqrt(c)*g - a*h)^3)
```

**maple [B]** time = 0.02, size = 5565, normalized size = 17.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x)

[Out] result too large to display
```

**maxima [B]** time = 0.84, size = 1772, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima")

[Out] -1/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^2*e*g^3
/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4
 + a^2*g*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*
h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x +
 c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/2*sqrt(c*x^2 + a)*c^2*f*g^3/(
c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^2*d*g^2/(c^2
*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*
g*h^5) + 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2
 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6
+ 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/2*sqrt(c*x^2 + a)*c^2*e*g^2/(c^2*g^4*h^2
 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c^2*g^4*h*x^2 +
2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h
^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1/2*sqrt(c*x^2 + a)*c^2*d*g
/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) - 1/3*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2
*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*
a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + sqrt(c*x^2 + a)*c*f*g^2/(c*g^2*h^4*x +
```

```
a*h^6*x + c*g^3*h^3 + a*g*h^5) + 1/3*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^3*x^3
+ a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x
 + c*g^5 + a*g^3*h^2) - 1/2*sqrt(c*x^2 + a)*c*e*g/(c*g^2*h^3*x + a*h^5*x +
c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(3/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2
*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) - sqrt(c*x^2 + a)*c*f*g/(
c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(3/2)*d/(c*g^2*h^2*x^3 + a*h^4*x^3 + 3
*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5/h + a*g^3*
h) - 1/2*(c*x^2 + a)^(3/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a
*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*sqrt(c*x^2 + a)*c*e/(c*g^2*h^2 + a*h^4)
 - sqrt(c*x^2 + a)*f/(h^4*x + g*h^3) + sqrt(c)*f*arcsinh(c*x/sqrt(a*c))/h^4
 - 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*ab
s(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 1/2*c^3*e*g^4*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^8) - 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/2*c^2*f*g^3*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) - c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) + 1/2*c^2*d*g*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) - 2*c*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 1/2*c*e*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^4)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

# 3.86 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

**Optimal**. Leaf size=313

$$-\frac{\sqrt{a+cx^2}\,(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(fg^2\right.\right.}{8\left(ah^2+cg^2\right)^{7/2}}$$

```
[Out] -1/4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^4+1/12*(4*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-5*d*h+e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c
*g^2)^2/(h*x+g)^3-1/8*a*c*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g
)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)
^(7/2)-1/8*(4*c^2*d*g^2+4*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+5*e*g)))*(-c*g*x+a*h
)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)^2
```

**Rubi [A]** time = 0.43, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{\sqrt{a+cx^2}\,(ah-cgx)\left(4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)+4c^2dg^2\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^3}-\frac{ac\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(4a^2fh^2-ac\left(fg^2\right.\right.}{8\left(ah^2+cg^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -((4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*(5*e*g - d*h)))*(a*h - c*g*x)
*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h
^2)*(a + c*x^2)^(3/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) + ((3*c*f*g^3 + c*
g*h*(e*g - 5*d*h) + 4*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3/2))/(12*h*(c*g^2
+ a*h^2)^2*(g + h*x)^3) - (a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 - h*
(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/(8*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\,\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh+c\left(eg+\frac{3fg^2}{h}-dh\right)\right)x\right)\sqrt{a+cx^2}}{(g+hx)^4}}{4\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} + \frac{\left(3cfg^3+cgh(eg-5dh)+4ah^2(2fg-eh)\right)(}{12h\left(cg^2+ah^2\right)^2(g+hx)^3}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh\right.}{4h\left(cg^2\right.}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh\right.}{4h\left(cg^2\right.}$$

$$= -\frac{\left(4c^2dg^2+4a^2fh^2-ac\left(fg^2-h(5eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^3(g+hx)^2} - \frac{\left(fg^2-egh\right.}{4h\left(cg^2\right.}$$

**Mathematica [A]** time = 1.31, size = 439, normalized size = 1.40

$$\frac{1}{24}\left(-\frac{3ac\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(4a^2fh^2-ac\left(h(dh-5eg)+fg^2\right)+4c^2dg^2\right)}{\left(ah^2+cg^2\right)^{7/2}} + \frac{3ac\log(g+hx)(}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) - 2*(c*
g^2 + a*h^2)^2*(9*c*f*g^3 + c*g*h*(-5*e*g + d*h) - 4*a*h^2*(-2*f*g + e*h))*
(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(9*f*g^4 - g^2*h*(e*g + d
*h)) + a*c*h^2*(35*f*g^2 + h*(-7*e*g + 3*d*h)))*(g + h*x)^2 - c*(4*a^2*h^4*
(7*f*g - 2*e*h) + a*c*g*h^2*(19*f*g^2 + h*(9*e*g - 13*d*h)) + 2*c^2*(3*f*g^
5 + g^3*h*(e*g + d*h)))*(g + h*x)^3))/((c*g^2*h + a*h^3)^3*(g + h*x)^4)) +
(3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g^2 + h*(-5*e*g + d*h)))*Log[g +
 h*x])/(c*g^2 + a*h^2)^(7/2) - (3*a*c*(4*c^2*d*g^2 + 4*a^2*f*h^2 - a*c*(f*g
^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^
2]])/(c*g^2 + a*h^2)^(7/2))/24
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 7237, normalized size = 23.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.14, size = 3404, normalized size = 10.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] -5/8*sqrt(c*x^2 + a)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c
*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 +
 a^3*g*h^9) + 5/8*sqrt(c*x^2 + a)*c^3*e*g^4/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^
5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2
*c*g^3*h^6 + a^3*g*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^3*x^2
+ 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x
 + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*
c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 5/8*sqrt(c*x^2 + a)*c^3*f*g^
4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 5/8*sqrt(c*
x^2 + a)*c^3*d*g^3/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x +
 a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 5
/8*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a
^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*
c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 +
```

a^3*g^2*h^6) - 5/8*sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^
4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 5/8*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^3*g^6*h
*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*
x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a
*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) + 5/8*sqrt(c*x^2 + a)*c^3*d*g^2
/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 5/12*(c*x^2 +
a)^(3/2)*c*f*g^3/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2
*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*
c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) +
9/8*sqrt(c*x^2 + a)*c^2*f*g^3/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
+ c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 5/12*(c*x^2 + a)^(3/2)*c*e*g^2
/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6
*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^
2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 5/8*sqrt(c*x^2 + a)*
c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*
c*g^3*h^4 + a^2*g*h^6) + 9/8*(c*x^2 + a)^(3/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2
*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*
g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 9/8*sqrt(c*x^2 + a)*c^
2*f*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 5/12*(c*x^2 + a)^(3/2)*c*
d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 +
6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2
*g^2*h^4*x + c^2*g^7/h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*sqrt(c*x^2 + a)*c
^2*d*g/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3
*h^3 + a^2*g*h^5) - 5/8*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^
2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x +
c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) + 5/8*sqrt(c*x^2 + a)*c^2*e*g/(c^2*
g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^5
*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*
g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 1/8*(c
*x^2 + a)^(3/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^
2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g
^2*h^3) - 1/8*sqrt(c*x^2 + a)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) +
1/4*(c*x^2 + a)^(3/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4
*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^
3*x + c*g^6 + a*g^4*h^2) + 2/3*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^4*x^3 + a*h^6
*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*
g^5*h + a*g^3*h^3) - 1/2*sqrt(c*x^2 + a)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g
^3*h^3 + a*g*h^5) - 1/4*(c*x^2 + a)^(3/2)*d/(c*g^2*h^3*x^4 + a*h^5*x^4 + 4*
c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x
+ 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) - 1/3*(c*x^2 + a)^(3/2)*e/(c*g^2*h^3*
x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h
^3*x + c*g^5 + a*g^3*h^2) - 1/2*(c*x^2 + a)^(3/2)*f/(c*g^2*h^3*x^2 + a*h^5*
x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*sqrt(c*x^2 +
a)*c*f/(c*g^2*h^3 + a*h^5) - 5/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*
x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^11) + 5/8*

```
c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(7/2)*h^10) - 5/8*c^4*d*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^9)
+ 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs
(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) - 5/4*c^3*e*g^3*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^8) + 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c
)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 13/8*c^2*f*g^2*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(3/2)*h^7) + 5/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6) - 1/8*c^2*d*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
3/2)*h^5) + 1/2*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**5,x)
[Out] Integral(sqrt(a + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

# 3.87 $\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

**Optimal**. Leaf size=433

$$-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18fg^2-h(33eg-8dh)\right)-c^2g^2\left(h(2eg-27dh)+3fg^2\right)\right)}{60h(g+hx)^3\left(ah^2+cg^2\right)^3}-\frac{c\sqrt{a+cx^2}\,(ah-cgx)\left(a\right.}{}$$

```
[Out] -1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)/(h*x+g)^5+1/20*(5*
a*h^2*(-e*h+2*f*g)+c*g*(3*f*g^2+h*(-7*d*h+2*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2
+c*g^2)^2/(h*x+g)^4-1/60*(20*a^2*f*h^4-c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-
a*c*h^2*(18*f*g^2-h*(-8*d*h+33*e*g)))*(c*x^2+a)^(3/2)/h/(a*h^2+c*g^2)^3/(h*
x+g)^3-1/8*a*c^2*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*
g^2)^(9/2)-1/8*c*(4*c^2*d*g^3+a^2*h^2*(-e*h+6*f*g)-a*c*g*(f*g^2-3*h*(-d*h+2
*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^4/(h*x+g)^2
```

**Rubi [A]** time = 0.74, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{c\sqrt{a+cx^2}\,(ah-cgx)\left(a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)+4c^2dg^3\right)}{8(g+hx)^2\left(ah^2+cg^2\right)^4}-\frac{\left(a+cx^2\right)^{3/2}\left(20a^2fh^4-ach^2\left(18f\right.\right.}{60h}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)
))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(8*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((f*g
^2 - e*g*h + d*h^2)*(a + c*x^2)^(3/2))/(5*h*(c*g^2 + a*h^2)*(g + h*x)^5) +
((3*c*f*g^3 + c*g*h*(2*e*g - 7*d*h) + 5*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(3
/2))/(20*h*(c*g^2 + a*h^2)^2*(g + h*x)^4) - ((20*a^2*f*h^4 - c^2*(3*f*g^4 +
 g^2*h*(2*e*g - 27*d*h)) - a*c*h^2*(18*f*g^2 - h*(33*e*g - 8*d*h)))*(a + c*
x^2)^(3/2))/(60*h*(c*g^2 + a*h^2)^3*(g + h*x)^3) - (a*c^2*(4*c^2*d*g^3 + a^
2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(2*e*g - d*h)))*ArcTanh[(a*h - c*g
*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(8*(c*g^2 + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-\left(5afh+c\left(2eg+\frac{3fg^2}{h}-2dh\right)\right)x\right)\sqrt{a+c\cdots}}{(g+hx)^5}}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\cdots}{20h\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{3/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} + \frac{\left(3cfg^3+cgh(2eg-7dh)+5ah^2(2fg-eh)\right)\cdots}{20h\left(cg^2+ah^2\right)^2(g+hx)^4}$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} \cdots$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} \cdots$$

$$= -\frac{c\left(4c^2dg^3+a^2h^2(6fg-eh)-acg\left(fg^2-3h(2eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{8\left(cg^2+ah^2\right)^4(g+hx)^2} \cdots$$

**Mathematica [A]** time = 1.62, size = 583, normalized size = 1.35

$$-\frac{ac^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(a^2h^2(6fg-eh)-acg\left(3h(dh-2eg)+fg^2\right)+4c^2dg^3\right)}{8\left(ah^2+cg^2\right)^{9/2}}+\frac{ac^2\log(g+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/120*(Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) -
6*(c*g^2 + a*h^2)^3*(11*c*f*g^3 + c*g*h*(-6*e*g + d*h) - 5*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(27*f*g^4 - g^2*h
*(2*e*g + 3*d*h)) + a*c*h^2*(54*f*g^2 + h*(-9*e*g + 4*d*h)))*(g + h*x)^2 -
c*(c*g^2 + a*h^2)*(5*a^2*h^4*(10*f*g - 3*e*h) + a*c*g*h^2*(21*f*g^2 + h*(24
*e*g - 29*d*h)) + c^2*(6*f*g^5 + 2*g^3*h*(2*e*g + 3*d*h)))*(g + h*x)^3 - c*
```

```
(-40*a^3*f*h^6 + a*c^2*g^2*h^2*(27*f*g^2 + h*(28*e*g - 83*d*h)) + c^3*(6*f*
g^6 + 2*g^4*h*(2*e*g + 3*d*h)) + a^2*c*h^4*(86*f*g^2 + h*(-81*e*g + 16*d*h)
))*(g + h*x)^4))/(h^3*(c*g^2 + a*h^2)^4*(g + h*x)^5) + (a*c^2*(4*c^2*d*g^3
+ a^2*h^2*(6*f*g - e*h) - a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[g + h*x])
/(8*(c*g^2 + a*h^2)^(9/2)) - (a*c^2*(4*c^2*d*g^3 + a^2*h^2*(6*f*g - e*h) -
a*c*g*(f*g^2 + 3*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*S
qrt[a + c*x^2]])/(8*(c*g^2 + a*h^2)^(9/2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.66, size = 4212, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(4*a*c^4*d*g^3 - a^2*c^3*f*g^3 - 3*a^2*c^3*d*g*h^2 + 6*a^3*c^2*f*g*h^2
 + 6*a^2*c^3*g^2*h*e - a^3*c^2*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))
*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c
^2*g^4*h^4 + 4*a^3*c*g^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/60*(60*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*d*g^3*h^8 - 15*(sqrt(c)*x - sqrt(c*x^2
+ a))^9*a^2*c^3*f*g^3*h^8 - 45*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*d*g*
h^10 + 90*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*f*g*h^10 + 90*(sqrt(c)*x
- sqrt(c*x^2 + a))^9*a^2*c^3*g^2*h^9*e - 15*(sqrt(c)*x - sqrt(c*x^2 + a))^9
*a^3*c^2*h^11*e - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 -
480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*f*g^6*h^5 + 540*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a*c^(9/2)*d*g^4*h^7 - 855*(sqrt(c)*x - sqrt(c*x^2 + a))^
8*a^2*c^(7/2)*f*g^4*h^7 - 405*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d
*g^2*h^9 + 330*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2)*f*h^11 + 810*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 135*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^3*c^(5/2)*g*h^10*e - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 9
60*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*f*g^7*h^4 + 1880*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a*c^5*d*g^5*h^6 - 1910*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c
^4*f*g^5*h^6 - 1690*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 + 193
0*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*f*g^3*h^8 + 210*(sqrt(c)*x - sqrt
(c*x^2 + a))^7*a^3*c^3*d*g*h^10 - 660*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c
```

^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 640*(sqrt
(c)*x - sqrt(c*x^2 + a))^7*a*c^5*g^6*h^5*e + 1860*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*a^2*c^4*g^4*h^7*e - 1530*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2
*h^9*e - 90*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e - 240*(sqrt(c)*x
 - sqrt(c*x^2 + a))^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))
^6*c^(13/2)*d*g^8*h^3 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^
8*h^3 + 2120*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 1250*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 - 5710*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^2*c^(9/2)*d*g^4*h^7 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^
6*a^3*c^(7/2)*f*g^4*h^7 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d
*g^2*h^9 - 2220*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 - 240
*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*d*h^11 + 240*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(
13/2)*g^9*h^2*e - 640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h^4*e
+ 3660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e - 4350*(sqrt(c
)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e + 330*(sqrt(c)*x - sqrt(c*x^
2 + a))^6*a^4*c^(5/2)*g*h^10*e - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*f*g
^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 + 48*(sqrt(c)*x - sq
rt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 1808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c
^6*d*g^7*h^4 + 604*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^4 - 7076
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 + 6710*(sqrt(c)*x - sqrt
(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 + 3770*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3
*c^4*d*g^3*h^8 - 5780*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^3*h^8 - 4
80*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 1200*(sqrt(c)*x - sqr
t(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*g
^10*h*e - 128*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3416*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e - 7320*(sqrt(c)*x - sqrt(c*x^2
+ a))^5*a^3*c^4*g^4*h^7*e + 2430*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*g^
2*h^9*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10*h + 240*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 - 5240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^2*c^(11/2)*d*g^6*h^5 + 2450*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*f
*g^6*h^5 + 5590*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d*g^4*h^7 - 766
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 - 2240*(sqrt(c)*x -
 sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 3440*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*f*g^2*h^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2
)*d*h^11 - 160*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*f*h^11 + 160*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1120*(sqrt(c)*x - sqrt(
c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 6140*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*g^5*h^6*e + 5650*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*g
^3*h^8*e - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g*h^10*e - 240*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 480*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^2*c^6*d*g^7*h^4 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5
*f*g^7*h^4 + 5000*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^5*d*g^5*h^6 - 3890*
(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 - 2910*(sqrt(c)*x - sqrt(

```
c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 + 4710*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*
c^3*f*g^3*h^8 + 430*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*d*g*h^10 - 940*
(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 160*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^2*c^6*g^8*h^3*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^
5*g^6*h^5*e + 5740*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*g^4*h^7*e - 1710
*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e + 90*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*a^6*c^2*h^11*e + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11
/2)*f*g^8*h^3 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*d*g^6*h^5
+ 570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*f*g^6*h^5 - 2810*(sqrt(c)
*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 + 2450*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 + 650*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c
^(7/2)*d*g^2*h^9 - 1700*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*f*g^2*h
^9 - 80*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*d*h^11 + 80*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^7*c^(3/2)*f*h^11 + 160*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*a^3*c^(11/2)*g^7*h^4*e + 1100*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2
)*g^5*h^6*e - 2570*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e +
270*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(5/2)*g*h^10*e - 60*(sqrt(c)*x -
sqrt(c*x^2 + a))*a^4*c^5*f*g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c
^5*d*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 + 770*(s
qrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*d*g^3*h^8 - 845*(sqrt(c)*x - sqrt(c*x^2
 + a))*a^6*c^3*f*g^3*h^8 - 115*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^
10 + 310*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 40*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^4*c^5*g^6*h^5*e - 280*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c
^4*g^4*h^7*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 15*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^7*c^2*h^11*e + 6*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5
*c^(9/2)*d*g^4*h^7 + 27*a^6*c^(7/2)*f*g^4*h^7 - 83*a^6*c^(7/2)*d*g^2*h^9 +
86*a^7*c^(5/2)*f*g^2*h^9 + 16*a^7*c^(5/2)*d*h^11 - 40*a^8*c^(3/2)*f*h^11 +
4*a^5*c^(9/2)*g^5*h^6*e + 28*a^6*c^(7/2)*g^3*h^8*e - 81*a^7*c^(5/2)*g*h^10*
e)/((c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 +
 a^4*h^12)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 +
 a))*sqrt(c)*g - a*h)^5)
```

**maple [B]** time = 0.02, size = 8546, normalized size = 19.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x)
[Out] result too large to display
```

**maxima [B]** time = 1.53, size = 5793, normalized size = 13.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima")

[Out] -7/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c
^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*
h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) + 7/8*sqrt(c*x^2 +
a)*c^4*e*g^5/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a
^3*c*g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5
*h^6 + 4*a^3*c*g^3*h^8 + a^4*g*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*f*g^5/(c^4
*g^8*h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^
9*x^2 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5
*h^6*x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3
+ 6*a^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) + 7/8*sqrt(c*x^2 + a)*
c^4*f*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*
h^9 + a^4*h^11) - 7/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^4*g^8*h^2*x + 4*a*c^3*g^
6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x + c^4*g^9*h
+ 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*g*h^9) + 7/8*
(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g^6*h^4*x^2 + 6*a^2*
c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2*c^4*g^9*h*x + 8*a*
c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x + 2*a^4*g*h^9*x +
c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c*g^4*h^6 + a^4*g^2*
h^8) - 7/8*sqrt(c*x^2 + a)*c^4*e*g^4/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2
*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/8*(c*x^2 + a)^(3/2)*c^3*d*g^
3/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^
2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*
h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*
a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 7/8*sqrt(c*x^2 + a)*c^4*
d*g^3/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 +
a^4*h^9) - 7/12*(c*x^2 + a)^(3/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*
h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*
g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a
*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*
g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 13/8*sqrt(c*x^2 + a)*c^3*f*g^4/(
c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^
7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/12*(c*x^2 + a)^(
3/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3
 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*
x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5
*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^
3*h^6) - sqrt(c*x^2 + a)*c^3*e*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a
^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^
6 + a^3*g*h^8) + 13/8*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^
2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c
^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*
h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) - 13/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^3*
g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/12*(c*x^2 + a)^(
```

3/2)*c^2*d*g^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3
 + a^3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^
2 + 3*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x +
 3*a^3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^
5) + 3/8*sqrt(c*x^2 + a)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a
^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5
+ a^3*g*h^7) - (c*x^2 + a)^(3/2)*c^2*e*g^2/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h
^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^
3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2
*c*g^4*h^4 + a^3*g^2*h^6) + sqrt(c*x^2 + a)*c^3*e*g^2/(c^3*g^6*h^2 + 3*a*c^
2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 7/20*(c*x^2 + a)^(3/2)*c*f*g^3/(c^
2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c
*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6
*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^
2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5) + 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^3
*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^
3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h
 + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 3/8*sqrt(c*x^2 + a)*c^3
*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) + 7/20*(c*x^
2 + a)^(3/2)*c*e*g^2/(c^2*g^4*h^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4
*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2
+ 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x
+ 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6*h^2 + a^2*g^4*h^4) + 29/30*(c*x^2 +
 a)^(3/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^
2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a
*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) -
 3/4*sqrt(c*x^2 + a)*c^2*f*g^2/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x
 + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 7/20*(c*x^2 + a)^(3/2)*c*d*g/
(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*
a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 +
6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g
^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) - 11/20*(c*x^2 + a)^(3/2)*c*e*g/(c^2*g^4*
h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h
^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*
x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) + 1/8*sqrt(c*x^2 + a)*c^2*e*g/(c
^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 +
a^2*g*h^6) - 3/4*(c*x^2 + a)^(3/2)*c*f*g/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x
^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*
g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 3/4*sqrt(c*x^2 + a)*c^2*f*g/(c^2*g^4
*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2 + a)^(3/2)*f*g^2/(c*g^2*h^6*x^
5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g
^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*
h^4*x + c*g^7*h + a*g^5*h^3) + 2/15*(c*x^2 + a)^(3/2)*c*d/(c^2*g^4*h^2*x^3
+ 2*a*c*g^2*h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3
*a^2*g*h^5*x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/

```
h + 2*a*c*g^5*h + a^2*g^3*h^3) + 1/8*(c*x^2 + a)^(3/2)*c*e/(c^2*g^4*h^2*x^2
 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^
2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h^4) - 1/8*sqrt(c*x^2 + a)*c^
2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/5*(c*x^2 + a)^(3/2)*e*g/(c*
g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^
3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x +
5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x^2 + a)^(3/2)*f*g/(c*g^2*h^5*x
^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^
2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) - 1/5*(c*x
^2 + a)^(3/2)*d/(c*g^2*h^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^
4 + 10*c*g^4*h^2*x^3 + 10*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2
 + 5*c*g^6*x + 5*a*g^4*h^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(3/2)*e
/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2
*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) -
 1/3*(c*x^2 + a)^(3/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a
*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 7/8*c^5
*f*g^7*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)
))/((a + c*g^2/h^2)^(9/2)*h^13) + 7/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*ab
s(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^12) -
7/8*c^5*d*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/((a + c*g^2/h^2)^(9/2)*h^11) + 5/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h
^11) - 15/8*c^4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^10) + 5/4*c^4*d*g^3*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(7/2)*h^9) - 19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) + 9/8*c^3*e*g^2*arcsin
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(5/2)*h^8) - 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*
h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) + 3/4*c^2*f*g*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g
^2/h^2)^(3/2)*h^7) - 1/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h
/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^6)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+a)**(1/2)/(h*x+g)**6,x)
[Out] Timed out
```

# 3.88 $\int (g+hx)^3 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=462

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\,\left(3a^2h^2(eh+3fg)-8acg\left(3\cdots\right.\right.}{128c^2}$$

```
[Out] 1/192*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*
(c*x^2+a)^(3/2)/c^2+1/504*(8*(-4*a*f+9*c*d)*h^2-3*c*g*(-9*e*h+5*f*g))*(h*x+
g)^2*(c*x^2+a)^(5/2)/c^2/h-1/72*(-9*e*h+5*f*g)*(h*x+g)^3*(c*x^2+a)^(5/2)/c/
h+1/9*f*(h*x+g)^4*(c*x^2+a)^(5/2)/c/h+1/5040*(128*a^2*f*h^4-32*a*c*h^2*(17*
f*g^2+9*h*(d*h+3*e*g))-12*c^2*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-5*c*h*(a*h^2
*(63*e*h+61*f*g)+2*c*g*(5*f*g^2-9*h*(12*d*h+e*g)))*x)*(c*x^2+a)^(5/2)/c^3/h
+1/128*a^2*(48*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)
))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^3+3*a^2*h
^2*(e*h+3*f*g)-8*a*c*g*(f*g^2+3*h*(d*h+e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

**Rubi [A]** time = 1.13, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{\left(a+cx^2\right)^{5/2}\left(4\left(32a^2fh^4-8ach^2\left(9h(dh+3eg)+17fg^2\right)-c^2\left(15fg^4-9g^2h(64dh+3eg)\right)\right)-5chx\left(ah^2(63eh+6\cdots\right.\right.}{5040c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*
h)))*x*Sqrt[a + c*x^2])/(128*c^2) + ((48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h
) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) + ((8
*(9*c*d - 4*a*f)*h^2 - 3*c*g*(5*f*g - 9*e*h))*(g + h*x)^2*(a + c*x^2)^(5/2)
)/(504*c^2*h) - ((5*f*g - 9*e*h)*(g + h*x)^3*(a + c*x^2)^(5/2))/(72*c*h) +
(f*(g + h*x)^4*(a + c*x^2)^(5/2))/(9*c*h) + ((4*(32*a^2*f*h^4 - 8*a*c*h^2*(
17*f*g^2 + 9*h*(3*e*g + d*h)) - c^2*(15*f*g^4 - 9*g^2*h*(3*e*g + 64*d*h)))
- 5*c*h*(a*h^2*(61*f*g + 63*e*h) + 2*c*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)))*x)
*(a + c*x^2)^(5/2))/(5040*c^3*h) + (a^2*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g +
e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(128*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
```

```
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3\left((9cd-4af)h^2-ch(5fg-9eh\right.}{9ch^2}$$

$$= -\frac{(5fg-9eh)(g+hx)^3\left(a+cx^2\right)^{5/2}}{72ch} + \frac{f(g+hx)^4\left(a+cx^2\right)^{5/2}}{9ch} + \frac{\int (g}{}$$

$$= \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h} - \frac{(5fg-}{}$$

$$= \frac{\left(8(9cd-4af)h^2-3cg(5fg-9eh)\right)(g+hx)^2\left(a+cx^2\right)^{5/2}}{504c^2h} - \frac{(5fg-}{}$$

$$= \frac{\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\left(a+cx^2\right.}{192c^2}$$

$$= \frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

$$= \frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

$$= \frac{a\left(48c^2dg^3+3a^2h^2(3fg+eh)-8acg\left(fg^2+3h(eg+dh)\right)\right)x\sqrt{a+c}}{128c^2}$$

**Mathematica [A]** time = 0.54, size = 481, normalized size = 1.04

$$\frac{315a^2\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2h^2(eh+3fg)-8acg\left(3h(dh+eg)+fg^2\right)+48c^2dg^3\right)+\sqrt{a+cx^2}\left(384c^2x^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(128*a^2*(8*a^2*f*h^3 + 63*c^2*g^2*(e*g + 3*d*h) - 18*a*c*
h*(3*f*g^2 + h*(3*e*g + d*h))) + 315*a*c*(80*c^2*d*g^3 - 3*a^2*h^2*(3*f*g +
 e*h) + 8*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*x + 128*a*c*(-4*a^2*f*h^3 + 126*
c^2*g^2*(e*g + 3*d*h) + 9*a*c*h*(3*f*g^2 + h*(3*e*g + d*h)))*x^2 + 210*c^2*
(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) + 56*a*c*g*(f*g^2 + 3*h*(e*g + d*h)
))*x^3 + 384*c^2*(a^2*f*h^3 + 21*c^2*g^2*(e*g + 3*d*h) + 24*a*c*h*(3*f*g^2
```

```
+ h*(3*e*g + d*h)))*x^4 + 840*c^3*(9*a*h^2*(3*f*g + e*h) + 8*c*(f*g^3 + 3*g
*h*(e*g + d*h)))*x^5 + 640*c^3*h*(10*a*f*h^2 + 9*c*(3*f*g^2 + h*(3*e*g + d*
h)))*x^6 + 5040*c^4*h^2*(3*f*g + e*h)*x^7 + 4480*c^4*f*h^3*x^8) + 315*a^2*S
qrt[c]*(48*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 8*a*c*g*(f*g^2 + 3*h*(e*g
+ d*h)))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(40320*c^3)
```

**fricas [A]** time = 1.48, size = 1177, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/80640*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*c*f)*
g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 +
a)*sqrt(c)*x - a) - 2*(4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c
*e*g*h^2 + 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*
c^4*e*g*h^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e
*g^2*h + 9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e
*g^3 + 72*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2
*c^2*f)*h^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d -
4*a^4*f)*h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*
c^3*f)*g^3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3
 + 27*a^2*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d -
4*a^3*c*f)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3
*d + a^2*c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a)
)/c^3, 1/40320*(315*(24*a^3*c*e*g^2*h - 3*a^4*e*h^3 - 8*(6*a^2*c^2*d - a^3*
c*f)*g^3 + 3*(8*a^3*c*d - 3*a^4*f)*g*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (4480*c^4*f*h^3*x^8 + 8064*a^2*c^2*e*g^3 - 6912*a^3*c*e*g*h^2
+ 5040*(3*c^4*f*g*h^2 + c^4*e*h^3)*x^7 + 640*(27*c^4*f*g^2*h + 27*c^4*e*g*h
^2 + (9*c^4*d + 10*a*c^3*f)*h^3)*x^6 + 840*(8*c^4*f*g^3 + 24*c^4*e*g^2*h +
9*a*c^3*e*h^3 + 3*(8*c^4*d + 9*a*c^3*f)*g*h^2)*x^5 + 384*(21*c^4*e*g^3 + 72
*a*c^3*e*g*h^2 + 9*(7*c^4*d + 8*a*c^3*f)*g^2*h + (24*a*c^3*d + a^2*c^2*f)*h
^3)*x^4 + 3456*(7*a^2*c^2*d - 2*a^3*c*f)*g^2*h - 256*(9*a^3*c*d - 4*a^4*f)*
h^3 + 210*(168*a*c^3*e*g^2*h + 3*a^2*c^2*e*h^3 + 8*(6*c^4*d + 7*a*c^3*f)*g^
3 + 3*(56*a*c^3*d + 3*a^2*c^2*f)*g*h^2)*x^3 + 128*(126*a*c^3*e*g^3 + 27*a^2
*c^2*e*g*h^2 + 27*(14*a*c^3*d + a^2*c^2*f)*g^2*h + (9*a^2*c^2*d - 4*a^3*c*f
)*h^3)*x^2 + 315*(24*a^2*c^2*e*g^2*h - 3*a^3*c*e*h^3 + 8*(10*a*c^3*d + a^2*
c^2*f)*g^3 + 3*(8*a^2*c^2*d - 3*a^3*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac [A]** time = 0.27, size = 652, normalized size = 1.41

$$\frac{1}{40320}\sqrt{cx^2+a}\Bigg(\Bigg(\Bigg(2\Bigg(\Bigg(\Bigg(4\Bigg(5\Bigg(2\Bigg(7\Bigg(8\,cfh^3x+\frac{9\left(3\,c^8fgh^2+c^8h^3e\right)}{c^7}\Bigg)x+\frac{8\,(27\,c^8fg^2h+9\,c^8dh^3+10\,ac^7fh^3+27\,c}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/40320*sqrt(c*x^2 + a)*((2*((4*(5*(2*(7*(8*c*f*h^3*x + 9*(3*c^8*f*g*h^2 +
c^8*h^3*e)/c^7)*x + 8*(27*c^8*f*g^2*h + 9*c^8*d*h^3 + 10*a*c^7*f*h^3 + 27*c
^8*g*h^2*e)/c^7)*x + 21*(8*c^8*f*g^3 + 24*c^8*d*g*h^2 + 27*a*c^7*f*g*h^2 +
24*c^8*g^2*h*e + 9*a*c^7*h^3*e)/c^7)*x + 48*(63*c^8*d*g^2*h + 72*a*c^7*f*g^
2*h + 24*a*c^7*d*h^3 + a^2*c^6*f*h^3 + 21*c^8*g^3*e + 72*a*c^7*g*h^2*e)/c^7
)*x + 105*(48*c^8*d*g^3 + 56*a*c^7*f*g^3 + 168*a*c^7*d*g*h^2 + 9*a^2*c^6*f*
g*h^2 + 168*a*c^7*g^2*h*e + 3*a^2*c^6*h^3*e)/c^7)*x + 64*(378*a*c^7*d*g^2*h
 + 27*a^2*c^6*f*g^2*h + 9*a^2*c^6*d*h^3 - 4*a^3*c^5*f*h^3 + 126*a*c^7*g^3*e
 + 27*a^2*c^6*g*h^2*e)/c^7)*x + 315*(80*a*c^7*d*g^3 + 8*a^2*c^6*f*g^3 + 24*
a^2*c^6*d*g*h^2 - 9*a^3*c^5*f*g*h^2 + 24*a^2*c^6*g^2*h*e - 3*a^3*c^5*h^3*e)
/c^7)*x + 128*(189*a^2*c^6*d*g^2*h - 54*a^3*c^5*f*g^2*h - 18*a^3*c^5*d*h^3
+ 8*a^4*c^4*f*h^3 + 63*a^2*c^6*g^3*e - 54*a^3*c^5*g*h^2*e)/c^7) - 1/128*(48
*a^2*c^2*d*g^3 - 8*a^3*c*f*g^3 - 24*a^3*c*d*g*h^2 + 9*a^4*f*g*h^2 - 24*a^3*
c*g^2*h*e + 3*a^4*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple [A]** time = 0.02, size = 794, normalized size = 1.72

$$\frac{\left(c\,x^2+a\right)^{\frac{5}{2}} f\,h^3 x^4}{9c}+\frac{3a^4 e\,h^3 \ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{128c^{\frac{5}{2}}}+\frac{9a^4 f g\,h^2 \ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{128c^{\frac{5}{2}}}-\frac{3a^3 d g\,h^2 \ln\left(\sqrt{c}\;x+\sqrt{c\,x^2}\right.}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] -3/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*g*h^2-3/16*a^3/c^(3/2)*ln
(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g^2*h+3/7*x^2*(c*x^2+a)^(5/2)/c*e*g*h^2+3/7*x
^2*(c*x^2+a)^(5/2)/c*f*g^2*h-6/35*a/c^2*(c*x^2+a)^(5/2)*e*g*h^2-6/35*a/c^2*
(c*x^2+a)^(5/2)*f*g^2*h-4/63*f*h^3*a/c^2*x^2*(c*x^2+a)^(5/2)+3/8*x^3*(c*x^2
+a)^(5/2)/c*f*g*h^2-1/16*a/c^2*x*(c*x^2+a)^(5/2)*e*h^3+1/64*a^2/c^2*x*(c*x^
2+a)^(3/2)*e*h^3+3/128*a^3/c^2*x*(c*x^2+a)^(1/2)*e*h^3+9/128*a^4/c^(5/2)*ln
(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g*h^2+1/2*x*(c*x^2+a)^(5/2)/c*d*g*h^2+1/2*x*(
c*x^2+a)^(5/2)/c*e*g^2*h-1/24*a/c*x*(c*x^2+a)^(3/2)*f*g^3-1/16*a^2/c*x*(c*x
^2+a)^(1/2)*f*g^3+1/5*(c*x^2+a)^(5/2)/c*e*g^3+1/4*d*g^3*x*(c*x^2+a)^(3/2)-3
/16*a^2/c*x*(c*x^2+a)^(1/2)*d*g*h^2-3/16*a^2/c*x*(c*x^2+a)^(1/2)*e*g^2*h-1/
8*a/c*x*(c*x^2+a)^(3/2)*e*g^2*h-3/16*a/c^2*x*(c*x^2+a)^(5/2)*f*g*h^2+3/64*a
^2/c^2*x*(c*x^2+a)^(3/2)*f*g*h^2+9/128*a^3/c^2*x*(c*x^2+a)^(1/2)*f*g*h^2-1/
8*a/c*x*(c*x^2+a)^(3/2)*d*g*h^2+1/9*f*h^3*x^4*(c*x^2+a)^(5/2)/c+8/315*f*h^3
*a^2/c^3*(c*x^2+a)^(5/2)+1/8*x^3*(c*x^2+a)^(5/2)/c*e*h^3+3/128*a^4/c^(5/2)*
ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h^3+1/7*x^2*(c*x^2+a)^(5/2)/c*d*h^3-2/35*a/
c^2*(c*x^2+a)^(5/2)*d*h^3+1/6*x*(c*x^2+a)^(5/2)/c*f*g^3-1/16*a^3/c^(3/2)*ln
```

```
(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^3+3/5*(c*x^2+a)^(5/2)/c*d*g^2*h+3/8*d*g^3*a
*x*(c*x^2+a)^(1/2)+3/8*d*g^3*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.46, size = 525, normalized size = 1.14

$$\frac{(cx^2+a)^{\frac{5}{2}}fh^3x^4}{9c} - \frac{4(cx^2+a)^{\frac{5}{2}}afh^3x^2}{63c^2} + \frac{1}{4}(cx^2+a)^{\frac{3}{2}}dg^3x + \frac{3}{8}\sqrt{cx^2+a}\,adg^3x + \frac{3a^2dg^3\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\sqrt{c}} + \frac{(cx^2+a)}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2 + a)^(5/2)*f*h^3*x^4/c - 4/63*(c*x^2 + a)^(5/2)*a*f*h^3*x^2/c^2
+ 1/4*(c*x^2 + a)^(3/2)*d*g^3*x + 3/8*sqrt(c*x^2 + a)*a*d*g^3*x + 3/8*a^2*d
*g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2 + a)^(5/2)*e*g^3/c + 3/5*(
c*x^2 + a)^(5/2)*d*g^2*h/c + 8/315*(c*x^2 + a)^(5/2)*a^2*f*h^3/c^3 + 1/8*(3
*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*x^3/c + 1/7*(3*f*g^2*h + 3*e*g*h^2 + d*
h^3)*(c*x^2 + a)^(5/2)*x^2/c - 1/16*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(5/2)*a
*x/c^2 + 1/64*(3*f*g*h^2 + e*h^3)*(c*x^2 + a)^(3/2)*a^2*x/c^2 + 3/128*(3*f*
g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a^3*x/c^2 + 1/6*(f*g^3 + 3*e*g^2*h + 3*d*g*h
^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*(c*x^2 + a
)^(3/2)*a*x/c - 1/16*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*a^2*x/
c + 3/128*(3*f*g*h^2 + e*h^3)*a^4*arcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/16*(f*
g^3 + 3*e*g^2*h + 3*d*g*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(3*f
*g^2*h + 3*e*g*h^2 + d*h^3)*(c*x^2 + a)^(5/2)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^3 (cx^2+a)^{3/2} (fx^2+ex+d)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 72.41, size = 1916, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*e*h**3*x/(128*c**2*sqrt(1 + c*x**2/a)) - 9*a**(7/2)*f*g*h**2*x/
(128*c**2*sqrt(1 + c*x**2/a)) + 3*a**(5/2)*d*g*h**2*x/(16*c*sqrt(1 + c*x**2
```

```
/a)) + 3*a**(5/2)*e*g**2*h*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*e*h**3*x*
*3/(128*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g**3*x/(16*c*sqrt(1 + c*x**2/a))
 - 3*a**(5/2)*f*g*h**2*x**3/(128*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*g**3*x*
sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**3*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/
2)*d*g*h**2*x**3/(16*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*e*g**2*h*x**3/(16*sq
rt(1 + c*x**2/a)) + 13*a**(3/2)*e*h**3*x**5/(64*sqrt(1 + c*x**2/a)) + 17*a*
*(3/2)*f*g**3*x**3/(48*sqrt(1 + c*x**2/a)) + 39*a**(3/2)*f*g*h**2*x**5/(64*
sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g**3*x**3/(8*sqrt(1 + c*x**2/a)) + 11*s
qrt(a)*c*d*g*h**2*x**5/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g**2*h*x**5/
(8*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*e*h**3*x**7/(16*sqrt(1 + c*x**2/a)) +
11*sqrt(a)*c*f*g**3*x**5/(24*sqrt(1 + c*x**2/a)) + 15*sqrt(a)*c*f*g*h**2*x*
*7/(16*sqrt(1 + c*x**2/a)) + 3*a**4*e*h**3*asinh(sqrt(c)*x/sqrt(a))/(128*c*
*(5/2)) + 9*a**4*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)) - 3*a**3*
d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - 3*a**3*e*g**2*h*asinh(sqr
t(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(16*c*
*(3/2)) + 3*a**2*d*g**3*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 3*a*d*g**2*h
*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) +
 a*d*h**3*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c
*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True))
 + a*e*g**3*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c
), True)) + 3*a*e*g*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + 3*a*f*g**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2
) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (s
qrt(a)*x**4/4, True)) + a*f*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c*
*3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35
*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 3*c*d*g
**2*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**
2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c
*d*h**3*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a
+ c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**
2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*e*g**3*Piecewise((-2*a**2*sqrt
(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x
**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + 3*c*e*g*h**2*Piecewise((8*a**3
*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*
x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)
*x**6/6, True)) + 3*c*f*g**2*h*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3
) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c
) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + c*f*h**3*
Piecewise((-16*a**4*sqrt(a + c*x**2)/(315*c**4) + 8*a**3*x**2*sqrt(a + c*x*
*2)/(315*c**3) - 2*a**2*x**4*sqrt(a + c*x**2)/(105*c**2) + a*x**6*sqrt(a +
c*x**2)/(63*c) + x**8*sqrt(a + c*x**2)/9, Ne(c, 0)), (sqrt(a)*x**8/8, True)
) + c**2*d*g**3*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*d*g*h**2*x**7/(2
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g**2*h*x**7/(2*sqrt(a)*sqrt(1 + c*x**
2/a)) + c**2*e*h**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**3*x**7/
```

```
(6*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*c**2*f*g*h**2*x**9/(8*sqrt(a)*sqrt(1 + c
*x**2/a))
```

# 3.89 $\int (g+hx)^2 \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=346

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+\right.}{128c^2}$$

```
[Out] 1/192*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(3
/2)/c^2-1/56*(-8*e*h+5*f*g)*(h*x+g)^2*(c*x^2+a)^(5/2)/c/h+1/8*f*(h*x+g)^3*(
c*x^2+a)^(5/2)/c/h-1/1680*(96*a*h^2*(e*h+2*f*g)+12*c*g*(5*f*g^2-8*h*(7*d*h+
e*g))-5*h*(7*(-3*a*f+8*c*d)*h^2-2*c*g*(-8*e*h+5*f*g))*x)*(c*x^2+a)^(5/2)/c^
2/h+1/128*a^2*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*c*(f*g^2+h*(d*h+2*e*g)))*arctan
h(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/128*a*(48*c^2*d*g^2+3*a^2*f*h^2-8*a*
c*(f*g^2+h*(d*h+2*e*g)))*x*(c*x^2+a)^(1/2)/c^2
```

**Rubi [A]** time = 0.52, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 833, 780, 195, 217, 206}

$$\frac{x\left(a+cx^2\right)^{3/2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+48c^2dg^2\right)}{192c^2}+\frac{ax\sqrt{a+cx^2}\left(3a^2fh^2-8ac\left(h(dh+2eg)+fg^2\right)+\right.}{128c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*x*Sqrt[a
 + c*x^2])/(128*c^2) + ((48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*
g + d*h)))*x*(a + c*x^2)^(3/2))/(192*c^2) - ((5*f*g - 8*e*h)*(g + h*x)^2*(a
 + c*x^2)^(5/2))/(56*c*h) + (f*(g + h*x)^3*(a + c*x^2)^(5/2))/(8*c*h) - ((1
2*(5*c*f*g^3 - 8*c*g*h*(e*g + 7*d*h) + 8*a*h^2*(2*f*g + e*h)) - 5*h*(7*(8*c
*d - 3*a*f)*h^2 - 2*c*g*(5*f*g - 8*e*h))*x)*(a + c*x^2)^(5/2))/(1680*c^2*h)
 + (a^2*(48*c^2*d*g^2 + 3*a^2*f*h^2 - 8*a*c*(f*g^2 + h*(2*e*g + d*h)))*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(5/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2\left((8cd-3af)h^2-ch(5fg-8e\right.}{8ch^2}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} + \frac{\int (g}{}$$

$$= -\frac{(5fg-8eh)(g+hx)^2\left(a+cx^2\right)^{5/2}}{56ch} + \frac{f(g+hx)^3\left(a+cx^2\right)^{5/2}}{8ch} - \frac{(12}{}$$

$$= \frac{\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\left(a+cx^2\right)^{3/2}}{192c^2} - \frac{(5f}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(48c}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(48c}{}$$

$$= \frac{a\left(48c^2dg^2+3a^2fh^2-8ac\left(fg^2+h(2eg+dh)\right)\right)x\sqrt{a+cx^2}}{128c^2} + \frac{(48c}{}$$

**Mathematica [A]** time = 1.09, size = 346, normalized size = 1.00

$$\sqrt{a+cx^2}\left(-\frac{280a\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{c}\,x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)\left(h(dh+2eg)+fg^2\right)}{c^{3/2}\sqrt{\frac{cx^2}{a}+1}} + 1680dg^2\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}} + 5ax + 2cx^3\right) + \frac{105}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((2688*g*(e*g + 2*d*h)*(a + c*x^2)^2)/c + (2240*(f*g^2 + h
*(2*e*g + d*h))*x*(a + c*x^2)^2)/c + (1680*f*h^2*x^3*(a + c*x^2)^2)/c + (38
4*h*(2*f*g + e*h)*(a + c*x^2)^2*(-2*a + 5*c*x^2))/c^2 - (280*a*(f*g^2 + h*(
2*e*g + d*h))*(Sqrt[c]*x*(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*Ar
cSinh[(Sqrt[c]*x)/Sqrt[a]]))/(c^(3/2)*Sqrt[1 + (c*x^2)/a]) + (105*a*f*h^2*(
-(Sqrt[c]*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4)) + (3*a^(5/2)*ArcSinh[(Sqrt[c]
*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/c^(5/2) + 1680*d*g^2*(5*a*x + 2*c*x^3 +
 (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/a]))))/
13440
```

**fricas [A]** time = 1.34, size = 831, normalized size = 2.40

$$\left[-\frac{105\left(16\,a^3cegh-8\left(6\,a^2c^2d-a^3cf\right)g^2+\left(8\,a^3cd-3\,a^4f\right)h^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(1680\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/26880*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d - a^3*c*f)*g^2 + (8*a^3*c*d
 - 3*a^4*f)*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) -
2*(1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1920*(2*c^4*
f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^4*d + 9*a*c
^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d + 8*a*c^3*f)
*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 + (56*a*c^3*d
 + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 384*(14*a*c^
3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 + 105*(16*a^2
*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a^3*c*f)*h^2
)*x)*sqrt(c*x^2 + a))/c^3, 1/13440*(105*(16*a^3*c*e*g*h - 8*(6*a^2*c^2*d -
a^3*c*f)*g^2 + (8*a^3*c*d - 3*a^4*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c
*x^2 + a)) + (1680*c^4*f*h^2*x^7 + 2688*a^2*c^2*e*g^2 - 768*a^3*c*e*h^2 + 1
920*(2*c^4*f*g*h + c^4*e*h^2)*x^6 + 280*(8*c^4*f*g^2 + 16*c^4*e*g*h + (8*c^
4*d + 9*a*c^3*f)*h^2)*x^5 + 384*(7*c^4*e*g^2 + 8*a*c^3*e*h^2 + 2*(7*c^4*d +
 8*a*c^3*f)*g*h)*x^4 + 70*(112*a*c^3*e*g*h + 8*(6*c^4*d + 7*a*c^3*f)*g^2 +
(56*a*c^3*d + 3*a^2*c^2*f)*h^2)*x^3 + 768*(7*a^2*c^2*d - 2*a^3*c*f)*g*h + 3
84*(14*a*c^3*e*g^2 + a^2*c^2*e*h^2 + 2*(14*a*c^3*d + a^2*c^2*f)*g*h)*x^2 +
105*(16*a^2*c^2*e*g*h + 8*(10*a*c^3*d + a^2*c^2*f)*g^2 + (8*a^2*c^2*d - 3*a
^3*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac [A]** time = 0.26, size = 452, normalized size = 1.31

$$\frac{1}{13440}\,\sqrt{cx^2+a}\left(\left(\left(2\left(\left(\left(4\left(5\left(6\left(7\,cfh^2x+\frac{8\left(2\,c^7fgh+c^7h^2e\right)}{c^6}\right)x+\frac{7\left(8\,c^7fg^2+8\,c^7dh^2+9\,ac^6fh^2+16\,c^7ghe\right)}{c^6}\right)x\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/13440*sqrt(c*x^2 + a)*((2*((4*(5*(6*(7*c*f*h^2*x + 8*(2*c^7*f*g*h + c^7*h
^2*e)/c^6)*x + 7*(8*c^7*f*g^2 + 8*c^7*d*h^2 + 9*a*c^6*f*h^2 + 16*c^7*g*h*e)
/c^6)*x + 48*(14*c^7*d*g*h + 16*a*c^6*f*g*h + 7*c^7*g^2*e + 8*a*c^6*h^2*e)/
c^6)*x + 35*(48*c^7*d*g^2 + 56*a*c^6*f*g^2 + 56*a*c^6*d*h^2 + 3*a^2*c^5*f*h
^2 + 112*a*c^6*g*h*e)/c^6)*x + 192*(28*a*c^6*d*g*h + 2*a^2*c^5*f*g*h + 14*a
```

```
*c^6*g^2*e + a^2*c^5*h^2*e)/c^6)*x + 105*(80*a*c^6*d*g^2 + 8*a^2*c^5*f*g^2
+ 8*a^2*c^5*d*h^2 - 3*a^3*c^4*f*h^2 + 16*a^2*c^5*g*h*e)/c^6)*x + 384*(14*a^
2*c^5*d*g*h - 4*a^3*c^4*f*g*h + 7*a^2*c^5*g^2*e - 2*a^3*c^4*h^2*e)/c^6) - 1
/128*(48*a^2*c^2*d*g^2 - 8*a^3*c*f*g^2 - 8*a^3*c*d*h^2 + 3*a^4*f*h^2 - 16*a
^3*c*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple** [A] time = 0.01, size = 552, normalized size = 1.60

$$\frac{3a^4 f h^2 \ln\left(\sqrt{c}\, x+\sqrt{c\, x^2+a}\right)}{128c^{\frac{5}{2}}}-\frac{a^3 d\, h^2 \ln\left(\sqrt{c}\, x+\sqrt{c\, x^2+a}\right)}{16c^{\frac{3}{2}}}-\frac{a^3 egh \ln\left(\sqrt{c}\, x+\sqrt{c\, x^2+a}\right)}{8c^{\frac{3}{2}}}-\frac{a^3 f\, g^2 \ln\left(\sqrt{c}\, x+\right.}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/3*x*(c*x^2+a)^(5/2)/c*e*g*h-1/24*a/c*x*(c*x^2+a)^(3/2)*d*h^2-1/24*a/c*x*(
c*x^2+a)^(3/2)*f*g^2-1/16*a^2/c*x*(c*x^2+a)^(1/2)*d*h^2-1/16*f*h^2*a/c^2*x*
(c*x^2+a)^(5/2)+1/64*f*h^2*a^2/c^2*x*(c*x^2+a)^(3/2)+3/128*f*h^2*a^3/c^2*x*
(c*x^2+a)^(1/2)+2/7*x^2*(c*x^2+a)^(5/2)/c*f*g*h-4/35*a/c^2*(c*x^2+a)^(5/2)*
f*g*h-1/8*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/16*a^2/c*x*(c*x
^2+a)^(1/2)*f*g^2+1/4*d*g^2*x*(c*x^2+a)^(3/2)+1/5*(c*x^2+a)^(5/2)/c*e*g^2+1
/8*f*h^2*x^3*(c*x^2+a)^(5/2)/c+3/128*f*h^2*a^4/c^(5/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))-1/12*a/c*x*(c*x^2+a)^(3/2)*e*g*h-1/8*a^2/c*x*(c*x^2+a)^(1/2)*e*g*
h+1/6*x*(c*x^2+a)^(5/2)/c*d*h^2+1/6*x*(c*x^2+a)^(5/2)/c*f*g^2-1/16*a^3/c^(3
/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2-1/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^
2+a)^(1/2))*f*g^2+2/5*(c*x^2+a)^(5/2)/c*d*g*h+3/8*d*g^2*a*x*(c*x^2+a)^(1/2)
+3/8*d*g^2*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/7*x^2*(c*x^2+a)^(5/2
)/c*e*h^2-2/35*a/c^2*(c*x^2+a)^(5/2)*e*h^2
```

**maxima** [A] time = 0.45, size = 380, normalized size = 1.10

$$\frac{\left(cx^2+a\right)^{\frac{5}{2}} fh^2x^3}{8\,c}+\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}} dg^2x+\frac{3}{8}\sqrt{cx^2+a}\,adg^2x-\frac{\left(cx^2+a\right)^{\frac{5}{2}} afh^2x}{16\,c^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}} a^2fh^2x}{64\,c^2}+\frac{3\sqrt{cx^2+a}\,a^3fh^2}{128\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2 + a)^(5/2)*f*h^2*x^3/c + 1/4*(c*x^2 + a)^(3/2)*d*g^2*x + 3/8*sqr
t(c*x^2 + a)*a*d*g^2*x - 1/16*(c*x^2 + a)^(5/2)*a*f*h^2*x/c^2 + 1/64*(c*x^2
 + a)^(3/2)*a^2*f*h^2*x/c^2 + 3/128*sqrt(c*x^2 + a)*a^3*f*h^2*x/c^2 + 3/8*a
^2*d*g^2*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/128*a^4*f*h^2*arcsinh(c*x/sqrt(
a*c))/c^(5/2) + 1/5*(c*x^2 + a)^(5/2)*e*g^2/c + 2/5*(c*x^2 + a)^(5/2)*d*g*h
/c + 1/7*(2*f*g*h + e*h^2)*(c*x^2 + a)^(5/2)*x^2/c + 1/6*(f*g^2 + 2*e*g*h +
```

```
d*h^2)*(c*x^2 + a)^(5/2)*x/c - 1/24*(f*g^2 + 2*e*g*h + d*h^2)*(c*x^2 + a)^
(3/2)*a*x/c - 1/16*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a)*a^2*x/c - 1/16
*(f*g^2 + 2*e*g*h + d*h^2)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/35*(2*f*g
*h + e*h^2)*(c*x^2 + a)^(5/2)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^2 \left(cx^2+a\right)^{3/2} \left(fx^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 54.50, size = 1304, normalized size = 3.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] -3*a**(7/2)*f*h**2*x/(128*c**2*sqrt(1 + c*x**2/a)) + a**(5/2)*d*h**2*x/(16*
c*sqrt(1 + c*x**2/a)) + a**(5/2)*e*g*h*x/(8*c*sqrt(1 + c*x**2/a)) + a**(5/2
)*f*g**2*x/(16*c*sqrt(1 + c*x**2/a)) - a**(5/2)*f*h**2*x**3/(128*c*sqrt(1 +
 c*x**2/a)) + a**(3/2)*d*g**2*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g**2*x/(8
*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*d*h**2*x**3/(48*sqrt(1 + c*x**2/a)) + 17
*a**(3/2)*e*g*h*x**3/(24*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*g**2*x**3/(48*
sqrt(1 + c*x**2/a)) + 13*a**(3/2)*f*h**2*x**5/(64*sqrt(1 + c*x**2/a)) + 3*s
qrt(a)*c*d*g**2*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*d*h**2*x**5/(24*
sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*e*g*h*x**5/(12*sqrt(1 + c*x**2/a)) + 11*
sqrt(a)*c*f*g**2*x**5/(24*sqrt(1 + c*x**2/a)) + 5*sqrt(a)*c*f*h**2*x**7/(16
*sqrt(1 + c*x**2/a)) + 3*a**4*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(128*c**(5/2)
) - a**3*d*h**2*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) - a**3*e*g*h*asinh(s
qrt(c)*x/sqrt(a))/(8*c**(3/2)) - a**3*f*g**2*asinh(sqrt(c)*x/sqrt(a))/(16*c
**(3/2)) + 3*a**2*d*g**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + 2*a*d*g*h*P
iecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + a
*e*g**2*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), T
rue)) + a*e*h**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqr
t(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4,
 True)) + 2*a*f*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x**2*
sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4
/4, True)) + 2*c*d*g*h*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*x*
*2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)*x
**4/4, True)) + c*e*g**2*Piecewise((-2*a**2*sqrt(a + c*x**2)/(15*c**2) + a*
```

```
x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(c, 0)), (sqrt(a)
*x**4/4, True)) + c*e*h**2*Piecewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) -
4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) +
x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) + 2*c*f*g*h*Pie
cewise((8*a**3*sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(
105*c**2) + a*x**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c,
 0)), (sqrt(a)*x**6/6, True)) + c**2*d*g**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**2
/a)) + c**2*d*h**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*g*h*x**7/(3
*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*f*g**2*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/
a)) + c**2*f*h**2*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

# 3.90 $\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

**Optimal**. Leaf size=213

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2+c\left(5fg^2-7h(dh+eg)\right)\right)+5chx(5fg-7eh)\right)}{210c^2h}$$

```
[Out] 1/24*(6*c*d*g-a*(e*h+f*g))*x*(c*x^2+a)^(3/2)/c+1/7*f*(h*x+g)^2*(c*x^2+a)^(5
/2)/c/h-1/210*(12*a*f*h^2+6*c*(5*f*g^2-7*h*(d*h+e*g))+5*c*h*(-7*e*h+5*f*g)*
x)*(c*x^2+a)^(5/2)/c^2/h+1/16*a^2*(-a*e*h-a*f*g+6*c*d*g)*arctanh(x*c^(1/2)/
(c*x^2+a)^(1/2))/c^(3/2)+1/16*a*(-a*e*h-a*f*g+6*c*d*g)*x*(c*x^2+a)^(1/2)/c
```

**Rubi [A]** time = 0.27, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1654, 780, 195, 217, 206}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(-aeh-afg+6cdg)}{16c^{3/2}}-\frac{\left(a+cx^2\right)^{5/2}\left(6\left(2afh^2-7ch(dh+eg)+5cfg^2\right)+5chx(5fg-7eh)\right)}{210c^2h}+$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d*g - a*f*g - a*e*h)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d*g - a*(f*g
 + e*h))*x*(a + c*x^2)^(3/2))/(24*c) + (f*(g + h*x)^2*(a + c*x^2)^(5/2))/(7
*c*h) - ((6*(5*c*f*g^2 + 2*a*f*h^2 - 7*c*h*(e*g + d*h)) + 5*c*h*(5*f*g - 7*
e*h)*x)*(a + c*x^2)^(5/2))/(210*c^2*h) + (a^2*(6*c*d*g - a*f*g - a*e*h)*Arc
Tanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left((7cd-2af)h^2-ch(5fg-7eh)x\right.}{7ch^2}$$

$$= \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{\left(6\left(5cfg^2+2afh^2-7ch(eg+dh)\right)+5ch(5\right.}{210c^2h}$$

$$= \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c} + \frac{f(g+hx)^2\left(a+cx^2\right)^{5/2}}{7ch} - \frac{\left(6\left(5c\right.\right.}{}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}$$

$$= \frac{a(6cdg-afg-aeh)x\sqrt{a+cx^2}}{16c} + \frac{(6cdg-a(fg+eh))x\left(a+cx^2\right)^{3/2}}{24c}$$

**Mathematica [A]** time = 0.63, size = 209, normalized size = 0.98

$$\frac{\sqrt{a+cx^2}\left(-\frac{105a^{5/2}\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(aeh+afg-6cdg)}{c^{3/2}\left(a+cx^2\right)}-\frac{96a^3fh}{c^2}+\frac{3a^2(112dh+7e(16g+5hx)+fx(35g+16hx))}{c}+2ax(21d(25g+16\ldots\right.}{168\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*((-96*a^3*f*h)/c^2 + (3*a^2*(112*d*h + 7*e*(16*g + 5*h*x)
+ f*x*(35*g + 16*h*x)))/c + 4*c*x^3*(21*d*(5*g + 4*h*x) + 2*x*(7*e*(6*g + 5
*h*x) + 5*f*x*(7*g + 6*h*x))) + 2*a*x*(21*d*(25*g + 16*h*x) + x*(7*e*(48*g
+ 35*h*x) + f*x*(245*g + 192*h*x))) - (105*a^(5/2)*(-6*c*d*g + a*f*g + a*e*
h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(c^(3/2)*(a + c*x^2)))
)/1680
```

**fricas [A]** time = 1.02, size = 477, normalized size = 2.24

$$\left[\frac{105\left(a^3eh-\left(6\,a^2cd-a^3f\right)g\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(240\,c^3fhx^6+280\left(c^3fg+c^3eh\right)x^5+3\ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*(a^3*e*h - (6*a^2*c*d - a^3*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqr
t(c*x^2 + a)*sqrt(c)*x - a) + 2*(240*c^3*f*h*x^6 + 280*(c^3*f*g + c^3*e*h)*
x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d + 8*a*c^2*f)*h)*x^4 + 70*(7*
a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14*a*c^2*e*g + (14*a*c^2*d +
 a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 105*(a^2*c*e*h + (10*a*c^2*
d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2, 1/1680*(105*(a^3*e*h - (6*a^2*c*d
- a^3*f)*g)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (240*c^3*f*h*x^6
+ 280*(c^3*f*g + c^3*e*h)*x^5 + 336*a^2*c*e*g + 48*(7*c^3*e*g + (7*c^3*d +
8*a*c^2*f)*h)*x^4 + 70*(7*a*c^2*e*h + (6*c^3*d + 7*a*c^2*f)*g)*x^3 + 48*(14
*a*c^2*e*g + (14*a*c^2*d + a^2*c*f)*h)*x^2 + 48*(7*a^2*c*d - 2*a^3*f)*h + 1
05*(a^2*c*e*h + (10*a*c^2*d + a^2*c*f)*g)*x)*sqrt(c*x^2 + a))/c^2]
```

**giac [A]** time = 0.25, size = 264, normalized size = 1.24

$$\frac{1}{1680}\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,cfhx+\frac{7\left(c^6fg+c^6he\right)}{c^5}\right)x+\frac{6\left(7\,c^6dh+8\,ac^5fh+7\,c^6ge\right)}{c^5}\right)x+\frac{35\left(6\,c^6dg+7\,ac^5f\ldots\right.}{c^5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1680*sqrt(c*x^2 + a)*((2*((4*(5*(6*c*f*h*x + 7*(c^6*f*g + c^6*h*e)/c^5)*x
 + 6*(7*c^6*d*h + 8*a*c^5*f*h + 7*c^6*g*e)/c^5)*x + 35*(6*c^6*d*g + 7*a*c^5
*f*g + 7*a*c^5*h*e)/c^5)*x + 24*(14*a*c^5*d*h + a^2*c^4*f*h + 14*a*c^5*g*e)
/c^5)*x + 105*(10*a*c^5*d*g + a^2*c^4*f*g + a^2*c^4*h*e)/c^5)*x + 48*(7*a^2
*c^4*d*h - 2*a^3*c^3*f*h + 7*a^2*c^4*g*e)/c^5) - 1/16*(6*a^2*c*d*g - a^3*f*
g - a^3*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.00, size = 287, normalized size = 1.35

$$-\frac{a^{3}eh\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{16c^{\frac{3}{2}}}-\frac{a^{3}fg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^{2}dg\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+a}\right)}{8\sqrt{c}}-\frac{\sqrt{c\,x^{2}+a}\;a^{2}ehx}{16c}-\frac{\sqrt{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/7*h*f*x^2*(c*x^2+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+a)^(5/2)+1/6*x*(c*x^2+a
)^(5/2)/c*e*h+1/6*x*(c*x^2+a)^(5/2)/c*f*g-1/24*a/c*x*(c*x^2+a)^(3/2)*e*h-1/
24*a/c*x*(c*x^2+a)^(3/2)*f*g-1/16*a^2/c*x*(c*x^2+a)^(1/2)*e*h-1/16*a^2/c*x*
(c*x^2+a)^(1/2)*f*g-1/16*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h-1/16
*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/5*(c*x^2+a)^(5/2)/c*d*h+1/
5*(c*x^2+a)^(5/2)/c*e*g+1/4*d*g*x*(c*x^2+a)^(3/2)+3/8*d*g*a*x*(c*x^2+a)^(1/
2)+3/8*d*g*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.45, size = 211, normalized size = 0.99

$$\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}fhx^{2}}{7\,c}+\frac{1}{4}\left(cx^{2}+a\right)^{\frac{3}{2}}dgx+\frac{3}{8}\sqrt{cx^{2}+a}\,adgx+\frac{3\,a^{2}dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}+\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}eg}{5\,c}+\frac{\left(cx^{2}+a\right)^{\frac{5}{2}}dh}{5\,c}-\frac{2\left(cx^{2}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2 + a)^(5/2)*f*h*x^2/c + 1/4*(c*x^2 + a)^(3/2)*d*g*x + 3/8*sqrt(c*
x^2 + a)*a*d*g*x + 3/8*a^2*d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 1/5*(c*x^2
+ a)^(5/2)*e*g/c + 1/5*(c*x^2 + a)^(5/2)*d*h/c - 2/35*(c*x^2 + a)^(5/2)*a*f
*h/c^2 + 1/6*(c*x^2 + a)^(5/2)*(f*g + e*h)*x/c - 1/24*(c*x^2 + a)^(3/2)*(f*
g + e*h)*a*x/c - 1/16*sqrt(c*x^2 + a)*(f*g + e*h)*a^2*x/c - 1/16*(f*g + e*h
)*a^3*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 27.89, size = 768, normalized size = 3.61

$$\frac{a^{\frac{5}{2}}ehx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{5}{2}}fgx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dgx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dgx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}ehx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fgx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,cdgx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,ceh}{24\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*e*h*x/(16*c*sqrt(1 + c*x**2/a)) + a**(5/2)*f*g*x/(16*c*sqrt(1 + c*
x**2/a)) + a**(3/2)*d*g*x*sqrt(1 + c*x**2/a)/2 + a**(3/2)*d*g*x/(8*sqrt(1 +
 c*x**2/a)) + 17*a**(3/2)*e*h*x**3/(48*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*
g*x**3/(48*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*c*d*g*x**3/(8*sqrt(1 + c*x**2/a)
) + 11*sqrt(a)*c*e*h*x**5/(24*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*g*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*e*h*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) -
a**3*f*g*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*a**2*d*g*asinh(sqrt(c)*
x/sqrt(a))/(8*sqrt(c)) + a*d*h*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)), ((a +
c*x**2)**(3/2)/(3*c), True)) + a*e*g*Piecewise((sqrt(a)*x**2/2, Eq(c, 0)),
((a + c*x**2)**(3/2)/(3*c), True)) + a*f*h*Piecewise((-2*a**2*sqrt(a + c*x*
*2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, N
e(c, 0)), (sqrt(a)*x**4/4, True)) + c*d*h*Piecewise((-2*a**2*sqrt(a + c*x**
2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne
(c, 0)), (sqrt(a)*x**4/4, True)) + c*e*g*Piecewise((-2*a**2*sqrt(a + c*x**2
)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c*x**2)/5, Ne(
c, 0)), (sqrt(a)*x**4/4, True)) + c*f*h*Piecewise((8*a**3*sqrt(a + c*x**2)/
(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x**4*sqrt(a + c*x*
*2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*x**6/6, True)) +
c**2*d*g*x**5/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + c**2*e*h*x**7/(6*sqrt(a)*sqr
t(1 + c*x**2/a)) + c**2*f*g*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

## 3.91 $\int \left(a+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx$

**Optimal**. Leaf size=137

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}\,(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

```
[Out] 1/24*(-a*f+6*c*d)*x*(c*x^2+a)^(3/2)/c+1/5*e*(c*x^2+a)^(5/2)/c+1/6*f*x*(c*x^
2+a)^(5/2)/c+1/16*a^2*(-a*f+6*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/
2)+1/16*a*(-a*f+6*c*d)*x*(c*x^2+a)^(1/2)/c
```

**Rubi [A]** time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{a^2(6cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{16c^{3/2}}+\frac{x\left(a+cx^2\right)^{3/2}(6cd-af)}{24c}+\frac{ax\sqrt{a+cx^2}\,(6cd-af)}{16c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (a*(6*c*d - a*f)*x*Sqrt[a + c*x^2])/(16*c) + ((6*c*d - a*f)*x*(a + c*x^2)^(
3/2))/(24*c) + (e*(a + c*x^2)^(5/2))/(5*c) + (f*x*(a + c*x^2)^(5/2))/(6*c)
+ (a^2*(6*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(16*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx &= \frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{\int (6cd-af+6cex)\left(a+cx^2\right)^{3/2}dx}{6c}\\
&= \frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(6cd-af)\int \left(a+cx^2\right)^{3/2}dx}{6c}\\
&= \frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a+cx^2\right)^{5/2}}{6c}+\frac{(a(6cd-af))}{8\ldots}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a\ldots\right.}{}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a\ldots\right.}{}\\
&= \frac{a(6cd-af)x\sqrt{a+cx^2}}{16c}+\frac{(6cd-af)x\left(a+cx^2\right)^{3/2}}{24c}+\frac{e\left(a+cx^2\right)^{5/2}}{5c}+\frac{fx\left(a\ldots\right.}{}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 125, normalized size = 0.91

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(3a^2(16e+5fx)+2acx(75d+x(48e+35fx))+4c^2x^3(15d+2x(6e+5fx))\right)-\frac{15a^{3/2}(af-6cd)\sinh^{-1}\ldots}{\sqrt{\frac{cx^2}{a}+1}}\ldots\right.}{240c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^2*(16*e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*
e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*e + 35*f*x))) - (15*a^(3/2)*(-6*c*d + a
*f)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(240*c^(3/2))
```

**fricas [A]** time = 1.19, size = 262, normalized size = 1.91

$$\left[-\frac{15\left(6\,a^2cd-a^3f\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(40\,c^3fx^5+48\,c^3ex^4+96\,ac^2ex^2+48\,a^2ce+10\left(6\right.\right.}{480\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/480*(15*(6*a^2*c*d - a^3*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sq
rt(c)*x - a) - 2*(40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e
 + 10*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 +
 a))/c^2, -1/240*(15*(6*a^2*c*d - a^3*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*
x^2 + a)) - (40*c^3*f*x^5 + 48*c^3*e*x^4 + 96*a*c^2*e*x^2 + 48*a^2*c*e + 10
*(6*c^3*d + 7*a*c^2*f)*x^3 + 15*(10*a*c^2*d + a^2*c*f)*x)*sqrt(c*x^2 + a))/
c^2]
```

**giac [A]** time = 0.23, size = 129, normalized size = 0.94

$$\frac{1}{240}\,\sqrt{cx^2+a}\left(\left(2\left(\left(4\left(5\,cfx+6\,ce\right)x+\frac{5\left(6\,c^5d+7\,ac^4f\right)}{c^4}\right)x+48\,ae\right)x+\frac{15\left(10\,ac^4d+a^2c^3f\right)}{c^4}\right)x+\frac{48\,a^2e}{c}\right)-\frac{\left(6\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/240*sqrt(c*x^2 + a)*((2*((4*(5*c*f*x + 6*c*e)*x + 5*(6*c^5*d + 7*a*c^4*f)
/c^4)*x + 48*a*e)*x + 15*(10*a*c^4*d + a^2*c^3*f)/c^4)*x + 48*a^2*e/c) - 1/
16*(6*a^2*c*d - a^3*f)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.01, size = 146, normalized size = 1.07

$$-\frac{a^3f\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{8\sqrt{c}}-\frac{\sqrt{c\,x^2+a}\;a^2fx}{16c}+\frac{3\sqrt{c\,x^2+a}\;adx}{8}-\frac{\left(c\,x^2+a\right)^{\frac{3}{2}}afx}{24c}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/6*f*x*(c*x^2+a)^(5/2)/c-1/24*f*a/c*x*(c*x^2+a)^(3/2)-1/16*f*a^2/c*x*(c*x^
2+a)^(1/2)-1/16*f*a^3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/5*e*(c*x^2+a)
^(5/2)/c+1/4*d*x*(c*x^2+a)^(3/2)+3/8*d*a*x*(c*x^2+a)^(1/2)+3/8*d*a^2/c^(1/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.45, size = 131, normalized size = 0.96

$$\frac{1}{4}\left(cx^2+a\right)^{\frac{3}{2}}dx+\frac{3}{8}\sqrt{cx^2+a}\,adx+\frac{\left(cx^2+a\right)^{\frac{5}{2}}fx}{6\,c}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}afx}{24\,c}-\frac{\sqrt{cx^2+a}\,a^2fx}{16\,c}+\frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{c}}-\frac{a^3f\,\mathrm{arsi}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2 + a)^(3/2)*d*x + 3/8*sqrt(c*x^2 + a)*a*d*x + 1/6*(c*x^2 + a)^(5/
2)*f*x/c - 1/24*(c*x^2 + a)^(3/2)*a*f*x/c - 1/16*sqrt(c*x^2 + a)*a^2*f*x/c
+ 3/8*a^2*d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/16*a^3*f*arcsinh(c*x/sqrt(a*
c))/c^(3/2) + 1/5*(c*x^2 + a)^(5/2)*e/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [A]** time = 17.01, size = 348, normalized size = 2.54

$$\frac{a^{\frac{5}{2}}fx}{16c\sqrt{1+\frac{cx^2}{a}}}+\frac{a^{\frac{3}{2}}dx\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a^{\frac{3}{2}}dx}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{17a^{\frac{3}{2}}fx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,cdx^3}{8\sqrt{1+\frac{cx^2}{a}}}+\frac{11\sqrt{a}\,cfx^5}{24\sqrt{1+\frac{cx^2}{a}}}-\frac{a^3f\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2d\,\mathrm{as}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] a**(5/2)*f*x/(16*c*sqrt(1 + c*x**2/a)) + a**(3/2)*d*x*sqrt(1 + c*x**2/a)/2
+ a**(3/2)*d*x/(8*sqrt(1 + c*x**2/a)) + 17*a**(3/2)*f*x**3/(48*sqrt(1 + c*x
**2/a)) + 3*sqrt(a)*c*d*x**3/(8*sqrt(1 + c*x**2/a)) + 11*sqrt(a)*c*f*x**5/(
24*sqrt(1 + c*x**2/a)) - a**3*f*asinh(sqrt(c)*x/sqrt(a))/(16*c**(3/2)) + 3*
```

```
a**2*d*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c)) + a*e*Piecewise((sqrt(a)*x**2/2
, Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + c*e*Piecewise((-2*a**2*sq
rt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a + c
*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + c**2*d*x**5/(4*sqrt(a)*sqrt(
1 + c*x**2/a)) + c**2*f*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a))
```

# 3.92 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

**Optimal**. Leaf size=326

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2g^3\left(fg^2-h(eg-dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{\left(ah^2+cg^2\right)^{3/2}\left(d\right.}{}$$

```
[Out] 1/12*(4*d*h^2-4*e*g*h+4*f*g^2-3*h*(-e*h+f*g)*x)*(c*x^2+a)^(3/2)/h^3+1/5*f*(
c*x^2+a)^(5/2)/c/h-(a*h^2+c*g^2)^(3/2)*(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+
a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*(3*a^2*h^4*(-e*h+f*g)+8*c
^2*g^3*(f*g^2-h*(-d*h+e*g))+12*a*c*g*h^2*(f*g^2-h*(-d*h+e*g)))*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+1/8*(8*(a*h^2+c*g^2)*(d*h^2-e*g*h+f*g^2)
-h*(4*c*d*g*h^2+(-e*h+f*g)*(3*a*h^2+4*c*g^2))*x)*(c*x^2+a)^(1/2)/h^5
```

**Rubi [A]** time = 0.77, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1654, 815, 844, 217, 206, 725}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^4(fg-eh)+12acgh^2\left(fg^2-h(eg-dh)\right)+8c^2\left(fg^5-g^3h(eg-dh)\right)\right)}{8\sqrt{c}\,h^6}+\frac{\left(a+cx^2\right)^{3/2}\left(4\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((8*(c*g^2 + a*h^2)*(f*g^2 - e*g*h + d*h^2) - h*(4*c*d*g*h^2 + (f*g - e*h)*
(4*c*g^2 + 3*a*h^2))*x)*Sqrt[a + c*x^2])/(8*h^5) + ((4*(f*g^2 - e*g*h + d*h
^2) - 3*h*(f*g - e*h)*x)*(a + c*x^2)^(3/2))/(12*h^3) + (f*(a + c*x^2)^(5/2)
)/(5*c*h) - ((3*a^2*h^4*(f*g - e*h) + 12*a*c*g*h^2*(f*g^2 - h*(e*g - d*h))
+ 8*c^2*(f*g^5 - g^3*h*(e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(8*Sqrt[c]*h^6) - ((c*g^2 + a*h^2)^(3/2)*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(a
*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{f(a+cx^2)^{5/2}}{5ch} + \frac{\int \frac{(5cdh^2-5ch(fg-eh)x)(a+cx^2)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= \frac{(4(fg^2-egh+dh^2)-3h(fg-eh)x)(a+cx^2)^{3/2}}{12h^3} + \frac{f(a+cx^2)^{5/2}}{5ch} + \frac{\int \frac{(5ac}{}}{}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2))x)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2))x)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2))x)}{8h^5}$$

$$= \frac{(8(cg^2+ah^2)(fg^2-egh+dh^2)-h(4cdgh^2+(fg-eh)(4cg^2+3ah^2))x)}{8h^5}$$

**Mathematica [A]** time = 1.21, size = 348, normalized size = 1.07

$$\frac{\sqrt{a+cx^2}\left(3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{c}\,x\left(5a+2cx^2\right)\sqrt{\frac{cx^2}{a}+1}\right)(eh-fg)}{8\sqrt{c}\,h^2\sqrt{\frac{cx^2}{a}+1}} - \frac{\left(h(dh-eg)+fg^2\right)\left(\sqrt{\frac{cx^2}{a}+1}\left(-h\sqrt{a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + c*x^2)^(5/2))/(5*c*h) + ((-(f*g) + e*h)*Sqrt[a + c*x^2]*(Sqrt[c]*x*
(5*a + 2*c*x^2)*Sqrt[1 + (c*x^2)/a] + 3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]
]))/(8*Sqrt[c]*h^2*Sqrt[1 + (c*x^2)/a]) - ((f*g^2 + h*(-(e*g) + d*h))*(3*Sq
rt[a]*Sqrt[c]*g*h^2*Sqrt[a + c*x^2]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]] + Sqrt[1 +
 (c*x^2)/a]*(-(h*Sqrt[a + c*x^2]*(6*c*g^2 + 8*a*h^2 - 3*c*g*h*x + 2*c*h^2*x
^2)) + 6*Sqrt[c]*g*(c*g^2 + a*h^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + 6
*(c*g^2 + a*h^2)^(3/2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a +
c*x^2])])))/(6*h^6*Sqrt[1 + (c*x^2)/a])
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.29, size = 551, normalized size = 1.69

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4cfx}{h}-\frac{5\left(c^4fgh^{19}-c^4h^{20}e\right)}{c^3h^{21}}\right)x+\frac{4\left(5c^4fg^2h^{18}+5c^4dh^{20}+6ac^3fh^{20}-5c^4gh^{19}e\right)}{c^3h^{21}}\right)x-\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*c*f*x/h - 5*(c^4*f*g*h^19 - c^4*h^20*e)/(c^
3*h^21))*x + 4*(5*c^4*f*g^2*h^18 + 5*c^4*d*h^20 + 6*a*c^3*f*h^20 - 5*c^4*g*
h^19*e)/(c^3*h^21))*x - 15*(4*c^4*f*g^3*h^17 + 4*c^4*d*g*h^19 + 5*a*c^3*f*g
*h^19 - 4*c^4*g^2*h^18*e - 5*a*c^3*h^20*e)/(c^3*h^21))*x + 8*(15*c^4*f*g^4*
h^16 + 15*c^4*d*g^2*h^18 + 20*a*c^3*f*g^2*h^18 + 20*a*c^3*d*h^20 + 3*a^2*c^
2*f*h^20 - 15*c^4*g^3*h^17*e - 20*a*c^3*g*h^19*e)/(c^3*h^21)) + 2*(c^2*f*g^
6 + c^2*d*g^4*h^2 + 2*a*c*f*g^4*h^2 + 2*a*c*d*g^2*h^4 + a^2*f*g^2*h^4 + a^2
*d*h^6 - c^2*g^5*h*e - 2*a*c*g^3*h^3*e - a^2*g*h^5*e)*arctan(-((sqrt(c)*x -
 sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2
)*h^6) + 1/8*(8*c^(5/2)*f*g^5 + 8*c^(5/2)*d*g^3*h^2 + 12*a*c^(3/2)*f*g^3*h^
2 + 12*a*c^(3/2)*d*g*h^4 + 3*a^2*sqrt(c)*f*g*h^4 - 8*c^(5/2)*g^4*h*e - 12*a
*c^(3/2)*g^2*h^3*e - 3*a^2*sqrt(c)*h^5*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/(c*h^6)
```

**maple [B]** time = 0.01, size = 2420, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] 1/5*f*(c*x^2+a)^(5/2)/c/h+1/2/h^3*c*g^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^
2+c*g^2)/h^2)^(1/2)*x*e-1/2/h^4*c*g^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2)*x*f-3/2/h^2*c^(1/2)*g*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*d-3/8/h^2*f*g*a*x*(c*x
^2+a)^(1/2)-3/8/h^2*f*g*a^2/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+3/2/h^3*c
^(1/2)*g^2*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h
^2+c*g^2)/h^2)^(1/2))*a*e-3/2/h^4*c^(1/2)*g^3*ln((-c*g/h+(x+g/h)*c)/c^(1/2)
+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*a*f+1/h^2/((a*h^2+
```

```
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*a^2*e*g-1/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g
^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2))/(x+g/h))*a^2*f*g^2+1/h^6/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*
(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^5*e-1/h^7/((a*h^2
+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2
)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h
))*c^2*g^6*f-1/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+
c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2
+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*d-1/2/h^2*c*g*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-1/3/h^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(3/2)*e*g+1/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2)*a*d+1/3/h^3*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(3/
2)*f*g^2+1/4/h*e*x*(c*x^2+a)^(3/2)-2/h^5/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(
x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^4*f-2/h^3/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*a*c*g^2*d+2/h^4/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c
*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+g/h))*a*c*g^3*e+1/3/h*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(
a*h^2+c*g^2)/h^2)^(3/2)*d+3/8/h*e*a*x*(c*x^2+a)^(1/2)+3/8/h*e*a^2/c^(1/2)*l
n(c^(1/2)*x+(c*x^2+a)^(1/2))-1/4/h^2*f*g*x*(c*x^2+a)^(3/2)+1/h^3*(-2*(x+g/h
)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*a*f*g^2-1/h^2*(-2*(x+g/h)*c*g/
h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*a*e*g+1/h^3*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*c*g^2*d-1/h^4*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+
(a*h^2+c*g^2)/h^2)^(1/2)*c*g^3*e+1/h^5*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2
+c*g^2)/h^2)^(1/2)*c*g^4*f-1/h^4*c^(3/2)*g^3*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+
(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*d+1/h^5*c^(3/2)*g^4
*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/
h^2)^(1/2))*e-1/h^6*c^(3/2)*g^5*ln((-c*g/h+(x+g/h)*c)/c^(1/2)+(-2*(x+g/h)*c
*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))*f-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*
ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*a^2*d
```

**maxima [B]** time = 0.79, size = 632, normalized size = 1.94

$$-\frac{\sqrt{cx^2+a}\,cfg^3x}{2\,h^4}+\frac{\sqrt{cx^2+a}\,ceg^2x}{2\,h^3}-\frac{\sqrt{cx^2+a}\,cdgx}{2\,h^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fgx}{4\,h^2}-\frac{3\,\sqrt{cx^2+a}\,afgx}{8\,h^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}ex}{4\,h}+\frac{3\,\sqrt{cx^2+}}{8\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(c*x^2 + a)*c*f*g^3*x/h^4 + 1/2*sqrt(c*x^2 + a)*c*e*g^2*x/h^3 - 1/
2*sqrt(c*x^2 + a)*c*d*g*x/h^2 - 1/4*(c*x^2 + a)^(3/2)*f*g*x/h^2 - 3/8*sqrt(
c*x^2 + a)*a*f*g*x/h^2 + 1/4*(c*x^2 + a)^(3/2)*e*x/h + 3/8*sqrt(c*x^2 + a)*
a*e*x/h - c^(3/2)*f*g^5*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*e*g^4*arcsinh(
c*x/sqrt(a*c))/h^5 - c^(3/2)*d*g^3*arcsinh(c*x/sqrt(a*c))/h^4 - 3/2*a*sqrt(
c)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*g^2*arcsinh(c*x/sqrt(
a*c))/h^3 - 3/2*a*sqrt(c)*d*g*arcsinh(c*x/sqrt(a*c))/h^2 - 3/8*a^2*f*g*arcs
inh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + 3/8*a^2*e*arcsinh(c*x/sqrt(a*c))/(sqrt(c
)*h) + (a + c*g^2/h^2)^(3/2)*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 - (a + c*g^2/h^2)^(3/2)*e*g*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^2 + (a + c*g^2
/h^2)^(3/2)*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/h + sqrt(c*x^2 + a)*c*f*g^4/h^5 - sqrt(c*x^2 + a)*c*e*g^3/h^4 + s
qrt(c*x^2 + a)*c*d*g^2/h^3 + 1/3*(c*x^2 + a)^(3/2)*f*g^2/h^3 + sqrt(c*x^2 +
 a)*a*f*g^2/h^3 - 1/3*(c*x^2 + a)^(3/2)*e*g/h^2 - sqrt(c*x^2 + a)*a*e*g/h^2
 + 1/3*(c*x^2 + a)^(3/2)*d/h + sqrt(c*x^2 + a)*a*d/h + 1/5*(c*x^2 + a)^(5/2
)*f/(c*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}\left(d + ex + fx^2\right)}{g + hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

## 3.93 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=432

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2g^2\left(5fg^2-h(4eg-3dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-eg\right.}{h(g+hx)\left(ah^2+\right.}$$

```
[Out] -1/12*(4*a*h^2*(-e*h+2*f*g)+4*c*g*(5*f*g^2-h*(-3*d*h+4*e*g))-3*h*(a*f*h^2+c
*(5*f*g^2-4*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)-(d*h^2-e*g*
h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)+1/8*(3*a^2*f*h^4+8*c^2*g^2
*(5*f*g^2-h*(-3*d*h+4*e*g))+12*a*c*h^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(x*
c^(1/2)/(c*x^2+a)^(1/2))/h^6/c^(1/2)+(a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2-h*(-3
*d*h+4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))*(a*
h^2+c*g^2)^(1/2)/h^6-1/8*(8*a*h^2*(-e*h+2*f*g)+8*c*g*(5*f*g^2-h*(-3*d*h+4*e
*g))-h*(3*a*f*h^2+12*c*d*h^2-16*c*e*g*h+20*c*f*g^2)*x)*(c*x^2+a)^(1/2)/h^5
```

**Rubi [A]** time = 0.90, antiderivative size = 428, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 815, 844, 217, 206, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^4+12ach^2\left(3fg^2-h(2eg-dh)\right)+8c^2\left(5fg^4-g^2h(4eg-3dh)\right)\right)}{8\sqrt{c}\,h^6}-\frac{\left(a+cx^2\right)^{5/2}\left(dh^2-eg\right.}{h(g+hx)\left(ah^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((8*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - h*(20*c*f*
g^2 - 16*c*e*g*h + 12*c*d*h^2 + 3*a*f*h^2)*x)*Sqrt[a + c*x^2])/(8*h^5) - ((
4*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2*(2*f*g - e*h)) - 3*h*(5*c*f*g^
2 + a*f*h^2 - 4*c*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*h
^2)) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(h*(c*g^2 + a*h^2)*(g +
h*x)) + ((3*a^2*f*h^4 + 8*c^2*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h)) + 12*a*c*h^
2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqr
t[c]*h^6) + (Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 - c*g*h*(4*e*g - 3*d*h) + a*h^2
*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])
])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{\left(-cdg+afg-aeh-\left(afh-c\left(4eg-\frac{5fg^2}{h}-4dh\right)\right)x\right)(a+c}{g+hx}}{cg^2+ah^2}$$

$$= -\frac{\left(4\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-3h\left(5cfg^2+afh^2-4ch(eg\right.\right.}{12h^3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12c\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12c\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12c\right.\right.}{8h^5}$$

$$= -\frac{\left(8\left(5cfg^3-cgh(4eg-3dh)+ah^2(2fg-eh)\right)-h\left(20cfg^2-16cegh+12c\right.\right.}{8h^5}$$

**Mathematica [A]** time = 0.53, size = 392, normalized size = 0.91

$$\frac{\frac{3\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2fh^4+12ach^2\left(h(dh-2eg)+3fg^2\right)+8c^2\left(g^2h(3dh-4eg)+5fg^4\right)\right)}{\sqrt{c}}+24\sqrt{ah^2+cg^2}\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] (h*Sqrt[a + c*x^2]*(8*(4*a*h^2*(-2*f*g + e*h) - 3*c*(4*f*g^3 + g*h*(-3*e*g
+ 2*d*h))) + 3*h*(5*a*f*h^2 + 4*c*(3*f*g^2 + h*(-2*e*g + d*h)))*x + 8*c*h^2
*(-2*f*g + e*h)*x^2 + 6*c*f*h^3*x^3 - (24*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g
) + d*h)))/(g + h*x)) - 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g^3 + c*g*h*(-4*e*g +
 3*d*h) + a*h^2*(2*f*g - e*h))*Log[g + h*x] + (3*(3*a^2*f*h^4 + 12*a*c*h^2*
(3*f*g^2 + h*(-2*e*g + d*h)) + 8*c^2*(5*f*g^4 + g^2*h*(-4*e*g + 3*d*h)))*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + 24*Sqrt[c*g^2 + a*h^2]*(5*c*f*g
```

```
^3 + c*g*h*(-4*e*g + 3*d*h) + a*h^2*(2*f*g - e*h))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(24*h^6)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas")
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
[Out] Timed out
```

**maple [B]** time = 0.02, size = 5121, normalized size = 11.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
[Out] result too large to display
```

**maxima [A]** time = 0.78, size = 708, normalized size = 1.64

$$-\frac{\left(cx^2+a\right)^{\frac{3}{2}}fg^2}{h^4x+gh^3}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}eg}{h^3x+gh^2}-\frac{\left(cx^2+a\right)^{\frac{3}{2}}d}{h^2x+gh}+\frac{5\sqrt{cx^2+a}\,cfg^2x}{2h^4}-\frac{2\sqrt{cx^2+a}\,cegx}{h^3}+\frac{3\sqrt{cx^2+a}\,cdx}{2h^2}+\frac{\left(cx^2+a\right)^{\frac{3}{2}}f}{4h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima")
[Out] -(c*x^2 + a)^(3/2)*f*g^2/(h^4*x + g*h^3) + (c*x^2 + a)^(3/2)*e*g/(h^3*x + g
*h^2) - (c*x^2 + a)^(3/2)*d/(h^2*x + g*h) + 5/2*sqrt(c*x^2 + a)*c*f*g^2*x/h
^4 - 2*sqrt(c*x^2 + a)*c*e*g*x/h^3 + 3/2*sqrt(c*x^2 + a)*c*d*x/h^2 + 1/4*(c
*x^2 + a)^(3/2)*f*x/h^2 + 3/8*sqrt(c*x^2 + a)*a*f*x/h^2 + 5*c^(3/2)*f*g^4*a
rcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g^3*arcsinh(c*x/sqrt(a*c))/h^5 + 3*
```

```
c^(3/2)*d*g^2*arcsinh(c*x/sqrt(a*c))/h^4 + 9/2*a*sqrt(c)*f*g^2*arcsinh(c*x/
sqrt(a*c))/h^4 - 3*a*sqrt(c)*e*g*arcsinh(c*x/sqrt(a*c))/h^3 + 3/2*a*sqrt(c)
*d*arcsinh(c*x/sqrt(a*c))/h^2 + 3/8*a^2*f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h
^2) - 3*sqrt(a + c*g^2/h^2)*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3*sqrt(a + c*g^2/h^2)*c*e*g^2*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 - 3*sqr
t(a + c*g^2/h^2)*c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a
*c)*abs(h*x + g)))/h^3 - 2*(a + c*g^2/h^2)^(3/2)*f*g*arcsinh(c*g*x/(sqrt(a*
c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + (a + c*g^2/h^2)^(3/2
)*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
h^2 - 5*sqrt(c*x^2 + a)*c*f*g^3/h^5 + 4*sqrt(c*x^2 + a)*c*e*g^2/h^4 - 3*sqr
t(c*x^2 + a)*c*d*g/h^3 - 2/3*(c*x^2 + a)^(3/2)*f*g/h^3 - 2*sqrt(c*x^2 + a)*
a*f*g/h^3 + 1/3*(c*x^2 + a)^(3/2)*e/h^2 + sqrt(c*x^2 + a)*a*e/h^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

## 3.94 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=488

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4+ach^2\left(19fg^2-3h(3eg-dh)\right)+2c^2g^2\left(10fg^2-3h(2eg-dh)\right)\right)}{2h^6\sqrt{ah^2+cg^2}}+\frac{\sqrt{a+cx^2}\ (2}{}$$

```
[Out] -1/6*(2*c*g*(6*e*g-10*f*g^2/h-3*d*h)-2*a*h*(-3*e*h+7*f*g)-(2*a*f*h^2+c*(5*f
*g^2-3*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)-1/2*(d*h
^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^2-1/2*(3*a*h^2*(-e*
h+3*f*g)+2*c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/
2))*c^(1/2)/h^6-1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+a*c*
h^2*(19*f*g^2-3*h*(-d*h+3*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(
c*x^2+a)^(1/2))/h^6/(a*h^2+c*g^2)^(1/2)+1/2*(2*a^2*f*h^4+2*c^2*g^2*(10*f*g^
2-3*h*(-d*h+2*e*g))+a*c*h^2*(19*f*g^2-3*h*(-d*h+3*e*g))-c*h*(a*h^2*(-3*e*h+
7*f*g)+c*g*(10*f*g^2-3*h*(-d*h+2*e*g)))*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2
)
```

**Rubi [A]** time = 0.92, antiderivative size = 480, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1651, 813, 815, 844, 217, 206, 725}

$$\frac{\sqrt{a+cx^2}\left(2a^2fh^3-cx\left(ah^2(7fg-3eh)-3cgh(2eg-dh)+10cfg^3\right)+ach\left(19fg^2-3h(3eg-dh)\right)-2c^2g^2\left(-3dh\right.\right.}{2h^4\left(ah^2+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((2*a^2*f*h^3 - 2*c^2*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) + a*c*h*(19*f*g^2
- 3*h*(3*e*g - d*h)) - c*(10*c*f*g^3 - 3*c*g*h*(2*e*g - d*h) + a*h^2*(7*f*g
 - 3*e*h))*x)*Sqrt[a + c*x^2])/(2*h^4*(c*g^2 + a*h^2)) - ((2*(c*g*(6*e*g -
(10*f*g^2)/h - 3*d*h) - a*h*(7*f*g - 3*e*h)) - (5*c*f*g^2 + 2*a*f*h^2 - 3*c
*h*(e*g - d*h))*x)*(a + c*x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)) - (
(f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) - (Sqrt[c]*(20*c*f*g^3 - 6*c*g*h*(2*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*h^6) - ((2*a^2*f*h^4 + 2*c^2*(10*f*
g^4 - 3*g^2*h*(2*e*g - d*h)) + a*c*h^2*(19*f*g^2 - 3*h*(3*e*g - d*h)))*ArcT
anh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(2*h^6*Sqrt[c*g^2
 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{2h\left(cg^2+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(-2(cdg-afg+aeh)-\left(2afh-c\left(3eg-\frac{5fg^2}{h}-3dh\right)\right)x\right)(a+}{(g+hx)^2}}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(2\left(cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-ah(7fg-3eh)\right)-\left(5cfg^2+2afh^2-3ch(eg-d\right.\right.}{6h^2\left(cg^2+ah^2\right)(g+hx)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cf\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cf\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cf\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

$$= \frac{\left(2a^2fh^3-2c^2g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)+ach\left(19fg^2-3h(3eg-dh)\right)-c\left(10cf\right.\right.}{2h^4\left(cg^2+ah^2\right)}$$

**Mathematica [A]** time = 0.65, size = 435, normalized size = 0.89

$$-\frac{3\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+19fg^2\right)+2c^2\left(3g^2h(dh-2eg)+10fg^4\right)\right)}{\sqrt{ah^2+cg^2}}+\frac{3\log(g+hx)\left(2a^2fh^4+ach^2\left(3h(dh-3eg)+19\right.\right.}{\sqrt{ah^2+cg}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((h*Sqrt[a + c*x^2]*(a*h^2*(-3*h*(e*g + d*h + 2*e*h*x) + f*(17*g^2 + 28*g*h
*x + 8*h^2*x^2)) + c*(f*(60*g^4 + 90*g^3*h*x + 20*g^2*h^2*x^2 - 5*g*h^3*x^3
 + 2*h^4*x^4) + 3*h*(d*h*(6*g^2 + 9*g*h*x + 2*h^2*x^2) + e*(-12*g^3 - 18*g^
2*h*x - 4*g*h^2*x^2 + h^3*x^3)))))/(g + h*x)^2 + (3*(2*a^2*f*h^4 + a*c*h^2*
(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 + 3*g^2*h*(-2*e*g + d*h))
)*Log[g + h*x])/Sqrt[c*g^2 + a*h^2] - 3*Sqrt[c]*(20*c*f*g^3 + 6*c*g*h*(-2*e
*g + d*h) - 3*a*h^2*(-3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] - (3
*(2*a^2*f*h^4 + a*c*h^2*(19*f*g^2 + 3*h*(-3*e*g + d*h)) + 2*c^2*(10*f*g^4 +
 3*g^2*h*(-2*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*
x^2]])/Sqrt[c*g^2 + a*h^2])/(6*h^6)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.43, size = 1036, normalized size = 2.12

$$\frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2cfx}{h^3}-\frac{3\left(3c^2fgh^{14}-c^2h^{15}e\right)}{ch^{18}}\right)+\frac{2\left(18c^2fg^2h^{13}+3c^2dh^{15}+4acfh^{15}-9c^2gh^{14}e\right)}{ch^{18}}\right)+\frac{\left(20c^{\frac{3}{2}}fg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*(x*(2*c*f*x/h^3 - 3*(3*c^2*f*g*h^14 - c^2*h^15*e)/(c*h^
18)) + 2*(18*c^2*f*g^2*h^13 + 3*c^2*d*h^15 + 4*a*c*f*h^15 - 9*c^2*g*h^14*e)
/(c*h^18)) + 1/2*(20*c^(3/2)*f*g^3 + 6*c^(3/2)*d*g*h^2 + 9*a*sqrt(c)*f*g*h^
```

```
2 - 12*c^(3/2)*g^2*h*e - 3*a*sqrt(c)*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2
 + a)))/h^6 + (20*c^2*f*g^4 + 6*c^2*d*g^2*h^2 + 19*a*c*f*g^2*h^2 + 3*a*c*d*
h^4 + 2*a^2*f*h^4 - 12*c^2*g^3*h*e - 9*a*c*g*h^3*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^6) + (10*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h + 6*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2
*h^3 + (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 8*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*c^2*g^3*h^2*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 1
8*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 10*(sqrt(c)*x - sqrt(c*x^
2 + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g
^3*h^2 - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 14*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*c^(5/2)*g^4*h*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e
 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 26*(sqrt(c)*x - sq
rt(c*x^2 + a))*a*c^2*f*g^4*h - 14*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2
*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt
(c*x^2 + a))*a^2*c*d*h^5 + 20*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e
 + 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 9*a^2*c^(3/2)*f*g^3*h^2
+ 5*a^2*c^(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 - 7*a^2*c^(3/2)*g^2*h^3*e -
 2*a^3*sqrt(c)*h^5*e)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x -
sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2*h^6)
```

**maple [B]** time = 0.02, size = 7817, normalized size = 16.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 0.88, size = 1299, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(c*x^2 + a)*c^2*f*g^4/(c*g^2*h^5 + a*h^7) - 3/2*sqrt(c*x^2 + a)*c^2
*f*g^3*x/(c*g^2*h^4 + a*h^6) - 3/2*sqrt(c*x^2 + a)*c^2*e*g^3/(c*g^2*h^4 + a
*h^6) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^3/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 +
a*g*h^5) + 3/2*sqrt(c*x^2 + a)*c^2*e*g^2*x/(c*g^2*h^3 + a*h^5) + 3/2*sqrt(c
*x^2 + a)*c^2*d*g^2/(c*g^2*h^3 + a*h^5) - 1/2*(c*x^2 + a)^(3/2)*c*e*g^2/(c*
g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f*g^2/(c
*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^
```

```
3) + 1/2*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a
)*c^2*d*g*x/(c*g^2*h^2 + a*h^4) + 1/2*(c*x^2 + a)^(3/2)*c*d*g/(c*g^2*h^2*x
+ a*h^4*x + c*g^3*h + a*g*h^3) + 1/2*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^2*x^2 +
 a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) - 1/2*(c*x^2 +
a)^(3/2)*c*e*g/(c*g^2*h^2 + a*h^4) - 1/2*(c*x^2 + a)^(5/2)*d/(c*g^2*h*x^2 +
 a*h^3*x^2 + 2*c*g^3*x + 2*a*g*h^2*x + c*g^4/h + a*g^2*h) + 1/2*(c*x^2 + a)
^(3/2)*c*d/(c*g^2*h + a*h^3) + 2*(c*x^2 + a)^(3/2)*f*g/(h^4*x + g*h^3) - (c
*x^2 + a)^(3/2)*e/(h^3*x + g*h^2) - 7/2*sqrt(c*x^2 + a)*c*f*g*x/h^4 + 3/2*s
qrt(c*x^2 + a)*c*e*x/h^3 - 10*c^(3/2)*f*g^3*arcsinh(c*x/sqrt(a*c))/h^6 + 6*
c^(3/2)*e*g^2*arcsinh(c*x/sqrt(a*c))/h^5 - 3*c^(3/2)*d*g*arcsinh(c*x/sqrt(a
*c))/h^4 - 9/2*a*sqrt(c)*f*g*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*e*a
rcsinh(c*x/sqrt(a*c))/h^3 + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) - 3/2*c^2*e
*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/(sqrt(a + c*g^2/h^2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
 + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^5) + 15/2*sqr
t(a + c*g^2/h^2)*c*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt
(a*c)*abs(h*x + g)))/h^5 - 9/2*sqrt(a + c*g^2/h^2)*c*e*g*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^4 + 3/2*sqrt(a + c*g
^2/h^2)*c*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/h^3 + (a + c*g^2/h^2)^(3/2)*f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^3 + 17/2*sqrt(c*x^2 + a)*c*f*g^2/h^5 -
9/2*sqrt(c*x^2 + a)*c*e*g/h^4 + 3/2*sqrt(c*x^2 + a)*c*d/h^3 + 1/3*(c*x^2 +
a)^(3/2)*f/h^3 + sqrt(c*x^2 + a)*a*f/h^3
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

# 3.95 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

**Optimal**. Leaf size=475

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2g^3\left(10fg^2-h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{(\ldots}{}$$

```
[Out] -1/6*(c*g*(4*e*g-10*f*g^2/h-d*h)-3*a*h*(-e*h+3*f*g)-(3*a*f*h^2+c*(5*f*g^2-2
*h*(-d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^2/(a*h^2+c*g^2)/(h*x+g)^2-1/3*(d*h^2-e
*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^3+1/2*c*(3*a^2*h^4*(-e*
h+4*f*g)+2*c^2*g^3*(10*f*g^2-h*(-d*h+4*e*g))+3*a*c*g*h^2*(11*f*g^2-h*(-d*h+
4*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a*h
^2+c*g^2)^(3/2)+1/2*(3*a*f*h^2+2*c*(10*f*g^2-h*(-d*h+4*e*g)))*arctanh(x*c^(
1/2)/(c*x^2+a)^(1/2))*c^(1/2)/h^6-1/2*((a*h^2+c*g^2)*(3*a*f*h^2+2*c*(10*f*g
^2-h*(-d*h+4*e*g)))+c*h*(3*a*h^2*(-e*h+3*f*g)+c*g*(10*f*g^2-h*(-d*h+4*e*g))
)*x)*(c*x^2+a)^(1/2)/h^5/(a*h^2+c*g^2)/(h*x+g)
```

**Rubi [A]** time = 0.84, antiderivative size = 469, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 813, 844, 217, 206, 725}

$$\frac{c\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(3a^2h^4(4fg-eh)+3acgh^2\left(11fg^2-h(4eg-dh)\right)+2c^2\left(10fg^5-g^3h(4eg-dh)\right)\right)}{2h^6\left(ah^2+cg^2\right)^{3/2}}-\frac{(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -(((c*g^2 + a*h^2)*(20*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h)) + c*h*(10
*c*f*g^3 - c*g*h*(4*e*g - d*h) + 3*a*h^2*(3*f*g - e*h))*x)*Sqrt[a + c*x^2])
/(2*h^5*(c*g^2 + a*h^2)*(g + h*x)) - ((c*g*(4*e*g - (10*f*g^2)/h - d*h) - 3
*a*h*(3*f*g - e*h) - (5*c*f*g^2 + 3*a*f*h^2 - 2*c*h*(e*g - d*h))*x)*(a + c*
x^2)^(3/2))/(6*h^2*(c*g^2 + a*h^2)*(g + h*x)^2) - ((f*g^2 - e*g*h + d*h^2)*
(a + c*x^2)^(5/2))/(3*h*(c*g^2 + a*h^2)*(g + h*x)^3) + (Sqrt[c]*(20*c*f*g^2
 + 3*a*f*h^2 - 2*c*h*(4*e*g - d*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(
2*h^6) + (c*(3*a^2*h^4*(4*f*g - e*h) + 3*a*c*g*h^2*(11*f*g^2 - h*(4*e*g - d
*h)) + 2*c^2*(10*f*g^5 - g^3*h*(4*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[
c*g^2 + a*h^2]*Sqrt[a + c*x^2]])])/(2*h^6*(c*g^2 + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{3h\left(cg^2+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-3(cdg-afg+aeh)-\left(3afh-c\left(2eg-\frac{5fg^2}{h}-2dh\right)\right)x\right)(}{(g+hx)^3}}{3\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(cg\left(4eg-\frac{10fg^2}{h}-dh\right)-3ah(3fg-eh)-\left(5cfg^2+3afh^2-2ch(eg-dh)\right)}{6h^2\left(cg^2+ah^2\right)(g+hx)^2}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

$$= -\frac{\left(\left(cg^2+ah^2\right)\left(20cfg^2+3afh^2-2ch(4eg-dh)\right)+ch\left(10cfg^3-cgh(4eg-\right.\right.}{2h^5\left(cg^2+ah^2\right)(g+hx)}$$

**Mathematica [A]** time = 1.23, size = 517, normalized size = 1.09

$$\frac{\frac{3c\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-3a^2h^4(eh-4fg)+3acgh^2\left(h(dh-4eg)+11fg^2\right)+2c^2\left(g^3h(dh-4eg)+10fg^5\right)\right)}{\left(ah^2+cg^2\right)^{3/2}} - \frac{3c\log(g+hx)\left(-3a^2h^4(eh-4fg)+3a\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(2*(c*g^2 + a*h^2)^2*(f*g^2 + h*(-(e*g) + d*h)) - (c*
g^2 + a*h^2)*(13*c*f*g^3 + c*g*h*(-10*e*g + 7*d*h) - 3*a*h^2*(-2*f*g + e*h)
)*(g + h*x) + (6*a^2*f*h^4 + a*c*h^2*(50*f*g^2 + h*(-23*e*g + 8*d*h)) + c^2
*(47*f*g^4 + g^2*h*(-26*e*g + 11*d*h)))*(g + h*x)^2 + 6*c*(4*f*g - e*h)*(c*
g^2 + a*h^2)*(g + h*x)^3 - 3*c*f*h*(c*g^2 + a*h^2)*x*(g + h*x)^3))/((c*g^2
+ a*h^2)*(g + h*x)^3)) - (3*c*(-3*a^2*h^4*(-4*f*g + e*h) + 3*a*c*g*h^2*(11*
```

```
f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^3*h*(-4*e*g + d*h)))*Log[g
+ h*x])/(c*g^2 + a*h^2)^(3/2) + 3*Sqrt[c]*(20*c*f*g^2 + 3*a*f*h^2 + 2*c*h*(
-4*e*g + d*h))*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(-3*a^2*h^4*(-4*f*
g + e*h) + 3*a*c*g*h^2*(11*f*g^2 + h*(-4*e*g + d*h)) + 2*c^2*(10*f*g^5 + g^
3*h*(-4*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]]
)/(c*g^2 + a*h^2)^(3/2))/(6*h^6)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.61, size = 1900, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(c*f*x/h^4 - 2*(4*c*f*g*h^10 - c*h^11*e)/h^15) - (20*c^
3*f*g^5 + 2*c^3*d*g^3*h^2 + 33*a*c^2*f*g^3*h^2 + 3*a*c^2*d*g*h^4 + 12*a^2*c
*f*g*h^4 - 8*c^3*g^4*h*e - 12*a*c^2*g^2*h^3*e - 3*a^2*c*h^5*e)*arctan(-((sq
rt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c*g^2*h^6
 + a*h^8)*sqrt(-c*g^2 - a*h^2)) - 1/3*(60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^3*f*g^5*h^2 + 18*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*d*g^3*h^4 + 69*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^2*f*g^3*h^4 + 15*(sqrt(c)*x - sqrt(c*x^2 + a)
)^5*a*c^2*d*g*h^6 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*f*g*h^6 - 36*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*c^3*g^4*h^3*e - 36*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a*c^2*g^2*h^5*e - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*h^7*e + 21
0*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*f*g^6*h + 54*(sqrt(c)*x - sqrt(c*
x^2 + a))^4*c^(7/2)*d*g^4*h^3 + 183*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/
2)*f*g^4*h^3 + 27*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*d*g^2*h^5 - 18*
(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(3/2)*f*g^2*h^5 - 12*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^2*c^(3/2)*d*h^7 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*s
qrt(c)*f*h^7 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(7/2)*g^5*h^2*e - 84*(
sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(5/2)*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(3/2)*g*h^6*e + 188*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*
f*g^7 + 44*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*d*g^5*h^2 - 82*(sqrt(c)*x -
sqrt(c*x^2 + a))^3*a*c^3*f*g^5*h^2 - 34*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c
^3*d*g^3*h^4 - 276*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*f*g^3*h^4 - 48*(
sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^2*d*g*h^6 - 36*(sqrt(c)*x - sqrt(c*x^2
```

```
 + a))^3*a^3*c*f*g*h^6 - 104*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^4*g^6*h*e +
64*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c^3*g^4*h^3*e + 138*(sqrt(c)*x - sqrt(
c*x^2 + a))^3*a^2*c^2*g^2*h^5*e - 354*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(
7/2)*f*g^6*h - 78*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(7/2)*d*g^4*h^3 - 276
*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*c^(5/2)*f*g^4*h^3 - 36*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*c^(5/2)*d*g^2*h^5 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^3*c^(3/2)*f*g^2*h^5 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d*h
^7 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*f*h^7 + 192*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a*c^(7/2)*g^5*h^2*e + 114*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^2*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*
g*h^6*e + 222*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c^3*f*g^5*h^2 + 48*(sqrt(c)
*x - sqrt(c*x^2 + a))*a^2*c^3*d*g^3*h^4 + 231*(sqrt(c)*x - sqrt(c*x^2 + a))
*a^3*c^2*f*g^3*h^4 + 33*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*d*g*h^6 + 24*
(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*f*g*h^6 - 120*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^2*c^3*g^4*h^3*e - 102*(sqrt(c)*x - sqrt(c*x^2 + a))*a^3*c^2*g^2*h^5*
e + 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*h^7*e - 47*a^3*c^(5/2)*f*g^4*h^3
- 11*a^3*c^(5/2)*d*g^2*h^5 - 50*a^4*c^(3/2)*f*g^2*h^5 - 8*a^4*c^(3/2)*d*h^7
 - 6*a^5*sqrt(c)*f*h^7 + 26*a^3*c^(5/2)*g^3*h^4*e + 23*a^4*c^(3/2)*g*h^6*e)
/((c*g^2*h^6 + a*h^8)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - s
qrt(c*x^2 + a))*sqrt(c)*g - a*h)^3) - 1/2*(20*c^(3/2)*f*g^2 + 2*c^(3/2)*d*h
^2 + 3*a*sqrt(c)*f*h^2 - 8*c^(3/2)*g*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 a)))/h^6
```

**maple [B]** time = 0.02, size = 9835, normalized size = 20.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.10, size = 2415, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*c^3*f*g^5/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) - 1/2
*sqrt(c*x^2 + a)*c^3*f*g^4*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) - 1/2*
sqrt(c*x^2 + a)*c^3*e*g^4/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) + 1/6*(c*
x^2 + a)^(3/2)*c^2*f*g^4/(c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2
*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*e*g^3*x/(c^
2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/2*sqrt(c*x^2 + a)*c^3*d*g^3/(c^2*g
```

```
^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^3/(c^2*g^
4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g
*h^6) - 1/6*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2
+ a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6
*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/6*(c*x^2 + a)^(3/2)*c^2*f*g^3/(c^2*g^
4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/2*sqrt(c*x^2 + a)*c^3*d*g^2*x/(c^2*g^4
*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) + 1/6*(c*x^2 + a)^(3/2)*c^2*d*g^2/(c^2*g^4*
h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5
) + 1/6*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^
2*h^6*x^2 + 2*c^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a
*c*g^4*h^2 + a^2*g^2*h^4) - 1/6*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^2 +
2*a*c*g^2*h^4 + a^2*h^6) - 9/2*sqrt(c*x^2 + a)*c^2*f*g^3/(c*g^2*h^5 + a*h^7
) + 4*sqrt(c*x^2 + a)*c^2*f*g^2*x/(c*g^2*h^4 + a*h^6) - 1/6*(c*x^2 + a)^(5/
2)*c*d*g/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4
*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a^2*g^2*h^3) + 1
/6*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2*h^5) + 3*sqrt
(c*x^2 + a)*c^2*e*g^2/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f*g^2/(c*
g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x +
 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) - 5/3*(c*x^2 + a)^(3/2)*c*f*g^2/(c*g^
2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^5) - 5/2*sqrt(c*x^2 + a)*c^2*e*g*x/(c
*g^2*h^3 + a*h^5) - 3/2*sqrt(c*x^2 + a)*c^2*d*g/(c*g^2*h^3 + a*h^5) + 1/3*(
c*x^2 + a)^(5/2)*e*g/(c*g^2*h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h
^4*x^2 + 3*c*g^4*h*x + 3*a*g^2*h^3*x + c*g^5 + a*g^3*h^2) + 7/6*(c*x^2 + a)
^(3/2)*c*e*g/(c*g^2*h^3*x + a*h^5*x + c*g^3*h^2 + a*g*h^4) + (c*x^2 + a)^(5
/2)*f*g/(c*g^2*h^3*x^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h
+ a*g^2*h^3) - (c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^3 + a*h^5) + sqrt(c*x^2 + a
)*c^2*d*x/(c*g^2*h^2 + a*h^4) - 1/3*(c*x^2 + a)^(5/2)*d/(c*g^2*h^2*x^3 + a*
h^4*x^3 + 3*c*g^3*h*x^2 + 3*a*g*h^3*x^2 + 3*c*g^4*x + 3*a*g^2*h^2*x + c*g^5
/h + a*g^3*h) - 2/3*(c*x^2 + a)^(3/2)*c*d/(c*g^2*h^2*x + a*h^4*x + c*g^3*h
+ a*g*h^3) - 1/2*(c*x^2 + a)^(5/2)*e/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h
*x + 2*a*g*h^3*x + c*g^4 + a*g^2*h^2) + 1/2*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^
2 + a*h^4) - (c*x^2 + a)^(3/2)*f/(h^4*x + g*h^3) + 3/2*sqrt(c*x^2 + a)*c*f*
x/h^4 + 10*c^(3/2)*f*g^2*arcsinh(c*x/sqrt(a*c))/h^6 - 4*c^(3/2)*e*g*arcsinh
(c*x/sqrt(a*c))/h^5 + c^(3/2)*d*arcsinh(c*x/sqrt(a*c))/h^4 + 3/2*a*sqrt(c)*
f*arcsinh(c*x/sqrt(a*c))/h^4 + 1/2*c^3*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h
*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 1/2*
c^3*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x +
 g)))/((a + c*g^2/h^2)^(3/2)*h^8) + 1/2*c^3*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*
abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) -
 9/2*c^2*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^7) + 3*c^2*e*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) -
3/2*c^2*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
 + g)))/(sqrt(a + c*g^2/h^2)*h^5) - 6*sqrt(a + c*g^2/h^2)*c*f*g*arcsinh(c*g
*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(
```

```
a + c*g^2/h^2)*c*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/h^4 - 6*sqrt(c*x^2 + a)*c*f*g/h^5 + 3/2*sqrt(c*x^2 + a)*c*e/
h^4
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

# 3.96 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

**Optimal**. Leaf size=511

$$\frac{\left(a+cx^2\right)^{3/2}\left(-3hx\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2g^2\left(5fg^2-h(dh+eg)\right)\right)+4a^2h^4(fg-2eh)-acgh\right.}{24h^3(g+hx)^3\left(ah^2+cg^2\right)^2}$$

```
[Out] 1/24*(4*a^2*h^4*(-2*e*h+f*g)-4*c^2*g^4*(-e*h+5*f*g)-a*c*g*h^2*(25*f*g^2-h*(
-9*d*h+5*e*g))-3*h*(4*a^2*f*h^4+a*c*h^2*(17*f*g^2-h*(-d*h+5*e*g))+2*c^2*g^2
*(5*f*g^2-h*(d*h+e*g)))*x)*(c*x^2+a)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^3-1/
4*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^4-c^(3/2)*(-e
*h+5*f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^6-1/8*c*(12*a^3*f*h^6+8*c^3*
g^5*(-e*h+5*f*g)+20*a*c^2*g^3*h^2*(-e*h+5*f*g)+3*a^2*c*h^4*(25*f*g^2-h*(-d*
h+5*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^6/(a
*h^2+c*g^2)^(5/2)+1/8*c*(8*(-e*h+5*f*g)*(a*h^2+c*g^2)^2+h*(12*a^2*f*h^4+4*c
^2*g^3*(-e*h+5*f*g)+a*c*h^2*(35*f*g^2-h*(-3*d*h+7*e*g)))*x)*(c*x^2+a)^(1/2)
/h^5/(a*h^2+c*g^2)^2/(h*x+g)
```

**Rubi [A]** time = 1.09, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1651, 811, 813, 844, 217, 206, 725}

$$\frac{\left(a+cx^2\right)^{3/2}\left(-3x\left(4a^2fh^4+ach^2\left(17fg^2-h(5eg-dh)\right)+2c^2\left(5fg^4-g^2h(dh+eg)\right)\right)+4a^2h^3(fg-2eh)-acgh\left(\right.\right.}{24h^2(g+hx)^3\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (c*(8*(5*f*g - e*h)*(c*g^2 + a*h^2)^2 + h*(12*a^2*f*h^4 + 4*c^2*g^3*(5*f*g
- e*h) + a*c*h^2*(35*f*g^2 - h*(7*e*g - 3*d*h)))*x)*Sqrt[a + c*x^2])/(8*h^5
*(c*g^2 + a*h^2)^2*(g + h*x)) + ((4*a^2*h^3*(f*g - 2*e*h) - (4*c^2*g^4*(5*f
*g - e*h))/h - a*c*g*h*(25*f*g^2 - h*(5*e*g - 9*d*h)) - 3*(4*a^2*f*h^4 + a*
c*h^2*(17*f*g^2 - h*(5*e*g - d*h)) + 2*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))*x
)*(a + c*x^2)^(3/2))/(24*h^2*(c*g^2 + a*h^2)^2*(g + h*x)^3) - ((f*g^2 - e*g
*h + d*h^2)*(a + c*x^2)^(5/2))/(4*h*(c*g^2 + a*h^2)*(g + h*x)^4) - (c^(3/2)
*(5*f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/h^6 - (c*(12*a^3*f*h^6
 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*c*h^4*(
25*f*g^2 - h*(5*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2]])/(8*h^6*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{4h\left(cg^2+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(-4(cdg-afg+aeh)-\left(4afh-c\left(eg-\frac{5fg^2}{h}-dh\right)\right)x\right)\left(a+cx \cdots\right.}{(g+hx)^4}}{4\left(cg^2+ah^2\right)} \cdots$$

$$= \frac{\left(4a^2h^3(fg-2eh)-\frac{4c^2g^4(5fg-eh)}{h}-acgh\left(25fg^2-h(5eg-9dh)\right)-3\left(4a^2fh^4+\cdots\right.\right.}{24h^2\left(cg^2+ah^2 \cdots\right.}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= \frac{c\left(8(5fg-eh)\left(cg^2+ah^2\right)^2+h\left(12a^2fh^4+4c^2g^3(5fg-eh)+ach^2\left(35fg^2-\cdots\right.\right.\right.}{8h^5\left(cg^2+ah^2\right)^2(g+hx)}$$

**Mathematica** [A] time = 2.15, size = 575, normalized size = 1.13

$$-\frac{h\sqrt{a+cx^2}\left((g+hx)^2\left(ah^2+cg^2\right)\left(12a^2fh^4+ach^2\left(h(15dh-43eg)+95fg^2\right)+2c^2\left(g^2h(9dh-23eg)+43fg^4\right)\right)-c(g+hx)^3\left(4a^2h^4(31fg-8eh)+acgh^2\left(h(15dh-91\right.\right.\right.}{(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] -1/24*((h*Sqrt[a + c*x^2]*(6*(c*g^2 + a*h^2)^3*(f*g^2 + h*(-(e*g) + d*h)) -
 2*(c*g^2 + a*h^2)^2*(17*c*f*g^3 + c*g*h*(-13*e*g + 9*d*h) - 4*a*h^2*(-2*f*
g + e*h))*(g + h*x) + (c*g^2 + a*h^2)*(12*a^2*f*h^4 + 2*c^2*(43*f*g^4 + g^2
*h*(-23*e*g + 9*d*h)) + a*c*h^2*(95*f*g^2 + h*(-43*e*g + 15*d*h)))*(g + h*x
)^2 - c*(4*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(77*f*g^5 + g^3*h*(-25*e*g + 3*
d*h)) + a*c*g*h^2*(287*f*g^2 + h*(-91*e*g + 15*d*h)))*(g + h*x)^3 - 24*c*f*
(c*g^2 + a*h^2)^2*(g + h*x)^4))/((c*g^2 + a*h^2)^2*(g + h*x)^4) - (3*c*(12*
a^3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^
2*c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[g + h*x])/(c*g^2 + a*h^2)^(5/2)
+ 24*c^(3/2)*(5*f*g - e*h)*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (3*c*(12*a^
3*f*h^6 + 8*c^3*g^5*(5*f*g - e*h) + 20*a*c^2*g^3*h^2*(5*f*g - e*h) + 3*a^2*
c*h^4*(25*f*g^2 + h*(-5*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(5/2))/h^6
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.02, size = 12481, normalized size = 24.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.50, size = 4326, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^4*f*g^6/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*
h^9 + a^3*h^11) - 3/8*sqrt(c*x^2 + a)*c^4*f*g^5*x/(c^3*g^6*h^4 + 3*a*c^2*g^
4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 3/8*sqrt(c*x^2 + a)*c^4*e*g^5/(c^3*g^
6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 1/8*(c*x^2 + a)^(3/
2)*c^3*f*g^5/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h
^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 3/8*
sqrt(c*x^2 + a)*c^4*e*g^4*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^
7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5
 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^3*e*g^4/(c^3*g^6*h^
3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a
*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) - 1/8*(c*x^2 + a)^(5/2)*c^2*f*g
^4/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x
^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*
x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) + 1/8*(c*x
^2 + a)^(3/2)*c^3*f*g^4/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 +
a^3*h^9) - 3/8*sqrt(c*x^2 + a)*c^4*d*g^3*x/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 +
 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^3/(c^3*g^6*h^2*
x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2
*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7) + 1/8*(c*x^2 + a)^(5/2)*c^2*e*g^3/(
c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 +
 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^
3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6) - 1/8*(c*x^2 + a)^
(3/2)*c^3*e*g^3/(c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)
 - 7/4*sqrt(c*x^2 + a)*c^3*f*g^4/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9) +
11/8*sqrt(c*x^2 + a)*c^3*f*g^3*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
1/8*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^
2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g
^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^
3*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h + 3*a*c^2*g^4*h^3 +
 3*a^2*c*g^2*h^5 + a^3*h^7) + 5/4*sqrt(c*x^2 + a)*c^3*e*g^3/(c^2*g^4*h^4 +
2*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^4*x^3 +
 2*a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 +
```

3*a^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2 *g^7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 17/24*(c*x^2 + a)^(3/2)*c^2*f*g^3/( c^2*g^4*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h^7) - 7/8*sqrt(c*x^2 + a)*c^3*e*g^2*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 3/4*sqrt(c*x^2 + a)*c^3*d*g^2/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a ^2*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^ 3 + a^2*h^7*x^3 + 3*c^2*g^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^ 2 + a^2*g^3*h^4) + 13/24*(c*x^2 + a)^(3/2)*c^2*e*g^2/(c^2*g^4*h^3*x + 2*a*c *g^2*h^5*x + a^2*h^7*x + c^2*g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 5/24*(c *x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4 *h^3 + a^2*g^2*h^5) - 5/24*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*h^3 + 2*a*c *g^2*h^5 + a^2*h^7) + 3/8*sqrt(c*x^2 + a)*c^3*d*g*x/(c^2*g^4*h^2 + 2*a*c*g^ 2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^2*x^3 + 2*a*c*g^2 *h^4*x^3 + a^2*h^6*x^3 + 3*c^2*g^5*h*x^2 + 6*a*c*g^3*h^3*x^2 + 3*a^2*g*h^5* x^2 + 3*c^2*g^6*x + 6*a*c*g^4*h^2*x + 3*a^2*g^2*h^4*x + c^2*g^7/h + 2*a*c*g ^5*h + a^2*g^3*h^3) - 3/8*(c*x^2 + a)^(3/2)*c^2*d*g/(c^2*g^4*h^2*x + 2*a*c* g^2*h^4*x + a^2*h^6*x + c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) - 1/24*(c*x^ 2 + a)^(5/2)*c*e*g/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c ^2*g^5*h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^ 2*g^2*h^4) + 1/24*(c*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) - 1/4*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3 *h^4*x^3 + 4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2* x + 4*a*g^3*h^4*x + c*g^6*h + a*g^4*h^3) + 39/8*sqrt(c*x^2 + a)*c^2*f*g^2/( c*g^2*h^5 + a*h^7) - 7/2*sqrt(c*x^2 + a)*c^2*f*g*x/(c*g^2*h^4 + a*h^6) - 1/ 8*(c*x^2 + a)^(5/2)*c*d/(c^2*g^4*h*x^2 + 2*a*c*g^2*h^3*x^2 + a^2*h^5*x^2 + 2*c^2*g^5*x + 4*a*c*g^3*h^2*x + 2*a^2*g*h^4*x + c^2*g^6/h + 2*a*c*g^4*h + a ^2*g^2*h^3) + 1/8*(c*x^2 + a)^(3/2)*c^2*d/(c^2*g^4*h + 2*a*c*g^2*h^3 + a^2* h^5) + 1/4*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^4*x^4 + a*h^6*x^4 + 4*c*g^3*h^3*x ^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 + 4*c*g^5*h*x + 4*a* g^3*h^3*x + c*g^6 + a*g^4*h^2) - 15/8*sqrt(c*x^2 + a)*c^2*e*g/(c*g^2*h^4 + a*h^6) + 2/3*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^4*x^3 + a*h^6*x^3 + 3*c*g^3*h^3 *x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*h + a*g^3*h^3) + 11/6*(c*x^2 + a)^(3/2)*c*f*g/(c*g^2*h^4*x + a*h^6*x + c*g^3*h^3 + a*g*h^ 5) + sqrt(c*x^2 + a)*c^2*e*x/(c*g^2*h^3 + a*h^5) - 1/4*(c*x^2 + a)^(5/2)*d/ (c*g^2*h^3*x^4 + a*h^5*x^4 + 4*c*g^3*h^2*x^3 + 4*a*g*h^4*x^3 + 6*c*g^4*h*x^ 2 + 6*a*g^2*h^3*x^2 + 4*c*g^5*x + 4*a*g^3*h^2*x + c*g^6/h + a*g^4*h) + 3/8* sqrt(c*x^2 + a)*c^2*d/(c*g^2*h^3 + a*h^5) - 1/3*(c*x^2 + a)^(5/2)*e/(c*g^2* h^3*x^3 + a*h^5*x^3 + 3*c*g^3*h^2*x^2 + 3*a*g*h^4*x^2 + 3*c*g^4*h*x + 3*a*g ^2*h^3*x + c*g^5 + a*g^3*h^2) - 2/3*(c*x^2 + a)^(3/2)*c*e/(c*g^2*h^3*x + a* h^5*x + c*g^3*h^2 + a*g*h^4) - 1/2*(c*x^2 + a)^(5/2)*f/(c*g^2*h^3*x^2 + a*h ^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*(c*x^2 + a)^(3/2)*c*f/(c*g^2*h^3 + a*h^5) - 5*c^(3/2)*f*g*arcsinh(c*x/sqrt(a*c))/h^6 + c^(3/2)*e*arcsinh(c*x/sqrt(a*c))/h^5 + 3/8*c^4*f*g^6*arcsinh(c*g*x/(sqrt

```
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*
h^11) - 3/8*c^4*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*
c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 3/8*c^4*d*g^4*arcsinh(c*g*
x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)
^(5/2)*h^9) - 7/4*c^3*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 5/4*c^3*e*g^3*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2
/h^2)^(3/2)*h^8) - 3/4*c^3*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a
*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 39/8*c^2*f*g^2*a
rcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt
(a + c*g^2/h^2)*h^7) - 15/8*c^2*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6) + 3/8*c^2*d*arcsi
nh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a +
 c*g^2/h^2)*h^5) + 3/2*sqrt(a + c*g^2/h^2)*c*f*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/h^5 + 3/2*sqrt(c*x^2 + a)*c*f/h^
5
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

# 3.97 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

**Optimal**. Leaf size=507

$$-\frac{\left(a+cx^2\right)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg^2\right.\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

```
[Out] -1/12*(4*c^2*f*g^5-a^2*h^4*(-3*e*h+2*f*g)+a*c*g*h^2*(3*d*h^2+5*f*g^2)+h*(4*
a^2*f*h^4+a*c*g*h^2*(-3*e*h+14*f*g)+c^2*(-3*d*g^2*h^2+7*f*g^4))*x)*(c*x^2+a
)^(3/2)/h^3/(a*h^2+c*g^2)^2/(h*x+g)^4-1/5*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5/
2)/h/(a*h^2+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h
^6+1/8*c^2*(8*c^3*f*g^7+28*a*c^2*f*g^5*h^2+3*a^3*h^6*(-e*h+6*f*g)+a^2*c*g*h
^4*(-3*d*h^2+35*f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^
(1/2))/h^6/(a*h^2+c*g^2)^(7/2)-1/8*c*(8*c^3*f*g^7+20*a*c^2*f*g^5*h^2-a^3*h^
6*(-3*e*h+2*f*g)+a^2*c*g*h^4*(3*d*h^2+13*f*g^2)+h*(12*c^3*f*g^6+8*a^3*f*h^6
+a^2*c*g*h^4*(-3*e*h+34*f*g)+a*c^2*g^2*h^2*(-3*d*h^2+35*f*g^2))*x)*(c*x^2+a
)^(1/2)/h^5/(a*h^2+c*g^2)^3/(h*x+g)^2
```

**Rubi** [A] time = 0.86, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 811, 844, 217, 206, 725}

$$-\frac{\left(a+cx^2\right)^{3/2}\left(hx\left(4a^2fh^4+acgh^2(14fg-3eh)+c^2\left(7fg^4-3dg^2h^2\right)\right)-a^2h^4(2fg-3eh)+acgh^2\left(3dh^2+5fg^2\right.\right.}{12h^3(g+hx)^4\left(ah^2+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(c*(8*c^3*f*g^7 + 20*a*c^2*f*g^5*h^2 - a^3*h^6*(2*f*g - 3*e*h) + a^2*c*g*h
^4*(13*f*g^2 + 3*d*h^2) + h*(12*c^3*f*g^6 + 8*a^3*f*h^6 + a^2*c*g*h^4*(34*f
*g - 3*e*h) + a*c^2*g^2*h^2*(35*f*g^2 - 3*d*h^2))*x)*Sqrt[a + c*x^2])/(8*h^
5*(c*g^2 + a*h^2)^3*(g + h*x)^2) - ((4*c^2*f*g^5 - a^2*h^4*(2*f*g - 3*e*h)
+ a*c*g*h^2*(5*f*g^2 + 3*d*h^2) + h*(4*a^2*f*h^4 + a*c*g*h^2*(14*f*g - 3*e*
h) + c^2*(7*f*g^4 - 3*d*g^2*h^2))*x)*(a + c*x^2)^(3/2))/(12*h^3*(c*g^2 + a*
h^2)^2*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(5*h*(c*g
^2 + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])
/h^6 + (c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 + 3*a^3*h^6*(6*f*g - e*h) + a
^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]
*Sqrt[a + c*x^2])])/(8*h^6*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{5h\left(cg^2+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-5(cdg-afg+aeh)-5f\left(\frac{cg^2}{h}+ah\right)x\right)\left(a+cx^2\right)^{3/2}}{(g+hx)^5}\,dx}{5\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(4c^2fg^5-a^2h^4(2fg-3eh)+acgh^2\left(5fg^2+3dh^2\right)+h\left(4a^2fh^4+acgh^2(1\cdots\right.\right.}{12h^3\left(cg^2+ah^2\right)^2(g+hx)\cdots}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(\cdots\right.\right.}{8h^5\left(cg^2\cdots\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(\cdots\right.\right.}{8h^5\left(cg^2\cdots\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(\cdots\right.\right.}{8h^5\left(cg^2\cdots\right.}$$

$$= -\frac{c\left(8c^3fg^7+20ac^2fg^5h^2-a^3h^6(2fg-3eh)+a^2cgh^4\left(13fg^2+3dh^2\right)+h\left(\cdots\right.\right.}{8h^5\left(cg^2\cdots\right.}$$

**Mathematica [A]** time = 2.27, size = 639, normalized size = 1.26

$$\frac{\frac{15c^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-3a^3h^6(eh-6fg)+a^2cgh^4\left(35fg^2-3dh^2\right)+28ac^2fg^5h^2+8c^3fg^7\right)}{\left(ah^2+cg^2\right)^{7/2}} - \frac{15c^2\log(g+hx)\left(-3a^3h^6(eh-6fg)+a^2cgh^4(\cdots\right.}{\left(ah^2+cg^2\right)\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] (-((h*Sqrt[a + c*x^2]*(24*(c*g^2 + a*h^2)^4*(f*g^2 + h*(-(e*g) + d*h)) - 6*
(c*g^2 + a*h^2)^3*(21*c*f*g^3 + c*g*h*(-16*e*g + 11*d*h) - 5*a*h^2*(-2*f*g
```

```
+ e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^2*(20*a^2*f*h^4 + c^2*(137*f*g^4 + 9*
g^2*h*(-8*e*g + 3*d*h)) + a*c*h^2*(154*f*g^2 + 3*h*(-23*e*g + 8*d*h)))*(g +
 h*x)^2 - c*(c*g^2 + a*h^2)*(5*a^2*h^4*(58*f*g - 15*e*h) + c^2*(326*f*g^5 +
 6*g^3*h*(-16*e*g + d*h)) + a*c*g*h^2*(631*f*g^2 + 3*h*(-62*e*g + 7*d*h)))*
(g + h*x)^3 + c*(160*a^3*f*h^6 + c^3*(274*f*g^6 - 6*g^4*h*(4*e*g + d*h)) +
3*a^2*c*h^4*(238*f*g^2 + h*(-33*e*g + 8*d*h)) + 3*a*c^2*g^2*h^2*(261*f*g^2
- h*(26*e*g + 9*d*h)))*(g + h*x)^4))/((c*g^2 + a*h^2)^3*(g + h*x)^5)) - (15
*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g
*h^4*(35*f*g^2 - 3*d*h^2))*Log[g + h*x])/(c*g^2 + a*h^2)^(7/2) + 120*c^(3/2
)*f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]] + (15*c^2*(8*c^3*f*g^7 + 28*a*c^2*f*
g^5*h^2 - 3*a^3*h^6*(-6*f*g + e*h) + a^2*c*g*h^4*(35*f*g^2 - 3*d*h^2))*Log[
a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/
(120*h^6)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 1.33, size = 4408, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] -1/4*(8*c^5*f*g^7 + 28*a*c^4*f*g^5*h^2 + 35*a^2*c^3*f*g^3*h^4 - 3*a^2*c^3*d
*g*h^6 + 18*a^3*c^2*f*g*h^6 - 3*a^3*c^2*h^7*e)*arctan(-((sqrt(c)*x - sqrt(c
*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^3*g^6*h^6 + 3*a*c^2*g^4
*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*sqrt(-c*g^2 - a*h^2)) - c^(3/2)*f*log(a
bs(-sqrt(c)*x + sqrt(c*x^2 + a)))/h^6 - 1/60*(600*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*c^5*f*g^7*h^4 + 1740*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*f*g^5*h^6
 + 1635*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^3*f*g^3*h^8 + 45*(sqrt(c)*x -
 sqrt(c*x^2 + a))^9*a^2*c^3*d*g*h^10 + 450*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^3*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^5*g^6*h^5*e - 360*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^4*g^4*h^7*e - 360*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^3*g^2*h^9*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^2*h
^11*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*f*g^8*h^3 - 120*(sqrt
(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*d*g^6*h^5 + 10020*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a*c^(9/2)*f*g^6*h^5 - 360*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9
/2)*d*g^4*h^7 + 8595*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*f*g^4*h^7
```

+ 45*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(7/2)*d*g^2*h^9 + 1530*(sqrt(c)*
x - sqrt(c*x^2 + a))^8*a^3*c^(5/2)*f*g^2*h^9 - 120*(sqrt(c)*x - sqrt(c*x^2
+ a))^8*a^3*c^(5/2)*d*h^11 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(3/2
)*f*h^11 - 480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(11/2)*g^7*h^4*e - 1440*(s
qrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(9/2)*g^5*h^6*e - 1440*(sqrt(c)*x - sqrt(
c*x^2 + a))^8*a^2*c^(7/2)*g^3*h^8*e - 75*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^
3*c^(5/2)*g*h^10*e + 8800*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*f*g^9*h^2 - 2
40*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^6*d*g^7*h^4 + 21240*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a*c^5*f*g^7*h^4 - 720*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^5*d
*g^5*h^6 + 11670*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*f*g^5*h^6 + 690*(s
qrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*d*g^3*h^8 - 4970*(sqrt(c)*x - sqrt(c*
x^2 + a))^7*a^3*c^3*f*g^3*h^8 - 450*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3
*d*g*h^10 - 2580*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^2*f*g*h^10 - 960*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqrt(c*x^2 +
 a))^7*a*c^5*g^6*h^5*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^4*g^4*h
^7*e + 1170*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^3*g^2*h^9*e + 30*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^2*h^11*e + 10000*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*c^(13/2)*f*g^10*h - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(13/2)*d*g^8*
h^3 + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*f*g^8*h^3 - 720*(sqr
t(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*d*g^6*h^5 - 14430*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^2*c^(9/2)*f*g^6*h^5 + 1590*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
a^2*c^(9/2)*d*g^4*h^7 - 28790*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*f
*g^4*h^7 - 1710*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*d*g^2*h^9 - 582
0*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(5/2)*f*g^2*h^9 + 720*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^5*c^(3/2)*f*h^11 - 960*(sqrt(c)*x - sqrt(c*x^2 + a))^6
*c^(13/2)*g^9*h^2*e - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(11/2)*g^7*h
^4*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(9/2)*g^5*h^6*e + 4950*(sq
rt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(7/2)*g^3*h^8*e - 270*(sqrt(c)*x - sqrt(
c*x^2 + a))^6*a^4*c^(5/2)*g*h^10*e + 4384*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c
^7*f*g^11 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c^7*d*g^9*h^2 - 9392*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a*c^6*f*g^9*h^2 + 48*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a*c^6*d*g^7*h^4 - 42996*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*f*g^7*h^
4 + 2364*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*d*g^5*h^6 - 31070*(sqrt(c)
*x - sqrt(c*x^2 + a))^5*a^3*c^4*f*g^5*h^6 - 2730*(sqrt(c)*x - sqrt(c*x^2 +
a))^5*a^3*c^4*d*g^3*h^8 + 8620*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*f*g^
3*h^8 + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^3*d*g*h^10 + 4800*(sqrt(c
)*x - sqrt(c*x^2 + a))^5*a^5*c^2*f*g*h^10 - 384*(sqrt(c)*x - sqrt(c*x^2 + a
))^5*c^7*g^10*h*e + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^6*g^8*h^3*e + 3
936*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^5*g^6*h^5*e + 5580*(sqrt(c)*x - s
qrt(c*x^2 + a))^5*a^3*c^4*g^4*h^7*e - 2970*(sqrt(c)*x - sqrt(c*x^2 + a))^5*
a^4*c^3*g^2*h^9*e - 11920*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*f*g^10
*h + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*d*g^8*h^3 - 15720*(sqrt
(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*f*g^8*h^3 + 720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^2*c^(11/2)*d*g^6*h^5 + 19670*(sqrt(c)*x - sqrt(c*x^2 + a))^4
*a^3*c^(9/2)*f*g^6*h^5 - 3510*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(9/2)*d

```
*g^4*h^7 + 36260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*f*g^4*h^7 + 14
40*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(7/2)*d*g^2*h^9 + 6240*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^5*c^(5/2)*f*g^2*h^9 - 240*(sqrt(c)*x - sqrt(c*x^2 +
a))^4*a^5*c^(5/2)*d*h^11 - 880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(3/2)*
f*h^11 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a*c^(13/2)*g^9*h^2*e + 1680*(s
qrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(11/2)*g^7*h^4*e - 480*(sqrt(c)*x - sqr
t(c*x^2 + a))^4*a^3*c^(9/2)*g^5*h^6*e - 6150*(sqrt(c)*x - sqrt(c*x^2 + a))^
4*a^4*c^(7/2)*g^3*h^8*e + 720*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(5/2)*g
*h^10*e + 13120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*f*g^9*h^2 - 240*(sq
rt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*d*g^7*h^4 + 30440*(sqrt(c)*x - sqrt(c*
x^2 + a))^3*a^3*c^5*f*g^7*h^4 - 1200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^
5*d*g^5*h^6 + 14130*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*f*g^5*h^6 + 231
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^4*d*g^3*h^8 - 10790*(sqrt(c)*x - sq
rt(c*x^2 + a))^3*a^5*c^3*f*g^3*h^8 - 510*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
5*c^3*d*g*h^10 - 3820*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*f*g*h^10 - 96
0*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^2*c^6*g^8*h^3*e - 2640*(sqrt(c)*x - sqr
t(c*x^2 + a))^3*a^3*c^5*g^6*h^5*e - 2640*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
4*c^4*g^4*h^7*e + 2790*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^3*g^2*h^9*e -
30*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^2*h^11*e - 7360*(sqrt(c)*x - sqrt(
c*x^2 + a))^2*a^3*c^(11/2)*f*g^8*h^3 + 120*(sqrt(c)*x - sqrt(c*x^2 + a))^2*
a^3*c^(11/2)*d*g^6*h^5 - 19930*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*
f*g^6*h^5 + 690*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*d*g^4*h^7 - 160
50*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*f*g^4*h^7 - 1050*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(7/2)*d*g^2*h^9 - 1300*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a^6*c^(5/2)*f*g^2*h^9 + 560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(3
/2)*f*h^11 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^3*c^(11/2)*g^7*h^4*e + 1
560*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(9/2)*g^5*h^6*e + 2130*(sqrt(c)*x
 - sqrt(c*x^2 + a))^2*a^5*c^(7/2)*g^3*h^8*e - 570*(sqrt(c)*x - sqrt(c*x^2 +
 a))^2*a^6*c^(5/2)*g*h^10*e + 2140*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*f*
g^7*h^4 - 60*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5*d*g^5*h^6 + 6090*(sqrt(c
)*x - sqrt(c*x^2 + a))*a^5*c^4*f*g^5*h^6 - 270*(sqrt(c)*x - sqrt(c*x^2 + a)
)*a^5*c^4*d*g^3*h^8 + 5505*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*f*g^3*h^8
+ 195*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^3*d*g*h^10 + 1150*(sqrt(c)*x - sq
rt(c*x^2 + a))*a^7*c^2*f*g*h^10 - 120*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c^5
*g^6*h^5*e - 420*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^4*g^4*h^7*e - 630*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^6*c^3*g^2*h^9*e + 75*(sqrt(c)*x - sqrt(c*x^2 +
a))*a^7*c^2*h^11*e - 274*a^5*c^(9/2)*f*g^6*h^5 + 6*a^5*c^(9/2)*d*g^4*h^7 -
783*a^6*c^(7/2)*f*g^4*h^7 + 27*a^6*c^(7/2)*d*g^2*h^9 - 714*a^7*c^(5/2)*f*g^
2*h^9 - 24*a^7*c^(5/2)*d*h^11 - 160*a^8*c^(3/2)*f*h^11 + 24*a^5*c^(9/2)*g^5
*h^6*e + 78*a^6*c^(7/2)*g^3*h^8*e + 99*a^7*c^(5/2)*g*h^10*e)/((c^3*g^6*h^6
+ 3*a*c^2*g^4*h^8 + 3*a^2*c*g^2*h^10 + a^3*h^12)*((sqrt(c)*x - sqrt(c*x^2 +
 a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^5)
```

**maple [B]** time = 0.03, size = 14169, normalized size = 27.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 1.89, size = 6650, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(c*x^2 + a)*c^5*f*g^7/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^
4*h^9 + 4*a^3*c*g^2*h^11 + a^4*h^13) - 3/8*sqrt(c*x^2 + a)*c^5*f*g^6*x/(c^4
*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^1
2) - 3/8*sqrt(c*x^2 + a)*c^5*e*g^6/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c
^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^6
/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^1
0*x + a^4*h^12*x + c^4*g^9*h^3 + 4*a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^
3*c*g^3*h^9 + a^4*g*h^11) + 3/8*sqrt(c*x^2 + a)*c^5*e*g^5*x/(c^4*g^8*h^3 +
4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 3/8*sqr
t(c*x^2 + a)*c^5*d*g^5/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 +
 4*a^3*c*g^2*h^9 + a^4*h^11) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^5/(c^4*g^8*h^3
*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*g^2*h^9*x + a^4*h^11
*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6 + 4*a^3*c*g^3*h^8 +
a^4*g*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^3*x^2 + 4*a*c^3*g^
6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2 + a^4*h^11*x^2 + 2*
c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*x + 8*a^3*c*g^3*h^8*
x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a^2*c^2*g^6*h^5 + 4*a
^3*c*g^4*h^7 + a^4*g^2*h^9) + 1/8*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^3
+ 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 3/8*s
qrt(c*x^2 + a)*c^5*d*g^4*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h
^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^4/(c^4*g^8
*h^2*x + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*
h^10*x + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7
+ a^4*g*h^9) + 1/8*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^2*x^2 + 4*a*c^3*g
^6*h^4*x^2 + 6*a^2*c^2*g^4*h^6*x^2 + 4*a^3*c*g^2*h^8*x^2 + a^4*h^10*x^2 + 2
*c^4*g^9*h*x + 8*a*c^3*g^7*h^3*x + 12*a^2*c^2*g^5*h^5*x + 8*a^3*c*g^3*h^7*x
 + 2*a^4*g*h^9*x + c^4*g^10 + 4*a*c^3*g^8*h^2 + 6*a^2*c^2*g^6*h^4 + 4*a^3*c
*g^4*h^6 + a^4*g^2*h^8) - 1/8*(c*x^2 + a)^(3/2)*c^4*e*g^4/(c^4*g^8*h^2 + 4*
a*c^3*g^6*h^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 3/2*sqrt(
c*x^2 + a)*c^4*f*g^5/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3
*h^11) + 9/8*sqrt(c*x^2 + a)*c^4*f*g^4*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3
*a^2*c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^3/(c^4*g^8*h*x^2
```

+ 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*
h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*
g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3
+ 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8
*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 9/8
*sqrt(c*x^2 + a)*c^4*e*g^4/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8
+ a^3*h^10) - 1/4*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^4*x^3 + 3*a*c^2*g
^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c
^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x +
9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c
^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) - 5/8*(c*x^2 + a)^(3/2)*c^3*f*g
^4/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^
3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) - 3/4*sqrt(c*x^2
+ a)*c^4*e*g^3*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^
9) - 3/4*sqrt(c*x^2 + a)*c^4*d*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c
*g^2*h^7 + a^3*h^9) + 1/4*(c*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^3*x^3 + 3*
a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 +
9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*
x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a
*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) + 1/2*(c*x^2 + a)^(3/2)*c^3*e
*g^3/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c
^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) + 1/8*(c*x^2 +
a)^(5/2)*c^2*f*g^3/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7
*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*
x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2
*h^7) - 1/8*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*
a^2*c*g^2*h^7 + a^3*h^9) + 3/8*sqrt(c*x^2 + a)*c^4*d*g^2*x/(c^3*g^6*h^2 + 3
*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) - 1/4*(c*x^2 + a)^(5/2)*c^2*d*g
^2/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^3*h^8*x
^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3*a^3*g*
h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^3*g^2*h
^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) - 3/8*(c*
x^2 + a)^(3/2)*c^3*d*g^2/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c*g^2*h
^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^
7) - 1/4*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^5*x^4 + 2*a*c*g^2*h^7*x^4 + a
^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^
2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^2*h^7*x^2 + 4*c^2*g^7*h^2*x +
8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h + 2*a*c*g^6*h^3 + a^2*g^4*h^5
) + 19/8*sqrt(c*x^2 + a)*c^3*f*g^3/(c^2*g^4*h^5 + 2*a*c*g^2*h^7 + a^2*h^9)
- 5/4*sqrt(c*x^2 + a)*c^3*f*g^2*x/(c^2*g^4*h^4 + 2*a*c*g^2*h^6 + a^2*h^8) -
1/8*(c*x^2 + a)^(5/2)*c^2*d*g/(c^3*g^6*h*x^2 + 3*a*c^2*g^4*h^3*x^2 + 3*a^2
*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*a*c^2*g^5*h^2*x + 6*a^2*c*g^
3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g^6*h + 3*a^2*c*g^4*h^3 + a^3
*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h + 3*a*c^2*g^4*h^3 + 3*
a^2*c*g^2*h^5 + a^3*h^7) + 1/4*(c*x^2 + a)^(5/2)*c*e*g^2/(c^2*g^4*h^4*x^4 +

```
 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^5*x^3 +
4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^2*h^6*x^
2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2*a*c*g^6
*h^2 + a^2*g^4*h^4) - 9/8*sqrt(c*x^2 + a)*c^3*e*g^2/(c^2*g^4*h^4 + 2*a*c*g^
2*h^6 + a^2*h^8) + 1/2*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h^4*x^3 + 2*a*c*g
^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a^2*g*
h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^7*h +
 2*a*c*g^5*h^3 + a^2*g^3*h^5) + 11/12*(c*x^2 + a)^(3/2)*c^2*f*g^2/(c^2*g^4*
h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*h
^7) + 3/8*sqrt(c*x^2 + a)*c^3*e*g*x/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7)
 - 1/4*(c*x^2 + a)^(5/2)*c*d*g/(c^2*g^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h
^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^
6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*
h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h + 2*a*c*g^6*h + a^2*g^4*h^3) + 3/8*sqrt
(c*x^2 + a)*c^3*d*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/4*(c*x^2 +
a)^(5/2)*c*e*g/(c^2*g^4*h^3*x^3 + 2*a*c*g^2*h^5*x^3 + a^2*h^7*x^3 + 3*c^2*g
^5*h^2*x^2 + 6*a*c*g^3*h^4*x^2 + 3*a^2*g*h^6*x^2 + 3*c^2*g^6*h*x + 6*a*c*g^
4*h^3*x + 3*a^2*g^2*h^5*x + c^2*g^7 + 2*a*c*g^5*h^2 + a^2*g^3*h^4) - 3/8*(c
*x^2 + a)^(3/2)*c^2*e*g/(c^2*g^4*h^3*x + 2*a*c*g^2*h^5*x + a^2*h^7*x + c^2*
g^5*h^2 + 2*a*c*g^3*h^4 + a^2*g*h^6) + 1/12*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^
4*h^3*x^2 + 2*a*c*g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h
^4*x + 2*a^2*g*h^6*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) - 1/12*(c*x
^2 + a)^(3/2)*c^2*f*g/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) - 1/5*(c*x^2
+ a)^(5/2)*f*g^2/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g*h^7*x
^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^3*h^5*
x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/8*(c*x^2 + a
)^(5/2)*c*e/(c^2*g^4*h^2*x^2 + 2*a*c*g^2*h^4*x^2 + a^2*h^6*x^2 + 2*c^2*g^5*
h*x + 4*a*c*g^3*h^3*x + 2*a^2*g*h^5*x + c^2*g^6 + 2*a*c*g^4*h^2 + a^2*g^2*h
^4) + 1/8*(c*x^2 + a)^(3/2)*c^2*e/(c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6) +
 1/5*(c*x^2 + a)^(5/2)*e*g/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3*h^4*x^4 + 5
*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h^2*x^2 + 10*
a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2) + 1/2*(c*x
^2 + a)^(5/2)*f*g/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 + 4*a*g*h^6*
x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3*h^4*x + c
*g^6*h + a*g^4*h^3) - 9/4*sqrt(c*x^2 + a)*c^2*f*g/(c*g^2*h^5 + a*h^7) + sqr
t(c*x^2 + a)*c^2*f*x/(c*g^2*h^4 + a*h^6) - 1/5*(c*x^2 + a)^(5/2)*d/(c*g^2*h
^4*x^5 + a*h^6*x^5 + 5*c*g^3*h^3*x^4 + 5*a*g*h^5*x^4 + 10*c*g^4*h^2*x^3 + 1
0*a*g^2*h^4*x^3 + 10*c*g^5*h*x^2 + 10*a*g^3*h^3*x^2 + 5*c*g^6*x + 5*a*g^4*h
^2*x + c*g^7/h + a*g^5*h) - 1/4*(c*x^2 + a)^(5/2)*e/(c*g^2*h^4*x^4 + a*h^6*
x^4 + 4*c*g^3*h^3*x^3 + 4*a*g*h^5*x^3 + 6*c*g^4*h^2*x^2 + 6*a*g^2*h^4*x^2 +
 4*c*g^5*h*x + 4*a*g^3*h^3*x + c*g^6 + a*g^4*h^2) + 3/8*sqrt(c*x^2 + a)*c^2
*e/(c*g^2*h^4 + a*h^6) - 1/3*(c*x^2 + a)^(5/2)*f/(c*g^2*h^4*x^3 + a*h^6*x^3
 + 3*c*g^3*h^3*x^2 + 3*a*g*h^5*x^2 + 3*c*g^4*h^2*x + 3*a*g^2*h^4*x + c*g^5*
h + a*g^3*h^3) - 2/3*(c*x^2 + a)^(3/2)*c*f/(c*g^2*h^4*x + a*h^6*x + c*g^3*h
^3 + a*g*h^5) + c^(3/2)*f*arcsinh(c*x/sqrt(a*c))/h^6 + 3/8*c^5*f*g^7*arcsin
```

```
h(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^
2/h^2)^(7/2)*h^13) - 3/8*c^5*e*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) + 3/8*c^5*d*g^5
*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
 + c*g^2/h^2)^(7/2)*h^11) - 3/2*c^4*f*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11) + 9/8*c^
4*e*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(5/2)*h^10) - 3/4*c^4*d*g^3*arcsinh(c*g*x/(sqrt(a*c)*a
bs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^9) +
19/8*c^3*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) - 9/8*c^3*e*g^2*arcsinh(c*g*x/(sqrt(
a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h
^8) + 3/8*c^3*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) - 9/4*c^2*f*g*arcsinh(c*g*x/(sqrt
(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^
7) + 3/8*c^2*e*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(
h*x + g)))/(sqrt(a + c*g^2/h^2)*h^6)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

```
[Out] Timed out
```

# 3.98 $\int \frac{(a+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx$

**Optimal**. Leaf size=404

$$-\frac{(a+cx^2)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}\,(ah-cgx)\left(6a^2fh^2-ac\left(fg^2\right.\right.}{16(g+hx)^2\left(ah^2+c\right.}$$

```
[Out] -1/24*(6*c^2*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*
x^2+a)^(3/2)/(a*h^2+c*g^2)^3/(h*x+g)^4-1/6*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(5
/2)/h/(a*h^2+c*g^2)/(h*x+g)^6+1/30*(6*a*h^2*(-e*h+2*f*g)+c*g*(5*f*g^2+h*(-7
*d*h+e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^5-1/16*a^2*c^2*(6*c^2
*d*g^2+6*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*arctanh((-c*g*x+a*h)/(a*h^2+
c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(9/2)-1/16*a*c*(6*c^2*d*g^2+6*a
^2*f*h^2-a*c*(f*g^2-h*(-d*h+7*e*g)))*(-c*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*
g^2)^4/(h*x+g)^2
```

**Rubi** [A] time = 0.55, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 807, 721, 725, 206}

$$-\frac{(a+cx^2)^{3/2}(ah-cgx)\left(6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)+6c^2dg^2\right)}{24(g+hx)^4\left(ah^2+cg^2\right)^3}-\frac{ac\sqrt{a+cx^2}\,(ah-cgx)\left(6a^2fh^2-ac\left(fg^2\right.\right.}{16(g+hx)^2\left(ah^2+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -(a*c*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*
g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^4*(g + h*x)^2) - ((6*c^2*d*g^2 +
6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d*h)))*(a*h - c*g*x)*(a + c*x^2)^(3/2
))/(24*(c*g^2 + a*h^2)^3*(g + h*x)^4) - ((f*g^2 - e*g*h + d*h^2)*(a + c*x^2
)^(5/2))/(6*h*(c*g^2 + a*h^2)*(g + h*x)^6) + ((5*c*f*g^3 + c*g*h*(e*g - 7*d
*h) + 6*a*h^2*(2*f*g - e*h))*(a + c*x^2)^(5/2))/(30*h*(c*g^2 + a*h^2)^2*(g
+ h*x)^5) - (a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 - h*(7*e*g - d
*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*
g^2 + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} - \frac{\int \frac{\left(-6(cdg-afg+aeh)-\left(6afh+c\left(eg+\frac{5fg^2}{h}-dh\right)\right)x\right)(a+}{(g+hx)^6}}{6\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{6h\left(cg^2+ah^2\right)(g+hx)^6} + \frac{\left(5cfg^3+cgh(eg-7dh)+6ah^2(2fg-eh\right.}{30h\left(cg^2+ah^2\right)^2(g+hx)^5}$$

$$= -\frac{\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\left(a+cx^2\right)^{3/2}}{24\left(cg^2+ah^2\right)^3(g+hx)^4} - \frac{(fg}{6}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{(6c^2}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{(6c^2}{}$$

$$= -\frac{ac\left(6c^2dg^2+6a^2fh^2-ac\left(fg^2-h(7eg-dh)\right)\right)(ah-cgx)\sqrt{a+cx^2}}{16\left(cg^2+ah^2\right)^4(g+hx)^2} - \frac{(6c^2}{}$$

**Mathematica [A]** time = 2.47, size = 696, normalized size = 1.72

$$\frac{1}{240}\left(-\frac{15a^2c^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(6a^2fh^2-ac\left(h(dh-7eg)+fg^2\right)+6c^2dg^2\right)}{\left(ah^2+cg^2\right)^{9/2}} + \frac{15a^2c^2\log}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] (-((Sqrt[a + c*x^2]*(40*(c*g^2 + a*h^2)^5*(f*g^2 + h*(-(e*g) + d*h)) - 8*(c
*g^2 + a*h^2)^4*(25*c*f*g^3 + c*g*h*(-19*e*g + 13*d*h) - 6*a*h^2*(-2*f*g +
e*h))*(g + h*x) + 2*(c*g^2 + a*h^2)^3*(30*a^2*f*h^4 + 2*c^2*(100*f*g^4 + g^
2*h*(-52*e*g + 19*d*h)) + a*c*h^2*(227*f*g^2 + h*(-101*e*g + 35*d*h)))*(g +
 h*x)^2 - 2*c*(c*g^2 + a*h^2)^2*(6*a^2*h^4*(31*f*g - 8*e*h) + 2*c^2*(100*f*
g^5 + g^3*h*(-28*e*g + d*h)) + 3*a*c*g*h^2*(131*f*g^2 + h*(-37*e*g + 3*d*h)
))*(g + h*x)^3 + c*(c*g^2 + a*h^2)*(150*a^3*f*h^6 + 4*c^3*(50*f*g^6 - g^4*h
*(2*e*g + d*h)) + 6*a*c^2*g^2*h^2*(99*f*g^2 - h*(5*e*g + 4*d*h)) + 3*a^2*c*
```

```
h^4*(193*f*g^2 + h*(-19*e*g + 5*d*h)))*(g + h*x)^4 - c^2*(6*a^3*h^6*(41*f*g
 - 8*e*h) + 3*a^2*c*g*h^4*(89*f*g^2 + h*(29*e*g - 27*d*h)) + 4*c^3*(10*f*g^
7 + g^5*h*(2*e*g + d*h)) + 2*a*c^2*g^3*h^2*(83*f*g^2 + h*(19*e*g + 14*d*h))
)*(g + h*x)^5))/(h^5*(c*g^2 + a*h^2)^4*(g + h*x)^6)) + (15*a^2*c^2*(6*c^2*d
*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7*e*g + d*h)))*Log[g + h*x])/(c*g^2 +
 a*h^2)^(9/2) - (15*a^2*c^2*(6*c^2*d*g^2 + 6*a^2*f*h^2 - a*c*(f*g^2 + h*(-7
*e*g + d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^
2 + a*h^2)^(9/2))/240
```

**<u>fricas</u> [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas")
```

```
[Out] Timed out
```

**<u>giac</u> [B]** time = 1.12, size = 6122, normalized size = 15.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] 1/8*(6*a^2*c^4*d*g^2 - a^3*c^3*f*g^2 - a^3*c^3*d*h^2 + 6*a^4*c^2*f*h^2 + 7*
a^3*c^3*g*h*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-
c*g^2 - a*h^2))/((c^4*g^8 + 4*a*c^3*g^6*h^2 + 6*a^2*c^2*g^4*h^4 + 4*a^3*c*g
^2*h^6 + a^4*h^8)*sqrt(-c*g^2 - a*h^2)) + 1/120*(240*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*c^6*f*g^8*h^5 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a*c^5*f*g^6
*h^7 + 1440*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c^4*f*g^4*h^9 - 90*(sqrt(c
)*x - sqrt(c*x^2 + a))^11*a^2*c^4*d*g^2*h^11 + 975*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^3*c^3*f*g^2*h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^3*d
*h^13 + 150*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^2*f*h^13 - 105*(sqrt(c)*
x - sqrt(c*x^2 + a))^11*a^3*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(13/2)*f*g^9*h^4 + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)
*f*g^7*h^6 + 7200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*f*g^5*h^8 -
990*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*d*g^3*h^10 + 4965*(sqrt(c)
*x - sqrt(c*x^2 + a))^10*a^3*c^(7/2)*f*g^3*h^10 + 165*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*d*g*h^12 + 210*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4
*c^(5/2)*f*g*h^12 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*c^(13/2)*g^8*h^5*e
 + 960*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(11/2)*g^6*h^7*e + 1440*(sqrt(c
)*x - sqrt(c*x^2 + a))^10*a^2*c^(9/2)*g^4*h^9*e - 195*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(7/2)*g^2*h^11*e + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a
^4*c^(5/2)*h^13*e + 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*f*g^10*h^3 + 3
```

20*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*d*g^8*h^5 + 12080*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a*c^6*f*g^8*h^5 + 1280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*
d*g^6*h^7 + 16320*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*f*g^6*h^7 - 2520*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^5*d*g^4*h^9 + 9220*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^3*c^4*f*g^4*h^9 + 2530*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*
c^4*d*g^2*h^11 - 4205*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*f*g^2*h^11 +
235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*d*h^13 - 210*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*a^5*c^2*f*h^13 + 640*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^7*g^9*
h^4*e + 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^6*g^7*h^6*e + 3840*(sqrt(c
)*x - sqrt(c*x^2 + a))^9*a^2*c^5*g^5*h^8*e - 2620*(sqrt(c)*x - sqrt(c*x^2 +
 a))^9*a^3*c^4*g^3*h^10*e + 1235*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^4*c^3*g*
h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*f*g^11*h^2 + 480*(sq
rt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c
*x^2 + a))^8*a*c^(13/2)*f*g^9*h^4 + 1920*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*
c^(13/2)*d*g^7*h^6 + 12480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*f*g
^7*h^6 - 7380*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(11/2)*d*g^5*h^8 - 3570
*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9/2)*f*g^5*h^8 + 8220*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*a^3*c^(9/2)*d*g^3*h^10 - 22545*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a^4*c^(7/2)*f*g^3*h^10 - 285*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(
7/2)*d*g*h^12 + 510*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*f*g*h^12 +
960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x -
 sqrt(c*x^2 + a))^8*a*c^(13/2)*g^8*h^5*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a
))^8*a^2*c^(11/2)*g^6*h^7*e - 9570*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(9
/2)*g^4*h^9*e + 5355*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^4*c^(7/2)*g^2*h^11*e
 - 240*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(5/2)*h^13*e + 3840*(sqrt(c)*x
 - sqrt(c*x^2 + a))^7*c^8*f*g^12*h + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*d*g^10*h^3 + 6336*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*f*g^10*h^3 + 1728
*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*d*g^8*h^5 - 11808*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^2*c^6*f*g^8*h^5 - 9456*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*
c^6*d*g^6*h^7 - 31704*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*f*g^6*h^7 + 2
0760*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^5*d*g^4*h^9 - 39960*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^4*c^4*f*g^4*h^9 - 2700*(sqrt(c)*x - sqrt(c*x^2 + a))^
7*a^4*c^4*d*g^2*h^11 + 12150*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*f*g^2*
h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*d*h^13 + 60*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*a^6*c^2*f*h^13 + 768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^
8*g^11*h^2*e + 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^7*g^9*h^4*e - 768*(
sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^6*g^7*h^6*e - 19608*(sqrt(c)*x - sqrt(
c*x^2 + a))^7*a^3*c^5*g^5*h^8*e + 14040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4
*c^4*g^3*h^10*e - 2730*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^5*c^3*g*h^12*e + 1
280*(sqrt(c)*x - sqrt(c*x^2 + a))^6*c^(17/2)*f*g^13 + 128*(sqrt(c)*x - sqrt
(c*x^2 + a))^6*c^(17/2)*d*g^11*h^2 - 4288*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a
*c^(15/2)*f*g^11*h^2 - 64*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*d*g^9*
h^4 - 24096*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*f*g^9*h^4 - 8592*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*d*g^7*h^6 - 26728*(sqrt(c)*x -
sqrt(c*x^2 + a))^6*a^3*c^(11/2)*f*g^7*h^6 + 24440*(sqrt(c)*x - sqrt(c*x^2 +

a))^6*a^3*c^(11/2)*d*g^5*h^8 - 12640*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c
^(9/2)*f*g^5*h^8 - 14860*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(9/2)*d*g^3*
h^10 + 41610*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*f*g^3*h^10 + 810*(
sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(7/2)*d*g*h^12 - 2460*(sqrt(c)*x - sqr
t(c*x^2 + a))^6*a^6*c^(5/2)*f*g*h^12 + 256*(sqrt(c)*x - sqrt(c*x^2 + a))^6*
c^(17/2)*g^12*h*e - 704*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(15/2)*g^10*h^3
*e - 4896*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(13/2)*g^8*h^5*e - 15656*(s
qrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(11/2)*g^6*h^7*e + 26800*(sqrt(c)*x - s
qrt(c*x^2 + a))^6*a^4*c^(9/2)*g^4*h^9*e - 9510*(sqrt(c)*x - sqrt(c*x^2 + a)
)^6*a^5*c^(7/2)*g^2*h^11*e + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(5/2
)*h^13*e - 3840*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*f*g^12*h - 384*(sqrt(
c)*x - sqrt(c*x^2 + a))^5*a*c^8*d*g^10*h^3 - 6336*(sqrt(c)*x - sqrt(c*x^2 +
 a))^5*a^2*c^7*f*g^10*h^3 - 1728*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^7*d*
g^8*h^5 + 11808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*f*g^8*h^5 + 19056*(
sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^6*d*g^6*h^7 + 29304*(sqrt(c)*x - sqrt(
c*x^2 + a))^5*a^4*c^5*f*g^6*h^7 - 21480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4
*c^5*d*g^4*h^9 + 46080*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*f*g^4*h^9 +
7020*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*d*g^2*h^11 - 17370*(sqrt(c)*x
- sqrt(c*x^2 + a))^5*a^6*c^3*f*g^2*h^11 + 390*(sqrt(c)*x - sqrt(c*x^2 + a))
^5*a^6*c^3*d*h^13 + 60*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^2*f*h^13 - 768
*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^8*g^11*h^2*e - 1728*(sqrt(c)*x - sqrt(
c*x^2 + a))^5*a^2*c^7*g^9*h^4*e - 192*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c
^6*g^7*h^6*e + 26808*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^5*g^5*h^8*e - 19
440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^4*g^3*h^10*e + 3030*(sqrt(c)*x -
sqrt(c*x^2 + a))^5*a^6*c^3*g*h^12*e + 4800*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^2*c^(15/2)*f*g^11*h^2 + 480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^2*c^(15/2)*
d*g^9*h^4 + 15120*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*f*g^9*h^4 +
3840*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(13/2)*d*g^7*h^6 + 12360*(sqrt(c
)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*f*g^7*h^6 - 18720*(sqrt(c)*x - sqrt(c
*x^2 + a))^4*a^4*c^(11/2)*d*g^5*h^8 + 1020*(sqrt(c)*x - sqrt(c*x^2 + a))^4*
a^5*c^(9/2)*f*g^5*h^8 + 11640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*d
*g^3*h^10 - 32490*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*f*g^3*h^10 -
930*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*d*g*h^12 + 3180*(sqrt(c)*x
- sqrt(c*x^2 + a))^4*a^7*c^(5/2)*f*g*h^12 + 960*(sqrt(c)*x - sqrt(c*x^2 + a
))^4*a^2*c^(15/2)*g^10*h^3*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(
13/2)*g^8*h^5*e + 7080*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(11/2)*g^6*h^7
*e - 22260*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(9/2)*g^4*h^9*e + 7470*(sq
rt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(7/2)*g^2*h^11*e - 480*(sqrt(c)*x - sqrt
(c*x^2 + a))^4*a^7*c^(5/2)*h^13*e - 3200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^
3*c^7*f*g^10*h^3 - 320*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c^7*d*g^8*h^5 -
12080*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^6*f*g^8*h^5 - 2960*(sqrt(c)*x -
 sqrt(c*x^2 + a))^3*a^4*c^6*d*g^6*h^7 - 16440*(sqrt(c)*x - sqrt(c*x^2 + a))
^3*a^5*c^5*f*g^6*h^7 + 12120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*d*g^4*
h^9 - 14120*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*f*g^4*h^9 - 2330*(sqrt(
c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*d*g^2*h^11 + 10555*(sqrt(c)*x - sqrt(c*x^

```
2 + a))^3*a^7*c^3*f*g^2*h^11 + 235*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*
d*h^13 - 210*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^2*f*h^13 - 640*(sqrt(c)*
x - sqrt(c*x^2 + a))^3*a^3*c^7*g^9*h^4*e - 3040*(sqrt(c)*x - sqrt(c*x^2 + a
))^3*a^4*c^6*g^7*h^6*e - 7800*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^5*g^5*h
^8*e + 10280*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^4*g^3*h^10*e - 1645*(sqr
t(c)*x - sqrt(c*x^2 + a))^3*a^7*c^3*g*h^12*e + 1200*(sqrt(c)*x - sqrt(c*x^2
 + a))^2*a^4*c^(13/2)*f*g^9*h^4 + 240*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c
^(13/2)*d*g^7*h^6 + 4920*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*f*g^7
*h^6 + 1656*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(11/2)*d*g^5*h^8 + 7824*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(9/2)*f*g^5*h^8 - 4038*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^6*c^(9/2)*d*g^3*h^10 + 8193*(sqrt(c)*x - sqrt(c*x^2 + a)
)^2*a^7*c^(7/2)*f*g^3*h^10 + 321*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(7/2
)*d*g*h^12 - 1686*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*f*g*h^12 + 24
0*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*c^(13/2)*g^8*h^5*e + 1272*(sqrt(c)*x
- sqrt(c*x^2 + a))^2*a^5*c^(11/2)*g^6*h^7*e + 3552*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a^6*c^(9/2)*g^4*h^9*e - 3207*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^
(7/2)*g^2*h^11*e + 48*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(5/2)*h^13*e -
240*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*f*g^8*h^5 - 48*(sqrt(c)*x - sqrt(
c*x^2 + a))*a^5*c^6*d*g^6*h^7 - 1032*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*
f*g^6*h^7 - 336*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*d*g^4*h^9 - 1764*(sqr
t(c)*x - sqrt(c*x^2 + a))*a^7*c^4*f*g^4*h^9 + 882*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^7*c^4*d*g^2*h^11 - 1977*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*f*g^2*
h^11 + 15*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^3*d*h^13 + 150*(sqrt(c)*x - s
qrt(c*x^2 + a))*a^9*c^2*f*h^13 - 96*(sqrt(c)*x - sqrt(c*x^2 + a))*a^5*c^6*g
^7*h^6*e - 456*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^5*g^5*h^8*e - 1044*(sqrt
(c)*x - sqrt(c*x^2 + a))*a^7*c^4*g^3*h^10*e + 471*(sqrt(c)*x - sqrt(c*x^2 +
 a))*a^8*c^3*g*h^12*e + 40*a^6*c^(11/2)*f*g^7*h^6 + 4*a^6*c^(11/2)*d*g^5*h^
8 + 166*a^7*c^(9/2)*f*g^5*h^8 + 28*a^7*c^(9/2)*d*g^3*h^10 + 267*a^8*c^(7/2)
*f*g^3*h^10 - 81*a^8*c^(7/2)*d*g*h^12 + 246*a^9*c^(5/2)*f*g*h^12 + 8*a^6*c^
(11/2)*g^6*h^7*e + 38*a^7*c^(9/2)*g^4*h^9*e + 87*a^8*c^(7/2)*g^2*h^11*e - 4
8*a^9*c^(5/2)*h^13*e)/((c^4*g^8*h^6 + 4*a*c^3*g^6*h^8 + 6*a^2*c^2*g^4*h^10
+ 4*a^3*c*g^2*h^12 + a^4*h^14)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt
(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^6)
```

**maple [B]** time = 0.04, size = 17026, normalized size = 42.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 2.73, size = 10724, normalized size = 26.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima")
```

```
[Out] 7/16*sqrt(c*x^2 + a)*c^6*f*g^8/(c^5*g^10*h^5 + 5*a*c^4*g^8*h^7 + 10*a^2*c^3
*g^6*h^9 + 10*a^3*c^2*g^4*h^11 + 5*a^4*c*g^2*h^13 + a^5*h^15) - 7/16*sqrt(c
*x^2 + a)*c^6*f*g^7*x/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8
+ 10*a^3*c^2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) - 7/16*sqrt(c*x^2 + a)
*c^6*e*g^7/(c^5*g^10*h^4 + 5*a*c^4*g^8*h^6 + 10*a^2*c^3*g^6*h^8 + 10*a^3*c^
2*g^4*h^10 + 5*a^4*c*g^2*h^12 + a^5*h^14) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^
7/(c^5*g^10*h^4*x + 5*a*c^4*g^8*h^6*x + 10*a^2*c^3*g^6*h^8*x + 10*a^3*c^2*g
^4*h^10*x + 5*a^4*c*g^2*h^12*x + a^5*h^14*x + c^5*g^11*h^3 + 5*a*c^4*g^9*h^
5 + 10*a^2*c^3*g^7*h^7 + 10*a^3*c^2*g^5*h^9 + 5*a^4*c*g^3*h^11 + a^5*g*h^13
) + 7/16*sqrt(c*x^2 + a)*c^6*e*g^6*x/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a
^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) + 7/16*s
qrt(c*x^2 + a)*c^6*d*g^6/(c^5*g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h
^7 + 10*a^3*c^2*g^4*h^9 + 5*a^4*c*g^2*h^11 + a^5*h^13) - 7/48*(c*x^2 + a)^(
3/2)*c^5*e*g^6/(c^5*g^10*h^3*x + 5*a*c^4*g^8*h^5*x + 10*a^2*c^3*g^6*h^7*x +
 10*a^3*c^2*g^4*h^9*x + 5*a^4*c*g^2*h^11*x + a^5*h^13*x + c^5*g^11*h^2 + 5*
a*c^4*g^9*h^4 + 10*a^2*c^3*g^7*h^6 + 10*a^3*c^2*g^5*h^8 + 5*a^4*c*g^3*h^10
+ a^5*g*h^12) - 7/48*(c*x^2 + a)^(5/2)*c^4*f*g^6/(c^5*g^10*h^3*x^2 + 5*a*c^
4*g^8*h^5*x^2 + 10*a^2*c^3*g^6*h^7*x^2 + 10*a^3*c^2*g^4*h^9*x^2 + 5*a^4*c*g
^2*h^11*x^2 + a^5*h^13*x^2 + 2*c^5*g^11*h^2*x + 10*a*c^4*g^9*h^4*x + 20*a^2
*c^3*g^7*h^6*x + 20*a^3*c^2*g^5*h^8*x + 10*a^4*c*g^3*h^10*x + 2*a^5*g*h^12*
x + c^5*g^12*h + 5*a*c^4*g^10*h^3 + 10*a^2*c^3*g^8*h^5 + 10*a^3*c^2*g^6*h^7
 + 5*a^4*c*g^4*h^9 + a^5*g^2*h^11) + 7/48*(c*x^2 + a)^(3/2)*c^5*f*g^6/(c^5*
g^10*h^3 + 5*a*c^4*g^8*h^5 + 10*a^2*c^3*g^6*h^7 + 10*a^3*c^2*g^4*h^9 + 5*a^
4*c*g^2*h^11 + a^5*h^13) - 7/16*sqrt(c*x^2 + a)*c^6*d*g^5*x/(c^5*g^10*h^2 +
 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6 + 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^
10 + a^5*h^12) + 7/48*(c*x^2 + a)^(3/2)*c^5*d*g^5/(c^5*g^10*h^2*x + 5*a*c^4
*g^8*h^4*x + 10*a^2*c^3*g^6*h^6*x + 10*a^3*c^2*g^4*h^8*x + 5*a^4*c*g^2*h^10
*x + a^5*h^12*x + c^5*g^11*h + 5*a*c^4*g^9*h^3 + 10*a^2*c^3*g^7*h^5 + 10*a^
3*c^2*g^5*h^7 + 5*a^4*c*g^3*h^9 + a^5*g*h^11) + 7/48*(c*x^2 + a)^(5/2)*c^4*
e*g^5/(c^5*g^10*h^2*x^2 + 5*a*c^4*g^8*h^4*x^2 + 10*a^2*c^3*g^6*h^6*x^2 + 10
*a^3*c^2*g^4*h^8*x^2 + 5*a^4*c*g^2*h^10*x^2 + a^5*h^12*x^2 + 2*c^5*g^11*h*x
 + 10*a*c^4*g^9*h^3*x + 20*a^2*c^3*g^7*h^5*x + 20*a^3*c^2*g^5*h^7*x + 10*a^
4*c*g^3*h^9*x + 2*a^5*g*h^11*x + c^5*g^12 + 5*a*c^4*g^10*h^2 + 10*a^2*c^3*g
^8*h^4 + 10*a^3*c^2*g^6*h^6 + 5*a^4*c*g^4*h^8 + a^5*g^2*h^10) - 7/48*(c*x^2
 + a)^(3/2)*c^5*e*g^5/(c^5*g^10*h^2 + 5*a*c^4*g^8*h^4 + 10*a^2*c^3*g^6*h^6
+ 10*a^3*c^2*g^4*h^8 + 5*a^4*c*g^2*h^10 + a^5*h^12) - 27/16*sqrt(c*x^2 + a)
*c^5*f*g^6/(c^4*g^8*h^5 + 4*a*c^3*g^6*h^7 + 6*a^2*c^2*g^4*h^9 + 4*a^3*c*g^2
*h^11 + a^4*h^13) + 5/4*sqrt(c*x^2 + a)*c^5*f*g^5*x/(c^4*g^8*h^4 + 4*a*c^3*
g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c*g^2*h^10 + a^4*h^12) - 7/48*(c*x^2 +
a)^(5/2)*c^4*d*g^4/(c^5*g^10*h*x^2 + 5*a*c^4*g^8*h^3*x^2 + 10*a^2*c^3*g^6*h
^5*x^2 + 10*a^3*c^2*g^4*h^7*x^2 + 5*a^4*c*g^2*h^9*x^2 + a^5*h^11*x^2 + 2*c^
```

```
5*g^11*x + 10*a*c^4*g^9*h^2*x + 20*a^2*c^3*g^7*h^4*x + 20*a^3*c^2*g^5*h^6*x
 + 10*a^4*c*g^3*h^8*x + 2*a^5*g*h^10*x + c^5*g^12/h + 5*a*c^4*g^10*h + 10*a
^2*c^3*g^8*h^3 + 10*a^3*c^2*g^6*h^5 + 5*a^4*c*g^4*h^7 + a^5*g^2*h^9) + 7/48
*(c*x^2 + a)^(3/2)*c^5*d*g^4/(c^5*g^10*h + 5*a*c^4*g^8*h^3 + 10*a^2*c^3*g^6
*h^5 + 10*a^3*c^2*g^4*h^7 + 5*a^4*c*g^2*h^9 + a^5*h^11) + 21/16*sqrt(c*x^2
+ a)*c^5*e*g^5/(c^4*g^8*h^4 + 4*a*c^3*g^6*h^6 + 6*a^2*c^2*g^4*h^8 + 4*a^3*c
*g^2*h^10 + a^4*h^12) - 7/24*(c*x^2 + a)^(5/2)*c^3*f*g^5/(c^4*g^8*h^4*x^3 +
 4*a*c^3*g^6*h^6*x^3 + 6*a^2*c^2*g^4*h^8*x^3 + 4*a^3*c*g^2*h^10*x^3 + a^4*h
^12*x^3 + 3*c^4*g^9*h^3*x^2 + 12*a*c^3*g^7*h^5*x^2 + 18*a^2*c^2*g^5*h^7*x^2
 + 12*a^3*c*g^3*h^9*x^2 + 3*a^4*g*h^11*x^2 + 3*c^4*g^10*h^2*x + 12*a*c^3*g^
8*h^4*x + 18*a^2*c^2*g^6*h^6*x + 12*a^3*c*g^4*h^8*x + 3*a^4*g^2*h^10*x + c^
4*g^11*h + 4*a*c^3*g^9*h^3 + 6*a^2*c^2*g^7*h^5 + 4*a^3*c*g^5*h^7 + a^4*g^3*
h^9) - 17/24*(c*x^2 + a)^(3/2)*c^4*f*g^5/(c^4*g^8*h^4*x + 4*a*c^3*g^6*h^6*x
 + 6*a^2*c^2*g^4*h^8*x + 4*a^3*c*g^2*h^10*x + a^4*h^12*x + c^4*g^9*h^3 + 4*
a*c^3*g^7*h^5 + 6*a^2*c^2*g^5*h^7 + 4*a^3*c*g^3*h^9 + a^4*g*h^11) - 7/8*sqr
t(c*x^2 + a)*c^5*e*g^4*x/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7
 + 4*a^3*c*g^2*h^9 + a^4*h^11) - 15/16*sqrt(c*x^2 + a)*c^5*d*g^4/(c^4*g^8*h
^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9 + a^4*h^11) + 7/
24*(c*x^2 + a)^(5/2)*c^3*e*g^4/(c^4*g^8*h^3*x^3 + 4*a*c^3*g^6*h^5*x^3 + 6*a
^2*c^2*g^4*h^7*x^3 + 4*a^3*c*g^2*h^9*x^3 + a^4*h^11*x^3 + 3*c^4*g^9*h^2*x^2
 + 12*a*c^3*g^7*h^4*x^2 + 18*a^2*c^2*g^5*h^6*x^2 + 12*a^3*c*g^3*h^8*x^2 + 3
*a^4*g*h^10*x^2 + 3*c^4*g^10*h*x + 12*a*c^3*g^8*h^3*x + 18*a^2*c^2*g^6*h^5*
x + 12*a^3*c*g^4*h^7*x + 3*a^4*g^2*h^9*x + c^4*g^11 + 4*a*c^3*g^9*h^2 + 6*a
^2*c^2*g^7*h^4 + 4*a^3*c*g^5*h^6 + a^4*g^3*h^8) + 7/12*(c*x^2 + a)^(3/2)*c^
4*e*g^4/(c^4*g^8*h^3*x + 4*a*c^3*g^6*h^5*x + 6*a^2*c^2*g^4*h^7*x + 4*a^3*c*
g^2*h^9*x + a^4*h^11*x + c^4*g^9*h^2 + 4*a*c^3*g^7*h^4 + 6*a^2*c^2*g^5*h^6
+ 4*a^3*c*g^3*h^8 + a^4*g*h^10) + 1/8*(c*x^2 + a)^(5/2)*c^3*f*g^4/(c^4*g^8*
h^3*x^2 + 4*a*c^3*g^6*h^5*x^2 + 6*a^2*c^2*g^4*h^7*x^2 + 4*a^3*c*g^2*h^9*x^2
 + a^4*h^11*x^2 + 2*c^4*g^9*h^2*x + 8*a*c^3*g^7*h^4*x + 12*a^2*c^2*g^5*h^6*
x + 8*a^3*c*g^3*h^8*x + 2*a^4*g*h^10*x + c^4*g^10*h + 4*a*c^3*g^8*h^3 + 6*a
^2*c^2*g^6*h^5 + 4*a^3*c*g^4*h^7 + a^4*g^2*h^9) - 1/8*(c*x^2 + a)^(3/2)*c^4
*f*g^4/(c^4*g^8*h^3 + 4*a*c^3*g^6*h^5 + 6*a^2*c^2*g^4*h^7 + 4*a^3*c*g^2*h^9
 + a^4*h^11) + 1/2*sqrt(c*x^2 + a)*c^5*d*g^3*x/(c^4*g^8*h^2 + 4*a*c^3*g^6*h
^4 + 6*a^2*c^2*g^4*h^6 + 4*a^3*c*g^2*h^8 + a^4*h^10) - 7/24*(c*x^2 + a)^(5/
2)*c^3*d*g^3/(c^4*g^8*h^2*x^3 + 4*a*c^3*g^6*h^4*x^3 + 6*a^2*c^2*g^4*h^6*x^3
 + 4*a^3*c*g^2*h^8*x^3 + a^4*h^10*x^3 + 3*c^4*g^9*h*x^2 + 12*a*c^3*g^7*h^3*
x^2 + 18*a^2*c^2*g^5*h^5*x^2 + 12*a^3*c*g^3*h^7*x^2 + 3*a^4*g*h^9*x^2 + 3*c
^4*g^10*x + 12*a*c^3*g^8*h^2*x + 18*a^2*c^2*g^6*h^4*x + 12*a^3*c*g^4*h^6*x
+ 3*a^4*g^2*h^8*x + c^4*g^11/h + 4*a*c^3*g^9*h + 6*a^2*c^2*g^7*h^3 + 4*a^3*
c*g^5*h^5 + a^4*g^3*h^7) - 11/24*(c*x^2 + a)^(3/2)*c^4*d*g^3/(c^4*g^8*h^2*x
 + 4*a*c^3*g^6*h^4*x + 6*a^2*c^2*g^4*h^6*x + 4*a^3*c*g^2*h^8*x + a^4*h^10*x
 + c^4*g^9*h + 4*a*c^3*g^7*h^3 + 6*a^2*c^2*g^5*h^5 + 4*a^3*c*g^3*h^7 + a^4*
g*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*f*g^4/(c^3*g^6*h^5*x^4 + 3*a*c^2*g^4*h^
7*x^4 + 3*a^2*c*g^2*h^9*x^4 + a^3*h^11*x^4 + 4*c^3*g^7*h^4*x^3 + 12*a*c^2*g
^5*h^6*x^3 + 12*a^2*c*g^3*h^8*x^3 + 4*a^3*g*h^10*x^3 + 6*c^3*g^8*h^3*x^2 +
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18*a*c^2*g^6*h^5*x^2 + 18*a^2*c*g^4*h^7*x^2 + 6*a^3*g^2*h^9*x^2 + 4*c^3*g^9*h^2*x + 12*a*c^2*g^7*h^4*x + 12*a^2*c*g^5*h^6*x + 4*a^3*g^3*h^8*x + c^3*g^10*h + 3*a*c^2*g^8*h^3 + 3*a^2*c*g^6*h^5 + a^3*g^4*h^7) + 39/16*sqrt(c*x^2 + a)*c^4*f*g^4/(c^3*g^6*h^5 + 3*a*c^2*g^4*h^7 + 3*a^2*c*g^2*h^9 + a^3*h^11) - 19/16*sqrt(c*x^2 + a)*c^4*f*g^3*x/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) - 1/8*(c*x^2 + a)^(5/2)*c^3*d*g^2/(c^4*g^8*h*x^2 + 4*a*c^3*g^6*h^3*x^2 + 6*a^2*c^2*g^4*h^5*x^2 + 4*a^3*c*g^2*h^7*x^2 + a^4*h^9*x^2 + 2*c^4*g^9*x + 8*a*c^3*g^7*h^2*x + 12*a^2*c^2*g^5*h^4*x + 8*a^3*c*g^3*h^6*x + 2*a^4*g*h^8*x + c^4*g^10/h + 4*a*c^3*g^8*h + 6*a^2*c^2*g^6*h^3 + 4*a^3*c*g^4*h^5 + a^4*g^2*h^7) + 1/8*(c*x^2 + a)^(3/2)*c^4*d*g^2/(c^4*g^8*h + 4*a*c^3*g^6*h^3 + 6*a^2*c^2*g^4*h^5 + 4*a^3*c*g^2*h^7 + a^4*h^9) + 7/24*(c*x^2 + a)^(5/2)*c^2*e*g^3/(c^3*g^6*h^4*x^4 + 3*a*c^2*g^4*h^6*x^4 + 3*a^2*c*g^2*h^8*x^4 + a^3*h^10*x^4 + 4*c^3*g^7*h^3*x^3 + 12*a*c^2*g^5*h^5*x^3 + 12*a^2*c*g^3*h^7*x^3 + 4*a^3*g*h^9*x^3 + 6*c^3*g^8*h^2*x^2 + 18*a*c^2*g^6*h^4*x^2 + 18*a^2*c*g^4*h^6*x^2 + 6*a^3*g^2*h^8*x^2 + 4*c^3*g^9*h*x + 12*a*c^2*g^7*h^3*x + 12*a^2*c*g^5*h^5*x + 4*a^3*g^3*h^7*x + c^3*g^10 + 3*a*c^2*g^8*h^2 + 3*a^2*c*g^6*h^4 + a^3*g^4*h^6) - 21/16*sqrt(c*x^2 + a)*c^4*e*g^3/(c^3*g^6*h^4 + 3*a*c^2*g^4*h^6 + 3*a^2*c*g^2*h^8 + a^3*h^10) + 13/24*(c*x^2 + a)^(5/2)*c^2*f*g^3/(c^3*g^6*h^4*x^3 + 3*a*c^2*g^4*h^6*x^3 + 3*a^2*c*g^2*h^8*x^3 + a^3*h^10*x^3 + 3*c^3*g^7*h^3*x^2 + 9*a*c^2*g^5*h^5*x^2 + 9*a^2*c*g^3*h^7*x^2 + 3*a^3*g*h^9*x^2 + 3*c^3*g^8*h^2*x + 9*a*c^2*g^6*h^4*x + 9*a^2*c*g^4*h^6*x + 3*a^3*g^2*h^8*x + c^3*g^9*h + 3*a*c^2*g^7*h^3 + 3*a^2*c*g^5*h^5 + a^3*g^3*h^7) + 15/16*(c*x^2 + a)^(3/2)*c^3*f*g^3/(c^3*g^6*h^4*x + 3*a*c^2*g^4*h^6*x + 3*a^2*c*g^2*h^8*x + a^3*h^10*x + c^3*g^7*h^3 + 3*a*c^2*g^5*h^5 + 3*a^2*c*g^3*h^7 + a^3*g*h^9) + 7/16*sqrt(c*x^2 + a)*c^4*e*g^2*x/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*d*g^2/(c^3*g^6*h^3*x^4 + 3*a*c^2*g^4*h^5*x^4 + 3*a^2*c*g^2*h^7*x^4 + a^3*h^9*x^4 + 4*c^3*g^7*h^2*x^3 + 12*a*c^2*g^5*h^4*x^3 + 12*a^2*c*g^3*h^6*x^3 + 4*a^3*g*h^8*x^3 + 6*c^3*g^8*h*x^2 + 18*a*c^2*g^6*h^3*x^2 + 18*a^2*c*g^4*h^5*x^2 + 6*a^3*g^2*h^7*x^2 + 4*c^3*g^9*x + 12*a*c^2*g^7*h^2*x + 12*a^2*c*g^5*h^4*x + 4*a^3*g^3*h^6*x + c^3*g^10/h + 3*a*c^2*g^8*h + 3*a^2*c*g^6*h^3 + a^3*g^4*h^5) + 9/16*sqrt(c*x^2 + a)*c^4*d*g^2/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/24*(c*x^2 + a)^(5/2)*c^2*e*g^2/(c^3*g^6*h^3*x^3 + 3*a*c^2*g^4*h^5*x^3 + 3*a^2*c*g^2*h^7*x^3 + a^3*h^9*x^3 + 3*c^3*g^7*h^2*x^2 + 9*a*c^2*g^5*h^4*x^2 + 9*a^2*c*g^3*h^6*x^2 + 3*a^3*g*h^8*x^2 + 3*c^3*g^8*h*x + 9*a*c^2*g^6*h^3*x + 9*a^2*c*g^4*h^5*x + 3*a^3*g^2*h^7*x + c^3*g^9 + 3*a*c^2*g^7*h^2 + 3*a^2*c*g^5*h^4 + a^3*g^3*h^6) - 7/16*(c*x^2 + a)^(3/2)*c^3*e*g^2/(c^3*g^6*h^3*x + 3*a*c^2*g^4*h^5*x + 3*a^2*c*g^2*h^7*x + a^3*h^9*x + c^3*g^7*h^2 + 3*a*c^2*g^5*h^4 + 3*a^2*c*g^3*h^6 + a^3*g*h^8) + 7/48*(c*x^2 + a)^(5/2)*c^2*f*g^2/(c^3*g^6*h^3*x^2 + 3*a*c^2*g^4*h^5*x^2 + 3*a^2*c*g^2*h^7*x^2 + a^3*h^9*x^2 + 2*c^3*g^7*h^2*x + 6*a*c^2*g^5*h^4*x + 6*a^2*c*g^3*h^6*x + 2*a^3*g*h^8*x + c^3*g^8*h + 3*a*c^2*g^6*h^3 + 3*a^2*c*g^4*h^5 + a^3*g^2*h^7) - 7/48*(c*x^2 + a)^(3/2)*c^3*f*g^2/(c^3*g^6*h^3 + 3*a*c^2*g^4*h^5 + 3*a^2*c*g^2*h^7 + a^3*h^9) - 7/30*(c*x^2 + a)^(5/2)*c*f*g^3/(c^2*g^4*h^6*x^5 + 2*a*c*g^2*h^8*x^5 + a^2*h^10*x^5 + 5*c^2*g^5*h^5*x^4 + 10

*a*c*g^3*h^7*x^4 + 5*a^2*g*h^9*x^4 + 10*c^2*g^6*h^4*x^3 + 20*a*c*g^4*h^6*x^
3 + 10*a^2*g^2*h^8*x^3 + 10*c^2*g^7*h^3*x^2 + 20*a*c*g^5*h^5*x^2 + 10*a^2*g
^3*h^7*x^2 + 5*c^2*g^8*h^2*x + 10*a*c*g^6*h^4*x + 5*a^2*g^4*h^6*x + c^2*g^9
*h + 2*a*c*g^7*h^3 + a^2*g^5*h^5) - 1/16*sqrt(c*x^2 + a)*c^4*d*g*x/(c^3*g^6
*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 1/24*(c*x^2 + a)^(5/2
)*c^2*d*g/(c^3*g^6*h^2*x^3 + 3*a*c^2*g^4*h^4*x^3 + 3*a^2*c*g^2*h^6*x^3 + a^
3*h^8*x^3 + 3*c^3*g^7*h*x^2 + 9*a*c^2*g^5*h^3*x^2 + 9*a^2*c*g^3*h^5*x^2 + 3
*a^3*g*h^7*x^2 + 3*c^3*g^8*x + 9*a*c^2*g^6*h^2*x + 9*a^2*c*g^4*h^4*x + 3*a^
3*g^2*h^6*x + c^3*g^9/h + 3*a*c^2*g^7*h + 3*a^2*c*g^5*h^3 + a^3*g^3*h^5) +
1/16*(c*x^2 + a)^(3/2)*c^3*d*g/(c^3*g^6*h^2*x + 3*a*c^2*g^4*h^4*x + 3*a^2*c
*g^2*h^6*x + a^3*h^8*x + c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^
3*g*h^7) - 7/48*(c*x^2 + a)^(5/2)*c^2*e*g/(c^3*g^6*h^2*x^2 + 3*a*c^2*g^4*h^
4*x^2 + 3*a^2*c*g^2*h^6*x^2 + a^3*h^8*x^2 + 2*c^3*g^7*h*x + 6*a*c^2*g^5*h^3
*x + 6*a^2*c*g^3*h^5*x + 2*a^3*g*h^7*x + c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*
c*g^4*h^4 + a^3*g^2*h^6) + 7/48*(c*x^2 + a)^(3/2)*c^3*e*g/(c^3*g^6*h^2 + 3*
a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8) + 7/30*(c*x^2 + a)^(5/2)*c*e*g^2
/(c^2*g^4*h^5*x^5 + 2*a*c*g^2*h^7*x^5 + a^2*h^9*x^5 + 5*c^2*g^5*h^4*x^4 + 1
0*a*c*g^3*h^6*x^4 + 5*a^2*g*h^8*x^4 + 10*c^2*g^6*h^3*x^3 + 20*a*c*g^4*h^5*x
^3 + 10*a^2*g^2*h^7*x^3 + 10*c^2*g^7*h^2*x^2 + 20*a*c*g^5*h^4*x^2 + 10*a^2*
g^3*h^6*x^2 + 5*c^2*g^8*h*x + 10*a*c*g^6*h^3*x + 5*a^2*g^4*h^5*x + c^2*g^9
+ 2*a*c*g^7*h^2 + a^2*g^5*h^4) + 13/24*(c*x^2 + a)^(5/2)*c*f*g^2/(c^2*g^4*h
^5*x^4 + 2*a*c*g^2*h^7*x^4 + a^2*h^9*x^4 + 4*c^2*g^5*h^4*x^3 + 8*a*c*g^3*h^
6*x^3 + 4*a^2*g*h^8*x^3 + 6*c^2*g^6*h^3*x^2 + 12*a*c*g^4*h^5*x^2 + 6*a^2*g^
2*h^7*x^2 + 4*c^2*g^7*h^2*x + 8*a*c*g^5*h^4*x + 4*a^2*g^3*h^6*x + c^2*g^8*h
+ 2*a*c*g^6*h^3 + a^2*g^4*h^5) - 25/16*sqrt(c*x^2 + a)*c^3*f*g^2/(c^2*g^4*
h^5 + 2*a*c*g^2*h^7 + a^2*h^9) + 3/8*sqrt(c*x^2 + a)*c^3*f*g*x/(c^2*g^4*h^4
+ 2*a*c*g^2*h^6 + a^2*h^8) + 1/48*(c*x^2 + a)^(5/2)*c^2*d/(c^3*g^6*h*x^2 +
3*a*c^2*g^4*h^3*x^2 + 3*a^2*c*g^2*h^5*x^2 + a^3*h^7*x^2 + 2*c^3*g^7*x + 6*
a*c^2*g^5*h^2*x + 6*a^2*c*g^3*h^4*x + 2*a^3*g*h^6*x + c^3*g^8/h + 3*a*c^2*g
^6*h + 3*a^2*c*g^4*h^3 + a^3*g^2*h^5) - 1/48*(c*x^2 + a)^(3/2)*c^3*d/(c^3*g
^6*h + 3*a*c^2*g^4*h^3 + 3*a^2*c*g^2*h^5 + a^3*h^7) - 7/30*(c*x^2 + a)^(5/2
)*c*d*g/(c^2*g^4*h^4*x^5 + 2*a*c*g^2*h^6*x^5 + a^2*h^8*x^5 + 5*c^2*g^5*h^3*
x^4 + 10*a*c*g^3*h^5*x^4 + 5*a^2*g*h^7*x^4 + 10*c^2*g^6*h^2*x^3 + 20*a*c*g^
4*h^4*x^3 + 10*a^2*g^2*h^6*x^3 + 10*c^2*g^7*h*x^2 + 20*a*c*g^5*h^3*x^2 + 10
*a^2*g^3*h^5*x^2 + 5*c^2*g^8*x + 10*a*c*g^6*h^2*x + 5*a^2*g^4*h^4*x + c^2*g
^9/h + 2*a*c*g^7*h + a^2*g^5*h^3) - 7/24*(c*x^2 + a)^(5/2)*c*e*g/(c^2*g^4*h
^4*x^4 + 2*a*c*g^2*h^6*x^4 + a^2*h^8*x^4 + 4*c^2*g^5*h^3*x^3 + 8*a*c*g^3*h^
5*x^3 + 4*a^2*g*h^7*x^3 + 6*c^2*g^6*h^2*x^2 + 12*a*c*g^4*h^4*x^2 + 6*a^2*g^
2*h^6*x^2 + 4*c^2*g^7*h*x + 8*a*c*g^5*h^3*x + 4*a^2*g^3*h^5*x + c^2*g^8 + 2
*a*c*g^6*h^2 + a^2*g^4*h^4) + 7/16*sqrt(c*x^2 + a)*c^3*e*g/(c^2*g^4*h^4 + 2
*a*c*g^2*h^6 + a^2*h^8) - 1/4*(c*x^2 + a)^(5/2)*c*f*g/(c^2*g^4*h^4*x^3 + 2*
a*c*g^2*h^6*x^3 + a^2*h^8*x^3 + 3*c^2*g^5*h^3*x^2 + 6*a*c*g^3*h^5*x^2 + 3*a
^2*g*h^7*x^2 + 3*c^2*g^6*h^2*x + 6*a*c*g^4*h^4*x + 3*a^2*g^2*h^6*x + c^2*g^
7*h + 2*a*c*g^5*h^3 + a^2*g^3*h^5) - 3/8*(c*x^2 + a)^(3/2)*c^2*f*g/(c^2*g^4
*h^4*x + 2*a*c*g^2*h^6*x + a^2*h^8*x + c^2*g^5*h^3 + 2*a*c*g^3*h^5 + a^2*g*

h^7) - 1/6*(c*x^2 + a)^(5/2)*f*g^2/(c*g^2*h^7*x^6 + a*h^9*x^6 + 6*c*g^3*h^6
*x^5 + 6*a*g*h^8*x^5 + 15*c*g^4*h^5*x^4 + 15*a*g^2*h^7*x^4 + 20*c*g^5*h^4*x
^3 + 20*a*g^3*h^6*x^3 + 15*c*g^6*h^3*x^2 + 15*a*g^4*h^5*x^2 + 6*c*g^7*h^2*x
 + 6*a*g^5*h^4*x + c*g^8*h + a*g^6*h^3) + 1/24*(c*x^2 + a)^(5/2)*c*d/(c^2*g
^4*h^3*x^4 + 2*a*c*g^2*h^5*x^4 + a^2*h^7*x^4 + 4*c^2*g^5*h^2*x^3 + 8*a*c*g^
3*h^4*x^3 + 4*a^2*g*h^6*x^3 + 6*c^2*g^6*h*x^2 + 12*a*c*g^4*h^3*x^2 + 6*a^2*
g^2*h^5*x^2 + 4*c^2*g^7*x + 8*a*c*g^5*h^2*x + 4*a^2*g^3*h^4*x + c^2*g^8/h +
 2*a*c*g^6*h + a^2*g^4*h^3) - 1/16*sqrt(c*x^2 + a)*c^3*d/(c^2*g^4*h^3 + 2*a
*c*g^2*h^5 + a^2*h^7) - 1/8*(c*x^2 + a)^(5/2)*c*f/(c^2*g^4*h^3*x^2 + 2*a*c*
g^2*h^5*x^2 + a^2*h^7*x^2 + 2*c^2*g^5*h^2*x + 4*a*c*g^3*h^4*x + 2*a^2*g*h^6
*x + c^2*g^6*h + 2*a*c*g^4*h^3 + a^2*g^2*h^5) + 1/8*(c*x^2 + a)^(3/2)*c^2*f
/(c^2*g^4*h^3 + 2*a*c*g^2*h^5 + a^2*h^7) + 1/6*(c*x^2 + a)^(5/2)*e*g/(c*g^2
*h^6*x^6 + a*h^8*x^6 + 6*c*g^3*h^5*x^5 + 6*a*g*h^7*x^5 + 15*c*g^4*h^4*x^4 +
 15*a*g^2*h^6*x^4 + 20*c*g^5*h^3*x^3 + 20*a*g^3*h^5*x^3 + 15*c*g^6*h^2*x^2
+ 15*a*g^4*h^4*x^2 + 6*c*g^7*h*x + 6*a*g^5*h^3*x + c*g^8 + a*g^6*h^2) + 2/5
*(c*x^2 + a)^(5/2)*f*g/(c*g^2*h^6*x^5 + a*h^8*x^5 + 5*c*g^3*h^5*x^4 + 5*a*g
*h^7*x^4 + 10*c*g^4*h^4*x^3 + 10*a*g^2*h^6*x^3 + 10*c*g^5*h^3*x^2 + 10*a*g^
3*h^5*x^2 + 5*c*g^6*h^2*x + 5*a*g^4*h^4*x + c*g^7*h + a*g^5*h^3) - 1/6*(c*x
^2 + a)^(5/2)*d/(c*g^2*h^5*x^6 + a*h^7*x^6 + 6*c*g^3*h^4*x^5 + 6*a*g*h^6*x^
5 + 15*c*g^4*h^3*x^4 + 15*a*g^2*h^5*x^4 + 20*c*g^5*h^2*x^3 + 20*a*g^3*h^4*x
^3 + 15*c*g^6*h*x^2 + 15*a*g^4*h^3*x^2 + 6*c*g^7*x + 6*a*g^5*h^2*x + c*g^8/
h + a*g^6*h) - 1/5*(c*x^2 + a)^(5/2)*e/(c*g^2*h^5*x^5 + a*h^7*x^5 + 5*c*g^3
*h^4*x^4 + 5*a*g*h^6*x^4 + 10*c*g^4*h^3*x^3 + 10*a*g^2*h^5*x^3 + 10*c*g^5*h
^2*x^2 + 10*a*g^3*h^4*x^2 + 5*c*g^6*h*x + 5*a*g^4*h^3*x + c*g^7 + a*g^5*h^2
) - 1/4*(c*x^2 + a)^(5/2)*f/(c*g^2*h^5*x^4 + a*h^7*x^4 + 4*c*g^3*h^4*x^3 +
4*a*g*h^6*x^3 + 6*c*g^4*h^3*x^2 + 6*a*g^2*h^5*x^2 + 4*c*g^5*h^2*x + 4*a*g^3
*h^4*x + c*g^6*h + a*g^4*h^3) + 3/8*sqrt(c*x^2 + a)*c^2*f/(c*g^2*h^5 + a*h^
7) + 7/16*c^6*f*g^8*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)
*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^15) - 7/16*c^6*e*g^7*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^
(9/2)*h^14) + 7/16*c^6*d*g^6*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(9/2)*h^13) - 27/16*c^5*f*g^6*arc
sinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c
*g^2/h^2)^(7/2)*h^13) + 21/16*c^5*e*g^5*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^12) - 15/16*c^
5*d*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g
)))/((a + c*g^2/h^2)^(7/2)*h^11) + 39/16*c^4*f*g^4*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^11)
 - 21/16*c^4*e*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*
abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^10) + 9/16*c^4*d*g^2*arcsinh(c*g*x/
(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(
5/2)*h^9) - 25/16*c^3*f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(s
qrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^9) + 7/16*c^3*e*g*arcsinh(
c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/
h^2)^(3/2)*h^8) - 1/16*c^3*d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(

```
sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^7) + 3/8*c^2*f*arcsinh(c*
g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^
2/h^2)*h^7)
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

**sympy** **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
[Out] Timed out
```

## 3.99 $\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

**Optimal**. Leaf size=532

$$-\frac{\left(a+cx^2\right)^{5/2}\left(42a^2fh^4-ach^2\left(26fg^2-h(61eg-12dh)\right)-c^2g^2\left(h(2eg-51dh)+5fg^2\right)\right)}{210h(g+hx)^5\left(ah^2+cg^2\right)^3}-\frac{ac^2\sqrt{a+cx^2}\,(ah-cgx)}{}$$

```
[Out] -1/24*c*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(
-c*g*x+a*h)*(c*x^2+a)^(3/2)/(a*h^2+c*g^2)^4/(h*x+g)^4-1/7*(d*h^2-e*g*h+f*g^
2)*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)/(h*x+g)^7+1/42*(7*a*h^2*(-e*h+2*f*g)+c*g
*(5*f*g^2+h*(-9*d*h+2*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^2/(h*x+g)^6-1/
210*(42*a^2*f*h^4-c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-a*c*h^2*(26*f*g^2-h*(
-12*d*h+61*e*g)))*(c*x^2+a)^(5/2)/h/(a*h^2+c*g^2)^3/(h*x+g)^5-1/16*a^2*c^3*
(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*arctanh((
-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(11/2)-1/16*
a*c^2*(6*c^2*d*g^3+a^2*h^2*(-e*h+8*f*g)-a*c*g*(f*g^2-h*(-3*d*h+8*e*g)))*(-c
*g*x+a*h)*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^5/(h*x+g)^2
```

**Rubi [A]** time = 0.89, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1651, 835, 807, 721, 725, 206}

$$-\frac{ac^2\sqrt{a+cx^2}\,(ah-cgx)\left(a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)+6c^2dg^3\right)}{16(g+hx)^2\left(ah^2+cg^2\right)^5}-\frac{c\left(a+cx^2\right)^{3/2}(ah-cgx)\left(a^2h^2(8\right.}{24(g}$$

Antiderivative was successfully verified.

```
[In] Int[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -(a*c^2*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*
d*h)))*(a*h - c*g*x)*Sqrt[a + c*x^2])/(16*(c*g^2 + a*h^2)^5*(g + h*x)^2) -
(c*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h))
)*(a*h - c*g*x)*(a + c*x^2)^(3/2))/(24*(c*g^2 + a*h^2)^4*(g + h*x)^4) - ((f
*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(5/2))/(7*h*(c*g^2 + a*h^2)*(g + h*x)^7)
+ ((5*c*f*g^3 + c*g*h*(2*e*g - 9*d*h) + 7*a*h^2*(2*f*g - e*h))*(a + c*x^2)^
(5/2))/(42*h*(c*g^2 + a*h^2)^2*(g + h*x)^6) - ((42*a^2*f*h^4 - c^2*(5*f*g^4
 + g^2*h*(2*e*g - 51*d*h)) - a*c*h^2*(26*f*g^2 - h*(61*e*g - 12*d*h)))*(a +
 c*x^2)^(5/2))/(210*h*(c*g^2 + a*h^2)^3*(g + h*x)^5) - (a^2*c^3*(6*c^2*d*g^
3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 - h*(8*e*g - 3*d*h)))*ArcTanh[(a*h
```

```
 - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(16*(c*g^2 + a*h^2)^(11/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} - \frac{\int \frac{\left(-7(cdg-afg+aeh)-\left(7afh+c\left(2eg+\frac{5fg^2}{h}-2dh\right)\right)x\right)(a\cdots}{(g+hx)^7}}{7\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh\cdots\right.}{42h\left(cg^2+ah^2\right)^2(g+hx)^6}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\left(a+cx^2\right)^{5/2}}{7h\left(cg^2+ah^2\right)(g+hx)^7} + \frac{\left(5cfg^3+cgh(2eg-9dh)+7ah^2(2fg-eh\cdots\right.}{42h\left(cg^2+ah^2\right)^2(g+hx)^6}$$

$$= -\frac{c\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\left(a+cx^2\cdots\right.}{24\left(cg^2+ah^2\right)^4(g+hx)^4}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+c\cdots}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+c\cdots}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

$$= -\frac{ac^2\left(6c^2dg^3+a^2h^2(8fg-eh)-acg\left(fg^2-h(8eg-3dh)\right)\right)(ah-cgx)\sqrt{a+c\cdots}}{16\left(cg^2+ah^2\right)^5(g+hx)^2}$$

**Mathematica [A]** time = 2.50, size = 863, normalized size = 1.62

$$\frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)g+a^2h^2(8fg-eh)\right)\log(g+hx)c^3}{16\left(cg^2+ah^2\right)^{11/2}} - \frac{a^2\left(6c^2dg^3-ac\left(fg^2+h(3dh-8eg)\right)g\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/1680*(Sqrt[a + c*x^2]*(240*(c*g^2 + a*h^2)^6*(f*g^2 + h*(-(e*g) + d*h))
- 40*(c*g^2 + a*h^2)^5*(29*c*f*g^3 + c*g*h*(-22*e*g + 15*d*h) - 7*a*h^2*(-2
*f*g + e*h))*(g + h*x) + 8*(c*g^2 + a*h^2)^4*(42*a^2*f*h^4 + a*c*h^2*(314*f
*g^2 + h*(-139*e*g + 48*d*h)) + c^2*(275*f*g^4 + g^2*h*(-142*e*g + 51*d*h))
)*(g + h*x)^2 - 2*c*(c*g^2 + a*h^2)^3*(7*a^2*h^4*(136*f*g - 35*e*h) + 2*c^2
*(500*f*g^5 + g^3*h*(-136*e*g + 3*d*h)) + a*c*g*h^2*(1979*f*g^2 + h*(-544*e
*g + 33*d*h)))*(g + h*x)^3 + 2*c*(c*g^2 + a*h^2)^2*(336*a^3*f*h^6 + c^3*(40
0*f*g^6 - 2*g^4*h*(4*e*g + 3*d*h)) + 3*a^2*c*h^4*(400*f*g^2 + h*(-29*e*g +
8*d*h)) + a*c^2*g^2*h^2*(1201*f*g^2 - h*(32*e*g + 45*d*h)))*(g + h*x)^4 - c
^2*(c*g^2 + a*h^2)*(21*a^3*h^6*(24*f*g - 5*e*h) + 2*a*c^2*g^3*h^2*(89*f*g^2
 + 44*e*g*h + 54*d*h^2) + 3*a^2*c*g*h^4*(109*f*g^2 + h*(94*e*g - 73*d*h)) +
 4*c^3*(10*f*g^7 + g^5*h*(4*e*g + 3*d*h)))*(g + h*x)^5 - c^2*(-336*a^4*f*h^
8 + 2*a*c^3*g^4*h^2*(109*f*g^2 + 52*e*g*h + 60*d*h^2) + a^2*c^2*g^2*h^4*(50
5*f*g^2 + h*(370*e*g - 741*d*h)) + 4*c^4*(10*f*g^8 + g^6*h*(4*e*g + 3*d*h))
 + 3*a^3*c*h^6*(312*f*g^2 + h*(-221*e*g + 32*d*h)))*(g + h*x)^6))/((c*g^2*h
 + a*h^3)^5*(g + h*x)^7) + (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) -
a*c*g*(f*g^2 + h*(-8*e*g + 3*d*h)))*Log[g + h*x])/(16*(c*g^2 + a*h^2)^(11/2
)) - (a^2*c^3*(6*c^2*d*g^3 + a^2*h^2*(8*f*g - e*h) - a*c*g*(f*g^2 + h*(-8*e
*g + 3*d*h)))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(16*(
c*g^2 + a*h^2)^(11/2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 1.12, size = 7936, normalized size = 14.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] -1/8*(6*a^2*c^5*d*g^3 - a^3*c^4*f*g^3 - 3*a^3*c^4*d*g*h^2 + 8*a^4*c^3*f*g*h
^2 + 8*a^3*c^4*g^2*h*e - a^4*c^3*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a
))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^5*g^10 + 5*a*c^4*g^8*h^2 + 10*a
^2*c^3*g^6*h^4 + 10*a^3*c^2*g^4*h^6 + 5*a^4*c*g^2*h^8 + a^5*h^10)*sqrt(-c*g
^2 - a*h^2)) - 1/840*(630*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^2*c^5*d*g^3*h^
12 - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*f*g^3*h^12 - 315*(sqrt(c)
```

*x - sqrt(c*x^2 + a))^13*a^3*c^4*d*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a
))^13*a^4*c^3*f*g*h^14 + 840*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^3*c^4*g^2*h
^13*e - 105*(sqrt(c)*x - sqrt(c*x^2 + a))^13*a^4*c^3*h^15*e - 1680*(sqrt(c)
*x - sqrt(c*x^2 + a))^12*c^(15/2)*f*g^10*h^5 - 8400*(sqrt(c)*x - sqrt(c*x^2
 + a))^12*a*c^(13/2)*f*g^8*h^7 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2
*c^(11/2)*f*g^6*h^9 + 8190*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^2*c^(11/2)*d*
g^4*h^11 - 18165*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*f*g^4*h^11 -
4095*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^3*c^(9/2)*d*g^2*h^13 + 2520*(sqrt(c
)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*f*g^2*h^13 - 1680*(sqrt(c)*x - sqrt(c
*x^2 + a))^12*a^5*c^(5/2)*f*h^15 + 10920*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a
^3*c^(9/2)*g^3*h^12*e - 1365*(sqrt(c)*x - sqrt(c*x^2 + a))^12*a^4*c^(7/2)*g
*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*f*g^11*h^4 - 28000*(sqr
t(c)*x - sqrt(c*x^2 + a))^11*a*c^7*f*g^9*h^6 - 56000*(sqrt(c)*x - sqrt(c*x^
2 + a))^11*a^2*c^6*f*g^7*h^8 + 44940*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^2*c
^6*d*g^5*h^10 - 63490*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*f*g^5*h^10 -
 26670*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*c^5*d*g^3*h^12 + 32620*(sqrt(c)
*x - sqrt(c*x^2 + a))^11*a^4*c^4*f*g^3*h^12 + 2100*(sqrt(c)*x - sqrt(c*x^2
+ a))^11*a^4*c^4*d*g*h^14 - 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*
f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^11*c^8*g^10*h^5*e - 11200*(sq
rt(c)*x - sqrt(c*x^2 + a))^11*a*c^7*g^8*h^7*e - 22400*(sqrt(c)*x - sqrt(c*x
^2 + a))^11*a^2*c^6*g^6*h^9*e + 37520*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^3*
c^5*g^4*h^11*e - 24290*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^4*c^4*g^2*h^13*e
- 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^11*a^5*c^3*h^15*e - 11200*(sqrt(c)*x -
 sqrt(c*x^2 + a))^10*c^(17/2)*f*g^12*h^3 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a
))^10*c^(17/2)*d*g^10*h^5 - 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/
2)*f*g^10*h^5 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c^(15/2)*d*g^8*h^7
 - 95200*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*f*g^8*h^7 + 100380*(
sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*d*g^6*h^9 - 100730*(sqrt(c)*x
- sqrt(c*x^2 + a))^10*a^3*c^(11/2)*f*g^6*h^9 - 146790*(sqrt(c)*x - sqrt(c*x
^2 + a))^10*a^3*c^(11/2)*d*g^4*h^11 + 163940*(sqrt(c)*x - sqrt(c*x^2 + a))^
10*a^4*c^(9/2)*f*g^4*h^11 + 6300*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/
2)*d*g^2*h^13 - 56000*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*f*g^2*h^
13 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*d*h^15 + 3360*(sqrt(
c)*x - sqrt(c*x^2 + a))^10*a^6*c^(5/2)*f*h^15 - 4480*(sqrt(c)*x - sqrt(c*x^
2 + a))^10*c^(17/2)*g^11*h^4*e - 22400*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a*c
^(15/2)*g^9*h^6*e - 44800*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^2*c^(13/2)*g^7
*h^8*e + 133840*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^3*c^(11/2)*g^5*h^10*e -
106330*(sqrt(c)*x - sqrt(c*x^2 + a))^10*a^4*c^(9/2)*g^3*h^12*e + 3220*(sqrt
(c)*x - sqrt(c*x^2 + a))^10*a^5*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(
c*x^2 + a))^9*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^9*c^9*d*g
^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*f*g^11*h^4 - 20160*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*d*g^9*h^6 - 52640*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*a^2*c^7*f*g^9*h^6 + 191016*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c
^7*d*g^7*h^8 - 9436*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*f*g^7*h^8 - 363
216*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*d*g^5*h^10 + 439306*(sqrt(c)*x

- sqrt(c*x^2 + a))^9*a^4*c^5*f*g^5*h^10 + 95340*(sqrt(c)*x - sqrt(c*x^2 + a
))^9*a^4*c^5*d*g^3*h^12 - 209965*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*f*
g^3*h^12 - 9975*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^5*c^4*d*g*h^14 + 32200*(s
qrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x
^2 + a))^9*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a*c^8*g^1
0*h^5*e - 49280*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^2*c^7*g^8*h^7*e + 263648*
(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^3*c^6*g^6*h^9*e - 332780*(sqrt(c)*x - sqr
t(c*x^2 + a))^9*a^4*c^5*g^4*h^11*e + 49490*(sqrt(c)*x - sqrt(c*x^2 + a))^9*
a^5*c^4*g^2*h^13*e - 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^9*a^6*c^3*h^15*e -
8960*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/2)*f*g^14*h - 2688*(sqrt(c)*x -
sqrt(c*x^2 + a))^8*c^(19/2)*d*g^12*h^3 - 15232*(sqrt(c)*x - sqrt(c*x^2 + a)
)^8*a*c^(17/2)*f*g^12*h^3 - 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2
)*d*g^10*h^5 + 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*f*g^10*h^
5 + 181104*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*d*g^8*h^7 + 143416*
(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*f*g^8*h^7 - 651924*(sqrt(c)*x
- sqrt(c*x^2 + a))^8*a^3*c^(13/2)*d*g^6*h^9 + 580034*(sqrt(c)*x - sqrt(c*x^
2 + a))^8*a^4*c^(11/2)*f*g^6*h^9 + 299460*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a
^4*c^(11/2)*d*g^4*h^11 - 568085*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)
*f*g^4*h^11 - 72975*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^5*c^(9/2)*d*g^2*h^13
+ 147000*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*f*g^2*h^13 - 3360*(sqr
t(c)*x - sqrt(c*x^2 + a))^8*a^6*c^(7/2)*d*h^15 - 5040*(sqrt(c)*x - sqrt(c*x
^2 + a))^8*a^7*c^(5/2)*f*h^15 - 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^8*c^(19/
2)*g^13*h^2*e - 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a*c^(17/2)*g^11*h^4*e
+ 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^2*c^(15/2)*g^9*h^6*e + 344512*(sqr
t(c)*x - sqrt(c*x^2 + a))^8*a^3*c^(13/2)*g^7*h^8*e - 613480*(sqrt(c)*x - sq
rt(c*x^2 + a))^8*a^4*c^(11/2)*g^5*h^10*e + 259210*(sqrt(c)*x - sqrt(c*x^2 +
 a))^8*a^5*c^(9/2)*g^3*h^12*e - 9765*(sqrt(c)*x - sqrt(c*x^2 + a))^8*a^6*c^
(7/2)*g*h^14*e - 2560*(sqrt(c)*x - sqrt(c*x^2 + a))^7*c^10*f*g^15 - 768*(sq
rt(c)*x - sqrt(c*x^2 + a))^7*c^10*d*g^13*h^2 + 12928*(sqrt(c)*x - sqrt(c*x^
2 + a))^7*a*c^9*f*g^13*h^2 + 384*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c^9*d*g^
11*h^4 + 80576*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*f*g^11*h^4 + 117984*
(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*d*g^9*h^6 + 101936*(sqrt(c)*x - sqr
t(c*x^2 + a))^7*a^3*c^7*f*g^9*h^6 - 603216*(sqrt(c)*x - sqrt(c*x^2 + a))^7*
a^3*c^7*d*g^7*h^8 + 256816*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*f*g^7*h^
8 + 703752*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^4*c^6*d*g^5*h^10 - 941332*(sqr
t(c)*x - sqrt(c*x^2 + a))^7*a^5*c^5*f*g^5*h^10 - 184380*(sqrt(c)*x - sqrt(c
*x^2 + a))^7*a^5*c^5*d*g^3*h^12 + 413280*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^
6*c^4*f*g^3*h^12 + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*d*g*h^14 -
 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^7*c^3*f*g*h^14 - 1024*(sqrt(c)*x -
 sqrt(c*x^2 + a))^7*c^10*g^14*h*e + 4096*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*
c^9*g^12*h^3*e + 32768*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^2*c^8*g^10*h^5*e +
 205952*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^3*c^7*g^8*h^7*e - 741776*(sqrt(c)
*x - sqrt(c*x^2 + a))^7*a^4*c^6*g^6*h^9*e + 608720*(sqrt(c)*x - sqrt(c*x^2
+ a))^7*a^5*c^5*g^4*h^11*e - 92820*(sqrt(c)*x - sqrt(c*x^2 + a))^7*a^6*c^4*
g^2*h^13*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*f*g^14*h + 268

8*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*d*g^12*h^3 + 15232*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*f*g^12*h^3 + 16800*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*d*g^10*h^5 - 53200*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*f*g^10*h^5 - 342384*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*d*g^8*h^7 - 103936*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*f*g^8*h^7 + 736344*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*d*g^6*h^9 - 726404*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*f*g^6*h^9 - 488460*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*d*g^4*h^11 + 764960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*f*g^4*h^11 + 33600*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*d*g^2*h^13 - 168000*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*f*g^2*h^13 - 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*d*h^15 + 6720*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^8*c^(5/2)*f*h^15 + 3584*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c^(19/2)*g^13*h^2*e + 9856*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*c^(17/2)*g^11*h^4*e + 8960*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^3*c^(15/2)*g^9*h^6*e - 487312*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^4*c^(13/2)*g^7*h^8*e + 807520*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^5*c^(11/2)*g^5*h^10*e - 310660*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^6*c^(9/2)*g^3*h^12*e + 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^7*c^(7/2)*g*h^14*e - 13440*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*f*g^13*h^2 - 4032*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*d*g^11*h^4 - 50848*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*f*g^11*h^4 - 47040*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*d*g^9*h^6 - 50960*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*f*g^9*h^6 + 438816*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*d*g^7*h^8 - 99736*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*f*g^7*h^8 - 556416*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*d*g^5*h^10 + 728756*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*f*g^5*h^10 + 167790*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*d*g^3*h^12 - 362915*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*f*g^3*h^12 - 10185*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*d*g*h^14 + 38360*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*f*g*h^14 - 5376*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c^9*g^12*h^3*e - 25984*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^3*c^8*g^10*h^5*e - 86240*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^4*c^7*g^8*h^7*e + 574448*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^5*c^6*g^6*h^9*e - 487480*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^6*c^5*g^4*h^11*e + 89740*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^7*c^4*g^2*h^13*e + 1085*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^8*c^3*h^15*e + 11200*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*f*g^12*h^3 + 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*d*g^10*h^5 + 52640*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*f*g^10*h^5 + 45360*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*d*g^8*h^7 + 96880*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*f*g^8*h^7 - 364728*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*d*g^6*h^9 + 215908*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*f*g^6*h^9 + 220710*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*d*g^4*h^11 - 406735*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*f*g^4*h^11 - 49581*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*d*g^2*h^13 + 104776*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*f*g^2*h^13 - 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*d*h^15 - 3696*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^9*c^(5/2)*f*h^15 + 4480*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*c^(17/2)*g^11*h^4*e + 29120*(

sqrt(c)*x - sqrt(c*x^2 + a))^4*a^4*c^(15/2)*g^9*h^6*e + 119056*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^5*c^(13/2)*g^7*h^8*e - 390656*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^6*c^(11/2)*g^5*h^10*e + 179900*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^7*c^(9/2)*g^3*h^12*e - 10703*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^8*c^(7/2)*g*h^14*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*f*g^11*h^4 - 3360*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*d*g^9*h^6 - 29680*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*f*g^9*h^6 - 32592*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*d*g^7*h^8 - 67088*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*f*g^7*h^8 + 172620*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*d*g^5*h^10 - 156170*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*f*g^5*h^10 - 62454*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*d*g^3*h^12 + 140084*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*f*g^3*h^12 + 5964*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*d*g*h^14 - 17024*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^3*f*g*h^14 - 2240*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^4*c^8*g^10*h^5*e - 16576*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^5*c^7*g^8*h^7*e - 72464*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^6*c^6*g^6*h^9*e + 179200*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^7*c^5*g^4*h^11*e - 31402*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^8*c^4*g^2*h^13*e + 1540*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^9*c^3*h^15*e + 1680*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*f*g^10*h^5 + 1008*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*d*g^8*h^7 + 9632*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*f*g^8*h^7 + 9996*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*d*g^6*h^9 + 24094*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*f*g^6*h^9 - 54894*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*d*g^4*h^11 + 56924*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*f*g^4*h^11 + 9156*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*d*g^2*h^13 - 32256*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*f*g^2*h^13 - 672*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*d*h^15 + 672*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^10*c^(5/2)*f*h^15 + 1344*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^5*c^(15/2)*g^9*h^6*e + 8624*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^6*c^(13/2)*g^7*h^8*e + 30352*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^7*c^(11/2)*g^5*h^10*e - 47362*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^8*c^(9/2)*g^3*h^12*e + 3276*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^9*c^(7/2)*g*h^14*e - 560*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*f*g^9*h^6 - 168*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*d*g^7*h^8 - 3052*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*f*g^7*h^8 - 1680*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*d*g^5*h^10 - 7070*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*f*g^5*h^10 + 9744*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*d*g^3*h^12 - 12999*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*f*g^3*h^12 - 1029*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*d*g*h^14 + 3864*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*f*g*h^14 - 224*(sqrt(c)*x - sqrt(c*x^2 + a))*a^6*c^7*g^8*h^7*e - 1456*(sqrt(c)*x - sqrt(c*x^2 + a))*a^7*c^6*g^6*h^9*e - 5180*(sqrt(c)*x - sqrt(c*x^2 + a))*a^8*c^5*g^4*h^11*e + 8442*(sqrt(c)*x - sqrt(c*x^2 + a))*a^9*c^4*g^2*h^13*e + 105*(sqrt(c)*x - sqrt(c*x^2 + a))*a^10*c^3*h^15*e + 40*a^7*c^(13/2)*f*g^8*h^7 + 12*a^7*c^(13/2)*d*g^6*h^9 + 218*a^8*c^(11/2)*f*g^6*h^9 + 120*a^8*c^(11/2)*d*g^4*h^11 + 505*a^9*c^(9/2)*f*g^4*h^11 - 741*a^9*c^(9/2)*d*g^2*h^13 + 936*a^10*c^(7/2)*f*g^2*h^13 + 96*a^10*c^(7/2)*d*h^15 - 336*a^11*c^(5/2)*f*h^15 + 16*a^7*c^(13/2)*g^7*h^8*e + 104*a^8*c^(11/2)*g^5*h^10*e + 370*a

```
^9*c^(9/2)*g^3*h^12*e - 663*a^10*c^(7/2)*g*h^14*e)/((c^5*g^10*h^6 + 5*a*c^4
*g^8*h^8 + 10*a^2*c^3*g^6*h^10 + 10*a^3*c^2*g^4*h^12 + 5*a^4*c*g^2*h^14 + a
^5*h^16)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a
))*sqrt(c)*g - a*h)^7)
```

**maple [B]** time = 0.05, size = 19093, normalized size = 35.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
[Out] result too large to display
```

**maxima [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima")
[Out] Timed out
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
[Out] int(((a + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
[Out] Timed out
```

# 3.100 $\int \left(a+cx^2\right)^{5/2} \left(A+Bx+Cx^2\right) dx$

**Optimal**. Leaf size=168

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\,(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac-}{192c}$$

```
[Out] 5/192*a*(8*A*c-C*a)*x*(c*x^2+a)^(3/2)/c+1/48*(8*A*c-C*a)*x*(c*x^2+a)^(5/2)/
c+1/7*B*(c*x^2+a)^(7/2)/c+1/8*C*x*(c*x^2+a)^(7/2)/c+5/128*a^3*(8*A*c-C*a)*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+5/128*a^2*(8*A*c-C*a)*x*(c*x^2+a)
^(1/2)/c
```

**Rubi [A]** time = 0.10, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1815, 641, 195, 217, 206}

$$\frac{5a^3(8Ac-aC)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{128c^{3/2}}+\frac{5a^2x\sqrt{a+cx^2}\,(8Ac-aC)}{128c}+\frac{x\left(a+cx^2\right)^{5/2}(8Ac-aC)}{48c}+\frac{5ax\left(a+cx^2\right)^{3/2}(8Ac-}{192c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (5*a^2*(8*A*c - a*C)*x*Sqrt[a + c*x^2])/(128*c) + (5*a*(8*A*c - a*C)*x*(a +
 c*x^2)^(3/2))/(192*c) + ((8*A*c - a*C)*x*(a + c*x^2)^(5/2))/(48*c) + (B*(a
 + c*x^2)^(7/2))/(7*c) + (C*x*(a + c*x^2)^(7/2))/(8*c) + (5*a^3*(8*A*c - a*
C)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(128*c^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+cx^2\right)^{5/2}\left(A+Bx+Cx^2\right)dx = \frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{\int (8Ac-aC+8Bcx)\left(a+cx^2\right)^{5/2}dx}{8c}$$

$$= \frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(8Ac-aC)\int\left(a+cx^2\right)^{5/2}dx}{8c}$$

$$= \frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+\frac{Cx\left(a+cx^2\right)^{7/2}}{8c}+\frac{(5a(8Ac-a}{}$$

$$= \frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a+cx^2\right)^{5/2}}{48c}+\frac{B\left(a+cx^2\right)^{7/2}}{7c}+$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}$$

$$= \frac{5a^2(8Ac-aC)x\sqrt{a+cx^2}}{128c}+\frac{5a(8Ac-aC)x\left(a+cx^2\right)^{3/2}}{192c}+\frac{(8Ac-aC)x\left(a\right.}{48c}$$

**Mathematica [A]** time = 0.31, size = 150, normalized size = 0.89

$$\frac{\sqrt{a+cx^2}\left(\sqrt{c}\left(3a^3(128B+35Cx)+2a^2cx(924A+x(576B+413Cx))+8ac^2x^3(182A+x(144B+119Cx))+16\right.\right.}{2688c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(5/2)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[a + c*x^2]*(Sqrt[c]*(3*a^3*(128*B + 35*C*x) + 16*c^3*x^5*(28*A + 3*x*
(8*B + 7*C*x)) + 8*a*c^2*x^3*(182*A + x*(144*B + 119*C*x)) + 2*a^2*c*x*(924
*A + x*(576*B + 413*C*x))) - (105*a^(5/2)*(-8*A*c + a*C)*ArcSinh[(Sqrt[c]*x
)/Sqrt[a]])/Sqrt[1 + (c*x^2)/a]))/(2688*c^(3/2))
```

**fricas [A]** time = 1.32, size = 333, normalized size = 1.98

$$\left[-\frac{105\left(Ca^4-8\,Aa^3c\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(336\,Cc^4x^7+384\,Bc^4x^6+1152\,Bac^3x^4+1152\right.}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] [-1/5376*(105*(C*a^4 - 8*A*a^3*c)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*
sqrt(c)*x - a) - 2*(336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152
*B*a^2*c^2*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2
*c^2 + 104*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))
/c^2, 1/2688*(105*(C*a^4 - 8*A*a^3*c)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) + (336*C*c^4*x^7 + 384*B*c^4*x^6 + 1152*B*a*c^3*x^4 + 1152*B*a^2*c^2
*x^2 + 56*(17*C*a*c^3 + 8*A*c^4)*x^5 + 384*B*a^3*c + 14*(59*C*a^2*c^2 + 104
*A*a*c^3)*x^3 + 21*(5*C*a^3*c + 88*A*a^2*c^2)*x)*sqrt(c*x^2 + a))/c^2]
```

**giac [A]** time = 0.21, size = 168, normalized size = 1.00

$$\frac{1}{2688}\left(\frac{384\,Ba^3}{c}+\left(2\left(576\,Ba^2+\left(4\left(144\,Bac+\left(6\left(7\,Cc^2x+8\,Bc^2\right)x+\frac{7\left(17\,Cac^7+8\,Ac^8\right)}{c^6}\right)x\right)x+\frac{7\left(59\,Ca^2c^6+\right.}{c^6}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/2688*(384*B*a^3/c + (2*(576*B*a^2 + (4*(144*B*a*c + (6*(7*C*c^2*x + 8*B*c
^2)*x + 7*(17*C*a*c^7 + 8*A*c^8)/c^6)*x)*x + 7*(59*C*a^2*c^6 + 104*A*a*c^7)
```

```
/c^6)*x)*x + 21*(5*C*a^3*c^5 + 88*A*a^2*c^6)/c^6)*x)*sqrt(c*x^2 + a) + 5/12
8*(C*a^4 - 8*A*a^3*c)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.01, size = 181, normalized size = 1.08

$$\frac{5A\,a^3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{16\sqrt{c}}-\frac{5C\,a^4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{128c^{\frac{3}{2}}}+\frac{5\sqrt{c\,x^2+a}\;A\,a^2x}{16}-\frac{5\sqrt{c\,x^2+a}\;C\,a^3x}{128c}+\frac{5\left(c\,x^2+a\right)^{\frac{3}{2}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x)
```

```
[Out] 1/8*C*x*(c*x^2+a)^(7/2)/c-1/48*C*a/c*x*(c*x^2+a)^(5/2)-5/192*C*a^2/c*x*(c*x
^2+a)^(3/2)-5/128*C*a^3/c*x*(c*x^2+a)^(1/2)-5/128*C*a^4/c^(3/2)*ln(c^(1/2)*
x+(c*x^2+a)^(1/2))+1/7*B*(c*x^2+a)^(7/2)/c+1/6*A*x*(c*x^2+a)^(5/2)+5/24*A*a
*x*(c*x^2+a)^(3/2)+5/16*A*a^2*x*(c*x^2+a)^(1/2)+5/16*A*a^3/c^(1/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))
```

**maxima [A]** time = 0.45, size = 166, normalized size = 0.99

$$\frac{1}{6}\left(cx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(cx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{cx^2+a}\,Aa^2x+\frac{\left(cx^2+a\right)^{\frac{7}{2}}Cx}{8c}-\frac{\left(cx^2+a\right)^{\frac{5}{2}}Cax}{48c}-\frac{5\left(cx^2+a\right)^{\frac{3}{2}}Ca^2x}{192c}-\frac{5\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(5/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2 + a)^(5/2)*A*x + 5/24*(c*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(c*x^2
+ a)*A*a^2*x + 1/8*(c*x^2 + a)^(7/2)*C*x/c - 1/48*(c*x^2 + a)^(5/2)*C*a*x/c
 - 5/192*(c*x^2 + a)^(3/2)*C*a^2*x/c - 5/128*sqrt(c*x^2 + a)*C*a^3*x/c - 5/
128*C*a^4*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 5/16*A*a^3*arcsinh(c*x/sqrt(a*c)
)/sqrt(c) + 1/7*(c*x^2 + a)^(7/2)*B/c
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2+a\right)^{5/2}\,\left(C\,x^2+B\,x+A\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2),x)
```

```
[Out] int((a + c*x^2)^(5/2)*(A + B*x + C*x^2), x)
```

**sympy [A]** time = 32.92, size = 510, normalized size = 3.04

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{cx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}cx^3}{48\sqrt{1+\frac{cx^2}{a}}}+\frac{17A\sqrt{a}\,c^2x^5}{24\sqrt{1+\frac{cx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{16\sqrt{c}}+\frac{Ac^3x^7}{6\sqrt{a}\sqrt{1+\frac{cx^2}{a}}}+Ba^2\left(\left\{\begin{array}{l}\frac{\sqrt{a}\,x^2}{2}\\ \frac{\left(a+cx^2\right)^{\frac{3}{2}}}{3c}\end{array}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(5/2)*(C*x**2+B*x+A),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + c*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + c*x**2/a))
+ 35*A*a**(3/2)*c*x**3/(48*sqrt(1 + c*x**2/a)) + 17*A*sqrt(a)*c**2*x**5/(24
*sqrt(1 + c*x**2/a)) + 5*A*a**3*asinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c)) + A*c
**3*x**7/(6*sqrt(a)*sqrt(1 + c*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(c, 0)), ((a + c*x**2)**(3/2)/(3*c), True)) + 2*B*a*c*Piecewise((-2*a**2
*sqrt(a + c*x**2)/(15*c**2) + a*x**2*sqrt(a + c*x**2)/(15*c) + x**4*sqrt(a
+ c*x**2)/5, Ne(c, 0)), (sqrt(a)*x**4/4, True)) + B*c**2*Piecewise((8*a**3*
sqrt(a + c*x**2)/(105*c**3) - 4*a**2*x**2*sqrt(a + c*x**2)/(105*c**2) + a*x
**4*sqrt(a + c*x**2)/(35*c) + x**6*sqrt(a + c*x**2)/7, Ne(c, 0)), (sqrt(a)*
x**6/6, True)) + 5*C*a**(7/2)*x/(128*c*sqrt(1 + c*x**2/a)) + 133*C*a**(5/2)
*x**3/(384*sqrt(1 + c*x**2/a)) + 127*C*a**(3/2)*c*x**5/(192*sqrt(1 + c*x**2
/a)) + 23*C*sqrt(a)*c**2*x**7/(48*sqrt(1 + c*x**2/a)) - 5*C*a**4*asinh(sqrt
(c)*x/sqrt(a))/(128*c**(3/2)) + C*c**3*x**9/(8*sqrt(a)*sqrt(1 + c*x**2/a))
```

# 3.101 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=325

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)}{8c^{5/2}}+\frac{\sqrt{a+cx^2}\,\left(4\left(16a^2fh^4-4ach^2\left(5h(dh\right.\right.\right.}{}$$

```
[Out] 1/8*(8*c^2*d*g^3+3*a^2*h^2*(e*h+3*f*g)-4*a*c*g*(f*g^2+3*h*(d*h+e*g)))*arcta
nh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)+1/60*(4*(-4*a*f+5*c*d)*h^2-3*c*g*(-5*
e*h+f*g))*(h*x+g)^2*(c*x^2+a)^(1/2)/c^2/h-1/20*(-5*e*h+f*g)*(h*x+g)^3*(c*x^
2+a)^(1/2)/c/h+1/5*f*(h*x+g)^4*(c*x^2+a)^(1/2)/c/h+1/120*(64*a^2*f*h^4-16*a
*c*h^2*(13*f*g^2+5*h*(d*h+3*e*g))-4*c^2*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-c*
h*(a*h^2*(45*e*h+71*f*g)+2*c*g*(3*f*g^2-5*h*(10*d*h+3*e*g)))*x)*(c*x^2+a)^(
1/2)/c^3/h
```

**Rubi [A]** time = 0.66, antiderivative size = 323, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\sqrt{a+cx^2}\,\left(4\left(16a^2fh^4-4ach^2\left(5h(dh+3eg)+13fg^2\right)-c^2g^2\left(3fg^2-5h(16dh+3eg)\right)\right)-chx\left(ah^2(45eh+71fg)\right.\right.}{120c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] ((4*(5*c*d - 4*a*f)*h^2 - 3*c*g*(f*g - 5*e*h))*(g + h*x)^2*Sqrt[a + c*x^2])
/(60*c^2*h) - ((f*g - 5*e*h)*(g + h*x)^3*Sqrt[a + c*x^2])/(20*c*h) + (f*(g
+ h*x)^4*Sqrt[a + c*x^2])/(5*c*h) + ((4*(16*a^2*f*h^4 - 4*a*c*h^2*(13*f*g^2
 + 5*h*(3*e*g + d*h)) - c^2*g^2*(3*f*g^2 - 5*h*(3*e*g + 16*d*h))) - c*h*(6*
c*f*g^3 - 10*c*g*h*(3*e*g + 10*d*h) + a*h^2*(71*f*g + 45*e*h))*x)*Sqrt[a +
c*x^2])/(120*c^3*h) + ((8*c^2*d*g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*
g^2 + 3*h*(e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3 \left(d+ex+fx^2\right)}{\sqrt{a+cx^2}} dx = \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left((5cd-4af)h^2-ch(fg-5eh)x\right)}{\sqrt{a+cx^2}} dx}{5ch^2}$$

$$= -\frac{(fg-5eh)(g+hx)^3\sqrt{a+cx^2}}{20ch} + \frac{f(g+hx)^4\sqrt{a+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^2\left(ch^2(20cdg-13af\right.}{}}{}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

$$= \frac{\left(4(5cd-4af)h^2-3cg(fg-5eh)\right)(g+hx)^2\sqrt{a+cx^2}}{60c^2h} - \frac{(fg-5eh)(g+hx)^3\sqrt{a}}{20ch}$$

**Mathematica [A]** time = 0.35, size = 252, normalized size = 0.78

$$15\sqrt{c}\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)\left(3a^2h^2(eh+3fg)-4acg\left(3h(dh+eg)+fg^2\right)+8c^2dg^3\right)+\sqrt{a+cx^2}\left(8\left(8a^2fh^3-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(8*(8*a^2*f*h^3 + 15*c^2*g^2*(e*g + 3*d*h) - 10*a*c*h*(3*f
*g^2 + h*(3*e*g + d*h))) + 15*c*(-3*a*h^2*(3*f*g + e*h) + 4*c*(f*g^3 + 3*g*
h*(e*g + d*h)))*x + 8*c*h*(-4*a*f*h^2 + 5*c*(3*f*g^2 + h*(3*e*g + d*h)))*x^
2 + 30*c^2*h^2*(3*f*g + e*h)*x^3 + 24*c^2*f*h^3*x^4) + 15*Sqrt[c]*(8*c^2*d*
g^3 + 3*a^2*h^2*(3*f*g + e*h) - 4*a*c*g*(f*g^2 + 3*h*(e*g + d*h)))*Log[c*x
+ Sqrt[c]*Sqrt[a + c*x^2]])/(120*c^3)
```

**fricas [A]** time = 1.22, size = 559, normalized size = 1.72

$$\left[-\frac{15\left(12\,aceg^2h-3\,a^2eh^3-4\left(2\,c^2d-acf\right)g^3+3\left(4\,acd-3\,a^2f\right)gh^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/240*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4*(2*c^2*d - a*c*f)*g^3 + 3*(4*
a*c*d - 3*a^2*f)*g*h^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x
- a) - 2*(24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^2 + 120*(3*c^2*d
 - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*f*g*h^2 + c^2*e*
h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d - 4*a*c*f)*h^3)*x^
2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c^2*d - 3*a*c*f)*
g*h^2)*x)*sqrt(c*x^2 + a))/c^3, 1/120*(15*(12*a*c*e*g^2*h - 3*a^2*e*h^3 - 4
*(2*c^2*d - a*c*f)*g^3 + 3*(4*a*c*d - 3*a^2*f)*g*h^2)*sqrt(-c)*arctan(sqrt(
-c)*x/sqrt(c*x^2 + a)) + (24*c^2*f*h^3*x^4 + 120*c^2*e*g^3 - 240*a*c*e*g*h^
2 + 120*(3*c^2*d - 2*a*c*f)*g^2*h - 16*(5*a*c*d - 4*a^2*f)*h^3 + 30*(3*c^2*
f*g*h^2 + c^2*e*h^3)*x^3 + 8*(15*c^2*f*g^2*h + 15*c^2*e*g*h^2 + (5*c^2*d -
4*a*c*f)*h^3)*x^2 + 15*(4*c^2*f*g^3 + 12*c^2*e*g^2*h - 3*a*c*e*h^3 + 3*(4*c
^2*d - 3*a*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac [A]** time = 0.26, size = 314, normalized size = 0.97

$$\frac{1}{120}\sqrt{cx^2+a}\left(\left(2\left(3\left(\frac{4fh^3x}{c}+\frac{5\left(3c^4fgh^2+c^4h^3e\right)}{c^5}\right)x+\frac{4\left(15c^4fg^2h+5c^4dh^3-4ac^3fh^3+15c^4gh^2e\right)}{c^5}\right)x+\frac{15}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(c*x^2 + a)*((2*(3*(4*f*h^3*x/c + 5*(3*c^4*f*g*h^2 + c^4*h^3*e)/c
^5)*x + 4*(15*c^4*f*g^2*h + 5*c^4*d*h^3 - 4*a*c^3*f*h^3 + 15*c^4*g*h^2*e)/c
^5)*x + 15*(4*c^4*f*g^3 + 12*c^4*d*g*h^2 - 9*a*c^3*f*g*h^2 + 12*c^4*g^2*h*e
 - 3*a*c^3*h^3*e)/c^5)*x + 8*(45*c^4*d*g^2*h - 30*a*c^3*f*g^2*h - 10*a*c^3*
d*h^3 + 8*a^2*c^2*f*h^3 + 15*c^4*g^3*e - 30*a*c^3*g*h^2*e)/c^5) - 1/8*(8*c^
2*d*g^3 - 4*a*c*f*g^3 - 12*a*c*d*g*h^2 + 9*a^2*f*g*h^2 - 12*a*c*g^2*h*e + 3
*a^2*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple [A]** time = 0.02, size = 528, normalized size = 1.62

$$\frac{\sqrt{cx^2+a}\;fh^3x^4}{5c}+\frac{\sqrt{cx^2+a}\;eh^3x^3}{4c}+\frac{3\sqrt{cx^2+a}\;fgh^2x^3}{4c}-\frac{4\sqrt{cx^2+a}\;afh^3x^2}{15c^2}+\frac{\sqrt{cx^2+a}\;dh^3x^2}{3c}+\frac{\sqrt{cx^2+a}\;eg}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/5*h^3*f*x^4/c*(c*x^2+a)^(1/2)-4/15*h^3*f*a/c^2*x^2*(c*x^2+a)^(1/2)+8/15*h
^3*f*a^2/c^3*(c*x^2+a)^(1/2)+1/4*x^3/c*(c*x^2+a)^(1/2)*h^3*e+3/4*x^3/c*(c*x
^2+a)^(1/2)*g*h^2*f-3/8*a/c^2*x*(c*x^2+a)^(1/2)*h^3*e-9/8*a/c^2*x*(c*x^2+a)
```

```
^(1/2)*g*h^2*f+3/8*a^2/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*h^3*e+9/8*a^2/
c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*f+1/3*x^2/c*(c*x^2+a)^(1/2)*h^3
*d+x^2/c*(c*x^2+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+a)^(1/2)*g^2*h*f-2/3*a/c^2*(c
*x^2+a)^(1/2)*h^3*d-2*a/c^2*(c*x^2+a)^(1/2)*g*h^2*e-2*a/c^2*(c*x^2+a)^(1/2)
*g^2*h*f+3/2*x/c*(c*x^2+a)^(1/2)*g*h^2*d+3/2*x/c*(c*x^2+a)^(1/2)*g^2*h*e+1/
2*x/c*(c*x^2+a)^(1/2)*g^3*f-3/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h
^2*d-3/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g^2*h*e-1/2*a/c^(3/2)*ln(c
^(1/2)*x+(c*x^2+a)^(1/2))*g^3*f+3/c*(c*x^2+a)^(1/2)*g^2*h*d+1/c*(c*x^2+a)^(
1/2)*g^3*e+g^3*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

**maxima [A]** time = 0.45, size = 349, normalized size = 1.07

$$\frac{\sqrt{cx^2+a}\,fh^3x^4}{5\,c}-\frac{4\,\sqrt{cx^2+a}\,afh^3x^2}{15\,c^2}+\frac{dg^3\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,eg^3}{c}+\frac{3\,\sqrt{cx^2+a}\,dg^2h}{c}+\frac{8\,\sqrt{cx^2+a}\,a^2fh^3}{15\,c^3}+\frac{(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c*x^2 + a)*f*h^3*x^4/c - 4/15*sqrt(c*x^2 + a)*a*f*h^3*x^2/c^2 + d*
g^3*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c*x^2 + a)*e*g^3/c + 3*sqrt(c*x^2
 + a)*d*g^2*h/c + 8/15*sqrt(c*x^2 + a)*a^2*f*h^3/c^3 + 1/4*(3*f*g*h^2 + e*h
^3)*sqrt(c*x^2 + a)*x^3/c + 1/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*x^2/c - 3/8*(3*f*g*h^2 + e*h^3)*sqrt(c*x^2 + a)*a*x/c^2 + 1/2*(f*g^3 +
 3*e*g^2*h + 3*d*g*h^2)*sqrt(c*x^2 + a)*x/c + 3/8*(3*f*g*h^2 + e*h^3)*a^2*a
rcsinh(c*x/sqrt(a*c))/c^(5/2) - 1/2*(f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*a*arcsi
nh(c*x/sqrt(a*c))/c^(3/2) - 2/3*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*sqrt(c*x^2
+ a)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

**sympy [A]** time = 22.20, size = 796, normalized size = 2.45

$$-\frac{3a^{\frac{3}{2}}eh^3x}{8c^2\sqrt{1+\frac{cx^2}{a}}}-\frac{9a^{\frac{3}{2}}fgh^2x}{8c^2\sqrt{1+\frac{cx^2}{a}}}+\frac{3\sqrt{a}\,dgh^2x\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{3\sqrt{a}\,eg^2hx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{\sqrt{a}\,eh^3x^3}{8c\sqrt{1+\frac{cx^2}{a}}}+\frac{\sqrt{a}\,fg^3x\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{3\cdot}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*e*h**3*x/(8*c**2*sqrt(1 + c*x**2/a)) - 9*a**(3/2)*f*g*h**2*x/(8
*c**2*sqrt(1 + c*x**2/a)) + 3*sqrt(a)*d*g*h**2*x*sqrt(1 + c*x**2/a)/(2*c) +
 3*sqrt(a)*e*g**2*h*x*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*e*h**3*x**3/(8*c*s
qrt(1 + c*x**2/a)) + sqrt(a)*f*g**3*x*sqrt(1 + c*x**2/a)/(2*c) - 3*sqrt(a)*
f*g*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) + 3*a**2*e*h**3*asinh(sqrt(c)*x/sqrt
(a))/(8*c**(5/2)) + 9*a**2*f*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) -
 3*a*d*g*h**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - 3*a*e*g**2*h*asinh(sq
rt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g**3*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/
2)) + d*g**3*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c
 < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a
/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + 3*d*g**2*h*Piecewis
e((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + d*h**3*Piecew
ise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)
), (x**4/(4*sqrt(a)), True)) + e*g**3*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)
), (sqrt(a + c*x**2)/c, True)) + 3*e*g*h**2*Piecewise((-2*a*sqrt(a + c*x**2
)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), Tru
e)) + 3*f*g**2*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a +
c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**3*Piecewise((8*a
**2*sqrt(a + c*x**2)/(15*c**3) - 4*a*x**2*sqrt(a + c*x**2)/(15*c**2) + x**4
*sqrt(a + c*x**2)/(5*c), Ne(c, 0)), (x**6/(6*sqrt(a)), True)) + e*h**3*x**5
/(4*sqrt(a)*sqrt(1 + c*x**2/a)) + 3*f*g*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x**
2/a))
```

# 3.102 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+cx^2}} dx$

**Optimal**. Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\,\left(4\left(4ah^2(eh+2fg)+cg\left(fg^2-4h(3dh\right.\right.\right.}{24c}$$

```
[Out] 1/8*(8*c^2*d*g^2+3*a^2*f*h^2-4*a*c*(f*g^2+h*(d*h+2*e*g)))*arctanh(x*c^(1/2)
/(c*x^2+a)^(1/2))/c^(5/2)-1/12*(-4*e*h+f*g)*(h*x+g)^2*(c*x^2+a)^(1/2)/c/h+1
/4*f*(h*x+g)^3*(c*x^2+a)^(1/2)/c/h-1/24*(16*a*h^2*(e*h+2*f*g)+4*c*g*(f*g^2-
4*h*(3*d*h+e*g))-h*(3*(-3*a*f+4*c*d)*h^2-2*c*g*(-4*e*h+f*g))*x)*(c*x^2+a)^(
1/2)/c^2/h
```

**Rubi [A]** time = 0.37, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 833, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)}{8c^{5/2}}-\frac{\sqrt{a+cx^2}\,\left(4\left(4ah^2(eh+2fg)-4cgh(3dh+eg)+\right.\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] -((f*g - 4*e*h)*(g + h*x)^2*Sqrt[a + c*x^2])/(12*c*h) + (f*(g + h*x)^3*Sqrt
[a + c*x^2])/(4*c*h) - ((4*(c*f*g^3 - 4*c*g*h*(e*g + 3*d*h) + 4*a*h^2*(2*f*
g + e*h)) - h*(3*(4*c*d - 3*a*f)*h^2 - 2*c*g*(f*g - 4*e*h))*x)*Sqrt[a + c*x
^2])/(24*c^2*h) + ((8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d
*h)))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left((4cd-3af)h^2-ch(fg-4eh)x\right)}{\sqrt{a+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} + \frac{\int \frac{(g+hx)\left(ch^2(12cdg-7afg-\right.}{}}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+\right.\right.}{}$$

$$= -\frac{(fg-4eh)(g+hx)^2\sqrt{a+cx^2}}{12ch} + \frac{f(g+hx)^3\sqrt{a+cx^2}}{4ch} - \frac{\left(4\left(cfg^3-4cgh(eg+\right.\right.}{}$$

**Mathematica [A]** time = 0.23, size = 164, normalized size = 0.74

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(3a^2fh^2-4ac\left(h(dh+2eg)+fg^2\right)+8c^2dg^2\right)+\sqrt{c}\,\sqrt{a+cx^2}\,\left(2c\left(6dh(4g+hx)+4e\left(3g^2+3g\right.\right.\right.}{24c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + c*x^2]*(-(a*h*(32*f*g + 16*e*h + 9*f*h*x)) + 2*c*(6*d*h*(
4*g + h*x) + 4*e*(3*g^2 + 3*g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2
*x^2))) + 3*(8*c^2*d*g^2 + 3*a^2*f*h^2 - 4*a*c*(f*g^2 + h*(2*e*g + d*h)))*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(24*c^(5/2))
```

**fricas [A]** time = 0.68, size = 381, normalized size = 1.71

$$\left[-\frac{3\left(8\,acegh-4\left(2\,c^2d-acf\right)g^2+\left(4\,acd-3\,a^2f\right)h^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(6\,c^2fh^2x^3+24\,c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2 + (4*a*c*d - 3*a^2*f)*h^2)
*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(6*c^2*f*h^2*x
```

```
^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h + 8*(2*c^2*f*
g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d - 3*a*c*f)*h
^2)*x)*sqrt(c*x^2 + a))/c^3, 1/24*(3*(8*a*c*e*g*h - 4*(2*c^2*d - a*c*f)*g^2
 + (4*a*c*d - 3*a^2*f)*h^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (
6*c^2*f*h^2*x^3 + 24*c^2*e*g^2 - 16*a*c*e*h^2 + 16*(3*c^2*d - 2*a*c*f)*g*h
+ 8*(2*c^2*f*g*h + c^2*e*h^2)*x^2 + 3*(4*c^2*f*g^2 + 8*c^2*e*g*h + (4*c^2*d
 - 3*a*c*f)*h^2)*x)*sqrt(c*x^2 + a))/c^3]
```

**giac** [A] time = 0.23, size = 206, normalized size = 0.92

$$\frac{1}{24}\sqrt{cx^2+a}\left(\left(2\left(\frac{3\,fh^2x}{c}+\frac{4\left(2\,c^3fgh+c^3h^2e\right)}{c^4}\right)x+\frac{3\left(4\,c^3fg^2+4\,c^3dh^2-3\,ac^2fh^2+8\,c^3ghe\right)}{c^4}\right)x+\frac{8\left(6\,c^3dgh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + a)*((2*(3*f*h^2*x/c + 4*(2*c^3*f*g*h + c^3*h^2*e)/c^4)*x
+ 3*(4*c^3*f*g^2 + 4*c^3*d*h^2 - 3*a*c^2*f*h^2 + 8*c^3*g*h*e)/c^4)*x + 8*(6
*c^3*d*g*h - 4*a*c^2*f*g*h + 3*c^3*g^2*e - 2*a*c^2*h^2*e)/c^4) - 1/8*(8*c^2
*d*g^2 - 4*a*c*f*g^2 - 4*a*c*d*h^2 + 3*a^2*f*h^2 - 8*a*c*g*h*e)*log(abs(-sq
rt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple** [A] time = 0.01, size = 339, normalized size = 1.52

$$\frac{\sqrt{c\,x^2+a}\;f\,h^2x^3}{4c}+\frac{\sqrt{c\,x^2+a}\;e\,h^2x^2}{3c}+\frac{2\sqrt{c\,x^2+a}\;fgh\,x^2}{3c}+\frac{3a^2f\,h^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{8c^{\frac{5}{2}}}-\frac{ad\,h^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x\right.}}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/4*h^2*f*x^3/c*(c*x^2+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+a)^(1/2)+3/8*h^2*f
*a^2/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+1/3*x^2/c*(c*x^2+a)^(1/2)*h^2*e+
2/3*x^2/c*(c*x^2+a)^(1/2)*g*h*f-2/3*a/c^2*(c*x^2+a)^(1/2)*h^2*e-4/3*a/c^2*(
c*x^2+a)^(1/2)*g*h*f+1/2*x/c*(c*x^2+a)^(1/2)*d*h^2+x/c*(c*x^2+a)^(1/2)*e*g*
h+1/2*x/c*(c*x^2+a)^(1/2)*f*g^2-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
*d*h^2-a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*g*h-1/2*a/c^(3/2)*ln(c^(1/
2)*x+(c*x^2+a)^(1/2))*f*g^2+2/c*(c*x^2+a)^(1/2)*g*h*d+1/c*(c*x^2+a)^(1/2)*g
^2*e+g^2*d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

**maxima** [A] time = 0.45, size = 230, normalized size = 1.03

$$\frac{\sqrt{cx^2+a}\,fh^2x^3}{4\,c}-\frac{3\,\sqrt{cx^2+a}\,afh^2x}{8\,c^2}+\frac{dg^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{3\,a^2fh^2\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{8\,c^{\frac{5}{2}}}+\frac{\sqrt{cx^2+a}\,eg^2}{c}+\frac{2\,\sqrt{cx^2+a}\,dgh}{c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c*x^2 + a)*f*h^2*x^3/c - 3/8*sqrt(c*x^2 + a)*a*f*h^2*x/c^2 + d*g^2
*arcsinh(c*x/sqrt(a*c))/sqrt(c) + 3/8*a^2*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5
/2) + sqrt(c*x^2 + a)*e*g^2/c + 2*sqrt(c*x^2 + a)*d*g*h/c + 1/3*(2*f*g*h +
e*h^2)*sqrt(c*x^2 + a)*x^2/c + 1/2*(f*g^2 + 2*e*g*h + d*h^2)*sqrt(c*x^2 + a
)*x/c - 1/2*(f*g^2 + 2*e*g*h + d*h^2)*a*arcsinh(c*x/sqrt(a*c))/c^(3/2) - 2/
3*(2*f*g*h + e*h^2)*sqrt(c*x^2 + a)*a/c^2
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(1/2), x)
```

**sympy [A]** time = 15.78, size = 518, normalized size = 2.32

$$-\frac{3a^{\frac{3}{2}} f h^2 x}{8c^2\sqrt{1+\frac{cx^2}{a}}} + \frac{\sqrt{a}\, d h^2 x\sqrt{1+\frac{cx^2}{a}}}{2c} + \frac{\sqrt{a}\, eghx\sqrt{1+\frac{cx^2}{a}}}{c} + \frac{\sqrt{a}\, f g^2 x\sqrt{1+\frac{cx^2}{a}}}{2c} - \frac{\sqrt{a}\, f h^2 x^3}{8c\sqrt{1+\frac{cx^2}{a}}} + \frac{3a^2 f h^2 \operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{8c^{\frac{5}{2}}} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*h**2*x/(8*c**2*sqrt(1 + c*x**2/a)) + sqrt(a)*d*h**2*x*sqrt(1
+ c*x**2/a)/(2*c) + sqrt(a)*e*g*h*x*sqrt(1 + c*x**2/a)/c + sqrt(a)*f*g**2*x
*sqrt(1 + c*x**2/a)/(2*c) - sqrt(a)*f*h**2*x**3/(8*c*sqrt(1 + c*x**2/a)) +
3*a**2*f*h**2*asinh(sqrt(c)*x/sqrt(a))/(8*c**(5/2)) - a*d*h**2*asinh(sqrt(c
)*x/sqrt(a))/(2*c**(3/2)) - a*e*g*h*asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - a*f
*g**2*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) + d*g**2*Piecewise((sqrt(-a/c)*
asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a
))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (
```

```
c > 0) & (a < 0))) + 2*d*g*h*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(
a + c*x**2)/c, True)) + e*g**2*Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqr
t(a + c*x**2)/c, True)) + e*h**2*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2)
+ x**2*sqrt(a + c*x**2)/(3*c), Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + 2*f*g
*h*Piecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c),
 Ne(c, 0)), (x**4/(4*sqrt(a)), True)) + f*h**2*x**5/(4*sqrt(a)*sqrt(1 + c*x
**2/a))
```

# 3.103 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=136

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}-\frac{\sqrt{a+cx^2}\,\left(2\left(2afh^2+c\left(fg^2-3h(dh+eg)\right)\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{f\sqrt{a+cx}}{3}$$

```
[Out] 1/2*(2*c*d*g-a*(e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+1/3*f*
(h*x+g)^2*(c*x^2+a)^(1/2)/c/h-1/6*(4*a*f*h^2+2*c*(f*g^2-3*h*(d*h+e*g))+c*h*
(-3*e*h+f*g)*x)*(c*x^2+a)^(1/2)/c^2/h
```

**Rubi [A]** time = 0.18, antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1654, 780, 217, 206}

$$-\frac{\sqrt{a+cx^2}\,\left(2\left(2afh^2-3ch(dh+eg)+cfg^2\right)+chx(fg-3eh)\right)}{6c^2h}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{2c^{3/2}}+\frac{f\sqrt{a+cx}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + c*x^2])/(3*c*h) - ((2*(c*f*g^2 + 2*a*f*h^2 - 3*c*h*
(e*g + d*h)) + c*h*(f*g - 3*e*h)*x)*Sqrt[a + c*x^2])/(6*c^2*h) + ((2*c*d*g
- a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
```

```
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left((3cd-2af)h^2-ch(fg-3eh)x\right)}{\sqrt{a+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a}}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a}}{6c^2h}$$

$$= \frac{f(g+hx)^2\sqrt{a+cx^2}}{3ch} - \frac{\left(2\left(cfg^2+2afh^2-3ch(eg+dh)\right)+ch(fg-3eh)x\right)\sqrt{a}}{6c^2h}$$

**Mathematica [A]** time = 0.10, size = 96, normalized size = 0.71

$$\frac{\sqrt{a+cx^2}\left(c\left(6dh+6eg+3ehx+3fgx+2fhx^2\right)-4afh\right)+3\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(2cdg-a(eh+fg))}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[a + c*x^2]*(-4*a*f*h + c*(6*e*g + 6*d*h + 3*f*g*x + 3*e*h*x + 2*f*h*x
^2)) + 3*Sqrt[c]*(2*c*d*g - a*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/(6*c^2)
```

**fricas [A]** time = 0.87, size = 199, normalized size = 1.46

$$\left[\frac{3\left(aeh-\left(2\,cd-af\right)g\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)+2\left(2\,cfhx^2+6\,ceg+2\left(3\,cd-2\,af\right)h+3\left(cfg+\right.\right.}{12\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)
*sqrt(c)*x - a) + 2*(2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d - 2*a*f)*h + 3*(c*f*g
 + c*e*h)*x)*sqrt(c*x^2 + a))/c^2, 1/6*(3*(a*e*h - (2*c*d - a*f)*g)*sqrt(-c
)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*f*h*x^2 + 6*c*e*g + 2*(3*c*d -
2*a*f)*h + 3*(c*f*g + c*e*h)*x)*sqrt(c*x^2 + a))/c^2]
```

**giac [A]** time = 0.21, size = 110, normalized size = 0.81

$$\frac{1}{6}\,\sqrt{cx^2+a}\left(\left(\frac{2\,fhx}{c}+\frac{3\left(c^2fg+c^2he\right)}{c^3}\right)x+\frac{2\left(3\,c^2dh-2\,acfh+3\,c^2ge\right)}{c^3}\right)-\frac{\left(2\,cdg-afg-ahe\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx\right.\right.}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2 + a)*((2*f*h*x/c + 3*(c^2*f*g + c^2*h*e)/c^3)*x + 2*(3*c^2*d
*h - 2*a*c*f*h + 3*c^2*g*e)/c^3) - 1/2*(2*c*d*g - a*f*g - a*h*e)*log(abs(-s
qrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.01, size = 172, normalized size = 1.26

$$\frac{\sqrt{c\,x^2+a}\;fh\,x^2}{3c}-\frac{aeh\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}-\frac{afg\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}+\frac{dg\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{\sqrt{c\,x^2+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+a)^(1/2)-2/3*h*f*a/c^2*(c*x^2+a)^(1/2)+1/2*x/c*(c*x^2+
a)^(1/2)*e*h+1/2*x/c*(c*x^2+a)^(1/2)*f*g-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+
a)^(1/2))*e*h-1/2*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g+1/c*(c*x^2+a)
^(1/2)*d*h+1/c*(c*x^2+a)^(1/2)*e*g+d*g*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2
)
```

**maxima** [A] time = 0.44, size = 126, normalized size = 0.93

$$\frac{\sqrt{cx^2+a}\,fhx^2}{3\,c}+\frac{dg\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}+\frac{\sqrt{cx^2+a}\,eg}{c}+\frac{\sqrt{cx^2+a}\,dh}{c}-\frac{2\,\sqrt{cx^2+a}\,afh}{3\,c^2}+\frac{\sqrt{cx^2+a}\left(fg+eh\right)x}{2\,c}-\frac{(fg+eh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2 + a)*f*h*x^2/c + d*g*arcsinh(c*x/sqrt(a*c))/sqrt(c) + sqrt(c
*x^2 + a)*e*g/c + sqrt(c*x^2 + a)*d*h/c - 2/3*sqrt(c*x^2 + a)*a*f*h/c^2 + 1
/2*sqrt(c*x^2 + a)*(f*g + e*h)*x/c - 1/2*(f*g + e*h)*a*arcsinh(c*x/sqrt(a*c
))/c^(3/2)
```

**mupad** [B] time = 5.17, size = 227, normalized size = 1.67

$$\begin{cases} \frac{2\,f\,g\,x^3+3\,e\,g\,x^2+6\,d\,g\,x}{6\,\sqrt{a}}+\frac{3\,f\,h\,x^4+4\,e\,h\,x^3+6\,d\,h\,x^2}{12\,\sqrt{a}} \\ \frac{d\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{d\,h\,\sqrt{c\,x^2+a}}{c}+\frac{e\,g\,\sqrt{c\,x^2+a}}{c}+\frac{e\,h\,x\,\sqrt{c\,x^2+a}}{2\,c}+\frac{f\,g\,x\,\sqrt{c\,x^2+a}}{2\,c}-\frac{f\,h\,\sqrt{c\,x^2+a}\,\left(2\,a-c\,x^2\right)}{3\,c^2}-\frac{a\,e\,h\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c}\right.}{2\,c^{3/2}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (3*e*g*x^2 + 2*f*g*x^3 + 6*d*g*x)/(6*a^(1/2)) + (6*d*h*x^
2 + 4*e*h*x^3 + 3*f*h*x^4)/(12*a^(1/2)), c ~= 0, (d*g*log(c^(1/2)*x + (a +
c*x^2)^(1/2)))/c^(1/2) + (d*h*(a + c*x^2)^(1/2))/c + (e*g*(a + c*x^2)^(1/2)
)/c + (e*h*x*(a + c*x^2)^(1/2))/(2*c) + (f*g*x*(a + c*x^2)^(1/2))/(2*c) - (
f*h*(a + c*x^2)^(1/2)*(2*a - c*x^2))/(3*c^2) - (a*e*h*log(2*c^(1/2)*x + 2*(
a + c*x^2)^(1/2)))/(2*c^(3/2)) - (a*f*g*log(2*c^(1/2)*x + 2*(a + c*x^2)^(1/
2)))/(2*c^(3/2)))
```

**sympy** [A] time = 9.02, size = 282, normalized size = 2.07

$$\frac{\sqrt{a}\,ehx\sqrt{1+\frac{cx^2}{a}}}{2c}+\frac{\sqrt{a}\,fgx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{aeh\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}-\frac{afg\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+dg\begin{cases}\frac{\sqrt{-\frac{a}{c}}\,\mathrm{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge \\ \frac{\sqrt{\frac{a}{c}}\,\mathrm{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge \\ \frac{\sqrt{-\frac{a}{c}}\,\mathrm{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*h*x*sqrt(1 + c*x**2/a)/(2*c) + sqrt(a)*f*g*x*sqrt(1 + c*x**2/a)/(
2*c) - a*e*h*asinh(sqrt(c)*x/sqrt(a))/(2*c**(3/2)) - a*f*g*asinh(sqrt(c)*x/
sqrt(a))/(2*c**(3/2)) + d*g*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a
), (a > 0) & (c < 0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c
> 0)), (sqrt(-a/c)*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + d*h*
Piecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + e*g*P
iecewise((x**2/(2*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True)) + f*h*Pi
ecewise((-2*a*sqrt(a + c*x**2)/(3*c**2) + x**2*sqrt(a + c*x**2)/(3*c), Ne(c
, 0)), (x**4/(4*sqrt(a)), True))
```

# 3.104 $\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=74

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

```
[Out] 1/2*(-a*f+2*c*d)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+e*(c*x^2+a)^(1/
2)/c+1/2*f*x*(c*x^2+a)^(1/2)/c
```

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1815, 641, 217, 206}

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}+\frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (e*Sqrt[a + c*x^2])/c + (f*x*Sqrt[a + c*x^2])/(2*c) + ((2*c*d - a*f)*ArcTan
h[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}}\,dx &= \frac{fx\sqrt{a+cx^2}}{2c}+\frac{\int \frac{2cd-af+2cex}{\sqrt{a+cx^2}}\,dx}{2c}\\ &= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2c}\\ &= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{2c}\\ &= \frac{e\sqrt{a+cx^2}}{c}+\frac{fx\sqrt{a+cx^2}}{2c}+\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 63, normalized size = 0.85

$$\frac{(2cd-af)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)+\sqrt{c}\,\sqrt{a+cx^2}\,(2e+fx)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + c*x^2],x]
```

```
[Out] (Sqrt[c]*(2*e + f*x)*Sqrt[a + c*x^2] + (2*c*d - a*f)*ArcTanh[(Sqrt[c]*x)/Sq
rt[a + c*x^2]])/(2*c^(3/2))
```

**fricas [A]** time = 0.60, size = 124, normalized size = 1.68

$$\left[-\frac{\left(2\,cd-af\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(cfx+2\,ce\right)\sqrt{cx^2+a}}{4\,c^2},-\frac{\left(2\,cd-af\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x}{\sqrt{cx^2+a}}\right.}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*c*d - a*f)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a
) - 2*(c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2, -1/2*((2*c*d - a*f)*sqrt(-c)*ar
ctan(sqrt(-c)*x/sqrt(c*x^2 + a)) - (c*f*x + 2*c*e)*sqrt(c*x^2 + a))/c^2]
```

**giac** [A] time = 0.20, size = 58, normalized size = 0.78

$$\frac{1}{2}\sqrt{cx^2+a}\left(\frac{fx}{c}+\frac{2e}{c}\right)-\frac{(2cd-af)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*(f*x/c + 2*e/c) - 1/2*(2*c*d - a*f)*log(abs(-sqrt(c)*x
+ sqrt(c*x^2 + a)))/c^(3/2)
```

**maple** [A] time = 0.01, size = 76, normalized size = 1.03

$$-\frac{af\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}+\frac{\sqrt{c\,x^2+a}\,fx}{2c}+\frac{\sqrt{c\,x^2+a}\,e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+a)^(1/2)/c-1/2*f*a/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+e*(
c*x^2+a)^(1/2)/c+d*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

**maxima** [A] time = 0.43, size = 61, normalized size = 0.82

$$\frac{\sqrt{cx^2+a}\,fx}{2c}+\frac{d\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}}-\frac{af\,\mathrm{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2c^{\frac{3}{2}}}+\frac{\sqrt{cx^2+a}\,e}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*f*x/c + d*arcsinh(c*x/sqrt(a*c))/sqrt(c) - 1/2*a*f*arcs
inh(c*x/sqrt(a*c))/c^(3/2) + sqrt(c*x^2 + a)*e/c
```

**mupad** [B] time = 4.56, size = 107, normalized size = 1.45

$$\begin{cases} \frac{2\,f\,x^3+3\,e\,x^2+6\,d\,x}{6\,\sqrt{a}} & \text{if } c=0 \\ \frac{e\,\sqrt{c\,x^2+a}}{c}+\frac{d\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{\sqrt{c}}-\frac{a\,f\,\ln\left(2\,\sqrt{c}\,x+2\,\sqrt{c\,x^2+a}\right)}{2\,c^{3/2}}+\frac{f\,x\,\sqrt{c\,x^2+a}}{2\,c} & \text{if } c\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(1/2),x)
```

```
[Out] piecewise(c == 0, (6*d*x + 3*e*x^2 + 2*f*x^3)/(6*a^(1/2)), c ~= 0, (e*(a +
c*x^2)^(1/2))/c + (d*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(1/2) - (a*f*log
(2*c^(1/2)*x + 2*(a + c*x^2)^(1/2)))/(2*c^(3/2)) + (f*x*(a + c*x^2)^(1/2))/
(2*c))
```

**sympy [A]** time = 3.50, size = 150, normalized size = 2.03

$$\frac{\sqrt{a}\,fx\sqrt{1+\frac{cx^2}{a}}}{2c}-\frac{af\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{2c^{\frac{3}{2}}}+d\left(\begin{cases}\frac{\sqrt{-\frac{a}{c}}\operatorname{asin}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c<0\\ \frac{\sqrt{\frac{a}{c}}\operatorname{asinh}\left(x\sqrt{\frac{c}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge c>0\\ \frac{\sqrt{-\frac{a}{c}}\operatorname{acosh}\left(x\sqrt{-\frac{c}{a}}\right)}{\sqrt{-a}} & \text{for } c>0\wedge a<0\end{cases}\right)+e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^2}}{c} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + c*x**2/a)/(2*c) - a*f*asinh(sqrt(c)*x/sqrt(a))/(2*c**(
3/2)) + d*Piecewise((sqrt(-a/c)*asin(x*sqrt(-c/a))/sqrt(a), (a > 0) & (c <
0)), (sqrt(a/c)*asinh(x*sqrt(c/a))/sqrt(a), (a > 0) & (c > 0)), (sqrt(-a/c)
*acosh(x*sqrt(-c/a))/sqrt(-a), (c > 0) & (a < 0))) + e*Piecewise((x**2/(2*s
qrt(a)), Eq(c, 0)), (sqrt(a + c*x**2)/c, True))
```

# 3.105 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=130

$$-\frac{(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

```
[Out] -(-e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g
^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*
g^2)^(1/2)+f*(c*x^2+a)^(1/2)/c/h
```

**Rubi** [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 217, 206, 725}

$$-\frac{(dh^2-egh+fg^2)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{h^2\sqrt{ah^2+cg^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(fg-eh)}{\sqrt{c}h^2}+\frac{f\sqrt{a+cx^2}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + c*x^2])/(c*h) - ((f*g - e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2
]])/(Sqrt[c]*h^2) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(Sqrt[c*
g^2 + a*h^2]*Sqrt[a + c*x^2])])/(h^2*Sqrt[c*g^2 + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+cx^2}}\,dx = \frac{f\sqrt{a+cx^2}}{ch} + \frac{\int \frac{cdh^2-ch(fg-eh)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{h^2}$$

$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(fg^2-egh+dh^2\right)\text{Subst}\left(\int \frac{}{cg}\right.}{h^2}$$

$$= \frac{f\sqrt{a+cx^2}}{ch} - \frac{(fg-eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{cg^2+ah^2}\,\sqrt{a+cx^2}}\right)}{h^2\sqrt{cg^2+ah^2}}$$

**Mathematica** [A] time = 0.22, size = 125, normalized size = 0.96

$$\frac{-\frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\sqrt{ah^2+cg^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(eh-fg)}{\sqrt{c}}+\frac{fh\sqrt{a+cx^2}}{c}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + c*x^2]),x]
```

```
[Out] ((f*h*Sqrt[a + c*x^2])/c + ((-(f*g) + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/Sqrt[c] - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g
^2 + a*h^2]*Sqrt[a + c*x^2])])/Sqrt[c*g^2 + a*h^2])/h^2
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [A] time = 0.22, size = 138, normalized size = 1.06

$$\frac{\sqrt{cx^2+a}\,f}{ch}+\frac{2\left(fg^2+dh^2-ghe\right)\arctan\left(-\frac{\left(\sqrt{c}\,x-\sqrt{cx^2+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^2-ah^2}}\right)}{\sqrt{-cg^2-ah^2}\,h^2}+\frac{\left(\sqrt{c}\,fg-\sqrt{c}\,he\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{ch^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + a)*f/(c*h) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c)*x - s
qrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/(sqrt(-c*g^2 - a*h^2)*
h^2) + (sqrt(c)*f*g - sqrt(c)*h*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/(
c*h^2)
```

**maple** [B] time = 0.01, size = 453, normalized size = 3.48

$$-\frac{d\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{a\,h^2+c\,g^2}{h^2}}}{x+\frac{g}{h}}\right)}{\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h}+\frac{eg\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+(}}{x+\frac{g}{h}}\right)}{\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x)
```

```
[Out] f*(c*x^2+a)^(1/2)/c/h+1/h*e*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/h^2*f*g
*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/h/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2
*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*
c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2+c*g^2)
/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^
(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e*g-
1/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*
((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2))/(x+g/h))*f*g^2
```

**maxima [A]** time = 0.56, size = 218, normalized size = 1.68

$$-\frac{fg\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h^2}+\frac{e\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}h}+\frac{fg^2\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^3}-\frac{eg\operatorname{arsinh}\left(\frac{cgx}{\sqrt{ac}|hx+g|}-\frac{ah}{\sqrt{ac}|hx+g|}\right)}{\sqrt{a+\frac{cg^2}{h^2}}h^2}+\frac{d\operatorname{arsi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -f*g*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^2) + e*arcsinh(c*x/sqrt(a*c))/(sqrt(
c)*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h
*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) - e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x
+ g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^2) + d*arcsinh
(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c
*g^2/h^2)*h) + sqrt(c*x^2 + a)*f/(c*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\,\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)), x)
```

## 3.106 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=168

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

```
[Out] (a*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^
(1/2)/(c*x^2+a)^(1/2))/h^2/(a*h^2+c*g^2)^(3/2)+f*arctanh(x*c^(1/2)/(c*x^2+a
)^(1/2))/h^2/c^(1/2)-(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h
*x+g)
```

**Rubi [A]** time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 217, 206, 725}

$$-\frac{\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{h(g+hx)\left(ah^2+cg^2\right)}+\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{h^2\left(ah^2+cg^2\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}h^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] -(((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(h*(c*g^2 + a*h^2)*(g + h*x)))
+ (f*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*h^2) + ((a*h^2*(2*f*g -
 e*h) + c*(f*g^3 - d*g*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqr
t[a + c*x^2]])])/(h^2*(c*g^2 + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} - \frac{\int \frac{-cdg+afg-aeh-f\left(\frac{cg^2}{h}+ah\right)x}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{h^2} + \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{h^2} - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\ldots}{\ldots}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{h\left(cg^2+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(cdg-2afg-\frac{cfg^3}{h^2}+aeh\right)\text{ta}\ldots}{\left(cg^2+ah^2\ldots\right.}$$

**Mathematica [A]** time = 0.36, size = 218, normalized size = 1.30

$$\frac{-\frac{h\sqrt{a+cx^2}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)}+\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(ah^2(2fg-eh)+c\left(fg^3-dgh^2\right)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{\log(g+hx)\left(ah^2(eh-2fg)+c\left(dgh^2-fg^3\right)\right)}{\left(ah^2+cg^2\right)^{3/2}}+\frac{f\log\left(\cdots\right.}{\cdots}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)*(g + h*x
))) + ((a*h^2*(-2*f*g + e*h) + c*(-(f*g^3) + d*g*h^2))*Log[g + h*x])/(c*g^2
 + a*h^2)^(3/2) + (f*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c] + ((a*h^2*
(2*f*g - e*h) + c*(f*g^3 - d*g*h^2))*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(3/2))/h^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 923, normalized size = 5.49

$$-\frac{cdg\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\,\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2c+\frac{a\,h^2+c\,g^2}{h^2}}}{x+\frac{g}{h}}\right)}{\left(a\,h^2+c\,g^2\right)\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h}+\frac{ce\,g^2\ln\left(\frac{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\frac{2a\,h^2+2c\,g^2}{h^2}+2\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\,\sqrt{-\frac{2\left(x+\frac{g}{h}\right)c\cdots}{h}\cdots}}{x+\frac{g}{h}}\right.}{\left(a\,h^2+c\,g^2\right)\sqrt{\frac{a\,h^2+c\,g^2}{h^2}}\;h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x)
```

```
[Out] f/h^2*ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)-1/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+
g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)
*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*
g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2-1
/h*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^
2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h
^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2*c*g^2/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h
^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1
/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-1/h^
3*c*g^3/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h
^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*
h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-1/h^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x
+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2+c*g^2)/h^
2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/
2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g
```

**maxima [B]** time = 0.58, size = 419, normalized size = 2.49

$$-\frac{\sqrt{cx^2+a}\,fg^2}{cg^2h^2x+ah^4x+cg^3h+agh^3}+\frac{\sqrt{cx^2+a}\,eg}{cg^2hx+ah^3x+cg^3+agh^2}-\frac{\sqrt{cx^2+a}\,d}{cg^2x+ah^2x+\frac{cg^3}{h}+agh}+\frac{f\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{c}\,h^2}+\frac{cfg^3\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^2*x + a*h^4*x + c*g^3*h + a*g*h^3) + sqrt(c
*x^2 + a)*e*g/(c*g^2*h*x + a*h^3*x + c*g^3 + a*g*h^2) - sqrt(c*x^2 + a)*d/(
c*g^2*x + a*h^2*x + c*g^3/h + a*g*h) + f*arcsinh(c*x/sqrt(a*c))/(sqrt(c)*h^
2) + c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*
x + g)))/((a + c*g^2/h^2)^(3/2)*h^5) - c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) + c*
d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/
((a + c*g^2/h^2)^(3/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g))
- a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3) + e*arcsinh(c*g*x
/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h
^2)*h^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+cx^2}\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**2), x)
```

# 3.107 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx$

**Optimal**. Leaf size=225

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\,\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\cdots}{\cdots}$$

```
[Out] -1/2*(2*c^2*d*g^2+2*a^2*f*h^2-a*c*(f*g^2-h*(-d*h+3*e*g)))*arctanh((-c*g*x+a
*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)-1/2*(d*h^2-e*g
*h+f*g^2)*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)/(h*x+g)^2+1/2*(2*a*h^2*(-e*h+2*f*
g)+c*g*(f*g^2+h*(-3*d*h+e*g)))*(c*x^2+a)^(1/2)/h/(a*h^2+c*g^2)^2/(h*x+g)
```

**Rubi** [A] time = 0.29, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\,\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^2-ac\left(fg^2-h(3eg-dh)\right)+2c^2dg^2\right)}{2\left(ah^2+cg^2\right)^{5/2}}-\frac{\sqrt{a+cx^2}\,\left(dh^2-egh+fg^2\right)}{2h(g+hx)^2\left(ah^2+cg^2\right)}+\frac{\sqrt{a+cx^2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/(2*h*(c*g^2 + a*h^2)*(g + h*x)^2
) + ((c*f*g^3 + c*g*h*(e*g - 3*d*h) + 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2
])/(2*h*(c*g^2 + a*h^2)^2*(g + h*x)) - ((2*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f
*g^2 - h*(3*e*g - d*h)))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a
+ c*x^2]])])/(2*(c*g^2 + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+cx^2}}\,dx = -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}-\frac{\int \frac{-2(cdg-afg+aeh)-\left(2afh+c\left(eg+\frac{fg^2}{h}-dh\right)\right)x}{(g+hx)^2\sqrt{a+cx^2}}\,dx}{2\left(cg^2+ah^2\right)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}+\frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}+\frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)(g+hx)^2}+\frac{\left(cfg^3+cgh(eg-3dh)+2ah^2(2fg-eh)\right)\sqrt{a+cx^2}}{2h\left(cg^2+ah^2\right)^2(g+hx)}$$

**Mathematica [A]** time = 0.45, size = 254, normalized size = 1.13

$$(g+hx)^2\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(-2a^2fh^2+ac\left(h(dh-3eg)+fg^2\right)-2c^2dg^2\right)+(g+hx)^2\log(g$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + c*x^2]),x]
```

```
[Out] (Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]*(c*g*(f*g^2*x + e*g*(2*g + h*x) - d*h*
(4*g + 3*h*x)) - a*h*(-(f*g*(3*g + 4*h*x)) + h*(d*h + e*(g + 2*h*x)))) + (2
*c^2*d*g^2 + 2*a^2*f*h^2 - a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g + h*x)^2*Log[
g + h*x] + (-2*c^2*d*g^2 - 2*a^2*f*h^2 + a*c*(f*g^2 + h*(-3*e*g + d*h)))*(g
 + h*x)^2*Log[a*h - c*g*x + Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(2*(c*g^2
 + a*h^2)^(5/2)*(g + h*x)^2)
```

**fricas [B]** time = 22.09, size = 1088, normalized size = 4.84

$$\left[\frac{\left(3\,aceg^3h + \left(2\,c^2d - acf\right)g^4 - \left(acd - 2\,a^2f\right)g^2h^2 + \left(3\,acegh^3 + \left(2\,c^2d - acf\right)g^2h^2 - \left(acd - 2\,a^2f\right)h^4\right)x^2 + 2\,\left(3\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2*f)*g^2*h^2 +
(3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f)*h^4)*x^2 + 2
*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*f)*g*h^3)*x)*s
qrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*
c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2
+ 2*g*h*x + g^2)) + 2*(2*c^2*e*g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^
5 - (4*c^2*d - 3*a*c*f)*g^4*h - (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 +
c^2*e*g^4*h - a*c*e*g^2*h^3 - 2*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (
3*a*c*d - 4*a^2*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 +
3*a^2*c*g^4*h^4 + a^3*g^2*h^6 + (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^
2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a
^3*g*h^7)*x), -1/2*((3*a*c*e*g^3*h + (2*c^2*d - a*c*f)*g^4 - (a*c*d - 2*a^2
*f)*g^2*h^2 + (3*a*c*e*g*h^3 + (2*c^2*d - a*c*f)*g^2*h^2 - (a*c*d - 2*a^2*f
)*h^4)*x^2 + 2*(3*a*c*e*g^2*h^2 + (2*c^2*d - a*c*f)*g^3*h - (a*c*d - 2*a^2*
f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a*h^2)*(c*g*x - a*h)
*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2)*x^2)) - (2*c^2*e*
g^5 + a*c*e*g^3*h^2 - a^2*e*g*h^4 - a^2*d*h^5 - (4*c^2*d - 3*a*c*f)*g^4*h -
 (5*a*c*d - 3*a^2*f)*g^2*h^3 + (c^2*f*g^5 + c^2*e*g^4*h - a*c*e*g^2*h^3 - 2
*a^2*e*h^5 - (3*c^2*d - 5*a*c*f)*g^3*h^2 - (3*a*c*d - 4*a^2*f)*g*h^4)*x)*sq
rt(c*x^2 + a))/(c^3*g^8 + 3*a*c^2*g^6*h^2 + 3*a^2*c*g^4*h^4 + a^3*g^2*h^6 +
 (c^3*g^6*h^2 + 3*a*c^2*g^4*h^4 + 3*a^2*c*g^2*h^6 + a^3*h^8)*x^2 + 2*(c^3*g
^7*h + 3*a*c^2*g^5*h^3 + 3*a^2*c*g^3*h^5 + a^3*g*h^7)*x)]
```

**giac [B]** time = 0.26, size = 848, normalized size = 3.77

$$-\frac{\left(2\,c^{2}dg^{2}-acfg^{2}-acdh^{2}+2\,a^{2}fh^{2}+3\,acghe\right)\arctan\left(\frac{\left(\sqrt{c}\,x-\sqrt{cx^{2}+a}\right)h+\sqrt{c}\,g}{\sqrt{-cg^{2}-ah^{2}}}\right)}{\left(c^{2}g^{4}+2\,acg^{2}h^{2}+a^{2}h^{4}\right)\sqrt{-cg^{2}-ah^{2}}}+\frac{2\left(\sqrt{c}\,x-\sqrt{cx^{2}+a}\right)^{3}c^{2}fg^{4}h-2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*c^2*d*g^2 - a*c*f*g^2 - a*c*d*h^2 + 2*a^2*f*h^2 + 3*a*c*g*h*e)*arctan((
(sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*g^2 - a*h^2))/((c^2*g^
4 + 2*a*c*g^2*h^2 + a^2*h^4)*sqrt(-c*g^2 - a*h^2)) + (2*(sqrt(c)*x - sqrt(c
*x^2 + a))^3*c^2*f*g^4*h - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3
+ 5*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
 + a))^3*a*c*d*h^5 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*g*h^4*e + 2*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + a))
^2*c^(5/2)*d*g^3*h^2 + 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*f*g^3*h^
2 + 3*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d*g*h^4 - 4*(sqrt(c)*x - sq
rt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 + 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^
(5/2)*g^4*h*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*g^2*h^3*e + 2*(
sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e - 2*(sqrt(c)*x - sqrt(c*x^
2 + a))*a*c^2*f*g^4*h + 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*d*g^2*h^3 -
11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^3 + (sqrt(c)*x - sqrt(c*x^2
+ a))*a^2*c*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*g^3*h^2*e + 5*(sq
rt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + a^2*c^(3/2)*f*g^3*h^2 - 3*a^2*c^
(3/2)*d*g*h^4 + 4*a^3*sqrt(c)*f*g*h^4 + a^2*c^(3/2)*g^2*h^3*e - 2*a^3*sqrt(
c)*h^5*e)/((c^2*g^4*h^2 + 2*a*c*g^2*h^4 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2
 + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

**maple [B]** time = 0.02, size = 1574, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/h/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2)*e+2/h^2/(a*h^2+c*g^2)/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c
*g^2)/h^2)^(1/2)*f*g-3/2/h^2*c*g/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln
((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+5/2/h^3*c*g^2/(a
*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/
```

```
h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)
/h^2)^(1/2))/(x+g/h))*f-f/h^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/
h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/2/h^2/(a*h^2+c*g^2)/(
x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3
/(a*h^2+c*g^2)/(x+g/h)^2*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2)*f*g^2-3/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*d+3/2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e-3/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x
+g/h)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f-3/2/h*c^2*g^
2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c
*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+
c*g^2)/h^2)^(1/2))/(x+g/h))*d+3/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/((a*h^2+c*g^2
)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)
^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-3
/2/h^3*c^2*g^4/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g
/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+1/2/h*c/(a*h^2+c*g^2)/((a*h^2+c*
g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h
^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*
d
```

**maxima [B]** time = 0.67, size = 896, normalized size = 3.98

$$-\frac{3\sqrt{cx^2+a}\,cfg^3}{2\left(c^2g^4h^2x+2\,acg^2h^4x+a^2h^6x+c^2g^5h+2\,acg^3h^3+a^2gh^5\right)}+\frac{3\sqrt{cx^2+a}\,ceg^2}{2\left(c^2g^4hx+2\,acg^2h^3x+a^2h^5x+c^2g^5+2\,acg^3h\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(c*x^2 + a)*c*f*g^3/(c^2*g^4*h^2*x + 2*a*c*g^2*h^4*x + a^2*h^6*x +
 c^2*g^5*h + 2*a*c*g^3*h^3 + a^2*g*h^5) + 3/2*sqrt(c*x^2 + a)*c*e*g^2/(c^2*
g^4*h*x + 2*a*c*g^2*h^3*x + a^2*h^5*x + c^2*g^5 + 2*a*c*g^3*h^2 + a^2*g*h^4
) - 3/2*sqrt(c*x^2 + a)*c*d*g/(c^2*g^4*x + 2*a*c*g^2*h^2*x + a^2*h^4*x + c^
2*g^5/h + 2*a*c*g^3*h + a^2*g*h^3) - 1/2*sqrt(c*x^2 + a)*f*g^2/(c*g^2*h^3*x
^2 + a*h^5*x^2 + 2*c*g^3*h^2*x + 2*a*g*h^4*x + c*g^4*h + a*g^2*h^3) + 1/2*s
qrt(c*x^2 + a)*e*g/(c*g^2*h^2*x^2 + a*h^4*x^2 + 2*c*g^3*h*x + 2*a*g*h^3*x +
 c*g^4 + a*g^2*h^2) + 2*sqrt(c*x^2 + a)*f*g/(c*g^2*h^2*x + a*h^4*x + c*g^3*
h + a*g*h^3) - 1/2*sqrt(c*x^2 + a)*d/(c*g^2*h*x^2 + a*h^3*x^2 + 2*c*g^3*x +
 2*a*g*h^2*x + c*g^4/h + a*g^2*h) - sqrt(c*x^2 + a)*e/(c*g^2*h*x + a*h^3*x
+ c*g^3 + a*g*h^2) + 3/2*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
 a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^7) - 3/2*c^2*e*g^3*
```

```
arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a
+ c*g^2/h^2)^(5/2)*h^6) + 3/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x +
g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 5/2*c*f*g
^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/(
(a + c*g^2/h^2)^(3/2)*h^5) + 3/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g
)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^4) - 1/2*c*d*ar
csinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a +
c*g^2/h^2)^(3/2)*h^3) + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqr
t(a*c)*abs(h*x + g)))/(sqrt(a + c*g^2/h^2)*h^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\sqrt{a + cx^{2}} \left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2)/(sqrt(a + c*x**2)*(g + h*x)**3), x)
```

# 3.108 $\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=229

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right.}{}$$

[Out] -1/2*(3*a*h^2*(e*h+3*f*g)-2*c*g*(f*g^2+3*h*(d*h+e*g)))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^3/a/c/(c*x^2+a)^(1/2)-1/3*(-4*a*f+3*c*d)*h*(h*x+g)^2*(c*x^2+a)^(1/2)/a/c^2-1/6*h*(12*c^2*d*g^2+16*a^2*f*h^2-4*a*c*(7*f*g^2+3*h*(d*h+3*e*g))+c*h*(-9*a*e*h-11*a*f*g+6*c*d*g)*x)*(c*x^2+a)^(1/2)/a/c^3

**Rubi** [A] time = 0.32, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1645, 833, 780, 217, 206}

$$-\frac{h\sqrt{a+cx^2}\left(4\left(4a^2fh^2-ac\left(3h(dh+3eg)+7fg^2\right)+3c^2dg^2\right)+chx(-9aeh-11afg+6cdg)\right)}{6ac^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^3)/(a*c*Sqrt[a + c*x^2])) - ((3*c*d - 4*
a*f)*h*(g + h*x)^2*Sqrt[a + c*x^2])/(3*a*c^2) - (h*(4*(3*c^2*d*g^2 + 4*a^2*
f*h^2 - a*c*(7*f*g^2 + 3*h*(3*e*g + d*h))) + c*h*(6*c*d*g - 11*a*f*g - 9*a*
e*h)*x)*Sqrt[a + c*x^2])/(6*a*c^3) + ((2*c*f*g^3 + 6*c*g*h*(e*g + d*h) - 3*
a*h^2*(3*f*g + e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)^2(-a(fg+3eh)+(3cd-4af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{\int \frac{(g+hx)\left(-a\left(2\right.\right.}{}}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2\right.\right.}{}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^3}{ac\sqrt{a+cx^2}} - \frac{(3cd-4af)h(g+hx)^2\sqrt{a+cx^2}}{3ac^2} - \frac{h\left(4\left(3c^2dg^2\right.\right.}{}$$

**<u>Mathematica</u> [A]** time = 0.45, size = 246, normalized size = 1.07

$$\frac{\frac{-16a^3fh^3+a^2ch\left(3h(4dh+3e(4g+hx))+f\left(36g^2+27ghx-8h^2x^2\right)\right)+ac^2\left(6dh\left(-3g^2-3ghx+h^2x^2\right)-3e\left(2g^3+6g^2hx-6gh^2x^2-h^3x^3\right)+fx\left(-6g^3+18g^2hx+9gh^2\right.\right.}{a\sqrt{a+cx^2}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] ((-16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2)
 - 3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h
*x + 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2)
 + 3*h*(4*d*h + 3*e*(4*g + h*x))))/(a*Sqrt[a + c*x^2]) + 3*Sqrt[c]*(2*c*f*g
^3 + 6*c*g*h*(e*g + d*h) - 3*a*h^2*(3*f*g + e*h))*Log[c*x + Sqrt[c]*Sqrt[a
+ c*x^2]])/(6*c^3)
```

**<u>fricas</u> [A]** time = 1.22, size = 758, normalized size = 3.31

$$\left[-\frac{3\left(2\,a^2cfg^3+6\,a^2ceg^2h-3\,a^3eh^3+3\left(2\,a^2cd-3\,a^3f\right)gh^2+\left(2\,ac^2fg^3+6\,ac^2eg^2h-3\,a^2ceh^3+3\left(2\,ac^2d-3\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a^2*c*d - 3
*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 + 3*(2*a*c
^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt
(c)*x - a) - 2*(2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(
a*c^2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h
^2 + a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d -
 4*a^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^
2*f)*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2
+ a^2*c^3), -1/6*(3*(2*a^2*c*f*g^3 + 6*a^2*c*e*g^2*h - 3*a^3*e*h^3 + 3*(2*a
^2*c*d - 3*a^3*f)*g*h^2 + (2*a*c^2*f*g^3 + 6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3
+ 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x
^2 + a)) - (2*a*c^2*f*h^3*x^4 - 6*a*c^2*e*g^3 + 36*a^2*c*e*g*h^2 - 18*(a*c^
2*d - 2*a^2*c*f)*g^2*h + 4*(3*a^2*c*d - 4*a^3*f)*h^3 + 3*(3*a*c^2*f*g*h^2 +
 a*c^2*e*h^3)*x^3 + 2*(9*a*c^2*f*g^2*h + 9*a*c^2*e*g*h^2 + (3*a*c^2*d - 4*a
^2*c*f)*h^3)*x^2 - 3*(6*a*c^2*e*g^2*h - 3*a^2*c*e*h^3 - 2*(c^3*d - a*c^2*f)
*g^3 + 3*(2*a*c^2*d - 3*a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^
2*c^3)]
```

**giac [A]** time = 0.25, size = 339, normalized size = 1.48

$$\frac{\left(\left(\left(\frac{2\,fh^3x}{c}+\frac{3\left(3\,ac^4fgh^2+ac^4h^3e\right)}{ac^5}\right)x+\frac{2\left(9\,ac^4fg^2h+3\,ac^4dh^3-4\,a^2c^3fh^3+9\,ac^4gh^2e\right)}{ac^5}\right)x+\frac{3\left(2\,c^5dg^3-2\,ac^4fg^3-6\,ac^4dgh^2+9\,a^2c^3fgh^2-6\,ac^4g^2h\right.}{ac^5}}{6\,\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*((((2*f*h^3*x/c + 3*(3*a*c^4*f*g*h^2 + a*c^4*h^3*e)/(a*c^5))*x + 2*(9*a
*c^4*f*g^2*h + 3*a*c^4*d*h^3 - 4*a^2*c^3*f*h^3 + 9*a*c^4*g*h^2*e)/(a*c^5))*
x + 3*(2*c^5*d*g^3 - 2*a*c^4*f*g^3 - 6*a*c^4*d*g*h^2 + 9*a^2*c^3*f*g*h^2 -
6*a*c^4*g^2*h*e + 3*a^2*c^3*h^3*e)/(a*c^5))*x - 2*(9*a*c^4*d*g^2*h - 18*a^2
*c^3*f*g^2*h - 6*a^2*c^3*d*h^3 + 8*a^3*c^2*f*h^3 + 3*a*c^4*g^3*e - 18*a^2*c
^3*g*h^2*e)/(a*c^5))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^3 + 6*c*d*g*h^2 - 9*a*f
*g*h^2 + 6*c*g^2*h*e - 3*a*h^3*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^
(5/2)
```

**maple [B]** time = 0.02, size = 516, normalized size = 2.25

$$\frac{f\,h^3x^4}{3\sqrt{c\,x^2+a}\;c}+\frac{e\,h^3x^3}{2\sqrt{c\,x^2+a}\;c}+\frac{3f\,g\,h^2x^3}{2\sqrt{c\,x^2+a}\;c}-\frac{4af\,h^3x^2}{3\sqrt{c\,x^2+a}\;c^2}+\frac{d\,h^3x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3eg\,h^2x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3f\,g^2h\,x^2}{\sqrt{c\,x^2+a}\;c}+\frac{3ae\,h^3x}{2\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/3*h^3*f*x^4/c/(c*x^2+a)^(1/2)-4/3*h^3*f*a/c^2*x^2/(c*x^2+a)^(1/2)-8/3*h^3
*f*a^2/c^3/(c*x^2+a)^(1/2)+1/2*x^3/c/(c*x^2+a)^(1/2)*h^3*e+3/2*x^3/c/(c*x^2
+a)^(1/2)*g*h^2*f+3/2*a/c^2*x/(c*x^2+a)^(1/2)*h^3*e+9/2*a/c^2*x/(c*x^2+a)^(
1/2)*g*h^2*f-3/2*a/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*h^3*e-9/2*a/c^(5/2
)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*f+x^2/c/(c*x^2+a)^(1/2)*h^3*d+3*x^2/c
/(c*x^2+a)^(1/2)*g*h^2*e+3*x^2/c/(c*x^2+a)^(1/2)*g^2*h*f+2*a/c^2/(c*x^2+a)^
(1/2)*h^3*d+6*a/c^2/(c*x^2+a)^(1/2)*g*h^2*e+6*a/c^2/(c*x^2+a)^(1/2)*g^2*h*f
-3*x/c/(c*x^2+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+a)^(1/2)*g^2*h*e-x/c/(c*x^2+a)^
(1/2)*g^3*f+3/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g*h^2*d+3/c^(3/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))*g^2*h*e+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*g^
3*f-3/c/(c*x^2+a)^(1/2)*g^2*h*d-1/c/(c*x^2+a)^(1/2)*g^3*e+g^3*d*x/a/(c*x^2+
a)^(1/2)
```

**maxima [A]** time = 0.46, size = 346, normalized size = 1.51

$$\frac{fh^3x^4}{3\sqrt{cx^2+a}\,c}-\frac{4\,afh^3x^2}{3\sqrt{cx^2+a}\,c^2}+\frac{dg^3x}{\sqrt{cx^2+a}\,a}-\frac{eg^3}{\sqrt{cx^2+a}\,c}-\frac{3\,dg^2h}{\sqrt{cx^2+a}\,c}-\frac{8\,a^2fh^3}{3\sqrt{cx^2+a}\,c^3}+\frac{\left(3\,fgh^2+eh^3\right)x^3}{2\sqrt{cx^2+a}\,c}+\frac{\left(3\,fg^2h+\right.}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*f*h^3*x^4/(sqrt(c*x^2 + a)*c) - 4/3*a*f*h^3*x^2/(sqrt(c*x^2 + a)*c^2) +
 d*g^3*x/(sqrt(c*x^2 + a)*a) - e*g^3/(sqrt(c*x^2 + a)*c) - 3*d*g^2*h/(sqrt(
c*x^2 + a)*c) - 8/3*a^2*f*h^3/(sqrt(c*x^2 + a)*c^3) + 1/2*(3*f*g*h^2 + e*h^
3)*x^3/(sqrt(c*x^2 + a)*c) + (3*f*g^2*h + 3*e*g*h^2 + d*h^3)*x^2/(sqrt(c*x^
2 + a)*c) + 3/2*(3*f*g*h^2 + e*h^3)*a*x/(sqrt(c*x^2 + a)*c^2) - (f*g^3 + 3*
e*g^2*h + 3*d*g*h^2)*x/(sqrt(c*x^2 + a)*c) - 3/2*(3*f*g*h^2 + e*h^3)*a*arcs
inh(c*x/sqrt(a*c))/c^(5/2) + (f*g^3 + 3*e*g^2*h + 3*d*g*h^2)*arcsinh(c*x/sq
rt(a*c))/c^(3/2) + 2*(3*f*g^2*h + 3*e*g*h^2 + d*h^3)*a/(sqrt(c*x^2 + a)*c^2
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^3\,\left(f\,x^2+e\,x+d\right)}{\left(c\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^{3}\left(d+ex+fx^{2}\right)}{\left(a+cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

# 3.109 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=149

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+hx}{}$$

```
[Out] 1/2*((-3*a*f+2*c*d)*h^2+2*c*g*(2*e*h+f*g))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2
))/c^(5/2)-(a*e-(-a*f+c*d)*x)*(h*x+g)^2/a/c/(c*x^2+a)^(1/2)-1/2*h*(4*c*d*g-
4*a*(e*h+2*f*g)+(-3*a*f+2*c*d)*h*x)*(c*x^2+a)^(1/2)/a/c^2
```

**Rubi [A]** time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1645, 780, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)\left(h^2(2cd-3af)+2cg(2eh+fg)\right)}{2c^{5/2}}-\frac{h\sqrt{a+cx^2}\,(4(cdg-a(eh+2fg))+hx(2cd-3af))}{2ac^2}-\frac{(g+hx}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x)^2)/(a*c*Sqrt[a + c*x^2])) - (h*(4*(c*d*g
 - a*(2*f*g + e*h)) + (2*c*d - 3*a*f)*h*x)*Sqrt[a + c*x^2])/(2*a*c^2) + (((
2*c*d - 3*a*f)*h^2 + 2*c*g*(f*g + 2*e*h))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^
2]])/(2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{\int \frac{(g+hx)(-a(fg+2eh)+(2cd-3af)hx)}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)^2}{ac\sqrt{a+cx^2}} - \frac{h(4(cdg-a(2fg+eh))+(2cd-3af)hx)\sqrt{a+cx}}{2ac^2}$$

**Mathematica [A]** time = 0.28, size = 177, normalized size = 1.19

$$\frac{\sqrt{c}\left(a^2h(4eh+8fg+3fhx)+ac\left(-2dh(2g+hx)-2e\left(g^2+2ghx-h^2x^2\right)+fx\left(-2g^2+4ghx+h^2x^2\right)\right)+2c^2dg^2x\right.}{2ac^{5/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h + 3*f*h*x) + a*c*(-2*d*h*(2*
g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-2*g^2 + 4*g*h*x + h^2*x^2)
)) - a^(3/2)*(3*a*f*h^2 - 2*c*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[1 + (c*x^2)/a
]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(2*a*c^(5/2)*Sqrt[a + c*x^2])
```

**fricas [A]** time = 0.89, size = 530, normalized size = 3.56

$$\left[-\frac{\left(2\,a^2cfg^2+4\,a^2cegh+\left(2\,a^2cd-3\,a^3f\right)h^2+\left(2\,ac^2fg^2+4\,ac^2egh+\left(2\,ac^2d-3\,a^2cf\right)h^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^2*c*f*g^2 + 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^
2*f*g^2 + 4*a*c^2*e*g*h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(c)*log(-2*
c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - 2*(a*c^2*f*h^2*x^3 - 2*a*c^2*e*g
^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2
*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2
*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^4*x^2 + a^2*c^3), -1/2*((2*a^2*c*f*g^2
+ 4*a^2*c*e*g*h + (2*a^2*c*d - 3*a^3*f)*h^2 + (2*a*c^2*f*g^2 + 4*a*c^2*e*g*
h + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2
 + a)) - (a*c^2*f*h^2*x^3 - 2*a*c^2*e*g^2 + 4*a^2*c*e*h^2 - 4*(a*c^2*d - 2*
a^2*c*f)*g*h + 2*(2*a*c^2*f*g*h + a*c^2*e*h^2)*x^2 - (4*a*c^2*e*g*h - 2*(c^
3*d - a*c^2*f)*g^2 + (2*a*c^2*d - 3*a^2*c*f)*h^2)*x)*sqrt(c*x^2 + a))/(a*c^
4*x^2 + a^2*c^3)]
```

**giac [A]** time = 0.25, size = 219, normalized size = 1.47

$$\frac{\left(\left(\frac{fh^2x}{c}+\frac{2\left(2\,ac^3fgh+ac^3h^2e\right)}{ac^4}\right)x+\frac{2\,c^4dg^2-2\,ac^3fg^2-2\,ac^3dh^2+3\,a^2c^2fh^2-4\,ac^3ghe}{ac^4}\right)x-\frac{2\left(2\,ac^3dgh-4\,a^2c^2fgh+ac^3g^2e-2\,a^2c^2h^2e\right)}{ac^4}}{2\,\sqrt{cx^2+a}}-\frac{\left(2\,cf\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((f*h^2*x/c + 2*(2*a*c^3*f*g*h + a*c^3*h^2*e)/(a*c^4))*x + (2*c^4*d*g^
2 - 2*a*c^3*f*g^2 - 2*a*c^3*d*h^2 + 3*a^2*c^2*f*h^2 - 4*a*c^3*g*h*e)/(a*c^4
))*x - 2*(2*a*c^3*d*g*h - 4*a^2*c^2*f*g*h + a*c^3*g^2*e - 2*a^2*c^2*h^2*e)/
(a*c^4))/sqrt(c*x^2 + a) - 1/2*(2*c*f*g^2 + 2*c*d*h^2 - 3*a*f*h^2 + 4*c*g*h
*e)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(5/2)
```

**maple [B]** time = 0.01, size = 327, normalized size = 2.19

$$\frac{f h^2 x^3}{2\sqrt{c x^2+a}\, c}+\frac{e h^2 x^2}{\sqrt{c x^2+a}\, c}+\frac{2 f g h x^2}{\sqrt{c x^2+a}\, c}+\frac{3 a f h^2 x}{2\sqrt{c x^2+a}\, c^2}+\frac{d g^2 x}{\sqrt{c x^2+a}\, a}-\frac{d h^2 x}{\sqrt{c x^2+a}\, c}-\frac{2 e g h x}{\sqrt{c x^2+a}\, c}-\frac{f g^2 x}{\sqrt{c x^2+a}\, c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] 1/2*h^2*f*x^3/c/(c*x^2+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+a)^(1/2)-3/2*h^2*f
*a/c^(5/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))+x^2/c/(c*x^2+a)^(1/2)*h^2*e+2*x^2/
c/(c*x^2+a)^(1/2)*g*h*f+2*a/c^2/(c*x^2+a)^(1/2)*h^2*e+4*a/c^2/(c*x^2+a)^(1/
2)*g*h*f-x/c/(c*x^2+a)^(1/2)*d*h^2-2*x/c/(c*x^2+a)^(1/2)*e*g*h-x/c/(c*x^2+a
)^(1/2)*f*g^2+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*d*h^2+2/c^(3/2)*ln(c^
(1/2)*x+(c*x^2+a)^(1/2))*e*g*h+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g^
2-2/c/(c*x^2+a)^(1/2)*g*h*d-1/c/(c*x^2+a)^(1/2)*g^2*e+g^2*d*x/a/(c*x^2+a)^(
1/2)
```

**maxima [A]** time = 0.45, size = 227, normalized size = 1.52

$$\frac{fh^2x^3}{2\sqrt{cx^2+a}\,c}+\frac{dg^2x}{\sqrt{cx^2+a}\,a}+\frac{3\,afh^2x}{2\sqrt{cx^2+a}\,c^2}-\frac{3\,afh^2\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{2\,c^{\frac{5}{2}}}-\frac{eg^2}{\sqrt{cx^2+a}\,c}-\frac{2\,dgh}{\sqrt{cx^2+a}\,c}+\frac{\left(2\,fgh+eh^2\right)x^2}{\sqrt{cx^2+a}\,c}-\frac{\left(fg^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*f*h^2*x^3/(sqrt(c*x^2 + a)*c) + d*g^2*x/(sqrt(c*x^2 + a)*a) + 3/2*a*f*h
^2*x/(sqrt(c*x^2 + a)*c^2) - 3/2*a*f*h^2*arcsinh(c*x/sqrt(a*c))/c^(5/2) - e
*g^2/(sqrt(c*x^2 + a)*c) - 2*d*g*h/(sqrt(c*x^2 + a)*c) + (2*f*g*h + e*h^2)*
x^2/(sqrt(c*x^2 + a)*c) - (f*g^2 + 2*e*g*h + d*h^2)*x/(sqrt(c*x^2 + a)*c) +
 (f*g^2 + 2*e*g*h + d*h^2)*arcsinh(c*x/sqrt(a*c))/c^(3/2) + 2*(2*f*g*h + e*
h^2)*a/(sqrt(c*x^2 + a)*c^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{\left(c\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^{2}\left(d+ex+fx^{2}\right)}{\left(a+cx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + c*x**2)**(3/2), x)
```

## 3.110 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{h\sqrt{a+cx^2}\,(cd-2af)}{ac^2}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

```
[Out] (e*h+f*g)*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)-(a*e-(-a*f+c*d)*x)*(h*
x+g)/a/c/(c*x^2+a)^(1/2)-(-2*a*f+c*d)*h*(c*x^2+a)^(1/2)/a/c^2
```

**Rubi [A]** time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1645, 641, 217, 206}

$$-\frac{h\sqrt{a+cx^2}\,(cd-2af)}{ac^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)(eh+fg)}{c^{3/2}}-\frac{(g+hx)(ae-x(cd-af))}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] -(((a*e - (c*d - a*f)*x)*(g + h*x))/(a*c*Sqrt[a + c*x^2])) - ((c*d - 2*a*f)
*h*Sqrt[a + c*x^2])/(a*c^2) + ((f*g + e*h)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x
^2]])/c^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+cx^2\right)^{3/2}}\,dx = -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{\int \frac{-a(fg+eh)+(cd-2af)hx}{\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\,\text{Subst}\left(\int \frac{1}{1-cx^2}\right.}{c}$$

$$= -\frac{(ae-(cd-af)x)(g+hx)}{ac\sqrt{a+cx^2}} - \frac{(cd-2af)h\sqrt{a+cx^2}}{ac^2} + \frac{(fg+eh)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right.}{c^{3/2}}$$

**Mathematica [A]** time = 0.14, size = 102, normalized size = 1.02

$$\frac{a^{3/2}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)(eh+fg)+2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx}{ac^2\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x]
```

```
[Out] (2*a^2*f*h + c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)) + a^(3/2)*
Sqrt[c]*(f*g + e*h)*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(a*c^
2*Sqrt[a + c*x^2])
```

**fricas [A]** time = 1.14, size = 278, normalized size = 2.78

$$\left[\frac{\left(a^2fg + a^2eh + \left(acfg + aceh\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2 - 2\,\sqrt{cx^2 + a}\,\sqrt{c}\,x - a\right) + 2\left(acfhx^2 - aceg - \left(acd - 2\,a^2f\right)h - \left(\right.\right.}{2\left(ac^3x^2 + a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(c)*log(-2*c*x^2 -
2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*(a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^
2*f)*h - (a*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2
*c^2), -((a^2*f*g + a^2*e*h + (a*c*f*g + a*c*e*h)*x^2)*sqrt(-c)*arctan(sqrt
(-c)*x/sqrt(c*x^2 + a)) - (a*c*f*h*x^2 - a*c*e*g - (a*c*d - 2*a^2*f)*h - (a
*c*e*h - (c^2*d - a*c*f)*g)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

**giac [A]** time = 0.25, size = 116, normalized size = 1.16

$$\frac{\left(\frac{fhx}{c} + \frac{c^3dg - ac^2fg - ac^2he}{ac^3}\right)x - \frac{ac^2dh - 2\,a^2cfh + ac^2ge}{ac^3}}{\sqrt{cx^2 + a}} - \frac{\left(fg + he\right)\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((f*h*x/c + (c^3*d*g - a*c^2*f*g - a*c^2*h*e)/(a*c^3))*x - (a*c^2*d*h - 2*a
^2*c*f*h + a*c^2*g*e)/(a*c^3))/sqrt(c*x^2 + a) - (f*g + h*e)*log(abs(-sqrt(
c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.00, size = 163, normalized size = 1.63

$$\frac{fh\,x^2}{\sqrt{c\,x^2 + a}\;c} + \frac{dgx}{\sqrt{c\,x^2 + a}\;a} - \frac{ehx}{\sqrt{c\,x^2 + a}\;c} - \frac{fgx}{\sqrt{c\,x^2 + a}\;c} + \frac{eh\ln\left(\sqrt{c}\;x + \sqrt{c\,x^2 + a}\right)}{c^{\frac{3}{2}}} + \frac{fg\ln\left(\sqrt{c}\;x + \sqrt{c\,x^2 + a}\right)}{c^{\frac{3}{2}}} + \frac{}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] h*f*x^2/c/(c*x^2+a)^(1/2)+2*h*f*a/c^2/(c*x^2+a)^(1/2)-x/c/(c*x^2+a)^(1/2)*e
*h-x/c/(c*x^2+a)^(1/2)*f*g+1/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*e*h+1/c^
(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))*f*g-1/c/(c*x^2+a)^(1/2)*d*h-1/c/(c*x^2+
a)^(1/2)*e*g+d*g*x/a/(c*x^2+a)^(1/2)
```

**maxima** [A] time = 0.44, size = 126, normalized size = 1.26

$$\frac{fhx^2}{\sqrt{cx^2+a}\,c}+\frac{dgx}{\sqrt{cx^2+a}\,a}-\frac{eg}{\sqrt{cx^2+a}\,c}-\frac{dh}{\sqrt{cx^2+a}\,c}+\frac{2\,afh}{\sqrt{cx^2+a}\,c^2}-\frac{\left(fg+eh\right)x}{\sqrt{cx^2+a}\,c}+\frac{\left(fg+eh\right)\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] f*h*x^2/(sqrt(c*x^2 + a)*c) + d*g*x/(sqrt(c*x^2 + a)*a) - e*g/(sqrt(c*x^2 +
 a)*c) - d*h/(sqrt(c*x^2 + a)*c) + 2*a*f*h/(sqrt(c*x^2 + a)*c^2) - (f*g + e
*h)*x/(sqrt(c*x^2 + a)*c) + (f*g + e*h)*arcsinh(c*x/sqrt(a*c))/c^(3/2)
```

**mupad** [B] time = 5.28, size = 151, normalized size = 1.51

$$\frac{e\,h\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}+\frac{f\,g\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}-\frac{d\,h}{c\,\sqrt{c\,x^2+a}}-\frac{e\,g}{c\,\sqrt{c\,x^2+a}}+\frac{d\,g\,x}{a\,\sqrt{c\,x^2+a}}-\frac{e\,h\,x}{c\,\sqrt{c\,x^2+a}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + c*x^2)^(3/2),x)
```

```
[Out] (e*h*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) + (f*g*log(c^(1/2)*x + (a
+ c*x^2)^(1/2)))/c^(3/2) - (d*h)/(c*(a + c*x^2)^(1/2)) - (e*g)/(c*(a + c*x^
2)^(1/2)) + (d*g*x)/(a*(a + c*x^2)^(1/2)) - (e*h*x)/(c*(a + c*x^2)^(1/2)) -
 (f*g*x)/(c*(a + c*x^2)^(1/2)) + (f*h*(2*a + c*x^2))/(c^2*(a + c*x^2)^(1/2)
)
```

**sympy** [A] time = 18.84, size = 209, normalized size = 2.09

$$dh\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eg\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+eh\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+fg\left(\frac{\operatorname{asinh}\left(\cdots\right.}{c^{\frac{3}{2}}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] d*h*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True
)) + e*g*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)),
 True)) + e*h*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*
x**2/a))) + f*g*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 +
c*x**2/a))) + f*h*Piecewise((2*a/(c**2*sqrt(a + c*x**2)) + x**2/(c*sqrt(a +
 c*x**2)), Ne(c, 0)), (x**4/(4*a**(3/2)), True)) + d*g*x/(a**(3/2)*sqrt(1 +
 c*x**2/a))
```

## 3.111 $\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

```
[Out] f*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/c^(3/2)+(-a*e+(-a*f+c*d)*x)/a/c/(c*x^2
+a)^(1/2)
```

**Rubi** [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 217, 206}

$$\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}-\frac{ae-x(cd-af)}{ac\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] -((a*e - (c*d - a*f)*x)/(a*c*Sqrt[a + c*x^2])) + (f*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex+fx^2}{\left(a+cx^2\right)^{3/2}}\,dx &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{\int \frac{af}{c\sqrt{a+cx^2}}\,dx}{a}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+cx^2}}\,dx}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{c}\\ &= -\frac{ae-(cd-af)x}{ac\sqrt{a+cx^2}}+\frac{f\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 74, normalized size = 1.21

$$\frac{a^{3/2}f\sqrt{\frac{cx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)+\sqrt{c}\,(cdx-a(e+fx))}{ac^{3/2}\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*(c*d*x - a*(e + f*x)) + a^(3/2)*f*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqr
t[c]*x)/Sqrt[a]])/(a*c^(3/2)*Sqrt[a + c*x^2])
```

**fricas [A]** time = 0.63, size = 181, normalized size = 2.97

$$\left[\frac{\left(acfx^2+a^2f\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{cx^2+a}\,\sqrt{c}\,x-a\right)-2\left(ace-\left(c^2d-acf\right)x\right)\sqrt{cx^2+a}}{2\left(ac^3x^2+a^2c^2\right)},-\frac{\left(acfx^2+a^2f\right)\sqrt{-c}\ \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*c*f*x^2 + a^2*f)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*
x - a) - 2*(a*c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^
2), -((a*c*f*x^2 + a^2*f)*sqrt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (a*
c*e - (c^2*d - a*c*f)*x)*sqrt(c*x^2 + a))/(a*c^3*x^2 + a^2*c^2)]
```

**giac [A]** time = 0.20, size = 63, normalized size = 1.03

$$-\frac{\frac{e}{c} - \frac{(c^2d-acf)x}{ac^2}}{\sqrt{cx^2+a}} - \frac{f\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+a}\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(e/c - (c^2*d - a*c*f)*x/(a*c^2))/sqrt(c*x^2 + a) - f*log(abs(-sqrt(c)*x +
 sqrt(c*x^2 + a)))/c^(3/2)
```

**maple [A]** time = 0.01, size = 69, normalized size = 1.13

$$\frac{dx}{\sqrt{c\,x^2+a}\;a} - \frac{fx}{\sqrt{c\,x^2+a}\;c} + \frac{f\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+a}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{c\,x^2+a}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+a)^(1/2)+f/c^(3/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))-e/c/(c*x^2+a
)^(1/2)+d*x/a/(c*x^2+a)^(1/2)
```

**maxima [A]** time = 0.43, size = 61, normalized size = 1.00

$$\frac{dx}{\sqrt{cx^2+a}\,a} - \frac{fx}{\sqrt{cx^2+a}\,c} + \frac{f\,\operatorname{arsinh}\left(\frac{cx}{\sqrt{ac}}\right)}{c^{\frac{3}{2}}} - \frac{e}{\sqrt{cx^2+a}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] d*x/(sqrt(c*x^2 + a)*a) - f*x/(sqrt(c*x^2 + a)*c) + f*arcsinh(c*x/sqrt(a*c)
)/c^(3/2) - e/(sqrt(c*x^2 + a)*c)
```

**mupad [B]** time = 4.33, size = 68, normalized size = 1.11

$$\frac{f\,\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+a}\right)}{c^{3/2}}-\frac{e}{c\,\sqrt{c\,x^2+a}}+\frac{d\,x}{a\,\sqrt{c\,x^2+a}}-\frac{f\,x}{c\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + c*x^2)^(3/2),x)
```

```
[Out] (f*log(c^(1/2)*x + (a + c*x^2)^(1/2)))/c^(3/2) - e/(c*(a + c*x^2)^(1/2)) +
(d*x)/(a*(a + c*x^2)^(1/2)) - (f*x)/(c*(a + c*x^2)^(1/2))
```

**sympy [A]** time = 8.86, size = 87, normalized size = 1.43

$$e\left(\begin{cases}-\frac{1}{c\sqrt{a+cx^2}} & \text{for } c\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+f\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{a}}\right)}{c^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,c\sqrt{1+\frac{cx^2}{a}}}\right)+\frac{dx}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] e*Piecewise((-1/(c*sqrt(a + c*x**2)), Ne(c, 0)), (x**2/(2*a**(3/2)), True))
 + f*(asinh(sqrt(c)*x/sqrt(a))/c**(3/2) - x/(sqrt(a)*c*sqrt(1 + c*x**2/a)))
 + d*x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

# 3.112 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

```
[Out] -(d*h^2-e*g*h+f*g^2)*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/
2))/(a*h^2+c*g^2)^(3/2)+(-a*(a*f*h-c*d*h+c*e*g)+c*(a*e*h-a*f*g+c*d*g)*x)/a/
c/(a*h^2+c*g^2)/(c*x^2+a)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{a(afh-cdh+ceg)-cx(aeh-afg+cdg)}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)}-\frac{\left(dh^2-egh+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*e*g - c*d*h + a*f*h) - c*(c*d*g - a*f*g + a*e*h)*x)/(a*c*(c*g^2 + a
*h^2)*Sqrt[a + c*x^2])) - ((f*g^2 - e*g*h + d*h^2)*ArcTanh[(a*h - c*g*x)/(S
qrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{ac\left(fg^2-egh+dh^2\right)}{\left(cg^2+ah^2\right)(g+hx)\sqrt{a+cx^2}}\,dx}{ac}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} + \frac{\left(fg^2-egh+dh^2\right)\int \frac{1}{(g+hx)\sqrt{a+cx^2}}\,dx}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} - \frac{\left(fg^2-egh+dh^2\right)\text{Subst}\left(\int \frac{1}{cg^2+ah^2-\cdots}\right.}{cg^2+ah^2}$$

$$= -\frac{a(ceg-cdh+afh)-c(cdg-afg+aeh)x}{ac\left(cg^2+ah^2\right)\sqrt{a+cx^2}} - \frac{\left(fg^2-egh+dh^2\right)\tanh^{-1}\left(\frac{ah-c\cdots}{\sqrt{cg^2+ah^2}\cdots}\right.}{\left(cg^2+ah^2\right)^{3/2}}$$

**Mathematica [A]** time = 0.18, size = 137, normalized size = 0.99

$$\frac{a^2(-f)h+ac(dh-eg+ehx-fgx)+c^2dgx}{ac\sqrt{a+cx^2}\left(ah^2+cg^2\right)} - \frac{\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)}{\left(ah^2+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a^2*f*h) + c^2*d*g*x + a*c*(-(e*g) + d*h - f*g*x + e*h*x))/(a*c*(c*g^2 +
 a*h^2)*Sqrt[a + c*x^2]) - ((f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(3/2)
```

**fricas [B]** time = 4.00, size = 721, normalized size = 5.22

$$\left[\frac{\left(a^2cfg^2 - a^2cegh + a^2cdh^2 + \left(ac^2fg^2 - ac^2egh + ac^2dh^2\right)x^2\right)\sqrt{cg^2 + ah^2}\log\left(\frac{2\,acghx - acg^2 - 2\,a^2h^2 - \left(2\,c^2g^2 + ach^2\right)x^2 - 2\,\sqrt{c}}{h^2x^2 + 2\,ghx + g^2}\right.}{2\left(a^2c^3g^4 + 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a*c^2*f*g^2 - a*c^2*e*g*h
 + a*c^2*d*h^2)*x^2)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g*h*x - a*c*g^2 - 2*a^2
*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g^2 + a*h^2)*(c*g*x - a*h)*sqrt
(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(a*c^2*e*g^3 + a^2*c*e*g*h^2 -
(a*c^2*d - a^2*c*f)*g^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*
e*h^3 + (c^3*d - a*c^2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 +
a))/(a^2*c^3*g^4 + 2*a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g
^2*h^2 + a^3*c^2*h^4)*x^2), -((a^2*c*f*g^2 - a^2*c*e*g*h + a^2*c*d*h^2 + (a
*c^2*f*g^2 - a*c^2*e*g*h + a*c^2*d*h^2)*x^2)*sqrt(-c*g^2 - a*h^2)*arctan(sq
rt(-c*g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*
g^2 + a*c*h^2)*x^2)) + (a*c^2*e*g^3 + a^2*c*e*g*h^2 - (a*c^2*d - a^2*c*f)*g
^2*h - (a^2*c*d - a^3*f)*h^3 - (a*c^2*e*g^2*h + a^2*c*e*h^3 + (c^3*d - a*c^
2*f)*g^3 + (a*c^2*d - a^2*c*f)*g*h^2)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^4 + 2*
a^3*c^2*g^2*h^2 + a^4*c*h^4 + (a*c^4*g^4 + 2*a^2*c^3*g^2*h^2 + a^3*c^2*h^4)
*x^2)]
```

**giac [B]** time = 0.29, size = 294, normalized size = 2.13

$$\frac{\frac{\left(c^3dg^3 - ac^2fg^3 + ac^2dgh^2 - a^2cfgh^2 + ac^2g^2he + a^2ch^3e\right)x}{ac^3g^4 + 2\,a^2c^2g^2h^2 + a^3ch^4} + \frac{ac^2dg^2h - a^2cfg^2h + a^2cdh^3 - a^3fh^3 - ac^2g^3e - a^2cgh^2e}{ac^3g^4 + 2\,a^2c^2g^2h^2 + a^3ch^4}}{\sqrt{cx^2 + a}} - \frac{2\left(fg^2 + dh^2 - ghe\right)\arctan\left(\right.}{\left(cg^2 + ah^2\right)\sqrt{-cg}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^3*d*g^3 - a*c^2*f*g^3 + a*c^2*d*g*h^2 - a^2*c*f*g*h^2 + a*c^2*g^2*h*e +
 a^2*c*h^3*e)*x/(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4) + (a*c^2*d*g^2*
h - a^2*c*f*g^2*h + a^2*c*d*h^3 - a^3*f*h^3 - a*c^2*g^3*e - a^2*c*g*h^2*e)/
(a*c^3*g^4 + 2*a^2*c^2*g^2*h^2 + a^3*c*h^4))/sqrt(c*x^2 + a) - 2*(f*g^2 + d
```

```
*h^2 - g*h*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 - a*h^2))/((c*g^2 + a*h^2)*sqrt(-c*g^2 - a*h^2))
```

**maple [B]** time = 0.02, size = 862, normalized size = 6.25

$$\frac{cdgx}{\left(a h^2+c g^2\right)\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2 c+\frac{a h^2+c g^2}{h^2}}\; a}-\frac{ce g^2 x}{\left(a h^2+c g^2\right)\sqrt{-\frac{2\left(x+\frac{g}{h}\right)cg}{h}+\left(x+\frac{g}{h}\right)^2 c+\frac{a h^2+c g^2}{h^2}}\; ah}+\frac{}{\left(a h^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x)
```

```
[Out] -1/h*f/c/(c*x^2+a)^(1/2)+1/h*e*x/a/(c*x^2+a)^(1/2)-1/h^2*f*g*x/a/(c*x^2+a)^
(1/2)+h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
)*d-1/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*
e*g+1/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2
)*f*g^2+g/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^
(1/2)*x*c*d-1/h*g^2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*x*c*e+1/h^2*g^3/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2
*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*f-h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/(a*h^2+c*g^
2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a
*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/
2))/(x+g/h))*e*g-1/h/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

**maxima [B]** time = 0.62, size = 453, normalized size = 3.28

$$\frac{cfg^3x}{\sqrt{cx^2+a}\,acg^2h^2+\sqrt{cx^2+a}\,a^2h^4}-\frac{ceg^2x}{\sqrt{cx^2+a}\,acg^2h+\sqrt{cx^2+a}\,a^2h^3}+\frac{cdgx}{\sqrt{cx^2+a}\,acg^2+\sqrt{cx^2+a}\,a^2h^2}+\frac{}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] c*f*g^3*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - c*e*g^2
*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + c*d*g*x/(sqrt(c*
x^2 + a)*a*c*g^2 + sqrt(c*x^2 + a)*a^2*h^2) + f*g^2/(sqrt(c*x^2 + a)*c*g^2*
```

```
h + sqrt(c*x^2 + a)*a*h^3) - e*g/(sqrt(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a
*h^2) + d/(sqrt(c*x^2 + a)*c*g^2/h + sqrt(c*x^2 + a)*a*h) - f*g*x/(sqrt(c*x
^2 + a)*a*h^2) + e*x/(sqrt(c*x^2 + a)*a*h) + f*g^2*arcsinh(c*g*x/(sqrt(a*c)
*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3)
- e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(3/2)*h^2) + d*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) -
a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h) - f/(sqrt(c*x^2 + a
)*c*h)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\left(a + cx^{2}\right)^{\frac{3}{2}} \left(g + hx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+a)**(3/2),x)
[Out] Integral((d + e*x + f*x**2)/((a + c*x**2)**(3/2)*(g + h*x)), x)
```

## 3.113 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=239

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}+$$

```
[Out] (a*h^2*(-e*h+2*f*g)-c*g*(f*g^2-h*(-3*d*h+2*e*g)))*arctanh((-c*g*x+a*h)/(a*h
^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h^2+c*g^2)^(5/2)+(-a*(c*g*(-2*d*h+e*g)+
a*h*(-e*h+2*f*g))+(c^2*d*g^2+a^2*f*h^2-a*c*(f*g^2-h*(-d*h+2*e*g)))*x)/a/(a*
h^2+c*g^2)^2/(c*x^2+a)^(1/2)-h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)/(a*h^2+c
*g^2)^2/(h*x+g)
```

**Rubi** [A] time = 0.42, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{a(ah(2fg-eh)+cg(eg-2dh))-x\left(a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)+c^2dg^2\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^2}-\frac{h\sqrt{a+cx^2}\left(dh^2-egh+fg^2\right)}{(g+hx)\left(ah^2+cg^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] -((a*(c*g*(e*g - 2*d*h) + a*h*(2*f*g - e*h)) - (c^2*d*g^2 + a^2*f*h^2 - a*c
*(f*g^2 - h*(2*e*g - d*h)))*x)/(a*(c*g^2 + a*h^2)^2*Sqrt[a + c*x^2])) - (h*
(f*g^2 - e*g*h + d*h^2)*Sqrt[a + c*x^2])/((c*g^2 + a*h^2)^2*(g + h*x)) - ((
c*f*g^3 - c*g*h*(2*e*g - 3*d*h) - a*h^2*(2*f*g - e*h))*ArcTanh[(a*h - c*g*x
)/(Sqrt[c*g^2 + a*h^2]*Sqrt[a + c*x^2])])/(c*g^2 + a*h^2)^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+cx^2\right)^{3/2}}\,dx = -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \cdots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \cdots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \cdots$$

$$= -\frac{a(cg(eg-2dh)+ah(2fg-eh))-\left(c^2dg^2+a^2fh^2-ac\left(fg^2-h(2eg-dh)\right)\right)x}{a\left(cg^2+ah^2\right)^2\sqrt{a+cx^2}} - \cdots$$

**Mathematica** [A] time = 0.61, size = 285, normalized size = 1.19

$$\frac{\left(ah^2+cg^2\right)^{3/2}\left(-a^2fh^2+ac(2h(dh-eg+ehx)+fg(g-2hx))+2c^2dghx\right)+2h\left(h\left(a+cx^2\right)\sqrt{ah^2+cg^2}\left(a^2fh^2\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-(a*f*(c*g^2 + a*h^2)^(5/2)) + (c*g^2 + a*h^2)^(3/2)*(-(a^2*f*h^2) + 2*c^2
*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(e*g) + d*h + e*h*x))) + 2*h*(h*Sqr
t[c*g^2 + a*h^2]*(c^2*d*g^2 + a^2*f*h^2 + a*c*(-2*f*g^2 + h*(3*e*g - 2*d*h)
))*(a + c*x^2) - a*c*(c*f*g^3 + c*g*h*(-2*e*g + 3*d*h) + a*h^2*(-2*f*g + e*
h))*(g + h*x)*Sqrt[a + c*x^2]*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*Sq
rt[a + c*x^2])]))/(2*a*c*h*(c*g^2 + a*h^2)^(5/2)*(g + h*x)*Sqrt[a + c*x^2])
```

**fricas** [B] time = 7.18, size = 1573, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((a^2*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)
*g^2*h^2 + (a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d -
2*a^2*c*f)*g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3
*a*c^2*d - 2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a
^3*e*h^4 + (3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(c*g^2 + a*h^2)*log((2*a*c*g
*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 + 2*sqrt(c*g^2 + a*h
^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) + 2*(a*c^2*e*
g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a*c^2*d - 3*a^2*c*f)
*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^2 + 3*a^2*c*e*g*h^4
 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h^3 - (2*a^2*c*d - a
^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^3*e*h^5 + (c^3*d -
a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d - a^3*f)*g*h^4)*x)*
sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^4*c*g^3*h^4 + a^5*g
*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*h^5 + a^4*c*h^7)*x^
3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 + a^4*c*g*h^6)*x^2 +
 (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 + a^5*h^7)*x), -((a^2
*c*f*g^4 - 2*a^2*c*e*g^3*h + a^3*e*g*h^3 + (3*a^2*c*d - 2*a^3*f)*g^2*h^2 +
(a*c^2*f*g^3*h - 2*a*c^2*e*g^2*h^2 + a^2*c*e*h^4 + (3*a*c^2*d - 2*a^2*c*f)*
g*h^3)*x^3 + (a*c^2*f*g^4 - 2*a*c^2*e*g^3*h + a^2*c*e*g*h^3 + (3*a*c^2*d -
2*a^2*c*f)*g^2*h^2)*x^2 + (a^2*c*f*g^3*h - 2*a^2*c*e*g^2*h^2 + a^3*e*h^4 +
```

```
(3*a^2*c*d - 2*a^3*f)*g*h^3)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*g^2 - a
*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 + a*c*h^2
)*x^2)) + (a*c^2*e*g^5 - a^2*c*e*g^3*h^2 - 2*a^3*e*g*h^4 + a^3*d*h^5 - (2*a
*c^2*d - 3*a^2*c*f)*g^4*h - (a^2*c*d - 3*a^3*f)*g^2*h^3 - (3*a*c^2*e*g^3*h^
2 + 3*a^2*c*e*g*h^4 + (c^3*d - 2*a*c^2*f)*g^4*h - (a*c^2*d + a^2*c*f)*g^2*h
^3 - (2*a^2*c*d - a^3*f)*h^5)*x^2 - (a*c^2*e*g^4*h + 2*a^2*c*e*g^2*h^3 + a^
3*e*h^5 + (c^3*d - a*c^2*f)*g^5 + 2*(a*c^2*d - a^2*c*f)*g^3*h^2 + (a^2*c*d
- a^3*f)*g*h^4)*x)*sqrt(c*x^2 + a))/(a^2*c^3*g^7 + 3*a^3*c^2*g^5*h^2 + 3*a^
4*c*g^3*h^4 + a^5*g*h^6 + (a*c^4*g^6*h + 3*a^2*c^3*g^4*h^3 + 3*a^3*c^2*g^2*
h^5 + a^4*c*h^7)*x^3 + (a*c^4*g^7 + 3*a^2*c^3*g^5*h^2 + 3*a^3*c^2*g^3*h^4 +
 a^4*c*g*h^6)*x^2 + (a^2*c^3*g^6*h + 3*a^3*c^2*g^4*h^3 + 3*a^4*c*g^2*h^5 +
a^5*h^7)*x)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 1663, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x)
```

```
[Out] f/h^2*x/a/(c*x^2+a)^(1/2)-1/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)
^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+1/h/(a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h
+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2+c*g^2)/(x+g/h)/(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f*g^2+3*h*c*g/(a*h^2+c*g^
2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d-3*c*g^2/(a*h^
2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+3/h*c*g
^3/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f
+3*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^
2)^(1/2)*x*d-3/h*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*x*e+3/h^2*c^2*g^4/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/
h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-3*h*c*g/(a*h^2+c*g^2)^2/((a*h^2+
c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)
/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h)
)*d+3*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+
2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2
```

```
*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e-3/h*c*g^3/(a*h^2+c*g^2)^2/((a*h^2+c
*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/
h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))
*f-2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*x*c*d+3/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2)*x*c*e*g-4/h^2/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c
*g^2)/h^2)^(1/2)*x*c*f*g^2+1/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*e-2/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*f*g-1/(a*h^2+c*g^2)/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2
*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*
c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h/(a*h^2+c*g^2)/((
a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+
c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(
x+g/h))*f*g
```

**maxima [B]** time = 0.77, size = 1085, normalized size = 4.54

$$\frac{3\,c^2fg^4x}{\sqrt{cx^2+a}\,ac^2g^4h^2+2\,\sqrt{cx^2+a}\,a^2cg^2h^4+\sqrt{cx^2+a}\,a^3h^6}-\frac{3\,c^2eg^3x}{\sqrt{cx^2+a}\,ac^2g^4h+2\,\sqrt{cx^2+a}\,a^2cg^2h^3+\sqrt{cx^2+a}\,a^3h^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 3*c^2*f*g^4*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*
h^4 + sqrt(c*x^2 + a)*a^3*h^6) - 3*c^2*e*g^3*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h
 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^2 + a)*a^3*h^5) + 3*c^2*d*g^2
*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x^
2 + a)*a^3*h^4) + 3*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*
a*c*g^2*h^3 + sqrt(c*x^2 + a)*a^2*h^5) - 4*c*f*g^2*x/(sqrt(c*x^2 + a)*a*c*g
^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 3*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4 + 2*
sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) + 3*c*e*g*x/(sqrt(c*
x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 3*c*d*g/(sqrt(c*x^2 + a)*c^
2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2 + a)*a^2*h^3) - f*g^2/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 2*c*d*x/(sqrt(c*x^2 + a)*a*c*g^2 + sqrt(c*
x^2 + a)*a^2*h^2) + e*g/(sqrt(c*x^2 + a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*
x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2 + a)*a*g*h^2) - 2*f*g/(sqrt(c*x^2 +
a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) - d/(sqrt(c*x^2 + a)*c*g^2*x + sqrt(c*x
^2 + a)*a*h^2*x + sqrt(c*x^2 + a)*c*g^3/h + sqrt(c*x^2 + a)*a*g*h) + e/(sqr
t(c*x^2 + a)*c*g^2 + sqrt(c*x^2 + a)*a*h^2) + f*x/(sqrt(c*x^2 + a)*a*h^2) +
 3*c*f*g^3*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(5/2)*h^5) - 3*c*e*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs
(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) + 3*
```

```
c*d*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g))
)/((a + c*g^2/h^2)^(5/2)*h^3) - 2*f*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)
) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(3/2)*h^3) + e*arcsinh(c
*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h
^2)^(3/2)*h^2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+a)**(3/2),x)
[Out] Timed out
```

# 3.114 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=374

$$-\frac{\tanh^{-1}\left(\frac{ah-cgx}{\sqrt{a+cx^2}\sqrt{ah^2+cg^2}}\right)\left(2a^2fh^4-ach^2\left(3dh^2-9egh+11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg^2\right)\right)}{2\left(ah^2+cg^2\right)^{7/2}}+\frac{a\left(a^2fh^3-ac\ldots\right.}{}$$

```
[Out] -1/2*(2*a^2*f*h^4-a*c*h^2*(3*d*h^2-9*e*g*h+11*f*g^2)+2*c^2*g^2*(6*d*h^2-3*e
*g*h+f*g^2))*arctanh((-c*g*x+a*h)/(a*h^2+c*g^2)^(1/2)/(c*x^2+a)^(1/2))/(a*h
^2+c*g^2)^(7/2)+(a*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(3*f*g^2-h*(-d*h+3
*e*g)))+c*(c^2*d*g^3+a^2*h^2*(-e*h+3*f*g)-a*c*g*(f*g^2-3*h*(-d*h+e*g)))*x)/
a/(a*h^2+c*g^2)^3/(c*x^2+a)^(1/2)-1/2*h*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1/2)
/(a*h^2+c*g^2)^2/(h*x+g)^2+1/2*h*(2*a*h^2*(-e*h+2*f*g)-c*g*(3*f*g^2-h*(-7*d
*h+5*e*g)))*(c*x^2+a)^(1/2)/(a*h^2+c*g^2)^3/(h*x+g)
```

**Rubi** **[A]** time = 1.03, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{cx\left(a^2h^2(3fg-eh)-acg\left(fg^2-3h(eg-dh)\right)+c^2dg^3\right)+a\left(a^2fh^3-ach\left(3fg^2-h(3eg-dh)\right)-c^2g^2(eg-3dh)\right)}{a\sqrt{a+cx^2}\left(ah^2+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (a*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*c*h*(3*f*g^2 - h*(3*e*g - d*h)))
+ c*(c^2*d*g^3 + a^2*h^2*(3*f*g - e*h) - a*c*g*(f*g^2 - 3*h*(e*g - d*h)))*x
)/(a*(c*g^2 + a*h^2)^3*Sqrt[a + c*x^2]) - (h*(f*g^2 - e*g*h + d*h^2)*Sqrt[a
 + c*x^2])/(2*(c*g^2 + a*h^2)^2*(g + h*x)^2) - (h*(3*c*f*g^3 - c*g*h*(5*e*g
 - 7*d*h) - 2*a*h^2*(2*f*g - e*h))*Sqrt[a + c*x^2])/(2*(c*g^2 + a*h^2)^3*(g
 + h*x)) - ((2*a^2*f*h^4 - a*c*h^2*(11*f*g^2 - 9*e*g*h + 3*d*h^2) + 2*c^2*g
^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*ArcTanh[(a*h - c*g*x)/(Sqrt[c*g^2 + a*h^2]*
Sqrt[a + c*x^2]])/(2*(c*g^2 + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+cx^2\right)^{3/2}}\,dx = \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-e\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-e\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-e\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-e\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

$$= \frac{a\left(a^2fh^3-c^2g^2(eg-3dh)-ach\left(3fg^2-h(3eg-dh)\right)\right)+c\left(c^2dg^3+a^2h^2(3fg-e\right.}{a\left(cg^2+ah^2\right)^3\sqrt{a+cx^2}}$$

**Mathematica [A]** time = 1.18, size = 404, normalized size = 1.08

$$\frac{1}{2}\left(-\frac{\log\left(\sqrt{a+cx^2}\sqrt{ah^2+cg^2}+ah-cgx\right)\left(2a^2fh^4+ach^2\left(-3dh^2+9egh-11fg^2\right)+2c^2g^2\left(6dh^2-3egh+fg\right.\right.}{\left(ah^2+cg^2\right)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x]
```

```
[Out] (-((Sqrt[a + c*x^2]*((h*(c*g^2 + a*h^2)*(f*g^2 + h*(-(e*g) + d*h)))/(g + h*
x)^2 + (h*(3*c*f*g^3 + c*g*h*(-5*e*g + 7*d*h) + 2*a*h^2*(-2*f*g + e*h)))/(g
 + h*x) + (2*(-(a^3*f*h^3) - c^3*d*g^3*x + a*c^2*g*(f*g^2*x + e*g*(g - 3*h*
x) + 3*d*h*(-g + h*x)) + a^2*c*h*(3*f*g*(g - h*x) + h*(-3*e*g + d*h + e*h*x
))))/(a*(a + c*x^2))))/(c*g^2 + a*h^2)^3) + ((2*a^2*f*h^4 + a*c*h^2*(-11*f*
g^2 + 9*e*g*h - 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[g + h
*x])/(c*g^2 + a*h^2)^(7/2) - ((2*a^2*f*h^4 + a*c*h^2*(-11*f*g^2 + 9*e*g*h -
 3*d*h^2) + 2*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2))*Log[a*h - c*g*x + Sqrt[c
*g^2 + a*h^2]*Sqrt[a + c*x^2]])/(c*g^2 + a*h^2)^(7/2))/2
```

**fricas [B]** time = 33.90, size = 2853, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a^2*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^
2*d - 11*a^3*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*
h^2 - 6*a*c^3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g
^2*h^4 - (3*a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*
e*g^4*h^2 + 9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*
a^2*c^2*d - 2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^
2*c^2*e*g^3*h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9
*(a^2*c^2*d - a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*
c^2*f*g^5*h - 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*
a^3*c*f)*g^3*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(c*g^2 + a*h^2)*log(
(2*a*c*g*h*x - a*c*g^2 - 2*a^2*h^2 - (2*c^2*g^2 + a*c*h^2)*x^2 - 2*sqrt(c*g
^2 + a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a))/(h^2*x^2 + 2*g*h*x + g^2)) - 2*(
2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4 + a^4*e*g*h^6 + a
^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2*d + 5*a^3*c*f)*g^
4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*h^3 + 7*a^2*c^2*e*
g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 - (11*a*c^3*d - 5*a
^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^3 - (16*a*c^3*e*g^
5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d - 2*a*c^3*f)*g^6*h
- (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a^3*c*f)*g^2*h^5 -
(3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^2*c^2*e*g^4*h^3 +
13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^7 + (8*a*c^3*d - 1
1*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 - (11*a^3*c*d - 8*a^
4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3*g^8*h^2 + 6*a^4*c
^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h^2 + 4*a^2*c^4*g^6
*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10)*x^4 + 2*(a*c^5*g
^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*g^3*h^7 + a^5*c*g*
h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g^6*h^4 + 10*a^4*c^
2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*g^9*h + 4*a^3*c^3*
g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9)*x), -1/2*((2*a^2
*c^2*f*g^6 - 6*a^2*c^2*e*g^5*h + 9*a^3*c*e*g^3*h^3 + (12*a^2*c^2*d - 11*a^3
*c*f)*g^4*h^2 - (3*a^3*c*d - 2*a^4*f)*g^2*h^4 + (2*a*c^3*f*g^4*h^2 - 6*a*c^
3*e*g^3*h^3 + 9*a^2*c^2*e*g*h^5 + (12*a*c^3*d - 11*a^2*c^2*f)*g^2*h^4 - (3*
a^2*c^2*d - 2*a^3*c*f)*h^6)*x^4 + 2*(2*a*c^3*f*g^5*h - 6*a*c^3*e*g^4*h^2 +
9*a^2*c^2*e*g^2*h^4 + (12*a*c^3*d - 11*a^2*c^2*f)*g^3*h^3 - (3*a^2*c^2*d -
2*a^3*c*f)*g*h^5)*x^3 + (2*a*c^3*f*g^6 - 6*a*c^3*e*g^5*h + 3*a^2*c^2*e*g^3*
h^3 + 9*a^3*c*e*g*h^5 + 3*(4*a*c^3*d - 3*a^2*c^2*f)*g^4*h^2 + 9*(a^2*c^2*d
- a^3*c*f)*g^2*h^4 - (3*a^3*c*d - 2*a^4*f)*h^6)*x^2 + 2*(2*a^2*c^2*f*g^5*h
```

```
- 6*a^2*c^2*e*g^4*h^2 + 9*a^3*c*e*g^2*h^4 + (12*a^2*c^2*d - 11*a^3*c*f)*g^3
*h^3 - (3*a^3*c*d - 2*a^4*f)*g*h^5)*x)*sqrt(-c*g^2 - a*h^2)*arctan(sqrt(-c*
g^2 - a*h^2)*(c*g*x - a*h)*sqrt(c*x^2 + a)/(a*c*g^2 + a^2*h^2 + (c^2*g^2 +
a*c*h^2)*x^2)) + (2*a*c^3*e*g^7 - 10*a^2*c^2*e*g^5*h^2 - 11*a^3*c*e*g^3*h^4
 + a^4*e*g*h^6 + a^4*d*h^7 - 2*(3*a*c^3*d - 5*a^2*c^2*f)*g^6*h + (4*a^2*c^2
*d + 5*a^3*c*f)*g^4*h^3 + (11*a^3*c*d - 5*a^4*f)*g^2*h^5 - (11*a*c^3*e*g^4*
h^3 + 7*a^2*c^2*e*g^2*h^5 - 4*a^3*c*e*h^7 + (2*c^4*d - 5*a*c^3*f)*g^5*h^2 -
 (11*a*c^3*d - 5*a^2*c^2*f)*g^3*h^4 - (13*a^2*c^2*d - 10*a^3*c*f)*g*h^6)*x^
3 - (16*a*c^3*e*g^5*h^2 + 17*a^2*c^2*e*g^3*h^4 + a^3*c*e*g*h^6 + 4*(c^4*d -
 2*a*c^3*f)*g^6*h - (10*a*c^3*d - a^2*c^2*f)*g^4*h^3 - (17*a^2*c^2*d - 11*a
^3*c*f)*g^2*h^5 - (3*a^3*c*d - 2*a^4*f)*h^7)*x^2 - (2*a*c^3*e*g^6*h + 17*a^
2*c^2*e*g^4*h^3 + 13*a^3*c*e*g^2*h^5 - 2*a^4*e*h^7 + 2*(c^4*d - a*c^3*f)*g^
7 + (8*a*c^3*d - 11*a^2*c^2*f)*g^5*h^2 - (5*a^2*c^2*d + a^3*c*f)*g^3*h^4 -
(11*a^3*c*d - 8*a^4*f)*g*h^6)*x)*sqrt(c*x^2 + a))/(a^2*c^4*g^10 + 4*a^3*c^3
*g^8*h^2 + 6*a^4*c^2*g^6*h^4 + 4*a^5*c*g^4*h^6 + a^6*g^2*h^8 + (a*c^5*g^8*h
^2 + 4*a^2*c^4*g^6*h^4 + 6*a^3*c^3*g^4*h^6 + 4*a^4*c^2*g^2*h^8 + a^5*c*h^10
)*x^4 + 2*(a*c^5*g^9*h + 4*a^2*c^4*g^7*h^3 + 6*a^3*c^3*g^5*h^5 + 4*a^4*c^2*
g^3*h^7 + a^5*c*g*h^9)*x^3 + (a*c^5*g^10 + 5*a^2*c^4*g^8*h^2 + 10*a^3*c^3*g
^6*h^4 + 10*a^4*c^2*g^4*h^6 + 5*a^5*c*g^2*h^8 + a^6*h^10)*x^2 + 2*(a^2*c^4*
g^9*h + 4*a^3*c^3*g^7*h^3 + 6*a^4*c^2*g^5*h^5 + 4*a^5*c*g^3*h^7 + a^6*g*h^9
)*x)]
```

**giac [B]** time = 0.39, size = 1440, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((c^6*d*g^9 - a*c^5*f*g^9 - 6*a^2*c^4*d*g^5*h^4 + 6*a^3*c^3*f*g^5*h^4 - 8*a
^3*c^3*d*g^3*h^6 + 8*a^4*c^2*f*g^3*h^6 - 3*a^4*c^2*d*g*h^8 + 3*a^5*c*f*g*h^
8 + 3*a*c^5*g^8*h*e + 8*a^2*c^4*g^6*h^3*e + 6*a^3*c^3*g^4*h^5*e - a^5*c*h^9
*e)*x/(a*c^6*g^12 + 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^
6*h^6 + 15*a^5*c^2*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12) + (3*a*c^5*d*g^8*
h - 3*a^2*c^4*f*g^8*h + 8*a^2*c^4*d*g^6*h^3 - 8*a^3*c^3*f*g^6*h^3 + 6*a^3*c
^3*d*g^4*h^5 - 6*a^4*c^2*f*g^4*h^5 - a^5*c*d*h^9 + a^6*f*h^9 - a*c^5*g^9*e
+ 6*a^3*c^3*g^5*h^4*e + 8*a^4*c^2*g^3*h^6*e + 3*a^5*c*g*h^8*e)/(a*c^6*g^12
+ 6*a^2*c^5*g^10*h^2 + 15*a^3*c^4*g^8*h^4 + 20*a^4*c^3*g^6*h^6 + 15*a^5*c^2
*g^4*h^8 + 6*a^6*c*g^2*h^10 + a^7*h^12))/sqrt(c*x^2 + a) - (2*c^2*f*g^4 + 1
2*c^2*d*g^2*h^2 - 11*a*c*f*g^2*h^2 - 3*a*c*d*h^4 + 2*a^2*f*h^4 - 6*c^2*g^3*
h*e + 9*a*c*g*h^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + a))*h + sqrt(c)*g)/s
qrt(-c*g^2 - a*h^2))/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*a^2*c*g^2*h^4 + a^3*h^
6)*sqrt(-c*g^2 - a*h^2)) - (2*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*f*g^4*h +
 6*(sqrt(c)*x - sqrt(c*x^2 + a))^3*c^2*d*g^2*h^3 - 5*(sqrt(c)*x - sqrt(c*x^
2 + a))^3*a*c*f*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + a))^3*a*c*d*h^5 - 4*(sq
```

```
rt(c)*x - sqrt(c*x^2 + a))^3*c^2*g^3*h^2*e + 3*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a*c*g*h^4*e + 6*(sqrt(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*f*g^5 + 14*(sqr
t(c)*x - sqrt(c*x^2 + a))^2*c^(5/2)*d*g^3*h^2 - 11*(sqrt(c)*x - sqrt(c*x^2
+ a))^2*a*c^(3/2)*f*g^3*h^2 - 7*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a*c^(3/2)*d
*g*h^4 + 4*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*f*g*h^4 - 10*(sqrt(c
)*x - sqrt(c*x^2 + a))^2*c^(5/2)*g^4*h*e + 9*(sqrt(c)*x - sqrt(c*x^2 + a))^
2*a*c^(3/2)*g^2*h^3*e - 2*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^2*sqrt(c)*h^5*e
 - 10*(sqrt(c)*x - sqrt(c*x^2 + a))*a*c^2*f*g^4*h - 22*(sqrt(c)*x - sqrt(c*
x^2 + a))*a*c^2*d*g^2*h^3 + 11*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*f*g^2*h^
3 - (sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*d*h^5 + 16*(sqrt(c)*x - sqrt(c*x^2
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + a))*a^2*c*g*h^4*e + 3*a
^2*c^(3/2)*f*g^3*h^2 + 7*a^2*c^(3/2)*d*g*h^4 - 4*a^3*sqrt(c)*f*g*h^4 - 5*a^
2*c^(3/2)*g^2*h^3*e + 2*a^3*sqrt(c)*h^5*e)/((c^3*g^6 + 3*a*c^2*g^4*h^2 + 3*
a^2*c*g^2*h^4 + a^3*h^6)*((sqrt(c)*x - sqrt(c*x^2 + a))^2*h + 2*(sqrt(c)*x
- sqrt(c*x^2 + a))*sqrt(c)*g - a*h)^2)
```

**maple [B]** time = 0.02, size = 2584, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x)
```

```
[Out] f/h/(a*h^2+c*g^2)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)+5/
2/h*c*g^2/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*e-5/2/h^2*c*g^3/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+
g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+15/2*c^3*g^3/(a*h^2+c*g^2)^3/a/(-2*(x+g
/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-15/2*h*c^2*g^2/(a*h^2+c*
g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2
*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)
^(1/2))/(x+g/h))*d-15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/((a*h^2+c*g^2)/h^2)^(1/2)
*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f-13/2*c^2*g/(a
*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*d-
2/h/(a*h^2+c*g^2)/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*
x*c*e+15/2/h*c*g^2/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)
*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x
+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*f+5/h^2/(a*h^2+c*g^2)/a/(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*c*f*g-15/2/h*c^3*g^4/(a*
h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e+1
5/2/h^2*c^3*g^5/(a*h^2+c*g^2)^3/a/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*x*f+19/2/h*c^2*g^2/(a*h^2+c*g^2)^2/a/(-2*(x+g/h)*c*g/h+(x+g/h
)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*e-25/2/h^2*c^2*g^3/(a*h^2+c*g^2)^2/a/(-2*(
x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*x*f-1/h/(a*h^2+c*g^2)/(x+
g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+9/2*c*g/(a*h^
```

```
2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e+2/h^2/(
a*h^2+c*g^2)/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)
*f*g-15/2/h*c*g^2/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^
2)/h^2)^(1/2)*f-9/2*c*g/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x
+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g
/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*e+1/2/h^2/(a*h^2+c*g^2)/(
x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3
/(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2)*f*g^2-5/2*c*g/(a*h^2+c*g^2)^2/(x+g/h)/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a
*h^2+c*g^2)/h^2)^(1/2)*d+15/2*h*c^2*g^2/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+(
x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d+15/2/h*c^2*g^4/(a*h^2+c*g^2)^3/(-2*(x
+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*f+15/2*c^2*g^3/(a*h^2+c*g^
2)^3/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*(
(a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(
1/2))/(x+g/h))*e+3/2*h*c/(a*h^2+c*g^2)^2/((a*h^2+c*g^2)/h^2)^(1/2)*ln((-2*(
x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*
g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+g/h))*d-f/h/(a*h^2+c*g^2)/((a*
h^2+c*g^2)/h^2)^(1/2)*ln((-2*(x+g/h)*c*g/h+2*(a*h^2+c*g^2)/h^2+2*((a*h^2+c*
g^2)/h^2)^(1/2)*(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2))/(x+
g/h))-1/2/h/(a*h^2+c*g^2)/(x+g/h)^2/(-2*(x+g/h)*c*g/h+(x+g/h)^2*c+(a*h^2+c*
g^2)/h^2)^(1/2)*d-15/2*c^2*g^3/(a*h^2+c*g^2)^3/(-2*(x+g/h)*c*g/h+(x+g/h)^2*
c+(a*h^2+c*g^2)/h^2)^(1/2)*e-3/2*h*c/(a*h^2+c*g^2)^2/(-2*(x+g/h)*c*g/h+(x+g
/h)^2*c+(a*h^2+c*g^2)/h^2)^(1/2)*d
```

**maxima [B]** time = 1.02, size = 2254, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 15/2*c^3*f*g^5*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h^2 + 3*sqrt(c*x^2 + a)*a^2*c^2
*g^4*h^4 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^6 + sqrt(c*x^2 + a)*a^4*h^8) - 15/
2*c^3*e*g^4*x/(sqrt(c*x^2 + a)*a*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*
h^3 + 3*sqrt(c*x^2 + a)*a^3*c*g^2*h^5 + sqrt(c*x^2 + a)*a^4*h^7) + 15/2*c^3
*d*g^3*x/(sqrt(c*x^2 + a)*a*c^3*g^6 + 3*sqrt(c*x^2 + a)*a^2*c^2*g^4*h^2 + 3
*sqrt(c*x^2 + a)*a^3*c*g^2*h^4 + sqrt(c*x^2 + a)*a^4*h^6) + 15/2*c^2*f*g^4/
(sqrt(c*x^2 + a)*c^3*g^6*h + 3*sqrt(c*x^2 + a)*a*c^2*g^4*h^3 + 3*sqrt(c*x^2
 + a)*a^2*c*g^2*h^5 + sqrt(c*x^2 + a)*a^3*h^7) - 25/2*c^2*f*g^3*x/(sqrt(c*x
^2 + a)*a*c^2*g^4*h^2 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a
^3*h^6) - 15/2*c^2*e*g^3/(sqrt(c*x^2 + a)*c^3*g^6 + 3*sqrt(c*x^2 + a)*a*c^2
*g^4*h^2 + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^4 + sqrt(c*x^2 + a)*a^3*h^6) + 19/
2*c^2*e*g^2*x/(sqrt(c*x^2 + a)*a*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^
3 + sqrt(c*x^2 + a)*a^3*h^5) + 15/2*c^2*d*g^2/(sqrt(c*x^2 + a)*c^3*g^6/h +
3*sqrt(c*x^2 + a)*a*c^2*g^4*h + 3*sqrt(c*x^2 + a)*a^2*c*g^2*h^3 + sqrt(c*x^
```

```
2 + a)*a^3*h^5) - 5/2*c*f*g^3/(sqrt(c*x^2 + a)*c^2*g^4*h^2*x + 2*sqrt(c*x^2
 + a)*a*c*g^2*h^4*x + sqrt(c*x^2 + a)*a^2*h^6*x + sqrt(c*x^2 + a)*c^2*g^5*h
 + 2*sqrt(c*x^2 + a)*a*c*g^3*h^3 + sqrt(c*x^2 + a)*a^2*g*h^5) - 13/2*c^2*d*
g*x/(sqrt(c*x^2 + a)*a*c^2*g^4 + 2*sqrt(c*x^2 + a)*a^2*c*g^2*h^2 + sqrt(c*x
^2 + a)*a^3*h^4) + 5/2*c*e*g^2/(sqrt(c*x^2 + a)*c^2*g^4*h*x + 2*sqrt(c*x^2
+ a)*a*c*g^2*h^3*x + sqrt(c*x^2 + a)*a^2*h^5*x + sqrt(c*x^2 + a)*c^2*g^5 +
2*sqrt(c*x^2 + a)*a*c*g^3*h^2 + sqrt(c*x^2 + a)*a^2*g*h^4) - 15/2*c*f*g^2/(
sqrt(c*x^2 + a)*c^2*g^4*h + 2*sqrt(c*x^2 + a)*a*c*g^2*h^3 + sqrt(c*x^2 + a)
*a^2*h^5) + 5*c*f*g*x/(sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^
4) - 5/2*c*d*g/(sqrt(c*x^2 + a)*c^2*g^4*x + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2*x
 + sqrt(c*x^2 + a)*a^2*h^4*x + sqrt(c*x^2 + a)*c^2*g^5/h + 2*sqrt(c*x^2 + a
)*a*c*g^3*h + sqrt(c*x^2 + a)*a^2*g*h^3) + 9/2*c*e*g/(sqrt(c*x^2 + a)*c^2*g
^4 + 2*sqrt(c*x^2 + a)*a*c*g^2*h^2 + sqrt(c*x^2 + a)*a^2*h^4) - 1/2*f*g^2/(
sqrt(c*x^2 + a)*c*g^2*h^3*x^2 + sqrt(c*x^2 + a)*a*h^5*x^2 + 2*sqrt(c*x^2 +
a)*c*g^3*h^2*x + 2*sqrt(c*x^2 + a)*a*g*h^4*x + sqrt(c*x^2 + a)*c*g^4*h + sq
rt(c*x^2 + a)*a*g^2*h^3) - 2*c*e*x/(sqrt(c*x^2 + a)*a*c*g^2*h + sqrt(c*x^2
+ a)*a^2*h^3) - 3/2*c*d/(sqrt(c*x^2 + a)*c^2*g^4/h + 2*sqrt(c*x^2 + a)*a*c*
g^2*h + sqrt(c*x^2 + a)*a^2*h^3) + 1/2*e*g/(sqrt(c*x^2 + a)*c*g^2*h^2*x^2 +
 sqrt(c*x^2 + a)*a*h^4*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*h*x + 2*sqrt(c*x^2 + a
)*a*g*h^3*x + sqrt(c*x^2 + a)*c*g^4 + sqrt(c*x^2 + a)*a*g^2*h^2) + 2*f*g/(s
qrt(c*x^2 + a)*c*g^2*h^2*x + sqrt(c*x^2 + a)*a*h^4*x + sqrt(c*x^2 + a)*c*g^
3*h + sqrt(c*x^2 + a)*a*g*h^3) - 1/2*d/(sqrt(c*x^2 + a)*c*g^2*h*x^2 + sqrt(
c*x^2 + a)*a*h^3*x^2 + 2*sqrt(c*x^2 + a)*c*g^3*x + 2*sqrt(c*x^2 + a)*a*g*h^
2*x + sqrt(c*x^2 + a)*c*g^4/h + sqrt(c*x^2 + a)*a*g^2*h) - e/(sqrt(c*x^2 +
a)*c*g^2*h*x + sqrt(c*x^2 + a)*a*h^3*x + sqrt(c*x^2 + a)*c*g^3 + sqrt(c*x^2
 + a)*a*g*h^2) + f/(sqrt(c*x^2 + a)*c*g^2*h + sqrt(c*x^2 + a)*a*h^3) + 15/2
*c^2*f*g^4*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x
+ g)))/((a + c*g^2/h^2)^(7/2)*h^7) - 15/2*c^2*e*g^3*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^6)
 + 15/2*c^2*d*g^2*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(7/2)*h^5) - 15/2*c*f*g^2*arcsinh(c*g*x/(sqr
t(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)
*h^5) + 9/2*c*e*g*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*a
bs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^4) - 3/2*c*d*arcsinh(c*g*x/(sqrt(a*c
)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))/((a + c*g^2/h^2)^(5/2)*h^3)
 + f*arcsinh(c*g*x/(sqrt(a*c)*abs(h*x + g)) - a*h/(sqrt(a*c)*abs(h*x + g)))
/((a + c*g^2/h^2)^(3/2)*h^3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,{\left(c\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)),x)
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+a)**(3/2),x)
[Out] Timed out
```

# 3.115 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

```
[Out] 1/3*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(3/2)+1/3*(2*A*c+C*a)*x/a^2/c/(c*x^2+a
)^(1/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1814, 12, 191}

$$\frac{x(aC+2Ac)}{3a^2c\sqrt{a+cx^2}}-\frac{aB-x(Ac-aC)}{3ac\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(3*a*c*(a + c*x^2)^(3/2)) + ((2*A*c + a*C)*x)/(3*a^2
*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{5/2}}\,dx = -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}} - \frac{\int \frac{-2A-\frac{aC}{c}}{\left(a+cx^2\right)^{3/2}}\,dx}{3a}$$

$$= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(2Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{3ac}$$

$$= -\frac{aB-(Ac-aC)x}{3ac\left(a+cx^2\right)^{3/2}} + \frac{(2Ac+aC)x}{3a^2c\sqrt{a+cx^2}}$$

**Mathematica [A]** time = 0.03, size = 50, normalized size = 0.75

$$\frac{-a^2B+acx\left(3A+Cx^2\right)+2Ac^2x^3}{3a^2c\left(a+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 2*A*c^2*x^3 + a*c*x*(3*A + C*x^2))/(3*a^2*c*(a + c*x^2)^(3/2))
```

**fricas [A]** time = 0.79, size = 68, normalized size = 1.01

$$\frac{\left(3\,Aacx+\left(Cac+2\,Ac^2\right)x^3-Ba^2\right)\sqrt{cx^2+a}}{3\left(a^2c^3x^4+2\,a^3c^2x^2+a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*a*c*x + (C*a*c + 2*A*c^2)*x^3 - B*a^2)*sqrt(c*x^2 + a)/(a^2*c^3*x^
4 + 2*a^3*c^2*x^2 + a^4*c)
```

**giac [A]** time = 0.21, size = 48, normalized size = 0.72

$$\frac{x\left(\frac{3\,A}{a}+\frac{\left(Cac+2\,Ac^2\right)x^2}{a^2c}\right)-\frac{B}{c}}{3\left(cx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x*(3*A/a + (C*a*c + 2*A*c^2)*x^2/(a^2*c)) - B/c)/(c*x^2 + a)^(3/2)
```

**maple [A]** time = 0.00, size = 47, normalized size = 0.70

$$\frac{2A\,c^{2}x^{3}+Cac\,x^{3}+3Axac-B\,a^{2}}{3\left(c\,x^{2}+a\right)^{\frac{3}{2}}a^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*(2*A*c^2*x^3+C*a*c*x^3+3*A*a*c*x-B*a^2)/(c*x^2+a)^(3/2)/a^2/c
```

**maxima [A]** time = 0.43, size = 83, normalized size = 1.24

$$\frac{2\,Ax}{3\,\sqrt{cx^{2}+a}\,a^{2}}+\frac{Ax}{3\left(cx^{2}+a\right)^{\frac{3}{2}}a}-\frac{Cx}{3\left(cx^{2}+a\right)^{\frac{3}{2}}c}+\frac{Cx}{3\,\sqrt{cx^{2}+a}\,ac}-\frac{B}{3\left(cx^{2}+a\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(c*x^2 + a)*a^2) + 1/3*A*x/((c*x^2 + a)^(3/2)*a) - 1/3*C*x/((c
*x^2 + a)^(3/2)*c) + 1/3*C*x/(sqrt(c*x^2 + a)*a*c) - 1/3*B/((c*x^2 + a)^(3/
2)*c)
```

**mupad [B]** time = 4.22, size = 59, normalized size = 0.88

$$\frac{2\,A\,c\,x\,\left(c\,x^{2}+a\right)-C\,a^{2}\,x-B\,a^{2}+C\,a\,x\,\left(c\,x^{2}+a\right)+A\,a\,c\,x}{3\,a^{2}\,c\,{\left(c\,x^{2}+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(5/2),x)
```

```
[Out] (2*A*c*x*(a + c*x^2) - C*a^2*x - B*a^2 + C*a*x*(a + c*x^2) + A*a*c*x)/(3*a^
2*c*(a + c*x^2)^(3/2))
```

**sympy** [A] time = 17.13, size = 194, normalized size = 2.90

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ac\sqrt{a+cx^2}+3c^2x^2\sqrt{a+cx^2}} & \text{for } c \neq \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherv}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 + c*x**2
/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
 c*x**2/a))) + B*Piecewise((-1/(3*a*c*sqrt(a + c*x**2) + 3*c**2*x**2*sqrt(a
 + c*x**2)), Ne(c, 0)), (x**2/(2*a**(5/2)), True)) + C*x**3/(3*a**(5/2)*sqr
t(1 + c*x**2/a) + 3*a**(3/2)*c*x**2*sqrt(1 + c*x**2/a))
```

# 3.116 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx$

**Optimal**. Leaf size=97

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

```
[Out] 1/5*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(5/2)+1/15*(4*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(3/2)+2/15*(4*A*c+C*a)*x/a^3/c/(c*x^2+a)^(1/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{2x(aC+4Ac)}{15a^3c\sqrt{a+cx^2}}+\frac{x(aC+4Ac)}{15a^2c\left(a+cx^2\right)^{3/2}}-\frac{aB-x(Ac-aC)}{5ac\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(5*a*c*(a + c*x^2)^(5/2)) + ((4*A*c + a*C)*x)/(15*a^
2*c*(a + c*x^2)^(3/2)) + (2*(4*A*c + a*C)*x)/(15*a^3*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{7/2}}\,dx &= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}-\frac{\int \frac{-4A-\frac{aC}{c}}{\left(a+cx^2\right)^{5/2}}\,dx}{5a}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{5ac}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{(2(4Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{15a^2c}\\
&= -\frac{aB-(Ac-aC)x}{5ac\left(a+cx^2\right)^{5/2}}+\frac{(4Ac+aC)x}{15a^2c\left(a+cx^2\right)^{3/2}}+\frac{2(4Ac+aC)x}{15a^3c\sqrt{a+cx^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 71, normalized size = 0.73

$$\frac{-3a^3B+5a^2cx\left(3A+Cx^2\right)+2ac^2x^3\left(10A+Cx^2\right)+8Ac^3x^5}{15a^3c\left(a+cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(7/2),x]
```

```
[Out] (-3*a^3*B + 8*A*c^3*x^5 + 5*a^2*c*x*(3*A + C*x^2) + 2*a*c^2*x^3*(10*A + C*x
^2))/(15*a^3*c*(a + c*x^2)^(5/2))
```

**fricas [A]** time = 0.89, size = 103, normalized size = 1.06

$$\frac{\left(2\left(Cac^2+4\,Ac^3\right)x^5+15\,Aa^2cx-3\,Ba^3+5\left(Ca^2c+4\,Aac^2\right)x^3\right)\sqrt{cx^2+a}}{15\left(a^3c^4x^6+3\,a^4c^3x^4+3\,a^5c^2x^2+a^6c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(C*a*c^2 + 4*A*c^3)*x^5 + 15*A*a^2*c*x - 3*B*a^3 + 5*(C*a^2*c + 4*A
*a*c^2)*x^3)*sqrt(c*x^2 + a)/(a^3*c^4*x^6 + 3*a^4*c^3*x^4 + 3*a^5*c^2*x^2 +
 a^6*c)
```

**giac [A]** time = 0.26, size = 80, normalized size = 0.82

$$\frac{\left(x^2\left(\frac{2\left(Cac^3+4\,Ac^4\right)x^2}{a^3c^2}+\frac{5\left(Ca^2c^2+4\,Aac^3\right)}{a^3c^2}\right)+\frac{15\,A}{a}\right)x-\frac{3\,B}{c}}{15\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((x^2*(2*(C*a*c^3 + 4*A*c^4)*x^2/(a^3*c^2) + 5*(C*a^2*c^2 + 4*A*a*c^3)
/(a^3*c^2)) + 15*A/a)*x - 3*B/c)/(c*x^2 + a)^(5/2)
```

**maple [A]** time = 0.00, size = 72, normalized size = 0.74

$$\frac{8A\,c^3x^5+2Ca\,c^2x^5+20Aa\,c^2x^3+5C\,a^2c\,x^3+15Ax\,a^2c-3B\,a^3}{15\left(c\,x^2+a\right)^{\frac{5}{2}}\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*(8*A*c^3*x^5+2*C*a*c^2*x^5+20*A*a*c^2*x^3+5*C*a^2*c*x^3+15*A*a^2*c*x-3
*B*a^3)/(c*x^2+a)^(5/2)/a^3/c
```

**maxima [A]** time = 0.44, size = 118, normalized size = 1.22

$$\frac{8\,Ax}{15\,\sqrt{cx^2+a}\,a^3}+\frac{4\,Ax}{15\left(cx^2+a\right)^{\frac{3}{2}}a^2}+\frac{Ax}{5\left(cx^2+a\right)^{\frac{5}{2}}a}-\frac{Cx}{5\left(cx^2+a\right)^{\frac{5}{2}}c}+\frac{2\,Cx}{15\,\sqrt{cx^2+a}\,a^2c}+\frac{Cx}{15\left(cx^2+a\right)^{\frac{3}{2}}ac}-\frac{B}{5\left(cx^2+a\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*A*x/(sqrt(c*x^2 + a)*a^3) + 4/15*A*x/((c*x^2 + a)^(3/2)*a^2) + 1/5*A*x
/((c*x^2 + a)^(5/2)*a) - 1/5*C*x/((c*x^2 + a)^(5/2)*c) + 2/15*C*x/(sqrt(c*x
```

```
^2 + a)*a^2*c) + 1/15*C*x/((c*x^2 + a)^(3/2)*a*c) - 1/5*B/((c*x^2 + a)^(5/2
)*c)
```

**mupad [B]** time = 4.28, size = 93, normalized size = 0.96

$$\frac{8\,A\,c\,x\,{\left(c\,x^2+a\right)}^2-3\,C\,a^3\,x-3\,B\,a^3+2\,C\,a\,x\,{\left(c\,x^2+a\right)}^2+C\,a^2\,x\,\left(c\,x^2+a\right)+3\,A\,a^2\,c\,x+4\,A\,a\,c\,x\,\left(c\,x^2+a\right)}{15\,a^3\,c\,{\left(c\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(7/2),x)
```

```
[Out] (8*A*c*x*(a + c*x^2)^2 - 3*C*a^3*x - 3*B*a^3 + 2*C*a*x*(a + c*x^2)^2 + C*a^
2*x*(a + c*x^2) + 3*A*a^2*c*x + 4*A*a*c*x*(a + c*x^2))/(15*a^3*c*(a + c*x^2
)^(5/2))
```

**sympy [B]** time = 37.22, size = 638, normalized size = 6.58

$$A\left(\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}+\frac{}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(7/2),x)
```

```
[Out] A*(15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1
+ c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3
*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a**(17/2)*sqrt(1 + c*x**2/a)
 + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 +
 c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5
/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a)
+ 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1
 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(
15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a))) + B*Piecewise((-1/(5*a**2*c*
sqrt(a + c*x**2) + 10*a*c**2*x**2*sqrt(a + c*x**2) + 5*c**3*x**4*sqrt(a + c
*x**2)), Ne(c, 0)), (x**2/(2*a**(7/2)), True)) + C*(5*a*x**3/(15*a**(9/2)*s
qrt(1 + c*x**2/a) + 30*a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**
2*x**4*sqrt(1 + c*x**2/a)) + 2*c*x**5/(15*a**(9/2)*sqrt(1 + c*x**2/a) + 30*
a**(7/2)*c*x**2*sqrt(1 + c*x**2/a) + 15*a**(5/2)*c**2*x**4*sqrt(1 + c*x**2/
a)))
```

# 3.117 $\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}} dx$

**Optimal**. Leaf size=127

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

```
[Out] 1/7*(-a*B+(A*c-C*a)*x)/a/c/(c*x^2+a)^(7/2)+1/35*(6*A*c+C*a)*x/a^2/c/(c*x^2+
a)^(5/2)+4/105*(6*A*c+C*a)*x/a^3/c/(c*x^2+a)^(3/2)+8/105*(6*A*c+C*a)*x/a^4/
c/(c*x^2+a)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1814, 12, 192, 191}

$$\frac{8x(aC+6Ac)}{105a^4c\sqrt{a+cx^2}}+\frac{4x(aC+6Ac)}{105a^3c\left(a+cx^2\right)^{3/2}}+\frac{x(aC+6Ac)}{35a^2c\left(a+cx^2\right)^{5/2}}-\frac{aB-x(Ac-aC)}{7ac\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] -(a*B - (A*c - a*C)*x)/(7*a*c*(a + c*x^2)^(7/2)) + ((6*A*c + a*C)*x)/(35*a^
2*c*(a + c*x^2)^(5/2)) + (4*(6*A*c + a*C)*x)/(105*a^3*c*(a + c*x^2)^(3/2))
+ (8*(6*A*c + a*C)*x)/(105*a^4*c*Sqrt[a + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+cx^2\right)^{9/2}}\,dx = -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} - \frac{\int \frac{-6A-\frac{aC}{c}}{\left(a+cx^2\right)^{7/2}}\,dx}{7a}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)\int \frac{1}{\left(a+cx^2\right)^{7/2}}\,dx}{7ac}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{(4(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)^{5/2}}\,dx}{35a^2c}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{(8(6Ac+aC))\int \frac{1}{\left(a+cx^2\right)}}{105a^3c}$$

$$= -\frac{aB-(Ac-aC)x}{7ac\left(a+cx^2\right)^{7/2}} + \frac{(6Ac+aC)x}{35a^2c\left(a+cx^2\right)^{5/2}} + \frac{4(6Ac+aC)x}{105a^3c\left(a+cx^2\right)^{3/2}} + \frac{8(6Ac+aC)x}{105a^4c\sqrt{a+cx^2}}$$

**Mathematica [A]** time = 0.06, size = 92, normalized size = 0.72

$$\frac{-15a^4B+35a^3cx\left(3A+Cx^2\right)+14a^2c^2x^3\left(15A+2Cx^2\right)+8ac^3x^5\left(21A+Cx^2\right)+48Ac^4x^7}{105a^4c\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + c*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*c^4*x^7 + 35*a^3*c*x*(3*A + C*x^2) + 8*a*c^3*x^5*(21*A +
C*x^2) + 14*a^2*c^2*x^3*(15*A + 2*C*x^2))/(105*a^4*c*(a + c*x^2)^(7/2))
```

**fricas [A]** time = 0.85, size = 137, normalized size = 1.08

$$\frac{\left(8\left(Cac^{3}+6\,Ac^{4}\right)x^{7}+105\,Aa^{3}cx+28\left(Ca^{2}c^{2}+6\,Aac^{3}\right)x^{5}-15\,Ba^{4}+35\left(Ca^{3}c+6\,Aa^{2}c^{2}\right)x^{3}\right)\sqrt{cx^{2}+a}}{105\left(a^{4}c^{5}x^{8}+4\,a^{5}c^{4}x^{6}+6\,a^{6}c^{3}x^{4}+4\,a^{7}c^{2}x^{2}+a^{8}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*c^3 + 6*A*c^4)*x^7 + 105*A*a^3*c*x + 28*(C*a^2*c^2 + 6*A*a*c^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*c + 6*A*a^2*c^2)*x^3)*sqrt(c*x^2 + a)/(a^4*c^
5*x^8 + 4*a^5*c^4*x^6 + 6*a^6*c^3*x^4 + 4*a^7*c^2*x^2 + a^8*c)
```

**giac [A]** time = 0.27, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^{2}\left(\frac{2\left(Cac^{5}+6\,Ac^{6}\right)x^{2}}{a^{4}c^{3}}+\frac{7\left(Ca^{2}c^{4}+6\,Aac^{5}\right)}{a^{4}c^{3}}\right)+\frac{35\left(Ca^{3}c^{3}+6\,Aa^{2}c^{4}\right)}{a^{4}c^{3}}\right)x^{2}+\frac{105\,A}{a}\right)x-\frac{15\,B}{c}}{105\left(cx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*c^5 + 6*A*c^6)*x^2/(a^4*c^3) + 7*(C*a^2*c^4 + 6*A*a*
c^5)/(a^4*c^3)) + 35*(C*a^3*c^3 + 6*A*a^2*c^4)/(a^4*c^3))*x^2 + 105*A/a)*x
- 15*B/c)/(c*x^2 + a)^(7/2)
```

**maple [A]** time = 0.01, size = 96, normalized size = 0.76

$$\frac{48A\,c^{4}x^{7}+8Ca\,c^{3}x^{7}+168Aa\,c^{3}x^{5}+28C\,a^{2}c^{2}x^{5}+210A\,a^{2}c^{2}x^{3}+35C\,a^{3}c\,x^{3}+105Ax\,a^{3}c-15B\,a^{4}}{105\left(c\,x^{2}+a\right)^{\frac{7}{2}}a^{4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/105*(48*A*c^4*x^7+8*C*a*c^3*x^7+168*A*a*c^3*x^5+28*C*a^2*c^2*x^5+210*A*a^
2*c^2*x^3+35*C*a^3*c*x^3+105*A*a^3*c*x-15*B*a^4)/(c*x^2+a)^(7/2)/a^4/c
```

**maxima [A]** time = 0.45, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\sqrt{cx^{2}+a}\,a^{4}}+\frac{8\,Ax}{35\left(cx^{2}+a\right)^{\frac{3}{2}}a^{3}}+\frac{6\,Ax}{35\left(cx^{2}+a\right)^{\frac{5}{2}}a^{2}}+\frac{Ax}{7\left(cx^{2}+a\right)^{\frac{7}{2}}a}-\frac{Cx}{7\left(cx^{2}+a\right)^{\frac{7}{2}}c}+\frac{8\,Cx}{105\sqrt{cx^{2}+a}\,a^{3}c}+\frac{4\,C}{105\left(cx^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(c*x^2 + a)*a^4) + 8/35*A*x/((c*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((c*x^2 + a)^(7/2)*a) - 1/7*C*x/((c*x^
2 + a)^(7/2)*c) + 8/105*C*x/(sqrt(c*x^2 + a)*a^3*c) + 4/105*C*x/((c*x^2 + a
)^(3/2)*a^2*c) + 1/35*C*x/((c*x^2 + a)^(5/2)*a*c) - 1/7*B/((c*x^2 + a)^(7/2
)*c)
```

**mupad [B]** time = 4.37, size = 115, normalized size = 0.91

$$\frac{x\,\left(6\,A\,c+C\,a\right)}{35\,a^2\,c\,{\left(c\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{7\,c}-x\,\left(\frac{A}{7\,a}-\frac{C}{7\,c}\right)}{{\left(c\,x^2+a\right)}^{7/2}}+\frac{x\,\left(24\,A\,c+4\,C\,a\right)}{105\,a^3\,c\,{\left(c\,x^2+a\right)}^{3/2}}+\frac{x\,\left(48\,A\,c+8\,C\,a\right)}{105\,a^4\,c\,\sqrt{c\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + c*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*c + C*a))/(35*a^2*c*(a + c*x^2)^(5/2)) - (B/(7*c) - x*(A/(7*a) - C/
(7*c)))/(a + c*x^2)^(7/2) + (x*(24*A*c + 4*C*a))/(105*a^3*c*(a + c*x^2)^(3/
2)) + (x*(48*A*c + 8*C*a))/(105*a^4*c*(a + c*x^2)^(1/2))
```

**sympy [B]** time = 78.30, size = 1880, normalized size = 14.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*
c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) +
 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt
(1 + c*x**2/a)) + 175*a**13*c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a
**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**
2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(
25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2*x**5/(35*a**(37/2)*sq
rt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*
c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) +
 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt
(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c*
*3*x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*
```

```
x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x
**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a
**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c
*x**2/a)) + 286*a**10*c**4*x**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(
35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a
) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sq
rt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/
2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**11/(35*a**(37/2)*sqrt(
1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**
2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 52
5*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1
+ c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x*
*13/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 + c*x**2
/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**(31/2)*c**3*x**6*
sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqrt(1 + c*x**2/a) + 210*a**(2
7/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**
2/a))) + B*Piecewise((-1/(7*a**3*c*sqrt(a + c*x**2) + 21*a**2*c**2*x**2*sqr
t(a + c*x**2) + 21*a*c**3*x**4*sqrt(a + c*x**2) + 7*c**4*x**6*sqrt(a + c*x*
*2)), Ne(c, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a**(15
/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x**2/
a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 63*a**4*c*x**5/(105*a**(
19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) + 630*a*
*(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(1 + c*x
**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 36*a**3*c**2*x**7/(1
05*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**2/a) +
 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6*sqrt(
1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)) + 8*a**2*c**3*x
**9/(105*a**(19/2)*sqrt(1 + c*x**2/a) + 420*a**(17/2)*c*x**2*sqrt(1 + c*x**
2/a) + 630*a**(15/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 420*a**(13/2)*c**3*x**6
*sqrt(1 + c*x**2/a) + 105*a**(11/2)*c**4*x**8*sqrt(1 + c*x**2/a)))
```

# 3.118 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx$

**Optimal**. Leaf size=106

$$\frac{2}{15}\sqrt{3x^2+2}\,(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}\,(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

```
[Out] 5/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)-19/540*(1+2*x)^2*(3*x^2+2)^(1/2)+13/60*(
1+2*x)^3*(3*x^2+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2+2)^(1/2)-1/810*(3937+2073*x)
*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 833, 780, 215}

$$\frac{2}{15}\sqrt{3x^2+2}\,(2x+1)^4+\frac{13}{60}\sqrt{3x^2+2}\,(2x+1)^3-\frac{19}{540}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{810}(2073x+3937)\sqrt{3x^2+2}+\frac{5\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-19*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/540 + (13*(1 + 2*x)^3*Sqrt[2 + 3*x^2])/60
 + (2*(1 + 2*x)^4*Sqrt[2 + 3*x^2])/15 - ((3937 + 2073*x)*Sqrt[2 + 3*x^2])/8
10 + (5*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-68+156x)}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{1}{720}\int \frac{(-2688-228x)(1+2x}{\sqrt{2+3x^2}}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} + \frac{\int}{}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{81}$$

$$= -\frac{19}{540}(1+2x)^2\sqrt{2+3x^2} + \frac{13}{60}(1+2x)^3\sqrt{2+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2+3x^2} - \frac{1}{81}$$

**Mathematica [A]** time = 0.07, size = 54, normalized size = 0.51

$$\frac{1}{405}\left(\sqrt{3x^2+2}\left(864x^4+2430x^3+2292x^2-135x-1841\right)+225\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-1841 - 135*x + 2292*x^2 + 2430*x^3 + 864*x^4) + 225*Sqrt
[3]*ArcSinh[Sqrt[3/2]*x])/405
```

**fricas [A]** time = 1.01, size = 60, normalized size = 0.57

$$\frac{1}{405}\left(864\,x^4+2430\,x^3+2292\,x^2-135\,x-1841\right)\sqrt{3\,x^2+2}+\frac{5}{18}\,\sqrt{3}\,\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/405*(864*x^4 + 2430*x^3 + 2292*x^2 - 135*x - 1841)*sqrt(3*x^2 + 2) + 5/18
*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

**giac [A]** time = 0.23, size = 54, normalized size = 0.51

$$\frac{1}{405}\,(3\,(2\,(9\,(16\,x+45)x+382)x-45)x-1841)\sqrt{3\,x^2+2}\,-\,\frac{5}{9}\,\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(3*(2*(9*(16*x + 45)*x + 382)*x - 45)*x - 1841)*sqrt(3*x^2 + 2) - 5/9
*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2))
```

**maple [A]** time = 0.01, size = 79, normalized size = 0.75

$$\frac{32\sqrt{3x^2+2}\;x^4}{15}+6\sqrt{3x^2+2}\;x^3+\frac{764\sqrt{3x^2+2}\;x^2}{135}-\frac{\sqrt{3x^2+2}\;x}{3}+\frac{5\sqrt{3}\;\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}-\frac{1841\sqrt{3x^2+2}}{405}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2+2)^(1/2)+764/135*x^2*(3*x^2+2)^(1/2)-1841/405*(3*x^2+2)^(1
/2)+6*x^3*(3*x^2+2)^(1/2)-1/3*x*(3*x^2+2)^(1/2)+5/9*arcsinh(1/2*x*6^(1/2))*
3^(1/2)
```

**maxima [A]** time = 0.95, size = 78, normalized size = 0.74

$$\frac{32}{15}\,\sqrt{3\,x^2+2}\,x^4+6\,\sqrt{3\,x^2+2}\,x^3+\frac{764}{135}\,\sqrt{3\,x^2+2}\,x^2-\frac{1}{3}\,\sqrt{3\,x^2+2}\,x+\frac{5}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{1841}{405}\,\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 + 2)*x^4 + 6*sqrt(3*x^2 + 2)*x^3 + 764/135*sqrt(3*x^2 + 2)
*x^2 - 1/3*sqrt(3*x^2 + 2)*x + 5/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 1841/40
5*sqrt(3*x^2 + 2)
```

**mupad [B]** time = 0.05, size = 45, normalized size = 0.42

$$\frac{5\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^4}{5}+18\,x^3+\frac{764\,x^2}{45}-x-\frac{1841}{135}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (5*3^(1/2)*asinh((6^(1/2)*x)/2))/9 + (3^(1/2)*(x^2 + 2/3)^(1/2)*((764*x^2)/
45 - x + 18*x^3 + (32*x^4)/5 - 1841/135))/3
```

**sympy [A]** time = 2.20, size = 94, normalized size = 0.89

$$\frac{32x^4\sqrt{3x^2+2}}{15}+6x^3\sqrt{3x^2+2}+\frac{764x^2\sqrt{3x^2+2}}{135}-\frac{x\sqrt{3x^2+2}}{3}-\frac{1841\sqrt{3x^2+2}}{405}+\frac{5\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 32*x**4*sqrt(3*x**2 + 2)/15 + 6*x**3*sqrt(3*x**2 + 2) + 764*x**2*sqrt(3*x**
2 + 2)/135 - x*sqrt(3*x**2 + 2)/3 - 1841*sqrt(3*x**2 + 2)/405 + 5*sqrt(3)*a
sinh(sqrt(6)*x/2)/9
```

# 3.119 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2+3x^2}}\,dx$

**Optimal**. Leaf size=82

$$\frac{1}{6}\sqrt{3x^2+2}\,(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

```
[Out] -arcsinh(1/2*x*6^(1/2))*3^(1/2)+5/18*(1+2*x)^2*(3*x^2+2)^(1/2)+1/6*(1+2*x)^
3*(3*x^2+2)^(1/2)-1/27*(61+3*x)*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 833, 780, 215}

$$\frac{1}{6}\sqrt{3x^2+2}\,(2x+1)^3+\frac{5}{18}\sqrt{3x^2+2}\,(2x+1)^2-\frac{1}{27}(3x+61)\sqrt{3x^2+2}-\sqrt{3}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (5*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/18 + ((1 + 2*x)^3*Sqrt[2 + 3*x^2])/6 - ((61
 + 3*x)*Sqrt[2 + 3*x^2])/27 - Sqrt[3]*ArcSinh[Sqrt[3/2]*x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
```

```
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{48}\int \frac{(1+2x)^2(-48+120x)}{\sqrt{2+3x^2}}\,dx \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}+\frac{1}{432}\int \frac{(-1392-144x)(1+2x)}{\sqrt{2+3x^2}} \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-3\int \frac{}{\sqrt{}} \\
&= \frac{5}{18}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{6}(1+2x)^3\sqrt{2+3x^2}-\frac{1}{27}(61+3x)\sqrt{2+3x^2}-\sqrt{3}\sin
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 48, normalized size = 0.59

$$\frac{1}{27}\sqrt{3x^2+2}\left(36x^3+84x^2+54x-49\right)-\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (Sqrt[2 + 3*x^2]*(-49 + 54*x + 84*x^2 + 36*x^3))/27 - Sqrt[3]*ArcSinh[Sqrt[
3/2]*x]
```

**fricas** [A] time = 0.90, size = 54, normalized size = 0.66

$$\frac{1}{27}\left(36\,x^{3}+84\,x^{2}+54\,x-49\right)\sqrt{3\,x^{2}+2}+\frac{1}{2}\,\sqrt{3}\,\log\left(\sqrt{3}\,\sqrt{3\,x^{2}+2}\,x-3\,x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/27*(36*x^3 + 84*x^2 + 54*x - 49)*sqrt(3*x^2 + 2) + 1/2*sqrt(3)*log(sqrt(3
)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1)
```

**giac** [A] time = 0.20, size = 48, normalized size = 0.59

$$\frac{1}{27}\,(6\,(2\,(3\,x+7)x+9)x-49)\sqrt{3\,x^{2}+2}+\sqrt{3}\,\log\left(-\sqrt{3}\,x+\sqrt{3\,x^{2}+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(6*(2*(3*x + 7)*x + 9)*x - 49)*sqrt(3*x^2 + 2) + sqrt(3)*log(-sqrt(3)*
x + sqrt(3*x^2 + 2))
```

**maple** [A] time = 0.00, size = 65, normalized size = 0.79

$$\frac{4\sqrt{3x^{2}+2}\;x^{3}}{3}+\frac{28\sqrt{3x^{2}+2}\;x^{2}}{9}+2\sqrt{3x^{2}+2}\;x-\sqrt{3}\;\operatorname{arcsinh}\left(\frac{\sqrt{6}\;x}{2}\right)-\frac{49\sqrt{3x^{2}+2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 4/3*(3*x^2+2)^(1/2)*x^3+2*(3*x^2+2)^(1/2)*x-arcsinh(1/2*6^(1/2)*x)*3^(1/2)+
28/9*(3*x^2+2)^(1/2)*x^2-49/27*(3*x^2+2)^(1/2)
```

**maxima** [A] time = 0.96, size = 64, normalized size = 0.78

$$\frac{4}{3}\,\sqrt{3\,x^{2}+2}\,x^{3}+\frac{28}{9}\,\sqrt{3\,x^{2}+2}\,x^{2}+2\,\sqrt{3\,x^{2}+2}\,x-\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{49}{27}\,\sqrt{3\,x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 + 2)*x^3 + 28/9*sqrt(3*x^2 + 2)*x^2 + 2*sqrt(3*x^2 + 2)*x -
sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 49/27*sqrt(3*x^2 + 2)
```

**mupad [B]** time = 4.10, size = 40, normalized size = 0.49

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(4\,x^3+\frac{28\,x^2}{3}+6\,x-\frac{49}{9}\right)}{3}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(6*x + (28*x^2)/3 + 4*x^3 - 49/9))/3 - 3^(1/2)*a
sinh((6^(1/2)*x)/2)
```

**sympy [A]** time = 1.19, size = 75, normalized size = 0.91

$$\frac{4x^3\sqrt{3x^2+2}}{3}+\frac{28x^2\sqrt{3x^2+2}}{9}+2x\sqrt{3x^2+2}-\frac{49\sqrt{3x^2+2}}{27}-\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 4*x**3*sqrt(3*x**2 + 2)/3 + 28*x**2*sqrt(3*x**2 + 2)/9 + 2*x*sqrt(3*x**2 +
2) - 49*sqrt(3*x**2 + 2)/27 - sqrt(3)*asinh(sqrt(6)*x/2)
```

## 3.120 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}} \, dx$

**Optimal**. Leaf size=62

$$\frac{2}{9}\sqrt{3x^2+2}\,(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -7/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2+2)^(1/2)+7/27*(1+3
*x)*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1654, 780, 215}

$$\frac{2}{9}\sqrt{3x^2+2}\,(2x+1)^2+\frac{7}{27}(3x+1)\sqrt{3x^2+2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 + 3*x^2])/9 + (7*(1 + 3*x)*Sqrt[2 + 3*x^2])/27 - (7*A
rcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2+3x^2}}\,dx &= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-28+84x)}{\sqrt{2+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2+3x^2}+\frac{7}{27}(1+3x)\sqrt{2+3x^2}-\frac{7\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 44, normalized size = 0.71

$$\frac{1}{27}\left(\sqrt{3x^2+2}\left(24x^2+45x+13\right)-21\sqrt{3}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 + 3*x^2],x]
[Out] (Sqrt[2 + 3*x^2]*(13 + 45*x + 24*x^2) - 21*Sqrt[3]*ArcSinh[Sqrt[3/2]*x])/27
```

**fricas [A]** time = 0.80, size = 49, normalized size = 0.79

$$\frac{1}{27}\left(24\,x^2+45\,x+13\right)\sqrt{3\,x^2+2}+\frac{7}{18}\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/27*(24*x^2 + 45*x + 13)*sqrt(3*x^2 + 2) + 7/18*sqrt(3)*log(sqrt(3)*sqrt(3
*x^2 + 2)*x - 3*x^2 - 1)
```

**giac [A]** time = 0.19, size = 44, normalized size = 0.71

$$\frac{1}{27}\left(3\left(8\,x+15\right)x+13\right)\sqrt{3\,x^2+2}+\frac{7}{9}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/27*(3*(8*x + 15)*x + 13)*sqrt(3*x^2 + 2) + 7/9*sqrt(3)*log(-sqrt(3)*x + s
qrt(3*x^2 + 2))
```

**maple** [A] time = 0.01, size = 51, normalized size = 0.82

$$\frac{8\sqrt{3x^2+2}\,x^2}{9}+\frac{5\sqrt{3x^2+2}\,x}{3}-\frac{7\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{13\sqrt{3x^2+2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2+2)^(1/2)*x^2+13/27*(3*x^2+2)^(1/2)+5/3*(3*x^2+2)^(1/2)*x-7/9*arc
sinh(1/2*6^(1/2)*x)*3^(1/2)
```

**maxima** [A] time = 0.96, size = 50, normalized size = 0.81

$$\frac{8}{9}\sqrt{3\,x^2+2}\,x^2+\frac{5}{3}\sqrt{3\,x^2+2}\,x-\frac{7}{9}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{13}{27}\sqrt{3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 + 2)*x^2 + 5/3*sqrt(3*x^2 + 2)*x - 7/9*sqrt(3)*arcsinh(1/2*s
qrt(6)*x) + 13/27*sqrt(3*x^2 + 2)
```

**mupad** [B] time = 0.03, size = 35, normalized size = 0.56

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{8\,x^2}{3}+5\,x+\frac{13}{9}\right)}{3}-\frac{7\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(1/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(5*x + (8*x^2)/3 + 13/9))/3 - (7*3^(1/2)*asinh((
6^(1/2)*x)/2))/9
```

**sympy** [A] time = 0.55, size = 63, normalized size = 1.02

$$\frac{8x^2\sqrt{3x^2+2}}{9}+\frac{5x\sqrt{3x^2+2}}{3}+\frac{13\sqrt{3x^2+2}}{27}-\frac{7\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(1/2),x)
```

```
[Out] 8*x**2*sqrt(3*x**2 + 2)/9 + 5*x*sqrt(3*x**2 + 2)/3 + 13*sqrt(3*x**2 + 2)/27
 - 7*sqrt(3)*asinh(sqrt(6)*x/2)/9
```

# 3.121 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx$

**Optimal**. Leaf size=67

$$\frac{2}{3}\sqrt{3x^2+2}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arcsinh(1/2*x*6^(1/2))*3^(1/2)-1/22*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+2/3*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1654, 844, 215, 725, 206}

$$\frac{2}{3}\sqrt{3x^2+2}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{3x^2+2}}\right)}{2\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 + 3*x^2])/3 + ArcSinh[Sqrt[3/2]*x]/(2*Sqrt[3]) - ArcTanh[(4 - 3*x
)/(Sqrt[11]*Sqrt[2 + 3*x^2])]/(2*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2+3x^2}}\,dx &= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{12}\int \frac{12+12x}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+3x^2}}\,dx+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= \frac{2}{3}\sqrt{2+3x^2}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{2\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{2\sqrt{11}}
\end{aligned}
$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.90

$$
\frac{1}{66}\left(44\sqrt{3x^2+2}-3\sqrt{11}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+11\sqrt{3}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (44*Sqrt[2 + 3*x^2] + 11*Sqrt[3]*ArcSinh[Sqrt[3/2]*x] - 3*Sqrt[11]*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/66
```

**fricas [A]** time = 0.91, size = 88, normalized size = 1.31

$$\frac{1}{12}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\frac{1}{44}\sqrt{11}\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}\,(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+\frac{2}{3}\sqrt{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + 1/44*sqrt(11)*lo
g(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x +
 1)) + 2/3*sqrt(3*x^2 + 2)
```

**giac [B]** time = 0.23, size = 99, normalized size = 1.48

$$-\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3x^2+2}\right)+\frac{1}{22}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}\,x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}\,x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/22*sqrt(11)*log(-abs(-2*
sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 + 2))/(2*sqrt(3)*x - sqrt(11)
 + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 2/3*sqrt(3*x^2 + 2)
```

**maple [A]** time = 0.01, size = 55, normalized size = 0.82

$$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}-\frac{\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{22}+\frac{2\sqrt{3x^2+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2+2)^(1/2)+1/6*arcsinh(1/2*6^(1/2)*x)*3^(1/2)-1/22*11^(1/2)*arctan
h(2/11*(4-3*x)*11^(1/2)/(12*(1/2+x)^2-12*x+5)^(1/2))
```

**maxima [A]** time = 0.96, size = 58, normalized size = 0.87

$$\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{1}{22}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2x+1|}-\frac{2\sqrt{6}}{3\,|2x+1|}\right)+\frac{2}{3}\sqrt{3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 1/22*sqrt(11)*arcsinh(1/2*sqrt(6)*x/ab
s(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) + 2/3*sqrt(3*x^2 + 2)
```

**mupad [B]** time = 0.19, size = 61, normalized size = 0.91

$$\frac{\sqrt{11}\left(2\ln\left(x+\frac{1}{2}\right)-2\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{44}+\frac{2\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{3}+\frac{\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/44 + (2*3^(1/2)*(x^2 + 2/3)^(1/2))/3 + (3^(1/2)*asinh((2^(1/2)*3
^(1/2)*x)/2))/6
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 + 2)), x)
```

## 3.122 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx$

**Optimal**. Leaf size=71

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

```
[Out] 1/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+4/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x
^2+2)^(1/2))*11^(1/2)-1/11*(3*x^2+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1651, 844, 215, 725, 206}

$$-\frac{\sqrt{3x^2+2}}{11(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}+\frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(11*(1 + 2*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh
[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(11*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{1}{11}\int \frac{-7-22x}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} - \frac{4}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx + \int \frac{1}{\sqrt{2+3x^2}}\,dx \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}} + \frac{4}{11}\,\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right) \\
&= -\frac{\sqrt{2+3x^2}}{11(1+2x)} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}$$

**Mathematica** [A] time = 0.10, size = 64, normalized size = 0.90

$$-\frac{\sqrt{3x^2+2}}{22x+11} + \frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{11\sqrt{11}} + \frac{\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 + 3*x^2]),x]

[Out] -(Sqrt[2 + 3*x^2]/(11 + 22*x)) + ArcSinh[Sqrt[3/2]*x]/Sqrt[3] + (4*ArcTanh[
(4 - 3*x)/Sqrt[22 + 33*x^2]])/(11*Sqrt[11])
```

**fricas** [A] time = 1.00, size = 106, normalized size = 1.49

$$\frac{121\sqrt{3}\,(2\,x+1)\log\left(-\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+12\,\sqrt{11}\,(2\,x+1)\log\left(\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)-21\,x^2+12\,x-19}{4\,x^2+4\,x+1}\right)-66\,\sqrt{3}}{726\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/726*(121*sqrt(3)*(2*x + 1)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
12*sqrt(11)*(2*x + 1)*log((sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) - 21*x^2 + 12
*x - 19)/(4*x^2 + 4*x + 1)) - 66*sqrt(3*x^2 + 2))/(2*x + 1)
```

**giac** [A] time = 0.32, size = 48, normalized size = 0.68

$$\frac{1}{22}\sqrt{3}\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)-\frac{\sqrt{-\frac{6}{2\,x+1}+\frac{11}{(2\,x+1)^2}+3}}{22\,\mathrm{sgn}\left(\frac{1}{2\,x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/22*sqrt(3)*sgn(1/(2*x + 1)) - 1/22*sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3
)/sgn(1/(2*x + 1))
```

**maple** [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{\sqrt{3}\ \mathrm{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}+\frac{4\sqrt{11}\ \mathrm{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121}-\frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{22\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x)
```

```
[Out] 1/3*arcsinh(1/2*6^(1/2)*x)*3^(1/2)+4/121*11^(1/2)*arctanh(2/11*(-3*x+4)*11^
(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)*(3*(x+1/2)^2-3*x+5/4)^(1/2
)
```

**maxima** [A] time = 0.97, size = 65, normalized size = 0.92

$$\frac{1}{3}\sqrt{3}\ \mathrm{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)-\frac{4}{121}\sqrt{11}\ \mathrm{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2+2}}{11\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 4/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/a
bs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1)) - 1/11*sqrt(3*x^2 + 2)/(2*x + 1)
```

**mupad [B]** time = 0.11, size = 68, normalized size = 0.96

$$\frac{\sqrt{3}\ \mathrm{asinh}\left(\frac{\sqrt{2}\ \sqrt{3}\ x}{2}\right)}{3}-\frac{4\sqrt{11}\ \ln\left(x+\frac{1}{2}\right)}{121}+\frac{4\sqrt{11}\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{121}-\frac{\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{22\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/3 - (4*11^(1/2)*log(x + 1/2))/121 +
(4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/121 - (3
^(1/2)*(x^2 + 2/3)^(1/2))/(22*(x + 1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 + 2)), x)
```

# 3.123 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx$

**Optimal**. Leaf size=77

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] -103/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)-1/22*(3*x
^2+2)^(1/2)/(1+2*x)^2+13/242*(3*x^2+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1651, 807, 725, 206}

$$\frac{13\sqrt{3x^2+2}}{242(2x+1)}-\frac{\sqrt{3x^2+2}}{22(2x+1)^2}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] -Sqrt[2 + 3*x^2]/(22*(1 + 2*x)^2) + (13*Sqrt[2 + 3*x^2])/(242*(1 + 2*x)) -
(103*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2+3x^2}}\,dx &= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}-\frac{1}{22}\int \frac{-14-41x}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}+\frac{103}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{\sqrt{2+3x^2}}{22(1+2x)^2}+\frac{13\sqrt{2+3x^2}}{242(1+2x)}-\frac{103\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 55, normalized size = 0.71

$$\frac{\frac{11(13x+1)\sqrt{3x^2+2}}{(2x+1)^2}-103\sqrt{11}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{1331}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 + 3*x^2]),x]
```

```
[Out] ((11*(1 + 13*x)*Sqrt[2 + 3*x^2])/(1 + 2*x)^2 - 103*Sqrt[11]*ArcTanh[(4 - 3*
x)/Sqrt[22 + 33*x^2]])/1331
```

**fricas [A]** time = 0.78, size = 89, normalized size = 1.16

$$\frac{103\,\sqrt{11}\left(4\,x^2+4\,x+1\right)\log\left(-\frac{\sqrt{11}\,\sqrt{3\,x^2+2}\,(3\,x-4)+21\,x^2-12\,x+19}{4\,x^2+4\,x+1}\right)+22\,\sqrt{3\,x^2+2}\,(13\,x+1)}{2662\left(4\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(103*sqrt(11)*(4*x^2 + 4*x + 1)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x
- 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 22*sqrt(3*x^2 + 2)*(13*x +
1))/(4*x^2 + 4*x + 1)
```

**giac [B]** time = 0.26, size = 180, normalized size = 2.34

$$\frac{103}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{72\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-13\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2}{484\left(\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^2+\sqrt{3}\left(\sqrt{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 103/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2
 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/484*(72*
(sqrt(3)*x - sqrt(3*x^2 + 2))^3 - 13*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^
2 - 168*sqrt(3)*x + 104*sqrt(3) + 168*sqrt(3*x^2 + 2))/((sqrt(3)*x - sqrt(3
*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2)) - 2)^2
```

**maple [A]** time = 0.01, size = 74, normalized size = 0.96

$$-\frac{103\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331}+\frac{13\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{484\left(x+\frac{1}{2}\right)}-\frac{\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{88\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x)
```

```
[Out] -103/1331*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1
/2))+13/484/(x+1/2)*(-3*x+3*(x+1/2)^2+5/4)^(1/2)-1/88/(x+1/2)^2*(-3*x+3*(x+
1/2)^2+5/4)^(1/2)
```

**maxima [A]** time = 0.97, size = 76, normalized size = 0.99

$$\frac{103}{1331}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)-\frac{\sqrt{3x^2+2}}{22\left(4x^2+4x+1\right)}+\frac{13\sqrt{3x^2+2}}{242(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 103/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) - 1/22*sqrt(3*x^2 + 2)/(4*x^2 + 4*x + 1) + 13/242*sqrt(3*x^2 + 2)/(2*
x + 1)
```

**mupad** [B] time = 0.11, size = 77, normalized size = 1.00

$$\frac{103\sqrt{11}\ln\left(x+\frac{1}{2}\right)}{1331}-\frac{103\sqrt{11}\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{88\left(x^2+x+\frac{1}{4}\right)}+\frac{13\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{484\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(1/2)),x)
```

```
[Out] (103*11^(1/2)*log(x + 1/2))/1331 - (103*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*
(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2))/(88*(x + x^
2 + 1/4)) + (13*3^(1/2)*(x^2 + 2/3)^(1/2))/(484*(x + 1/2))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(1/2),x)
```

```
[Out] Timed out
```

# 3.124 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2+3x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$\frac{32}{27}\sqrt{3x^2+2}\,x^2+4\sqrt{3x^2+2}\,x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] -38/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(398+279*x)/(3*x^2+2)^(1/2)+292/8
1*(3*x^2+2)^(1/2)+4*x*(3*x^2+2)^(1/2)+32/27*x^2*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1814, 1815, 641, 215}

$$\frac{32}{27}\sqrt{3x^2+2}\,x^2+4\sqrt{3x^2+2}\,x+\frac{292}{81}\sqrt{3x^2+2}+\frac{279x+398}{54\sqrt{3x^2+2}}-\frac{38\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (398 + 279*x)/(54*Sqrt[2 + 3*x^2]) + (292*Sqrt[2 + 3*x^2])/81 + 4*x*Sqrt[2
+ 3*x^2] + (32*x^2*Sqrt[2 + 3*x^2])/27 - (38*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{398+279x}{54\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{\frac{28}{3}-\frac{280x}{9}-48x^2-\frac{64x^3}{3}}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{18}\int \frac{84-\frac{584x}{3}-432x^2}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{1}{108}\int \frac{1368-1168x}{\sqrt{2+3x^2}}\,dx$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38}{3}\int \frac{1}{\sqrt{2+3x^2}}$$

$$= \frac{398+279x}{54\sqrt{2+3x^2}}+\frac{292}{81}\sqrt{2+3x^2}+4x\sqrt{2+3x^2}+\frac{32}{27}x^2\sqrt{2+3x^2}-\frac{38\sinh^{-1}\left(\sqrt{\right.}}{3\sqrt{3}}$$

**Mathematica [A]** time = 0.05, size = 58, normalized size = 0.67

$$\frac{576x^4+1944x^3+2136x^2-684\sqrt{9x^2+6}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)+2133x+2362}{162\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2362 + 2133*x + 2136*x^2 + 1944*x^3 + 576*x^4 - 684*Sqrt[6 + 9*x^2]*ArcSin
h[Sqrt[3/2]*x])/(162*Sqrt[2 + 3*x^2])
```

**fricas** [A] time = 0.85, size = 76, normalized size = 0.87

$$\frac{342\,\sqrt{3}\left(3\,x^2+2\right)\log\left(\sqrt{3}\,\sqrt{3\,x^2+2}\,x-3\,x^2-1\right)+\left(576\,x^4+1944\,x^3+2136\,x^2+2133\,x+2362\right)\sqrt{3\,x^2+2}}{162\left(3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/162*(342*sqrt(3)*(3*x^2 + 2)*log(sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
 (576*x^4 + 1944*x^3 + 2136*x^2 + 2133*x + 2362)*sqrt(3*x^2 + 2))/(3*x^2 +
2)
```

**giac** [A] time = 0.21, size = 54, normalized size = 0.62

$$\frac{38}{9}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3\,x^2+2}\right)+\frac{3\,(8\,(3\,(8\,x+27)x+89)x+711)x+2362}{162\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 38/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(3*(8*(3*(8*x + 27)*
x + 89)*x + 711)*x + 2362)/sqrt(3*x^2 + 2)
```

**maple** [A] time = 0.01, size = 79, normalized size = 0.91

$$\frac{32x^4}{9\sqrt{3x^2+2}}+\frac{12x^3}{\sqrt{3x^2+2}}+\frac{356x^2}{27\sqrt{3x^2+2}}+\frac{79x}{6\sqrt{3x^2+2}}-\frac{38\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{1181}{81\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 32/9*x^4/(3*x^2+2)^(1/2)+356/27*x^2/(3*x^2+2)^(1/2)+1181/81/(3*x^2+2)^(1/2)
+12*x^3/(3*x^2+2)^(1/2)+79/6*x/(3*x^2+2)^(1/2)-38/9*arcsinh(1/2*6^(1/2)*x)*
3^(1/2)
```

**maxima** [A] time = 0.96, size = 78, normalized size = 0.90

$$\frac{32\,x^4}{9\,\sqrt{3\,x^2+2}}+\frac{12\,x^3}{\sqrt{3\,x^2+2}}+\frac{356\,x^2}{27\,\sqrt{3\,x^2+2}}-\frac{38}{9}\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{79\,x}{6\,\sqrt{3\,x^2+2}}+\frac{1181}{81\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 32/9*x^4/sqrt(3*x^2 + 2) + 12*x^3/sqrt(3*x^2 + 2) + 356/27*x^2/sqrt(3*x^2 +
 2) - 38/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) + 79/6*x/sqrt(3*x^2 + 2) + 1181/8
1/sqrt(3*x^2 + 2)
```

**mupad [B]** time = 0.06, size = 110, normalized size = 1.26

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{32\,x^2}{9}+12\,x+\frac{292}{27}\right)}{3}-\frac{38\,\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-1194+\sqrt{6}\,279\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\,\sqrt{6}\,(1\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(12*x + (32*x^2)/9 + 292/27))/3 - (38*3^(1/2)*as
inh((2^(1/2)*3^(1/2)*x)/2))/9 - (3^(1/2)*6^(1/2)*(6^(1/2)*279i - 1194)*(x^2
 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*2
79i + 1194)*(x^2 + 2/3)^(1/2)*1i)/(1944*(x - (6^(1/2)*1i)/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

## 3.125 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=71

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] 4/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(70-47*x)/(3*x^2+2)^(1/2)+28/9*(3*x
^2+2)^(1/2)+8/9*x*(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1814, 1815, 641, 215}

$$\frac{70-47x}{18\sqrt{3x^2+2}}+\frac{8}{9}x\sqrt{3x^2+2}+\frac{28}{9}\sqrt{3x^2+2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (70 - 47*x)/(18*Sqrt[2 + 3*x^2]) + (28*Sqrt[2 + 3*x^2])/9 + (8*x*Sqrt[2 + 3
*x^2])/9 + (4*ArcSinh[Sqrt[3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx &= \frac{70-47x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{56}{9}-\frac{56x}{3}-\frac{32x^2}{3}}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{8}{9}x\sqrt{2+3x^2}-\frac{1}{12}\int \frac{-16-112x}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{18\sqrt{2+3x^2}}+\frac{28}{9}\sqrt{2+3x^2}+\frac{8}{9}x\sqrt{2+3x^2}+\frac{4\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 53, normalized size = 0.75

$$\frac{48x^3+168x^2+8\sqrt{9x^2+6}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)-15x+182}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (182 - 15*x + 168*x^2 + 48*x^3 + 8*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(1
8*Sqrt[2 + 3*x^2])
```

**fricas [A]** time = 0.86, size = 72, normalized size = 1.01

$$\frac{4\sqrt{3}\left(3x^2+2\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(48x^3+168x^2-15x+182\right)\sqrt{3x^2+2}}{18\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*(4*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) + (
48*x^3 + 168*x^2 - 15*x + 182)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

**giac** [A] time = 0.20, size = 49, normalized size = 0.69

$$-\frac{4}{9}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{3(8(2x+7)x-5)x+182}{18\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -4/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(8*(2*x + 7)*x - 5
)*x + 182)/sqrt(3*x^2 + 2)
```

**maple** [A] time = 0.01, size = 65, normalized size = 0.92

$$\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{28x^2}{3\sqrt{3x^2+2}}-\frac{5x}{6\sqrt{3x^2+2}}+\frac{4\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{91}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2+2)^(1/2)*x^3-5/6/(3*x^2+2)^(1/2)*x+4/9*arcsinh(1/2*6^(1/2)*x)*3^
(1/2)+28/3/(3*x^2+2)^(1/2)*x^2+91/9/(3*x^2+2)^(1/2)
```

**maxima** [A] time = 0.96, size = 64, normalized size = 0.90

$$\frac{8x^3}{3\sqrt{3x^2+2}}+\frac{28x^2}{3\sqrt{3x^2+2}}+\frac{4}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}x\right)-\frac{5x}{6\sqrt{3x^2+2}}+\frac{91}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 + 2) + 28/3*x^2/sqrt(3*x^2 + 2) + 4/9*sqrt(3)*arcsinh(1/
2*sqrt(6)*x) - 5/6*x/sqrt(3*x^2 + 2) + 91/9/sqrt(3*x^2 + 2)
```

**mupad** [B] time = 4.07, size = 105, normalized size = 1.48

$$\frac{4\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{3}x}{2}\right)}{9}+\frac{\sqrt{3}\left(\frac{8x}{3}+\frac{28}{3}\right)\sqrt{x^2+\frac{2}{3}}}{3}+\frac{\sqrt{3}\sqrt{6}\left(-630+\sqrt{6}\,141\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{1944\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\sqrt{6}\left(630+\sqrt{6}\ 1\right.}{1944\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (4*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/9 + (3^(1/2)*((8*x)/3 + 28/3)*(x^2
 + 2/3)^(1/2))/3 + (3^(1/2)*6^(1/2)*(6^(1/2)*141i - 630)*(x^2 + 2/3)^(1/2)*
1i)/(1944*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*141i + 630)*(x^
2 + 2/3)^(1/2)*1i)/(1944*(x + (6^(1/2)*1i)/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(3/2), x)
```

# 3.126 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=55

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

```
[Out] 10/9*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/18*(2-51*x)/(3*x^2+2)^(1/2)+8/9*(3*x^
2+2)^(1/2)
```

**Rubi [A]** time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 641, 215}

$$\frac{2-51x}{18\sqrt{3x^2+2}}+\frac{8}{9}\sqrt{3x^2+2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
```

```
[Out] (2 - 51*x)/(18*Sqrt[2 + 3*x^2]) + (8*Sqrt[2 + 3*x^2])/9 + (10*ArcSinh[Sqrt[
3/2]*x])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{3/2}}\,dx = \frac{2-51x}{18\sqrt{2+3x^2}}-\frac{1}{2}\int \frac{-\frac{20}{3}-\frac{16x}{3}}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10}{3}\int \frac{1}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{2-51x}{18\sqrt{2+3x^2}}+\frac{8}{9}\sqrt{2+3x^2}+\frac{10\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{3\sqrt{3}}$$

**Mathematica [A]** time = 0.03, size = 48, normalized size = 0.87

$$\frac{48x^2+20\sqrt{9x^2+6}\,\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)-51x+34}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(3/2),x]
[Out] (34 - 51*x + 48*x^2 + 20*Sqrt[6 + 9*x^2]*ArcSinh[Sqrt[3/2]*x])/(18*Sqrt[2 +
 3*x^2])
```

**fricas [A]** time = 0.76, size = 67, normalized size = 1.22

$$\frac{10\sqrt{3}\left(3x^2+2\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(48x^2-51x+34\right)\sqrt{3x^2+2}}{18\left(3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] 1/18*(10*sqrt(3)*(3*x^2 + 2)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^2 - 1) +
(48*x^2 - 51*x + 34)*sqrt(3*x^2 + 2))/(3*x^2 + 2)
```

**giac [A]** time = 0.23, size = 44, normalized size = 0.80

$$-\frac{10}{9}\sqrt{3}\log\left(-\sqrt{3}\,x+\sqrt{3x^2+2}\right)+\frac{3\,(16x-17)x+34}{18\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/18*(3*(16*x - 17)*x + 3
4)/sqrt(3*x^2 + 2)
```

**maple** [A] time = 0.00, size = 51, normalized size = 0.93

$$\frac{8x^2}{3\sqrt{3x^2+2}}-\frac{17x}{6\sqrt{3x^2+2}}+\frac{10\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{9}+\frac{17}{9\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2+2)^(1/2)*x^2+17/9/(3*x^2+2)^(1/2)-17/6/(3*x^2+2)^(1/2)*x+10/9*ar
csinh(1/2*6^(1/2)*x)*3^(1/2)
```

**maxima** [A] time = 0.96, size = 50, normalized size = 0.91

$$\frac{8\,x^2}{3\,\sqrt{3\,x^2+2}}+\frac{10}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{17\,x}{6\,\sqrt{3\,x^2+2}}+\frac{17}{9\,\sqrt{3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 + 2) + 10/9*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 17/6*x/sqrt
(3*x^2 + 2) + 17/9/sqrt(3*x^2 + 2)
```

**mupad** [B] time = 0.04, size = 100, normalized size = 1.82

$$\frac{8\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{9}+\frac{10\,\sqrt{3}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{9}+\frac{\sqrt{3}\,\sqrt{6}\,\left(-6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{648\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{\sqrt{3}\,\sqrt{6}\,\left(6+\sqrt{6}\,51\mathrm{i}\right)\,\sqrt{x^2+\frac{2}{3}}\,1}{648\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(3/2),x)
```

```
[Out] (8*3^(1/2)*(x^2 + 2/3)^(1/2))/9 + (10*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))
/9 + (3^(1/2)*6^(1/2)*(6^(1/2)*51i - 6)*(x^2 + 2/3)^(1/2)*1i)/(648*(x - (6^
(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*51i + 6)*(x^2 + 2/3)^(1/2)*1i)/(6
48*(x + (6^(1/2)*1i)/3))
```

**sympy [B]** time = 15.96, size = 114, normalized size = 2.07

$$\frac{30\sqrt{3}\,x^{2}\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27x^{2}+18}+\frac{8x^{2}}{3\sqrt{3x^{2}+2}}-\frac{30x\sqrt{3x^{2}+2}}{27x^{2}+18}+\frac{x}{2\sqrt{3x^{2}+2}}+\frac{20\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27x^{2}+18}+\frac{17}{9\sqrt{3x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(3/2),x)
```

```
[Out] 30*sqrt(3)*x**2*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 8*x**2/(3*sqrt(3*x**2 +
 2)) - 30*x*sqrt(3*x**2 + 2)/(27*x**2 + 18) + x/(2*sqrt(3*x**2 + 2)) + 20*s
qrt(3)*asinh(sqrt(6)*x/2)/(27*x**2 + 18) + 17/(9*sqrt(3*x**2 + 2))
```

## 3.127 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=53

$$\frac{21x-38}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

```
[Out] -2/121*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/66*(-38+21
*x)/(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 12, 725, 206}

$$-\frac{38-21x}{66\sqrt{3x^2+2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{11\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -(38 - 21*x)/(66*Sqrt[2 + 3*x^2]) - (2*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 +
 3*x^2])])/(11*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{1}{6}\int -\frac{12}{11(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}+\frac{2}{11}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2}{11}\,\mathrm{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{38-21x}{66\sqrt{2+3x^2}}-\frac{2\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\,\sqrt{2+3x^2}}\right)}{11\sqrt{11}}
\end{aligned}
$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 0.96

$$
\frac{-12\sqrt{33x^2+22}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)+231x-418}{726\sqrt{3x^2+2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (-418 + 231*x - 12*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/
(726*Sqrt[2 + 3*x^2])
```

**fricas [A]** time = 0.73, size = 83, normalized size = 1.57

$$
\frac{6\sqrt{11}\left(3x^2+2\right)\log\left(-\frac{\sqrt{11}\,\sqrt{3x^2+2}\,(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\sqrt{3x^2+2}\,(21x-38)}{726\left(3x^2+2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/726*(6*sqrt(11)*(3*x^2 + 2)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21
*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*sqrt(3*x^2 + 2)*(21*x - 38))/(3*x
^2 + 2)
```

**giac [A]** time = 0.21, size = 82, normalized size = 1.55

$$\frac{2}{121}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{21x-38}{66\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/121*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/66*(21*x -
38)/sqrt(3*x^2 + 2)
```

**maple [B]** time = 0.01, size = 88, normalized size = 1.66

$$\frac{x}{4\sqrt{3x^2+2}}+\frac{3x}{44\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{2\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{121}-\frac{2}{3\sqrt{3x^2+2}}+\frac{1}{11\sqrt{-3x+3\left(x+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2+2)^(1/2)+1/4/(3*x^2+2)^(1/2)*x+1/11/(-3*x+3*(x+1/2)^2+5/4)^(1/2
)+3/44*x/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-2/121*11^(1/2)*arctanh(2/11*(-3*x+4)*
11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.96, size = 58, normalized size = 1.09

$$\frac{2}{121}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)+\frac{7x}{22\sqrt{3x^2+2}}-\frac{19}{33\sqrt{3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/121*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x + 1
)) + 7/22*x/sqrt(3*x^2 + 2) - 19/33/sqrt(3*x^2 + 2)
```

**mupad [B]** time = 0.14, size = 106, normalized size = 2.00

$$\frac{\sqrt{11}\left(2\ln\left(x+\frac{1}{2}\right)-2\ln\left(x-\frac{\sqrt{3}\sqrt{11}\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{121}-\frac{\sqrt{3}\sqrt{6}\left(-114+\sqrt{6}\,21\mathrm{i}\right)\sqrt{x^2+\frac{2}{3}}\,1\mathrm{i}}{2376\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\sqrt{6}\left(114+\sqrt{6}\,2\right.}{2376\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (11^(1/2)*(2*log(x + 1/2) - 2*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/121 - (3^(1/2)*6^(1/2)*(6^(1/2)*21i - 114)*(x^2 + 2/3)^(1/2)*1i)
/(2376*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*21i + 114)*(x^2 +
2/3)^(1/2)*1i)/(2376*(x + (6^(1/2)*1i)/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 + 2)**(3/2)), x)
```

# 3.128 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=75

$$\frac{97x-10}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] 4/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/242*(-10+9
7*x)/(3*x^2+2)^(1/2)-4/121*(3*x^2+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{242\sqrt{3x^2+2}}-\frac{4\sqrt{3x^2+2}}{121(2x+1)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] -(10 - 97*x)/(242*Sqrt[2 + 3*x^2]) - (4*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)) +
(4*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx &= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{1}{6}\int \frac{-\frac{72}{121}+\frac{120x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx \\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}-\frac{4}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx \\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right) \\
&= -\frac{10-97x}{242\sqrt{2+3x^2}}-\frac{4\sqrt{2+3x^2}}{121(1+2x)}+\frac{4\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 71, normalized size = 0.95

$$\frac{11\left(170x^2+77x-26\right)+8(2x+1)\sqrt{33x^2+22}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{2662(2x+1)\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(-26 + 77*x + 170*x^2) + 8*(1 + 2*x)*Sqrt[22 + 33*x^2]*ArcTanh[(4 - 3*x
)/Sqrt[22 + 33*x^2]])/(2662*(1 + 2*x)*Sqrt[2 + 3*x^2])
```

**fricas [A]** time = 1.12, size = 103, normalized size = 1.37

$$\frac{4\sqrt{11}\left(6x^3+3x^2+4x+2\right)\log\left(\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)-21x^2+12x-19}{4x^2+4x+1}\right)+11\left(170x^2+77x-26\right)\sqrt{3x^2+2}}{2662\left(6x^3+3x^2+4x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2662*(4*sqrt(11)*(6*x^3 + 3*x^2 + 4*x + 2)*log((sqrt(11)*sqrt(3*x^2 + 2)*
(3*x - 4) - 21*x^2 + 12*x - 19)/(4*x^2 + 4*x + 1)) + 11*(170*x^2 + 77*x - 2
6)*sqrt(3*x^2 + 2))/(6*x^3 + 3*x^2 + 4*x + 2)
```

**giac [B]** time = 0.27, size = 168, normalized size = 2.24

$$-\frac{1}{7986}\sqrt{11}\left(85\sqrt{11}\sqrt{3}+24\log\left(\sqrt{11}\sqrt{3}-3\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{\frac{\frac{93}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{44}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{85}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{242\sqrt{-\frac{6}{2x+1}+\frac{11}{(2x+1)^2}+3}}+\frac{4\sqrt{11}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/7986*sqrt(11)*(85*sqrt(11)*sqrt(3) + 24*log(sqrt(11)*sqrt(3) - 3))*sgn(1
/(2*x + 1)) - 1/242*((93/sgn(1/(2*x + 1)) + 44/((2*x + 1)*sgn(1/(2*x + 1)))
)/(2*x + 1) - 85/sgn(1/(2*x + 1)))/sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3)
+ 4/1331*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2*x + 1)^2 + 3) + s
qrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1))
```

**maple [A]** time = 0.01, size = 98, normalized size = 1.31

$$\frac{x}{2\sqrt{3x^2+2}}-\frac{18x}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{4\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{1331}-\frac{2}{121\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x)
```

```
[Out] 1/2/(3*x^2+2)^(1/2)*x-2/121/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-18/121/(-3*x+3*(x+
1/2)^2+5/4)^(1/2)*x+4/1331*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+1
2*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(1/2)
```

**maxima [A]** time = 0.97, size = 84, normalized size = 1.12

$$-\frac{4}{1331}\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{85\,x}{242\,\sqrt{3\,x^2+2}}-\frac{2}{121\,\sqrt{3\,x^2+2}}-\frac{1}{11\left(2\,\sqrt{3\,x^2+2}\,x+\sqrt{3\,x^2+2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
 1)) + 85/242*x/sqrt(3*x^2 + 2) - 2/121/sqrt(3*x^2 + 2) - 1/11/(2*sqrt(3*x^
2 + 2)*x + sqrt(3*x^2 + 2))
```

**mupad [B]** time = 4.14, size = 157, normalized size = 2.09

$$\frac{4\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}-\frac{4\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{1331}+\frac{97\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{97\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{1452\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}-\frac{2\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{121\left(x+\frac{1}{2}\right)}+\frac{\sqrt{3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (4*11^(1/2)*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331 - (
4*11^(1/2)*log(x + 1/2))/1331 + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x - (
6^(1/2)*1i)/3)) + (97*3^(1/2)*(x^2 + 2/3)^(1/2))/(1452*(x + (6^(1/2)*1i)/3)
) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(121*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 +
 2/3)^(1/2)*5i)/(1452*(x - (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^
(1/2)*5i)/(1452*(x + (6^(1/2)*1i)/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

# 3.129 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=97

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] -322/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^2+2)^(1/2))*11^(1/2)+1/2662*(
358+351*x)/(3*x^2+2)^(1/2)-2/121*(3*x^2+2)^(1/2)/(1+2*x)^2+2/1331*(3*x^2+2)
^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.12, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{2\sqrt{3x^2+2}}{1331(2x+1)}-\frac{2\sqrt{3x^2+2}}{121(2x+1)^2}-\frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (358 + 351*x)/(2662*Sqrt[2 + 3*x^2]) - (2*Sqrt[2 + 3*x^2])/(121*(1 + 2*x)^2
) + (2*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (322*ArcTanh[(4 - 3*x)/(Sqrt[11]
*Sqrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx = \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{1}{6}\int \frac{-\frac{2940}{1331}-\frac{7272x}{1331}-\frac{8592x^2}{1331}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{1}{132}\int \frac{\frac{3768}{121}+\frac{7800x}{121}}{(1+2x)^2\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} + \frac{322\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx, x, \frac{4-3x}{\sqrt{2+3x^2}}\right)}{1331}$$

$$= \frac{358+351x}{2662\sqrt{2+3x^2}} - \frac{2\sqrt{2+3x^2}}{121(1+2x)^2} + \frac{2\sqrt{2+3x^2}}{1331(1+2x)} - \frac{322\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}$$

**Mathematica [A]** time = 0.08, size = 78, normalized size = 0.80

$$\frac{11\left(1428x^3+2716x^2+1799x+278\right)-644(2x+1)^2\sqrt{33x^2+22}\,\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{29282(2x+1)^2\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(3/2)),x]
```

```
[Out] (11*(278 + 1799*x + 2716*x^2 + 1428*x^3) - 644*(1 + 2*x)^2*Sqrt[22 + 33*x^2
]*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(29282*(1 + 2*x)^2*Sqrt[2 + 3*x^2])
```

**fricas [A]** time = 0.82, size = 119, normalized size = 1.23

$$\frac{322\sqrt{11}\left(12x^4+12x^3+11x^2+8x+2\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(1428x^3+2716x^2+1799x\right.}{29282\left(12x^4+12x^3+11x^2+8x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/29282*(322*sqrt(11)*(12*x^4 + 12*x^3 + 11*x^2 + 8*x + 2)*log(-(sqrt(11)*s
qrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(142
```

```
8*x^3 + 2716*x^2 + 1799*x + 278)*sqrt(3*x^2 + 2))/(12*x^4 + 12*x^3 + 11*x^2
 + 8*x + 2)
```

**giac [B]** time = 0.24, size = 196, normalized size = 2.02

$$\frac{322}{14641}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{351x+358}{2662\sqrt{3x^2+2}}+\frac{36\left(\sqrt{3}x-\sqrt{3x^2+2}\right)^3-\sqrt{3}\left(\sqrt{3}x\right.}{1331\left(\left(\sqrt{3}x-\sqrt{3x^2+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] 322/14641*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^
2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2662*(3
51*x + 358)/sqrt(3*x^2 + 2) + 1/1331*(36*(sqrt(3)*x - sqrt(3*x^2 + 2))^3 -
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 48*sqrt(3)*x + 8*sqrt(3) - 48*sqr
t(3*x^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(
3*x^2 + 2)) - 2)^2
```

**maple [A]** time = 0.01, size = 107, normalized size = 1.10

$$\frac{357x}{2662\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{322\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{14641}+\frac{161}{1331\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{}{484\left(x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x)
```

```
[Out] 161/1331/(-3*x+3*(x+1/2)^2+5/4)^(1/2)+357/2662/(-3*x+3*(x+1/2)^2+5/4)^(1/2)
*x-322/14641*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)
^(1/2))+7/484/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-1/88/(x+1/2)^2/(-3*x+3*(
x+1/2)^2+5/4)^(1/2)
```

**maxima [A]** time = 0.98, size = 124, normalized size = 1.28

$$\frac{322}{14641}\sqrt{11}\operatorname{arsinh}\left(\frac{\sqrt{6}x}{2|2x+1|}-\frac{2\sqrt{6}}{3|2x+1|}\right)+\frac{357x}{2662\sqrt{3x^2+2}}+\frac{161}{1331\sqrt{3x^2+2}}-\frac{1}{22\left(4\sqrt{3x^2+2}x^2+4\sqrt{3x^2+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 322/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
 + 1)) + 357/2662*x/sqrt(3*x^2 + 2) + 161/1331/sqrt(3*x^2 + 2) - 1/22/(4*sq
rt(3*x^2 + 2)*x^2 + 4*sqrt(3*x^2 + 2)*x + sqrt(3*x^2 + 2)) + 7/242/(2*sqrt(
3*x^2 + 2)*x + sqrt(3*x^2 + 2))
```

**mupad** [B] time = 4.17, size = 180, normalized size = 1.86

$$\frac{322\sqrt{11}\ \ln\left(x+\frac{1}{2}\right)}{14641}-\frac{322\sqrt{11}\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{14641}+\frac{117\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{5324\left(x-\frac{\sqrt{6}\ 1\mathrm{i}}{3}\right)}+\frac{117\sqrt{3}\ \sqrt{x^2+\frac{2}{3}}}{5324\left(x+\frac{\sqrt{6}\ 1\mathrm{i}}{3}\right)}-\frac{\sqrt{3}\ \sqrt{x^2+}}{242\left(x^2+x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(3/2)),x)
```

```
[Out] (322*11^(1/2)*log(x + 1/2))/14641 - (322*11^(1/2)*log(x - (3^(1/2)*11^(1/2)
*(x^2 + 2/3)^(1/2))/3 - 4/3))/14641 + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324
*(x - (6^(1/2)*1i)/3)) + (117*3^(1/2)*(x^2 + 2/3)^(1/2))/(5324*(x + (6^(1/2
)*1i)/3)) - (3^(1/2)*(x^2 + 2/3)^(1/2))/(242*(x + x^2 + 1/4)) + (3^(1/2)*(x
^2 + 2/3)^(1/2))/(1331*(x + 1/2)) - (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i
)/(15972*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/2)*179i)/(
15972*(x + (6^(1/2)*1i)/3))
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

# 3.130 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=73

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

```
[Out] 1/162*(398+279*x)/(3*x^2+2)^(3/2)+8/3*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(
-152-465*x)/(3*x^2+2)^(1/2)+32/27*(3*x^2+2)^(1/2)
```

**Rubi** [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 641, 215}

$$\frac{279x+398}{162\left(3x^2+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2+2}-\frac{465x+152}{54\sqrt{3x^2+2}}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (398 + 279*x)/(162*(2 + 3*x^2)^(3/2)) - (152 + 465*x)/(54*Sqrt[2 + 3*x^2])
+ (32*Sqrt[2 + 3*x^2])/27 + (8*ArcSinh[Sqrt[3/2]*x])/Sqrt[3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{\frac{22}{3}-\frac{280x}{3}-144x^2-64x^3}{\left(2+3x^2\right)^{3/2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{96+\frac{128x}{3}}{\sqrt{2+3x^2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+8\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{398+279x}{162\left(2+3x^2\right)^{3/2}}-\frac{152+465x}{54\sqrt{2+3x^2}}+\frac{32}{27}\sqrt{2+3x^2}+\frac{8\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 63, normalized size = 0.86

$$\frac{1728x^4-4185x^3+936x^2+432\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)-2511x+254}{162\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (254 - 2511*x + 936*x^2 - 4185*x^3 + 1728*x^4 + 432*Sqrt[3]*(2 + 3*x^2)^(3/
2)*ArcSinh[Sqrt[3/2]*x])/(162*(2 + 3*x^2)^(3/2))
```

**fricas [A]** time = 0.85, size = 87, normalized size = 1.19

$$\frac{216\sqrt{3}\left(9x^4+12x^2+4\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)+\left(1728x^4-4185x^3+936x^2-2511x+254\right)\sqrt{3x^2}}{162\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/162*(216*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*
x^2 - 1) + (1728*x^4 - 4185*x^3 + 936*x^2 - 2511*x + 254)*sqrt(3*x^2 + 2))/
(9*x^4 + 12*x^2 + 4)
```

**giac [A]** time = 0.18, size = 53, normalized size = 0.73

$$-\frac{8}{3}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)+\frac{9\left(\left(3\left(64x-155\right)x+104\right)x-279\right)x+254}{162\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -8/3*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) + 1/162*(9*((3*(64*x - 155)*
x + 104)*x - 279)*x + 254)/(3*x^2 + 2)^(3/2)
```

**maple [A]** time = 0.01, size = 91, normalized size = 1.25

$$\frac{32x^4}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{8x^3}{\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{52x^2}{9\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{107x}{18\sqrt{3x^2+2}}-\frac{65x}{18\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{8\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{3}+\frac{127}{81\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 32/3*x^4/(3*x^2+2)^(3/2)+52/9*x^2/(3*x^2+2)^(3/2)+127/81/(3*x^2+2)^(3/2)-8*
x^3/(3*x^2+2)^(3/2)-107/18/(3*x^2+2)^(1/2)*x+8/3*arcsinh(1/2*6^(1/2)*x)*3^(
1/2)-65/18*x/(3*x^2+2)^(3/2)
```

**maxima [A]** time = 0.95, size = 105, normalized size = 1.44

$$\frac{32\,x^4}{3\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{8}{3}\,x\left(\frac{9\,x^2}{\left(3\,x^2+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3\,x^2+2\right)^{\frac{3}{2}}}\right)+\frac{8}{3}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{2}\,\sqrt{6}\,x\right)-\frac{11\,x}{18\,\sqrt{3\,x^2+2}}+\frac{52\,x^2}{9\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{65\,x}{18\left(3\,x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 + 2)^(3/2) - 8/3*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)
^(3/2)) + 8/3*sqrt(3)*arcsinh(1/2*sqrt(6)*x) - 11/18*x/sqrt(3*x^2 + 2) + 52
/9*x^2/(3*x^2 + 2)^(3/2) - 65/18*x/(3*x^2 + 2)^(3/2) + 127/81/(3*x^2 + 2)^(
3/2)
```

**mupad [B]** time = 0.06, size = 212, normalized size = 2.90

$$\frac{32\sqrt{3}\sqrt{x^2+\frac{2}{3}}}{27}+\frac{8\sqrt{3}\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{3}\,x}{2}\right)}{3}-\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{31}{16}+\frac{\sqrt{6}\,199\mathrm{i}}{144}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(-\frac{31}{24}+\frac{\sqrt{6}\,199\mathrm{i}}{216}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{31}{16}+\frac{\sqrt{6}}{14}}{x+\frac{\sqrt{6}}{3}}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (32*3^(1/2)*(x^2 + 2/3)^(1/2))/27 + (8*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2)
)/3 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 - 31/16)/(x - (6^(1/2
)*1i)/3) - (6^(1/2)*((6^(1/2)*199i)/216 - 31/24)*1i)/(2*(x - (6^(1/2)*1i)/3
)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*199i)/144 + 31/16)/(x + (
6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*199i)/216 + 31/24)*1i)/(2*(x + (6^(1/2)
*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i - 1824)*(x^2 + 2/3)^(1/2)
*1i)/(7776*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(6^(1/2)*1953i + 1824)*
(x^2 + 2/3)^(1/2)*1i)/(7776*(x - (6^(1/2)*1i)/3))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

## 3.131 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}} \, dx$

**Optimal**. Leaf size=60

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

```
[Out] 1/54*(70-47*x)/(3*x^2+2)^(3/2)+16/27*arcsinh(1/2*x*6^(1/2))*3^(1/2)+1/54*(-
168-59*x)/(3*x^2+2)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1814, 12, 215}

$$\frac{70-47x}{54\left(3x^2+2\right)^{3/2}}-\frac{59x+168}{54\sqrt{3x^2+2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (70 - 47*x)/(54*(2 + 3*x^2)^(3/2)) - (168 + 59*x)/(54*Sqrt[2 + 3*x^2]) + (1
6*ArcSinh[Sqrt[3/2]*x])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx &= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{1}{6}\int \frac{-\frac{74}{9}-56x-32x^2}{\left(2+3x^2\right)^{3/2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{1}{12}\int \frac{64}{3\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2+3x^2}}\,dx\\
&= \frac{70-47x}{54\left(2+3x^2\right)^{3/2}}-\frac{168+59x}{54\sqrt{2+3x^2}}+\frac{16\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{9\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 58, normalized size = 0.97

$$\frac{-177x^3-504x^2+32\sqrt{3}\left(3x^2+2\right)^{3/2}\sinh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)-165x-266}{54\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
```

```
[Out] (-266 - 165*x - 504*x^2 - 177*x^3 + 32*Sqrt[3]*(2 + 3*x^2)^(3/2)*ArcSinh[Sq
rt[3/2]*x])/(54*(2 + 3*x^2)^(3/2))
```

**fricas [A]** time = 0.78, size = 83, normalized size = 1.38

$$\frac{16\sqrt{3}\left(9x^4+12x^2+4\right)\log\left(-\sqrt{3}\sqrt{3x^2+2}\,x-3x^2-1\right)-\left(177x^3+504x^2+165x+266\right)\sqrt{3x^2+2}}{54\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/54*(16*sqrt(3)*(9*x^4 + 12*x^2 + 4)*log(-sqrt(3)*sqrt(3*x^2 + 2)*x - 3*x^
2 - 1) - (177*x^3 + 504*x^2 + 165*x + 266)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2
 + 4)
```

**giac [A]** time = 0.19, size = 48, normalized size = 0.80

$$-\frac{16}{27}\sqrt{3}\log\left(-\sqrt{3}x+\sqrt{3x^2+2}\right)-\frac{3\left((59x+168)x+55\right)x+266}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-sqrt(3)*x + sqrt(3*x^2 + 2)) - 1/54*(3*((59*x + 168)*x
+ 55)*x + 266)/(3*x^2 + 2)^(3/2)
```

**maple [A]** time = 0.01, size = 77, normalized size = 1.28

$$-\frac{16x^3}{9\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{28x^2}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{x}{2\sqrt{3x^2+2}}-\frac{37x}{18\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{16\sqrt{3}\ \operatorname{arcsinh}\left(\frac{\sqrt{6}\,x}{2}\right)}{27}-\frac{133}{27\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] -16/9/(3*x^2+2)^(3/2)*x^3-1/2/(3*x^2+2)^(1/2)*x+16/27*arcsinh(1/2*6^(1/2)*x
)*3^(1/2)-28/3/(3*x^2+2)^(3/2)*x^2-133/27/(3*x^2+2)^(3/2)-37/18/(3*x^2+2)^(
3/2)*x
```

**maxima [B]** time = 0.96, size = 91, normalized size = 1.52

$$-\frac{16}{27}x\left(\frac{9x^2}{\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4}{\left(3x^2+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{2}\sqrt{6}\,x\right)+\frac{37x}{54\sqrt{3x^2+2}}-\frac{28x^2}{3\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{37x}{18\left(3x^2+2\right)^{\frac{3}{2}}}-\frac{}{27\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -16/27*x*(9*x^2/(3*x^2 + 2)^(3/2) + 4/(3*x^2 + 2)^(3/2)) + 16/27*sqrt(3)*ar
csinh(1/2*sqrt(6)*x) + 37/54*x/sqrt(3*x^2 + 2) - 28/3*x^2/(3*x^2 + 2)^(3/2)
 - 37/18*x/(3*x^2 + 2)^(3/2) - 133/27/(3*x^2 + 2)^(3/2)
```

**mupad [B]** time = 0.05, size = 200, normalized size = 3.33

$$\frac{16\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{3}\,x}{2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{-\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\left(-\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\left(\frac{\frac{47}{48}+\frac{\sqrt{6}\,35\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\left(\frac{47}{72}+\frac{\sqrt{6}\,35\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (16*3^(1/2)*asinh((2^(1/2)*3^(1/2)*x)/2))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(
((6^(1/2)*35i)/48 - 47/48)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*35i)/7
2 - 47/72)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*(x^2 + 2/3)^(1/2)
*(((6^(1/2)*35i)/48 + 47/48)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*35i)
/72 + 47/72)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*6^(1/2)*(6^(1/2
)*63i - 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6
^(1/2)*(6^(1/2)*63i + 672)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1/2)*1i)/3)
)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 + 2)**(5/2), x)
```

## 3.132 $\int \frac{(1+2x)(1+3x+4x^2)}{(2+3x^2)^{5/2}}\,dx$

**Optimal**. Leaf size=41

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

[Out] 1/54*(2-51*x)/(3*x^2+2)^(3/2)+1/18*(-16+13*x)/(3*x^2+2)^(1/2)

**Rubi** [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1814, 637}

$$\frac{2-51x}{54\left(3x^2+2\right)^{3/2}}-\frac{16-13x}{18\sqrt{3x^2+2}}$$

Antiderivative was successfully verified.

[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]

[Out] (2 - 51*x)/(54*(2 + 3*x^2)^(3/2)) - (16 - 13*x)/(18*Sqrt[2 + 3*x^2])

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2+3x^2\right)^{5/2}}\,dx = \frac{2-51x}{54\left(2+3x^2\right)^{3/2}} - \frac{1}{6}\int \frac{-\frac{26}{3}-16x}{\left(2+3x^2\right)^{3/2}}\,dx$$
$$= \frac{2-51x}{54\left(2+3x^2\right)^{3/2}} - \frac{16-13x}{18\sqrt{2+3x^2}}$$

**Mathematica** [A] time = 0.02, size = 30, normalized size = 0.73

$$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 + 3*x^2)^(5/2),x]
[Out] (-94 + 27*x - 144*x^2 + 117*x^3)/(54*(2 + 3*x^2)^(3/2))
```

**fricas** [A] time = 0.77, size = 40, normalized size = 0.98

$$\frac{\left(117\,x^3-144\,x^2+27\,x-94\right)\sqrt{3\,x^2+2}}{54\left(9\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="fricas")
[Out] 1/54*(117*x^3 - 144*x^2 + 27*x - 94)*sqrt(3*x^2 + 2)/(9*x^4 + 12*x^2 + 4)
```

**giac** [A] time = 0.34, size = 25, normalized size = 0.61

$$\frac{9\left((13\,x-16)x+3\right)x-94}{54\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="giac")
[Out] 1/54*(9*((13*x - 16)*x + 3)*x - 94)/(3*x^2 + 2)^(3/2)
```

**maple** [A] time = 0.00, size = 27, normalized size = 0.66

$$\frac{117x^3-144x^2+27x-94}{54\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/54*(117*x^3-144*x^2+27*x-94)/(3*x^2+2)^(3/2)
```

**maxima [A]** time = 0.42, size = 50, normalized size = 1.22

$$\frac{13\,x}{18\,\sqrt{3\,x^2+2}}-\frac{8\,x^2}{3\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{17\,x}{18\,(3\,x^2+2)^{\frac{3}{2}}}-\frac{47}{27\,(3\,x^2+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 13/18*x/sqrt(3*x^2 + 2) - 8/3*x^2/(3*x^2 + 2)^(3/2) - 17/18*x/(3*x^2 + 2)^(
3/2) - 47/27/(3*x^2 + 2)^(3/2)
```

**mupad [B]** time = 4.11, size = 185, normalized size = 4.51

$$\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{-\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{\frac{17}{16}+\frac{\sqrt{6}\,1\mathrm{i}}{48}}{x-\frac{\sqrt{6}\,1\mathrm{i}}{3}}-\frac{\sqrt{6}\,\left(\frac{17}{24}+\frac{\sqrt{6}\,1\mathrm{i}}{72}\right)1\mathrm{i}}{2\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}-\frac{\sqrt{3}\,\sqrt{6}\,\left(-192+\sqrt{6}\,69\mathrm{i}\right.}{2592\left(x-\frac{\sqrt{6}}{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 + 2)^(5/2),x)
```

```
[Out] (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 - 17/16)/(x + (6^(1/2)*1i)/3)
+ (6^(1/2)*((6^(1/2)*1i)/72 - 17/24)*1i)/(2*(x + (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*1i)/48 + 17/16)/(x - (6^(1/2)*1i)/3)
- (6^(1/2)*((6^(1/2)*1i)/72 + 17/24)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 -
(3^(1/2)*6^(1/2)*(6^(1/2)*69i - 192)*(x^2 + 2/3)^(1/2)*1i)/(2592*(x - (6^(1
/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*69i + 192)*(x^2 + 2/3)^(1/2)*1i)/(2
592*(x + (6^(1/2)*1i)/3))
```

**sympy [B]** time = 77.50, size = 180, normalized size = 4.39

$$\frac{10x^3}{18x^2\sqrt{3x^2+2}+12\sqrt{3x^2+2}}+\frac{x^3}{6x^2\sqrt{3x^2+2}+4\sqrt{3x^2+2}}-\frac{72x^2}{81x^2\sqrt{3x^2+2}+54\sqrt{3x^2+2}}+\frac{x}{6x^2\sqrt{3x^2+2}+4\sqrt{3\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2+2)**(5/2),x)
```

```
[Out] 10*x**3/(18*x**2*sqrt(3*x**2 + 2) + 12*sqrt(3*x**2 + 2)) + x**3/(6*x**2*sqr
t(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 72*x**2/(81*x**2*sqrt(3*x**2 + 2) + 5
4*sqrt(3*x**2 + 2)) + x/(6*x**2*sqrt(3*x**2 + 2) + 4*sqrt(3*x**2 + 2)) - 32
/(81*x**2*sqrt(3*x**2 + 2) + 54*sqrt(3*x**2 + 2)) - 5/(27*x**2*sqrt(3*x**2
+ 2) + 18*sqrt(3*x**2 + 2))
```

## 3.133 $\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=73

$$\frac{21x-38}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

```
[Out] 1/198*(-38+21*x)/(3*x^2+2)^(3/2)-8/1331*arctanh(1/11*(4-3*x)*11^(1/2)/(3*x^
2+2)^(1/2))*11^(1/2)+1/726*(24+95*x)/(3*x^2+2)^(1/2)
```

**Rubi** [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 823, 12, 725, 206}

$$-\frac{38-21x}{198\left(3x^2+2\right)^{3/2}}+\frac{95x+24}{726\sqrt{3x^2+2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -(38 - 21*x)/(198*(2 + 3*x^2)^(3/2)) + (24 + 95*x)/(726*Sqrt[2 + 3*x^2]) -
(8*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(121*Sqrt[11])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{78}{11}-\frac{84x}{11}}{(1+2x)\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{\int \frac{864}{11(1+2x)\sqrt{2+3x^2}}\,dx}{1188}\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}+\frac{8}{121}\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8}{121}\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-3x}{\sqrt{2+3x^2}}\right)\\
&= -\frac{38-21x}{198\left(2+3x^2\right)^{3/2}}+\frac{24+95x}{726\sqrt{2+3x^2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{121\sqrt{11}}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 58, normalized size = 0.79

$$\frac{855x^3+216x^2+801x-274}{2178\left(3x^2+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (-274 + 801*x + 216*x^2 + 855*x^3)/(2178*(2 + 3*x^2)^(3/2)) - (8*ArcTanh[(4
 - 3*x)/Sqrt[22 + 33*x^2]])/(121*Sqrt[11])
```

**fricas** [A] time = 0.58, size = 103, normalized size = 1.41

$$\frac{72\sqrt{11}\left(9x^4+12x^2+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(855x^3+216x^2+801x-274\right)\sqrt{3x^2+2}}{23958\left(9x^4+12x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23958*(72*sqrt(11)*(9*x^4 + 12*x^2 + 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3
*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) + 11*(855*x^3 + 216*x^2 +
801*x - 274)*sqrt(3*x^2 + 2))/(9*x^4 + 12*x^2 + 4)
```

**giac** [A] time = 0.21, size = 91, normalized size = 1.25

$$\frac{8}{1331}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9((95x+24)x+89)x-274}{2178\left(3x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/1331*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*x^2 +
 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/2178*(9*((
95*x + 24)*x + 89)*x - 274)/(3*x^2 + 2)^(3/2)
```

**maple** [B] time = 0.01, size = 133, normalized size = 1.82

$$\frac{x}{12\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{x}{12\sqrt{3x^2+2}}+\frac{x}{44\left(-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{23x}{484\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{8\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(-}{11\sqrt{-12}}\right.}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2+2)^(3/2)+1/12/(3*x^2+2)^(3/2)*x+1/12/(3*x^2+2)^(1/2)*x+1/33/(-3
*x+3*(x+1/2)^2+5/4)^(3/2)+1/44*x/(-3*x+3*(x+1/2)^2+5/4)^(3/2)+23/484/(-3*x+
3*(x+1/2)^2+5/4)^(1/2)*x+4/121/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-8/1331*11^(1/2)
*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.97, size = 81, normalized size = 1.11

$$\frac{8}{1331}\,\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{95\,x}{726\,\sqrt{3\,x^2+2}}+\frac{4}{121\,\sqrt{3\,x^2+2}}+\frac{7\,x}{66\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{19}{99\,\left(3\,x^2+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/1331*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x +
1)) + 95/726*x/sqrt(3*x^2 + 2) + 4/121/sqrt(3*x^2 + 2) + 7/66*x/(3*x^2 + 2)
^(3/2) - 19/99/(3*x^2 + 2)^(3/2)
```

**mupad [B]** time = 0.13, size = 218, normalized size = 2.99

$$\frac{\sqrt{11}\,\left(8\,\ln\left(x+\frac{1}{2}\right)-8\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{1331}-\frac{\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}\,\left(\frac{-\frac{21}{176}+\frac{\sqrt{6}\,19\mathrm{i}}{176}}{x+\frac{\sqrt{6}\,1\mathrm{i}}{3}}+\frac{\sqrt{6}\,\left(-\frac{7}{88}+\frac{\sqrt{6}\,19\mathrm{i}}{264}\right)\,1\mathrm{i}}{2\,\left(x+\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)^2}\right)}{27}+\frac{\sqrt{3}\,\sqrt{x^2+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/1331 - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 - 21/176)/
(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*19i)/264 - 7/88)*1i)/(2*(x + (6^(
1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*19i)/176 + 21/1
76)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*19i)/264 + 7/88)*1i)/(2*(x -
(6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*303i - 288)*(x^2 + 2/3)
^(1/2)*1i)/(104544*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2)*(6^(1/2)*303i +
 288)*(x^2 + 2/3)^(1/2)*1i)/(104544*(x - (6^(1/2)*1i)/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

## 3.134 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{97x-10}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

```
[Out] 1/726*(-10+97*x)/(3*x^2+2)^(3/2)-32/14641*arctanh(1/11*(4-3*x)*11^(1/2)/(3*
x^2+2)^(1/2))*11^(1/2)+1/7986*(24+887*x)/(3*x^2+2)^(1/2)-16/1331*(3*x^2+2)^
(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.17, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1647, 807, 725, 206}

$$-\frac{10-97x}{726\left(3x^2+2\right)^{3/2}}-\frac{16\sqrt{3x^2+2}}{1331(2x+1)}+\frac{887x+24}{7986\sqrt{3x^2+2}}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{1331\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] -(10 - 97*x)/(726*(2 + 3*x^2)^(3/2)) + (24 + 887*x)/(7986*Sqrt[2 + 3*x^2])
- (16*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)) - (32*ArcTanh[(4 - 3*x)/(Sqrt[11]*S
qrt[2 + 3*x^2])])/(1331*Sqrt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
```

```
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2+3x^2\right)^{5/2}}\,dx &= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}-\frac{1}{18}\int \frac{-\frac{798}{121}-\frac{1968x}{121}-\frac{2328x^2}{121}}{(1+2x)^2\left(2+3x^2\right)^{3/2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}+\frac{1}{108}\int \frac{\frac{10368}{1331}+\frac{1728x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}+\frac{32\int \frac{1}{(1+2x)\sqrt{2+3x^2}}\,dx}{1331}\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\,\text{Subst}\left(\int \frac{1}{11-x^2}\,dx,x,\frac{4-}{\sqrt{2-}}\right.}{1331}\\
&= -\frac{10-97x}{726\left(2+3x^2\right)^{3/2}}+\frac{24+887x}{7986\sqrt{2+3x^2}}-\frac{16\sqrt{2+3x^2}}{1331(1+2x)}-\frac{32\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{2+3x^2}}\right)}{1331\sqrt{11}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 91, normalized size = 0.96

$$\frac{11\left(4458x^4+2805x^3+4602x^2+2717x-446\right)-192\sqrt{33x^2+22}\left(6x^3+3x^2+4x+2\right)\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{87846(2x+1)\left(3x^2+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (11*(-446 + 2717*x + 4602*x^2 + 2805*x^3 + 4458*x^4) - 192*Sqrt[22 + 33*x^2
]*(2 + 4*x + 3*x^2 + 6*x^3)*ArcTanh[(4 - 3*x)/Sqrt[22 + 33*x^2]])/(87846*(1
 + 2*x)*(2 + 3*x^2)^(3/2))
```

**fricas [A]** time = 0.59, size = 134, normalized size = 1.41

$$\frac{96 \sqrt{11} \left(18 x^5 + 9 x^4 + 24 x^3 + 12 x^2 + 8 x + 4\right) \log\left(-\frac{\sqrt{11} \sqrt{3 x^2+2} (3 x-4)+21 x^2-12 x+19}{4 x^2+4 x+1}\right) + 11 \left(4458 x^4 + 2805 x^3 + 4\right.}{87846 \left(18 x^5 + 9 x^4 + 24 x^3 + 12 x^2 + 8 x + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/87846*(96*sqrt(11)*(18*x^5 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)*log(-(sqr
t(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2 + 4*x + 1)) +
11*(4458*x^4 + 2805*x^3 + 4602*x^2 + 2717*x - 446)*sqrt(3*x^2 + 2))/(18*x^5
 + 9*x^4 + 24*x^3 + 12*x^2 + 8*x + 4)
```

**giac [B]** time = 0.52, size = 233, normalized size = 2.45

$$-\frac{1}{263538} \sqrt{11} \left(743 \sqrt{11} \sqrt{3} - 576 \log\left(\sqrt{11} \sqrt{3} - 3\right)\right) \operatorname{sgn}\left(\frac{1}{2 x + 1}\right) - \frac{32 \sqrt{11} \log\left(\sqrt{11} \left(\sqrt{-\frac{6}{2 x+1} + \frac{11}{(2 x+1)^2} + 3}\right.\right.}{14641 \operatorname{sgn}\left(\frac{1}{2 x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/263538*sqrt(11)*(743*sqrt(11)*sqrt(3) - 576*log(sqrt(11)*sqrt(3) - 3))*s
gn(1/(2*x + 1)) - 32/14641*sqrt(11)*log(sqrt(11)*(sqrt(-6/(2*x + 1) + 11/(2
*x + 1)^2 + 3) + sqrt(11)/(2*x + 1)) - 3)/sgn(1/(2*x + 1)) + 1/7986*(((11*(
731/sgn(1/(2*x + 1)) + 528/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1) - 14163/
sgn(1/(2*x + 1)))/(2*x + 1) + 6111/sgn(1/(2*x + 1)))/(2*x + 1) - 2229/sgn(1
/(2*x + 1)))/((6/(2*x + 1) - 11/(2*x + 1)^2 - 3)*sqrt(-6/(2*x + 1) + 11/(2*
x + 1)^2 + 3))
```

**maple [A]** time = 0.01, size = 143, normalized size = 1.51

$$\frac{x}{6\left(3x^{2}+2\right)^{\frac{3}{2}}}+\frac{x}{6\sqrt{3x^{2}+2}}-\frac{10x}{121\left(-3x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{98x}{1331\sqrt{-3x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}}}-\frac{32\sqrt{11}\ \operatorname{arctanh}\left(\frac{2(}{11\sqrt{-1}}\right.}{14641}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x)
```

```
[Out] 1/6/(3*x^2+2)^(3/2)*x+1/6/(3*x^2+2)^(1/2)*x+4/363/(-3*x+3*(x+1/2)^2+5/4)^(3
/2)-10/121/(-3*x+3*(x+1/2)^2+5/4)^(3/2)*x-98/1331/(-3*x+3*(x+1/2)^2+5/4)^(1
/2)*x+16/1331/(-3*x+3*(x+1/2)^2+5/4)^(1/2)-32/14641*11^(1/2)*arctanh(2/11*(
-3*x+4)*11^(1/2)/(-12*x+12*(x+1/2)^2+5)^(1/2))-1/22/(x+1/2)/(-3*x+3*(x+1/2)
^2+5/4)^(3/2)
```

**maxima [A]** time = 0.98, size = 107, normalized size = 1.13

$$\frac{32}{14641}\sqrt{11}\ \operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{743\,x}{7986\,\sqrt{3\,x^{2}+2}}+\frac{16}{1331\,\sqrt{3\,x^{2}+2}}+\frac{61\,x}{726\left(3\,x^{2}+2\right)^{\frac{3}{2}}}-\frac{}{11\left(2\left(3\,x^{2}+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/14641*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2*x
+ 1)) + 743/7986*x/sqrt(3*x^2 + 2) + 16/1331/sqrt(3*x^2 + 2) + 61/726*x/(3*
x^2 + 2)^(3/2) - 1/11/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)) + 4/363/(
3*x^2 + 2)^(3/2)
```

**mupad [B]** time = 4.31, size = 270, normalized size = 2.84

$$\frac{\sqrt{11}\left(8\ \ln\left(x+\frac{1}{2}\right)-8\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^{2}+\frac{2}{3}}}{3}-\frac{4}{3}\right)\right)}{14641}+\frac{\sqrt{11}\left(\frac{48\ \ln\left(x+\frac{1}{2}\right)}{1331}-\frac{48\ \ln\left(x-\frac{\sqrt{3}\ \sqrt{11}\ \sqrt{x^{2}+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{1331}\right)}{22}-\frac{8\sqrt{3}\ \sqrt{x^{2}+\frac{2}{3}}}{1331\left(x+\frac{1}{2}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (11^(1/2)*(8*log(x + 1/2) - 8*log(x - (3^(1/2)*11^(1/2)*(x^2 + 2/3)^(1/2))/
3 - 4/3)))/14641 + (11^(1/2)*((48*log(x + 1/2))/1331 - (48*log(x - (3^(1/2)
*11^(1/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/1331))/22 - (8*3^(1/2)*(x^2 + 2/3)^(
1/2))/(1331*(x + 1/2)) - (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)/1936 -
291/1936)/(x + (6^(1/2)*1i)/3) + (6^(1/2)*((6^(1/2)*5i)/968 - 97/968)*1i)/(
2*(x + (6^(1/2)*1i)/3)^2)))/27 + (3^(1/2)*(x^2 + 2/3)^(1/2)*(((6^(1/2)*15i)
/1936 + 291/1936)/(x - (6^(1/2)*1i)/3) - (6^(1/2)*((6^(1/2)*5i)/968 + 97/96
8)*1i)/(2*(x - (6^(1/2)*1i)/3)^2)))/27 - (3^(1/2)*6^(1/2)*(6^(1/2)*2481i -
288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x + (6^(1/2)*1i)/3)) - (3^(1/2)*6^(1/2
)*(6^(1/2)*2481i + 288)*(x^2 + 2/3)^(1/2)*1i)/(1149984*(x - (6^(1/2)*1i)/3)
)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

## 3.135 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=117

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

```
[Out] 1/7986*(358+351*x)/(3*x^2+2)^(3/2)-1216/161051*arctanh(1/11*(4-3*x)*11^(1/2
)/(3*x^2+2)^(1/2))*11^(1/2)+1/29282*(1216+2133*x)/(3*x^2+2)^(1/2)-8/1331*(3
*x^2+2)^(1/2)/(1+2*x)^2-8/1331*(3*x^2+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1647, 1651, 807, 725, 206}

$$\frac{351x+358}{7986\left(3x^2+2\right)^{3/2}}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)}-\frac{8\sqrt{3x^2+2}}{1331(2x+1)^2}+\frac{2133x+1216}{29282\sqrt{3x^2+2}}-\frac{1216\tanh^{-1}\left(\frac{4-3x}{\sqrt{11}\sqrt{3x^2+2}}\right)}{14641\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] (358 + 351*x)/(7986*(2 + 3*x^2)^(3/2)) + (1216 + 2133*x)/(29282*Sqrt[2 + 3*
x^2]) - (8*Sqrt[2 + 3*x^2])/(1331*(1 + 2*x)^2) - (8*Sqrt[2 + 3*x^2])/(1331*
(1 + 2*x)) - (1216*ArcTanh[(4 - 3*x)/(Sqrt[11]*Sqrt[2 + 3*x^2])])/(14641*Sq
rt[11])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2+3x^2\right)^{5/2}}\,dx = \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} - \frac{1}{18}\int \frac{-\frac{10926}{1331}-\frac{3132x}{121}-\frac{51048x^2}{1331}-\frac{16848x^3}{1331}}{(1+2x)^3\left(2+3x^2\right)^{3/2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} + \frac{1}{108}\int \frac{\frac{245376}{14641}+\frac{544320x}{14641}+\frac{525312x^2}{14641}}{(1+2x)^3\sqrt{2+3x^2}}\,dx$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{\int \frac{-\frac{338688}{1331}-\frac{468288x}{1331}}{(1+2x)^2\sqrt{2+3x^2}}\,dx}{2376}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} + \frac{1216\int -\ldots}{1\ldots}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\,\mathrm{Sub}\ldots}{\ldots}$$

$$= \frac{358+351x}{7986\left(2+3x^2\right)^{3/2}} + \frac{1216+2133x}{29282\sqrt{2+3x^2}} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)^2} - \frac{8\sqrt{2+3x^2}}{1331(1+2x)} - \frac{1216\tanh\ldots}{14\ldots}$$

**Mathematica** [A] time = 0.10, size = 75, normalized size = 0.64

$$\frac{\frac{11\left(67284x^5+111060x^4+116937x^3+109844x^2+57371x+7010\right)}{(2x+1)^2\left(3x^2+2\right)^{3/2}} - 7296\sqrt{11}\tanh^{-1}\left(\frac{4-3x}{\sqrt{33x^2+22}}\right)}{966306}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 + 3*x^2)^(5/2)),x]
```

```
[Out] ((11*(7010 + 57371*x + 109844*x^2 + 116937*x^3 + 111060*x^4 + 67284*x^5))/(
(1 + 2*x)^2*(2 + 3*x^2)^(3/2)) - 7296*Sqrt[11]*ArcTanh[(4 - 3*x)/Sqrt[22 +
33*x^2]])/966306
```

**fricas** [A] time = 0.90, size = 149, normalized size = 1.27

$$\frac{3648\sqrt{11}\left(36x^6+36x^5+57x^4+48x^3+28x^2+16x+4\right)\log\left(-\frac{\sqrt{11}\sqrt{3x^2+2}(3x-4)+21x^2-12x+19}{4x^2+4x+1}\right)+11\left(67284x^5\ldots\right.}{966306\left(36x^6+36x^5+57x^4+48x^3+28x^2+16\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/966306*(3648*sqrt(11)*(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16*x
+ 4)*log(-(sqrt(11)*sqrt(3*x^2 + 2)*(3*x - 4) + 21*x^2 - 12*x + 19)/(4*x^2
+ 4*x + 1)) + 11*(67284*x^5 + 111060*x^4 + 116937*x^3 + 109844*x^2 + 57371*
x + 7010)*sqrt(3*x^2 + 2))/(36*x^6 + 36*x^5 + 57*x^4 + 48*x^3 + 28*x^2 + 16
*x + 4)
```

**giac [A]** time = 0.29, size = 183, normalized size = 1.56

$$\frac{1216}{161051}\sqrt{11}\log\left(-\frac{\left|-2\sqrt{3}x-\sqrt{11}-\sqrt{3}+2\sqrt{3x^2+2}\right|}{2\sqrt{3}x-\sqrt{11}+\sqrt{3}-2\sqrt{3x^2+2}}\right)+\frac{9((2133x+1216)x+1851)x+11234}{87846\left(3x^2+2\right)^{\frac{3}{2}}}+\frac{4\left(\sqrt{3}\left(\sqrt{3}x\right.\right.}{1331\left(\left(\sqrt{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1216/161051*sqrt(11)*log(-abs(-2*sqrt(3)*x - sqrt(11) - sqrt(3) + 2*sqrt(3*
x^2 + 2))/(2*sqrt(3)*x - sqrt(11) + sqrt(3) - 2*sqrt(3*x^2 + 2))) + 1/87846
*(9*((2133*x + 1216)*x + 1851)*x + 11234)/(3*x^2 + 2)^(3/2) + 4/1331*(sqrt(
3)*(sqrt(3)*x - sqrt(3*x^2 + 2))^2 + 24*sqrt(3)*x - 8*sqrt(3) - 24*sqrt(3*x
^2 + 2))/((sqrt(3)*x - sqrt(3*x^2 + 2))^2 + sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 + 2)) - 2)^2
```

**maple [A]** time = 0.01, size = 140, normalized size = 1.20

$$\frac{87x}{2662\left(-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{1869x}{29282\sqrt{-3x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{1216\sqrt{11}\,\operatorname{arctanh}\left(\frac{2(-3x+4)\sqrt{11}}{11\sqrt{-12x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{161051}+\frac{}{399\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x)
```

```
[Out] 152/3993/(-3*x+3*(x+1/2)^2+5/4)^(3/2)+87/2662/(-3*x+3*(x+1/2)^2+5/4)^(3/2)*
x+1869/29282/(-3*x+3*(x+1/2)^2+5/4)^(1/2)*x+608/14641/(-3*x+3*(x+1/2)^2+5/4
)^(1/2)-1216/161051*11^(1/2)*arctanh(2/11*(-3*x+4)*11^(1/2)/(-12*x+12*(x+1/
2)^2+5)^(1/2))+1/484/(x+1/2)/(-3*x+3*(x+1/2)^2+5/4)^(3/2)-1/88/(x+1/2)^2/(-
3*x+3*(x+1/2)^2+5/4)^(3/2)
```

**maxima [A]** time = 0.99, size = 147, normalized size = 1.26

$$\frac{1216}{161051}\,\sqrt{11}\,\operatorname{arsinh}\left(\frac{\sqrt{6}\,x}{2\,|2\,x+1|}-\frac{2\,\sqrt{6}}{3\,|2\,x+1|}\right)+\frac{1869\,x}{29282\,\sqrt{3\,x^2+2}}+\frac{608}{14641\,\sqrt{3\,x^2+2}}+\frac{87\,x}{2662\,\left(3\,x^2+2\right)^{\frac{3}{2}}}-\frac{}{22\left(4\left(3\,x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1216/161051*sqrt(11)*arcsinh(1/2*sqrt(6)*x/abs(2*x + 1) - 2/3*sqrt(6)/abs(2
*x + 1)) + 1869/29282*x/sqrt(3*x^2 + 2) + 608/14641/sqrt(3*x^2 + 2) + 87/26
62*x/(3*x^2 + 2)^(3/2) - 1/22/(4*(3*x^2 + 2)^(3/2)*x^2 + 4*(3*x^2 + 2)^(3/2
)*x + (3*x^2 + 2)^(3/2)) + 1/242/(2*(3*x^2 + 2)^(3/2)*x + (3*x^2 + 2)^(3/2)
) + 152/3993/(3*x^2 + 2)^(3/2)
```

**mupad [B]** time = 4.19, size = 301, normalized size = 2.57

$$\frac{1216\,\sqrt{11}\,\ln\left(x+\frac{1}{2}\right)}{161051}-\frac{1216\,\sqrt{11}\,\ln\left(x-\frac{\sqrt{3}\,\sqrt{11}\,\sqrt{x^2+\frac{2}{3}}}{3}-\frac{4}{3}\right)}{161051}-\frac{179\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{95832\,\left(x^2+\frac{2\mathrm{i}\,\sqrt{6}\,x}{3}-\frac{2}{3}\right)}+\frac{711\,\sqrt{3}\,\sqrt{x^2+\frac{2}{3}}}{58564\,\left(x-\frac{\sqrt{6}\,1\mathrm{i}}{3}\right)}+\frac{711}{5856}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 + 2)^(5/2)),x)
```

```
[Out] (1216*11^(1/2)*log(x + 1/2))/161051 - (1216*11^(1/2)*log(x - (3^(1/2)*11^(1
/2)*(x^2 + 2/3)^(1/2))/3 - 4/3))/161051 - (179*3^(1/2)*(x^2 + 2/3)^(1/2))/(
95832*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58
564*(x - (6^(1/2)*1i)/3)) + (711*3^(1/2)*(x^2 + 2/3)^(1/2))/(58564*(x + (6^
(1/2)*1i)/3)) - (2*3^(1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + x^2 + 1/4)) + (179
*3^(1/2)*(x^2 + 2/3)^(1/2))/(95832*((6^(1/2)*x*2i)/3 - x^2 + 2/3)) - (4*3^(
1/2)*(x^2 + 2/3)^(1/2))/(1331*(x + 1/2)) + (3^(1/2)*6^(1/2)*(x^2 + 2/3)^(1/
2)*13i)/(21296*((6^(1/2)*x*2i)/3 + x^2 - 2/3)) - (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x - (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*9265i)/(2108304*(x + (6^(1/2)*1i)/3)) + (3^(1/2)*6^(1/2)*(x^2 + 2/
3)^(1/2)*13i)/(21296*((6^(1/2)*x*2i)/3 - x^2 + 2/3))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2+2)**(5/2),x)
```

```
[Out] Timed out
```

# 3.136 $\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=420

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1} \left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} \left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)\right.}{ch^3(m+1)(m+2p+3)}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(1+p)/c/h/(3+m+2*p)-(a*f*h^2*(1+m)-c*(2*f*g^2*(1+
p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+a)^p*AppellF1(1+m,-p,-p,2+
m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/c/h^3/
(1+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h
*(-a)^(1/2)/c^(1/2)))^p)-(2*f*g*(1+p)-e*h*(3+m+2*p))*(h*x+g)^(2+m)*(c*x^2+a
)^p*AppellF1(2+m,-p,-p,3+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-
a)^(1/2)/c^(1/2)))/h^3/(2+m)/(3+m+2*p)/((1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)
))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)
```

**Rubi [A]** time = 0.60, antiderivative size = 417, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1654, 844, 760, 133}

$$-\frac{\left(a+cx^2\right)^p (g+hx)^{m+1} \left(1-\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}}\right)^{-p} \left(1-\frac{g+hx}{\frac{\sqrt{-a}h}{\sqrt{c}}+g}\right)^{-p} F_1\left(m+1;-p,-p;m+2;\frac{g+hx}{g-\frac{\sqrt{-a}h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}h}{\sqrt{c}}}\right)\left(afh^2(m+1)\right.}{ch^3(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - ((a*f*h^2*(
1 + m) - 2*c*f*g^2*(1 + p) + c*h*(e*g - d*h)*(3 + m + 2*p))*(g + h*x)^(1 +
m)*(a + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)
/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(3 + m + 2
*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[
-a]*h)/Sqrt[c]))^p) - ((2*f*g*(1 + p) - e*h*(3 + m + 2*p))*(g + h*x)^(2 + m
)*(a + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/
Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(3 + m + 2*p)
*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1 - (g + h*x)/(g + (Sqrt[-a]
*h)/Sqrt[c]))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
```

```
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-cd(3+\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\int (g+hx)^{1+m}\left(a+cx^2\right.}{h^2}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(\left(eh-\frac{2fg(1+p)}{3+m+2p}\right)\left(a+cx^2\right)^p\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}}{\sqrt{c}}}\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\left(afh^2(1+m)-2cfg^2(1+p)+ch(eg\right.}{}$$

**Mathematica [F]** time = 1.15, size = 0, normalized size = 0.00

$$\int (g+hx)^m \left(a+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

**fricas [F]** time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

**maple [F]** time = 0.11, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+ex+d\right)\left(c\,x^{2}+a\right)^{p}\left(hx+g\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^{2}+ex+d\right)\left(cx^{2}+a\right)^{p}\left(hx+g\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^m, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,{\left(c\,x^2+a\right)}^p\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^p*(d + e*x + f*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

# 3.137 $\int (g+hx)^m \sqrt{a+cx^2}\left(d+ex+fx^2\right)\,dx$

**Optimal**. Leaf size=403

$$-\frac{\sqrt{a+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)\left(afh^2(m+1)-c\left(3fg^2-h(m+4)(eg-dh)\right)\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+a)^(3/2)/c/h/(4+m)-(a*f*h^2*(1+m)-c*(3*f*g^2-h*(-d*h
+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,(h*x+g)/(g-h*(-a)^(1
/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^2+a)^(1/2)/c/h^3/(1+m)/
(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(-h*x-g)/(g+h*(-a)^(1/
2)/c^(1/2)))^(1/2)-(3*f*g-e*h*(4+m))*(h*x+g)^(2+m)*AppellF1(2+m,-1/2,-1/2,3
+m,(h*x+g)/(g-h*(-a)^(1/2)/c^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))*(c*x^
2+a)^(1/2)/h^3/(2+m)/(4+m)/(1+(-h*x-g)/(g-h*(-a)^(1/2)/c^(1/2)))^(1/2)/(1+(
-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^(1/2)
```

**Rubi [A]** time = 0.45, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1654, 844, 760, 133}

$$\frac{\sqrt{a+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)\left(-afh^2(m+1)-ch(m+4)(eg-dh)+3cfg^2\right)}{ch^3(m+1)(m+4)\sqrt{1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}}\sqrt{1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 - a*f*h
^2*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[a + c*x^2]*App
ellF1[1 + m, -1/2, -1/2, 2 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g +
h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (g + h*x)
/(g - (Sqrt[-a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
) - ((3*f*g - e*h*(4 + m))*(g + h*x)^(2 + m)*Sqrt[a + c*x^2]*AppellF1[2 + m
, -1/2, -1/2, 3 + m, (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (
Sqrt[-a]*h)/Sqrt[c])])/(h^3*(2 + m)*(4 + m)*Sqrt[1 - (g + h*x)/(g - (Sqrt[-
a]*h)/Sqrt[c])]*Sqrt[1 - (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m \left(-h^2(af(1+m)-cd(4+m \right.}{ch^2(4}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{(3fg-eh(4+m)) \int (g+hx)^{1+m}\sqrt{a+c}}{h^2(4+m)}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} - \frac{\left((3fg-eh(4+m))\sqrt{a+cx^2}\right)\text{Subst}\left(\int\right.}{h^3(4+m)\sqrt{1-}}$$

$$= \frac{f(g+hx)^{1+m}\left(a+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2-afh^2(1+m)-ch(eg-dh)(4\right.}{ch^3(1+}$$

**Mathematica** [F] time = 0.69, size = 0, normalized size = 0.00

$$\int (g+hx)^m \sqrt{a+cx^2} \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*Sqrt[a + c*x^2]*(d + e*x + f*x^2), x]
```

**fricas** [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+a}\left(fx^2+ex+d\right)(hx+g)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**maple [F]** time = 0.03, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+e x +d \right) \sqrt{c\,x^{2}+a}\, \left(h x +g \right)^{m}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c x^{2} + a} {\left(f x^{2} + e x + d\right)} {\left(h x + g\right)}^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + c x^{2}} \left(g + h x\right)^{m} \left(d + e x + f x^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+a)**(1/2),x)
[Out] Integral(sqrt(a + c*x**2)*(g + h*x)**m*(d + e*x + f*x**2), x)
```

# 3.138 $\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=474

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)}{2h^3p}-\frac{\left(a+cx^2\right)^{p+1}}{2}$$

```
[Out] -1/2*(d*h^2-e*g*h+f*g^2)*(c*x^2+a)^(1+p)/h/(a*h^2+c*g^2)/(1+p)/((h*x+g)^(2+
2*p))-1/2*f*(c*x^2+a)^p*AppellF1(-2*p,-p,-p,1-2*p,(h*x+g)/(g-h*(-a)^(1/2)/c
^(1/2)),(h*x+g)/(g+h*(-a)^(1/2)/c^(1/2)))/h^3/p/((h*x+g)^(2*p))/((1+(-h*x-g
)/(g-h*(-a)^(1/2)/c^(1/2)))^p)/((1+(-h*x-g)/(g+h*(-a)^(1/2)/c^(1/2)))^p)+(a
*h^2*(-e*h+2*f*g)+c*(-d*g*h^2+f*g^3))*(h*x+g)^(-1-2*p)*(c*x^2+a)^p*hypergeo
m([-p, -1-2*p],[-2*p],2*(h*x+g)*(-a)^(1/2)*c^(1/2)/(-h*(-a)^(1/2)+g*c^(1/2)
)/((-a)^(1/2)-x*c^(1/2)))*((-a)^(1/2)-x*c^(1/2))/h^2/(a*h^2+c*g^2)/(1+2*p)/
(h*(-a)^(1/2)+g*c^(1/2))/((-(h*(-a)^(1/2)+g*c^(1/2))*((-a)^(1/2)+x*c^(1/2))
/(-h*(-a)^(1/2)+g*c^(1/2))/((-a)^(1/2)-x*c^(1/2)))^p)
```

**Rubi [A]** time = 0.52, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1656, 760, 133, 807, 727}

$$-\frac{f\left(a+cx^2\right)^p(g+hx)^{-2p}\left(1-\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)^{-p}\left(1-\frac{g+hx}{\frac{\sqrt{-a}\,h}{\sqrt{c}}+g}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{g+hx}{g-\frac{\sqrt{-a}\,h}{\sqrt{c}}},\frac{g+hx}{g+\frac{\sqrt{-a}\,h}{\sqrt{c}}}\right)}{2h^3p}+\frac{\left(\sqrt{-a}-\sqrt{c}\,x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - e*g*h + d*h^2)*(a + c*x^2)^(1 + p))/(2*h*(c*g^2 + a*h^2)*(1 + p)
*(g + h*x)^(2*(1 + p))) - (f*(a + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
(g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]), (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c])]
)/(2*h^3*p*(g + h*x)^(2*p)*(1 - (g + h*x)/(g - (Sqrt[-a]*h)/Sqrt[c]))^p*(1
- (g + h*x)/(g + (Sqrt[-a]*h)/Sqrt[c]))^p) + ((a*h^2*(2*f*g - e*h) + c*(f*g
^3 - d*g*h^2))*(Sqrt[-a] - Sqrt[c]*x)*(g + h*x)^(-1 - 2*p)*(a + c*x^2)^p*Hy
pergeometric2F1[-1 - 2*p, -p, -2*p, (2*Sqrt[-a]*Sqrt[c]*(g + h*x))/((Sqrt[c
]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))])/(h^2*(Sqrt[c]*g + Sqrt[-a]*h)*(
c*g^2 + a*h^2)*(1 + 2*p)*(-(((Sqrt[c]*g + Sqrt[-a]*h)*(Sqrt[-a] + Sqrt[c]*x
))/((Sqrt[c]*g - Sqrt[-a]*h)*(Sqrt[-a] - Sqrt[c]*x))))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 727

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((Rt[-(a*c), 2] - c*x)*(d + e*x)^(m + 1)*(a + c*x^2)^p*Hypergeometric2F1[m
+ 1, -p, m + 2, (2*c*Rt[-(a*c), 2]*(d + e*x))/((c*d - e*Rt[-(a*c), 2])*(Rt[
-(a*c), 2] - c*x))])/((m + 1)*(c*d + e*Rt[-(a*c), 2])*(((c*d + e*Rt[-(a*c),
 2])*(Rt[-(a*c), 2] + c*x))/((c*d - e*Rt[-(a*c), 2])*(-Rt[-(a*c), 2] + c*x)
))^p), x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Int
egerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1656

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d + e*x)^(m + q)
*(a + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(a + c*x^2)^p*ExpandTo
Sum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x], x]] /; FreeQ[{a, c, d, e,
 m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(IGtQ[m, 0] && Rat
ionalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx = \frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+cx^2\right)^p\,dx}{h^2} + \frac{f}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)} - \frac{\left(ah^2(2fg-e\right.}{}$$

$$= -\frac{\left(fg^2-egh+dh^2\right)(g+hx)^{-2(1+p)}\left(a+cx^2\right)^{1+p}}{2h\left(cg^2+ah^2\right)(1+p)} - \frac{f(g+hx)^{-2p}}{}$$

**Mathematica [F]** time = 2.85, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + c*x^2)^p*(d + e*x + f*x^2), x]
```

**fricas [F]** time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+a\right)^p\left(hx+g\right)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+ex+d\right)\left(c\,x^{2}+a\right)^{p}\left(hx+g\right)^{-2p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^{2}+ex+d\right)\left(cx^{2}+a\right)^{p}\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + a)^p*(h*x + g)^(-2*p - 3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^{2}+a\right)^{p}\,\left(f\,x^{2}+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

```
[Out] int(((a + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

# 3.139 $\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f \right.$

**Optimal**. Leaf size=222

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right.}{}$$

```
[Out] g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)-(b
*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^m*(c*(e*x+d)/(-b*e+2*c*d)
)^(-m-p)*(-c*e*x-b*e+c*d)^2*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^p*hypergeom([
-m-p, 2+p],[3+p],(-c*e*x-b*e+c*d)/(-b*e+2*c*d))/c^2/e^2/(2+p)/(3+m+2*p)
```

**Rubi [A]** time = 0.41, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 70, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1632, 794, 679, 677, 70, 69}

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(-be+cd-cex)^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*(-((c*d - b*e)*f
) + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] (g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c*
e^2*(3 + m + 2*p)) - ((b*e*g*(1 + m + p) + c*(d*g*(1 - m) - e*f*(3 + m + 2*
p)))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(c*d - b*e - c*e*x)
^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F1[-m - p, 2 +
 p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*e^2*(2 + p)*(3 + m + 2*
p))
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
```

```
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 677

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^m*(a + b*x + c*x^2)^FracPart[p])/((1 + (e*x)/d)^FracPart[p
]*(a/d + (c*x)/e)^FracPart[p]), Int[(1 + (e*x)/d)^(m + p)*(a/d + (c*x)/e)^p
, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^
2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && (IntegerQ[m] || GtQ[d, 0]) && !(
IGtQ[m, 0] && (IntegerQ[3*p] || IntegerQ[4*p]))
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(d^IntPart[m]*(d + e*x)^FracPart[m])/(1 + (e*x)/d)^FracPart[m
], Int[(1 + (e*x)/d)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !Intege
rQ[p] && !(IntegerQ[m] || GtQ[d, 0])
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1
))/(c*(m + 2*p + 2)), x] + Dist[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d
^2 - b*d*e + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && (NeQ[m, 2] || EqQ[d, 0])
```

Rule 1632

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e
 + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rubi steps

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left(-(cd-be)f+(cef-cdg+beg)x+cegx^2\right) dx = (de)\int (d+ex)^{-1+m}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(c\right.}{ce^2(3}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(c\right.}{ce^2(3}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(c\right.}{ce^2(3}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(c\right.}{ce^2(3}$$

$$= \frac{g(d+ex)^{-1+m}\left(-d(c\right.}{ce^2(3}$$

**Mathematica [A]** time = 0.32, size = 165, normalized size = 0.74

$$\frac{(d+ex)^m(be-cd+cex)^2(-((d+ex)(c(d-ex)-be)))^p\left(\frac{e\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(-beg(m+p+1)+cdg(m-1)+cef(m+2p+3))\,{}_2F_1\left(-m-p,p+2;p\right.}{p+2}\right.}{c^2e^3(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) +
 b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

```
[Out] ((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x))))
^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 + m
 + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p, 2
 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3*(3
+ m + 2*p))
```

**fricas [F]** time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(cegx^2-(cd-be)f+\left(cef-(cd-be)g\right)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^2
 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cegx^2 - (cd - be)f + \left(cef - cdg + beg\right)x\right)\left(ce^2x^2 + be^2x - cd^2 + bde\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

**maple [F]** time = 0.18, size = 0, normalized size = 0.00

$$\int \left(ceg\,x^2 - (-be + cd)\,f + \left(beg - cdg + cef\right)x\right)(ex + d)^m \left(c\,e^2x^2 + b\,e^2x + bde - c\,d^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

```
[Out] int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g-c*d*g
+c*e*f)*x+c*e*g*x^2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cegx^2 - (cd - be)f + \left(cef - cdg + beg\right)x\right)\left(ce^2x^2 + be^2x - cd^2 + bde\right)^p (ex + d)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*(-(-b*e+c*d)*f+(b*e*g
-c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (d + e\,x)^m \left(c\,e\,g\,x^2 + \left(b\,e\,g - c\,d\,g + c\,e\,f\right)\,x + f\,\left(b\,e - c\,d\right)\right)\left(-c\,d^2 + b\,d\,e + c\,e^2\,x^2 + b\,e^2\,x\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

```
[Out] int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c*
e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

**sympy** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*(-(-b*e+c*d)*f+
(b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

## 3.140 $\int \left(a+bx+cx^2\right)^4 \left(A+Cx^2\right) dx$

**Optimal**. Leaf size=254

$$a^4Ax+2a^3Abx^2+abx^4\left(a^2C+A\left(3ac+b^2\right)\right)+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)+\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\right.$$

```
[Out] a^4*A*x+2*a^3*A*b*x^2+1/3*a^2*(4*A*a*c+6*A*b^2+C*a^2)*x^3+a*b*(A*(3*a*c+b^2
)+a^2*C)*x^4+1/5*(A*(6*a^2*c^2+12*a*b^2*c+b^4)+2*a^2*(2*a*c+3*b^2)*C)*x^5+2
/3*b*(3*a*c+b^2)*(A*c+C*a)*x^6+1/7*(2*A*c^2*(2*a*c+3*b^2)+(6*a^2*c^2+12*a*b
^2*c+b^4)*C)*x^7+1/2*b*c*(A*c^2+(3*a*c+b^2)*C)*x^8+1/9*c^2*(A*c^2+4*C*a*c+6
*C*b^2)*x^9+2/5*b*c^3*C*x^10+1/11*c^4*C*x^11
```

**Rubi [A]** time = 0.33, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$\frac{1}{7}x^7\left(C\left(6a^2c^2+12ab^2c+b^4\right)+2Ac^2\left(2ac+3b^2\right)\right)+\frac{1}{5}x^5\left(A\left(6a^2c^2+12ab^2c+b^4\right)+2a^2C\left(2ac+3b^2\right)\right)+abx^4\left(a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
(b^2 + 3*a*c) + a^2*C)*x^4 + ((A*(b^4 + 12*a*b^2*c + 6*a^2*c^2) + 2*a^2*(3*
b^2 + 2*a*c)*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((2*A*c^2*
(3*b^2 + 2*a*c) + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*C)*x^7)/7 + (b*c*(A*c^2 +
(b^2 + 3*a*c)*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*b*c^
3*C*x^10)/5 + (c^4*C*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^4\left(A+Cx^2\right)dx=\int \left(a^4A+4a^3Abx+a^2\left(6Ab^2+4aAc+a^2C\right)x^2+4ab\left(A\left(b^2+3ac\right)+a^2C\right)\right.$$

$$=a^4Ax+2a^3Abx^2+\frac{1}{3}a^2\left(6Ab^2+4aAc+a^2C\right)x^3+ab\left(A\left(b^2+3ac\right)+a^2C\right)x$$

**Mathematica [A]** time = 0.09, size = 256, normalized size = 1.01

$$a^4Ax+2a^3Abx^2+abx^4\left(a^2C+3aAc+Ab^2\right)+\frac{1}{3}a^2x^3\left(a^2C+4aAc+6Ab^2\right)+\frac{1}{7}x^7\left(6a^2c^2C+4aAc^3+12ab^2cC+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^4*(A + C*x^2),x]
```

```
[Out] a^4*A*x + 2*a^3*A*b*x^2 + (a^2*(6*A*b^2 + 4*a*A*c + a^2*C)*x^3)/3 + a*b*(A*
b^2 + 3*a*A*c + a^2*C)*x^4 + ((A*b^4 + 12*a*A*b^2*c + 6*a^2*A*c^2 + 6*a^2*b
^2*C + 4*a^3*c*C)*x^5)/5 + (2*b*(b^2 + 3*a*c)*(A*c + a*C)*x^6)/3 + ((6*A*b^
2*c^2 + 4*a*A*c^3 + b^4*C + 12*a*b^2*c*C + 6*a^2*c^2*C)*x^7)/7 + (b*c*(A*c^
2 + b^2*C + 3*a*c*C)*x^8)/2 + (c^2*(A*c^2 + 6*b^2*C + 4*a*c*C)*x^9)/9 + (2*
b*c^3*C*x^10)/5 + (c^4*C*x^11)/11
```

**fricas [A]** time = 0.71, size = 308, normalized size = 1.21

$$\frac{1}{11}x^{11}c^4C+\frac{2}{5}x^{10}c^3bC+\frac{2}{3}x^9c^2b^2C+\frac{4}{9}x^9c^3aC+\frac{1}{9}x^9c^4A+\frac{1}{2}x^8cb^3C+\frac{3}{2}x^8c^2baC+\frac{1}{2}x^8c^3bA+\frac{1}{7}x^7b^4C+\frac{12}{7}x^7cb^2aC+\frac{6}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^4*C + 2/5*x^10*c^3*b*C + 2/3*x^9*c^2*b^2*C + 4/9*x^9*c^3*a*C +
1/9*x^9*c^4*A + 1/2*x^8*c*b^3*C + 3/2*x^8*c^2*b*a*C + 1/2*x^8*c^3*b*A + 1/7
*x^7*b^4*C + 12/7*x^7*c*b^2*a*C + 6/7*x^7*c^2*a^2*C + 6/7*x^7*c^2*b^2*A + 4
/7*x^7*c^3*a*A + 2/3*x^6*b^3*a*C + 2*x^6*c*b*a^2*C + 2/3*x^6*c*b^3*A + 2*x^
6*c^2*b*a*A + 6/5*x^5*b^2*a^2*C + 4/5*x^5*c*a^3*C + 1/5*x^5*b^4*A + 12/5*x^
5*c*b^2*a*A + 6/5*x^5*c^2*a^2*A + x^4*b*a^3*C + x^4*b^3*a*A + 3*x^4*c*b*a^2
*A + 1/3*x^3*a^4*C + 2*x^3*b^2*a^2*A + 4/3*x^3*c*a^3*A + 2*x^2*b*a^3*A + x*
a^4*A
```

**giac [A]** time = 0.15, size = 308, normalized size = 1.21

$$\frac{1}{11}Cc^4x^{11}+\frac{2}{5}Cbc^3x^{10}+\frac{2}{3}Cb^2c^2x^9+\frac{4}{9}Cac^3x^9+\frac{1}{9}Ac^4x^9+\frac{1}{2}Cb^3cx^8+\frac{3}{2}Cabc^2x^8+\frac{1}{2}Abc^3x^8+\frac{1}{7}Cb^4x^7+\frac{12}{7}Cab^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 2/3*C*b^2*c^2*x^9 + 4/9*C*a*c^3*x^9 +
1/9*A*c^4*x^9 + 1/2*C*b^3*c*x^8 + 3/2*C*a*b*c^2*x^8 + 1/2*A*b*c^3*x^8 + 1/7
*C*b^4*x^7 + 12/7*C*a*b^2*c*x^7 + 6/7*C*a^2*c^2*x^7 + 6/7*A*b^2*c^2*x^7 + 4
/7*A*a*c^3*x^7 + 2/3*C*a*b^3*x^6 + 2*C*a^2*b*c*x^6 + 2/3*A*b^3*c*x^6 + 2*A*
a*b*c^2*x^6 + 6/5*C*a^2*b^2*x^5 + 1/5*A*b^4*x^5 + 4/5*C*a^3*c*x^5 + 12/5*A*
```

```
a*b^2*c*x^5 + 6/5*A*a^2*c^2*x^5 + C*a^3*b*x^4 + A*a*b^3*x^4 + 3*A*a^2*b*c*x
^4 + 1/3*C*a^4*x^3 + 2*A*a^2*b^2*x^3 + 4/3*A*a^3*c*x^3 + 2*A*a^3*b*x^2 + A*
a^4*x
```

**maple [A]** time = 0.00, size = 343, normalized size = 1.35

$$\frac{C c^4 x^{11}}{11}+\frac{2Cb\,c^3x^{10}}{5}+\frac{\left(A\,c^4+\left(4b^2c^2+2\left(2ac+b^2\right)c^2\right)C\right)x^9}{9}+\frac{\left(4Ab\,c^3+\left(4ab\,c^2+4\left(2ac+b^2\right)bc\right)C\right)x^8}{8}+2A\,a^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^4*(C*x^2+A),x)
```

```
[Out] 1/11*c^4*C*x^11+2/5*b*c^3*C*x^10+1/9*((2*(2*a*c+b^2)*c^2+4*b^2*c^2)*C+c^4*A
)*x^9+1/8*((4*a*b*c^2+4*(2*a*c+b^2)*b*c)*C+4*b*c^3*A)*x^8+1/7*((2*a^2*c^2+8
*a*b^2*c+(2*a*c+b^2)^2)*C+(2*(2*a*c+b^2)*c^2+4*b^2*c^2)*A)*x^7+1/6*((4*a^2*
b*c+4*a*b*(2*a*c+b^2))*C+(4*a*b*c^2+4*(2*a*c+b^2)*b*c)*A)*x^6+1/5*((2*a^2*(
2*a*c+b^2)+4*a^2*b^2)*C+(2*a^2*c^2+8*a*b^2*c+(2*a*c+b^2)^2)*A)*x^5+1/4*(4*a
^3*b*C+(4*a^2*b*c+4*a*b*(2*a*c+b^2))*A)*x^4+1/3*(a^4*C+(2*a^2*(2*a*c+b^2)+4
*a^2*b^2)*A)*x^3+2*a^3*A*b*x^2+a^4*A*x
```

**maxima [A]** time = 0.44, size = 263, normalized size = 1.04

$$\frac{1}{11}\,Cc^4x^{11}+\frac{2}{5}\,Cbc^3x^{10}+\frac{1}{9}\left(6\,Cb^2c^2+4\,Cac^3+Ac^4\right)x^9+\frac{1}{2}\left(Cb^3c+3\,Cabc^2+Abc^3\right)x^8+\frac{1}{7}\left(Cb^4+12\,Cab^2c+4\,Aa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^4*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/11*C*c^4*x^11 + 2/5*C*b*c^3*x^10 + 1/9*(6*C*b^2*c^2 + 4*C*a*c^3 + A*c^4)*
x^9 + 1/2*(C*b^3*c + 3*C*a*b*c^2 + A*b*c^3)*x^8 + 1/7*(C*b^4 + 12*C*a*b^2*c
 + 4*A*a*c^3 + 6*(C*a^2 + A*b^2)*c^2)*x^7 + 2*A*a^3*b*x^2 + 2/3*(C*a*b^3 +
3*A*a*b*c^2 + (3*C*a^2*b + A*b^3)*c)*x^6 + A*a^4*x + 1/5*(6*C*a^2*b^2 + A*b
^4 + 6*A*a^2*c^2 + 4*(C*a^3 + 3*A*a*b^2)*c)*x^5 + (C*a^3*b + A*a*b^3 + 3*A*
a^2*b*c)*x^4 + 1/3*(C*a^4 + 6*A*a^2*b^2 + 4*A*a^3*c)*x^3
```

**mupad [B]** time = 0.13, size = 244, normalized size = 0.96

$$x^5\left(\frac{4\,C\,a^3\,c}{5}+\frac{6\,C\,a^2\,b^2}{5}+\frac{6\,A\,a^2\,c^2}{5}+\frac{12\,A\,a\,b^2\,c}{5}+\frac{A\,b^4}{5}\right)+x^7\left(\frac{6\,C\,a^2\,c^2}{7}+\frac{12\,C\,a\,b^2\,c}{7}+\frac{4\,A\,a\,c^3}{7}+\frac{C\,b^4}{7}+\frac{6\,A\,b}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^4,x)
```

```
[Out] x^5*((A*b^4)/5 + (6*A*a^2*c^2)/5 + (6*C*a^2*b^2)/5 + (4*C*a^3*c)/5 + (12*A*
a*b^2*c)/5) + x^7*((C*b^4)/7 + (6*A*b^2*c^2)/7 + (6*C*a^2*c^2)/7 + (4*A*a*c
^3)/7 + (12*C*a*b^2*c)/7) + x^3*((C*a^4)/3 + 2*A*a^2*b^2 + (4*A*a^3*c)/3) +
 x^9*((A*c^4)/9 + (2*C*b^2*c^2)/3 + (4*C*a*c^3)/9) + (C*c^4*x^11)/11 + A*a^
4*x + (2*b*x^6*(3*a*c + b^2)*(A*c + C*a))/3 + a*b*x^4*(A*b^2 + C*a^2 + 3*A*
a*c) + (b*c*x^8*(A*c^2 + C*b^2 + 3*C*a*c))/2 + 2*A*a^3*b*x^2 + (2*C*b*c^3*x
^10)/5
```

**sympy [A]** time = 0.14, size = 320, normalized size = 1.26

$$Aa^4x+2Aa^3bx^2+\frac{2Cbc^3x^{10}}{5}+\frac{Cc^4x^{11}}{11}+x^9\left(\frac{Ac^4}{9}+\frac{4Cac^3}{9}+\frac{2Cb^2c^2}{3}\right)+x^8\left(\frac{Abc^3}{2}+\frac{3Cabc^2}{2}+\frac{Cb^3c}{2}\right)+x^7\left(\frac{4Aac^3}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**4*(C*x**2+A),x)
```

```
[Out] A*a**4*x + 2*A*a**3*b*x**2 + 2*C*b*c**3*x**10/5 + C*c**4*x**11/11 + x**9*(A
*c**4/9 + 4*C*a*c**3/9 + 2*C*b**2*c**2/3) + x**8*(A*b*c**3/2 + 3*C*a*b*c**2
/2 + C*b**3*c/2) + x**7*(4*A*a*c**3/7 + 6*A*b**2*c**2/7 + 6*C*a**2*c**2/7 +
 12*C*a*b**2*c/7 + C*b**4/7) + x**6*(2*A*a*b*c**2 + 2*A*b**3*c/3 + 2*C*a**2
*b*c + 2*C*a*b**3/3) + x**5*(6*A*a**2*c**2/5 + 12*A*a*b**2*c/5 + A*b**4/5 +
 4*C*a**3*c/5 + 6*C*a**2*b**2/5) + x**4*(3*A*a**2*b*c + A*a*b**3 + C*a**3*b
) + x**3*(4*A*a**3*c/3 + 2*A*a**2*b**2 + C*a**4/3)
```

# 3.141 $\int \left(a+bx+cx^2\right)^3 \left(A+Cx^2\right) \, dx$

**Optimal**. Leaf size=161

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\right.$$

```
[Out] a^3*A*x+3/2*a^2*A*b*x^2+1/3*a*(3*A*(a*c+b^2)+a^2*C)*x^3+1/4*b*(A*(6*a*c+b^2
)+3*a^2*C)*x^4+3/5*(a*c+b^2)*(A*c+C*a)*x^5+1/6*b*(3*A*c^2+(6*a*c+b^2)*C)*x^
6+1/7*c*(A*c^2+3*(a*c+b^2)*C)*x^7+3/8*b*c^2*C*x^8+1/9*c^3*C*x^9
```

**Rubi [A]** time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$\frac{1}{4}bx^4\left(3a^2C+A\left(6ac+b^2\right)\right)+\frac{1}{3}ax^3\left(a^2C+3A\left(ac+b^2\right)\right)+\frac{3}{2}a^2Abx^2+a^3Ax+\frac{1}{7}cx^7\left(3C\left(ac+b^2\right)+Ac^2\right)+\frac{1}{6}bx^6\left(C\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*(b^2 + a*c) + a^2*C)*x^3)/3 + (b*(A*(
b^2 + 6*a*c) + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(3*
A*c^2 + (b^2 + 6*a*c)*C)*x^6)/6 + (c*(A*c^2 + 3*(b^2 + a*c)*C)*x^7)/7 + (3*
b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^3\left(A+Cx^2\right)dx=\int \left(a^3A+3a^2Abx+a\left(3A\left(b^2+ac\right)+a^2C\right)x^2+b\left(A\left(b^2+6ac\right)+3a^2C\right)x^3\right.$$
$$=a^3Ax+\frac{3}{2}a^2Abx^2+\frac{1}{3}a\left(3A\left(b^2+ac\right)+a^2C\right)x^3+\frac{1}{4}b\left(A\left(b^2+6ac\right)+3a^2C\right)$$

**Mathematica [A]** time = 0.05, size = 163, normalized size = 1.01

$$a^3Ax+\frac{1}{4}bx^4\left(3a^2C+6aAc+Ab^2\right)+\frac{1}{3}ax^3\left(a^2C+3aAc+3Ab^2\right)+\frac{3}{2}a^2Abx^2+\frac{1}{7}cx^7\left(3acC+Ac^2+3b^2C\right)+\frac{1}{6}bx^6\left(6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^3*(A + C*x^2),x]
```

```
[Out] a^3*A*x + (3*a^2*A*b*x^2)/2 + (a*(3*A*b^2 + 3*a*A*c + a^2*C)*x^3)/3 + (b*(A
*b^2 + 6*a*A*c + 3*a^2*C)*x^4)/4 + (3*(b^2 + a*c)*(A*c + a*C)*x^5)/5 + (b*(
3*A*c^2 + b^2*C + 6*a*c*C)*x^6)/6 + (c*(A*c^2 + 3*b^2*C + 3*a*c*C)*x^7)/7 +
 (3*b*c^2*C*x^8)/8 + (c^3*C*x^9)/9
```

**fricas [A]** time = 0.65, size = 187, normalized size = 1.16

$$\frac{1}{9}x^9c^3C+\frac{3}{8}x^8c^2bC+\frac{3}{7}x^7cb^2C+\frac{3}{7}x^7c^2aC+\frac{1}{7}x^7c^3A+\frac{1}{6}x^6b^3C+x^6cbaC+\frac{1}{2}x^6c^2bA+\frac{3}{5}x^5b^2aC+\frac{3}{5}x^5ca^2C+\frac{3}{5}x^5cb^2A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*C + 3/8*x^8*c^2*b*C + 3/7*x^7*c*b^2*C + 3/7*x^7*c^2*a*C + 1/7*x
^7*c^3*A + 1/6*x^6*b^3*C + x^6*c*b*a*C + 1/2*x^6*c^2*b*A + 3/5*x^5*b^2*a*C
+ 3/5*x^5*c*a^2*C + 3/5*x^5*c*b^2*A + 3/5*x^5*c^2*a*A + 3/4*x^4*b*a^2*C + 1
/4*x^4*b^3*A + 3/2*x^4*c*b*a*A + 1/3*x^3*a^3*C + x^3*b^2*a*A + x^3*c*a^2*A
+ 3/2*x^2*b*a^2*A + x*a^3*A
```

**giac [A]** time = 0.17, size = 187, normalized size = 1.16

$$\frac{1}{9}\,Cc^3x^9+\frac{3}{8}\,Cbc^2x^8+\frac{3}{7}\,Cb^2cx^7+\frac{3}{7}\,Cac^2x^7+\frac{1}{7}\,Ac^3x^7+\frac{1}{6}\,Cb^3x^6+Cabcx^6+\frac{1}{2}\,Abc^2x^6+\frac{3}{5}\,Cab^2x^5+\frac{3}{5}\,Ca^2cx^5+\frac{3}{5}\,Ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 3/7*C*b^2*c*x^7 + 3/7*C*a*c^2*x^7 + 1/7*A
*c^3*x^7 + 1/6*C*b^3*x^6 + C*a*b*c*x^6 + 1/2*A*b*c^2*x^6 + 3/5*C*a*b^2*x^5
+ 3/5*C*a^2*c*x^5 + 3/5*A*b^2*c*x^5 + 3/5*A*a*c^2*x^5 + 3/4*C*a^2*b*x^4 + 1
/4*A*b^3*x^4 + 3/2*A*a*b*c*x^4 + 1/3*C*a^3*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3
+ 3/2*A*a^2*b*x^2 + A*a^3*x
```

**maple [A]** time = 0.00, size = 223, normalized size = 1.39

$$\frac{C\,c^3x^9}{9}+\frac{3Cb\,c^2x^8}{8}+\frac{\left(A\,c^3+\left(a\,c^2+2b^2c+\left(2ac+b^2\right)c\right)C\right)x^7}{7}+\frac{3A\,a^2b\,x^2}{2}+\frac{\left(3Ab\,c^2+\left(4abc+\left(2ac+b^2\right)b\right)C\right)x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^3*(C*x^2+A),x)
```

```
[Out] 1/9*C*c^3*x^9+3/8*b*c^2*C*x^8+1/7*((a*c^2+2*b^2*c+c*(2*a*c+b^2))*C+c^3*A)*x
^7+1/6*((4*a*b*c+b*(2*a*c+b^2))*C+3*b*c^2*A)*x^6+1/5*((a*(2*a*c+b^2)+2*b^2*
a+c*a^2)*C+(a*c^2+2*b^2*c+c*(2*a*c+b^2))*A)*x^5+1/4*(3*a^2*b*C+(4*a*b*c+b*(
2*a*c+b^2))*A)*x^4+1/3*(a^3*C+(a*(2*a*c+b^2)+2*b^2*a+c*a^2)*A)*x^3+3/2*a^2*
A*b*x^2+A*a^3*x
```

**maxima [A]** time = 0.43, size = 165, normalized size = 1.02

$$\frac{1}{9}\,Cc^3x^9+\frac{3}{8}\,Cbc^2x^8+\frac{1}{7}\left(3\,Cb^2c+3\,Cac^2+Ac^3\right)x^7+\frac{1}{6}\left(Cb^3+6\,Cabc+3\,Abc^2\right)x^6+\frac{3}{2}\,Aa^2bx^2+\frac{3}{5}\left(Cab^2+Aac^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^3*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/9*C*c^3*x^9 + 3/8*C*b*c^2*x^8 + 1/7*(3*C*b^2*c + 3*C*a*c^2 + A*c^3)*x^7 +
 1/6*(C*b^3 + 6*C*a*b*c + 3*A*b*c^2)*x^6 + 3/2*A*a^2*b*x^2 + 3/5*(C*a*b^2 +
 A*a*c^2 + (C*a^2 + A*b^2)*c)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + A*b^3 + 6*A*
a*b*c)*x^4 + 1/3*(C*a^3 + 3*A*a*b^2 + 3*A*a^2*c)*x^3
```

**mupad [B]** time = 0.07, size = 149, normalized size = 0.93

$$x^3\left(\frac{C\,a^3}{3}+A\,c\,a^2+A\,a\,b^2\right)+x^7\left(\frac{3\,C\,b^2\,c}{7}+\frac{A\,c^3}{7}+\frac{3\,C\,a\,c^2}{7}\right)+\frac{b\,x^4\,\left(3\,C\,a^2+6\,A\,c\,a+A\,b^2\right)}{4}+\frac{b\,x^6\,\left(C\,b^2+3\,A\,c^2\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^3,x)
```

```
[Out] x^3*((C*a^3)/3 + A*a*b^2 + A*a^2*c) + x^7*((A*c^3)/7 + (3*C*a*c^2)/7 + (3*C
*b^2*c)/7) + (b*x^4*(A*b^2 + 3*C*a^2 + 6*A*a*c))/4 + (b*x^6*(3*A*c^2 + C*b^
2 + 6*C*a*c))/6 + (C*c^3*x^9)/9 + A*a^3*x + (3*x^5*(a*c + b^2)*(A*c + C*a))
/5 + (3*A*a^2*b*x^2)/2 + (3*C*b*c^2*x^8)/8
```

**sympy [A]** time = 0.11, size = 197, normalized size = 1.22

$$Aa^3x+\frac{3Aa^2bx^2}{2}+\frac{3Cbc^2x^8}{8}+\frac{Cc^3x^9}{9}+x^7\left(\frac{Ac^3}{7}+\frac{3Cac^2}{7}+\frac{3Cb^2c}{7}\right)+x^6\left(\frac{Abc^2}{2}+Cabc+\frac{Cb^3}{6}\right)+x^5\left(\frac{3Aac^2}{5}+\frac{3Ab^2c}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**3*(C*x**2+A),x)
```

```
[Out] A*a**3*x + 3*A*a**2*b*x**2/2 + 3*C*b*c**2*x**8/8 + C*c**3*x**9/9 + x**7*(A*
c**3/7 + 3*C*a*c**2/7 + 3*C*b**2*c/7) + x**6*(A*b*c**2/2 + C*a*b*c + C*b**3
/6) + x**5*(3*A*a*c**2/5 + 3*A*b**2*c/5 + 3*C*a**2*c/5 + 3*C*a*b**2/5) + x*
*4*(3*A*a*b*c/2 + A*b**3/4 + 3*C*a**2*b/4) + x**3*(A*a**2*c + A*a*b**2 + C*
a**3/3)
```

# 3.142 $\int \left(a+bx+cx^2\right)^2 \left(A+Cx^2\right) \, dx$

**Optimal**. Leaf size=96

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

```
[Out] a^2*A*x+a*A*b*x^2+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+1/2*b*(A*c+C*a)*x^4+1/5*(A*
c^2+(2*a*c+b^2)*C)*x^5+1/3*b*c*C*x^6+1/7*c^2*C*x^7
```

**Rubi [A]** time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1657}

$$\frac{1}{3}x^3\left(a^2C+A\left(2ac+b^2\right)\right)+a^2Ax+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+Ac^2\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*(b^2 + 2*a*c) + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + (b^2 + 2*a*c)*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^2\left(A+Cx^2\right)\,dx=\int \left(a^2A+2aAbx+\left(A\left(b^2+2ac\right)+a^2C\right)x^2+2b(Ac+aC)x^3+\left(Ac^2+\left(b^2\right.\right.\right.$$

$$=a^2Ax+aAbx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+a^2C\right)x^3+\frac{1}{2}b(Ac+aC)x^4+\frac{1}{5}\left(Ac^2+\left(\right.\right.$$

**Mathematica [A]** time = 0.02, size = 96, normalized size = 1.00

$$\frac{1}{3}x^3\left(a^2C+2aAc+Ab^2\right)+a^2Ax+\frac{1}{5}x^5\left(2acC+Ac^2+b^2C\right)+\frac{1}{2}bx^4(aC+Ac)+aAbx^2+\frac{1}{3}bcCx^6+\frac{1}{7}c^2Cx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^2*(A + C*x^2),x]
```

```
[Out] a^2*A*x + a*A*b*x^2 + ((A*b^2 + 2*a*A*c + a^2*C)*x^3)/3 + (b*(A*c + a*C)*x^
4)/2 + ((A*c^2 + b^2*C + 2*a*c*C)*x^5)/5 + (b*c*C*x^6)/3 + (c^2*C*x^7)/7
```

**fricas [A]** time = 0.83, size = 99, normalized size = 1.03

$$\frac{1}{7}x^7c^2C+\frac{1}{3}x^6cbC+\frac{1}{5}x^5b^2C+\frac{2}{5}x^5caC+\frac{1}{5}x^5c^2A+\frac{1}{2}x^4baC+\frac{1}{2}x^4cbA+\frac{1}{3}x^3a^2C+\frac{1}{3}x^3b^2A+\frac{2}{3}x^3caA+x^2baA+xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^2*C + 1/3*x^6*c*b*C + 1/5*x^5*b^2*C + 2/5*x^5*c*a*C + 1/5*x^5*c^2
*A + 1/2*x^4*b*a*C + 1/2*x^4*c*b*A + 1/3*x^3*a^2*C + 1/3*x^3*b^2*A + 2/3*x^
3*c*a*A + x^2*b*a*A + x*a^2*A
```

**giac [A]** time = 0.15, size = 99, normalized size = 1.03

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}Cb^2x^5+\frac{2}{5}Cacx^5+\frac{1}{5}Ac^2x^5+\frac{1}{2}Cabx^4+\frac{1}{2}Abcx^4+\frac{1}{3}Ca^2x^3+\frac{1}{3}Ab^2x^3+\frac{2}{3}Aacx^3+Aabx^2+Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 1/5*A*c^2*x
^5 + 1/2*C*a*b*x^4 + 1/2*A*b*c*x^4 + 1/3*C*a^2*x^3 + 1/3*A*b^2*x^3 + 2/3*A*
a*c*x^3 + A*a*b*x^2 + A*a^2*x
```

**maple [A]** time = 0.00, size = 90, normalized size = 0.94

$$\frac{C\,c^2x^7}{7}+\frac{Cbc\,x^6}{3}+Aab\,x^2+\frac{\left(A\,c^2+\left(2ac+b^2\right)C\right)x^5}{5}+A\,a^2x+\frac{\left(2bcA+2abC\right)x^4}{4}+\frac{\left(C\,a^2+\left(2ac+b^2\right)A\right)x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^2*(C*x^2+A),x)
```

```
[Out] 1/7*C*c^2*x^7+1/3*b*c*C*x^6+1/5*(A*c^2+(2*a*c+b^2)*C)*x^5+1/4*(2*A*b*c+2*C*
a*b)*x^4+1/3*(A*(2*a*c+b^2)+a^2*C)*x^3+a*A*b*x^2+A*a^2*x
```

**maxima [A]** time = 0.43, size = 87, normalized size = 0.91

$$\frac{1}{7}Cc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{5}\left(Cb^2+2\,Cac+Ac^2\right)x^5+Aabx^2+\frac{1}{2}\left(Cab+Abc\right)x^4+Aa^2x+\frac{1}{3}\left(Ca^2+Ab^2+2\,Aac\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^2*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/3*C*b*c*x^6 + 1/5*(C*b^2 + 2*C*a*c + A*c^2)*x^5 + A*a*b*x
^2 + 1/2*(C*a*b + A*b*c)*x^4 + A*a^2*x + 1/3*(C*a^2 + A*b^2 + 2*A*a*c)*x^3
```

**mupad** [B] time = 4.09, size = 88, normalized size = 0.92

$$x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,c\,a}{3}+\frac{A\,b^2}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{A\,c^2}{5}+\frac{2\,C\,a\,c}{5}\right)+\frac{C\,c^2\,x^7}{7}+A\,a^2\,x+\frac{b\,x^4\,\left(A\,c+C\,a\right)}{2}+A\,a\,b\,x^2+\frac{C\,b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^2,x)
```

```
[Out] x^3*((A*b^2)/3 + (C*a^2)/3 + (2*A*a*c)/3) + x^5*((A*c^2)/5 + (C*b^2)/5 + (2
*C*a*c)/5) + (C*c^2*x^7)/7 + A*a^2*x + (b*x^4*(A*c + C*a))/2 + A*a*b*x^2 +
(C*b*c*x^6)/3
```

**sympy** [A] time = 0.09, size = 102, normalized size = 1.06

$$Aa^2x+Aabx^2+\frac{Cbcx^6}{3}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Abc}{2}+\frac{Cab}{2}\right)+x^3\left(\frac{2Aac}{3}+\frac{Ab^2}{3}+\frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**2*(C*x**2+A),x)
```

```
[Out] A*a**2*x + A*a*b*x**2 + C*b*c*x**6/3 + C*c**2*x**7/7 + x**5*(A*c**2/5 + 2*C
*a*c/5 + C*b**2/5) + x**4*(A*b*c/2 + C*a*b/2) + x**3*(2*A*a*c/3 + A*b**2/3
+ C*a**2/3)
```

# 3.143 $\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx$

**Optimal**. Leaf size=46

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

```
[Out] a*A*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*c*C*x^5
```

**Rubi** [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx+cx^2\right)\left(A+Cx^2\right)\,dx&=\int \left(aA+Abx+(Ac+aC)x^2+bCx^3+cCx^4\right)\,dx\\&=aAx+\frac{1}{2}Abx^2+\frac{1}{3}(Ac+aC)x^3+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 46, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ac)+aAx+\frac{1}{2}Abx^2+\frac{1}{4}bCx^4+\frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)*(A + C*x^2),x]
```

```
[Out] a*A*x + (A*b*x^2)/2 + ((A*c + a*C)*x^3)/3 + (b*C*x^4)/4 + (c*C*x^5)/5
```

**fricas [A]** time = 0.73, size = 40, normalized size = 0.87

$$\frac{1}{5}x^5cC + \frac{1}{4}x^4bC + \frac{1}{3}x^3aC + \frac{1}{3}x^3cA + \frac{1}{2}x^2bA + xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="fricas")
[Out] 1/5*x^5*c*C + 1/4*x^4*b*C + 1/3*x^3*a*C + 1/3*x^3*c*A + 1/2*x^2*b*A + x*a*A
```

**giac [A]** time = 0.18, size = 40, normalized size = 0.87

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{3}\,Cax^3 + \frac{1}{3}\,Acx^3 + \frac{1}{2}\,Abx^2 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="giac")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/3*C*a*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2 + A*a*x
```

**maple [A]** time = 0.00, size = 39, normalized size = 0.85

$$\frac{Cc\,x^5}{5} + \frac{Cb\,x^4}{4} + \frac{Ab\,x^2}{2} + Aax + \frac{(Ac + aC)\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)*(C*x^2+A),x)
[Out] A*a*x+1/2*A*b*x^2+1/3*(A*c+C*a)*x^3+1/4*b*C*x^4+1/5*C*c*x^5
```

**maxima [A]** time = 0.43, size = 38, normalized size = 0.83

$$\frac{1}{5}\,Ccx^5 + \frac{1}{4}\,Cbx^4 + \frac{1}{2}\,Abx^2 + \frac{1}{3}\,(Ca + Ac)x^3 + Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)*(C*x^2+A),x, algorithm="maxima")
[Out] 1/5*C*c*x^5 + 1/4*C*b*x^4 + 1/2*A*b*x^2 + 1/3*(C*a + A*c)*x^3 + A*a*x
```

**mupad [B]** time = 0.03, size = 39, normalized size = 0.85

$$\frac{C\,c\,x^5}{5} + \frac{C\,b\,x^4}{4} + \left(\frac{A\,c}{3} + \frac{C\,a}{3}\right)\,x^3 + \frac{A\,b\,x^2}{2} + A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2),x)
```

```
[Out] x^3*((A*c)/3 + (C*a)/3) + A*a*x + (A*b*x^2)/2 + (C*b*x^4)/4 + (C*c*x^5)/5
```

**sympy [A]** time = 0.07, size = 42, normalized size = 0.91

$$Aax + \frac{Abx^2}{2} + \frac{Cbx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)*(C*x**2+A),x)
```

```
[Out] A*a*x + A*b*x**2/2 + C*b*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 + C*a/3)
```

# 3.144 $\int \frac{A+Cx^2}{a+bx+cx^2} \, dx$

**Optimal**. Leaf size=81

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

```
[Out] C*x/c-1/2*b*C*ln(c*x^2+b*x+a)/c^2-(2*A*c^2+(-2*a*c+b^2)*C)*arctanh((2*c*x+b
)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\left(C\left(b^2-2ac\right)+2Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c - ((2*A*c^2 + (b^2 - 2*a*c)*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c^2*Sqrt[b^2 - 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Cx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{C}{c}+\frac{Ac-aC-bCx}{c\left(a+bx+cx^2\right)}\right)dx\\
&= \frac{Cx}{c}+\frac{\int \frac{Ac-aC-bCx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{Cx}{c}-\frac{(bC)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{1}{2}\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\int \frac{1}{a+bx+cx^2}\,dx\\
&= \frac{Cx}{c}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\left(-2A-\frac{\left(b^2-2ac\right)C}{c^2}\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right.\\
&= \frac{Cx}{c}-\frac{\left(2A+\frac{\left(b^2-2ac\right)C}{c^2}\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 84, normalized size = 1.04

$$\frac{\left(-2acC+2Ac^2+b^2C\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{bC\log\left(a+bx+cx^2\right)}{2c^2}+\frac{Cx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (C*x)/c + ((2*A*c^2 + b^2*C - 2*a*c*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/(c^2*Sqrt[-b^2 + 4*a*c]) - (b*C*Log[a + b*x + c*x^2])/(2*c^2)
```

**fricas [A]** time = 0.77, size = 265, normalized size = 3.27

$$\left[\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\left(Cb^2c-4\,Cac^2\right)x-\left(Cb^3-4\,Cabc\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(C*
b^2*c - 4*C*a*c^2)*x - (C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 -
 4*a*c^3), -1/2*(2*(C*b^2 - 2*C*a*c + 2*A*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(C*b^2*c - 4*C*a*c^2)*x +
(C*b^3 - 4*C*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

**giac [A]** time = 0.15, size = 78, normalized size = 0.96

$$\frac{Cx}{c}-\frac{Cb\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(Cb^2-2\,Cac+2\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] C*x/c - 1/2*C*b*log(c*x^2 + b*x + a)/c^2 + (C*b^2 - 2*C*a*c + 2*A*c^2)*arct
an((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

**maple [A]** time = 0.01, size = 140, normalized size = 1.73

$$\frac{2A\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{2Ca\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c}+\frac{C\,b^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}-\frac{Cb\ln\left(c\,x^2+bx+a\right)}{2c^2}+\frac{Cx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a),x)
```

```
[Out] C/c*x-1/2*b*C*ln(c*x^2+b*x+a)/c^2+2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*A-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*C+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*C
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 0.19, size = 224, normalized size = 2.77

$$\frac{2\,A\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}+\frac{C\,x}{c}+\frac{C\,b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,C\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{C\,b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}\right.}{c^2\,\sqrt{4\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (2*A*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^
2)^(1/2) + (C*x)/c + (C*b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) -
 (2*C*a*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*
c - b^2)^(1/2)) + (C*b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)
^(1/2)))/(c^2*(4*a*c - b^2)^(1/2)) - (2*C*a*b*c*log(a + b*x + c*x^2))/(4*a*
c^3 - b^2*c^2)
```

**sympy [B]** time = 1.21, size = 413, normalized size = 5.10

$$\frac{Cx}{c}+\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-Abc-Cab-4ac^2\left(-\frac{Cb}{2c^2}-\frac{\sqrt{-4ac+b^2}\left(-2Ac^2+2Cac-Cb^2\right)}{2c^2\left(4ac-b^2\right)}\right.}{-2Ac^2+2Cac-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a),x)
```

```
[Out] C*x/c + (-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)
/(2*c**2*(4*a*c - b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2)
 - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2
))) + b**2*c*(-C*b/(2*c**2) - sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*
b**2)/(2*c**2*(4*a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2)) + (-C*b/(2*
c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c -
 b**2)))*log(x + (-A*b*c - C*a*b - 4*a*c**2*(-C*b/(2*c**2) + sqrt(-4*a*c +
b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-C*
b/(2*c**2) + sqrt(-4*a*c + b**2)*(-2*A*c**2 + 2*C*a*c - C*b**2)/(2*c**2*(4*
a*c - b**2))))/(-2*A*c**2 + 2*C*a*c - C*b**2))
```

# 3.145 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=100

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*
(A*c+C*a)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1660, 12, 618, 206}

$$\frac{4(aC+Ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] -((b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(c*(b^2 - 4*a*c)*(a +
 b*x + c*x^2))) + (4*(A*c + a*C)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b
^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^2}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2(Ac+aC)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2(Ac+aC))\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4(Ac+aC))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c\right.}{b^2-4ac}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4(Ac+aC)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

**Mathematica [A]** time = 0.08, size = 98, normalized size = 0.98

$$\frac{aC(b-2cx)+Ac(b+2cx)+b^2Cx}{c\left(4ac-b^2\right)(a+x(b+cx))}+\frac{4(aC+Ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b +
 c*x))) + (4*(A*c + a*C)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*
a*c)^(3/2)
```

**fricas [B]** time = 1.26, size = 511, normalized size = 5.11

$$\left[-\frac{Cab^3-4\,Aabc^2+2\left(Ca^2c+Aac^2+\left(Cac^2+Ac^3\right)x^2+\left(Cabc+Abc^2\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b}}{cx^2+bx+a}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-(C*a*b^3 - 4*A*a*b*c^2 + 2*(C*a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (
C*a*b*c + A*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (4*C*a^2*b - A*b^
3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x)/(a*b^
4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2
 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), -(C*a*b^3 - 4*A*a*b*c^2 - 4*(C*
a^2*c + A*a*c^2 + (C*a*c^2 + A*c^3)*x^2 + (C*a*b*c + A*b*c^2)*x)*sqrt(-b^2
+ 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (4*C*a^2*b
 - A*b^3)*c + (C*b^4 - 6*C*a*b^2*c - 8*A*a*c^3 + 2*(4*C*a^2 + A*b^2)*c^2)*x
)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x)]
```

**giac [A]** time = 0.16, size = 108, normalized size = 1.08

$$-\frac{4\,(Ca+Ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{Cb^2x-2\,Cacx+2\,Ac^2x+Cab+Abc}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*(C*a + A*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (C*b^2*x - 2*C*a*c*x + 2*A*c^2*x + C*a*b + A*b*c)/((b^2*c -
 4*a*c^2)*(c*x^2 + b*x + a))
```

**maple [A]** time = 0.01, size = 146, normalized size = 1.46

$$\frac{4Ac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4Ca\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{\frac{(Ac+aC)b}{\left(4ac-b^2\right)c}+\frac{\left(2A\,c^2-2Cac+C\,b^2\right)x}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((2*A*c^2-2*C*a*c+C*b^2)/c/(4*a*c-b^2)*x+b/c*(A*c+C*a)/(4*a*c-b^2))/(c*x^2+
b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c+4/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*C
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 4.53, size = 172, normalized size = 1.72

$$\frac{\frac{A\,b\,c+C\,a\,b}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(C\,b^2+2\,A\,c^2-2\,C\,a\,c\right)}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,\mathrm{atan}\left(\frac{\left(\frac{2\,\left(A\,c+C\,a\right)\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c\,x\,\left(A\,c+C\,a\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,A\,c+2\,C\,a}\right)\,\left(A\,c+C\,a\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((A*b*c + C*a*b)/(c*(4*a*c - b^2)) + (x*(2*A*c^2 + C*b^2 - 2*C*a*c))/(c*(4*
a*c - b^2)))/(a + b*x + c*x^2) - (4*atan((((2*(A*c + C*a)*(b^3 - 4*a*b*c))/
(4*a*c - b^2)^(5/2) - (4*c*x*(A*c + C*a))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2
))/(2*A*c + 2*C*a))*(A*c + C*a))/(4*a*c - b^2)^(3/2)
```

**sympy [B]** time = 1.21, size = 376, normalized size = 3.76

$$-2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(Ac+Ca)\log\left(x+\frac{2Abc+2Cab-32a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(Ac+Ca)+16ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\,(Ac+}{4Ac^2+4Cac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2*A*b*c + 2*C*a*b - 32*a
**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 16*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(A*c + C*a) - 2*b**4*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a))/
(4*A*c**2 + 4*C*a*c)) + 2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a)*log(x + (2
*A*b*c + 2*C*a*b + 32*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) - 16
*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(A*c + C*a) + 2*b**4*sqrt(-1/(4*a*c -
b**2)**3)*(A*c + C*a))/(4*A*c**2 + 4*C*a*c)) + (A*b*c + C*a*b + x*(2*A*c**2
 - 2*C*a*c + C*b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2)
 + x*(4*a*b*c**2 - b**3*c))
```

## 3.146 $\int \frac{A+Cx^2}{(a+bx+cx^2)^3}\,dx$

**Optimal**. Leaf size=161

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\frac{b^2}{c}\right.}{2\left(b^2-4ac\right)^2\left(a+bx+cx\right.}$$

```
[Out] 1/2*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^2+1/2*(6*A*c+2*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-2*(6*A
*c^2+(2*a*c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2
)
```

**Rubi [A]** time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{2\left(C\left(2ac+b^2\right)+6Ac^2\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{(b+2cx)\left(2aC+6Ac+\frac{b^2}{c}\right.}{2\left(b^2-4ac\right)^2\left(a+bx+cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(2*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^2) + ((6*A*c + 2*a*C + (b^2*C)/c)*(b + 2*c*x))/(2*(b^2 - 4*a
*c)^2*(a + b*x + c*x^2)) - (2*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^3}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\int \frac{6Ac+2aC+\frac{b^2C}{c}}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{\left(6Ac^2+\left(b^2\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{\left(2\left(6Ac^2+\left(\right.\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{2c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{\left(6Ac+2aC+\frac{b^2C}{c}\right)(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{2\left(6Ac^2+\left(b\right.\right.}{}$$

**Mathematica** [**A**] time = 0.20, size = 160, normalized size = 0.99

$$\frac{1}{2}\left(\frac{(b+2cx)\left(C\left(2ac+b^2\right)+6Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{4\left(C\left(2ac+b^2\right)+6Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{aC(b-2cx)+Ac(b+2cx)+b}{c\left(4ac-b^2\right)(a+x(b+cx))^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^3,x]
```

```
[Out] (((6*A*c^2 + (b^2 + 2*a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) + (4*(6*A*c^2 + (b^2 + 2*a*c)*C)*ArcTan[(b + 2*c*x)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

**fricas** [**B**] time = 0.89, size = 1199, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 - 2
4*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A*
a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 + 2*(C*a^2*b^2 + 2*C*a^3*c + 6*A
*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^2
 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b^
2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) - 2*(12*C*a^3*b - 7*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^
3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128
*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)
, 1/2*(6*C*a^2*b^3 - A*b^5 - 40*A*a^2*b*c^2 + 2*(C*b^4*c - 2*C*a*b^2*c^2 -
24*A*a*c^4 - 2*(4*C*a^2 - 3*A*b^2)*c^3)*x^3 + 3*(C*b^5 - 2*C*a*b^3*c - 24*A
*a*b*c^3 - 2*(4*C*a^2*b - 3*A*b^3)*c^2)*x^2 - 4*(C*a^2*b^2 + 2*C*a^3*c + 6*
A*a^2*c^2 + (C*b^2*c^2 + 2*C*a*c^3 + 6*A*c^4)*x^4 + 2*(C*b^3*c + 2*C*a*b*c^
2 + 6*A*b*c^3)*x^3 + (C*b^4 + 4*C*a*b^2*c + 12*A*a*c^3 + 2*(2*C*a^2 + 3*A*b
^2)*c^2)*x^2 + 2*(C*a*b^3 + 2*C*a^2*b*c + 6*A*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(12*C*a^3*b - 7
*A*a*b^3)*c + 2*(5*C*a*b^4 - 40*A*a^2*c^3 + 2*(4*C*a^3 + A*a*b^2)*c^2 - 2*(
11*C*a^2*b^2 - A*b^4)*c)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

**giac [A]** time = 0.18, size = 217, normalized size = 1.35

$$\frac{2\left(Cb^2+2\,Cac+6\,Ac^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Cb^2cx^3+4\,Cac^2x^3+12\,Ac^3x^3+3\,Cb^3x^2+6\,Cabcx^2+18\,Abc^2x}{2\left(b^4-8\,ab^2c+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*(C*b^2 + 2*C*a*c + 6*A*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*C*b^2*c*x^3 + 4*C*a*
c^2*x^3 + 12*A*c^3*x^3 + 3*C*b^3*x^2 + 6*C*a*b*c*x^2 + 18*A*b*c^2*x^2 + 10*
C*a*b^2*x - 4*C*a^2*c*x + 4*A*b^2*c*x + 20*A*a*c^2*x + 6*C*a^2*b - A*b^3 +
10*A*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

**maple [B]** time = 0.01, size = 373, normalized size = 2.32

$$\frac{12A\,c^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{4Cac\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{2C\,b^{2}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}+\frac{(6A\,c^{2}+\cdots}{16a^{2}c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^3,x)
```

```
[Out] (c*(6*A*c^2+2*C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(6*A*c^2+2*
C*a*c+C*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+(10*A*a*c^2+2*A*b^2*c-2*C*a^2*c
+5*C*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*(10*A*a*c-A*b^2+6*C*a^2)/(16
*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2+4/(16*a^2*c^2-8*a*b
^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*a*c+2/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*C*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 4.17, size = 401, normalized size = 2.49

$$\frac{\frac{6\,C\,a^{2}\,b+10\,A\,c\,a\,b-A\,b^{3}}{2\left(16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}\right)}+\frac{x\left(-2\,C\,a^{2}\,c+5\,C\,a\,b^{2}+10\,A\,a\,c^{2}+2\,A\,b^{2}\,c\right)}{16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}}+\frac{3\,b\,x^{2}\left(C\,b^{2}+6\,A\,c^{2}+2\,C\,a\,c\right)}{2\left(16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}\right)}+\frac{c\,x^{3}\left(C\,b^{2}+6\,A\,c^{2}+2\,C\,a\,c\right)}{16\,a^{2}\,c^{2}-8\,a\,b^{2}\,c+b^{4}}}{x^{2}\left(b^{2}+2\,a\,c\right)+a^{2}+c^{2}\,x^{4}+2\,a\,b\,x+2\,b\,c\,x^{3}}+\frac{2\,\mathrm{atan}\left(\left(\left(\frac{(16\cdots}{\cdots}\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^3,x)
```

```
[Out] ((6*C*a^2*b - A*b^3 + 10*A*a*b*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(
10*A*a*c^2 + 2*A*b^2*c + 5*C*a*b^2 - 2*C*a^2*c))/(b^4 + 16*a^2*c^2 - 8*a*b^
```

```
2*c) + (3*b*x^2*(6*A*c^2 + C*b^2 + 2*C*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2
*c)) + (c*x^3*(6*A*c^2 + C*b^2 + 2*C*a*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/
(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((b^5
 + 16*a^2*b*c^2 - 8*a*b^3*c)*(6*A*c^2 + C*b^2 + 2*C*a*c))/((4*a*c - b^2)^(5
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (2*c*x*(6*A*c^2 + C*b^2 + 2*C*a*c))/(
4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*A*c^2 + C*b^2 + 2*C*
a*c))*(6*A*c^2 + C*b^2 + 2*C*a*c))/(4*a*c - b^2)^(5/2)
```

**sympy [B]** time = 2.36, size = 774, normalized size = 4.81

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+Cb^2\right)\log\left(x+\frac{6Abc^2+2Cabc+Cb^3-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(6Ac^2+2Cac+C\right.}{\phantom{x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c*
*2 + 2*C*a*b*c + C*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2
 + 2*C*a*c + C*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c*
*2 + 2*C*a*c + C*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 +
 2*C*a*c + C*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c +
C*b**2))/(12*A*c**3 + 4*C*a*c**2 + 2*C*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5
)*(6*A*c**2 + 2*C*a*c + C*b**2)*log(x + (6*A*b*c**2 + 2*C*a*b*c + C*b**3 +
64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - 48*
a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) + 1
2*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2) - b**6*
sqrt(-1/(4*a*c - b**2)**5)*(6*A*c**2 + 2*C*a*c + C*b**2))/(12*A*c**3 + 4*C*
a*c**2 + 2*C*b**2*c)) + (10*A*a*b*c - A*b**3 + 6*C*a**2*b + x**3*(12*A*c**3
 + 4*C*a*c**2 + 2*C*b**2*c) + x**2*(18*A*b*c**2 + 6*C*a*b*c + 3*C*b**3) + x
*(20*A*a*c**2 + 4*A*b**2*c - 4*C*a**2*c + 10*C*a*b**2))/(32*a**4*c**2 - 16*
a**3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c*
*2) + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**
3 - 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

## 3.147 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx$

**Optimal**. Leaf size=206

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b^2}}\right.}{\left(b^2-4ac\right)^{7/2}}$$

```
[Out] 1/3*(-b*c*(A+a*C/c)-(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^3+1/3*(5*A*c+(a+1/c*b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^2-2*(5
*A*c^2+(a*c+b^2)*C)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*x+a)+8*c*(5*A*c^2+(a*
c+b^2)*C)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(7/2)
```

**Rubi [A]** time = 0.19, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1660, 12, 614, 618, 206}

$$-\frac{2(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)}-\frac{x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3}+\frac{8c\left(C\left(ac+b^2\right)+5Ac^2\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b^2}}\right.}{\left(b^2-4ac\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] -(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x)/(3*c*(b^2 - 4*a*c)*(a
+ b*x + c*x^2)^3) + ((5*A*c + (a + b^2/c)*C)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^
2*(a + b*x + c*x^2)^2) - (2*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 -
4*a*c)^3*(a + b*x + c*x^2)) + (8*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(7/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^4}\,dx = -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\int \frac{2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)}{\left(a+bx+cx^2\right)^3}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} - \frac{\left(2\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)\right)\int \frac{1}{\left(a+bx+cx^2\right)^3}dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} + \frac{\left(2\left(5Ac^2+\right.\right.}{}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^3} + \frac{\left(5Ac+\left(a+\frac{b^2}{c}\right)C\right)(b+2cx)}{3\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^2} - \frac{2\left(5Ac^2+\left(b\right.\right.}{\left(b^2-4ac\right.}$$

**Mathematica [A]** time = 0.41, size = 204, normalized size = 0.99

$$\frac{1}{3}\left(-\frac{6(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{\left(b^2-4ac\right)^3(a+x(b+cx))} + \frac{(b+2cx)\left(C\left(ac+b^2\right)+5Ac^2\right)}{c\left(b^2-4ac\right)^2(a+x(b+cx))^2} + \frac{24c\left(C\left(ac+b^2\right)+5Ac^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b}}\right.}{\left(4ac-b^2\right)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^4,x]
```

```
[Out] (((5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x
))^2) - (6*(5*A*c^2 + (b^2 + a*c)*C)*(b + 2*c*x))/((b^2 - 4*a*c)^3*(a + x*(
b + c*x))) + (b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))/(c*(-b^2 + 4*a*c
)*(a + x*(b + c*x))^3) + (24*c*(5*A*c^2 + (b^2 + a*c)*C)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(7/2))/3
```

**fricas [B]** time = 0.97, size = 2103, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/3*(C*a^2*b^5 + A*b^7 - 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4
- 20*A*a*c^6 - (4*C*a^2 - 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3
 - 20*A*a*b*c^5 - (4*C*a^2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b
^4*c^2 - 320*A*a^2*c^5 - 4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55
*A*b^4)*c^3)*x^3 - 2*(52*C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^
5*c - 320*A*a^2*b*c^4 - 4*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5
*A*b^5)*c^2)*x^2 + 12*(C*a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 +
 C*a*c^5 + 5*A*c^6)*x^6 + 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*
b^4*c^2 + 2*C*a*b^2*c^3 + 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c
 + 7*C*a*b^3*c^2 + 30*A*a*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b
^4*c + 2*C*a^2*b^2*c^2 + 5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*
a^2*b^3*c + C*a^3*b*c^2 + 5*A*a^2*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c^4 + 4*(4*C*a^
4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (18*C*a^2*b^4 -
A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3 + 2
56*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6 + 2
56*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4 - 256*a^3*b^3*
c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*b^6*c^3 - 160*
a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3*b^5*c^3 - 128
0*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8*c + 80*a^3*b^6
*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a^3*b^7*c + 96*
a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x), -1/3*(C*a^2*b^5 + A*b^7
- 264*A*a^3*b*c^3 + 12*(C*b^4*c^3 - 3*C*a*b^2*c^4 - 20*A*a*c^6 - (4*C*a^2 -
 5*A*b^2)*c^5)*x^5 + 30*(C*b^5*c^2 - 3*C*a*b^3*c^3 - 20*A*a*b*c^5 - (4*C*a^
2*b - 5*A*b^3)*c^4)*x^4 + 2*(11*C*b^6*c - 17*C*a*b^4*c^2 - 320*A*a^2*c^5 -
4*(16*C*a^3 + 35*A*a*b^2)*c^4 - (92*C*a^2*b^2 - 55*A*b^4)*c^3)*x^3 - 2*(52*
C*a^4*b - 59*A*a^2*b^3)*c^2 + 3*(C*b^7 + 13*C*a*b^5*c - 320*A*a^2*b*c^4 - 4
*(16*C*a^3*b - 15*A*a*b^3)*c^3 - (52*C*a^2*b^3 - 5*A*b^5)*c^2)*x^2 - 24*(C*
a^3*b^2*c + C*a^4*c^2 + 5*A*a^3*c^3 + (C*b^2*c^4 + C*a*c^5 + 5*A*c^6)*x^6 +
 3*(C*b^3*c^3 + C*a*b*c^4 + 5*A*b*c^5)*x^5 + 3*(C*b^4*c^2 + 2*C*a*b^2*c^3 +
 5*A*a*c^5 + (C*a^2 + 5*A*b^2)*c^4)*x^4 + (C*b^5*c + 7*C*a*b^3*c^2 + 30*A*a
*b*c^4 + (6*C*a^2*b + 5*A*b^3)*c^3)*x^3 + 3*(C*a*b^4*c + 2*C*a^2*b^2*c^2 +
5*A*a^2*c^4 + (C*a^3 + 5*A*a*b^2)*c^3)*x^2 + 3*(C*a^2*b^3*c + C*a^3*b*c^2 +
 5*A*a^2*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) + (22*C*a^3*b^3 - 17*A*a*b^5)*c + 3*(C*a*b^6 - 176*A*a^3*c
^4 + 4*(4*C*a^4 - 7*A*a^2*b^2)*c^3 - 2*(46*C*a^3*b^2 - 11*A*a*b^4)*c^2 + (1
8*C*a^2*b^4 - A*b^6)*c)*x)/(a^3*b^8 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3 + 256*a^7*c^4 + (b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6 + 256*a^4*c^7)*x^6 + 3*(b^9*c^2 - 16*a*b^7*c^3 + 96*a^2*b^5*c^4
- 256*a^3*b^3*c^5 + 256*a^4*b*c^6)*x^5 + 3*(b^10*c - 15*a*b^8*c^2 + 80*a^2*
```

b^6*c^3 - 160*a^3*b^4*c^4 + 256*a^5*c^6)*x^4 + (b^11 - 10*a*b^9*c + 320*a^3 *b^5*c^3 - 1280*a^4*b^3*c^4 + 1536*a^5*b*c^5)*x^3 + 3*(a*b^10 - 15*a^2*b^8* c + 80*a^3*b^6*c^2 - 160*a^4*b^4*c^3 + 256*a^6*c^5)*x^2 + 3*(a^2*b^9 - 16*a ^3*b^7*c + 96*a^4*b^5*c^2 - 256*a^5*b^3*c^3 + 256*a^6*b*c^4)*x)]

**giac [B]** time = 0.17, size = 407, normalized size = 1.98

$$-\frac{8\left(Cb^2c + Cac^2 + 5\,Ac^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^6 - 12\,ab^4c + 48\,a^2b^2c^2 - 64\,a^3c^3\right)\sqrt{-b^2+4\,ac}} - \frac{12\,Cb^2c^3x^5 + 12\,Cac^4x^5 + 60\,Ac^5x^5 + 30\,Cb^3c^2x^4 + 30\,Cabc^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8*(C*b^2*c + C*a*c^2 + 5*A*c^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-b^2 + 4*a*c)) - 1/3*(1
2*C*b^2*c^3*x^5 + 12*C*a*c^4*x^5 + 60*A*c^5*x^5 + 30*C*b^3*c^2*x^4 + 30*C*a
*b*c^3*x^4 + 150*A*b*c^4*x^4 + 22*C*b^4*c*x^3 + 54*C*a*b^2*c^2*x^3 + 32*C*a
^2*c^3*x^3 + 110*A*b^2*c^3*x^3 + 160*A*a*c^4*x^3 + 3*C*b^5*x^2 + 51*C*a*b^3
*c*x^2 + 48*C*a^2*b*c^2*x^2 + 15*A*b^3*c^2*x^2 + 240*A*a*b*c^3*x^2 + 3*C*a*
b^4*x + 66*C*a^2*b^2*c*x - 3*A*b^4*c*x - 12*C*a^3*c^2*x + 54*A*a*b^2*c^2*x
+ 132*A*a^2*c^3*x + C*a^2*b^3 + A*b^5 + 26*C*a^3*b*c - 13*A*a*b^3*c + 66*A*
a^2*b*c^2)/((b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3)*(c*x^2 + b*x +
 a)^3)
```

**maple [B]** time = 0.02, size = 643, normalized size = 3.12

$$\frac{40A\,c^3\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\sqrt{4ac-b^2}}+\frac{8Ca\,c^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(64c^3a^3-48a^2b^2c^2+12a\,b^4c-b^6\right)\sqrt{4ac-b^2}}+\frac{8C\,b^2}{\left(64c^3a^3-48a^2b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^4,x)
```

```
[Out] (4*c^3*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x^5
+10*c^2*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*b*
x^4+2/3*(16*a*c+11*b^2)*c*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2*b^2*c^2+
12*a*b^4*c-b^6)*x^3+b*(16*a*c+b^2)*(5*A*c^2+C*a*c+C*b^2)/(64*a^3*c^3-48*a^2
*b^2*c^2+12*a*b^4*c-b^6)*x^2+(44*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-4*C*a^3*c
^2+22*C*a^2*b^2*c+C*a*b^4)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)*x+1/3
*(66*A*a^2*c^2-13*A*a*b^2*c+A*b^4+26*C*a^3*c+C*a^2*b^2)*b/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6))/(c*x^2+b*x+a)^3+40*c^3/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A+8
```

```
*c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*C*a+8*c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*C*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 4.36, size = 698, normalized size = 3.39

$$-\frac{\frac{26\,C\,a^3\,b\,c+C\,a^2\,b^3+66\,A\,a^2\,b\,c^2-13\,A\,a\,b^3\,c+A\,b^5}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}+\frac{x\,\left(-4\,C\,a^3\,c^2+22\,C\,a^2\,b^2\,c+44\,A\,a^2\,c^3+C\,a\,b^4+18\,A\,a\,b^2\,c^2-A\,b^4\,c\right)}{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}+\frac{2\,x^3\,\left(11\,b^2\,c+16\,a\,c^2\right)\,(}{3\,\left(-64\,a^3\,c^3+48\,a^2\,b^2}}{x^2\,\left(3\,c\,a^2+3\,a\,b^2\right)+x^4\,\left(3\,b^2\,c+3\,a\,c^2\right)+a^3+x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^4,x)
```

```
[Out] - ((A*b^5 + C*a^2*b^3 - 13*A*a*b^3*c + 26*C*a^3*b*c + 66*A*a^2*b*c^2)/(3*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(44*A*a^2*c^3 - 4*C*a^
3*c^2 - A*b^4*c + C*a*b^4 + 18*A*a*b^2*c^2 + 22*C*a^2*b^2*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (2*x^3*(16*a*c^2 + 11*b^2*c)*(5*A*c^2
 + C*b^2 + C*a*c))/(3*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (
x^2*(b^3 + 16*a*b*c)*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c) + (4*c^3*x^5*(5*A*c^2 + C*b^2 + C*a*c))/(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (10*b*c^2*x^4*(5*A*c^2 + C*b^2 + C*a*
c))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(x^2*(3*a*b^2 + 3*a^2
*c) + x^4*(3*a*c^2 + 3*b^2*c) + a^3 + x^3*(b^3 + 6*a*b*c) + c^3*x^6 + 3*b*c
^2*x^5 + 3*a^2*b*x) - (8*c*atan((((8*c^2*x*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*
c - b^2)^(7/2) + (4*c*(5*A*c^2 + C*b^2 + C*a*c)*(b^7 - 64*a^3*b*c^3 + 48*a^
2*b^3*c^2 - 12*a*b^5*c))/((4*a*c - b^2)^(7/2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)))*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(20
*A*c^3 + 4*C*a*c^2 + 4*C*b^2*c))*(5*A*c^2 + C*b^2 + C*a*c))/(4*a*c - b^2)^(
7/2)
```

**sympy [B]** time = 4.22, size = 1224, normalized size = 5.94

$$-4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}\left(5Ac^2+Cac+Cb^2\right)\log\left(x+\frac{20Abc^3+4Cabc^2+4Cb^3c-1024a^4c^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^7}}\left(5Ac^2+Ca\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**4,x)
```

```
[Out] -4*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b
*c**3 + 4*C*a*b*c**2 + 4*C*b**3*c - 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) + 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)
**7)*(5*A*c**2 + C*a*c + C*b**2) - 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2
)**7)*(5*A*c**2 + C*a*c + C*b**2) + 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**
7)*(5*A*c**2 + C*a*c + C*b**2) - 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + 4*c*sqrt
(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a*c + C*b**2)*log(x + (20*A*b*c**3 + 4
*C*a*b*c**2 + 4*C*b**3*c + 1024*a**4*c**5*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) - 1024*a**3*b**2*c**4*sqrt(-1/(4*a*c - b**2)**7)*(5*A
*c**2 + C*a*c + C*b**2) + 384*a**2*b**4*c**3*sqrt(-1/(4*a*c - b**2)**7)*(5*
A*c**2 + C*a*c + C*b**2) - 64*a*b**6*c**2*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c
**2 + C*a*c + C*b**2) + 4*b**8*c*sqrt(-1/(4*a*c - b**2)**7)*(5*A*c**2 + C*a
*c + C*b**2))/(40*A*c**4 + 8*C*a*c**3 + 8*C*b**2*c**2)) + (66*A*a**2*b*c**2
 - 13*A*a*b**3*c + A*b**5 + 26*C*a**3*b*c + C*a**2*b**3 + x**5*(60*A*c**5 +
 12*C*a*c**4 + 12*C*b**2*c**3) + x**4*(150*A*b*c**4 + 30*C*a*b*c**3 + 30*C*
b**3*c**2) + x**3*(160*A*a*c**4 + 110*A*b**2*c**3 + 32*C*a**2*c**3 + 54*C*a
*b**2*c**2 + 22*C*b**4*c) + x**2*(240*A*a*b*c**3 + 15*A*b**3*c**2 + 48*C*a*
*2*b*c**2 + 51*C*a*b**3*c + 3*C*b**5) + x*(132*A*a**2*c**3 + 54*A*a*b**2*c*
*2 - 3*A*b**4*c - 12*C*a**3*c**2 + 66*C*a**2*b**2*c + 3*C*a*b**4))/(192*a**
6*c**3 - 144*a**5*b**2*c**2 + 36*a**4*b**4*c - 3*a**3*b**6 + x**6*(192*a**3
*c**6 - 144*a**2*b**2*c**5 + 36*a*b**4*c**4 - 3*b**6*c**3) + x**5*(576*a**3
*b*c**5 - 432*a**2*b**3*c**4 + 108*a*b**5*c**3 - 9*b**7*c**2) + x**4*(576*a
**4*c**5 + 144*a**3*b**2*c**4 - 324*a**2*b**4*c**3 + 99*a*b**6*c**2 - 9*b**
8*c) + x**3*(1152*a**4*b*c**4 - 672*a**3*b**3*c**3 + 72*a**2*b**5*c**2 + 18
*a*b**7*c - 3*b**9) + x**2*(576*a**5*c**4 + 144*a**4*b**2*c**3 - 324*a**3*b
**4*c**2 + 99*a**2*b**6*c - 9*a*b**8) + x*(576*a**5*b*c**3 - 432*a**4*b**3*
c**2 + 108*a**3*b**5*c - 9*a**2*b**7))
```

# 3.148 $\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2} \, dx$

**Optimal**. Leaf size=591

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(\cdots\right.\right.\right.}{2c^5}$$

```
[Out] -(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+c
^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+c^
2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b*e*
h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e^3*h
-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g)+b^2
*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2*h+3*d
*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e^2*(5*a*e*h
+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2*f))-b*c^2*e
*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))+c^3*(2*a^2*e
^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h+3*d*e*g+e^2*f
)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

**Rubi** [A] time = 1.43, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(\cdots\right.\right.\right.}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] -(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d*
h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))*x
)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g + 3*b
*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^2) +
 (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h
 + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
 b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^
2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a
*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int\left(-\frac{b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg\right.}{c^4}\right.$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3\right.\right.}{c^4}$$

$$= -\frac{\left(b^3e^3h-c^3d\left(3e^2f+3deg+d^2h\right)-bce^2(beg+3bdh+2aeh)+c^2e\left(ae(eg+3\right.\right.}{c^4}$$

**Mathematica** [**A**] time = 0.65, size = 585, normalized size = 0.99

$$\frac{6\log(a+x(b+cx))\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*
h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g
+ 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c^4*e
^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h + 5*a*
e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) - b*c^2*
e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) +
 c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) + 3*a*b*e
*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt
[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2*(b*e*g +
3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f
+ 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) + a*e*(e^2*f +
 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

**fricas** [**A**] time = 2.91, size = 2150, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*
(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4
 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^
3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c^5*d^
3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3
)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*
e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 -
 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2
- (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*((3*(b^2*c^4
 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*c^5)*d
^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*
e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*
e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2
 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*d^3 -
3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2
 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4)*d^3
- 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^2 + 8*
a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h
*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c
^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 -
4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*
e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3
)*h)*x^2 - 12*sqrt(-b^2 + 4*a*c)*((2*c^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 -
 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 -
2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a
^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e +
 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*e^3)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 12*((3*(b
^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4 - 4*a*
c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2
*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3
*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)
*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4 - 4*a*c^5)*
d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)
*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^3 - 4*a*b*c^4
)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b^5*c - 6*a*b^3*c^
```

```
2 + 8*a^2*b*c^3)*d*e^2 - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*e^3
)*h)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

**giac [A]** time = 0.17, size = 771, normalized size = 1.30

$$\frac{3\,c^3hx^4e^3 + 12\,c^3dhx^3e^2 + 18\,c^3d^2hx^2e + 12\,c^3d^3hx + 4\,c^3gx^3e^3 - 4\,bc^2hx^3e^3 + 18\,c^3dgx^2e^2 - 18\,bc^2dhx^2e^2 + 36\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*h*x^4*e^3 + 12*c^3*d*h*x^3*e^2 + 18*c^3*d^2*h*x^2*e + 12*c^3*d^
3*h*x + 4*c^3*g*x^3*e^3 - 4*b*c^2*h*x^3*e^3 + 18*c^3*d*g*x^2*e^2 - 18*b*c^2
*d*h*x^2*e^2 + 36*c^3*d^2*g*x*e - 36*b*c^2*d^2*h*x*e + 6*c^3*f*x^2*e^3 - 6*
b*c^2*g*x^2*e^3 + 6*b^2*c*h*x^2*e^3 - 6*a*c^2*h*x^2*e^3 + 36*c^3*d*f*x*e^2
- 36*b*c^2*d*g*x*e^2 + 36*b^2*c*d*h*x*e^2 - 36*a*c^2*d*h*x*e^2 - 12*b*c^2*f
*x*e^3 + 12*b^2*c*g*x*e^3 - 12*a*c^2*g*x*e^3 - 12*b^3*h*x*e^3 + 24*a*b*c*h*
x*e^3)/c^4 + 1/2*(c^4*d^3*g - b*c^3*d^3*h + 3*c^4*d^2*f*e - 3*b*c^3*d^2*g*e
 + 3*b^2*c^2*d^2*h*e - 3*a*c^3*d^2*h*e - 3*b*c^3*d*f*e^2 + 3*b^2*c^2*d*g*e^
2 - 3*a*c^3*d*g*e^2 - 3*b^3*c*d*h*e^2 + 6*a*b*c^2*d*h*e^2 + b^2*c^2*f*e^3 -
 a*c^3*f*e^3 - b^3*c*g*e^3 + 2*a*b*c^2*g*e^3 + b^4*h*e^3 - 3*a*b^2*c*h*e^3
+ a^2*c^2*h*e^3)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - b*c^4*d^3*g + b^
2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b*c^4*d^2*f*e + 3*b^2*c^3*d^2*g*e - 6*a*c^4
*d^2*g*e - 3*b^3*c^2*d^2*h*e + 9*a*b*c^3*d^2*h*e + 3*b^2*c^3*d*f*e^2 - 6*a*
c^4*d*f*e^2 - 3*b^3*c^2*d*g*e^2 + 9*a*b*c^3*d*g*e^2 + 3*b^4*c*d*h*e^2 - 12*
a*b^2*c^2*d*h*e^2 + 6*a^2*c^3*d*h*e^2 - b^3*c^2*f*e^3 + 3*a*b*c^3*f*e^3 + b
^4*c*g*e^3 - 4*a*b^2*c^2*g*e^3 + 2*a^2*c^3*g*e^3 - b^5*h*e^3 + 5*a*b^3*c*h*
e^3 - 5*a^2*b*c^2*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*c^5)
```

**maple [B]** time = 0.01, size = 1738, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/4*e^3*h*x^4/c+1/2/c*ln(c*x^2+b*x+a)*d^3*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*d^3*f+1/3/c*x^3*e^3*g+1/2/c*x^2*e^3*f+1/c*d^3*h*x+
9/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e^2*g+9/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2*e*h-12/c^3
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d*e^2*h-1/2/c^
4*ln(c*x^2+b*x+a)*b^3*e^3*g+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e^3*f-1/2/c^2*ln(c*
```

```
x^2+b*x+a)*b*d^3*h+3/2/c*ln(c*x^2+b*x+a)*d^2*e*f-1/3/c^2*x^3*b*e^3*h+1/c*x^
3*d*e^2*h-1/2/c^2*x^2*a*e^3*h+1/2/c^3*x^2*b^2*e^3*h-1/2/c^2*x^2*b*e^3*g+3/2
/c*x^2*d^2*e*h+3/2/c*x^2*d*e^2*g+1/c^3*ln(c*x^2+b*x+a)*a*b*e^3*g-3/2/c^2*ln
(c*x^2+b*x+a)*a*d^2*e*h-3/2/c^2*ln(c*x^2+b*x+a)*a*d*e^2*g-3/2/c^4*ln(c*x^2+
b*x+a)*b^3*d*e^2*h+3/2/c^3*ln(c*x^2+b*x+a)*b^2*d^2*e*h+3/2/c^3*ln(c*x^2+b*x
+a)*b^2*d*e^2*g-3/2/c^2*ln(c*x^2+b*x+a)*b*d^2*e*g-3/2/c^2*ln(c*x^2+b*x+a)*b
*d*e^2*f+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*e^
3*g+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*e^3*h
+3/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4*d*e^2*h-3/
c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*e*f+3/c^3*ln(
c*x^2+b*x+a)*a*b*d*e^2*h+6/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*a^2*d*e^2*h-6/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*a*d^2*e*g-6/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*
d*e^2*f-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e
^3*g+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^3*f-
5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h-3/c
^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d^2*e*h-3/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g+3/c^2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*e*g+3/c^2/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d*e^2*f-1/c^2*a*e^3*g*x
-1/c^4*b^3*e^3*h*x+1/c^3*b^2*e^3*g*x-1/c^2*b*e^3*f*x+3/c*d^2*e*g*x+3/c*d*e^
2*f*x+1/2/c^3*ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/c^2*ln(c*x^2+b*x+a)*a*e^3*f+1/2
/c^5*ln(c*x^2+b*x+a)*b^4*e^3*h-3/2/c^2*x^2*b*d*e^2*h+2/c^3*a*b*e^3*h*x-3/c^
2*a*d*e^2*h*x+3/c^3*b^2*d*e^2*h*x-3/c^2*b*d^2*e*h*x-3/c^2*b*d*e^2*g*x-1/c^3
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+1/c^2/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^3*h-1/c/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^3*g-1/c^5/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*e^3*h+2/c^2/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^3*g-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*a*d^3*h-3/2/c^4*ln(c*x^2+b*x+a)*a*b^2*e^3*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 5.46, size = 967, normalized size = 1.64

$$x^3\left(\frac{g\,e^3+3\,d\,h\,e^2}{3\,c}-\frac{b\,e^3\,h}{3\,c^2}\right)+x\left(\frac{h\,d^3+3\,g\,d^2\,e+3\,f\,d\,e^2}{c}+\frac{b\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{c}+\frac{a\,e^3\,h}{c^2}\right)}{c}-\frac{a\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*f
 + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^3*
f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*h)/
c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/(
2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (log(a
+ b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2*e^3*f
 - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4*a*b*c^
4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*a*b^2*c^
3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2*g + 3*b^
2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^3*d^2*e*g
+ 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h + 12*a*b*c^4
*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2*c^3*d^2*e*h
 + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 - b^2*c^5)) +
(e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/
2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3*f + b^2*c^3*d^
3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3*e^3*f + 5*a*b^3
*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2*e*f + 3*b^4*c*d*
e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3*d*e^2*f + 6*a^2*c
^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b^3*c^2*d^2*e*h + 9*
a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d*e^2*h))/(c^5*(4*a*c -
b^2)^(1/2))
```

**sympy [B]** time = 118.42, size = 4972, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2*
c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2) +
```

```
3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c**3
- 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*h/c*
*3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3*f/c*
*2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*(5*a**
2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3
*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3
*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e*
*2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c
**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d
**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**
2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d*
*2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b*
*3*c*d*e**2*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*
e**2*g + a*b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b
*c**3*d*e**2*f - 4*a*c**5*(-sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a
**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2
*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*
g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e*
*2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e
*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c*
*3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*
c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*
h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*
d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g
+ 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d
**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e
*f)/(2*c**5)) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(-sqrt(-4*a
*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*
g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*
a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3
*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*
h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2
*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f +
b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2))
+ (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*
e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*
h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*
d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**
```

2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e*
*3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h
 - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**
2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g
 + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c*
*3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3
*b*c**4*d**2*e*f - 2*c**5*d**3*f)) + (sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e*
*3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a
*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c*
*3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a
*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c
**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h -
 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**
2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b*
*2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h -
 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3
*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f
- b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c
**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e**3*h - 4*a**2*b**2*c*e**3*h
+ 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3*g - 6*a**2*c**3*d**2*e*h - 6*
a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f + a*b**4*e**3*h - 3*a*b**3*c*d*e**2
*h - a*b**3*c*e**3*g + 3*a*b**2*c**2*d**2*e*h + 3*a*b**2*c**2*d*e**2*g + a*
b**2*c**2*e**3*f - a*b*c**3*d**3*h - 3*a*b*c**3*d**2*e*g - 3*a*b*c**3*d*e**
2*f - 4*a*c**5*(sqrt(-4*a*c + b**2)*(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e
**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h +
4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c*
*3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*
e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*
c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g
- 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)
/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c*
*2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a
*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**
2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*
c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)
) + 2*a*c**4*d**3*g + 6*a*c**4*d**2*e*f + b**2*c**4*(sqrt(-4*a*c + b**2)*(5
*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*
c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2
*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4
*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e*
*3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**
2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g
 + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2*
e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**3*g - 3*a*
c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h - 3*b**3*c*

```
d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d*e**2*g + b*
*2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*d*e**2*f + c*
*4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5)) - b*c**4*d**3*f)/(5*a**2*b*c**2*e**3
*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b**3*c*e**3*h + 12*a*b
**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*d**2*e*h - 9*a*b*c**3
*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a*c**4*d**2*e*g + 6*a*c
**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**4*c*e**3*g + 3*b**3*c**
2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*f - b**2*c**3*d**3*h - 3
*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**4*d**3*g + 3*b*c**4*d**2*
e*f - 2*c**5*d**3*f)) + e**3*h*x**4/(4*c)
```

# 3.149 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2} dx$

**Optimal**. Leaf size=348

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(a\right.\right.\right.}{c^4\sqrt{b^2-4ac}}$$

```
[Out] (b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*e
*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c+1/2*(c^3*d*(d*g+2*e*f)-b^3*
e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h+2*d*e*g+e
^2*f)))*ln(c*x^2+b*x+a)/c^4-(2*c^4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b*d*h
+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2*h+3*
a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2
)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.68, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(a\right.\right.\right.}{c^4\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h)
)*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c) -
((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e
*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h))))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e^2*h +
 b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*
d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)}{c^3}+\frac{e(ceg+2cdh-beh}{c^2}\right.$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

$$= \frac{\left(b^2e^2h+c^2\left(e^2f+2deg+d^2h\right)-ce(beg+2bdh+aeh)\right)x}{c^3}+\frac{e(ceg+2cdh-beh}{2c^2}$$

**Mathematica [A]** time = 0.38, size = 345, normalized size = 0.99

$$\frac{\frac{6\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3\left(2a\left(d^2h+2deg+e^2f\right)+bd(dg+2ef)\right)+b^4e^2h+2c^4d^2f\right)}{\sqrt{4ac-b^2}}+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*
e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2*c
^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2*e*
f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*(e*g
 + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*c*e*(b
*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*e*g + d
^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

**fricas [A]** time = 1.11, size = 1273, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*(
b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 - 4
*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^2 -
 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2*a*c
^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^2)*h)
*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*
e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 -
4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c
^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2
 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g -
 ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^
5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c
^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g +
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 - 6*sqrt(-b^
2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*
d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 -
2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*((b^2*c^3 -
```

```
 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g
 + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^
2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*((2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4
*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e +
 (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*
(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2
)*h)*log(c*x^2 + b*x + a))/(b^2*c^4 - 4*a*c^5)]
```

**giac [A]** time = 0.16, size = 426, normalized size = 1.22

$$\frac{2\,c^2hx^3e^2 + 6\,c^2dhx^2e + 6\,c^2d^2hx + 3\,c^2gx^2e^2 - 3\,bchx^2e^2 + 12\,c^2dgxe - 12\,bcdhxe + 6\,c^2fxe^2 - 6\,bcgxe^2 + 6\,b^2hxe}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*h*x^3*e^2 + 6*c^2*d*h*x^2*e + 6*c^2*d^2*h*x + 3*c^2*g*x^2*e^2 -
3*b*c*h*x^2*e^2 + 12*c^2*d*g*x*e - 12*b*c*d*h*x*e + 6*c^2*f*x*e^2 - 6*b*c*g
*x*e^2 + 6*b^2*h*x*e^2 - 6*a*c*h*x*e^2)/c^3 + 1/2*(c^3*d^2*g - b*c^2*d^2*h
+ 2*c^3*d*f*e - 2*b*c^2*d*g*e + 2*b^2*c*d*h*e - 2*a*c^2*d*h*e - b*c^2*f*e^2
 + b^2*c*g*e^2 - a*c^2*g*e^2 - b^3*h*e^2 + 2*a*b*c*h*e^2)*log(c*x^2 + b*x +
 a)/c^4 + (2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*
c^3*d*f*e + 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h
*e + b^2*c^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*
e^2 - 4*a*b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a
*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

**maple [B]** time = 0.01, size = 1028, normalized size = 2.95

$$\frac{e^2h\,x^3}{3c}+\frac{2a^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}-\frac{4a\,b^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3}+\frac{6abdeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}+\frac{3ab\,e^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/3*e^2*h*x^3/c+1/2/c*ln(c*x^2+b*x+a)*d^2*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+1/2/c*x^2*e^2*g+1/c*d^2*h*x+1/c*e^2*f*x+6/c^
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h-2/c^2*b*d
*e*h*x-1/c^2*ln(c*x^2+b*x+a)*b*d*e*g+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*a*d^2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*a*e^2*f+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
```

```
^4*e^2*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^
2*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2*f-1
/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^2*g-1/c^3/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+1/c^3*ln(c*x^2+
b*x+a)*a*b*e^2*h-1/c^2*ln(c*x^2+b*x+a)*a*d*e*h+1/c^3*ln(c*x^2+b*x+a)*b^2*d*
e*h-1/2/c^4*ln(c*x^2+b*x+a)*b^3*e^2*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e^2*g-1/2
/c^2*ln(c*x^2+b*x+a)*b*d^2*h-1/2/c^2*ln(c*x^2+b*x+a)*a*e^2*g-1/2/c^2*x^2*b*
e^2*h+1/c*x^2*d*e*h-1/c^2*a*e^2*h*x+1/c^3*b^2*e^2*h*x-1/c^2*b*e^2*g*x+2/c*d
*e*g*x-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^
2*h-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f+3/c^2
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g-4/c/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e*g-1/2/c^2*ln(c*x^2+b*
x+a)*b*e^2*f+1/c*ln(c*x^2+b*x+a)*d*e*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h+2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*b^2*d*e*g
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad** [**B**] time = 4.68, size = 557, normalized size = 1.60

$$x^2\left(\frac{g\,e^2+2\,d\,h\,e}{2\,c}-\frac{b\,e^2\,h}{2\,c^2}\right)-x\left(\frac{b\left(\frac{g\,e^2+2\,d\,h\,e}{c}-\frac{b\,e^2\,h}{c^2}\right)}{c}-\frac{h\,d^2+2\,g\,d\,e+f\,e^2}{c}+\frac{a\,e^2\,h}{c^2}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-8\,h\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*h
)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) - (l
og(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c^2*
e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f + 4*a
*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2*b^3*
c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2*b^4*c
*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^4)) + (
```

```
e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2
))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2
*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*
e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

**sympy [B]** time = 47.19, size = 2839, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b*
*2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c + e*
*2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g +
 a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*a
*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6
*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*
a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d*
*2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*
f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e
*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*
d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c*
*4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*
d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**
2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b
**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4
*d**2*f)) + (sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h +
6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2
```

```
*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d
**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e
*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*
e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2
*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c
**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**2*e**2*g
 + a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d**2*h + 2*
a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(sqrt(-4*a*c + b**2)*(2*a**2*c*
*2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*
c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e
*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*
f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2))
 + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*
c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f +
c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g - 4*a*c**3*d*e*f -
b**2*c**3*(sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*
a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a
*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**
2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f
 + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*
h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d
**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**
4)) + b*c**3*d**2*f)/(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d
*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2
*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b*
*2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*
d**2*f)) + e**2*h*x**3/(3*c)
```

## 3.150 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

**Optimal**. Leaf size=177

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef\right.}{c^3\sqrt{b^2-}}$$

```
[Out] (-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e*
h+b*d*h+b*e*g))*ln(c*x^2+b*x+a)/c^3-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2*a*e*g
+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.35, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef\right.}{c^3\sqrt{b^2-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] ((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h -
 c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(e*f
 + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2*c^3
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \int \left(\frac{ceg+cdh-beh}{c^2}+\frac{ehx}{c}+\frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh\right.}{c^2\left(a+bx+cx^2\right)}\right.$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\int \frac{c^2df+abeh-ac(eg+dh)+\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)x}{a+bx+cx^2}}{c^2}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\int \frac{b+2}{a+bx}}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}+\frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(a\right.}{2c^3}$$

$$= \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c}-\frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+b\right.}{c^3\sqrt{b^2-4a}}$$

**Mathematica [A]** time = 0.20, size = 173, normalized size = 0.98

$$\frac{\log(a+x(b+cx))\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c^2(2adh+2aeg+bdg+bef)-bc(3aeh+bdh+be\right.}{\sqrt{4ac-b^2}}}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

```
[Out] (2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h + c
^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*g)
+ b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

**fricas [A]** time = 0.93, size = 654, normalized size = 3.69

$$\left[\frac{\left(b^2c^2 - 4\,ac^3\right)ehx^2 + \sqrt{b^2 - 4\,ac}\left(\left(2\,c^3d - bc^2e\right)f - \left(bc^2d - \left(b^2c - 2\,ac^2\right)e\right)g + \left(\left(b^2c - 2\,ac^2\right)d - \left(b^3 - 3\,abc\right)e\right)h\right) \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)*
f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b
*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x
 + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^
3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 -
 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*(
(b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^2*e)*f -
 (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)
*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*((b^2*c^2
- 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b
^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g -
 ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b
*x + a))/(b^2*c^3 - 4*a*c^4)]
```

**giac [A]** time = 0.19, size = 201, normalized size = 1.14

$$\frac{chx^2e + 2\,cdhx + 2\,cgxe - 2\,bhxe}{2\,c^2} + \frac{\left(c^2dg - bcdh + c^2fe - bcge + b^2he - ache\right)\log\left(cx^2 + bx + a\right)}{2\,c^3} + \frac{\left(2\,c^3df - bc^2dg \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*h*x^2*e + 2*c*d*h*x + 2*c*g*x*e - 2*b*h*x*e)/c^2 + 1/2*(c^2*d*g - b*
c*d*h + c^2*f*e - b*c*g*e + b^2*h*e - a*c*h*e)*log(c*x^2 + b*x + a)/c^3 + (
2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e - 2
```

```
*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*c^3)
```

**maple [B]** time = 0.01, size = 510, normalized size = 2.88

$$\frac{3abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{2adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}-\frac{2aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}-\frac{b^3eh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^3}+\frac{b^2dh\arctan\left(\right.}{\sqrt{4ac-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2*e*h*x^2/c-1/c^2*b*e*h*x+1/c*d*h*x+1/c*e*g*x-1/2/c^2*ln(c*x^2+b*x+a)*a*e
*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*e*h-1/2/c^2*ln(c*x^2+b*x+a)*b*d*h-1/2/c^2*ln
(c*x^2+b*x+a)*b*e*g+1/2/c*ln(c*x^2+b*x+a)*d*g+1/2/c*ln(c*x^2+b*x+a)*e*f+3/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h-2/c/(4*a*c
-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h-2/c/(4*a*c-b^2)^(1/2)
*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g+2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*d*f-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^3*e*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b^2*d*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b
^2*e*g-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-1/c/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 0.53, size = 273, normalized size = 1.54

$$x\left(\frac{d\,h+e\,g}{c}-\frac{b\,e\,h}{c^2}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(b^4\,e\,h-4\,a\,c^3\,d\,g-4\,a\,c^3\,e\,f-b^3\,c\,d\,h-b^3\,c\,e\,g+b^2\,c^2\,d\,g+b^2\,c^2\,e\,f+\right.}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

```
[Out] x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3*
d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4*a
^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4 -
b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^3
*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*
c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^2)/(2*
c)
```

**sympy [B]** time = 14.46, size = 1265, normalized size = 7.15

$$x\left(-\frac{beh}{c^2}+\frac{dh}{c}+\frac{eg}{c}\right)+\left(-\frac{\sqrt{-4ac+b^2}\left(3abceh-2ac^2dh-2ac^2eg-b^3eh+b^2cdh+b^2ceg-bc^2dg-bc^2ef+2c^3df\right.}{2c^3\left(4ac-b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*
c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g -
 b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b
*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a
*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c
*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b
**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g
 - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d
*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**
2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h
 + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*
a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d
*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h - a*b**
2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h
- 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2
*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e
*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*c**2*d*g - 2*
a*c**2*e*f - b**2*c**2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2
*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c**2*e*f
+ 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h + b*c*d*h + b*c
*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*b*c*e*h - 2*a*c**2
*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g - b*c
**2*e*f + 2*c**3*d*f)) + e*h*x**2/(2*c)
```

# 3.151 $\int \frac{f+gx+hx^2}{a+bx+cx^2} \, dx$

**Optimal**. Leaf size=92

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

```
[Out] h*x/c+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*arc
tanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(2
*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx &= \int \left(\frac{h}{c}+\frac{cf-ah+(cg-bh)x}{c\left(a+bx+cx^2\right)}\right)dx \\
&= \frac{hx}{c}+\frac{\int \frac{cf-ah+(cg-bh)x}{a+bx+cx^2}\,dx}{c} \\
&= \frac{hx}{c}+\frac{(cg-bh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{\left(2c^2f-bcg+b^2h-2ach\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2} \\
&= \frac{hx}{c}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b-\right.}{c^2} \\
&= \frac{hx}{c}-\frac{\left(2c^2f-bcg+b^2h-2ach\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 95, normalized size = 1.03

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{4ac-b^2}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

```
[Out] (h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/(
2*c^2)
```

**fricas [A]** time = 0.77, size = 302, normalized size = 3.28

$$\left[\frac{2\left(b^2c-4\,ac^2\right)hx-\left(2\,c^2f-bcg+\left(b^2-2\,ac\right)h\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(\left(b^2c-4\,ac\right.\right.}{2\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*log(
c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2*(2*
c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*
log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

**giac [A]** time = 0.16, size = 89, normalized size = 0.97

$$\frac{hx}{c}+\frac{\left(cg-bh\right)\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(2\,c^2f-bcg+b^2h-2\,ach\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*h
 - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

**maple [B]** time = 0.00, size = 196, normalized size = 2.13

$$-\frac{2ah\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{b^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2}-\frac{bg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c}+\frac{2f\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{bh\ln\left(c\,x^2+bx+a\right.}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

```
[Out] h*x/c-1/2/c^2*ln(c*x^2+b*x+a)*h*b+1/2/c*ln(c*x^2+b*x+a)*g-2/c/(4*a*c-b^2)^(
1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*h+2/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*f+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^2*h-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 0.25, size = 132, normalized size = 1.43

$$\frac{h\,x}{c}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\right.}{c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

```
[Out] (h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))/
(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^
2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

**sympy [B]** time = 2.14, size = 488, normalized size = 5.30

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\left(2ach-b^2h+bcg-2c^2f\right)}{2c^2\left(4ac-b^2\right)}-\frac{bh-cg}{2c^2}\right)}{2ach-b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
- b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c +
 b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h
 - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*
h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c
*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*a*c*h
- b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)
)*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g
```

```
 - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a*c*g + b*
*2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*
a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2
*c**2*f)) + h*x/c
```

# 3.152 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx$

**Optimal**. Leaf size=196

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg}{2c\left(ae^2-bde+cd^2\right)}$$

```
[Out] (d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*g
+c*e*f)*ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)-(2*c^2*d*f+b*(-a*e+b*d)*h-c*(
2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*e^
2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.35, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg}{2c\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] -(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e^
2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^2 -
b*d*e + a*e^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{cdf-bef+aeg-adh-(cef-cdg+bdh-ae}{\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}\right.$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}+\frac{\int \frac{cdf-bef+aeg-adh-(cef-cdg+bdh-aeh)x}{a+bx+cx^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c\left(cd^2-bde+ae^2\right)}+\frac{\left(2c^2d\right.}{}$$

$$= \frac{\left(e^2f-deg+d^2h\right)\log(d+ex)}{e\left(cd^2-bde+ae^2\right)}-\frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\left(cd^2-bde+ae^2\right)}-$$

$$= -\frac{\left(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\left(e^2f\right.}{}$$

**Mathematica [A]** time = 0.21, size = 193, normalized size = 0.98

$$\frac{-2e\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(c(2adh-2aeg+bdg+bef)+bh(ae-bd)-2c^2df\right)+2c\sqrt{4ac-b^2}\log(d+ex)\left(d^2h-deg\right.}{2ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

```
[Out] (-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d
*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2*f
 - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b*d*
h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*
d) + a*e)))
```

**fricas [A]** time = 83.61, size = 625, normalized size = 3.19

$$\left[-\frac{\sqrt{b^2-4\,ac}\left(\left(2\,c^2de-bce^2\right)f-\left(bcde-2\,ace^2\right)g-\left(abe^2-\left(b^2-2\,ac\right)de\right)h\right)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right.}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2 - 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g
 - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^
2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)
*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e
- (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*sqrt(-b^2
 + 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b*e^2 - (
b^2 - 2*a*c)*d*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e
 - (a*b^2 - 4*a^2*c)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^
2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^
2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*
e^3)]
```

**giac [A]** time = 0.16, size = 204, normalized size = 1.04

$$\frac{\left(cdg-bdh-cfe+ahe\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^2-bcde+ace^2\right)}+\frac{\left(d^2h-dge+fe^2\right)\log\left(|xe+d|\right)}{cd^2e-bde^2+ae^3}+\frac{\left(2\,c^2df-bcdg+b^2dh-2\,acdh-\cdots\right.}{\left(c^2d^2-bcde+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c*d*g - b*d*h - c*f*e + a*h*e)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e
 + a*c*e^2) + (d^2*h - d*g*e + f*e^2)*log(abs(x*e + d))/(c*d^2*e - b*d*e^2
+ a*e^3) + (2*c^2*d*f - b*c*d*g + b^2*d*h - 2*a*c*d*h - b*c*f*e + 2*a*c*g*e
 - a*b*h*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*
c*e^2)*sqrt(-b^2 + 4*a*c))
```

**maple [B]** time = 0.01, size = 622, normalized size = 3.17

$$-\frac{abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\sqrt{4ac-b^2}\; c}-\frac{2adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\sqrt{4ac-b^2}}+\frac{2aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a e^2-bde+c d^2\right)\sqrt{4ac-b^2}}+\frac{b^2dh\,\mathrm{arcta}}{\left(a e^2-bde+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^2+b*x+a)*a*e*h-1/2/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^2+b*x+a)*b*d*h+1/2/(a*e^2-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*d*g-1/2/(a*e^2
-b*d*e+c*d^2)*ln(c*x^2+b*x+a)*e*f-2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-1/(a*e^2-b*d*e+c*d^2)/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*e*f+2/(a*e^2-b*d*e+c*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*d*f-1/(a*e^2-b*d*
e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))/c*b*a*e*h+1/
(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))/c
*b^2*d*h-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*d*g+1/(a*e^2-b*d*e+c*d^2)/e*ln(e*x+d)*d^2*h-1/(a*e^2-b*d*e+c*d^
2)*ln(e*x+d)*d*g+1/(a*e^2-b*d*e+c*d^2)*e*ln(e*x+d)*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 10.45, size = 2467, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)

[Out] (log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^
4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)^(
1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h -
10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*
c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(
1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) -
2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 2*c^2
*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2
*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*(b^2 - 4*
a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*
e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*d^2*e^2
*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*a*c*d*e^3*f*
(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*c*d^3*e*g*(b^
2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^4*f*x*(b^2 - 4
*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x -
10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*
x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/
2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2)
+ 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*
e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) - 2*b
*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x - 5*a*c*d*e^3*g*x*(b
^2 - 4*a*c)^(1/2) + 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) + b*c*d^3*e*h*x*(b^
2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(b^3*d*h + 4*a*c^2*d*g - 4*a*c^2*e*f
- a*b^2*e*h - b^2*c*d*g + b^2*c*e*f + 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)
^(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^
(1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c
*d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) - (
log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d^4
*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x - a^2*e^4*g*(b^2 - 4*a*c)^(1
/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h - 1
0*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*a^2*c
*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x + 3*a^2*d*e^3*h*(b^2 - 4*a*c)^(1
/2) + c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + b^2*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2
*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) + a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 2*c^2*
d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2*h + b^2*
c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x + 2*a*b*e^4*f*(b^2 - 4*a
*c)^(1/2) + b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*a*b*c*d^3*e
*h + 7*a*b*c*e^4*f*x + 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*d^2*e^2*
h*x*(b^2 - 4*a*c)^(1/2) - a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 7*a*c*d*e^3*f*(
b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) - 2*b*c*d^3*e*g*(b^2
 - 4*a*c)^(1/2) - a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*a*c*e^4*f*x*(b^2 - 4*
a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2*c*d*e^3*f*x - 1
```

```
0*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x + 3*b^2*c*d^3*e*h*x
 - 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + 7*a*c*d^2*e^2*g*(b^2 - 4*a*c)^(1/2
) + b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - b^2*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) -
 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2*g*x - 3*b*c^2*d^2*e
^2*f*x - 2*b^2*c*d^2*e^2*g*x - 5*a*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 7*a*b*c*d*e^3*g*x + 5*a*c*d*e^3*g*x*(b^
2 - 4*a*c)^(1/2) - 5*b*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - b*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*h*x)*(4*a*c^2*e*f - 4*a*c^2*d*g - b^3*d*h +
 a*b^2*e*h + b^2*c*d*g - b^2*c*e*f - 4*a^2*c*e*h - 2*c^2*d*f*(b^2 - 4*a*c)^
(1/2) - b^2*d*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*d*h + a*b*e*h*(b^2 - 4*a*c)^(
1/2) + 2*a*c*d*h*(b^2 - 4*a*c)^(1/2) - 2*a*c*e*g*(b^2 - 4*a*c)^(1/2) + b*c*
d*g*(b^2 - 4*a*c)^(1/2) + b*c*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^2 + 4
*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e)) + (l
og(d + e*x)*(e^2*f + d^2*h - d*e*g))/(a*e^3 - b*d*e^2 + c*d^2*e)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

## 3.153 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx$

**Optimal**. Leaf size=316

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)+(c*d*(-d*g+2*e*f)+a*e*(-
2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d*g
+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^2-(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(
d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(a*e^2-b*d*e+c*d^2)^2/(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 0.76, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] -((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2*
d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f - d
*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e +
 a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Lo
g[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)}\,dx = \int \left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^2}+\frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right.\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)}\right.$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)\right)}{\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e^2f-deg+d^2h}{e\left(cd^2-bde+ae^2\right)(d+ex)}-\frac{\left(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2\left(e^2f+\right.\right.}{\sqrt{b^2}}$$

**Mathematica [A]** time = 0.54, size = 281, normalized size = 0.89

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)}{\sqrt{4ac-b^2}}-\frac{2\left(e(ae-bd)+cd^2\right)\left(d^2h-deg+e^2f\right)}{e(d+ex)}+2\log(d}{2\left(e(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

```
[Out] ((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (2
*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c
*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g - 2
*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e*(-(e
*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.20, size = 449, normalized size = 1.42

$$\frac{\left(2\,c^2d^2fe^2-bcd^2ge^2+b^2d^2he^2-2\,acd^2he^2-2\,bcdfe^3+4\,acdge^3-2\,abdhe^3+b^2fe^4-2\,acfe^4-abge^4+2\,a^2he^4\right.}{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*f*e^2 - b*c*d^2*g*e^2 + b^2*d^2*h*e^2 - 2*a*c*d^2*h*e^2 - 2*b*c*
d*f*e^3 + 4*a*c*d*g*e^3 - 2*a*b*d*h*e^3 + b^2*f*e^4 - 2*a*c*f*e^4 - a*b*g*e
^4 + 2*a^2*h*e^4)*arctan((2*c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e +
d) - 2*a*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((c^2*d^4 - 2*b*c
*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(c*d^2*g - b*d^2*h - 2*c*d*f*e + 2*a*d*h*e + b*f*e^2 - a*g*e^2
```

```
)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e
+ d)^2 + a*e^2/(x*e + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^
2*e^2 - 2*a*b*d*e^3 + a^2*e^4) - (d^2*h*e/(x*e + d) - d*g*e^2/(x*e + d) + f
*e^3/(x*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4)
```

**maple [B]** time = 0.01, size = 1125, normalized size = 3.56

$$\frac{2a^2e^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{2abdeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{ab\,e^2g\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(a\,e^2-bde+c\,d^2\right)^2\sqrt{4ac-b^2}}-\frac{2ac\,d^2h\arctan}{\left(a\,e^2-bde+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x)
```

```
[Out] -1/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)*f+1/(a*e^2-b*d*e+c*d^2)/(e*x+d)*d*g-2/(a*e
^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c
*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*b*c*d^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a*b*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c*d^2*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c*d*e*f-2/(a*e^2-b*d*e+c
*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e*h+4/(
a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*
a*c*d*e*g-1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*g*e^2*a-1/2/(a*e^2-b*d*
e+c*d^2)^2*ln(c*x^2+b*x+a)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*e^2*f+2/(a*e^2-b*d*e+c*d^2)^2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^2*f-2/(a*e^2-b*d*e
+c*d^2)^2*ln(e*x+d)*a*d*e*h+2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*c*d*f*e+1/(a*
e^2-b*d*e+c*d^2)^2*ln(c*x^2+b*x+a)*a*d*e*h-1/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x
^2+b*x+a)*d*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))*a^2*e^2*h+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*d^2*h+1/2/(a*e^2-b*d*e+c*d^2)^2*ln(c*x^
2+b*x+a)*f*e^2*b+1/2/(a*e^2-b*d*e+c*d^2)^2*c*ln(c*x^2+b*x+a)*g*d^2-1/(a*e^2
-b*d*e+c*d^2)/e/(e*x+d)*d^2*h+1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*a*e^2*g+1/(
a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*d^2*h-1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*b*
e^2*f-1/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*c*d^2*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 14.71, size = 3991, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(a
^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)
 + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g +
 b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x
+ 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a*c)^(
1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^
3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x +
 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) - a^
2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d
^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*b^3*e^4*f*
x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^
3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e
^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d
^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^2*e^2*f*(b^2 -
 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*f*(b^2
- 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*h*x*(
b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*
b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h -
 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e
*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x - 8*a*c^2*
d^3*e*g*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) + 2*a^2*b
*d*e^3*h*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) - 2*a*b^
2*e^4*g*x*(b^2 - 4*a*c)^(1/2) + a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c
*e^4*g*x*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 8*c^3*
d^3*e*f*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g
+ 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^
2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x + 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2)
 + 2*a*b^2*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^
(1/2) + 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^3*e*g*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e*h*x*(b^2
 - 4*a*c)^(1/2) - 10*a*b*c^2*d^2*e^2*g*x - 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*
c)^(1/2) + 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^2*g*x*(b^
2 - 4*a*c)^(1/2) - 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 10*a*b*c*d^3*e*h*
```

(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^3*f*x + 6*a*b^2*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x + 6*a*b*c*d*e^3*g*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2))*(b^3*d^2*h - b^3*e^2*f + a*b^2*e^2*g + 4*a*c^2*d^2*g - 4*a^2*c*e^2*g - b^2*c*d^2*g - b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*e^2*f - 4*a*b*c*d^2*h - 8*a*c^2*d*e*f - 2*a*b^2*d*e*h + 2*b^2*c*d*e*f + 8*a^2*c*d*e*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*c)^(1/2) + 2*a*c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*b*d*e*h*(b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c*d*e*f*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*g + b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*f*x + 2*c^4*d^4*f*x + c^3*d^4*f*(b^2 - 4*a*c)^(1/2) - a^3*e^4*h*(b^2 - 4*a*c)^(1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*c*d*e^3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^4*h*x - 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) + 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1/2) + a^2*c*e^4*f*(b^2 - 4*a*c)^(1/2) - a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b*c^2*d^4*g*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) - 2*b^3*e^4*f*x*(b^2 - 4*a*c)^(1/2) - 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d*e^3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f - b^3*c*d^2*e^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*x + b^2*c^2*d^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g - 14*a*c^2*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d^2*e^2*h*x - 6*a*b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3*c*d^3*e*h*x + 8*a*c^2*d^3*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d*e^3*h*(b^2 - 4*a*c)^(1/2) - 8*a^2*c*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*e^4*g*x*(b^2 - 4*a*c)^(1/2) - a^2*b*e^4*h*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*b*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) + 8*c^3*d^3*e*f*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2*f + 2*a*b^2*c*d^2*e^2*g + 10*a^2*b*c*d^2*e^2*h - 28*a*c^3*d^2*e^2*f*x - 16*a^2*c^2*d*e^3*g*x - 2*b^2*c^2*d^3*e*g*x + b^3*c*d^2*e^2*g*x - 8*a*c^2*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) + 8*b^2*c*d*e^3*f*x*(b^2 - 4*a*c)^(1/2) - 8*a*c^2*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) + 2*b*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d*e^3*h*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^3*e*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*b*c^2*d^2*e^2*g*x + 10*a*c^2*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) - 12*b*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) - b^2*c*d^2*e^2*g*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) - 10*a*b*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 4*a*b*c*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^2*d*e^3*f*x + 6*a*b^2

```
*c*d*e^3*g*x - 12*a*b*c^2*d^3*e*h*x - 12*a^2*b*c*d*e^3*h*x - 6*a*b*c*d*e^3*
g*x*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2))*(b^3*e^2
*f - b^3*d^2*h - a*b^2*e^2*g - 4*a*c^2*d^2*g + 4*a^2*c*e^2*g + b^2*c*d^2*g
- b^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 2*c^2*d^2*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*e
^2*h*(b^2 - 4*a*c)^(1/2) - b^2*d^2*h*(b^2 - 4*a*c)^(1/2) - 4*a*b*c*e^2*f +
4*a*b*c*d^2*h + 8*a*c^2*d*e*f + 2*a*b^2*d*e*h - 2*b^2*c*d*e*f - 8*a^2*c*d*e
*h + a*b*e^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*c*e^2*f*(b^2 - 4*a*c)^(1/2) + 2*a*
c*d^2*h*(b^2 - 4*a*c)^(1/2) + b*c*d^2*g*(b^2 - 4*a*c)^(1/2) + 2*a*b*d*e*h*(
b^2 - 4*a*c)^(1/2) - 4*a*c*d*e*g*(b^2 - 4*a*c)^(1/2) + 2*b*c*d*e*f*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b
^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*
d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2)) - (e^2*f + d^2*h - d*e*g)/(e*
(d + e*x)*(a*e^2 + c*d^2 - b*d*e))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

# 3.154 $\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx$

**Optimal**. Leaf size=509

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-c\left(ae\left(3d^2h-3deg+e^2f\right)+b\left(3de^2f-d^3h\right)\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\ldots$$

```
[Out] 1/2*(-d^2*h+d*e*g-e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2+(-c*d*(-d*g+2*e*f)
-a*e*(-2*d*h+e*g)+b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(c^2*d^2*
(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3
*h+3*d*e^2*f)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*(c^2*d^2*(-d*g+3*e*f)+e
^3*(a^2*h-a*b*g+b^2*f)-c*(a*e*(3*d^2*h-3*d*e*g+e^2*f)+b*(-d^3*h+3*d*e^2*f))
)*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3-(2*c^3*d^3*f-b*e^3*(a^2*h-a*b*g+b^2
*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3*e^2*f))-c*(2*a^2*e^2*(-3*d*
h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h+3*d*e^2*f)))*arctanh((2*c*x+
b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)
```

**Rubi** [A] time = 1.25, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -(e^2*f - d*e*g + d^2*h)/(2*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*d*(
2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e + a*e
^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2*d*(b
*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e*g - 3
*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*ArcTanh
[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*e*(e^2*f
- 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*d^2 - b*d*
e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*
e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a + b*x + c*x^
2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}\,dx = \int\left(\frac{e^2f-deg+d^2h}{\left(cd^2-bde+ae^2\right)(d+ex)^3} + \frac{e\left(cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2\right.\right.}{\left(cd^2-bde+ae^2\right)^2(d+ex)^2}\right.$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$= -\frac{e^2f-deg+d^2h}{2e\left(cd^2-bde+ae^2\right)(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b\left(e^2f-d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

**Mathematica [A]** time = 0.77, size = 504, normalized size = 0.99

$$-\frac{\log(d+ex)\left(-\left(e^3\left(a^2h-abg+b^2f\right)\right)+ace\left(3d^2h-3deg+e^2f\right)+bc\left(3de^2f-d^3h\right)+c^2d^2(dg-3ef)\right)}{\left(e(ae-bd)+cd^2\right)^3}+\frac{\log(a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

```
[Out] -1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) + (
c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 + e*
(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a^2*
h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-2*a^
2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f + d^
3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*g + a^2
*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*Log[d +
e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f
- a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3
*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [A]** time = 0.21, size = 1002, normalized size = 1.97

$$\frac{\left(c^2d^3g - bcd^3h - 3\,c^2d^2fe + 3\,acd^2he + 3\,bcdfe^2 - 3\,acdge^2 - b^2fe^3 + acfe^3 + abge^3 - a^2he^3\right)\log\left(cx^2 + bx + a\right)}{2\left(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 + 3\,ac^2d^4e^2 - b^3d^3e^3 - 6\,abcd^3e^3 + 3\,ab^2d^2e^4 + 3\,a^2cd^2e^4 - 3\,a^2bde^5 + a^3e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(c^2*d^3*g - b*c*d^3*h - 3*c^2*d^2*f*e + 3*a*c*d^2*h*e + 3*b*c*d*f*e^2
- 3*a*c*d*g*e^2 - b^2*f*e^3 + a*c*f*e^3 + a*b*g*e^3 - a^2*h*e^3)*log(c*x^2
+ b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b
^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*
d*e^5 + a^3*e^6) - (c^2*d^3*g*e - b*c*d^3*h*e - 3*c^2*d^2*f*e^2 + 3*a*c*d^2
*h*e^2 + 3*b*c*d*f*e^3 - 3*a*c*d*g*e^3 - b^2*f*e^4 + a*c*f*e^4 + a*b*g*e^4
- a^2*h*e^4)*log(abs(x*e + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e
^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*
a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - b*c^2*d^3*g + b^2
*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*d^2*f*e + 6*a*c^2*d^2*g*e - 3*a*b*c*d^2*
h*e + 3*b^2*c*d*f*e^2 - 6*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 6*a^2*c*d*h*e^2
 - b^3*f*e^3 + 3*a*b*c*f*e^3 + a*b^2*g*e^3 - 2*a^2*c*g*e^3 - a^2*b*h*e^3)*a
rctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d
^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4
+ 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(c^2
*d^6*h - 3*c^2*d^5*g*e + 5*c^2*d^4*f*e^2 + 4*b*c*d^4*g*e^2 - b^2*d^4*h*e^2
- 2*a*c*d^4*h*e^2 - 8*b*c*d^3*f*e^3 - b^2*d^3*g*e^3 - 2*a*c*d^3*g*e^3 + 4*a
*b*d^3*h*e^3 + 3*b^2*d^2*f*e^4 + 6*a*c*d^2*f*e^4 - 3*a^2*d^2*h*e^4 - 4*a*b*
d*f*e^5 + a^2*d*g*e^5 + a^2*f*e^6 - 2*(c^2*d^4*g*e^2 - b*c*d^4*h*e^2 - 2*c^
2*d^3*f*e^3 - b*c*d^3*g*e^3 + b^2*d^3*h*e^3 + 2*a*c*d^3*h*e^3 + 3*b*c*d^2*f
*e^4 - 3*a*b*d^2*h*e^4 - b^2*d*f*e^5 - 2*a*c*d*f*e^5 + a*b*d*g*e^5 + 2*a^2*
d*h*e^5 + a*b*f*e^6 - a^2*g*e^6)*x)*e^(-1)/((c*d^2 - b*d*e + a*e^2)^3*(x*e
+ d)^2)
```

**maple [B]** time = 0.02, size = 1945, normalized size = 3.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x)
```

```
[Out] -3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*b*c^2*d^2*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*a^2*c*d*e^2*h+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f+6/(a*e^2-b*d*e+c*d^2)^3/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g-6/(a*e^2
-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2
*d*e^2*f+3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b^2*c*d*e^2*f-1/2/(a*e^2-b*d*e+c*d^2)*e/(e*x+d)^2*f+1/2/(a*e^2-
b*d*e+c*d^2)/(e*x+d)^2*d*g+1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*g*e^3*
b*a+1/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)*f*e^3*a-1/2/(a*e^2-b*d*e+c*
d^2)^3*c*ln(c*x^2+b*x+a)*b*d^3*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*
arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^2*g-3/(a*e^2-b*d*e+c*d^2)^3/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d^2*e*h+1/(a*e^2
-b*d*e+c*d^2)^3*ln(e*x+d)*a^2*e^3*h+1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b^2*e
^3*f-1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c^2*d^3*g-1/(a*e^2-b*d*e+c*d^2)^2/(e
*x+d)*a*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*b*d^2*h+1/(a*e^2-b*d*e+c*d^2)
^2/(e*x+d)*b*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*g*d^2-3/2/(a*e^2-b*d*e
+c*d^2)^3*c*ln(c*x^2+b*x+a)*d*g*e^2*a+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+
b*x+a)*d*f*e^2*b-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a
*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^3*g-2/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^3*
h+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g+3/2/(a*e^2-b*d*e+c*d^2)^3*c*ln(c*x^2+b*x+a)
*a*d^2*e*h-3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*c*d^2*e*h+3/(a*e^2-b*d*e+c*d
^2)^3*ln(e*x+d)*a*c*d*e^2*g-3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d*e^2*f-3
/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2+b*x+a)*d^2*f*e-1/(a*e^2-b*d*e+c*d^2)^
3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+2/(a*e^2-
b*d*e+c*d^2)^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^
3*f-1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*b*e^3*g-1/(a*e^2-b*d*e+c*d^2)^3*ln(
e*x+d)*a*c*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*c*d^3*h+3/(a*e^2-b*d*e
+c*d^2)^3*ln(e*x+d)*c^2*d^2*f*e+2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*a*d*e*h-2/(
a*e^2-b*d*e+c*d^2)^2/(e*x+d)*c*d*e*f+1/2/(a*e^2-b*d*e+c*d^2)^3*c^2*ln(c*x^2
+b*x+a)*g*d^3-1/2/(a*e^2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*a^2*e^3*h-1/2/(a*e^
2-b*d*e+c*d^2)^3*ln(c*x^2+b*x+a)*f*e^3*b^2-1/2/(a*e^2-b*d*e+c*d^2)/e/(e*x+d
)^2*d^2*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 6.82, size = 12784, normalized size = 25.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

```
[Out] symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c
^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 12
0*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2
*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d
^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^
2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z
^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4
*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d
*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e
^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*
e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^1
0*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d
^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^
6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e
*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2
*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^
2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*d*e^5*f*g*z +
 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*
e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^
3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*
z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*
d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*
g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^
2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d
^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^
3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e^5*g^2*z - 66
*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*
z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^
2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*
e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^
6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*
```

h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k)*(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^4*c*

d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z - 3*a
^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3*e^3*f
*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93*a^2*b*c^3
*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c^3*d^3*e^3*f*g*z +
27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^4*b*c*e^6*g*h*z - 7
*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*d^3*e^3*g*h*z + 3*b
^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3*c^3*d^2*e^4*f*h*z
- 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 21*a^2*b*c^3*d^5*e*h
^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*z + 6*a^2*b^3*c*d*e
^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5*f^2*z + 27*a^2*b*c
^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b^2*c^2*d^2*e^4*g^2*
z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*b^3*c^3*d^5*e*f*h*z
 - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*c^3*d*e^5*f*g*z + 9
*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*c^2*e^6*f*g*z + 3*a
^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^2*d^3*e^3*h^2*z + 3
*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*a^2*b*c^3*d^3*e^3*g
^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2*z + 63*a*b^2*c^3*d
^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z + 2*a^3*b^3*e^6*g*
h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*d*e^5*f^2*z + 3*b*c
^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f*g*z + 33*a^3*c^3*d
^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2*e^4*g^2*z - 27*a^2
*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c^2*d^2*e^4*f^2*z -
12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*a^2*b^2*c^2*e^6*f^2
*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^4*e^2*f^2*z + 4*a^3
*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h^2*z - b^2*c^4*d^6*
g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*d^6*h^2*z + 3*a^3*c
^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a^5*c*e^6*h^2*z + 3*
a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^2*c^2*d*e^2*f*g*h -
 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b*c^2*d*e^2*g^2*h -
3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^2*c^3*d*e^2*f*g*h +
 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c^3*d*e^2*f*g^2 - 2*
b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*f^2*g + 3*a*c^4*d^2
*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2*a*b*c^3*d^3*g^2*h
- 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*e^2*f^2*h + 3*b^2*c
^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e*g^2*h + 9*a^2*c^3*
d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^2*h + 2*a^2*b^2*c*e
^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 - 3*a^3*c^2*e^3*f*h^
2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^3*g*h^2 - a^2*c^3*e
^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a^2*b*c^2*e^3*g^3 -
3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^2*h + b^3*c^2*d^3*f
*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f^2*h + b*c^4*d^3*f*
g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 - a*c^4*d^3*g^3 + b^2
*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k)*((8*a*c^6*d^9*e^2 + 8*a^5*c^2*d*e^10 -
b^6*c*d^5*e^6 + 32*a^2*c^5*d^7*e^4 + 48*a^3*c^4*d^5*e^6 + 32*a^4*c^3*d^3*e^
8 + 3*b^2*c^5*d^9*e^2 - 2*b^3*c^4*d^8*e^3 - 2*b^4*c^3*d^7*e^4 + 3*b^5*c^2*d

^6*e^5 - a^5*b*c*e^11 - b*c^6*d^10*e + 114*a^2*b^2*c^3*d^5*e^6 - 38*a^2*b^3
*c^2*d^4*e^7 + 60*a^3*b^2*c^2*d^3*e^8 - 37*a*b*c^5*d^8*e^3 + 3*a*b^5*c*d^4*
e^7 + 3*a^4*b^2*c*d*e^10 + 60*a*b^2*c^4*d^7*e^4 - 38*a*b^3*c^3*d^6*e^5 + 4*
a*b^4*c^2*d^5*e^6 - 106*a^2*b*c^4*d^6*e^5 - 2*a^2*b^4*c*d^3*e^8 - 106*a^3*b
*c^3*d^4*e^7 - 2*a^3*b^3*c*d^2*e^9 - 37*a^4*b*c^2*d^2*e^9)/(a^4*e^8 + c^4*d
^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*
b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 -
4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 -
12*a^2*b*c*d^3*e^5) + (x*(6*a^5*c^2*e^11 - 2*c^7*d^10*e - 2*a^4*b^2*c*e^11
- 2*a*c^6*d^8*e^3 + 10*b*c^6*d^9*e^2 - 2*b^6*c*d^4*e^7 + 12*a^2*c^5*d^6*e^5
+ 28*a^3*c^4*d^4*e^7 + 22*a^4*c^3*d^2*e^9 - 22*b^2*c^5*d^8*e^3 + 28*b^3*c^
4*d^7*e^4 - 22*b^4*c^3*d^6*e^5 + 10*b^5*c^2*d^5*e^6 + 24*a^2*b^2*c^3*d^4*e^
7 + 12*a^2*b^3*c^2*d^3*e^8 + 20*a^3*b^2*c^2*d^2*e^9 + 8*a*b*c^5*d^7*e^4 + 8
*a*b^5*c*d^3*e^8 + 8*a^3*b^3*c*d*e^10 - 22*a^4*b*c^2*d*e^10 - 20*a*b^2*c^4*
d^6*e^5 + 32*a*b^3*c^3*d^5*e^6 - 26*a*b^4*c^2*d^4*e^7 - 36*a^2*b*c^4*d^5*e^
6 - 12*a^2*b^4*c*d^2*e^9 - 56*a^3*b*c^3*d^3*e^8))/(a^4*e^8 + c^4*d^8 + b^4*
d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5
*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*
d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*
c*d^3*e^5)) + (a^4*c^2*e^8*g + c^6*d^7*e*f + a^4*b*c*e^8*h - a*c^5*d^7*e*h
- b*c^5*d^7*e*g + a^2*b^3*c*e^8*f - 2*a^3*b*c^2*e^8*f - a^3*b^2*c*e^8*g + 3
*a*c^5*d^5*e^3*f + a^3*c^3*d*e^7*f + a*c^5*d^6*e^2*g - b*c^5*d^6*e^2*f + b^
5*c*d^2*e^6*f - a^4*c^2*d*e^7*h + b^2*c^4*d^7*e*h + 3*a^2*c^4*d^3*e^5*f + 3
*a^2*c^4*d^4*e^4*g + 3*a^3*c^3*d^2*e^6*g - 3*b^2*c^4*d^5*e^3*f + 6*b^3*c^3*
d^4*e^4*f - 4*b^4*c^2*d^3*e^5*f - 3*a^2*c^4*d^5*e^3*h - 3*a^3*c^3*d^3*e^5*h
+ 2*b^2*c^4*d^6*e^2*g - b^3*c^3*d^5*e^3*g - 2*b^3*c^3*d^6*e^2*h + b^4*c^2*
d^5*e^3*h - a*b^2*c^3*d^3*e^5*f + 4*a*b^3*c^2*d^2*e^6*f - 5*a^2*b*c^3*d^2*e
^6*f + 2*a^2*b^2*c^2*d*e^7*f - 2*a*b^2*c^3*d^4*e^4*g + 4*a*b^3*c^2*d^3*e^5*
g - a^2*b*c^3*d^3*e^5*g + 5*a*b^2*c^3*d^5*e^3*h - 4*a*b^3*c^2*d^4*e^4*h + a
^2*b*c^3*d^4*e^4*h + a^2*b^3*c*d^2*e^6*h + 2*a^3*b*c^2*d^2*e^6*h - 2*a*b^4*
c*d*e^7*f - 5*a^2*b^2*c^2*d^2*e^6*g + 2*a^2*b^2*c^2*d^3*e^5*h - 4*a*b*c^4*d
^4*e^4*f - 2*a*b*c^4*d^5*e^3*g - a*b^4*c*d^2*e^6*g + 2*a^2*b^3*c*d*e^7*g -
2*a^3*b^2*c*d*e^7*h)/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*
a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*
b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5) + (x*(3*a^4*c^2*e^
8*h - 3*a^3*c^3*e^8*f + 5*c^6*d^6*e^2*f - c^6*d^7*e*g + b*c^5*d^7*e*h - 2*a
^3*b*c^2*e^8*g + 7*a*c^5*d^4*e^4*f + 5*a*c^5*d^5*e^3*g - 15*b*c^5*d^5*e^3*f
+ 7*a^3*c^3*d*e^7*g - 5*a*c^5*d^6*e^2*h + b*c^5*d^6*e^2*g + 2*a^2*b^2*c^2*
e^8*f - a^2*c^4*d^2*e^6*f + 13*a^2*c^4*d^3*e^5*g + 17*b^2*c^4*d^4*e^4*f - 9
*b^3*c^3*d^3*e^5*f + 2*b^4*c^2*d^2*e^6*f - 7*a^2*c^4*d^4*e^4*h + a^3*c^3*d^
2*e^6*h + b^2*c^4*d^5*e^3*g - b^3*c^3*d^4*e^4*g - b^2*c^4*d^6*e^2*h - b^3*c
^3*d^5*e^3*h + b^4*c^2*d^4*e^4*h + 11*a*b^2*c^3*d^2*e^6*f + 13*a*b^2*c^3*d^
3*e^5*g - 2*a*b^3*c^2*d^2*e^6*g - 19*a^2*b*c^3*d^2*e^6*g + 4*a^2*b^2*c^2*d*
e^7*g - a*b^2*c^3*d^4*e^4*h - 4*a*b^3*c^2*d^3*e^5*h + a^2*b*c^3*d^3*e^5*h +

8*a^2*b^2*c^2*d^2*e^6*h - 14*a*b*c^4*d^3*e^5*f - 4*a*b^3*c^2*d*e^7*f + a^2
*b*c^3*d*e^7*f - 16*a*b*c^4*d^4*e^4*g + 10*a*b*c^4*d^5*e^3*h - 8*a^3*b*c^2*
d*e^7*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*
e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4
*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e
^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5)) - (2*c^5*d^3*e^2*f^2 - b^3*c
^2*e^5*f^2 - c^5*d^4*e*f*g + 2*a^2*c^3*d^3*e^2*h^2 + a*b*c^3*e^5*f^2 - 2*a*
c^4*d*e^4*f^2 - a^2*c^3*e^5*f*g + a^3*c^2*e^5*g*h - a^2*b*c^2*e^5*g^2 - 2*a
*c^4*d^3*e^2*g^2 - 5*b*c^4*d^2*e^3*f^2 + 2*a^2*c^3*d*e^4*g^2 + 4*b^2*c^3*d*
e^4*f^2 - 2*a^3*c^2*d*e^4*h^2 + a*b*c^3*d^2*e^3*g^2 - b^2*c^3*d^2*e^3*f*g -
6*a^2*c^3*d^2*e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + a*c^
4*d^4*e*g*h + b*c^4*d^4*e*f*h + a^2*b*c^2*d^2*e^3*h^2 - a*b*c^3*d^4*e*h^2 +
2*a*b^2*c^2*e^5*f*g - a^2*b*c^2*e^5*f*h + 6*a*c^4*d^2*e^3*f*g - 4*a*c^4*d^
3*e^2*f*h + 2*b*c^4*d^3*e^2*f*g + 4*a^2*c^3*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*f
*h - 2*a*b^2*c^2*d*e^4*f*h + 2*a*b*c^3*d^3*e^2*g*h + 2*a^2*b*c^2*d*e^4*g*h
- a*b^2*c^2*d^2*e^3*g*h - 6*a*b*c^3*d*e^4*f*g)/(a^4*e^8 + c^4*d^8 + b^4*d^4
*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^
3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4 + 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e
^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 + 12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d
^3*e^5) + (x*(c^5*d^4*e*g^2 + a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 + 4*c^5*d^2
*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 - 4*b*c^4*d*e^4*f^2 - 4*c^5*d^3*e^2*f*g -
2*a*c^4*d^2*e^3*g^2 + b^2*c^3*d^4*e*h^2 - 4*a*b*c^3*d^3*e^2*h^2 - 2*b^2*c^3
*d^2*e^3*f*h - 2*a*b*c^3*e^5*f*g + 4*a*c^4*d*e^4*f*g - 2*b*c^4*d^4*e*g*h -
8*a*c^4*d^2*e^3*f*h + 2*b*c^4*d^2*e^3*f*g + 4*a*c^4*d^3*e^2*g*h + 4*b*c^4*d
^3*e^2*f*h - 4*a^2*c^3*d*e^4*g*h + 2*a*b*c^3*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*
f*h))/(a^4*e^8 + c^4*d^8 + b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 4*a*c^3*d^6*e^2
+ 4*a^3*c*d^2*e^6 - 4*b^3*c*d^5*e^3 + 6*a^2*b^2*d^2*e^6 + 6*a^2*c^2*d^4*e^4
+ 6*b^2*c^2*d^6*e^2 - 4*a^3*b*d*e^7 - 4*b*c^3*d^7*e - 12*a*b*c^2*d^5*e^3 +
12*a*b^2*c*d^4*e^4 - 12*a^2*b*c*d^3*e^5))*root(24*a^6*b*c*d*e^11*z^3 + 24*
a*b*c^6*d^11*e*z^3 + 240*a^4*b*c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3
+ 120*a^5*b*c^2*d^3*e^9*z^3 + 120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*
e^10*z^3 - 54*a*b^2*c^5*d^10*e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*
c^4*d^9*e^3*z^3 - 36*a^2*b^5*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*
a*b^6*c*d^6*e^6*z^3 - 340*a^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8
*z^3 - 225*a^2*b^2*c^4*d^8*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*
b^3*c^3*d^7*e^5*z^3 - 30*a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*
b^3*c^5*d^11*e*z^3 - 6*a^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^
3*d^9*e^3*z^3 + 15*b^6*c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c
^4*d^6*e^6*z^3 - 60*a^5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c
^2*d^2*e^10*z^3 - 24*a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4
*b^4*d^2*e^10*z^3 + 15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^1
2*z^3 + b^8*d^6*e^6*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c
*d*e^5*g*h*z - 9*a*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b
^3*c*d*e^5*f*h*z + 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a
^2*b^2*c^2*d^3*e^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*

f*h*z - 12*a*b^4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*
d^4*e^2*g*h*z - 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*
a^2*b*c^3*d^3*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f
*h*z - 93*a^2*b*c^3*d^2*e^4*f*g*z + 51*a^2*b^2*c^2*d*e^5*f*g*z - 34*a*b^2*c
^3*d^3*e^3*f*g*z + 27*a*b^3*c^2*d^2*e^4*f*g*z - 24*a*c^5*d^5*e*f*g*z - 7*a^
4*b*c*e^6*g*h*z - 7*a*b*c^4*d^6*g*h*z + a*b^4*c*d^3*e^3*g*h*z - 80*a^3*c^3*
d^3*e^3*g*h*z + 3*b^4*c^2*d^4*e^2*f*h*z - 66*a^2*c^4*d^4*e^2*f*h*z + 54*a^3
*c^3*d^2*e^4*f*h*z - 3*b^3*c^3*d^4*e^2*f*g*z + 80*a^2*c^4*d^3*e^3*f*g*z - 2
1*a^2*b*c^3*d^5*e*h^2*z + 6*a*b^3*c^2*d^5*e*h^2*z - 21*a^3*b*c^2*d*e^5*g^2*
z + 6*a^2*b^3*c*d*e^5*g^2*z - 66*a*b*c^4*d^3*e^3*f^2*z - 30*a*b^3*c^2*d*e^5
*f^2*z + 27*a^2*b*c^3*d*e^5*f^2*z - 12*a^2*b^2*c^2*d^4*e^2*h^2*z - 12*a^2*b
^2*c^2*d^2*e^4*g^2*z + 24*a^4*c^2*d*e^5*g*h*z + 24*a^2*c^4*d^5*e*g*h*z - 3*
b^3*c^3*d^5*e*f*h*z - b^5*c*d^3*e^3*f*h*z + 3*b^2*c^4*d^5*e*f*g*z - 24*a^3*
c^3*d*e^5*f*g*z + 9*a^3*b^2*c*e^6*f*h*z - 10*a^2*b^3*c*e^6*f*g*z + 9*a^3*b*
c^2*e^6*f*g*z + 3*a^4*b*c*d*e^5*h^2*z + 3*a*b*c^4*d^5*e*g^2*z + 14*a^3*b*c^
2*d^3*e^3*h^2*z + 3*a^3*b^2*c*d^2*e^4*h^2*z - a^2*b^3*c*d^3*e^3*h^2*z + 14*
a^2*b*c^3*d^3*e^3*g^2*z + 3*a*b^2*c^3*d^4*e^2*g^2*z - a*b^3*c^2*d^3*e^3*g^2
*z + 63*a*b^2*c^3*d^2*e^4*f^2*z + 2*b^3*c^3*d^6*g*h*z - 6*a^4*c^2*e^6*f*h*z
 + 2*a^3*b^3*e^6*g*h*z - b^2*c^4*d^6*f*h*z - 2*a^2*b^4*e^6*f*h*z + 6*b^5*c*
d*e^5*f^2*z + 3*b*c^5*d^5*e*f^2*z + 6*a*b^4*c*e^6*f^2*z + b^4*c^2*d^3*e^3*f
*g*z + 33*a^3*c^3*d^4*e^2*h^2*z - 27*a^4*c^2*d^2*e^4*h^2*z + 33*a^3*c^3*d^2
*e^4*g^2*z - 27*a^2*c^4*d^4*e^2*g^2*z + 19*b^3*c^3*d^3*e^3*f^2*z - 15*b^4*c
^2*d^2*e^4*f^2*z - 12*b^2*c^4*d^4*e^2*f^2*z - 27*a^2*c^4*d^2*e^4*f^2*z - 9*
a^2*b^2*c^2*e^6*f^2*z + 2*a*c^5*d^6*f*h*z + 2*a*b^5*e^6*f*g*z + 33*a*c^5*d^
4*e^2*f^2*z + 4*a^3*b^2*c*e^6*g^2*z + 4*a*b^2*c^3*d^6*h^2*z - b^4*c^2*d^6*h
^2*z - b^2*c^4*d^6*g^2*z - a^4*c^2*e^6*g^2*z - a^4*b^2*e^6*h^2*z - a^2*c^4*
d^6*h^2*z + 3*a^3*c^3*e^6*f^2*z - a^2*b^4*e^6*g^2*z + b*c^5*d^6*f*g*z + 3*a
^5*c*e^6*h^2*z + 3*a*c^5*d^6*g^2*z - c^6*d^6*f^2*z - b^6*e^6*f^2*z + 6*a*b^
2*c^2*d*e^2*f*g*h - 2*a*b^3*c*e^3*f*g*h + 3*a^2*b*c^2*d^2*e*g*h^2 - 3*a^2*b
*c^2*d*e^2*g^2*h - 3*a^2*b*c^2*d*e^2*f*h^2 - 3*a*b^2*c^2*d^2*e*f*h^2 - 6*a^
2*c^3*d*e^2*f*g*h + 2*a^2*b*c^2*e^3*f*g*h + 6*a*b*c^3*d*e^2*f^2*h - 6*a*b*c
^3*d*e^2*f*g^2 - 2*b^2*c^3*d^3*f*g*h - 9*a*c^4*d^2*e*f^2*h - 3*b*c^4*d^2*e*
f^2*g + 3*a*c^4*d^2*e*f*g^2 + 3*a*c^4*d*e^2*f^2*g - 2*a^3*b*c*e^3*g*h^2 + 2
*a*b*c^3*d^3*g^2*h - 2*a*b*c^3*d^3*f*h^2 + 2*a*c^4*d^3*f*g*h - 3*b^3*c^2*d*
e^2*f^2*h + 3*b^2*c^3*d^2*e*f^2*h + 3*a^3*c^2*d*e^2*g*h^2 - 3*a^2*c^3*d^2*e
*g^2*h + 9*a^2*c^3*d^2*e*f*h^2 + 3*b^2*c^3*d*e^2*f^2*g - 3*a*b^2*c^2*e^3*f^
2*h + 2*a^2*b^2*c*e^3*f*h^2 - a*b^2*c^2*d^3*g*h^2 + 2*a*b^2*c^2*e^3*f*g^2 -
 3*a^3*c^2*e^3*f*h^2 + 3*a^2*c^3*e^3*f^2*h - b^3*c^2*e^3*f^2*g - a^2*c^3*d^
3*g*h^2 - a^2*c^3*e^3*f*g^2 - 3*a^3*c^2*d^2*e*h^3 + 3*a^2*c^3*d*e^2*g^3 - a
^2*b*c^2*e^3*g^3 - 3*b*c^4*d*e^2*f^3 + a^2*b^2*c*e^3*g^2*h + a^3*c^2*e^3*g^
2*h + b^3*c^2*d^3*f*h^2 + a^2*b*c^2*d^3*h^3 + b^4*c*e^3*f^2*h + b*c^4*d^3*f
^2*h + b*c^4*d^3*f*g^2 - c^5*d^3*f^2*g + 3*c^5*d^2*e*f^3 - a*c^4*e^3*f^3 -
a*c^4*d^3*g^3 + b^2*c^3*e^3*f^3 + a^4*c*e^3*h^3, z, k), k, 1, 3) - ((a*e^4*
f + c*d^4*h + a*d*e^3*g - 3*b*d*e^3*f + b*d^3*e*h - 3*c*d^3*e*g - 3*a*d^2*e
^2*h + b*d^2*e^2*g + 5*c*d^2*e^2*f)/(2*e*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 -

```
 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2)) + (x*(a*e^3*g - b*e^3*f - 2*a*
d*e^2*h + 2*c*d*e^2*f + b*d^2*e*h - c*d^2*e*g))/(a^2*e^4 + c^2*d^4 + b^2*d^
2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))/(d^2 + e^2*x^2 + 2*d*e*
x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

```
[Out] Timed out
```

## 3.155 $\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=288

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{e^2x\left(-6ach+2b^2h-bcg+2c^2f\right)}{c^2\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b}{\sqrt{}}\right.}{}$$

```
[Out] e^2*(2*c^2*f+2*b^2*h-c*(6*a*h+b*g))*x/c^2/(-4*a*c+b^2)+(e*x+d)^2*(c*(2*a*g-
b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)
+(4*c^4*d^2*f-2*b^4*e^2*h-6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*a*e
*h+2*b*d*h+b*e*g)-c^3*(2*b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h+2*c*
d*h+c*e*g)*ln(c*x^2+b*x+a)/c^3
```

**Rubi [A]** time = 0.70, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1644, 773, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bdh\right.}{c^3\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (e^2*(2*c^2*f - b*c*g + 2*b^2*h - 6*a*c*h)*x)/(c^2*(b^2 - 4*a*c)) + ((d + e
*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)
)/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c
^2*e*(b*e*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c
^3*(2*b*d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) + (e*(c*e*g + 2*c*d*h - 2*
b*e*h)*Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}dx = \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \int \frac{(d+ex)\left(2cdf\cdots\right.}{\cdots}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-b\cdots\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\cdots\right.}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-b\cdots\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\cdots\right.}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-b\cdots\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\cdots\right.}$$

$$= \frac{e^2\left(2c^2f-bcg+2b^2h-6ach\right)x}{c^2\left(b^2-4ac\right)} + \frac{(d+ex)^2\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-b\cdots\right.\right.}{c\left(b^2-4ac\right)\left(a+bx+cx^2\cdots\right.}$$

**Mathematica [A]** time = 0.81, size = 398, normalized size = 1.38

$$-\frac{\frac{2\left(bc\left(-3a^2e^2h+ac\left(d^2h+2de(g+3hx)+e^2(f+3gx)\right)+c^2d(d(f-gx)-2efx)\right)+2c^2\left(a^2e(2dh+e(g+hx))-ac\left(d^2(g+hx)+2de(f+gx)+e^2fx\right)+c^2d^2fx\right)+b^3e(aeh-\cdots\right.}{\left(b^2-4ac\right)(a+x(b+cx))}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] (2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c*
(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^2*
d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h +
e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*c*(d
^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c
*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h + 2*a*e*
h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g) + 4*a
*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)])/(2*c^3
)
```

**fricas [B]** time = 2.48, size = 2771, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f -
(2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d^2
+ (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)*x^2
 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^2*c^3
*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c - 6*a^2
*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d^2 - b^
2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*c - 6*a*
b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b^5 - 6*a
*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
 + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^
3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2
*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*
c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4
*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*
a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c
^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^
3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7
*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^
2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*
b^2*c^2 - 24*a^3*c^3)*e^2)*h)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e
^2*g + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*d*e - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 +
2*((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a
^3*b*c^2)*e^2)*h + ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)
*h)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*x), 1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8
*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + 2*((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e
^2)*f - (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*
c^4*d^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2
)*h)*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8
*a^2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c
 - 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*
d^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (b
^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(
a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2
*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*
```

```
b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*((a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c
 - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^
2)*h - 2*((2*(b^2*c^4 - 4*a*c^5)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c
^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*
c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e
^2)*g + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 +
 12*a^2*b*c^3)*d*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*e^2)*h
)*x + ((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e^2*g + ((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*e^2*g + 2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*e - (b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2)*h)*x^2 + 2*((a*b^4*c - 8*a^2*b^2*c^2
+ 16*a^3*c^3)*d*e - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e^2)*h + ((b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*e^2*g + 2*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*d*e - (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*e^2)*h)*x)*log(c*x^2 + b*x + a))
/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*
c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

**giac [A]** time = 0.18, size = 540, normalized size = 1.88

$$\frac{hxe^2}{c^2} - \frac{\left(4\,c^4d^2f - 2\,bc^3d^2g + 4\,ac^3d^2h - 4\,bc^3dfe + 8\,ac^3dge + 2\,b^3cdhe - 12\,abc^2dhe + 4\,ac^3fe^2 + b^3cge^2 - 6\,abc\right.}{\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] h*x*e^2/c^2 - (4*c^4*d^2*f - 2*b*c^3*d^2*g + 4*a*c^3*d^2*h - 4*b*c^3*d*f*e
+ 8*a*c^3*d*g*e + 2*b^3*c*d*h*e - 12*a*b*c^2*d*h*e + 4*a*c^3*f*e^2 + b^3*c*
g*e^2 - 6*a*b*c^2*g*e^2 - 2*b^4*h*e^2 + 12*a*b^2*c*h*e^2 - 12*a^2*c^2*h*e^2
)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4
*a*c)) + 1/2*(2*c*d*h*e + c*g*e^2 - 2*b*h*e^2)*log(c*x^2 + b*x + a)/c^3 - (
(2*c^4*d^2*f - b*c^3*d^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b*c^3*d*f*e
+ 2*b^2*c^2*d*g*e - 4*a*c^3*d*g*e - 2*b^3*c*d*h*e + 6*a*b*c^2*d*h*e + b^2*c
^2*f*e^2 - 2*a*c^3*f*e^2 - b^3*c*g*e^2 + 3*a*b*c^2*g*e^2 + b^4*h*e^2 - 4*a*
b^2*c*h*e^2 + 2*a^2*c^2*h*e^2)*x/c + (b*c^3*d^2*f - 2*a*c^3*d^2*g + a*b*c^2
*d^2*h - 4*a*c^3*d*f*e + 2*a*b*c^2*d*g*e - 2*a*b^2*c*d*h*e + 4*a^2*c^2*d*h*
e + a*b*c^2*f*e^2 - a*b^2*c*g*e^2 + 2*a^2*c^2*g*e^2 + a*b^3*h*e^2 - 3*a^2*b
*c*h*e^2)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

**maple [B]** time = 0.02, size = 1712, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] e^2*h/c^2*x+6/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*d*e*h-4/(c*x^2+b*x+a)/(4*a*
c-b^2)*x*a*d*e*g+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4*e^2*h+1/c/(c*x^2+b*x
+a)/(4*a*c-b^2)*x*b^2*d^2*h+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*e^2*h+1/c/(
c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*e^2*f+4/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*d*e*
h+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)
*a*b*d^2*h+1/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*e^2*f-1/c^2/(c*x^2+b*x+a)/(4*a
*c-b^2)*x*b^3*e^2*g-3/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^2*h-1/c^2/(c*x^
2+b*x+a)/(4*a*c-b^2)*a*b^2*e^2*g-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*b*e^2*g+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*e^2*f-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d^
2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d^2*g+1/(c*x^2+b*x+a)/(4*a*c-b^2)*b*d^2*f
+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d^2*f+4/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d^2*h-4/(4*a*c-b^2)^(3/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*e*f-12/c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2*h-2/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^4*e^2*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^2
*g+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2*g+2/
c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e^2*g-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*e^2*f
-2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*d^2*h-1/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d^2*
g-4/(c*x^2+b*x+a)/(4*a*c-b^2)*a*d*e*f+1/c^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^3
*e^2*h+2*c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*d^2*f+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*
a^2*e^2*g+8/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*e*g+3
/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*e^2*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^
2*d*e*g+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*d*e*g-4/c^2/(c*x^2+b*x+a)/(4*a*c-
b^2)*x*a*b^2*e^2*h-2/c^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d*e*h-2/c^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*a*b^2*d*e*h-12/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*
a*c-b^2)^(1/2))*a*b*d*e*h+4/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*d*e*h+12/c^2/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2*h+2/c^2/(4*a*
c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e*h-4/c^2/(4*a*c-b^2
)*ln(c*x^2+b*x+a)*a*b*e^2*h-1/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d*e*h-2/(
c*x^2+b*x+a)/(4*a*c-b^2)*x*b*d*e*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 5.78, size = 742, normalized size = 2.58

$$\frac{\frac{-3\,h\,a^2\,b\,c\,e^2+4\,h\,a^2\,c^2\,d\,e+2\,g\,a^2\,c^2\,e^2+h\,a\,b^3\,e^2-2\,h\,a\,b^2\,c\,d\,e-g\,a\,b^2\,c\,e^2+h\,a\,b\,c^2\,d^2+2\,g\,a\,b\,c^2\,d\,e+f\,a\,b\,c^2\,e^2-2\,g\,a\,c^3\,d^2-4\,f\,a\,c^3\,d\,e+f\,b\,c^3\,d^2}{c\left(4\,a\,c-b^2\right)}+\frac{x\left(2\,h\right.}{}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^2
*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h - 4*
a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*(2*
c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h - 2
*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g
- 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f - 2*b^3
*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x)
 + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^2*g - 24
*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*b*c^3*e^2
*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h + 24*a*b^4
*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*
c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 12*a^2*c^2*e^2
*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^2*g - 6*a*b*c^
2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f + 2*b^3*c*d*e*h
- 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/c^2
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.156 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=178

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (e*x+d)*(c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+
b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*f)-2*a*(d
*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*
e*h*ln(c*x^2+b*x+a)/c^2
```

**Rubi [A]** time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1644, 634, 618, 206, 628}

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg\right.}{c^2\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*
h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((4*c^3*d*f + b^3*e*h - 6*a*b*
c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + (e*h*Log[a + b*x + c*x^2])/(2*c^2
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeqQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\int \frac{2cdf-b(ef+dg)-\cdots}{\cdots}}{\cdots}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{(eh)\int \frac{b+2cx}{a+bx+cx^2}\,d\cdots}{2c^2}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{eh\log\left(a+bx+\cdots\right.}{2c^2}$$

$$= \frac{(d+ex)\left(c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)} + \frac{\left(4c^3df+b^3eh-\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.46, size = 225, normalized size = 1.26

$$\frac{-\frac{2\left(2c\left(a^2eh-ac(d(g+hx)+e(f+gx))+c^2dfx\right)+b^2(cx(dh+eg)-aeh)+bc(adh+ae(g+3hx)+cd(f-gx)-cefx)+b^3(-e)hx\right)}{\left(b^2-4ac\right)(a+x(b+cx))}+\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-2c^2(b(dg+ef\cdots\right.}{\left(4ac\cdots\right.}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

```
[Out] ((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f*
x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*(f
 + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*d*f
 + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*(b +
c*x)])/(2*c^2)
```

**fricas [B]** time = 1.27, size = 1413, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
(b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d
```

```
- 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d
- b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*
b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c +
sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a*b*c^3
)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - (a*
b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 - 6*a^2*b
^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3
)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e)*g
+ ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*e
)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*log(c*x^2 + b*
x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 +
16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x), 1/2*(2*((2*(2*
c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d + (b^3*c - 6*a*
b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*d - 2*a^2*c^2*e
)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3*d - b^2*c^2*e)*
f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 - 6*a*b^2*c)*e)*h)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 - 2*((b^3*c^2 - 4*a*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^
2*c^2 - 4*a^2*c^3)*d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b
*c^2)*d - (a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4
)*d - (b^3*c^2 - 4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*
b^2*c^2 + 8*a^2*c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7
*a*b^3*c + 12*a^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*
x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)*e*h)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 +
 (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x)]
```

**giac [A]** time = 0.17, size = 285, normalized size = 1.60

$$\frac{he\log\left(cx^2+bx+a\right)}{2\,c^2}-\frac{\left(4\,c^3df-2\,bc^2dg+4\,ac^2dh-2\,bc^2fe+4\,ac^2ge+b^3he-6\,abche\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}-\frac{bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*h*e*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*d*g + 4*a*c^2*d*h -
 2*b*c^2*f*e + 4*a*c^2*g*e + b^3*h*e - 6*a*b*c*h*e)*arctan((2*c*x + b)/sqrt
(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f - 2*a
*c^2*d*g + a*b*c*d*h - 2*a*c^2*f*e + a*b*c*g*e - a*b^2*h*e + 2*a^2*c*h*e +
(2*c^3*d*f - b*c^2*d*g + b^2*c*d*h - 2*a*c^2*d*h - b*c^2*f*e + b^2*c*g*e -
2*a*c^2*g*e - b^3*h*e + 3*a*b*c*h*e)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^
2)
```

**maple [B]** time = 0.01, size = 500, normalized size = 2.81

$$-\frac{6abeh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}+\frac{4adh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4aeg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{b^3eh\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c^2}-\frac{2bdg\arctan\left(\right.}{\left(4ac-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] ((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*g
-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+a*
b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a)+2
/c/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*e*h-1/2/c^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^
2*e*h-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*e*h+4/(
4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*d*h+4/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*e*g-2/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))*b*d*g-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*e*f+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*d*f+1/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 5.04, size = 376, normalized size = 2.11

$$\frac{\frac{b\,c^2\,d\,f-2\,a\,c^2\,e\,f-2\,a\,c^2\,d\,g-a\,b^2\,e\,h+2\,a^2\,c\,e\,h+a\,b\,c\,d\,h+a\,b\,c\,e\,g}{c^2\,\left(4\,a\,c-b^2\right)}-\frac{x\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g-3\,a\,b\,c\,e\,h\right)}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*d
*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h
 + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*
h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6*e*h
 - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c^5 -
b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^2)^(1/
2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e*h + 4*
a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))/(c^2*(4
*a*c - b^2)^(3/2))
```

**sympy [B]** time = 60.26, size = 1535, normalized size = 8.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] (e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*
c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2*c*
*3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4
*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*e*h +
8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c
**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*
f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - a*b*
*2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(-(4*a*c -
b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*
d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d*f)/(6*a*b*c*
e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f
- 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*
a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3
*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log
(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h -
 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c
**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
+ 8*a**2*c*e*h + 8*a*b**2*c**2*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6*
a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**
2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2)
+ sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**
3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c*
*2*d*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*
```

```
g + 2*b*c**2*e*f - 4*c**3*d*f)) + (2*a**2*c*e*h - a*b**2*e*h + a*b*c*d*h +
a*b*c*e*g - 2*a*c**2*d*g - 2*a*c**2*e*f + b*c**2*d*f + x*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
- b*c**2*e*f + 2*c**3*d*f))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b
**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

# 3.157 $\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=118

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] (c*(2*a*g-b*(f+a*h/c))-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*
x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(3/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 206}

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(b
^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{\left(a+bx+cx^2\right)^2}\,dx = \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{\int \frac{2cf-bg+2ah}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2cf-bg+2ah)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(2(2cf-bg+2ah))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4\ldots}\right.}{b^2-4ac}$$

$$= \frac{c\left(2ag-b\left(f+\frac{ah}{c}\right)\right)-\left(2c^2f-bcg+b^2h-2ach\right)x}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{2(2cf-bg+2ah)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{3/2}}$$

**Mathematica [A]** time = 0.11, size = 114, normalized size = 0.97

$$\frac{abh-2ac(g+hx)+b^2hx+bc(f-gx)+2c^2fx}{c\left(4ac-b^2\right)(a+x(b+cx))}-\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)(-2ah+bg-2cf)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

```
[Out] (a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

**fricas [B]** time = 0.94, size = 632, normalized size = 5.36

$$\left[-\frac{\left(2\,ac^2f-abcg+2\,a^2ch+\left(2\,c^3f-bc^2g+2\,ac^2h\right)x^2+\left(2\,bc^2f-b^2cg+2\,abch\right)x\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b}{\cdots}\right.}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 +
 (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*
b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (
b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*h +
(2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2
*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3
+ 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a*c^2*f
- a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*c^2*f -
b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^2*c^2)*g -
 (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g +
 (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
+ (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*x)]
```

**giac [A]** time = 0.16, size = 125, normalized size = 1.06

$$-\frac{2\left(2\,cf-bg+2\,ah\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{\left(b^2c-4\,ac^2\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*c*
f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

**maple [A]** time = 0.01, size = 194, normalized size = 1.64

$$\frac{4ah\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}-\frac{2bg\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{4cf\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{\left(4ac-b^2\right)c}+\frac{abh-2acg+bcf}{\left(4ac-b^2\right)c}}{c\,x^2+bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

```
[Out] (-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/(
4*a*c-b^2))/(c*x^2+b*x+a)+4/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*h-2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*g+4/(4
*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 3.90, size = 203, normalized size = 1.72

$$\frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^3-4\,a\,b\,c\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{2\,c\,x\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a\,h-b\,g+2\,c\,f}\right)\,\left(2\,a\,h-b\,g+2\,c\,f\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

```
[Out] ((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c*
h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*b*
c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*c*f
))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h - b*g
+ 2*c*f))/(4*a*c - b^2)^(3/2)
```

**sympy [B]** time = 2.24, size = 459, normalized size = 3.89

$$-\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-bg+2cf\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2ah-b\right.}{4ach-2bc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*sq
rt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2)**
3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*f)
) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*c**2*
sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c - b**2)
**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**2*
f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*c**2*f))/
(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*c**2 - b**
3*c))
```

## 3.158 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2} \, dx$

**Optimal**. Leaf size=407

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

```
[Out] (b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-(2*c^2*d*f+b*(-a*e+
b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e*g+
e^2*f))-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^2*d^
2*g+2*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g+e^2*f
)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2+b*x+a)
/(a*e^2-b*d*e+c*d^2)^2
```

**Rubi [A]** time = 1.09, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-\right.\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^2
*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2 -
4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) + ((4*c^3*d^3*f + b*e*(4*
a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*d*(3*e*
f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*a^2*e*(e
*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + (e*(e^2*f - d*e*
g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^2 - (e*(e^2*f - d*e*g + d^
2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-\left(2c^2df+b(bd-ae)h-c(be\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)}$$

**Mathematica [A]** time = 0.90, size = 405, normalized size = 1.00

$$\frac{-2a^2eh+ab(dh+e(g-hx))+2ac(e(f+gx)-d(g+hx))+b^2(dhx-ef)+bc(d(f-gx)-efx)+2c^2dfx}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(bd-ae)-cd^2\right)}-\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

```
[Out] (-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f - g
*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2 -
 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f + 2*
c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b*d*e
*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2*g +
2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^2) +
 (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^2 - (e
```

```
*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e)
)^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.19, size = 860, normalized size = 2.11

$$-\frac{\left(d^2he-dge^2+fe^3\right)\log\left(cx^2+bx+a\right)}{2\left(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4\right)}+\frac{\left(d^2he^2-dge^3+fe^4\right)\log\left(|xe+d|\right)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\,abde^4+a^2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d^2*h*e - d*g*e^2 + f*e^3)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3*
e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (d^2*h*e^2 - d*g
*e^3 + f*e^4)*log(abs(x*e + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3 +
2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 2*b*c^2*d^3*g + 4*a
*c^2*d^3*h - 6*b*c^2*d^2*f*e + 4*b^2*c*d^2*g*e - 4*a*c^2*d^2*g*e - b^3*d^2*
h*e - 2*a*b*c*d^2*h*e + 12*a*c^2*d*f*e^2 - b^3*d*g*e^2 - 2*a*b*c*d*g*e^2 +
4*a*b^2*d*h*e^2 - 4*a^2*c*d*h*e^2 + b^3*f*e^3 - 6*a*b*c*f*e^3 + 4*a^2*c*g*e
^3 - 2*a^2*b*h*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2*d^4 -
4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2
*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 -
4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*a*c^2*d^3*g + a*b*c*d^3
*h - 2*b^2*c*d^2*f*e + 2*a*c^2*d^2*f*e + 3*a*b*c*d^2*g*e - a*b^2*d^2*h*e -
2*a^2*c*d^2*h*e + b^3*d*f*e^2 - a*b*c*d*f*e^2 - a*b^2*d*g*e^2 - 2*a^2*c*d*g
*e^2 + 3*a^2*b*d*h*e^2 - a*b^2*f*e^3 + 2*a^2*c*f*e^3 + a^2*b*g*e^3 - 2*a^3*
h*e^3 + (2*c^3*d^3*f - b*c^2*d^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*b*c^2*
d^2*f*e + b^2*c*d^2*g*e + 2*a*c^2*d^2*g*e - b^3*d^2*h*e + a*b*c*d^2*h*e + b
^2*c*d*f*e^2 + 2*a*c^2*d*f*e^2 - 3*a*b*c*d*g*e^2 + 2*a*b^2*d*h*e^2 - 2*a^2*
c*d*h*e^2 - a*b*c*f*e^3 + 2*a^2*c*g*e^3 - a^2*b*h*e^3)*x)/((c*d^2 - b*d*e +
 a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

**maple [B]** time = 0.02, size = 3202, normalized size = 7.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x)
```

```
[Out] 2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*d*e^2*g-4/(a*e^2-b*
d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*d*
e^2*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b^2*d*e^2*h-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*e^3*f-4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d^2*e*g+12/(a*e^2-b*d*e+c*d^
2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^2*d*e^2*f+4/
(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*b^2*c*d^2*e*g-6/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))*b*c^2*d^2*e*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(c*
x^2+b*x+a)*a*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^
2*b*e^3*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2*c*e^3*g-2
/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^3*h-1/(a*e^2-b*d
*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^3*d^2*e*h+1/(a*e^2-b*d*e+c*d^2)^2
/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^2*c*d^3*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x
+a)/(4*a*c-b^2)*x*b*c^2*d^3*g+e^3/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*f+3/(a*e^
2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*d*e^2*h-2/(a*e^2-b*d*e+c*d
^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c*d^2*e*h-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^
2+b*x+a)/(4*a*c-b^2)*a^2*c*d*e^2*g-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4
*a*c-b^2)*a*b^2*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a
*b^2*d*e^2*g+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^3*h+
2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^2*d^2*e*f-2/(a*e^2-b*
d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c*d^2*e*f+2/(a*e^2-b*d*e+c*d^2)^
2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+
b*x+a)/(4*a*c-b^2)*x*b^2*c*d^2*e*g+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4
*a*c-b^2)*x*b^2*c*d*e^2*f-3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)
*x*b*c^2*d^2*e*f+3/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c*d^
2*e*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b*c*d^2*e*h-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+
a)/(4*a*c-b^2)*x*a^2*c*d*e^2*h+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c
-b^2)*x*a*b^2*d*e^2*h-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b
*c*d*e^2*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*e^3*f+
2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*c^2*d^2*e*g+1/(a*e^2-
b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d^2*e*h-3/(a*e^2-b*d*e+c*d
^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a*b*c*d*e^2*g+1/2/(a*e^2-b*d*e+c*d^2)^2/(
4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*d^2*e*h-1/2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2
)*ln(c*x^2+b*x+a)*b^2*d*e^2*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^3*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c*e^3*g+2/(a*e^2-b*d*e+
c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*x*c^3*d^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x
```

```
^2+b*x+a)/(4*a*c-b^2)*a^2*b*e^3*g+2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*a^2*c*e^3*f-1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^
2*e^3*f-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^2*d^3*g+4/(a*
e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*
c^2*d^3*h-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b^3*d^2*e*h-1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*d*e^2*g+e/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d
^2*h-e^2/(a*e^2-b*d*e+c*d^2)^2*ln(e*x+d)*d*g-2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*a^3*e^3*h+4/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^3*d^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^3*f+1/2/(a*e^2-b*d*e+c*
d^2)^2/(4*a*c-b^2)*ln(c*x^2+b*x+a)*b^2*e^3*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2
+b*x+a)/(4*a*c-b^2)*b^3*d*e^2*f+1/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)/(4*a*
c-b^2)*b*c^2*d^3*f-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b*c^2*d^3*g-2/(a*e^2-b*d*e+c*d^2)^2/(4*a*c-b^2)*c*ln(
c*x^2+b*x+a)*a*e^3*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 6.70, size = 13698, normalized size = 33.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)
```

```
[Out] symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192
*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z
^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5
*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c
*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3
*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^
3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5
*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4
*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^
```

5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3
- 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 38
4*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 +
6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 -
64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^
3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z -
10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f
*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2
*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^
2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h
*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*
d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 3
6*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e
^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z
+ 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z
+ 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^
2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^
4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c
*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b
*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 4
5*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^
2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h
*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z
+ 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z -
6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*
a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2
*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*
g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2
*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^
2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^
2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2
*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^
2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z
+ 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z +
2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^
4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z -
4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e
^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a
*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*
d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2
*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^
3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*
b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^
2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f
*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 1

2*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*((a*b^5*c*e^6*f - 8*a^4*c^3*e^6*g + 8*a*c^6*d^5*e*f - b^6*c*d*e^5*f + 20*a^3*b*c^3*e^6*f - a^3*b^3*c*e^6*h + 8*a^3*c^4*d*e^5*f + 4*a^4*b*c^2*e^6*h - 2*b^2*c^5*d^5*e*f + 8*a^2*c^5*d^5*e*h + 8*a^4*c^3*d*e^5*h + b^3*c^4*d^5*e*g + b^6*c*d^2*e^4*g - b^6*c*d^3*e^3*h - 9*a^2*b^3*c^2*e^6*f + 2*a^3*b^2*c^2*e^6*g + 16*a^2*c^5*d^3*e^3*f - 8*a^2*c^5*d^4*e^2*g - 16*a^3*c^4*d^2*e^4*g + 3*b^3*c^4*d^4*e^2*f + 16*a^3*c^4*d^3*e^3*h - 2*b^4*c^3*d^4*e^2*g + b^5*c^2*d^4*e^2*h - 4*a*b^2*c^4*d^3*e^3*f - 2*a*b^3*c^3*d^2*e^4*f + 8*a^2*b*c^4*d^2*e^4*f - 26*a^2*b^2*c^3*d*e^5*f + 10*a*b^2*c^4*d^4*e^2*g + 2*a*b^3*c^3*d^3*e^3*g - 8*a*b^4*c^2*d^2*e^4*g - 8*a^2*b*c^4*d^3*e^3*g + 5*a^2*b^3*c^2*d*e^5*g - 5*a*b^3*c^3*d^4*e^2*h + 8*a*b^4*c^2*d^3*e^3*h + 4*a^2*b*c^4*d^4*e^2*h + 8*a^3*b*c^3*d^2*e^4*h - 10*a^3*b^2*c^2*d*e^5*h - 4*a*b*c^5*d^5*e*g - a*b^5*c*d*e^5*g + 20*a^2*b^2*c^3*d^2*e^4*g - 20*a^2*b^2*c^3*d^3*e^3*h - 2*a^2*b^3*c^2*d^2*e^4*h - 12*a*b*c^5*d^4*e^2*f + 10*a*b^4*c^2*d*e^5*f - 4*a^3*b*c^3*d*e^5*g - 2*a*b^2*c^4*d^5*e*h + 2*a^2*b^4*c*d*e^5*h)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z

+ 84*a^2*b^2*c^2*d^3*e^3*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^
4*d^5*e*f*h*z - 20*a*b^4*c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^
3*c^2*d^4*e^2*g*h*z - 4*a^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z
+ 16*a^2*b*c^3*d^3*e^3*f*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d
^4*e^2*f*h*z + 66*a^2*b^2*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20
*a*b^3*c^2*d^2*e^4*f*g*z + 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z
+ 4*a^4*b*c*e^6*g*h*z - 2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^
3*c^3*d^3*e^3*g*h*z - 3*b^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z -
16*a^2*c^4*d^4*e^2*f*h*z + 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*
g*z + 16*a^2*c^4*d^3*e^3*f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*
e*h^2*z + 2*a*b^3*c^2*d^5*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d
*e^5*g^2*z + 2*a^2*b^3*c*d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*
c^3*d*e^5*f^2*z - 6*a*b^3*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z -
45*a^2*b^2*c^2*d^2*e^4*g^2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z
+ 8*a^4*c^2*d*e^5*g*h*z + 8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 1
4*b^2*c^4*d^5*e*f*g*z - 2*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2
*b^4*d*e^5*g*h*z - 120*a^3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3
*b*c^2*e^6*f*g*z - 2*a^2*b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^
4*d^5*e*g^2*z + 6*a^3*b^2*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a
*b^2*c^3*d^4*e^2*g^2*z + 2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f
^2*z - b^6*d^2*e^4*f*h*z + 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56
*a^3*c^3*d^4*e^2*h^2*z - 5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z
+ 56*a^3*c^3*d^2*e^4*g^2*z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^
2*z - 4*a^2*c^4*d^4*e^2*g^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3
*f^2*z - 36*a^2*c^4*d^2*e^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*
f*g*z - 8*a*c^5*d^6*f*h*z + 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b
^5*c*d^3*e^3*g^2*z + 4*a^3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c
^5*d^4*e^2*f^2*z + b^6*d^3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*
z - b^6*d^2*e^4*g^2*z - 4*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4
*d^6*g^2*z - a^4*b^2*e^6*h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b
^6*e^6*f^2*z - 16*a*b*c^2*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d
^4*e*f*g*h - 4*a^2*b*c*e^5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*
e^3*g*h^2 + 8*a*b*c^2*d^3*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d
^2*e^3*g^2*h + 6*a*b^2*c*d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d
^3*e^2*f*g*h - 8*a^2*c^2*d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^
4*e*g*h^2 + 4*a^2*b*c*d*e^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*
f^2*h + 8*a*c^3*d^4*e*f*h^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g -
2*a*b^3*d*e^4*f*h^2 + 6*a*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c
^2*d^3*e^2*g*h^2 + 12*a^2*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b
^2*c^2*d^2*e^3*f*g^2 + 4*a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*
c*d^3*e^2*g^2*h - 4*b*c^3*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*
d^2*e^3*f^2*h + 9*b^2*c^2*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2
*e^3*g*h^2 - a^2*b^2*d*e^4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^
2*h^3 - 4*a*b*c^2*d^2*e^3*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b
^4*d^2*e^3*f*h^2 + 4*a^2*c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*

e^3*g^3 - 4*a^2*c^2*d*e^4*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2
*b^3*c*e^5*f^2*g - 4*b*c^3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^
2*g^3 + b^4*d^3*e^2*g*h^2 + a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c
^2*e^5*f^3 + a^2*b^2*d^2*e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k)*
((128*a^5*c^4*d*e^6 - 16*a^5*b*c^3*e^7 - a^3*b^5*c*e^7 - b^5*c^4*d^6*e - b^
8*c*d^3*e^4 + 8*a^4*b^3*c^2*e^7 + 128*a^3*c^6*d^5*e^2 + 256*a^4*c^5*d^3*e^4
 + b^6*c^3*d^5*e^2 + b^7*c^2*d^4*e^3 - 48*a^2*b^2*c^5*d^5*e^2 + 168*a^2*b^3
*c^4*d^4*e^3 - 80*a^2*b^4*c^3*d^3*e^4 - 27*a^2*b^5*c^2*d^2*e^5 + 32*a^3*b^2
*c^4*d^3*e^4 + 168*a^3*b^3*c^3*d^2*e^5 + 8*a*b^3*c^5*d^6*e + a*b^7*c*d^2*e^
5 - 16*a^2*b*c^6*d^6*e + a^2*b^6*c*d*e^6 - 27*a*b^5*c^3*d^4*e^3 + 18*a*b^6*
c^2*d^3*e^4 - 304*a^3*b*c^5*d^4*e^3 - 304*a^4*b*c^4*d^2*e^5 - 48*a^4*b^2*c^
3*d*e^6)/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6
*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5
*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*
c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) - (x*(2*a^2*b^6*c*e^7
 - 96*a^5*c^4*e^7 + 32*a^2*c^7*d^6*e + 2*b^4*c^5*d^6*e + 2*b^8*c*d^2*e^5 - 2
2*a^3*b^4*c^2*e^7 + 80*a^4*b^2*c^3*e^7 - 32*a^3*c^6*d^4*e^3 - 160*a^4*c^5*d
^2*e^5 - 6*b^5*c^4*d^5*e^2 + 8*b^6*c^3*d^4*e^3 - 6*b^7*c^2*d^3*e^4 - 4*a*b^
7*c*d*e^6 + 144*a^2*b^2*c^5*d^4*e^3 - 128*a^2*b^3*c^4*d^3*e^4 + 6*a^2*b^4*c
^3*d^2*e^5 + 112*a^3*b^2*c^4*d^2*e^5 - 16*a*b^2*c^6*d^6*e + 160*a^4*b*c^4*d
*e^6 + 48*a*b^3*c^5*d^5*e^2 - 66*a*b^4*c^4*d^4*e^3 + 52*a*b^5*c^3*d^3*e^4 -
 14*a*b^6*c^2*d^2*e^5 - 96*a^2*b*c^6*d^5*e^2 + 42*a^2*b^5*c^2*d*e^6 + 64*a^
3*b*c^5*d^3*e^4 - 144*a^3*b^3*c^3*d*e^6))/(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 1
6*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e
^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*
e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*
c^2*d*e^3)) - (x*(8*a^3*b*c^3*e^6*g - 2*a*b^4*c^2*e^6*f - 48*a^3*c^4*e^6*f
 - 16*a*c^6*d^4*e^2*f + a^2*b^4*c*e^6*h + 32*a^3*c^4*d*e^5*g + 2*b^5*c^2*d*e
^5*f + b^6*c*d^2*e^4*h + 20*a^2*b^2*c^3*e^6*f - 2*a^2*b^3*c^2*e^6*g - 64*a^
2*c^5*d^2*e^4*f - 4*a^3*b^2*c^2*e^6*h + 32*a^2*c^5*d^3*e^3*g + 4*b^2*c^5*d^
4*e^2*f - 8*b^3*c^4*d^3*e^3*f + 2*b^4*c^3*d^2*e^4*f - 32*a^2*c^5*d^4*e^2*h
 - 32*a^3*c^4*d^2*e^4*h - 2*b^3*c^4*d^4*e^2*g + 6*b^4*c^3*d^3*e^3*g - 4*b^5*
c^2*d^2*e^4*g - b^4*c^3*d^4*e^2*h + 8*a*b^2*c^4*d^2*e^4*f - 32*a*b^2*c^4*d^
3*e^3*g + 20*a*b^3*c^3*d^2*e^4*g - 16*a^2*b*c^4*d^2*e^4*g - 32*a^2*b^2*c^3*
d*e^5*g + 12*a*b^2*c^4*d^4*e^2*h - 8*a*b^3*c^3*d^3*e^3*h - 4*a*b^4*c^2*d^2*
e^4*h + 32*a^2*b*c^4*d^3*e^3*h + 8*a^2*b^3*c^2*d*e^5*h - 2*a*b^5*c*d*e^5*h
 + 8*a^2*b^2*c^3*d^2*e^4*h + 32*a*b*c^5*d^3*e^3*f - 24*a*b^3*c^3*d*e^5*f + 6
4*a^2*b*c^4*d*e^5*f + 8*a*b*c^5*d^4*e^2*g + 6*a*b^4*c^2*d*e^5*g))/(a^2*b^4*
e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2
*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d
^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2
*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)) - (4*a^2*c^3*d^3*e^2*h^2 - 4*c^5*d^3*e^
2*f^2 - b^3*c^2*e^5*f^2 - b^2*c^3*d^3*e^2*g^2 + b^3*c^2*d^2*e^3*g^2 + 4*a*b
*c^3*e^5*f^2 - 8*a*c^4*d*e^4*f^2 - 8*a^2*c^3*e^5*f*g + 4*b*c^4*d^2*e^3*f^2
+ 4*a^2*c^3*d*e^4*g^2 + b^2*c^3*d*e^4*f^2 - 2*a*b^2*c^2*d*e^4*g^2 + a*b^3*c

*d^2*e^3*h^2 - a^2*b^2*c*d*e^4*h^2 - 4*b^2*c^3*d^2*e^3*f*g - 8*a^2*c^3*d^2*
e^3*g*h - 2*b^2*c^3*d^3*e^2*f*h + b^3*c^2*d^2*e^3*f*h + b^3*c^2*d^3*e^2*g*h
 - a*b^3*c*e^5*f*h + b^4*c*d*e^4*f*h - 2*a*b^2*c^2*d^3*e^2*h^2 + 2*a*b^2*c^
2*e^5*f*g + 4*a^2*b*c^2*e^5*f*h + 4*b*c^4*d^3*e^2*f*g + 8*a^2*c^3*d*e^4*f*h
 - b^4*c*d^2*e^3*g*h + 4*a*b*c^3*d^2*e^3*f*h - 8*a*b^2*c^2*d*e^4*f*h + 2*a*
b^2*c^2*d^2*e^3*g*h + 4*a*b*c^3*d*e^4*f*g + a*b^3*c*d*e^4*g*h)/(a^2*b^4*e^4
 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^
3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*
e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^
3*c*d*e^3 - 32*a^3*b*c^2*d*e^3) + (x*(4*a^2*c^3*e^5*g^2 + b^2*c^3*e^5*f^2 +
 4*c^5*d^2*e^3*f^2 + 4*a^2*c^3*d^2*e^3*h^2 + b^2*c^3*d^2*e^3*g^2 - 4*b*c^4*
d*e^4*f^2 + a^2*b^2*c*e^5*h^2 + b^4*c*d^2*e^3*h^2 + 4*a^2*b*c^2*d*e^4*h^2 +
 4*b^2*c^3*d^2*e^3*f*h - 2*b^3*c^2*d^2*e^3*g*h - 4*a*b*c^3*e^5*f*g + 8*a*c^
4*d*e^4*f*g - 4*a*b^2*c^2*d^2*e^3*h^2 - 4*a*b*c^3*d*e^4*g^2 - 2*a*b^3*c*d*e
^4*h^2 + 2*a*b^2*c^2*e^5*f*h - 4*a^2*b*c^2*e^5*g*h - 8*a*c^4*d^2*e^3*f*h -
4*b*c^4*d^2*e^3*f*g + 2*b^2*c^3*d*e^4*f*g - 8*a^2*c^3*d*e^4*g*h - 2*b^3*c^2
*d*e^4*f*h + 4*a*b*c^3*d^2*e^3*g*h + 6*a*b^2*c^2*d*e^4*g*h))/(a^2*b^4*e^4 +
 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*
d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e
+ 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*
c*d*e^3 - 32*a^3*b*c^2*d*e^3))*root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c
^5*d^5*e^3*z^3 - 192*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68
*a^3*b^6*c*d^2*e^6*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^
3 + 36*a*b^7*c^2*d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*
e*z^3 + 48*a^4*b^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d
^4*e^4*z^3 + 440*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 32
0*a^3*b^3*c^4*d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d
^6*e^2*z^3 - 192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90
*a^2*b^6*c^2*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*
e^3*z^3 - 4*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 -
 4*a*b^9*d^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*
c^2*d^6*e^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a
^4*c^6*d^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^
2*b^2*c^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*
z^3 + b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3
*b^2*c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z +
14*a^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3
*g*h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*
c*d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a
^3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^3*f
*h*z + 16*a*b^3*c^2*d^3*e^3*f*h*z - 14*a*b^2*c^3*d^4*e^2*f*h*z + 66*a^2*b^2
*c^2*d*e^5*f*g*z - 36*a*b^2*c^3*d^3*e^3*f*g*z + 20*a*b^3*c^2*d^2*e^4*f*g*z
+ 12*a^2*b*c^3*d^2*e^4*f*g*z + 8*a*c^5*d^5*e*f*g*z + 4*a^4*b*c*e^6*g*h*z -
2*a*b^5*d*e^5*f*h*z + 4*a*b*c^4*d^6*g*h*z - 112*a^3*c^3*d^3*e^3*g*h*z - 3*b
^4*c^2*d^4*e^2*f*h*z + 120*a^3*c^3*d^2*e^4*f*h*z - 16*a^2*c^4*d^4*e^2*f*h*z

+ 14*b^3*c^3*d^4*e^2*f*g*z - 2*b^4*c^2*d^3*e^3*f*g*z + 16*a^2*c^4*d^3*e^3*
f*g*z + 8*a*b^4*c*d^4*e^2*h^2*z + 4*a^2*b*c^3*d^5*e*h^2*z + 2*a*b^3*c^2*d^5
*e*h^2*z + 8*a*b^4*c*d^2*e^4*g^2*z + 4*a^3*b*c^2*d*e^5*g^2*z + 2*a^2*b^3*c*
d*e^5*g^2*z + 48*a*b*c^4*d^3*e^3*f^2*z + 36*a^2*b*c^3*d*e^5*f^2*z - 6*a*b^3
*c^2*d*e^5*f^2*z - 45*a^2*b^2*c^2*d^4*e^2*h^2*z - 45*a^2*b^2*c^2*d^2*e^4*g^
2*z + 2*b^5*c*d^4*e^2*g*h*z - b^4*c^2*d^5*e*g*h*z + 8*a^4*c^2*d*e^5*g*h*z +
8*a^2*c^4*d^5*e*g*h*z + 2*b^3*c^3*d^5*e*f*h*z - 14*b^2*c^4*d^5*e*f*g*z - 2
*b^5*c*d^2*e^4*f*g*z + 2*a*b^5*d^2*e^4*g*h*z - a^2*b^4*d*e^5*g*h*z - 120*a^
3*c^3*d*e^5*f*g*z - 6*a^3*b^2*c*e^6*f*h*z + 12*a^3*b*c^2*e^6*f*g*z - 2*a^2*
b^3*c*e^6*f*g*z - 4*a^4*b*c*d*e^5*h^2*z - 4*a*b*c^4*d^5*e*g^2*z + 6*a^3*b^2
*c*d^2*e^4*h^2*z + 2*a^2*b^3*c*d^3*e^3*h^2*z + 6*a*b^2*c^3*d^4*e^2*g^2*z +
2*a*b^3*c^2*d^3*e^3*g^2*z - 18*a*b^2*c^3*d^2*e^4*f^2*z - b^6*d^2*e^4*f*h*z
+ 12*b*c^5*d^5*e*f^2*z + 12*a*b^4*c*e^6*f^2*z + 56*a^3*c^3*d^4*e^2*h^2*z -
5*b^4*c^2*d^4*e^2*g^2*z - 4*a^4*c^2*d^2*e^4*h^2*z + 56*a^3*c^3*d^2*e^4*g^2*
z - 9*b^2*c^4*d^4*e^2*f^2*z - 5*a^2*b^4*d^2*e^4*h^2*z - 4*a^2*c^4*d^4*e^2*g
^2*z + 3*b^4*c^2*d^2*e^4*f^2*z - 2*b^3*c^3*d^3*e^3*f^2*z - 36*a^2*c^4*d^2*e
^4*f^2*z - 45*a^2*b^2*c^2*e^6*f^2*z + 2*b^6*d*e^5*f*g*z - 8*a*c^5*d^6*f*h*z
+ 4*b*c^5*d^6*f*g*z + 4*b^3*c^3*d^5*e*g^2*z + 2*b^5*c*d^3*e^3*g^2*z + 4*a^
3*b^3*d*e^5*h^2*z + 2*a*b^5*d^3*e^3*h^2*z - 24*a*c^5*d^4*e^2*f^2*z + b^6*d^
3*e^3*g*h*z + a^2*b^4*e^6*f*h*z - b^6*d^4*e^2*h^2*z - b^6*d^2*e^4*g^2*z - 4
*a^4*c^2*e^6*g^2*z - 4*a^2*c^4*d^6*h^2*z - b^2*c^4*d^6*g^2*z - a^4*b^2*e^6*
h^2*z + 48*a^3*c^3*e^6*f^2*z - 4*c^6*d^6*f^2*z - b^6*e^6*f^2*z - 16*a*b*c^2
*d^2*e^3*f*g*h - 4*a*b^2*c*d*e^4*f*g*h - 4*b*c^3*d^4*e*f*g*h - 4*a^2*b*c*e^
5*f*g*h + 6*b^2*c^2*d^3*e^2*f*g*h - 8*a^2*b*c*d^2*e^3*g*h^2 + 8*a*b*c^2*d^3
*e^2*g^2*h + 2*a*b^2*c*d^3*e^2*g*h^2 - 2*a*b^2*c*d^2*e^3*g^2*h + 6*a*b^2*c*
d^2*e^3*f*h^2 + 4*b^3*c*d^2*e^3*f*g*h - 16*a*c^3*d^3*e^2*f*g*h - 8*a^2*c^2*
d*e^4*f*g*h + 4*a^2*b*c*d*e^4*g^2*h - 4*a*b*c^2*d^4*e*g*h^2 + 4*a^2*b*c*d*e
^4*f*h^2 + 16*a*b*c^2*d*e^4*f*g^2 - 2*b^3*c*d*e^4*f^2*h + 8*a*c^3*d^4*e*f*h
^2 - 4*b^3*c*d*e^4*f*g^2 - 24*a*c^3*d*e^4*f^2*g - 2*a*b^3*d*e^4*f*h^2 + 6*a
*b^2*c*e^5*f^2*h - 12*a*b*c^2*e^5*f^2*g - 12*a^2*c^2*d^3*e^2*g*h^2 + 12*a^2
*c^2*d^2*e^3*g^2*h - 3*b^2*c^2*d^2*e^3*f^2*h - 5*b^2*c^2*d^2*e^3*f*g^2 + 4*
a^2*c^2*d^2*e^3*f*h^2 + 2*b^4*d*e^4*f*g*h - 2*b^3*c*d^3*e^2*g^2*h - 4*b*c^3
*d^3*e^2*f^2*h - 2*b^3*c*d^3*e^2*f*h^2 + 24*a*c^3*d^2*e^3*f^2*h + 9*b^2*c^2
*d*e^4*f^2*g + 4*b*c^3*d^3*e^2*f*g^2 + 2*a*b^3*d^2*e^3*g*h^2 - a^2*b^2*d*e^
4*g*h^2 + 8*a*c^3*d^2*e^3*f*g^2 + 4*a^2*b*c*d^3*e^2*h^3 - 4*a*b*c^2*d^2*e^3
*g^3 - b^4*d^2*e^3*g^2*h - 4*c^4*d^3*e^2*f^2*g - b^4*d^2*e^3*f*h^2 + 4*a^2*
c^2*e^5*f*g^2 + 4*a^2*c^2*d^4*e*h^3 + 2*b^3*c*d^2*e^3*g^3 - 4*a^2*c^2*d*e^4
*g^3 - 2*a*b^3*d^3*e^2*h^3 + 4*c^4*d^4*e*f^2*h + 2*b^3*c*e^5*f^2*g - 4*b*c^
3*d*e^4*f^3 + b^2*c^2*d^4*e*g^2*h - b^2*c^2*d^3*e^2*g^3 + b^4*d^3*e^2*g*h^2
+ a^2*b^2*e^5*f*h^2 + 4*c^4*d^2*e^3*f^3 - 3*b^2*c^2*e^5*f^3 + a^2*b^2*d^2*
e^3*h^3 - b^4*e^5*f^2*h + 16*a*c^3*e^5*f^3, z, k), k, 1, 3) - ((a*b*d*h - 2
*a^2*e*h - b^2*e*f + a*b*e*g - 2*a*c*d*g + 2*a*c*e*f + b*c*d*f)/(a*b^2*e^2
- 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (x*(a*b*
e*h - b^2*d*h - 2*c^2*d*f + 2*a*c*d*h - 2*a*c*e*g + b*c*d*g + b*c*e*f))/(a*
b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e))/(

```
a + b*x + c*x^2)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
[Out] Timed out
```

## 3.159 $\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=673

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h\cdots}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(\cdots\right.}$$

```
[Out] -e*(d^2*h-d*e*g+e^2*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)+(-b^3*e^2*f+b^2*e*(a*e
*g+2*c*d*f)-2*a*c*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g))-b*(c^2*d^2*f+a^2*e^2*
h-a*c*(-d^2*h-2*d*e*g+3*e^2*f))-c*(2*c^2*d^2*f+2*a^2*e^2*h-a*b*e*(2*d*h+e*g
)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*g+e^2*f)))*x)/(-4*a
*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*d*h-a
*e*g-b*d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e^2*f))
-6*c^2*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c*e*(6*a
^2*b*e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(2*d^2*h-
d*e*g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(
a*e^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*
e*g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*h-a*e*g-b*d
*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^3
```

**Rubi** [A] time = 2.56, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1646, 1628, 634, 618, 206, 628}

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h\cdots}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) - (b^3
*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e*g - 2*
d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h)) + c*(2*
c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*
d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*c)*(c*d^2
- b*d*e + a*e^2)^2*(a + b*x + c*x^2)) + ((4*c^4*d^4*f - b^3*e^3*(2*b*e*f -
b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f - 2*
d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2*d
*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h - b^3*d*(4*e^2*f - 3*d*
```

```
e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^3) - (e*
(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h
))*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 + (e*(e^2*(2*b*e*f - b*d*g - a*e
*g + 2*a*d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2
*(c*d^2 - b*d*e + a*e^2)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
```

```
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+a^2\right.}{}$$

$$= -\frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+ae(eg-2dh))+b\left(c^2d^2f+a^2\right.}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+}{}$$

$$= -\frac{e\left(e^2f-deg+d^2h\right)}{\left(cd^2-bde+ae^2\right)^2(d+ex)} - \frac{b^3e^2f-b^2e(2cdf+aeg)+2ac(cd(2ef-dg)+}{}$$

**Mathematica [A]** time = 2.21, size = 650, normalized size = 0.97

$$\frac{b\left(-a^2e^2h+ac\left(d^2(-h)-2de(g-hx)+e^2(3f+gx)\right)+c^2d(-df+dgx+2efx)\right)+2c\left(a^2(-e)(e(g+hx)-2dh)+\right.}{\left(b^2-4ac\right)(a+x(b+cx))\left(e(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

```
[Out] -((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) + (
-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^2*
e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g*x)
 - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f + g*x)
 + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x)))))/((b^2 - 4*a*c)*(c*d^2 +
e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*b*e*f
+ b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e^2*f -
2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e^2*f - 2
*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*e^2*f - 3
*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTan[(b + 2*c
*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*d - a*e))^3
) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g +
 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*(-2*b*e*f + b
*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + x*(b
 + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [B]** time = 0.33, size = 1437, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(4*c^4*d^4*f*e^2 - 2*b*c^3*d^4*g*e^2 + 4*a*c^3*d^4*h*e^2 - 8*b*c^3*d^3*f*e
^3 + 6*b^2*c^2*d^3*g*e^3 - 8*a*c^3*d^3*g*e^3 - 2*b^3*c*d^3*h*e^3 + 24*a*c^3
*d^2*f*e^4 - 3*b^3*c*d^2*g*e^4 + 12*a*b^2*c*d^2*h*e^4 - 24*a^2*c^2*d^2*h*e^
4 + 4*b^3*c*d*f*e^5 - 24*a*b*c^2*d*f*e^5 + b^4*d*g*e^5 - 6*a*b^2*c*d*g*e^5
+ 24*a^2*c^2*d*g*e^5 - 2*a*b^3*d*h*e^5 - 2*b^4*f*e^6 + 12*a*b^2*c*f*e^6 - 1
2*a^2*c^2*f*e^6 + a*b^3*g*e^6 - 6*a^2*b*c*g*e^6 + 4*a^3*c*h*e^6)*arctan((2*
c*d - 2*c*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*a*e^2/(x*e + d))*e^(-
1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e
 + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^
4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^
2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3
*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^3*
h*e - 3*c*d^2*g*e^2 + 4*c*d*f*e^3 + b*d*g*e^3 - 2*a*d*h*e^3 - 2*b*f*e^4 + a
```

```
*g*e^4)*log(c - 2*c*d/(x*e + d) + c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e
/(x*e + d)^2 + a*e^2/(x*e + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a
^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (d^2*h*e^5/(x*e + d) - d*g*e^6/(x
*e + d) + f*e^7/(x*e + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6 + 2*a
*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - ((2*c^4*d^3*f*e - b*c^3*d^3*g*e + b^2
*c^2*d^3*h*e - 2*a*c^3*d^3*h*e - 3*b*c^3*d^2*f*e^2 + 6*a*c^3*d^2*g*e^2 - 3*
a*b*c^2*d^2*h*e^2 + 3*b^2*c^2*d*f*e^3 - 6*a*c^3*d*f*e^3 - 3*a*b*c^2*d*g*e^3
 + 6*a^2*c^2*d*h*e^3 - b^3*c*f*e^4 + 3*a*b*c^2*f*e^4 + a*b^2*c*g*e^4 - 2*a^
2*c^2*g*e^4 - a^2*b*c*h*e^4)/(c*d^2 - b*d*e + a*e^2) - (2*c^4*d^4*f*e^2 - b
*c^3*d^4*g*e^2 + b^2*c^2*d^4*h*e^2 - 2*a*c^3*d^4*h*e^2 - 4*b*c^3*d^3*f*e^3
+ 8*a*c^3*d^3*g*e^3 - 4*a*b*c^2*d^3*h*e^3 + 6*b^2*c^2*d^2*f*e^4 - 12*a*c^3*
d^2*f*e^4 - 6*a*b*c^2*d^2*g*e^4 + 12*a^2*c^2*d^2*h*e^4 - 4*b^3*c*d*f*e^5 +
12*a*b*c^2*d*f*e^5 + 4*a*b^2*c*d*g*e^5 - 8*a^2*c^2*d*g*e^5 - 4*a^2*b*c*d*h*
e^5 + b^4*f*e^6 - 4*a*b^2*c*f*e^6 + 2*a^2*c^2*f*e^6 - a*b^3*g*e^6 + 3*a^2*b
*c*g*e^6 + a^2*b^2*h*e^6 - 2*a^3*c*h*e^6)*e^(-1)/((c*d^2 - b*d*e + a*e^2)*(
x*e + d)))/((c*d^2 - b*d*e + a*e^2)^2*(b^2 - 4*a*c)*(c - 2*c*d/(x*e + d) +
c*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + a*e^2/(x*e + d)^2))
```

**maple [B]** time = 0.04, size = 4716, normalized size = 7.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

```
[Out] -e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*f-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*a*b*c^2*d^2*e^2*f-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^
2+b*x+a)*a*b*d*e^3*g+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d^2*e^2*h-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*d*e^3*g-24/(a*e^2-b*d
*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*c^2*d
*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a^2*b*e^4*g+4/
(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*a^2*d*e^3*g+1/(a*e^2-
b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*a*b^2*e^4*f+4/(a*e^2-b*d*e+c*d
^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*a*d^3*e*g-1/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)*c/(4*a*c-b^2)*x*b^3*d^3*e*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a
)*c/(4*a*c-b^2)*x*b^3*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*
a*c-b^2)*x*b^2*d^3*e*g+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2
)*x*b^2*d^2*e^2*f-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b
*d^3*e*f+6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*d^2*e^2*
h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^3*e*h-3/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d^2*e^2*g+1/(a*e^2-b*d*e
+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^2*c*d*e^3*f+4/(a*e^2-b*d*e+c*d^2)^3
/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^3*e*g-6/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b
```

```
*x+a)*c^2/(4*a*c-b^2)*x*a*b*d^2*e^2*g-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)
*c/(4*a*c-b^2)*x*a^2*b*d*e^3*h+3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a
*c-b^2)*x*a*b^2*d^2*e^2*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c/(4*a*c-b^
2)*x*a*b^2*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*b^4*e^4*f-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^
2+b*x+a)*b^3*e^4*f+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*c^4*d^4*f-2*e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*d*h
+e^3/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*d*g+2*e/(a*e^2-b*d*e+c*d^2)^3*ln(e*x
+d)*c*d^3*h-3*e^2/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*c*d^2*g+4*e^3/(a*e^2-b*d*
e+c*d^2)^3*ln(e*x+d)*c*d*f+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b
*x+a)*a^2*d*e^3*h+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*a*b
*e^4*f+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c^2*ln(c*x^2+b*x+a)*a*d^2*e^2*g+
1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b*c^2*d^4*h+4/(a*e^2-b*
d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^3*e*f+3/(a*e^2-b*d*e+c*d^2)^
3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^3*c*d^2*e^2*f-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-
b^2)*c^2*ln(c*x^2+b*x+a)*a*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln
(c*x^2+b*x+a)*b^2*d^3*e*h-3/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+
b*x+a)*b^2*d^2*e^2*g+2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x+a)*
b^2*d*e^3*f-1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(c*x^2+b*x+a)*a*b^2*d*e^3
*h-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))*b*c^3*d^3*e*f-6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a^2*b*c*e^4*g-24/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(
3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d^2*e^2*h+24/(a*e^2-b*d*e+
c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*d*e^
3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*a*b^3*d*e^3*h+12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*c*e^4*f-8/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^3*e*g+24/(a*e^2-b*d*e+c*d
^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*c^3*d^2*e^2*f
-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*b^3*c*d^3*e*h-3/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))*b^3*c*d^2*e^2*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*e^3*f-3/(a*e^2-b*d*e+c*d^2)
^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c^2*d^3*e*f-4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)*c^2*ln(c*x^2+b*x+a)*a*d^3*e*h+6/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/
2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d^3*e*g-4/(a*e^2-b*d*e+c*d^2
)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d^3*e*h+4/(a*e^2-b*d*e+c*d^2)^3/(c*x^
2+b*x+a)/(4*a*c-b^2)*a^2*c^2*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/
(4*a*c-b^2)*a*b^3*d*e^3*g-4/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)
*a^3*c*d*e^3*h-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*d*
e^3*h-3/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b*c*e^4*f+1/(a*
e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*b^3*e^4*f-2/(a*e^2-b*d*e+c*d
^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a*c^3*d^4*g-1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+
b*x+a)/(4*a*c-b^2)*b^4*d*e^3*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c
-b^2)*b*c^3*d^4*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^4/(4*a*c-b^2)*x*d
```

```
^4*f+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*b*e^4*h+2/(a*e^2
-b*d*e+c*d^2)^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^3*c*e^4*g-1/(a*e^2-b*d*e+c*d^2)
^3/(c*x^2+b*x+a)/(4*a*c-b^2)*a^2*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)*ln(c*x^2+b*x+a)*a*b^2*e^4*g+1/2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*ln(
c*x^2+b*x+a)*b^3*d*e^3*g-2/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)*c*ln(c*x^2+b*x
+a)*a^2*e^4*g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a*b^3*e^4*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*c*e^4*h-12/(a*e^2-b*d*e+c*d^2)^3/(4*a*c
-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*c^2*e^4*f-2/(a*e^2-b*d*
e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*d^4*
g+1/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))*b^4*d*e^3*g+4/(a*e^2-b*d*e+c*d^2)^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*a*c^3*d^4*h-e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d^2*h+e^2/(
a*e^2-b*d*e+c*d^2)^2/(e*x+d)*d*g+e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*a*g-2*
e^4/(a*e^2-b*d*e+c*d^2)^3*ln(e*x+d)*b*f+2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+
a)*c/(4*a*c-b^2)*x*a^3*e^4*h-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a
*c-b^2)*x*a^2*e^4*f-2/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x
*a*d^4*h+1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x*b^2*d^4*h-
1/(a*e^2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x*b*d^4*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 8.93, size = 26278, normalized size = 39.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f -
 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^2*
c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*e^2
*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^
4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b
*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f + 2*c^3
```

*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2*a^2*c*e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f + 2*a*c^2*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b*c*e^3*f + 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) - (x^2*(6*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f - 8*a*c^2*d*e^2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2*c*d*e^2*g - 2*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + symsum(log((x*(36*a^2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 + 4*c^7*d^4*e^3*f^2 + a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b^2*c^5*d^2*e^5*f^2 + 36*a^2*c^5*d^4*e^3*h^2 - 24*a^3*c^4*d^2*e^5*h^2 + b^2*c^5*d^4*e^3*g^2 + 2*b^3*c^4*d^3*e^4*g^2 + b^4*c^3*d^2*e^5*g^2 + 4*b^4*c^3*d^4*e^3*h^2 - 24*a^3*c^4*e^7*f*h - 24*a*b^2*c^4*e^7*f^2 - 24*a*c^6*d^2*e^5*f^2 - 8*b*c^6*d^3*e^4*f^2 - 8*b^3*c^4*d*e^6*f^2 - 16*a*b*c^5*d^3*e^4*g^2 + 2*a*b^3*c^3*d*e^6*g^2 - 16*a^2*b*c^4*d*e^6*g^2 - 8*a^3*b*c^3*d*e^6*h^2 + 8*a^2*b^2*c^3*e^7*f*h + 80*a^2*c^5*d^2*e^5*f*h - 96*a^2*c^5*d^3*e^4*g*h + 8*b^2*c^5*d^4*e^3*f*h - 8*b^3*c^4*d^3*e^4*f*h + 8*b^4*c^3*d^2*e^5*f*h - 4*b^3*c^4*d^4*e^3*g*h - 4*b^4*c^3*d^3*e^4*g*h - 14*a*b^2*c^4*d^2*e^5*g^2 - 24*a*b^2*c^4*d^4*e^3*h^2 - 8*a*b^3*c^3*d^3*e^4*h^2 + 24*a^2*b*c^4*d^3*e^4*h^2 + 24*a*b*c^5*d*e^6*f^2 - 4*a*b^3*c^3*e^7*f*g + 12*a^2*b*c^4*e^7*f*g + 32*a*c^6*d^3*e^4*f*g - 96*a^2*c^5*d*e^6*f*g - 4*a^3*b*c^3*e^7*g*h - 24*a*c^6*d^4*e^3*f*h - 4*b*c^6*d^4*e^3*f*g - 4*b^4*c^3*d*e^6*f*g + 32*a^3*c^4*d*e^6*g*h + 12*a^2*b^2*c^3*d^2*e^5*h^2 - 24*a*b*c^5*d^2*e^5*f*g + 48*a*b^2*c^4*d*e^6*f*g + 16*a*b*c^5*d^3*e^4*f*h - 8*a*b^3*c^3*d*e^6*f*h + 16*a^2*b*c^4*d*e^6*f*h + 12*a*b*c^5*d^4*e^3*g*h - 40*a*b^2*c^4*d^2*e^5*f*h + 48*a*b^2*c^4*d^3*e^4*g*h - 24*a^2*b*c^4*d^2*e^5*g*h))/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) - root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3

+ 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^
4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3
 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b
^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7
*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3
*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10
*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*
a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6
*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*
a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e
^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3
 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 2
0*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 -
 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z
^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e
^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2
*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12
*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*
c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a
^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z
- 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4
*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*
h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a
^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*
d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g
*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^
6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a
*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*
g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b
*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*
h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^
5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z
 - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^
3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z
 - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z
+ 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z +
 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4
*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4
*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b
^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*
z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e
^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*
d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b
^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z
+ 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c

^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*
h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a
^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*
d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2
*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*
z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z
- 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z +
4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z -
2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z -
48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z
+ 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d
^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z -
102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5
*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*
a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e
^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*
e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2
*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*
e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c
^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 4
32*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^
2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e
^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*
d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*
a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a
*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^
2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2
*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2
*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2
*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*
b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h -
48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3
*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*
b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*
h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*
b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h
+ 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5
*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2
*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c
^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12
*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2
+ 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g
- 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6
*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h
- 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4

*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4
*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c
^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24
*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3
+ 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h
- 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h -
168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 +
12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g
+ 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h
+ 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h -
8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g +
12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 -
6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4
*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5
*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h +
8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d
^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2
*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3
+ 16*a^3*c^3*e^6*g^3, z, k)*((8*a^6*c^3*e^9*h - 24*a^5*c^4*e^9*f - 8*a*c^8*
d^8*e*f + 2*a^2*b^6*c*e^9*f - a^3*b^5*c*e^9*g - 20*a^5*b*c^3*e^9*g + 16*a^5
*c^4*d*e^8*g + 2*b^2*c^7*d^8*e*f + 2*b^8*c*d^2*e^7*f - 8*a^2*c^7*d^8*e*h -
b^3*c^6*d^8*e*g - b^8*c*d^3*e^6*g - 18*a^3*b^4*c^2*e^9*f + 46*a^4*b^2*c^3*e
^9*f + 9*a^4*b^3*c^2*e^9*g - 48*a^2*c^7*d^6*e^3*f - 96*a^3*c^6*d^4*e^5*f -
80*a^4*c^5*d^2*e^7*f - 2*a^5*b^2*c^2*e^9*h + 16*a^2*c^7*d^7*e^2*g + 48*a^3*
c^6*d^5*e^4*g + 48*a^4*c^5*d^3*e^6*g - 6*b^3*c^6*d^7*e^2*f + 4*b^4*c^5*d^6*
e^3*f + 4*b^6*c^3*d^4*e^5*f - 6*b^7*c^2*d^3*e^6*f - 16*a^3*c^6*d^6*e^3*h +
16*a^5*c^4*d^2*e^7*h + 4*b^4*c^5*d^7*e^2*g - 3*b^5*c^4*d^6*e^3*g - 3*b^6*c^
3*d^5*e^4*g + 4*b^7*c^2*d^4*e^5*g - 2*b^5*c^4*d^7*e^2*h + 4*b^6*c^3*d^6*e^3
*h - 2*b^7*c^2*d^5*e^4*h - 4*a*b^2*c^6*d^6*e^3*f - 14*a*b^3*c^5*d^5*e^4*f -
38*a*b^4*c^4*d^4*e^5*f + 54*a*b^5*c^3*d^3*e^6*f - 10*a*b^6*c^2*d^2*e^7*f +
56*a^2*b*c^6*d^5*e^4*f + 34*a^2*b^5*c^2*d*e^8*f + 40*a^3*b*c^5*d^3*e^6*f -
74*a^3*b^3*c^3*d*e^8*f - 20*a*b^2*c^6*d^7*e^2*g + 10*a*b^3*c^5*d^6*e^3*g +
34*a*b^4*c^4*d^5*e^4*g - 33*a*b^5*c^3*d^4*e^5*g + 4*a*b^6*c^2*d^3*e^6*g +
8*a^2*b*c^6*d^6*e^3*g - 16*a^3*b*c^5*d^4*e^5*g - 10*a^3*b^4*c^2*d*e^8*g - 4
0*a^4*b*c^4*d^2*e^7*g + 20*a^4*b^2*c^3*d*e^8*g + 10*a*b^3*c^5*d^7*e^2*h - 2
6*a*b^4*c^4*d^6*e^3*h + 12*a*b^5*c^3*d^5*e^4*h - 8*a^2*b*c^6*d^7*e^2*h - 4*
a^2*b^6*c*d^2*e^7*h - 8*a^3*b*c^5*d^5*e^4*h + 8*a^4*b*c^4*d^3*e^6*h - 10*a^
4*b^3*c^2*d*e^8*h - 4*a*b^7*c*d*e^8*f + 4*a*b*c^7*d^8*e*g + 112*a^2*b^2*c^5
*d^4*e^5*f - 130*a^2*b^3*c^4*d^3*e^6*f - 28*a^2*b^4*c^3*d^2*e^7*f + 164*a^3
*b^2*c^4*d^2*e^7*f - 100*a^2*b^2*c^5*d^5*e^4*g + 72*a^2*b^3*c^4*d^4*e^5*g +
12*a^2*b^4*c^3*d^3*e^6*g - 7*a^2*b^5*c^2*d^2*e^7*g - 60*a^3*b^2*c^4*d^3*e^
6*g + 22*a^3*b^3*c^3*d^2*e^7*g + 44*a^2*b^2*c^5*d^6*e^3*h - 14*a^2*b^3*c^4*
d^5*e^4*h - 12*a^2*b^5*c^2*d^3*e^6*h + 14*a^3*b^3*c^3*d^3*e^6*h + 26*a^3*b^
4*c^2*d^2*e^7*h - 44*a^4*b^2*c^3*d^2*e^7*h + 24*a*b*c^7*d^7*e^2*f + 8*a^4*b
*c^4*d*e^8*f + a*b^7*c*d^2*e^7*g + a^2*b^6*c*d*e^8*g + 2*a*b^2*c^6*d^8*e*h

+ 2*a*b^7*c*d^3*e^6*h + 2*a^3*b^5*c*d*e^8*h + 8*a^5*b*c^3*d*e^8*h)/(16*a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5) + root(3840*a^6*b*c^5*d^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 - 120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 - 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 - 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 792*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z + 156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c

^5*d^6*e^2*g*h*z + 10*a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z
 - 308*a^3*b^3*c^2*d*e^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2
*d^3*e^5*f*h*z + 68*a*b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z -
24*a*b^2*c^5*d^6*e^2*f*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3
*d*e^7*f*g*z + 306*a^2*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z +
222*a*b^5*c^2*d^2*e^6*f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^
5*e^3*f*g*z + 16*a*c^7*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*
h*z + 6*b^6*c^2*d^5*e^3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*
e^5*g*h*z - 720*a^3*c^5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4
*d^6*e^2*f*h*z + 1072*a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 3
0*b^6*c^2*d^3*e^5*f*g*z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g
*z - 10*b^4*c^4*d^5*e^3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5
*e^3*f*g*z - 24*a^4*b^2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b
^3*c*d*e^7*h^2*z + 4*a*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*
a^4*b*c^3*d*e^7*g^2*z + 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z
 - 912*a^3*b*c^4*d*e^7*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*
e^3*f^2*z + 432*a^3*b^2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z -
 168*a^2*b^2*c^4*d^6*e^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3
*c^2*d^3*e^5*h^2*z - 20*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4
*g^2*z + 336*a^3*b^2*c^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 12
0*a^2*b^4*c^2*d^2*e^6*g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e
^4*g*h*z - 2*b^4*c^4*d^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7
*e*g*h*z + 4*b^7*c*d^3*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6
*f*g*z - 20*b^2*c^6*d^7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g
*h*z + 528*a^4*c^4*d*e^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g
*h*z + 4*a^3*b^4*c*e^8*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f
*g*z - 8*a*b*c^6*d^7*e*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^
6*e^2*h^2*z + 12*a^2*b^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312
*a^3*b*c^4*d^3*e^5*g^2*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*
e^7*g^2*z - 66*a*b^5*c^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*
b^2*c^5*d^6*e^2*g^2*z - 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*
f^2*z + 240*a^2*b*c^5*d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b
^2*c^5*d^4*e^4*f^2*z + 24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c
^7*d^7*e*f^2*z - 2*a*b^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6
*e^2*h^2*z - 536*a^4*c^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^
3*c^5*d^6*e^2*h^2*z - 12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z +
 10*b^5*c^3*d^5*e^3*g^2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6
*g^2*z + 20*b^4*c^4*d^4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d
^6*e^2*f^2*z - 16*a^2*c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c
^5*d^5*e^3*f^2*z - 4*a^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 12
0*a^2*c^6*d^4*e^4*f^2*z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^
2*z - 192*a^2*b^4*c^2*e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4
*b*c^7*d^8*f*g*z + 4*a*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^
7*e*g^2*z - 48*a*c^7*d^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g
^2*z - 4*a^6*c^2*e^8*h^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b

^2*c^6*d^8*g^2*z - 36*a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2
*z - 4*b^8*e^8*f^2*z - 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h
+ 16*a*b^3*c^2*d*e^5*f*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*
e^3*g*h^2 + 16*a*b^3*c^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2
*b^2*c^2*d*e^5*g^2*h - 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*
h^2 + 24*a^2*b*c^3*d^2*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^
5*f*g*h - 8*b*c^5*d^5*e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*
g*h + 16*b^4*c^2*d^2*e^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*
e^4*f*g*h + 48*a^2*b^2*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^
4*d^4*e^2*g^2*h - 8*a^2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*
b^2*c^3*d*e^5*f^2*h + 24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 +
 96*a*b*c^4*d^2*e^4*f*g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e
^4*g*h^2 - 56*a*c^5*d^4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5
*g^2*h + 16*a*b^4*c*d*e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f
*g*h + 16*a*c^5*d^5*e*f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g
^2*h + 4*b^4*c^2*d^4*e^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*
e^2*g*h^2 + 24*a^3*c^3*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^
4*d^3*e^3*f^2*h - 8*b^4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^
2*c^4*d^2*e^4*f^2*g - 48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 -
 24*b^2*c^4*d^3*e^3*f*g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2
*h^3 + 8*a^2*b^3*c*d^2*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2
*e^4*g^3 + 2*b^5*c*d^2*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5
*g^2*h - 8*b^4*c^2*d*e^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*
f^2*h + 96*a*c^5*d^3*e^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*
f*g^2 + 12*b*c^5*d^4*e^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5
*f*g^2 + 48*a*c^5*d^3*e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*
g^2*h + 48*a^2*b*c^3*e^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f
*h^2 - 60*a^2*b*c^3*e^6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f
*g^2 + 24*a^2*b*c^3*d*e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*
g^3 - 12*c^6*d^4*e^2*f^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 3
6*a^2*c^4*e^6*f^2*g - 8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^
4*d*e^5*f^3 - 24*b*c^5*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h
+ 144*a*c^5*d*e^5*f^3 - 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4
*c^2*d^2*e^4*g^3 - 3*b^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b
^2*c^2*e^6*g^3 + 16*c^6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g
^3, z, k)*((a^5*b^5*c*e^11 + 16*a^7*b*c^3*e^11 - 128*a^7*c^4*d*e^10 + b^5*c
^6*d^10*e + b^10*c*d^5*e^6 - 8*a^6*b^3*c^2*e^11 - 128*a^3*c^8*d^9*e^2 - 512
*a^4*c^7*d^7*e^4 - 768*a^5*c^6*d^5*e^6 - 512*a^6*c^5*d^3*e^8 - 3*b^6*c^5*d^
9*e^2 + 2*b^7*c^4*d^8*e^3 + 2*b^8*c^3*d^7*e^4 - 3*b^9*c^2*d^6*e^5 + 16*a^2*
b^2*c^7*d^9*e^2 - 264*a^2*b^3*c^6*d^8*e^3 + 480*a^2*b^4*c^5*d^7*e^4 - 246*a
^2*b^5*c^4*d^6*e^5 - 66*a^2*b^6*c^3*d^5*e^6 + 62*a^2*b^7*c^2*d^4*e^7 - 704*
a^3*b^2*c^6*d^7*e^4 - 240*a^3*b^3*c^5*d^6*e^5 + 800*a^3*b^4*c^4*d^5*e^6 - 2
46*a^3*b^5*c^3*d^4*e^7 - 76*a^3*b^6*c^2*d^3*e^8 - 1440*a^4*b^2*c^5*d^5*e^6
- 240*a^4*b^3*c^4*d^4*e^7 + 480*a^4*b^4*c^3*d^3*e^8 + 21*a^4*b^5*c^2*d^2*e^
9 - 704*a^5*b^2*c^4*d^3*e^8 - 264*a^5*b^3*c^3*d^2*e^9 - 8*a*b^3*c^7*d^10*e

- 3*a*b^9*c*d^4*e^7 + 16*a^2*b*c^8*d^10*e - 3*a^4*b^6*c*d*e^10 + 16*a*b^4*c
^6*d^9*e^2 + 21*a*b^5*c^5*d^8*e^3 - 76*a*b^6*c^4*d^7*e^4 + 62*a*b^7*c^3*d^6
*e^5 - 12*a*b^8*c^2*d^5*e^6 + 2*a^2*b^8*c*d^3*e^8 + 592*a^3*b*c^7*d^8*e^3 +
2*a^3*b^7*c*d^2*e^9 + 1696*a^4*b*c^6*d^6*e^5 + 1696*a^5*b*c^5*d^4*e^7 + 16
*a^5*b^4*c^2*d*e^10 + 592*a^6*b*c^4*d^2*e^9 + 16*a^6*b^2*c^3*d*e^10)/(16*a^
2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*
b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c
^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^
4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6
*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^
4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*
e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*
c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*
e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5)
+ (x*(2*a^4*b^6*c*e^11 - 96*a^7*c^4*e^11 + 32*a^2*c^9*d^10*e + 2*b^4*c^7*d
^10*e + 2*b^10*c*d^4*e^7 - 22*a^5*b^4*c^2*e^11 + 80*a^6*b^2*c^3*e^11 + 32*a
^3*c^8*d^8*e^3 - 192*a^4*c^7*d^6*e^5 - 448*a^5*c^6*d^4*e^7 - 352*a^6*c^5*d^
2*e^9 - 10*b^5*c^6*d^9*e^2 + 22*b^6*c^5*d^8*e^3 - 28*b^7*c^4*d^7*e^4 + 22*b
^8*c^3*d^6*e^5 - 10*b^9*c^2*d^5*e^6 + 336*a^2*b^2*c^7*d^8*e^3 - 384*a^2*b^3
*c^6*d^7*e^4 + 180*a^2*b^4*c^5*d^6*e^5 + 132*a^2*b^5*c^4*d^5*e^6 - 200*a^2*
b^6*c^3*d^4*e^7 + 52*a^2*b^7*c^2*d^3*e^8 + 416*a^3*b^2*c^6*d^6*e^5 - 800*a^
3*b^3*c^5*d^5*e^6 + 580*a^3*b^4*c^4*d^4*e^7 + 24*a^3*b^5*c^3*d^3*e^8 - 116*
a^3*b^6*c^2*d^2*e^9 - 160*a^4*b^2*c^5*d^4*e^7 - 640*a^4*b^3*c^4*d^3*e^8 + 3
30*a^4*b^4*c^3*d^2*e^9 - 144*a^5*b^2*c^4*d^2*e^9 - 16*a*b^2*c^8*d^10*e - 8*
a*b^9*c*d^3*e^8 - 8*a^3*b^7*c*d*e^10 + 352*a^6*b*c^4*d*e^10 + 80*a*b^3*c^7*
d^9*e^2 - 174*a*b^4*c^6*d^8*e^3 + 216*a*b^5*c^5*d^7*e^4 - 156*a*b^6*c^4*d^6
*e^5 + 48*a*b^7*c^3*d^5*e^6 + 10*a*b^8*c^2*d^4*e^7 - 160*a^2*b*c^8*d^9*e^2
+ 12*a^2*b^8*c*d^2*e^9 - 128*a^3*b*c^7*d^7*e^4 + 576*a^4*b*c^6*d^5*e^6 + 86
*a^4*b^5*c^2*d*e^10 + 896*a^5*b*c^5*d^3*e^8 - 304*a^5*b^3*c^3*d*e^10))/(16*
a^2*c^6*d^8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*
a*b^2*c^5*d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5
*c^3*d^7*e - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*
a^4*c^4*d^4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d
^6*e^2 + 32*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*
d^4*e^4 + 32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^
7*e + 4*a*b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*
b*c^2*d*e^7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^
3*e^5 - 192*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^
5)) - (x*(48*a^5*c^4*e^9*g - 72*a^4*b*c^4*e^9*f + 16*a*c^8*d^7*e^2*f + 144*
a^4*c^5*d*e^8*f - 8*a^5*b*c^3*e^9*h - 80*a^5*c^4*d*e^8*h - 4*a^2*b^5*c^2*e^
9*f + 34*a^3*b^3*c^3*e^9*f + 2*a^3*b^4*c^2*e^9*g - 20*a^4*b^2*c^3*e^9*g + 1
76*a^2*c^7*d^5*e^4*f + 304*a^3*c^6*d^3*e^6*f + 2*a^4*b^3*c^2*e^9*h - 80*a^2
*c^7*d^6*e^3*g - 112*a^3*c^6*d^4*e^5*g + 16*a^4*c^5*d^2*e^7*g - 4*b^2*c^7*d
^7*e^2*f + 14*b^3*c^6*d^6*e^3*f - 10*b^4*c^5*d^5*e^4*f - 10*b^5*c^4*d^4*e^5
*f + 14*b^6*c^3*d^3*e^6*f - 4*b^7*c^2*d^2*e^7*f + 48*a^2*c^7*d^7*e^2*h + 16

*a^3*c^6*d^5*e^4*h - 112*a^4*c^5*d^3*e^6*h + 2*b^3*c^6*d^7*e^2*g - 12*b^4*c
^5*d^6*e^3*g + 20*b^5*c^4*d^5*e^4*g - 12*b^6*c^3*d^4*e^5*g + 2*b^7*c^2*d^3*
e^6*g + 2*b^4*c^5*d^7*e^2*h - 2*b^5*c^4*d^6*e^3*h - 2*b^6*c^3*d^5*e^4*h + 2
*b^7*c^2*d^4*e^5*h - 4*a*b^2*c^6*d^5*e^4*f + 150*a*b^3*c^5*d^4*e^5*f - 128*
a*b^4*c^4*d^3*e^6*f + 14*a*b^5*c^3*d^2*e^7*f - 440*a^2*b*c^6*d^4*e^5*f - 62
*a^2*b^4*c^3*d*e^8*f - 456*a^3*b*c^5*d^2*e^7*f + 84*a^3*b^2*c^4*d*e^8*f + 6
8*a*b^2*c^6*d^6*e^3*g - 118*a*b^3*c^5*d^5*e^4*g + 54*a*b^4*c^4*d^4*e^5*g +
6*a*b^5*c^3*d^3*e^6*g - 2*a*b^6*c^2*d^2*e^7*g + 152*a^2*b*c^6*d^5*e^4*g - 2
*a^2*b^5*c^2*d*e^8*g + 72*a^3*b*c^5*d^3*e^6*g + 30*a^3*b^3*c^3*d*e^8*g - 20
*a*b^2*c^6*d^7*e^2*h + 30*a*b^3*c^5*d^6*e^3*h - 4*a*b^4*c^4*d^5*e^4*h + 6*a
*b^5*c^3*d^4*e^5*h - 12*a*b^6*c^2*d^3*e^6*h - 88*a^2*b*c^6*d^6*e^3*h + 72*a
^3*b*c^5*d^4*e^5*h - 12*a^3*b^4*c^2*d*e^8*h + 152*a^4*b*c^4*d^2*e^7*h + 68*
a^4*b^2*c^3*d*e^8*h + 212*a^2*b^2*c^5*d^3*e^6*f + 122*a^2*b^3*c^4*d^2*e^7*f
 + 4*a^2*b^2*c^5*d^4*e^5*g - 74*a^2*b^3*c^4*d^3*e^6*g - 4*a^2*b^4*c^3*d^2*e
^7*g + 44*a^3*b^2*c^4*d^2*e^7*g + 44*a^2*b^2*c^5*d^5*e^4*h - 74*a^2*b^3*c^4
*d^4*e^5*h + 54*a^2*b^4*c^3*d^3*e^6*h + 20*a^2*b^5*c^2*d^2*e^7*h + 4*a^3*b^
2*c^4*d^3*e^6*h - 118*a^3*b^3*c^3*d^2*e^7*h - 56*a*b*c^7*d^6*e^3*f + 8*a*b^
6*c^2*d*e^8*f - 8*a*b*c^7*d^7*e^2*g - 88*a^4*b*c^4*d*e^8*g))/(16*a^2*c^6*d^
8 + a^4*b^4*e^8 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*
d^8 - 8*a^5*b^2*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e
 - 4*b^7*c*d^5*e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^
4*e^4 + 64*a^5*c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 3
2*a^2*b^3*c^3*d^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 +
32*a^3*b^3*c^2*d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*
b^6*c*d^4*e^4 - 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^
7 - 44*a*b^4*c^3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 19
2*a^3*b*c^4*d^5*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5)) - (32*
a^2*c^5*d^3*e^4*g^2 - 4*c^7*d^5*e^2*f^2 - a^2*b^3*c^2*e^7*g^2 - 4*b^5*c^2*e
^7*f^2 - 4*b^2*c^5*d^3*e^4*f^2 - 4*b^3*c^4*d^2*e^5*f^2 + 12*a^2*c^5*d^5*e^2
*h^2 - 40*a^3*c^4*d^3*e^4*h^2 - b^2*c^5*d^5*e^2*g^2 + b^3*c^4*d^4*e^3*g^2 +
 b^4*c^3*d^3*e^4*g^2 - b^5*c^2*d^2*e^5*g^2 + 24*a^3*c^4*e^7*f*g - 8*a^4*c^3
*e^7*g*h + 28*a*b^3*c^3*e^7*f^2 - 48*a^2*b*c^4*e^7*f^2 + 4*a^3*b*c^3*e^7*g^
2 - 8*a*c^6*d^3*e^4*f^2 + 60*a^2*c^5*d*e^6*f^2 + 8*b*c^6*d^4*e^3*f^2 - 32*a
^3*c^4*d*e^6*g^2 + 8*b^4*c^3*d*e^6*f^2 + 12*a^4*c^3*d*e^6*h^2 + 24*a*b*c^5*
d^2*e^5*f^2 - 48*a*b^2*c^4*d*e^6*f^2 + 4*a*b*c^5*d^4*e^3*g^2 - 2*a*b^4*c^2*
d*e^6*g^2 - 22*a^2*b^2*c^3*e^7*f*g - 4*a^2*b^3*c^2*e^7*f*h - 112*a^2*c^5*d^
2*e^5*f*g + 2*a^3*b^2*c^2*e^7*g*h + 80*a^2*c^5*d^3*e^4*f*h - 6*b^2*c^5*d^4*
e^3*f*g + 4*b^3*c^4*d^3*e^4*f*g - 6*b^4*c^3*d^2*e^5*f*g - 40*a^2*c^5*d^4*e^
3*g*h + 80*a^3*c^4*d^2*e^5*g*h - 4*b^2*c^5*d^5*e^2*f*h + 4*b^3*c^4*d^4*e^3*
f*h + 4*b^4*c^3*d^3*e^4*f*h - 4*b^5*c^2*d^2*e^5*f*h + 2*b^3*c^4*d^5*e^2*g*h
 - 4*b^4*c^3*d^4*e^3*g*h + 2*b^5*c^2*d^3*e^4*g*h - 18*a*b^2*c^4*d^3*e^4*g^2
 + 12*a*b^3*c^3*d^2*e^5*g^2 - 24*a^2*b*c^4*d^2*e^5*g^2 + 15*a^2*b^2*c^3*d*e
^6*g^2 - 4*a*b^2*c^4*d^5*e^2*h^2 + 4*a*b^3*c^3*d^4*e^3*h^2 - 4*a*b^4*c^2*d^
3*e^4*h^2 - 8*a^2*b*c^4*d^4*e^3*h^2 - 8*a^3*b*c^3*d^2*e^5*h^2 - 4*a^3*b^2*c
^2*d*e^6*h^2 + 4*a*b^4*c^2*e^7*f*g + 16*a^3*b*c^3*e^7*f*h - 8*a*c^6*d^4*e^3

*f*g + 8*a*c^6*d^5*e^2*f*h + 4*b*c^6*d^5*e^2*f*g - 56*a^3*c^4*d*e^6*f*h + 4
*b^5*c^2*d*e^6*f*g + 20*a^2*b^2*c^3*d^3*e^4*h^2 + 4*a^2*b^3*c^2*d^2*e^5*h^2
 + 8*a*b*c^5*d^3*e^4*f*g - 40*a*b^3*c^3*d*e^6*f*g + 100*a^2*b*c^4*d*e^6*f*g
 - 4*a*b*c^5*d^5*e^2*g*h - 4*a^3*b*c^3*d*e^6*g*h + 44*a*b^2*c^4*d^2*e^5*f*g
 - 48*a*b^2*c^4*d^3*e^4*f*h + 32*a*b^3*c^3*d^2*e^5*f*h - 48*a^2*b*c^4*d^2*e
^5*f*h + 12*a^2*b^2*c^3*d*e^6*f*h + 18*a*b^2*c^4*d^4*e^3*g*h - 8*a*b^3*c^3*
d^3*e^4*g*h + 2*a*b^4*c^2*d^2*e^5*g*h + 24*a^2*b*c^4*d^3*e^4*g*h + 2*a^2*b^
3*c^2*d*e^6*g*h - 36*a^2*b^2*c^3*d^2*e^5*g*h)/(16*a^2*c^6*d^8 + a^4*b^4*e^8
 + 16*a^6*c^2*e^8 + b^4*c^4*d^8 + b^8*d^4*e^4 - 8*a*b^2*c^5*d^8 - 8*a^5*b^2
*c*e^8 - 4*a*b^7*d^3*e^5 - 4*a^3*b^5*d*e^7 - 4*b^5*c^3*d^7*e - 4*b^7*c*d^5*
e^3 + 6*a^2*b^6*d^2*e^6 + 64*a^3*c^5*d^6*e^2 + 96*a^4*c^4*d^4*e^4 + 64*a^5*
c^3*d^2*e^6 + 6*b^6*c^2*d^6*e^2 + 64*a^2*b^2*c^4*d^6*e^2 + 32*a^2*b^3*c^3*d
^5*e^3 - 74*a^2*b^4*c^2*d^4*e^4 + 144*a^3*b^2*c^3*d^4*e^4 + 32*a^3*b^3*c^2*
d^3*e^5 + 64*a^4*b^2*c^2*d^2*e^6 + 32*a*b^3*c^4*d^7*e + 4*a*b^6*c*d^4*e^4 -
 64*a^2*b*c^5*d^7*e + 32*a^4*b^3*c*d*e^7 - 64*a^5*b*c^2*d*e^7 - 44*a*b^4*c^
3*d^6*e^2 + 20*a*b^5*c^2*d^5*e^3 + 20*a^2*b^5*c*d^3*e^5 - 192*a^3*b*c^4*d^5
*e^3 - 44*a^3*b^4*c*d^2*e^6 - 192*a^4*b*c^3*d^3*e^5))*root(3840*a^6*b*c^5*d
^5*e^7*z^3 + 3840*a^5*b*c^6*d^7*e^5*z^3 + 1920*a^7*b*c^4*d^3*e^9*z^3 + 1920
*a^4*b*c^7*d^9*e^3*z^3 - 288*a^7*b^3*c^2*d*e^11*z^3 - 288*a^2*b^3*c^7*d^11*
e*z^3 + 210*a^4*b^7*c*d^3*e^9*z^3 + 210*a*b^7*c^4*d^9*e^3*z^3 - 174*a^5*b^6
*c*d^2*e^10*z^3 - 174*a*b^6*c^5*d^10*e^2*z^3 - 120*a^3*b^8*c*d^4*e^8*z^3 -
120*a*b^8*c^3*d^8*e^4*z^3 + 12*a^2*b^9*c*d^5*e^7*z^3 + 12*a*b^9*c^2*d^7*e^5
*z^3 + 384*a^8*b*c^3*d*e^11*z^3 + 384*a^3*b*c^8*d^11*e*z^3 + 72*a^6*b^5*c*d
*e^11*z^3 + 72*a*b^5*c^6*d^11*e*z^3 + 18*a*b^10*c*d^6*e^6*z^3 - 4800*a^5*b^
2*c^5*d^6*e^6*z^3 - 3120*a^6*b^2*c^4*d^4*e^8*z^3 - 3120*a^4*b^2*c^6*d^8*e^4
*z^3 + 2160*a^4*b^4*c^4*d^6*e^6*z^3 - 1776*a^4*b^5*c^3*d^5*e^7*z^3 - 1776*a
^3*b^5*c^4*d^7*e^5*z^3 + 1740*a^5*b^4*c^3*d^4*e^8*z^3 + 1740*a^3*b^4*c^5*d^
8*e^4*z^3 + 960*a^5*b^3*c^4*d^5*e^7*z^3 + 960*a^4*b^3*c^5*d^7*e^5*z^3 - 672
*a^7*b^2*c^3*d^2*e^10*z^3 - 672*a^3*b^2*c^7*d^10*e^2*z^3 + 648*a^6*b^4*c^2*
d^2*e^10*z^3 + 648*a^2*b^4*c^6*d^10*e^2*z^3 - 600*a^5*b^5*c^2*d^3*e^9*z^3 -
 600*a^2*b^5*c^5*d^9*e^3*z^3 + 372*a^3*b^7*c^2*d^5*e^7*z^3 + 372*a^2*b^7*c^
3*d^7*e^5*z^3 + 316*a^3*b^6*c^3*d^6*e^6*z^3 - 222*a^2*b^8*c^2*d^6*e^6*z^3 -
 160*a^6*b^3*c^3*d^3*e^9*z^3 - 160*a^3*b^3*c^6*d^9*e^3*z^3 + 15*a^4*b^6*c^2
*d^4*e^8*z^3 + 15*a^2*b^6*c^4*d^8*e^4*z^3 - 6*b^11*c*d^7*e^5*z^3 - 6*b^7*c^
5*d^11*e*z^3 - 6*a^5*b^7*d*e^11*z^3 - 6*a*b^11*d^5*e^7*z^3 - 12*a^7*b^4*c*e
^12*z^3 - 12*a*b^4*c^7*d^12*z^3 - 20*b^9*c^3*d^9*e^3*z^3 + 15*b^10*c^2*d^8*
e^4*z^3 + 15*b^8*c^4*d^10*e^2*z^3 - 1280*a^6*c^6*d^6*e^6*z^3 - 960*a^7*c^5*
d^4*e^8*z^3 - 960*a^5*c^7*d^8*e^4*z^3 - 384*a^8*c^4*d^2*e^10*z^3 - 384*a^4*
c^8*d^10*e^2*z^3 - 20*a^3*b^9*d^3*e^9*z^3 + 15*a^4*b^8*d^2*e^10*z^3 + 15*a^
2*b^10*d^4*e^8*z^3 + 48*a^8*b^2*c^2*e^12*z^3 + 48*a^2*b^2*c^8*d^12*z^3 - 64
*a^9*c^3*e^12*z^3 - 64*a^3*c^9*d^12*z^3 + b^12*d^6*e^6*z^3 + b^6*c^6*d^12*z
^3 + a^6*b^6*e^12*z^3 - 44*a^3*b^4*c*d*e^7*g*h*z - 20*a*b^6*c*d^3*e^5*g*h*z
 - 12*a*b^2*c^5*d^7*e*g*h*z + 432*a^4*b*c^3*d*e^7*f*h*z + 84*a^2*b^5*c*d*e^
7*f*h*z + 28*a*b^6*c*d^2*e^6*f*h*z - 8*a*b*c^6*d^6*e^2*f*g*z - 804*a^3*b^2*
c^3*d^3*e^5*g*h*z + 564*a^2*b^2*c^4*d^5*e^3*g*h*z + 222*a^3*b^3*c^2*d^2*e^6

*g*h*z + 186*a^2*b^4*c^2*d^3*e^5*g*h*z - 166*a^2*b^3*c^3*d^4*e^4*g*h*z + 79
2*a^3*b^2*c^3*d^2*e^6*f*h*z - 744*a^2*b^2*c^4*d^4*e^4*f*h*z + 492*a^2*b^3*c
^3*d^3*e^5*f*h*z - 264*a^2*b^4*c^2*d^2*e^6*f*h*z + 996*a^2*b^2*c^4*d^3*e^5*
f*g*z - 870*a^2*b^3*c^3*d^2*e^6*f*g*z + 16*a*b*c^6*d^7*e*f*h*z - 56*a*b^6*c
*d*e^7*f*g*z - 264*a^4*b*c^3*d^2*e^6*g*h*z + 208*a^3*b*c^4*d^4*e^4*g*h*z +
156*a^4*b^2*c^2*d*e^7*g*h*z - 148*a*b^4*c^3*d^5*e^3*g*h*z + 54*a*b^5*c^2*d^
4*e^4*g*h*z - 48*a^2*b^5*c*d^2*e^6*g*h*z - 24*a^2*b*c^5*d^6*e^2*g*h*z + 10*
a*b^3*c^4*d^6*e^2*g*h*z - 656*a^3*b*c^4*d^3*e^5*f*h*z - 308*a^3*b^3*c^2*d*e
^7*f*h*z + 116*a*b^4*c^3*d^4*e^4*f*h*z - 84*a*b^5*c^2*d^3*e^5*f*h*z + 68*a*
b^3*c^4*d^5*e^3*f*h*z - 48*a^2*b*c^5*d^5*e^3*f*h*z - 24*a*b^2*c^5*d^6*e^2*f
*h*z + 1320*a^3*b*c^4*d^2*e^6*f*g*z - 732*a^3*b^2*c^3*d*e^7*f*g*z + 306*a^2
*b^4*c^2*d*e^7*f*g*z - 304*a*b^4*c^3*d^3*e^5*f*g*z + 222*a*b^5*c^2*d^2*e^6*
f*g*z + 110*a*b^3*c^4*d^4*e^4*f*g*z - 84*a*b^2*c^5*d^5*e^3*f*g*z + 16*a*c^7
*d^7*e*f*g*z - 8*a*b^7*d*e^7*f*h*z + 4*a*b*c^6*d^8*g*h*z + 6*b^6*c^2*d^5*e^
3*g*h*z + 6*b^5*c^3*d^6*e^2*g*h*z + 1072*a^4*c^4*d^3*e^5*g*h*z - 720*a^3*c^
5*d^5*e^3*g*h*z - 8*b^6*c^2*d^4*e^4*f*h*z - 8*b^4*c^4*d^6*e^2*f*h*z + 1072*
a^3*c^5*d^4*e^4*f*h*z - 960*a^4*c^4*d^2*e^6*f*h*z + 30*b^6*c^2*d^3*e^5*f*g*
z + 30*b^3*c^5*d^6*e^2*f*g*z - 10*b^5*c^3*d^4*e^4*f*g*z - 10*b^4*c^4*d^5*e^
3*f*g*z - 1488*a^3*c^5*d^3*e^5*f*g*z + 48*a^2*c^6*d^5*e^3*f*g*z - 24*a^4*b^
2*c^2*e^8*f*h*z + 186*a^3*b^3*c^2*e^8*f*g*z + 4*a^4*b^3*c*d*e^7*h^2*z + 4*a
*b^6*c*d^4*e^4*h^2*z + 4*a*b^3*c^4*d^7*e*h^2*z + 168*a^4*b*c^3*d*e^7*g^2*z
+ 24*a^2*b^5*c*d*e^7*g^2*z + 18*a*b^6*c*d^2*e^6*g^2*z - 912*a^3*b*c^4*d*e^7
*f^2*z - 192*a*b^5*c^2*d*e^7*f^2*z + 144*a*b*c^6*d^5*e^3*f^2*z + 432*a^3*b^
2*c^3*d^4*e^4*h^2*z - 168*a^4*b^2*c^2*d^2*e^6*h^2*z - 168*a^2*b^2*c^4*d^6*e
^2*h^2*z - 108*a^2*b^4*c^2*d^4*e^4*h^2*z - 20*a^3*b^3*c^2*d^3*e^5*h^2*z - 2
0*a^2*b^3*c^3*d^5*e^3*h^2*z - 426*a^2*b^2*c^4*d^4*e^4*g^2*z + 336*a^3*b^2*c
^3*d^2*e^6*g^2*z + 274*a^2*b^3*c^3*d^3*e^5*g^2*z - 120*a^2*b^4*c^2*d^2*e^6*
g^2*z - 864*a^2*b^2*c^4*d^2*e^6*f^2*z - 2*b^7*c*d^4*e^4*g*h*z - 2*b^4*c^4*d
^7*e*g*h*z - 240*a^5*c^3*d*e^7*g*h*z + 16*a^2*c^6*d^7*e*g*h*z + 4*b^7*c*d^3
*e^5*f*h*z + 4*b^3*c^5*d^7*e*f*h*z - 20*b^7*c*d^2*e^6*f*g*z - 20*b^2*c^6*d^
7*e*f*g*z + 4*a^2*b^6*d*e^7*g*h*z + 4*a*b^7*d^2*e^6*g*h*z + 528*a^4*c^4*d*e
^7*f*g*z + 12*a^5*b*c^2*e^8*g*h*z - 2*a^4*b^3*c*e^8*g*h*z + 4*a^3*b^4*c*e^8
*f*h*z - 228*a^4*b*c^3*e^8*f*g*z - 48*a^2*b^5*c*e^8*f*g*z - 8*a*b*c^6*d^7*e
*g^2*z + 36*a^3*b^4*c*d^2*e^6*h^2*z + 36*a*b^4*c^3*d^6*e^2*h^2*z + 12*a^2*b
^5*c*d^3*e^5*h^2*z + 12*a*b^5*c^2*d^5*e^3*h^2*z - 312*a^3*b*c^4*d^3*e^5*g^2
*z + 104*a*b^4*c^3*d^4*e^4*g^2*z - 102*a^3*b^3*c^2*d*e^7*g^2*z - 66*a*b^5*c
^2*d^3*e^5*g^2*z + 24*a^2*b*c^5*d^5*e^3*g^2*z + 24*a*b^2*c^5*d^6*e^2*g^2*z
- 18*a*b^3*c^4*d^5*e^3*g^2*z + 744*a^2*b^3*c^3*d*e^7*f^2*z + 240*a^2*b*c^5*
d^3*e^5*f^2*z + 216*a*b^4*c^3*d^2*e^6*f^2*z - 120*a*b^2*c^5*d^4*e^4*f^2*z +
24*a^5*c^3*e^8*f*h*z + 16*b^7*c*d*e^7*f^2*z + 16*b*c^7*d^7*e*f^2*z - 2*a*b
^7*d*e^7*g^2*z + 48*a*b^6*c*e^8*f^2*z - 4*b^6*c^2*d^6*e^2*h^2*z - 536*a^4*c
^4*d^4*e^4*h^2*z + 240*a^5*c^3*d^2*e^6*h^2*z + 240*a^3*c^5*d^6*e^2*h^2*z -
12*b^6*c^2*d^4*e^4*g^2*z - 12*b^4*c^4*d^6*e^2*g^2*z + 10*b^5*c^3*d^5*e^3*g^
2*z + 528*a^3*c^5*d^4*e^4*g^2*z - 432*a^4*c^4*d^2*e^6*g^2*z + 20*b^4*c^4*d^
4*e^4*f^2*z - 16*b^6*c^2*d^2*e^6*f^2*z - 16*b^2*c^6*d^6*e^2*f^2*z - 16*a^2*

```
c^6*d^6*e^2*g^2*z - 8*b^5*c^3*d^3*e^5*f^2*z - 8*b^3*c^5*d^5*e^3*f^2*z - 4*a
^2*b^6*d^2*e^6*h^2*z + 912*a^3*c^5*d^2*e^6*f^2*z - 120*a^2*c^6*d^4*e^4*f^2*
z - 45*a^4*b^2*c^2*e^8*g^2*z + 264*a^3*b^2*c^3*e^8*f^2*z - 192*a^2*b^4*c^2*
e^8*f^2*z + 4*b^8*d*e^7*f*g*z - 8*a*c^7*d^8*f*h*z + 4*b*c^7*d^8*f*g*z + 4*a
*b^7*e^8*f*g*z + 6*b^7*c*d^3*e^5*g^2*z + 6*b^3*c^5*d^7*e*g^2*z - 48*a*c^7*d
^6*e^2*f^2*z + 12*a^3*b^4*c*e^8*g^2*z - b^8*d^2*e^6*g^2*z - 4*a^6*c^2*e^8*h
^2*z + 48*a^5*c^3*e^8*g^2*z - 4*a^2*c^6*d^8*h^2*z - b^2*c^6*d^8*g^2*z - 36*
a^4*c^4*e^8*f^2*z - a^2*b^6*e^8*g^2*z - 4*c^8*d^8*f^2*z - 4*b^8*e^8*f^2*z -
 80*a*b*c^4*d^3*e^3*f*g*h + 24*a^2*b*c^3*d*e^5*f*g*h + 16*a*b^3*c^2*d*e^5*f
*g*h - 72*a*b^2*c^3*d^2*e^4*f*g*h - 48*a^2*b*c^3*d^3*e^3*g*h^2 + 16*a*b^3*c
^2*d^3*e^3*g*h^2 - 12*a*b^2*c^3*d^3*e^3*g^2*h - 6*a^2*b^2*c^2*d*e^5*g^2*h -
 72*a^2*b^2*c^2*d*e^5*f*h^2 + 48*a*b^2*c^3*d^3*e^3*f*h^2 + 24*a^2*b*c^3*d^2
*e^4*f*h^2 - 8*a*b^3*c^2*d^2*e^4*f*h^2 - 8*b^5*c*d*e^5*f*g*h - 8*b*c^5*d^5*
e*f*g*h - 8*a*b^4*c*e^6*f*g*h + 24*b^3*c^3*d^3*e^3*f*g*h + 16*b^4*c^2*d^2*e
^4*f*g*h + 16*b^2*c^4*d^4*e^2*f*g*h + 48*a^2*c^4*d^2*e^4*f*g*h + 48*a^2*b^2
*c^2*e^6*f*g*h + 40*a^3*b*c^2*d*e^5*g*h^2 + 28*a*b*c^4*d^4*e^2*g^2*h - 8*a^
2*b^3*c*d*e^5*g*h^2 - 8*a*b^4*c*d^2*e^4*g*h^2 + 96*a*b^2*c^3*d*e^5*f^2*h +
24*a*b*c^4*d^2*e^4*f^2*h + 16*a*b*c^4*d^4*e^2*f*h^2 + 96*a*b*c^4*d^2*e^4*f*
g^2 - 48*a*b^2*c^3*d*e^5*f*g^2 + 12*a^2*b^2*c^2*d^2*e^4*g*h^2 - 56*a*c^5*d^
4*e^2*f*g*h - 8*a*b*c^4*d^5*e*g*h^2 + 4*a*b^4*c*d*e^5*g^2*h + 16*a*b^4*c*d*
e^5*f*h^2 - 48*a*b*c^4*d*e^5*f^2*g - 24*a^3*c^3*e^6*f*g*h + 16*a*c^5*d^5*e*
f*h^2 - 6*b^4*c^2*d^3*e^3*g^2*h - 6*b^3*c^3*d^4*e^2*g^2*h + 4*b^4*c^2*d^4*e
^2*g*h^2 + 80*a^2*c^4*d^3*e^3*g^2*h - 44*a^2*c^4*d^4*e^2*g*h^2 + 24*a^3*c^3
*d^2*e^4*g*h^2 - 16*b^3*c^3*d^2*e^4*f^2*h - 16*b^2*c^4*d^3*e^3*f^2*h - 8*b^
4*c^2*d^3*e^3*f*h^2 - 8*b^3*c^3*d^4*e^2*f*h^2 + 60*b^2*c^4*d^2*e^4*f^2*g -
48*a^2*c^4*d^3*e^3*f*h^2 - 24*b^3*c^3*d^2*e^4*f*g^2 - 24*b^2*c^4*d^3*e^3*f*
g^2 - 24*a^3*b*c^2*d^2*e^4*h^3 + 24*a^2*b*c^3*d^4*e^2*h^3 + 8*a^2*b^3*c*d^2
*e^4*h^3 - 8*a*b^3*c^2*d^4*e^2*h^3 + 18*a*b^2*c^3*d^2*e^4*g^3 + 2*b^5*c*d^2
*e^4*g^2*h + 2*b^2*c^4*d^5*e*g^2*h - 48*a^3*c^3*d*e^5*g^2*h - 8*b^4*c^2*d*e
^5*f^2*h - 8*b*c^5*d^4*e^2*f^2*h - 168*a^2*c^4*d*e^5*f^2*h + 96*a*c^5*d^3*e
^3*f^2*h + 64*a^3*c^3*d*e^5*f*h^2 + 12*b^4*c^2*d*e^5*f*g^2 + 12*b*c^5*d^4*e
^2*f*g^2 - 168*a*c^5*d^2*e^4*f^2*g + 48*a^2*c^4*d*e^5*f*g^2 + 48*a*c^5*d^3*
e^3*f*g^2 - 12*a^3*b*c^2*e^6*g^2*h + 2*a^2*b^3*c*e^6*g^2*h + 48*a^2*b*c^3*e
^6*f^2*h - 48*a*b^3*c^2*e^6*f^2*h - 8*a^3*b*c^2*e^6*f*h^2 - 60*a^2*b*c^3*e^
6*f*g^2 + 48*a*b^2*c^3*e^6*f^2*g + 12*a*b^3*c^2*e^6*f*g^2 + 24*a^2*b*c^3*d*
e^5*g^3 - 24*a*b*c^4*d^3*e^3*g^3 - 6*a*b^3*c^2*d*e^5*g^3 - 12*c^6*d^4*e^2*f
^2*g + 4*a^4*c^2*e^6*g*h^2 - 12*b^4*c^2*e^6*f^2*g + 36*a^2*c^4*e^6*f^2*g -
8*a^4*c^2*d*e^5*h^3 + 8*a^2*c^4*d^5*e*h^3 - 24*b^2*c^4*d*e^5*f^3 - 24*b*c^5
*d^2*e^4*f^3 + 8*c^6*d^5*e*f^2*h + 8*b^5*c*e^6*f^2*h + 144*a*c^5*d*e^5*f^3
- 72*a*b*c^4*e^6*f^3 + 10*b^3*c^3*d^3*e^3*g^3 - 3*b^4*c^2*d^2*e^4*g^3 - 3*b
^2*c^4*d^4*e^2*g^3 - 48*a^2*c^4*d^2*e^4*g^3 - 3*a^2*b^2*c^2*e^6*g^3 + 16*c^
6*d^3*e^3*f^3 + 16*b^3*c^3*e^6*f^3 + 16*a^3*c^3*e^6*g^3, z, k), k, 1, 3)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

```
[Out] Timed out
```

# 3.160 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx$

**Optimal**. Leaf size=62

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log\left(x^2-x+1\right)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] 3*x+1/2*x^2+2/3*(2-x)/(x^2-x+1)+2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2*x)*3^(1/
2))*3^(1/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{x^2}{2}+\frac{2(2-x)}{3(x^2-x+1)}+2\log\left(x^2-x+1\right)+3x+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{-2+6x+6x^2+3x^3}{1-x+x^2}\,dx\\ &= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(9+3x-\frac{11-12x}{1-x+x^2}\right)dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{11-12x}{1-x+x^2}\,dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx+2\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+2\log\left(1-x+x^2\right)+\frac{10}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= 3x+\frac{x^2}{2}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.97

$$\frac{x^2}{2}-\frac{2(x-2)}{3\left(x^2-x+1\right)}+2\log\left(x^2-x+1\right)+3x-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] 3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

**fricas [A]** time = 1.36, size = 75, normalized size = 1.21

$$\frac{9\,x^4+45\,x^3-20\,\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-45\,x^2+36\left(x^2-x+1\right)\log\left(x^2-x+1\right)+42\,x+24}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1
)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1)
```

**giac** [A] time = 0.16, size = 51, normalized size = 0.82

$$\frac{1}{2}\,x^{2} - \frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,\left(x^{2}-x+1\right)} + 2\,\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

**maple** [A] time = 0.01, size = 53, normalized size = 0.85

$$\frac{x^{2}}{2} + 3x - \frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9} + 2\ln\left(x^{2}-x+1\right) + \frac{-\frac{2x}{3}+\frac{4}{3}}{x^{2}-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] 1/2*x^2+3*x+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2
*x-1)*3^(1/2))
```

**maxima** [A] time = 0.96, size = 51, normalized size = 0.82

$$\frac{1}{2}\,x^{2} - \frac{10}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,\left(x^{2}-x+1\right)} + 2\,\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(x
^2 - x + 1) + 2*log(x^2 - x + 1)
```

**mupad** [B] time = 0.04, size = 55, normalized size = 0.89

$$3\,x + 2\,\ln\left(x^{2}-x+1\right) - \frac{\frac{2\,x}{3}-\frac{4}{3}}{x^{2}-x+1} - \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] 3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*atan
((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

**sympy [A]** time = 0.16, size = 60, normalized size = 0.97

$$\frac{x^2}{2} + 3x + \frac{4-2x}{3x^2-3x+3} + 2\log\left(x^2-x+1\right) - \frac{10\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqrt
(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

# 3.161 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} \, dx$

**Optimal**. Leaf size=55

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log(x^2-x+1)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x+2/3*(1-2*x)/(x^2-x+1)+3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(
1/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{2(1-2x)}{3(x^2-x+1)}+\frac{3}{2}\log(x^2-x+1)+x-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x + (2*(1 - 2*x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + (3*Log[1 - x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{2+6x+3x^2}{1-x+x^2}\,dx\\
&= \frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(3-\frac{1-9x}{1-x+x^2}\right)dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{1}{3}\int \frac{1-9x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx+\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}+\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= x+\frac{2(1-2x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{3}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 55, normalized size = 1.00

$$-\frac{2(2x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\log\left(x^2-x+1\right)+x+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sq
rt[3]) + (3*Log[1 - x + x^2])/2
```

**fricas** [A] time = 0.81, size = 70, normalized size = 1.27

$$\frac{18\,x^3+14\,\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-18\,x^2+27\left(x^2-x+1\right)\log\left(x^2-x+1\right)-6\,x+12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18*
x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

**giac** [A] time = 0.16, size = 46, normalized size = 0.84

$$\frac{7}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\left(x^2-x+1\right)}+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

**maple** [A] time = 0.01, size = 46, normalized size = 0.84

$$x+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

**maxima [A]** time = 0.95, size = 46, normalized size = 0.84

$$\frac{7}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\,\left(x^2-x+1\right)}+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1)
 + 3/2*log(x^2 - x + 1)
```

**mupad [B]** time = 0.04, size = 48, normalized size = 0.87

$$x+\frac{3\,\ln\left(x^2-x+1\right)}{2}-\frac{\frac{4\,x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{7\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

**sympy [A]** time = 0.15, size = 54, normalized size = 0.98

$$x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(2
*sqrt(3)*x/3 - sqrt(3)/3)/9
```

# 3.162 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx$

**Optimal**. Leaf size=52

$$-\frac{2(x+1)}{3(x^2-x+1)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(1+x)/(x^2-x+1)+1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/
2)
```

**Rubi [A]** time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1660, 634, 618, 204, 628}

$$-\frac{2(x+1)}{3(x^2-x+1)}+\frac{1}{2}\log\left(x^2-x+1\right)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
 + Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(1+x+x^2\right)}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{4+3x}{1-x+x^2}\,dx \\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{11}{6}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{2}\log\left(1-x+x^2\right)-\frac{11}{3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\log\left(1-x+x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 52, normalized size = 1.00

$$-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)+\frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
) + Log[1 - x + x^2]/2
```

**fricas [A]** time = 1.35, size = 60, normalized size = 1.15

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+9\left(x^2-x+1\right)\log\left(x^2-x+1\right)-12x-12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x +
 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

**giac [A]** time = 0.15, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

**maple [A]** time = 0.00, size = 45, normalized size = 0.87

$$\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{\ln\left(x^2-x+1\right)}{2}+\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

**maxima [A]** time = 0.96, size = 43, normalized size = 0.83

$$\frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}+\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1/
2*log(x^2 - x + 1)
```

**mupad** [B] time = 3.84, size = 59, normalized size = 1.13

$$\frac{\ln\left(x^2-x+1\right)}{2}-\frac{2\,x}{3\,\left(x^2-x+1\right)}-\frac{2}{3\,\left(x^2-x+1\right)}+\frac{11\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

```
[Out] log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3^
(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

**sympy** [A] time = 0.15, size = 53, normalized size = 1.02

$$\frac{-2x-2}{3x^2-3x+3}+\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sqr
t(3)*x/3 - sqrt(3)/3)/9
```

## 3.163 $\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx$

**Optimal**. Leaf size=41

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(2-x)/(x^2-x+1)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

**Rubi [A]** time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1660, 12, 618, 204}

$$-\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (-2*(2 - x))/(3*(1 - x + x^2)) - (10*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx &= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{5}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right) \\
&= -\frac{2(2-x)}{3\left(1-x+x^2\right)}-\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 39, normalized size = 0.95

$$\frac{2(x-2)}{3\left(x^2-x+1\right)}+\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

```
[Out] (2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]
)
```

**fricas [A]** time = 1.27, size = 41, normalized size = 1.00

$$\frac{2\left(5\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
- x + 1)
```

**giac [A]** time = 0.16, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(x-2)}{3\,\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

**maple [A]** time = 0.00, size = 34, normalized size = 0.83

$$\frac{10\sqrt{3}\;\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/(x^2-x+1)^2,x)
```

```
[Out] (2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

**maxima [A]** time = 0.95, size = 32, normalized size = 0.78

$$\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(x-2)}{3\,\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

**mupad [B]** time = 3.83, size = 35, normalized size = 0.85

$$\frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
```

```
[Out] ((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/
3))/9
```

**sympy [A]** time = 0.14, size = 41, normalized size = 1.00

$$\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2-x+1)**2,x)
```

```
[Out] (2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

# 3.164 $\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2} \, dx$

**Optimal**. Leaf size=56

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/3*(1-2*x)/(x^2-x+1)+ln(x)-1/2*ln(x^2-x+1)-11/9*arctan(1/3*(1-2*x)*3^(1/2
))*3^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{2(1-2x)}{3\left(x^2-x+1\right)}-\frac{1}{2}\log\left(x^2-x+1\right)+\log(x)-\frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (-2*(1 - 2*x))/(3*(1 - x + x^2)) - (11*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[x] - Log[1 - x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \frac{1}{3}\int \frac{3+4x}{x\left(1-x+x^2\right)}\,dx$$
$$= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \frac{1}{3}\int \left(\frac{3}{x} + \frac{7-3x}{1-x+x^2}\right)dx$$
$$= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) + \frac{1}{3}\int \frac{7-3x}{1-x+x^2}\,dx$$
$$= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) - \frac{1}{2}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{11}{6}\int \frac{1}{1-x+x^2}\,dx$$
$$= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right) - \frac{11}{3}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)$$
$$= -\frac{2(1-2x)}{3\left(1-x+x^2\right)} - \frac{11\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)$$

**Mathematica [A]** time = 0.03, size = 56, normalized size = 1.00

$$\frac{2(2x-1)}{3\left(x^2-x+1\right)} - \frac{1}{2}\log\left(x^2-x+1\right) + \log(x) + \frac{11\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

```
[Out] (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[
3]) + Log[x] - Log[1 - x + x^2]/2
```

**fricas [A]** time = 1.09, size = 72, normalized size = 1.29

$$\frac{22\sqrt{3}\left(x^2-x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right) - 9\left(x^2-x+1\right)\log\left(x^2-x+1\right) + 18\left(x^2-x+1\right)\log(x) + 24x - 12}{18\left(x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x +
 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

**giac** [A] time = 0.15, size = 48, normalized size = 0.86

$$\frac{11}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(2\,x-1)}{3\,\left(x^2-x+1\right)}-\frac{1}{2}\,\log\left(x^2-x+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(abs(x))
```

**maple** [A] time = 0.01, size = 48, normalized size = 0.86

$$\frac{11\sqrt{3}\ \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\ln(x)-\frac{\ln\left(x^2-x+1\right)}{2}-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x/(x^2-x+1)^2,x)
```

```
[Out] -(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))+ln(x)
```

**maxima** [A] time = 0.95, size = 47, normalized size = 0.84

$$\frac{11}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{2\,(2\,x-1)}{3\,\left(x^2-x+1\right)}-\frac{1}{2}\,\log\left(x^2-x+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
1/2*log(x^2 - x + 1) + log(x)
```

**mupad** [B] time = 0.10, size = 58, normalized size = 1.04

$$\ln(x)+\frac{\frac{4\,x}{3}-\frac{2}{3}}{x^2-x+1}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

```
[Out] log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 - 1/
2)
```

**sympy [A]** time = 0.18, size = 54, normalized size = 0.96

$$\frac{4x-2}{3x^2-3x+3}+\log(x)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

```
[Out] (4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*at
an(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

# 3.165 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2} \, dx$

**Optimal**. Leaf size=61

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/x+2/3*(1+x)/(x^2-x+1)+3*ln(x)-3/2*ln(x^2-x+1)-7/9*arctan(1/3*(1-2*x)*3^(
1/2))*3^(1/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/(3*
Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+2x^2}{x^2\left(1-x+x^2\right)}\,dx$$
$$= \frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^2}+\frac{9}{x}+\frac{8-9x}{1-x+x^2}\right)dx$$
$$= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{1}{3}\int \frac{8-9x}{1-x+x^2}\,dx$$
$$= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)+\frac{7}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx$$
$$= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)-\frac{7}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+\right.$$
$$= -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

**Mathematica [A]** time = 0.02, size = 61, normalized size = 1.00

$$\frac{2(x+1)}{3\left(x^2-x+1\right)}-\frac{3}{2}\log\left(x^2-x+1\right)-\frac{1}{x}+3\log(x)+\frac{7\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

```
[Out] -x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3
*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

**fricas [A]** time = 1.20, size = 85, normalized size = 1.39

$$\frac{14\sqrt{3}\left(x^3-x^2+x\right)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-6x^2-27\left(x^3-x^2+x\right)\log\left(x^2-x+1\right)+54\left(x^3-x^2+x\right)\log(x)}{18\left(x^3-x^2+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 27
*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18)/
(x^3 - x^2 + x)
```

**giac** [A] time = 0.15, size = 55, normalized size = 0.90

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

**maple** [A] time = 0.01, size = 55, normalized size = 0.90

$$\frac{7\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+3\ln(x)-\frac{3\ln\left(x^2-x+1\right)}{2}-\frac{1}{x}-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^2/(x^2-x+1)^2,x)
```

```
[Out] -(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))-1/x+3*ln(x)
```

**maxima** [A] time = 0.96, size = 54, normalized size = 0.89

$$\frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{x^2-5x+3}{3\left(x^3-x^2+x\right)}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] 7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
+ x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

**mupad** [B] time = 4.13, size = 68, normalized size = 1.11

$$3\ln(x)-\frac{\frac{x^2}{3}-\frac{5x}{3}+1}{x^3-x^2+x}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

```
[Out] 3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7i)
/18 - 3/2)
```

**sympy [A]** time = 0.20, size = 65, normalized size = 1.07

$$\frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log(x)-\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

```
[Out] (-x**2 + 5*x - 3)/(3*x**3 - 3*x**2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)/
2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

## 3.166 $\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx$

**Optimal**. Leaf size=68

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/2/x^2-3/x+2/3*(2-x)/(x^2-x+1)+4*ln(x)-2*ln(x^2-x+1)+10/9*arctan(1/3*(1-2
*x)*3^(1/2))*3^(1/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{2(2-x)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/(2*x^2) - 3/x + (2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(1 - 2*x)/Sqr
t[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \frac{3+6x+6x^2-2x^3}{x^3\left(1-x+x^2\right)}\,dx$$

$$= \frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{1}{3}\int \left(\frac{3}{x^3}+\frac{9}{x^2}+\frac{12}{x}+\frac{1-12x}{1-x+x^2}\right)dx$$

$$= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)+\frac{1}{3}\int \frac{1-12x}{1-x+x^2}\,dx$$

$$= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-\frac{5}{3}\int \frac{1}{1-x+x^2}\,dx-2\int \frac{-1+2x}{1-x+x^2}\,dx$$

$$= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+4\log(x)-2\log\left(1-x+x^2\right)+\frac{10}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.$$

$$= -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3\left(1-x+x^2\right)}+\frac{10\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+4\log(x)-2\log\left(1-x+x^2\right)$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.97

$$-\frac{2(x-2)}{3\left(x^2-x+1\right)}-\frac{1}{2x^2}-2\log\left(x^2-x+1\right)-\frac{3}{x}+4\log(x)-\frac{10\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

```
[Out] -1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/S
qrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

**fricas [A]** time = 0.92, size = 98, normalized size = 1.44

$$-\frac{66\,x^3+20\,\sqrt{3}\left(x^4-x^3+x^2\right)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-69\,x^2+36\left(x^4-x^3+x^2\right)\log\left(x^2-x+1\right)-72\left(x^4-x^3+\right.}{18\left(x^4-x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
- 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*log
(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

**giac** [A] time = 0.16, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{22x^3-23x^2+15x+3}{6(x^2-x+1)x^2}-2\log\left(x^2-x+1\right)+4\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

**maple** [A] time = 0.01, size = 60, normalized size = 0.88

$$-\frac{10\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+4\ln(x)-2\ln\left(x^2-x+1\right)-\frac{3}{x}-\frac{1}{2x^2}-\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)/x^3/(x^2-x+1)^2,x)
```

```
[Out] -(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))-1/2/x^2-3/x+4*ln(x)
```

**maxima** [A] time = 0.95, size = 63, normalized size = 0.93

$$-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{22x^3-23x^2+15x+3}{6(x^4-x^3+x^2)}-2\log\left(x^2-x+1\right)+4\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

```
[Out] -10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x +
 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

**mupad** [B] time = 0.10, size = 75, normalized size = 1.10

$$4\ln(x)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\frac{\frac{11x^3}{3}-\frac{23x^2}{6}+\frac{5x}{2}+\frac{1}{2}}{x^4-x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

```
[Out] 4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3)/
3 + 1/2)/(x^2 - x^3 + x^4)
```

**sympy [A]** time = 0.22, size = 71, normalized size = 1.04

$$4\log(x) - 2\log\left(x^2 - x + 1\right) - \frac{10\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} - \frac{\sqrt{3}}{3}\right)}{9} + \frac{-22x^3 + 23x^2 - 15x - 3}{6x^4 - 6x^3 + 6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

```
[Out] 4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)
/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

# 3.167 $\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx$

**Optimal**. Leaf size=10

$$\frac{x}{x^2+x+1}$$

[Out] x/(x^2+x+1)

**Rubi [A]** time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+x+x^2\right)^2} \, dx = \frac{x}{1+x+x^2}$$

**Mathematica [A]** time = 0.01, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + x + x^2)^2,x]

[Out] x/(1 + x + x^2)

**fricas** [A] time = 0.82, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="fricas")
[Out] x/(x^2 + x + 1)
```

**giac** [A] time = 0.15, size = 8, normalized size = 0.80

$$\frac{1}{x+\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="giac")
[Out] 1/(x + 1/x + 1)
```

**maple** [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+x+1)^2,x)
[Out] x/(x^2+x+1)
```

**maxima** [A] time = 0.43, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+x+1)^2,x, algorithm="maxima")
[Out] x/(x^2 + x + 1)
```

**mupad** [B] time = 0.05, size = 10, normalized size = 1.00

$$\frac{x}{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x + x^2 + 1)^2,x)
```

```
[Out] x/(x + x^2 + 1)
```

**sympy [A]** time = 0.10, size = 7, normalized size = 0.70

$$\frac{x}{x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+x+1)**2,x)
```

```
[Out] x/(x**2 + x + 1)
```

# 3.168 $\int \frac{1+x^2}{1+x+x^2} \, dx$

**Optimal**. Leaf size=31

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

**Rubi [A]** time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x + x^2),x]
```

```
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2}{1+x+x^2}\,dx &= \int \left(1-\frac{x}{1+x+x^2}\right)dx\\
&= x-\int \frac{x}{1+x+x^2}\,dx\\
&= x+\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\
&= x-\frac{1}{2}\log\left(1+x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= x+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}
$$

**Mathematica** [A]  time = 0.01, size = 31, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2+x+1\right)+x+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x + x^2),x]
[Out] x + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x + x^2]/2
```

**fricas** [A]  time = 1.03, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="fricas")
```

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)

**giac** [A] time = 0.17, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)

**maple** [A] time = 0.00, size = 28, normalized size = 0.90

$$x+\frac{\sqrt{3}\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)}{3}-\frac{\ln\left(x^2+x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)/(x^2+x+1),x)

[Out] x-1/2*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)

**maxima** [A] time = 0.95, size = 27, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x-\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)/(x^2+x+1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + x - 1/2*log(x^2 + x + 1)

**mupad** [B] time = 0.03, size = 29, normalized size = 0.94

$$x-\frac{\ln\left(x^2+x+1\right)}{2}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)/(x + x^2 + 1),x)

[Out] x - log(x + x^2 + 1)/2 + (3^(1/2)*atan((2*3^(1/2)*x)/3 + 3^(1/2)/3))/3

**sympy** [A] time = 0.12, size = 36, normalized size = 1.16

$$x - \frac{\log\left(x^2 + x + 1\right)}{2} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2+x+1),x)
```

```
[Out] x - log(x**2 + x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

# 3.169 $\int \frac{-1+x^2}{25-6x+x^2} \, dx$

**Optimal**. Leaf size=23

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

```
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

**Rubi [A]** time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1657, 634, 618, 204, 628}

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{25-6x+x^2}\,dx &= \int \left(1-\frac{2(13-3x)}{25-6x+x^2}\right)dx\\ &= x-2\int \frac{13-3x}{25-6x+x^2}\,dx\\ &= x+3\int \frac{-6+2x}{25-6x+x^2}\,dx-8\int \frac{1}{25-6x+x^2}\,dx\\ &= x+3\log\left(25-6x+x^2\right)+16\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,-6+2x\right)\\ &= x-2\tan^{-1}\left(\frac{1}{4}(-3+x)\right)+3\log\left(25-6x+x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 23, normalized size = 1.00

$$3\log\left(x^2-6x+25\right)+x-2\tan^{-1}\left(\frac{x-3}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(25 - 6*x + x^2),x]
```

```
[Out] x - 2*ArcTan[(-3 + x)/4] + 3*Log[25 - 6*x + x^2]
```

**fricas [A]** time = 0.66, size = 21, normalized size = 0.91

$$x-2\,\arctan\left(\frac{1}{4}\,x-\frac{3}{4}\right)+3\,\log\left(x^2-6\,x+25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

**giac** [A] time = 0.15, size = 21, normalized size = 0.91

$$x - 2\ \arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\ \log\left(x^2 - 6\,x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="giac")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

**maple** [A] time = 0.00, size = 22, normalized size = 0.96

$$x - 2\arctan\left(\frac{x}{4} - \frac{3}{4}\right) + 3\ln\left(x^2 - 6x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2-6*x+25),x)
[Out] x-2*arctan(-3/4+1/4*x)+3*ln(x^2-6*x+25)
```

**maxima** [A] time = 0.95, size = 21, normalized size = 0.91

$$x - 2\ \arctan\left(\frac{1}{4}\,x - \frac{3}{4}\right) + 3\ \log\left(x^2 - 6\,x + 25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2-6*x+25),x, algorithm="maxima")
[Out] x - 2*arctan(1/4*x - 3/4) + 3*log(x^2 - 6*x + 25)
```

**mupad** [B] time = 0.04, size = 21, normalized size = 0.91

$$x + 3\ \ln\left(x^2 - 6\,x + 25\right) - 2\,\mathrm{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 - 6*x + 25),x)
[Out] x + 3*log(x^2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

**sympy** [A] time = 0.11, size = 22, normalized size = 0.96

$$x + 3\log\left(x^2 - 6x + 25\right) - 2\,\mathrm{atan}\left(\frac{x}{4} - \frac{3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2-6*x+25),x)
```

```
[Out] x + 3*log(x**2 - 6*x + 25) - 2*atan(x/4 - 3/4)
```

# 3.170 $\int \frac{-10+3x^2}{4-4x+x^2} \, dx$

**Optimal**. Leaf size=21

$$3x+\frac{2}{2-x}+12\log(2-x)$$

[Out] 2/(2-x)+3*x+12*ln(2-x)

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {27, 697}

$$3x+\frac{2}{2-x}+12\log(2-x)$$

Antiderivative was successfully verified.

[In] Int[(-10 + 3*x^2)/(4 - 4*x + x^2),x]

[Out] 2/(2 - x) + 3*x + 12*Log[2 - x]

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-10+3x^2}{4-4x+x^2} \, dx &= \int \frac{-10+3x^2}{(-2+x)^2} \, dx\\
&= \int \left(3+\frac{2}{(-2+x)^2}+\frac{12}{-2+x}\right) dx\\
&= \frac{2}{2-x}+3x+12\log(2-x)
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 19, normalized size = 0.90

$$3(x-2)-\frac{2}{x-2}+12\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + 3*x^2)/(4 - 4*x + x^2),x]
[Out] -2/(-2 + x) + 3*(-2 + x) + 12*Log[-2 + x]
```

**fricas** [A] time = 0.78, size = 25, normalized size = 1.19

$$\frac{3\,x^2+12\,(x-2)\log\,(x-2)-6\,x-2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="fricas")
[Out] (3*x^2 + 12*(x - 2)*log(x - 2) - 6*x - 2)/(x - 2)
```

**giac** [A] time = 0.17, size = 18, normalized size = 0.86

$$3\,x-\frac{2}{x-2}+12\,\log\,(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="giac")
[Out] 3*x - 2/(x - 2) + 12*log(abs(x - 2))
```

**maple** [A] time = 0.01, size = 18, normalized size = 0.86

$$3x+12\ln\,(x-2)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-10)/(x^2-4*x+4),x)
[Out] 3*x+12*ln(x-2)-2/(x-2)
```

**maxima** [A] time = 0.42, size = 17, normalized size = 0.81

$$3\,x-\frac{2}{x-2}+12\,\log\,(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-10)/(x^2-4*x+4),x, algorithm="maxima")
```

```
[Out] 3*x - 2/(x - 2) + 12*log(x - 2)
```

**mupad [B]** time = 0.04, size = 17, normalized size = 0.81

$$3\,x+12\,\ln\left(x-2\right)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 10)/(x^2 - 4*x + 4),x)
```

```
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

**sympy [A]** time = 0.09, size = 14, normalized size = 0.67

$$3x+12\log\left(x-2\right)-\frac{2}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-10)/(x**2-4*x+4),x)
```

```
[Out] 3*x + 12*log(x - 2) - 2/(x - 2)
```

# 3.171 $\int \frac{8+x^2}{6-5x+x^2} dx$

**Optimal**. Leaf size=18

$$x - 12\log(2 - x) + 17\log(3 - x)$$

```
[Out] x-12*ln(2-x)+17*ln(3-x)
```

**Rubi [A]** time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1657, 632, 31}

$$x - 12\log(2 - x) + 17\log(3 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(8 + x^2)/(6 - 5*x + x^2),x]
```

```
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{8+x^2}{6-5x+x^2}\,dx &= \int \left(1+\frac{2+5x}{6-5x+x^2}\right)dx \\
&= x+\int \frac{2+5x}{6-5x+x^2}\,dx \\
&= x-12\int \frac{1}{-2+x}\,dx+17\int \frac{1}{-3+x}\,dx \\
&= x-12\log(2-x)+17\log(3-x)
\end{aligned}
$$

**Mathematica** [**A**] time = 0.00, size = 18, normalized size = 1.00

$$x-12\log(2-x)+17\log(3-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 + x^2)/(6 - 5*x + x^2),x]
[Out] x - 12*Log[2 - x] + 17*Log[3 - x]
```

**fricas** [**A**] time = 0.69, size = 14, normalized size = 0.78

$$x-12\,\log\left(x-2\right)+17\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="fricas")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

**giac** [**A**] time = 0.15, size = 16, normalized size = 0.89

$$x-12\,\log\left(|x-2|\right)+17\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="giac")
[Out] x - 12*log(abs(x - 2)) + 17*log(abs(x - 3))
```

**maple** [**A**] time = 0.01, size = 15, normalized size = 0.83

$$x+17\ln\left(x-3\right)-12\ln\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+8)/(x^2-5*x+6),x)
[Out] x-12*ln(x-2)+17*ln(x-3)
```

**maxima [A]** time = 0.43, size = 14, normalized size = 0.78

$$x - 12 \log(x - 2) + 17 \log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+8)/(x^2-5*x+6),x, algorithm="maxima")
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

**mupad [B]** time = 3.92, size = 14, normalized size = 0.78

$$x - 12 \ln(x - 2) + 17 \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 8)/(x^2 - 5*x + 6),x)
[Out] x - 12*log(x - 2) + 17*log(x - 3)
```

**sympy [A]** time = 0.11, size = 14, normalized size = 0.78

$$x + 17 \log(x - 3) - 12 \log(x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+8)/(x**2-5*x+6),x)
[Out] x + 17*log(x - 3) - 12*log(x - 2)
```

# 3.172 $\int \frac{-4+3x+x^2}{-8-2x+x^2} \, dx$

**Optimal**. Leaf size=14

$$x + 4\log(4 - x) + \log(x + 2)$$

[Out] x+4*ln(4-x)+ln(2+x)

**Rubi [A]** time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x + 4\log(4 - x) + \log(x + 2)$$

Antiderivative was successfully verified.

[In] Int[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]

[Out] x + 4*Log[4 - x] + Log[2 + x]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{-8-2x+x^2}\,dx &= \int \left(1+\frac{4+5x}{-8-2x+x^2}\right)dx\\ &= x+\int \frac{4+5x}{-8-2x+x^2}\,dx\\ &= x+4\int \frac{1}{-4+x}\,dx+\int \frac{1}{2+x}\,dx\\ &= x+4\log(4-x)+\log(2+x)\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 14, normalized size = 1.00

$$x+4\log(4-x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/(-8 - 2*x + x^2),x]
```

```
[Out] x + 4*Log[4 - x] + Log[2 + x]
```

**fricas** [A] time = 0.86, size = 12, normalized size = 0.86

$$x+\log(x+2)+4\,\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="fricas")
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

**giac** [A] time = 0.16, size = 14, normalized size = 1.00

$$x+\log(|x+2|)+4\,\log(|x-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="giac")
```

```
[Out] x + log(abs(x + 2)) + 4*log(abs(x - 4))
```

**maple** [A] time = 0.01, size = 13, normalized size = 0.93

$$x+\ln(x+2)+4\ln(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(x^2-2*x-8),x)
```

```
[Out] x+ln(2+x)+4*ln(x-4)
```

**maxima [A]** time = 0.43, size = 12, normalized size = 0.86

$$x + \log(x + 2) + 4 \log(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(x^2-2*x-8),x, algorithm="maxima")
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

**mupad [B]** time = 3.85, size = 12, normalized size = 0.86

$$x + \ln(x + 2) + 4 \ln(x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + x^2 - 4)/(2*x - x^2 + 8),x)
```

```
[Out] x + log(x + 2) + 4*log(x - 4)
```

**sympy [A]** time = 0.11, size = 12, normalized size = 0.86

$$x + 4\log(x - 4) + \log(x + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(x**2-2*x-8),x)
```

```
[Out] x + 4*log(x - 4) + log(x + 2)
```

# 3.173 $\int \frac{7+5x+4x^2}{5+4x+4x^2} dx$

**Optimal**. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x+3/8*arctan(1/2+x)+1/8*ln(4*x^2+4*x+5)
```

**Rubi [A]** time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right)dx \\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx \\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right) \\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

**fricas [A]** time = 0.61, size = 21, normalized size = 0.78

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{1}{8}\log\left(4x^2+4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

**giac** [A] time = 0.15, size = 21, normalized size = 0.78

$$x + \frac{3}{8}\,\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\,\log\left(4\,x^2 + 4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

**maple** [A] time = 0.00, size = 22, normalized size = 0.81

$$x + \frac{3\arctan\left(x + \frac{1}{2}\right)}{8} + \frac{\ln\left(4x^2 + 4x + 5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

**maxima** [A] time = 0.94, size = 21, normalized size = 0.78

$$x + \frac{3}{8}\,\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\,\log\left(4\,x^2 + 4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

**mupad** [B] time = 3.80, size = 17, normalized size = 0.63

$$x + \frac{\ln\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 4*x^2 + 7)/(4*x + 4*x^2 + 5),x)
[Out] x + log(x + x^2 + 5/4)/8 + (3*atan(x + 1/2))/8
```

**sympy** [A] time = 0.12, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3\,\mathrm{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

# 3.174 $\int \frac{2-x+x^2}{-5+2x+x^2}\,dx$

**Optimal**. Leaf size=48

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

```
[Out] x-1/6*ln(1+x-6^(1/2))*(9-5*6^(1/2))-1/6*ln(1+x+6^(1/2))*(9+5*6^(1/2))
```

**Rubi [A]** time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1657, 632, 31}

$$x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(x-\sqrt{6}+1\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x - ((9 - 5*Sqrt[6])*Log[1 - Sqrt[6] + x])/6 - ((9 + 5*Sqrt[6])*Log[1 + Sqr
t[6] + x])/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2-x+x^2}{-5+2x+x^2}\,dx = \int \left(1+\frac{7-3x}{-5+2x+x^2}\right)dx$$
$$= x+\int \frac{7-3x}{-5+2x+x^2}\,dx$$
$$= x+\frac{1}{6}\left(-9+5\sqrt{6}\right)\int \frac{1}{1-\sqrt{6}+x}\,dx-\frac{1}{6}\left(9+5\sqrt{6}\right)\int \frac{1}{1+\sqrt{6}+x}\,dx$$
$$= x-\frac{1}{6}\left(9-5\sqrt{6}\right)\log\left(1-\sqrt{6}+x\right)-\frac{1}{6}\left(9+5\sqrt{6}\right)\log\left(1+\sqrt{6}+x\right)$$

**Mathematica [A]** time = 0.08, size = 48, normalized size = 1.00

$$x+\frac{1}{6}\left(5\sqrt{6}-9\right)\log\left(-x+\sqrt{6}-1\right)+\frac{1}{6}\left(-9-5\sqrt{6}\right)\log\left(x+\sqrt{6}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(-5 + 2*x + x^2),x]
```

```
[Out] x + ((-9 + 5*Sqrt[6])*Log[-1 + Sqrt[6] - x])/6 + ((-9 - 5*Sqrt[6])*Log[1 +
Sqrt[6] + x])/6
```

**fricas [A]** time = 0.90, size = 55, normalized size = 1.15

$$\frac{5}{6}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}(x+1)-x^2-2x-7}{x^2+2x-5}\right)+x-\frac{3}{2}\log\left(x^2+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="fricas")
```

```
[Out] 5/6*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(x + 1) - x^2 - 2*x - 7)/(x^2 +
 2*x - 5)) + x - 3/2*log(x^2 + 2*x - 5)
```

**giac [A]** time = 0.19, size = 45, normalized size = 0.94

$$\frac{5}{6}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+2\right|}{\left|2x+2\sqrt{6}+2\right|}\right)+x-\frac{3}{2}\log\left(\left|x^2+2x-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="giac")
```

```
[Out] 5/6*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2)/abs(2*x + 2*sqrt(6) + 2)) + x - 3/
2*log(abs(x^2 + 2*x - 5))
```

**maple [A]** time = 0.00, size = 30, normalized size = 0.62

$$x-\frac{5\sqrt{6}\ \operatorname{arctanh}\left(\frac{(2x+2)\sqrt{6}}{12}\right)}{3}-\frac{3\ln\left(x^{2}+2x-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^2+2*x-5),x)
```

```
[Out] x-3/2*ln(x^2+2*x-5)-5/3*6^(1/2)*arctanh(1/12*(2*x+2)*6^(1/2))
```

**maxima [A]** time = 0.96, size = 36, normalized size = 0.75

$$\frac{5}{6}\sqrt{6}\log\left(\frac{x-\sqrt{6}+1}{x+\sqrt{6}+1}\right)+x-\frac{3}{2}\log\left(x^{2}+2x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^2+2*x-5),x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(6)*log((x - sqrt(6) + 1)/(x + sqrt(6) + 1)) + x - 3/2*log(x^2 + 2*
x - 5)
```

**mupad [B]** time = 0.11, size = 35, normalized size = 0.73

$$x-\ln\left(x+\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}+\frac{3}{2}\right)+\ln\left(x-\sqrt{6}+1\right)\left(\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x + 2)/(2*x + x^2 - 5),x)
```

```
[Out] x - log(x + 6^(1/2) + 1)*((5*6^(1/2))/6 + 3/2) + log(x - 6^(1/2) + 1)*((5*6
^(1/2))/6 - 3/2)
```

**sympy [A]** time = 0.12, size = 46, normalized size = 0.96

$$x+\left(-\frac{5\sqrt{6}}{6}-\frac{3}{2}\right)\log\left(x+1+\sqrt{6}\right)+\left(-\frac{3}{2}+\frac{5\sqrt{6}}{6}\right)\log\left(x-\sqrt{6}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**2+2*x-5),x)
```

```
[Out] x + (-5*sqrt(6)/6 - 3/2)*log(x + 1 + sqrt(6)) + (-3/2 + 5*sqrt(6)/6)*log(x
- sqrt(6) + 1)
```

# 3.175 $\int \frac{1+4x+3x^2}{(4+7x+2x^2)^2}\,dx$

**Optimal**. Leaf size=21

$$-\frac{3x+2}{2(2x^2+7x+4)}$$

```
[Out] 1/2*(-2-3*x)/(2*x^2+7*x+4)
```

**Rubi [A]** time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1660, 8}

$$-\frac{3x+2}{2(2x^2+7x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
```

```
[Out] -(2 + 3*x)/(2*(4 + 7*x + 2*x^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1+4x+3x^2}{\left(4+7x+2x^2\right)^2}\,dx = -\frac{2+3x}{2\left(4+7x+2x^2\right)} - \frac{\int 0\,dx}{17}$$
$$= -\frac{2+3x}{2\left(4+7x+2x^2\right)}$$

**Mathematica** [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{-3x-2}{2\left(2x^2+7x+4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x + 3*x^2)/(4 + 7*x + 2*x^2)^2,x]
[Out] (-2 - 3*x)/(2*(4 + 7*x + 2*x^2))
```

**fricas** [A] time = 0.78, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="fricas")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

**giac** [A] time = 0.16, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="giac")
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

**maple** [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{-\frac{3x}{4}-\frac{1}{2}}{x^2+\frac{7}{2}x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x)
```

```
[Out] (-3/4*x-1/2)/(x^2+7/2*x+2)
```

**maxima** [A] time = 0.43, size = 19, normalized size = 0.90

$$-\frac{3\,x+2}{2\,\left(2\,x^2+7\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+4*x+1)/(2*x^2+7*x+4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*x + 2)/(2*x^2 + 7*x + 4)
```

**mupad** [B] time = 3.84, size = 17, normalized size = 0.81

$$-\frac{\frac{3\,x}{4}+\frac{1}{2}}{x^2+\frac{7\,x}{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 3*x^2 + 1)/(7*x + 2*x^2 + 4)^2,x)
```

```
[Out] -((3*x)/4 + 1/2)/((7*x)/2 + x^2 + 2)
```

**sympy** [A] time = 0.12, size = 15, normalized size = 0.71

$$\frac{-3x-2}{4x^2+14x+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+4*x+1)/(2*x**2+7*x+4)**2,x)
```

```
[Out] (-3*x - 2)/(4*x**2 + 14*x + 8)
```

# 3.176 $\int \frac{1+x+x^2}{(3+2x+x^2)^2} \, dx$

**Optimal**. Leaf size=39

$$\frac{1-x}{4(x^2+2x+3)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*(1-x)/(x^2+2*x+3)+3/8*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)
```

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1660, 12, 618, 204}

$$\frac{1-x}{4(x^2+2x+3)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
```

```
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
```

```
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x+x^2}{\left(3+2x+x^2\right)^2}\,dx &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{1}{8}\int \frac{6}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3}{4}\int \frac{1}{3+2x+x^2}\,dx\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2+2x\right)\\ &= \frac{1-x}{4\left(3+2x+x^2\right)}+\frac{3\tan^{-1}\left(\frac{1+x}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 39, normalized size = 1.00

$$\frac{1-x}{4\left(x^2+2x+3\right)}+\frac{3\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)/(3 + 2*x + x^2)^2,x]
[Out] (1 - x)/(4*(3 + 2*x + x^2)) + (3*ArcTan[(1 + x)/Sqrt[2]])/(4*Sqrt[2])
```

**fricas [A]** time = 0.64, size = 39, normalized size = 1.00

$$\frac{3\sqrt{2}\left(x^2+2\,x+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\,(x+1)\right)-2\,x+2}{8\left(x^2+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="fricas")
```

[Out] 1/8*(3*sqrt(2)*(x^2 + 2*x + 3)*arctan(1/2*sqrt(2)*(x + 1)) - 2*x + 2)/(x^2 + 2*x + 3)

**giac [A]** time = 0.16, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="giac")

[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)

**maple [A]** time = 0.01, size = 34, normalized size = 0.87

$$\frac{3\sqrt{2}\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{8}+\frac{-\frac{x}{4}+\frac{1}{4}}{x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x+1)/(x^2+2*x+3)^2,x)

[Out] (-1/4*x+1/4)/(x^2+2*x+3)+3/8*2^(1/2)*arctan(1/4*(2*x+2)*2^(1/2))

**maxima [A]** time = 0.96, size = 30, normalized size = 0.77

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right)-\frac{x-1}{4\left(x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x+1)/(x^2+2*x+3)^2,x, algorithm="maxima")

[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - 1/4*(x - 1)/(x^2 + 2*x + 3)

**mupad [B]** time = 3.84, size = 36, normalized size = 0.92

$$\frac{3\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8}-\frac{\frac{x}{4}-\frac{1}{4}}{x^2+2x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 + 1)/(2*x + x^2 + 3)^2,x)

[Out] (3*2^(1/2)*atan((2^(1/2)*x)/2 + 2^(1/2)/2))/8 - (x/4 - 1/4)/(2*x + x^2 + 3)

**sympy [A]** time = 0.14, size = 37, normalized size = 0.95

$$\frac{1-x}{4x^2+8x+12}+\frac{3\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}+\frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)/(x**2+2*x+3)**2,x)
```

```
[Out] (1 - x)/(4*x**2 + 8*x + 12) + 3*sqrt(2)*atan(sqrt(2)*x/2 + sqrt(2)/2)/8
```

# 3.177 $\int \frac{-1+2x+5x^2}{\left(1+x+x^2\right)^4} dx$

**Optimal**. Leaf size=11

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

```
[Out] -x/(x^2+x+1)^3
```

**Rubi [A]** time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
```

```
[Out] -(x/(1 + x + x^2)^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+2x+5x^2}{\left(1+x+x^2\right)^4} dx = -\frac{x}{\left(1+x+x^2\right)^3}$$

**Mathematica [A]** time = 0.01, size = 11, normalized size = 1.00

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + 5*x^2)/(1 + x + x^2)^4,x]
[Out] -(x/(1 + x + x^2)^3)
```

**fricas [B]** time = 0.83, size = 33, normalized size = 3.00

$$-\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="fricas")
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

**giac [A]** time = 0.15, size = 11, normalized size = 1.00

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="giac")
[Out] -x/(x^2 + x + 1)^3
```

**maple [A]** time = 0.00, size = 12, normalized size = 1.09

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x-1)/(x^2+x+1)^4,x)
[Out] -x/(x^2+x+1)^3
```

**maxima [B]** time = 0.44, size = 33, normalized size = 3.00

$$-\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x-1)/(x^2+x+1)^4,x, algorithm="maxima")
[Out] -x/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

**mupad [B]** time = 3.80, size = 11, normalized size = 1.00

$$-\frac{x}{\left(x^2+x+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 - 1)/(x + x^2 + 1)^4,x)
```

```
[Out] -x/(x + x^2 + 1)^3
```

**sympy [B]** time = 0.13, size = 31, normalized size = 2.82

$$-\frac{x}{x^6+3x^5+6x^4+7x^3+6x^2+3x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x-1)/(x**2+x+1)**4,x)
```

```
[Out] -x/(x**6 + 3*x**5 + 6*x**4 + 7*x**3 + 6*x**2 + 3*x + 1)
```

# 3.178 $\int \left(a+bx+cx^2\right)^{5/2} \left(A+Cx^2\right) dx$

**Optimal**. Leaf size=267

$$-\frac{5\left(b^2-4ac\right)^3\left(-4acC+32Ac^2+9b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{11/2}}+\frac{5\left(b^2-4ac\right)^2(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC\right.}{16384c^5}$$

```
[Out] -5/6144*(-4*a*c+b^2)*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/
2)/c^4+1/384*(32*A*c^2-4*C*a*c+9*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2)/c^3-9
/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-5/32768*(-4*
a*c+b^2)^3*(32*A*c^2-4*C*a*c+9*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))/c^(11/2)+5/16384*(-4*a*c+b^2)^2*(32*A*c^2-4*C*a*c+9*C*b^2)*(2
*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

**Rubi [A]** time = 0.24, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{5/2}\left(-4acC+32Ac^2+9b^2C\right)}{384c^3}-\frac{5\left(b^2-4ac\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+32Ac^2+\right.}{6144c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (5*(b^2 - 4*a*c)^2*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x
+ c*x^2])/(16384*c^5) - (5*(b^2 - 4*a*c)*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^4) + ((32*A*c^2 + 9*b^2*C - 4*a*c
*C)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2))/(384*c^3) - (9*b*C*(a + b*x + c*x^
2)^(7/2))/(112*c^2) + (C*x*(a + b*x + c*x^2)^(7/2))/(8*c) - (5*(b^2 - 4*a*c
)^3*(32*A*c^2 + 9*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(32768*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{5/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\int \left(8Ac - aC - \frac{9bCx}{2}\right)\left(a+bx+cx^2\right)^{5/2}dx}{8c}$$

$$= -\frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2} + \frac{Cx\left(a+bx+cx^2\right)^{7/2}}{8c} + \frac{\left(\frac{9b^2C}{2} + 2c(8Ac - aC)\right)\int (}{16c^2}$$

$$= \frac{\left(32Ac^2 + 9b^2C - 4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{5/2}}{384c^3} - \frac{9bC\left(a+bx+cx^2\right)^{7/2}}{112c^2}$$

$$= -\frac{5\left(b^2-4ac\right)\left(32Ac^2 + 9b^2C - 4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{6144c^4} + \frac{\left(32Ac^2\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2 + 9b^2C - 4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2 + 9b^2C - 4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right.}{}$$

$$= \frac{5\left(b^2-4ac\right)^2\left(32Ac^2 + 9b^2C - 4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{16384c^5} - \frac{5\left(b^2-4ac\right.}{}$$

**Mathematica [A]** time = 0.86, size = 344, normalized size = 1.29

$$-\frac{1120A\left(b^2-4ac\right)\left(16c^{3/2}(b+2cx)(a+x(b+cx))^{3/2} - 3\left(b^2-4ac\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)} - \left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\right)}{c^{5/2}} + 57344A(b+2cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(5/2)*(A + C*x^2),x]
```

```
[Out] (57344*A*(b + 2*c*x)*(a + x*(b + c*x))^(5/2) - (55296*b*C*(a + x*(b + c*x))
^(7/2))/c + 86016*C*x*(a + x*(b + c*x))^(7/2) - (1120*A*(b^2 - 4*a*c)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)]])]))/c^(5/2) + (7*(9*b^2 - 4*a*c)*C*(256*c^(5/2)*(
b + 2*c*x)*(a + x*(b + c*x))^(5/2) - 5*(b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x
)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a +
 x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])]))))/c^(9/2))/(688128*c)
```

**fricas [B]** time = 1.20, size = 953, normalized size = 3.57

$$\left[\frac{105\left(9\,Cb^8 - 112\,Cab^6c - 2048\,Aa^3c^5 + 256\left(Ca^4 + 6\,Aa^2b^2\right)c^4 - 384\left(2\,Ca^3b^2 + Aab^4\right)c^3 + 32\left(15\,Ca^2b^4 + Ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/1376256*(105*(9*C*b^8 - 112*C*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*
A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6
)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) + 4*(43008*C*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C
*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*
C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^
3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)
*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 +
 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3 - 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 +
 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 - 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3
- 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 7
0*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/688128*(105*(9*C*b^8 - 112*C
*a*b^6*c - 2048*A*a^3*c^5 + 256*(C*a^4 + 6*A*a^2*b^2)*c^4 - 384*(2*C*a^3*b^
2 + A*a*b^4)*c^3 + 32*(15*C*a^2*b^4 + A*b^6)*c^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(43008*C
*c^8*x^7 + 101376*C*b*c^7*x^6 + 945*C*b^7*c - 10500*C*a*b^5*c^2 + 118272*A*
a^2*b*c^5 + 256*(243*C*b^2*c^6 + 476*C*a*c^7 + 224*A*c^8)*x^5 - 64*(663*C*a
^3*b + 560*A*a*b^3)*c^4 + 128*(3*C*b^3*c^5 + 1228*C*a*b*c^6 + 1120*A*b*c^7)
*x^4 + 112*(337*C*a^2*b^3 + 30*A*b^5)*c^3 - 16*(27*C*b^4*c^4 - 216*C*a*b^2*
c^5 - 11648*A*a*c^7 - 112*(59*C*a^2 + 54*A*b^2)*c^6)*x^3 + 8*(63*C*b^5*c^3
- 568*C*a*b^3*c^4 + 34944*A*a*b*c^6 + 16*(87*C*a^2*b + 14*A*b^3)*c^5)*x^2 -
 2*(315*C*b^6*c^2 - 3164*C*a*b^4*c^3 - 118272*A*a^2*c^6 - 1344*(5*C*a^3 + 8
*A*a*b^2)*c^5 + 16*(597*C*a^2*b^2 + 70*A*b^4)*c^4)*x)*sqrt(c*x^2 + b*x + a)
)/c^6]
```

**giac [B]** time = 0.27, size = 482, normalized size = 1.81

$$\frac{1}{344064}\sqrt{cx^2 + bx + a}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Cc^2x + 33\,Cbc\right)x + \frac{243\,Cb^2c^7 + 476\,Cac^8 + 224\,Ac^9}{c^7}\right)x + \frac{3\,Cb^3c^6}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/344064*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(2*(12*(14*C*c^2*x + 33*C*b*c)*x
 + (243*C*b^2*c^7 + 476*C*a*c^8 + 224*A*c^9)/c^7)*x + (3*C*b^3*c^6 + 1228*C
*a*b*c^7 + 1120*A*b*c^8)/c^7)*x - (27*C*b^4*c^5 - 216*C*a*b^2*c^6 - 6608*C*
a^2*c^7 - 6048*A*b^2*c^7 - 11648*A*a*c^8)/c^7)*x + (63*C*b^5*c^4 - 568*C*a*
b^3*c^5 + 1392*C*a^2*b*c^6 + 224*A*b^3*c^6 + 34944*A*a*b*c^7)/c^7)*x - (315
*C*b^6*c^3 - 3164*C*a*b^4*c^4 + 9552*C*a^2*b^2*c^5 + 1120*A*b^4*c^5 - 6720*
C*a^3*c^6 - 10752*A*a*b^2*c^6 - 118272*A*a^2*c^7)/c^7)*x + (945*C*b^7*c^2 -
 10500*C*a*b^5*c^3 + 37744*C*a^2*b^3*c^4 + 3360*A*b^5*c^4 - 42432*C*a^3*b*c
^5 - 35840*A*a*b^3*c^5 + 118272*A*a^2*b*c^6)/c^7) + 5/32768*(9*C*b^8 - 112*
C*a*b^6*c + 480*C*a^2*b^4*c^2 + 32*A*b^6*c^2 - 768*C*a^3*b^2*c^3 - 384*A*a*
b^4*c^3 + 256*C*a^4*c^4 + 1536*A*a^2*b^2*c^4 - 2048*A*a^3*c^5)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

**maple [B]** time = 0.01, size = 997, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x)
```

```
[Out] -9/112*b*C*(c*x^2+b*x+a)^(7/2)/c^2+1/8*C*x*(c*x^2+b*x+a)^(7/2)/c-95/2048*C/
c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*a-5/32*A/c*(c*x^2+b*x+a)^(1/2)*x*a*b^2+55/512
*C/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a^2+25/384*C/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x
*a+5/16*A/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-5/1024*A/
c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^6-5/128*C*a^4/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-45/32768*C/c^(11/2)*b^8*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/128*C/c^3*b^3*(c*x^2+b*x+a)^(5/2)-1
5/2048*C/c^4*b^5*(c*x^2+b*x+a)^(3/2)+45/16384*C/c^5*b^7*(c*x^2+b*x+a)^(1/2)
+1/12*A/c*(c*x^2+b*x+a)^(5/2)*b+5/24*A*(c*x^2+b*x+a)^(3/2)*x*a-5/192*A/c^2*
(c*x^2+b*x+a)^(3/2)*b^3+5/16*A*(c*x^2+b*x+a)^(1/2)*x*a^2+5/512*A/c^3*(c*x^2
+b*x+a)^(1/2)*b^5+1/6*A*(c*x^2+b*x+a)^(5/2)*x-15/1024*C/c^3*b^4*(c*x^2+b*x+
a)^(3/2)*x+25/768*C/c^3*b^3*(c*x^2+b*x+a)^(3/2)*a-5/96*A/c*(c*x^2+b*x+a)^(3
/2)*x*b^2+5/48*A/c*(c*x^2+b*x+a)^(3/2)*b*a+5/256*A/c^2*(c*x^2+b*x+a)^(1/2)*
x*b^4+35/2048*C/c^(9/2)*b^6*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1
/48*C*a/c*(c*x^2+b*x+a)^(5/2)*x-1/96*C*a/c^2*(c*x^2+b*x+a)^(5/2)*b-5/192*C*
a^2/c*(c*x^2+b*x+a)^(3/2)*x+45/8192*C/c^4*b^6*(c*x^2+b*x+a)^(1/2)*x+55/1024
*C/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a^2-95/4096*C/c^4*b^5*(c*x^2+b*x+a)^(1/2)*a-
5/384*C*a^2/c^2*(c*x^2+b*x+a)^(3/2)*b-5/128*C*a^3/c*(c*x^2+b*x+a)^(1/2)*x-5
/256*C*a^3/c^2*(c*x^2+b*x+a)^(1/2)*b+3/64*C/c^2*b^2*(c*x^2+b*x+a)^(5/2)*x+1
5/128*C/c^(5/2)*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^3-75/1024
*C/c^(7/2)*b^4*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+5/32*A/c*(c*
x^2+b*x+a)^(1/2)*b*a^2-5/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3*a-15/64*A/c^(3/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a^2+15/256*A/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4*a
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(5/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+b\,x+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(5/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C x^{2}\right) \left(a + b x + c x^{2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(5/2), x)
```

# 3.179 $\int \left(a+bx+cx^2\right)^{3/2} \left(A+Cx^2\right) dx$

**Optimal**. Leaf size=212

$$\frac{\left(b^2-4ac\right)^2 \left(-4acC+24Ac^2+7b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24A\right.}{512c^4}$$

```
[Out] 1/192*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^3-7/60*b*C
*(c*x^2+b*x+a)^(5/2)/c^2+1/6*C*x*(c*x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^
2*(24*A*c^2-4*C*a*c+7*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*A*c^2-4*C*a*c+7*C*b^2)*(2*c*x+b)*(c*x^2
+b*x+a)^(1/2)/c^4
```

**Rubi [A]** time = 0.18, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acC+24Ac^2+7b^2C\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+24Ac^2+7b^2\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c
*x^2])/(512*c^4) + ((24*A*c^2 + 7*b^2*C - 4*a*c*C)*(b + 2*c*x)*(a + b*x + c
*x^2)^(3/2))/(192*c^3) - (7*b*C*(a + b*x + c*x^2)^(5/2))/(60*c^2) + (C*x*(a
 + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*c)^2*(24*A*c^2 + 7*b^2*C - 4*a*c
*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int \left(6Ac-aC-\frac{7bCx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx}{6c}$$

$$= -\frac{7bC\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{Cx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(\frac{7b^2C}{2}+2c(6Ac-aC)\right)\int}{12c}$$

$$= \frac{\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}-\frac{7bC\left(a+bx+cx^2\right)}{60c^2}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24Ac^2+7b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+\frac{\left(24Ac^2+\right.}{}$$

**Mathematica [A]** time = 0.58, size = 267, normalized size = 1.26

$$\frac{\frac{360A(b^2-4ac)\left((b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}+1920A(b+2cx)(a+x(b+cx))^{3/2}+\frac{C\left(5(7b^2-4ac)\left(\frac{3(b^2-\cdots}{\cdots}\right.\right.}{\cdots}}{15360\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(A + C*x^2),x]
```

```
[Out] (1920*A*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 2560*C*x*(a + x*(b + c*x))^(5
/2) + (360*A*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] +
(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3
/2) + (C*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*S
qrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

**fricas [A]** time = 1.09, size = 605, normalized size = 2.85

$$\left[\frac{15\left(7Cb^6-60Cab^4c+384Aa^2c^4-64\left(Ca^3+3Aab^2\right)c^3+24\left(6Ca^2b^2+Ab^4\right)c^2\right)\sqrt{c}\log\left(-8c^2x^2-8bcx-b^2\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [1/30720*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*a^3 + 3*A*a*b^
2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b
^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(1280*C*c^6*x
^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*c^4 - 72
*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A*c^6)*x^
3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*c^2 - 21
6*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c*x^2 + b
*x + a))/c^5, -1/15360*(15*(7*C*b^6 - 60*C*a*b^4*c + 384*A*a^2*c^4 - 64*(C*
a^3 + 3*A*a*b^2)*c^3 + 24*(6*C*a^2*b^2 + A*b^4)*c^2)*sqrt(-c)*arctan(1/2*sq
rt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280
*C*c^6*x^5 + 1664*C*b*c^5*x^4 - 105*C*b^5*c + 760*C*a*b^3*c^2 + 2400*A*a*b*
c^4 - 72*(18*C*a^2*b + 5*A*b^3)*c^3 + 16*(3*C*b^2*c^4 + 140*C*a*c^5 + 120*A
*c^6)*x^3 - 8*(7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*b*c^5)*x^2 + 2*(35*C*b^4*
c^2 - 216*C*a*b^2*c^3 + 2400*A*a*c^5 + 120*(2*C*a^2 + A*b^2)*c^4)*x)*sqrt(c
*x^2 + b*x + a))/c^5]
```

**giac [A]** time = 0.28, size = 297, normalized size = 1.40

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,Ccx+13\,Cb\right)x+\frac{3\,Cb^2c^4+140\,Cac^5+120\,Ac^6}{c^5}\right)x-\frac{7\,Cb^3c^3-36\,Cabc^4-3}{c^5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*C*c*x + 13*C*b)*x + (3*C*b^2*c
^4 + 140*C*a*c^5 + 120*A*c^6)/c^5)*x - (7*C*b^3*c^3 - 36*C*a*b*c^4 - 360*A*
b*c^5)/c^5)*x + (35*C*b^4*c^2 - 216*C*a*b^2*c^3 + 240*C*a^2*c^4 + 120*A*b^2
*c^4 + 2400*A*a*c^5)/c^5)*x - (105*C*b^5*c - 760*C*a*b^3*c^2 + 1296*C*a^2*b
*c^3 + 360*A*b^3*c^3 - 2400*A*a*b*c^4)/c^5) - 1/1024*(7*C*b^6 - 60*C*a*b^4*
c + 144*C*a^2*b^2*c^2 + 24*A*b^4*c^2 - 64*C*a^3*c^3 - 192*A*a*b^2*c^3 + 384
*A*a^2*c^4)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^
(9/2)
```

**maple [B]** time = 0.01, size = 613, normalized size = 2.89

$$\frac{3A\,a^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}-\frac{3Aa\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{3A\,b^4\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{128c^{\frac{5}{2}}}-\frac{C\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x)
```

```
[Out] 1/8*C/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a-7/60*b*C*(c*x^2+b*x+a)^(5/2)/c^2+1/6*
C*x*(c*x^2+b*x+a)^(5/2)/c+3/8*A/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a^2+3/128*A/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^
4+7/192*C/c^3*b^3*(c*x^2+b*x+a)^(3/2)-7/512*C/c^4*b^5*(c*x^2+b*x+a)^(1/2)+7
/1024*C/c^(9/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*C*a^3/
c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*A/c*(c*x^2+b*x+a)^(
3/2)*b+3/8*A*(c*x^2+b*x+a)^(1/2)*x*a-3/64*A/c^2*(c*x^2+b*x+a)^(1/2)*b^3-3/3
2*A/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*A/c*(c*x^2+b*x+a)^(1/2)*b*a+1/4*A*(c*x
^2+b*x+a)^(3/2)*x-1/32*C*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b+7/96*C/c^2*b^2*(c*x^
2+b*x+a)^(3/2)*x-7/256*C/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x+1/16*C/c^3*b^3*(c*x^
2+b*x+a)^(1/2)*a-3/16*A/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*b^2*a+9/64*C/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-1
5/256*C/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/24*C*a/
c*(c*x^2+b*x+a)^(3/2)*x-1/48*C*a/c^2*(c*x^2+b*x+a)^(3/2)*b-1/16*C*a^2/c*(c*
x^2+b*x+a)^(1/2)*x
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(C\,x^2+A\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int((A + C*x^2)*(a + b*x + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + C x^{2}\right) \left(a + b x + c x^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*(a + b*x + c*x**2)**(3/2), x)
```

# 3.180 $\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx$

**Optimal**. Leaf size=157

$$-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right.}{64c^3}$$

```
[Out] -5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-1/128*(-4*a
*c+b^2)*(16*A*c^2-4*C*a*c+5*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/c^(7/2)+1/64*(16*A*c^2-4*C*a*c+5*C*b^2)*(2*c*x+b)*(c*x^2+b*x+a)^
(1/2)/c^3
```

**Rubi** [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acC+16Ac^2+5b^2C\right)}{64c^3}-\frac{\left(b^2-4ac\right)\left(-4acC+16Ac^2+5b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] ((16*A*c^2 + 5*b^2*C - 4*a*c*C)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(64*c^3)
 - (5*b*C*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (C*x*(a + b*x + c*x^2)^(3/2))
/(4*c) - ((b^2 - 4*a*c)*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(A+Cx^2\right)dx = \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4Ac-aC-\frac{5bCx}{2}\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= -\frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{Cx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(\frac{5b^2C}{2}+2c(4Ac-aC)\right)\int \sqrt{a}}{8c^2}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{C}{}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{C}{}$$

$$= \frac{\left(16Ac^2+5b^2C-4acC\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} - \frac{5bC\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{C}{}$$

**Mathematica [A]** time = 0.21, size = 144, normalized size = 0.92

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(C\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)+15b^3-10b^2cx\right)+48Ac^2(b+2cx)\right)-3\left(b^2-4ac\right)(}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(A + C*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(48*A*c^2*(b + 2*c*x) + C*(15*b^3 - 10*b^2
*c*x + 24*c^2*x*(a + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2))) - 3*(b^2 - 4*a*c)
*(16*A*c^2 + 5*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])])/(384*c^(7/2))
```

**fricas [A]** time = 0.88, size = 355, normalized size = 2.26

$$\left[-\frac{3\left(5\,Cb^4-24\,Cab^2c-64\,Aac^3+16\left(Ca^2+Ab^2\right)c^2\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 + A*b^2)*c^2)*s
qrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)
*sqrt(c) - 4*a*c) - 4*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c - 52*C*a*b
*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt(c*x^2 +
 b*x + a))/c^4, 1/384*(3*(5*C*b^4 - 24*C*a*b^2*c - 64*A*a*c^3 + 16*(C*a^2 +
 A*b^2)*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)
/(c^2*x^2 + b*c*x + a*c)) + 2*(48*C*c^4*x^3 + 8*C*b*c^3*x^2 + 15*C*b^3*c -
52*C*a*b*c^2 + 48*A*b*c^3 - 2*(5*C*b^2*c^2 - 12*C*a*c^3 - 48*A*c^4)*x)*sqrt
(c*x^2 + b*x + a))/c^4]
```

**giac [A]** time = 0.22, size = 160, normalized size = 1.02

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,Cx+\frac{Cb}{c}\right)x-\frac{5\,Cb^2c-12\,Cac^2-48\,Ac^3}{c^3}\right)x+\frac{15\,Cb^3-52\,Cabc+48\,Abc^2}{c^3}\right)+\frac{\left(5\,Cb^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*C*x + C*b/c)*x - (5*C*b^2*c - 12*C*a*c
^2 - 48*A*c^3)/c^3)*x + (15*C*b^3 - 52*C*a*b*c + 48*A*b*c^2)/c^3) + 1/128*(
5*C*b^4 - 24*C*a*b^2*c + 16*C*a^2*c^2 + 16*A*b^2*c^2 - 64*A*a*c^3)*log(abs(
-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

**maple [B]** time = 0.01, size = 327, normalized size = 2.08

$$\frac{Aa\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\frac{A\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}-\frac{C\,a^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}+\frac{3Ca\,b^2\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x)
```

```
[Out] 1/4*C*x*(c*x^2+b*x+a)^(3/2)/c-5/24*b*C*(c*x^2+b*x+a)^(3/2)/c^2+5/32*C/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*C/c^3*b^3*(c*x^2+b*x+a)^(1/2)+3/16*C/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*C/c^(7/2)*b^4*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*C*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*C*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*C*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/2*A*(c*x^2+b*x+a)^(1/2)*x+1/4*A/c*(c*x^2+b*x+a)^(1/2)*b
+1/2*A/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*A/c^(3/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+A),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 4.26, size = 240, normalized size = 1.53

$$A\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{C\,a\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] A*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (C*a*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
```

```
 b^2/4))/(2*c^(3/2))))/(4*c) + (A*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) - (5*C*b*((log((b + 2*c*x)/c^(1/2) + 2
*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (C*x*(a + b
*x + c*x^2)^(3/2))/(4*c)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + Cx^2\right)\sqrt{a + bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+A),x)
```

```
[Out] Integral((A + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

# 3.181 $\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=104

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] 1/8*(8*A*c^2-4*C*a*c+3*C*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+1/2*C*x*(c*x^2+b*x+a)^(1/2)/c
```

**Rubi [A]** time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1661, 640, 621, 206}

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}-\frac{3bC\sqrt{a+bx+cx^2}}{4c^2}+\frac{Cx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-3*b*C*Sqrt[a + b*x + c*x^2])/(4*c^2) + (C*x*Sqrt[a + b*x + c*x^2])/(2*c)
+ ((8*A*c^2 + 3*b^2*C - 4*a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2Ac-aC-\frac{3bCx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{4c^2}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(\frac{3b^2C}{2}+2c(2Ac-aC)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x\right.}{2c^2}$$

$$= -\frac{3bC\sqrt{a+bx+cx^2}}{4c^2} + \frac{Cx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8Ac^2+3b^2C-4acC\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+c}}\right.}{8c^{5/2}}$$

**Mathematica [A]** time = 0.15, size = 86, normalized size = 0.83

$$\frac{\left(-4acC+8Ac^2+3b^2C\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{8c^{5/2}} + \frac{C(2cx-3b)\sqrt{a+x(b+cx)}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (C*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*A*c^2 + 3*b^2*C - 4*
a*c*C)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(5/2))
```

**fricas [A]** time = 1.02, size = 203, normalized size = 1.95

$$\left[\frac{\left(3\,Cb^2-4\,Cac+8\,Ac^2\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)+4\left(2\,Cc^2x-3\,Cbc\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*C*c^2*x - 3*
C*b*c)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((3*C*b^2 - 4*C*a*c + 8*A*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(2*C*c^2*x - 3*C*b*c)*sqrt(c*x^2 + b*x + a))/c^3]
```

**giac [A]** time = 0.25, size = 84, normalized size = 0.81

$$\frac{1}{4}\,\sqrt{cx^2+bx+a}\left(\frac{2\,Cx}{c}-\frac{3\,Cb}{c^2}\right)-\frac{\left(3\,Cb^2-4\,Cac+8\,Ac^2\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}\,-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*C*x/c - 3*C*b/c^2) - 1/8*(3*C*b^2 - 4*C*a*c +
8*A*c^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/
2)
```

**maple [A]** time = 0.01, size = 136, normalized size = 1.31

$$\frac{A\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{Ca\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3C\,b^2\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}+\frac{\sqrt{c\,x^2+bx+}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*C*x*(c*x^2+b*x+a)^(1/2)/c-3/4*b*C*(c*x^2+b*x+a)^(1/2)/c^2+3/8*C/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*C*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+A*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{C\,x^2+A}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((A + C*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Cx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.182 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{3/2}} \, dx$

**Optimal**. Leaf size=98

$$\frac{C \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} - \frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)-2*(b*c*(A+a*C/
c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 621, 206}

$$\frac{C \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}} - \frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(c*(b^2 - 4*a*c)*S
qrt[a + b*x + c*x^2]) + (C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)C}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2C)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{C\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.73, size = 104, normalized size = 1.06

$$\frac{\frac{2\sqrt{c}\left(aC(b-2cx)+Ac(b+2cx)+b^2Cx\right)}{\sqrt{a+x(b+cx)}}-C\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x)))/Sqrt[a + x*(b +
c*x)] - (b^2 - 4*a*c)*C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(c^(3/2)*(-b^2 + 4*a*c))
```

**fricas [B]** time = 1.22, size = 403, normalized size = 4.11

$$\left[\frac{\left(Cab^2 - 4\,Ca^2c + \left(Cb^2c - 4\,Cac^2\right)x^2 + \left(Cb^3 - 4\,Cabc\right)x\right)\sqrt{c}\,\log\left(-8\,c^2x^2 - 8\,bcx - b^2 - 4\,\sqrt{cx^2 + bx + a}\,(2\,cx + \right.}{2\left(ab^2c^2 - 4\,a^2c^3 + \left(b^2c^3 - 4\,ac^4\right)x^2 + \left(b^3c^2 - \right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a*c^2)*x^2 + (C*b^3 - 4*C*a*b*c
)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) - 4*(C*a*b*c + A*b*c^2 + (C*b^2*c - 2*C*a*c^2 + 2*A
*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -((C*a*b^2 - 4*C*a^2*c + (C*b^2*c - 4*C*a
*c^2)*x^2 + (C*b^3 - 4*C*a*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(C*a*b*c + A*b*c^2 + (C
*b^2*c - 2*C*a*c^2 + 2*A*c^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x)]
```

**giac [A]** time = 0.27, size = 110, normalized size = 1.12

$$-\frac{2\left(\frac{\left(Cb^2 - 2\,Cac + 2\,Ac^2\right)x}{b^2c - 4\,ac^2} + \frac{Cab + Abc}{b^2c - 4\,ac^2}\right)}{\sqrt{cx^2 + bx + a}} - \frac{C\log\left(\left|-2\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((C*b^2 - 2*C*a*c + 2*A*c^2)*x/(b^2*c - 4*a*c^2) + (C*a*b + A*b*c)/(b^2*
c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - C*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*sqrt(c) - b))/c^(3/2)
```

**maple [A]** time = 0.01, size = 169, normalized size = 1.72

$$\frac{C\,b^2x}{\left(4ac - b^2\right)\sqrt{c\,x^2 + bx + a}\;c} + \frac{C\,b^3}{2\left(4ac - b^2\right)\sqrt{c\,x^2 + bx + a}\;c^2} + \frac{2\,(2cx + b)\,A}{\left(4ac - b^2\right)\sqrt{c\,x^2 + bx + a}} - \frac{Cx}{\sqrt{c\,x^2 + bx + a}\;c} + \frac{C\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -C*x/c/(c*x^2+b*x+a)^(1/2)+1/2*C/c^2*b/(c*x^2+b*x+a)^(1/2)+C/c*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*C/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+C/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2*A*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [B]** time = 4.21, size = 108, normalized size = 1.10

$$\frac{C\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}+\frac{A\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{C\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (C*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) + (A*(b/2 +
c*x))/((a*c - b^2/4)*(a + b*x + c*x^2)^(1/2)) + (C*((a*b)/2 - x*(a*c - b^2/
2)))/(c*(a*c - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Cx^2}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((A + C*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

# 3.183 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=114

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] -2/3*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(3/2)+2/3*(8*A*c+4*a*C+b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1
/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1660, 12, 613}

$$\frac{2(b+2cx)\left(4aC+8Ac+\frac{b^2C}{c}\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(3*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(3/2)) + (2*(8*A*c + 4*a*C + (b^2*C)/c)*(b + 2*c*x))/(3*
(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{2\int \frac{8Ac+4aC+\frac{b^2C}{c}}{2\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}-\frac{\left(8Ac+4aC+\frac{b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{3\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2\left(8Ac+4aC+\frac{b^2C}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

**Mathematica [A]** time = 0.89, size = 107, normalized size = 0.94

$$\frac{2C\left(8a^2b+4ax\left(3b^2+3bcx+2c^2x^2\right)+b^2x^2(3b+2cx)\right)-2A(b+2cx)\left(-4c\left(3a+2cx^2\right)+b^2-8bcx\right)}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*A*(b + 2*c*x)*(b^2 - 8*b*c*x - 4*c*(3*a + 2*c*x^2)) + 2*C*(8*a^2*b + b^
2*x^2*(3*b + 2*c*x) + 4*a*x*(3*b^2 + 3*b*c*x + 2*c^2*x^2)))/(3*(b^2 - 4*a*c
)^2*(a + x*(b + c*x))^(3/2))
```

**fricas [B]** time = 2.34, size = 242, normalized size = 2.12

$$\frac{2\left(8\,Ca^2b-Ab^3+12\,Aabc+2\left(Cb^2c+4\,Cac^2+8\,Ac^3\right)x^3+3\left(Cb^3+4\,Cabc+8\,Abc^2\right)x^2+6\left(2\,Cab^2+\right.\right.}{3\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2+\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)x^4+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^3+\left(b^6-6\,ab^4c+32\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*C*a^2*b - A*b^3 + 12*A*a*b*c + 2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x^3
 + 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)*x^2 + 6*(2*C*a*b^2 + A*b^2*c + 4*A*a*c
^2)*x)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x
^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b
*c^2)*x)
```

**giac [A]** time = 0.26, size = 193, normalized size = 1.69

$$\frac{2\left(\left(\left(\frac{2\left(Cb^2c+4\,Cac^2+8\,Ac^3\right)x}{b^4-8\,ab^2c+16\,a^2c^2}+\frac{3\left(Cb^3+4\,Cabc+8\,Abc^2\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{6\left(2\,Cab^2+Ab^2c+4\,Aac^2\right)}{b^4-8\,ab^2c+16\,a^2c^2}\right)x+\frac{8\,Ca^2b-Ab^3+12\,Aabc}{b^4-8\,ab^2c+16\,a^2c^2}\right)}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(((2*(C*b^2*c + 4*C*a*c^2 + 8*A*c^3)*x/(b^4 - 8*a*b^2*c + 16*a^2*c^2) +
 3*(C*b^3 + 4*C*a*b*c + 8*A*b*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + 6*(2
*C*a*b^2 + A*b^2*c + 4*A*a*c^2)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))*x + (8*C*a^
2*b - A*b^3 + 12*A*a*b*c)/(b^4 - 8*a*b^2*c + 16*a^2*c^2))/(c*x^2 + b*x + a)
^(3/2)
```

**maple [A]** time = 0.01, size = 137, normalized size = 1.20

$$\frac{\frac{32}{3}A\,c^3x^3+\frac{16}{3}Ca\,c^2x^3+\frac{4}{3}C\,b^2c\,x^3+16Ab\,c^2x^2+8Cabc\,x^2+2C\,b^3x^2+16Aa\,c^2x+4A\,b^2cx+8Ca\,b^2x+8Aabc+}{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(16a^2c^2-8a\,b^2c+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(16*A*c^3*x^3+8*C*a*c^2*x^3+2*C*b^2*c*x^3+24*A*b*c^
2*x^2+12*C*a*b*c*x^2+3*C*b^3*x^2+24*A*a*c^2*x+6*A*b^2*c*x+12*C*a*b^2*x+12*A
*a*b*c-A*b^3+8*C*a^2*b)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [B]** time = 4.14, size = 127, normalized size = 1.11

$$\frac{2\left(8\,C\,a^2\,b+12\,C\,a\,b^2\,x+12\,C\,a\,b\,c\,x^2+12\,A\,a\,b\,c+8\,C\,a\,c^2\,x^3+24\,A\,a\,c^2\,x+3\,C\,b^3\,x^2-A\,b^3+2\,C\,b^2\,c\,x^3+6\right.}{3\left(4\,a\,c-b^2\right)^2\left(c\,x^2+b\,x+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] (2*(16*A*c^3*x^3 - A*b^3 + 3*C*b^3*x^2 + 8*C*a^2*b + 24*A*a*c^2*x + 6*A*b^2
*c*x + 12*C*a*b^2*x + 24*A*b*c^2*x^2 + 8*C*a*c^2*x^3 + 2*C*b^2*c*x^3 + 12*A
*a*b*c + 12*C*a*b*c*x^2))/(3*(4*a*c - b^2)^2*(a + b*x + c*x^2)^(3/2))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

# 3.184 $\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx$

**Optimal**. Leaf size=167

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\frac{3b\cdots}{}\right.}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\cdots\right.}$$

```
[Out] -2/5*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(5/2)+2/15*(16*A*c+4*a*C+3*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(3/2)-16/15*(16*A*c^2+4*C*a*c+3*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b*
x+a)^(1/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$-\frac{16(b+2cx)\left(4acC+16Ac^2+3b^2C\right)}{15\left(b^2-4ac\right)^3\sqrt{a+bx+cx^2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\left(\frac{aC}{c}+A\right)\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2(b+2cx)\left(4aC+16Ac+\frac{3b\cdots}{}\right.}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(5*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(5/2)) + (2*(16*A*c + 4*a*C + (3*b^2*C)/c)*(b + 2*c*x))/
(15*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(3/2)) - (16*(16*A*c^2 + 3*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(15*(b^2 - 4*a*c)^3*Sqrt[a + b*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{7/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{2\int \frac{16Ac+4aC+\frac{3b^2C}{c}}{2\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}-\frac{\left(16Ac+4aC+\frac{3b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{5/2}}\,dx}{5\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}+$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{5c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{5/2}}+\frac{2\left(16Ac+4aC+\frac{3b^2C}{c}\right)(b+2cx)}{15\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{3/2}}-$$

**Mathematica [A]** time = 1.93, size = 148, normalized size = 0.89

$$\frac{2\left(\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))\left(4acC+16Ac^2+3b^2C\right)-8c(b+2cx)(a+x(b+cx))^2\left(4acC+16Ac^2+3\right.\right.}{15c\left(b^2-4ac\right)^3(a+x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(7/2),x]
```

```
[Out] (2*((b^2 - 4*a*c)*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*
x)) - 8*c*(16*A*c^2 + 3*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b + c*x))^2 -
3*(b^2 - 4*a*c)^2*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2*c*x))))/(15*c*(b^
2 - 4*a*c)^3*(a + x*(b + c*x))^(5/2))
```

**fricas [B]** time = 17.77, size = 563, normalized size = 3.37

$$-\frac{2\left(8\,Ca^{2}b^{3}+3\,Ab^{5}+240\,Aa^{2}bc^{2}+16\left(3\,Cb^{2}c^{3}+4\,Cac^{4}+16\,Ac^{5}\right)x^{5}+40\left(3\,Cb^{3}c^{2}+4\,Cabc^{3}+16\,Abc^{4}\right)\cdots\right.}{15\left(a^{3}b^{6}-12\,a^{4}b^{4}c+48\,a^{5}b^{2}c^{2}-64\,a^{6}c^{3}+\left(b^{6}c^{3}-12\,ab^{4}c^{4}+48\,a^{2}b^{2}c^{5}-64\,a^{3}c^{6}\right)x^{6}+3\left(b^{7}c^{2}-12\,ab^{5}c^{3}+48\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*C*a^2*b^3 + 3*A*b^5 + 240*A*a^2*b*c^2 + 16*(3*C*b^2*c^3 + 4*C*a*c^
4 + 16*A*c^5)*x^5 + 40*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)*x^4 + 10*(9
*C*b^4*c + 24*C*a*b^2*c^2 + 64*A*a*c^4 + 16*(C*a^2 + 3*A*b^2)*c^3)*x^3 + 5*
(3*C*b^5 + 40*C*a*b^3*c + 192*A*a*b*c^3 + 16*(3*C*a^2*b + A*b^3)*c^2)*x^2 +
 8*(12*C*a^3*b - 5*A*a*b^3)*c + 10*(2*C*a*b^4 + 24*A*a*b^2*c^2 + 48*A*a^2*c
^3 + (24*C*a^2*b^2 - A*b^4)*c)*x)*sqrt(c*x^2 + b*x + a)/(a^3*b^6 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c
^5 - 64*a^3*c^6)*x^6 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*
b*c^5)*x^5 + 3*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64
*a^4*c^5)*x^4 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a
^4*b*c^4)*x^3 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 -
 64*a^5*c^4)*x^2 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^
3)*x)
```

**giac [B]** time = 0.28, size = 452, normalized size = 2.71

$$-\frac{2\left(\left(\left(2\left(4\left(\frac{2\left(3\,Cb^{2}c^{3}+4\,Cac^{4}+16\,Ac^{5}\right)x}{b^{6}-12\,ab^{4}c+48\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}}+\frac{5\left(3\,Cb^{3}c^{2}+4\,Cabc^{3}+16\,Abc^{4}\right)}{b^{6}-12\,ab^{4}c+48\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}}\right)x+\frac{5\left(9\,Cb^{4}c+24\,Cab^{2}c^{2}+16\,Ca^{2}c^{3}+48\,Ab^{2}c^{3}+64\,Aac^{4}\right)}{b^{6}-12\,ab^{4}c+48\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}}\right)x+\cdots\right.\right.\right.}{1\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*(((2*(4*(2*(3*C*b^2*c^3 + 4*C*a*c^4 + 16*A*c^5)*x/(b^6 - 12*a*b^4*c +
 48*a^2*b^2*c^2 - 64*a^3*c^3) + 5*(3*C*b^3*c^2 + 4*C*a*b*c^3 + 16*A*b*c^4)/
(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(9*C*b^4*c + 24*C*a
```

```
*b^2*c^2 + 16*C*a^2*c^3 + 48*A*b^2*c^3 + 64*A*a*c^4)/(b^6 - 12*a*b^4*c + 48
*a^2*b^2*c^2 - 64*a^3*c^3))*x + 5*(3*C*b^5 + 40*C*a*b^3*c + 48*C*a^2*b*c^2
+ 16*A*b^3*c^2 + 192*A*a*b*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3
*c^3))*x + 10*(2*C*a*b^4 + 24*C*a^2*b^2*c - A*b^4*c + 24*A*a*b^2*c^2 + 48*A
*a^2*c^3)/(b^6 - 12*a*b^4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))*x + (8*C*a^2*b^
3 + 3*A*b^5 + 96*C*a^3*b*c - 40*A*a*b^3*c + 240*A*a^2*b*c^2)/(b^6 - 12*a*b^
4*c + 48*a^2*b^2*c^2 - 64*a^3*c^3))/(c*x^2 + b*x + a)^(5/2)
```

**maple [B]** time = 0.01, size = 316, normalized size = 1.89

$$\frac{\frac{128}{15}Ca\,c^4x^5+\frac{32}{5}C\,b^2c^3x^5+\frac{256}{3}Ab\,c^4x^4+16C\,b^3c^2x^4+\frac{256}{3}Aa\,c^4x^3+64A\,b^2c^3x^3+\frac{64}{3}C\,a^2c^3x^3+12C\,b^4c\,x^3+\frac{32}{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x)
```

```
[Out] 2/15/(c*x^2+b*x+a)^(5/2)*(256*A*c^5*x^5+64*C*a*c^4*x^5+48*C*b^2*c^3*x^5+640
*A*b*c^4*x^4+160*C*a*b*c^3*x^4+120*C*b^3*c^2*x^4+640*A*a*c^4*x^3+480*A*b^2*
c^3*x^3+160*C*a^2*c^3*x^3+240*C*a*b^2*c^2*x^3+90*C*b^4*c*x^3+960*A*a*b*c^3*
x^2+80*A*b^3*c^2*x^2+240*C*a^2*b*c^2*x^2+200*C*a*b^3*c*x^2+15*C*b^5*x^2+480
*A*a^2*c^3*x+240*A*a*b^2*c^2*x-10*A*b^4*c*x+240*C*a^2*b^2*c*x+20*C*a*b^4*x+
240*A*a^2*b*c^2-40*A*a*b^3*c+3*A*b^5+96*C*a^3*b*c+8*C*a^2*b^3)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [B]** time = 4.53, size = 578, normalized size = 3.46

$$\frac{\frac{b\,c\,\left(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{2\,c^2\,x\,\left(56\,C\,b^2+256\,A\,c^2+32\,C\,a\,c\right)}{15\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}}{\sqrt{c\,x^2+b\,x+a}}+\frac{\frac{8\,C\,b\,c}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}+\frac{16\,C\,c^2\,x}{15\,\left(4\,a\,c^2-b^2\,c\right)\,\left(4\,a\,c-b^2\right)}}{\sqrt{c\,x^2+b\,x+a}}-\frac{\frac{4\,C\,x}{15\,\left(4\,a\,c-b^2\right)}-}{\left(c\,x^2+b\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(7/2),x)
```

```
[Out] ((b*c*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2
)^2) + (2*c^2*x*(256*A*c^2 + 56*C*b^2 + 32*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4
*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((8*C*b*c)/(15*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)) + (16*C*c^2*x)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*
x + c*x^2)^(1/2) - ((4*C*x)/(15*(4*a*c - b^2)) - (2*C*b)/(15*c*(4*a*c - b^2
)))/(a + b*x + c*x^2)^(3/2) + (x*((4*A*c^2)/(5*(4*a*c^2 - b^2*c)) + (2*C*b^
2)/(5*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(5*(4*a*c^2 - b^2*c))) + (2*A*b*c)/(5*
(4*a*c^2 - b^2*c)) + (2*C*a*b)/(5*(4*a*c^2 - b^2*c)))/(a + b*x + c*x^2)^(5/
2) + (x*((2*c*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(15*(4*a*c^2 - b^2*c)*(4*a*c
- b^2)) + (16*C*a*c^2)/(15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*b^2*c)/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(32*A*c^2 + 8*C*b^2 + 8*C*a*c))/(
15*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*a*b*c)/(15*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)))/(a + b*x + c*x^2)^(3/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

# 3.185 $\int \frac{A+Cx^2}{(a+bx+cx^2)^{9/2}}\,dx$

**Optimal**. Leaf size=220

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\right.}{7c\left(b^2-4ac\right)\left(a+bx+cx^{2}\right.}$$

```
[Out] -2/7*(b*c*(A+a*C/c)+(2*A*c^2+(-2*a*c+b^2)*C)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a
)^(7/2)+2/35*(24*A*c+4*a*C+5*b^2*C/c)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^2+b*x+a
)^(5/2)-32/105*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^2+b
*x+a)^(3/2)+256/105*c*(24*A*c^2+4*C*a*c+5*C*b^2)*(2*c*x+b)/(-4*a*c+b^2)^4/(
c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1660, 12, 614, 613}

$$\frac{256c(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^4\sqrt{a+bx+cx^2}}-\frac{32(b+2cx)\left(4acC+24Ac^2+5b^2C\right)}{105\left(b^2-4ac\right)^3\left(a+bx+cx^2\right)^{3/2}}-\frac{2\left(x\left(C\left(b^2-2ac\right)+2Ac^2\right)+bc\right.}{7c\left(b^2-4ac\right)\left(a+bx+cx^{2}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (-2*(b*c*(A + (a*C)/c) + (2*A*c^2 + (b^2 - 2*a*c)*C)*x))/(7*c*(b^2 - 4*a*c)
*(a + b*x + c*x^2)^(7/2)) + (2*(24*A*c + 4*a*C + (5*b^2*C)/c)*(b + 2*c*x))/
(35*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)^(5/2)) - (32*(24*A*c^2 + 5*b^2*C + 4*
a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^3*(a + b*x + c*x^2)^(3/2)) + (256*c*
(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x))/(105*(b^2 - 4*a*c)^4*Sqrt[a + b
*x + c*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Cx^2}{\left(a+bx+cx^2\right)^{9/2}}\,dx = -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{2\int \frac{24Ac+4aC+\frac{5b^2C}{c}}{2\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}-\frac{\left(24Ac+4aC+\frac{5b^2C}{c}\right)\int \frac{1}{\left(a+bx+cx^2\right)^{7/2}}\,dx}{7\left(b^2-4ac\right)}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}+\frac{(16\cdots}{\cdots}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32\cdots}{1\cdots}$$

$$= -\frac{2\left(bc\left(A+\frac{aC}{c}\right)+\left(2Ac^2+\left(b^2-2ac\right)C\right)x\right)}{7c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{7/2}}+\frac{2\left(24Ac+4aC+\frac{5b^2C}{c}\right)(b+2cx)}{35\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)^{5/2}}-\frac{32\cdots}{1\cdots}$$

**Mathematica [A]** time = 1.74, size = 199, normalized size = 0.90

$$\frac{2\left(3\left(b^2-4ac\right)^2(b+2cx)(a+x(b+cx))\left(4acC+24Ac^2+5b^2C\right)-16c\left(b^2-4ac\right)(b+2cx)(a+x(b+cx))^2\left(4ac\right.\right.}{105c\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*x^2)/(a + b*x + c*x^2)^(9/2),x]
```

```
[Out] (2*(3*(b^2 - 4*a*c)^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a + x*(b
+ c*x)) - 16*c*(b^2 - 4*a*c)*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a
+ x*(b + c*x))^2 + 128*c^2*(24*A*c^2 + 5*b^2*C + 4*a*c*C)*(b + 2*c*x)*(a +
x*(b + c*x))^3 - 15*(b^2 - 4*a*c)^3*(b^2*C*x + a*C*(b - 2*c*x) + A*c*(b + 2
*c*x))))/(105*c*(b^2 - 4*a*c)^4*(a + x*(b + c*x))^(7/2))
```

**fricas [B]** time = 52.07, size = 978, normalized size = 4.45

$$-\frac{2\left(8\,Ca^2b^5+15\,Ab^7-6720\,Aa^3bc^3-256\left(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7\right)x^7-896\left(5\,Cb^3c^4+4\,C\right.\right.}{105\left(a^4b^8-16\,a^5b^6c+96\,a^6b^4c^2-256\,a^7b^2c^3+256\,a^8c^4+\left(b^8c^4-16\,ab^6c^5+96\,a^2b^4c^6-256\,a^3b^2c^7+256\,a^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*C*a^2*b^5 + 15*A*b^7 - 6720*A*a^3*b*c^3 - 256*(5*C*b^2*c^5 + 4*C*
a*c^6 + 24*A*c^7)*x^7 - 896*(5*C*b^3*c^4 + 4*C*a*b*c^5 + 24*A*b*c^6)*x^6 -
224*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 96*A*a*c^6 + 8*(2*C*a^2 + 15*A*b^2)*c^
5)*x^5 - 560*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 96*A*a*b*c^5 + 8*(2*C*a^2*b +
3*A*b^3)*c^4)*x^4 - 70*(5*C*b^6*c + 124*C*a*b^4*c^2 + 384*A*a^2*c^5 + 64*(C
*a^3 + 9*A*a*b^2)*c^4 + 8*(22*C*a^2*b^2 + 3*A*b^4)*c^3)*x^3 - 240*(8*C*a^4*
b - 7*A*a^2*b^3)*c^2 + 7*(5*C*b^7 - 196*C*a*b^5*c - 5760*A*a^2*b*c^4 - 960*
(C*a^3*b + A*a*b^3)*c^3 - 8*(170*C*a^2*b^3 - 3*A*b^5)*c^2)*x^2 - 4*(80*C*a^
3*b^3 + 63*A*a*b^5)*c + 14*(2*C*a*b^6 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4 -
 60*(8*C*a^3*b^2 - A*a*b^4)*c^2 - (80*C*a^2*b^4 + 3*A*b^6)*c)*x)*sqrt(c*x^2
 + b*x + a)/(a^4*b^8 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3 + 25
6*a^8*c^4 + (b^8*c^4 - 16*a*b^6*c^5 + 96*a^2*b^4*c^6 - 256*a^3*b^2*c^7 + 25
6*a^4*c^8)*x^8 + 4*(b^9*c^3 - 16*a*b^7*c^4 + 96*a^2*b^5*c^5 - 256*a^3*b^3*c
^6 + 256*a^4*b*c^7)*x^7 + 2*(3*b^10*c^2 - 46*a*b^8*c^3 + 256*a^2*b^6*c^4 -
576*a^3*b^4*c^5 + 256*a^4*b^2*c^6 + 512*a^5*c^7)*x^6 + 4*(b^11*c - 13*a*b^9
*c^2 + 48*a^2*b^7*c^3 + 32*a^3*b^5*c^4 - 512*a^4*b^3*c^5 + 768*a^5*b*c^6)*x
^5 + (b^12 - 4*a*b^10*c - 90*a^2*b^8*c^2 + 800*a^3*b^6*c^3 - 2240*a^4*b^4*c
^4 + 1536*a^5*b^2*c^5 + 1536*a^6*c^6)*x^4 + 4*(a*b^11 - 13*a^2*b^9*c + 48*a
^3*b^7*c^2 + 32*a^4*b^5*c^3 - 512*a^5*b^3*c^4 + 768*a^6*b*c^5)*x^3 + 2*(3*a
^2*b^10 - 46*a^3*b^8*c + 256*a^4*b^6*c^2 - 576*a^5*b^4*c^3 + 256*a^6*b^2*c^
```

```
4 + 512*a^7*c^5)*x^2 + 4*(a^3*b^9 - 16*a^4*b^7*c + 96*a^5*b^5*c^2 - 256*a^6
*b^3*c^3 + 256*a^7*b*c^4)*x)
```

**giac [B]** time = 0.31, size = 805, normalized size = 3.66

$$2\left(\left(\left(2\left(8\left(2\left(4\left(\frac{2\left(5\,Cb^2c^5+4\,Cac^6+24\,Ac^7\right)x}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4}+\frac{7\left(5\,Cb^3c^4+4\,Cabc^5+24\,Abc^6\right)}{b^8-16\,ab^6c+96\,a^2b^4c^2-256\,a^3b^2c^3+256\,a^4c^4}\right)x+\frac{7\left(25\,Cb^4c^3+40\,Cab^2c^4+16\,C\cdots\right.}{b^8-16\,ab^6c+96\,a^2b^4c^2-\cdots}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(((2*(8*(2*(4*(2*(5*C*b^2*c^5 + 4*C*a*c^6 + 24*A*c^7)*x/(b^8 - 16*a*b
^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4) + 7*(5*C*b^3*c^4 + 4
*C*a*b*c^5 + 24*A*b*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c
^3 + 256*a^4*c^4))*x + 7*(25*C*b^4*c^3 + 40*C*a*b^2*c^4 + 16*C*a^2*c^5 + 12
0*A*b^2*c^5 + 96*A*a*c^6)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*
c^3 + 256*a^4*c^4))*x + 35*(5*C*b^5*c^2 + 24*C*a*b^3*c^3 + 16*C*a^2*b*c^4 +
 24*A*b^3*c^4 + 96*A*a*b*c^5)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*
b^2*c^3 + 256*a^4*c^4))*x + 35*(5*C*b^6*c + 124*C*a*b^4*c^2 + 176*C*a^2*b^2
*c^3 + 24*A*b^4*c^3 + 64*C*a^3*c^4 + 576*A*a*b^2*c^4 + 384*A*a^2*c^5)/(b^8
- 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x - 7*(5*C*
b^7 - 196*C*a*b^5*c - 1360*C*a^2*b^3*c^2 + 24*A*b^5*c^2 - 960*C*a^3*b*c^3 -
 960*A*a*b^3*c^3 - 5760*A*a^2*b*c^4)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 2
56*a^3*b^2*c^3 + 256*a^4*c^4))*x - 14*(2*C*a*b^6 - 80*C*a^2*b^4*c - 3*A*b^6
*c - 480*C*a^3*b^2*c^2 + 60*A*a*b^4*c^2 - 720*A*a^2*b^2*c^3 - 960*A*a^3*c^4
)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))*x -
(8*C*a^2*b^5 + 15*A*b^7 - 320*C*a^3*b^3*c - 252*A*a*b^5*c - 1920*C*a^4*b*c^
2 + 1680*A*a^2*b^3*c^2 - 6720*A*a^3*b*c^3)/(b^8 - 16*a*b^6*c + 96*a^2*b^4*c
^2 - 256*a^3*b^2*c^3 + 256*a^4*c^4))/(c*x^2 + b*x + a)^(7/2)
```

**maple [B]** time = 0.01, size = 555, normalized size = 2.52

$$\frac{\frac{2048}{105}Ca\,c^6x^7+\frac{512}{21}C\,b^2c^5x^7+\frac{2048}{5}Ab\,c^6x^6+\frac{256}{3}C\,b^3c^4x^6+\frac{2048}{5}Aa\,c^6x^5+512A\,b^2c^5x^5+\frac{1024}{15}C\,a^2c^5x^5+\frac{320}{3}C\,b^4c^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x)
```

```
[Out] 2/105/(c*x^2+b*x+a)^(7/2)*(6144*A*c^7*x^7+1024*C*a*c^6*x^7+1280*C*b^2*c^5*x
^7+21504*A*b*c^6*x^6+3584*C*a*b*c^5*x^6+4480*C*b^3*c^4*x^6+21504*A*a*c^6*x^
5+26880*A*b^2*c^5*x^5+3584*C*a^2*c^5*x^5+8960*C*a*b^2*c^4*x^5+5600*C*b^4*c^
```

```
3*x^5+53760*A*a*b*c^5*x^4+13440*A*b^3*c^4*x^4+8960*C*a^2*b*c^4*x^4+13440*C*
a*b^3*c^3*x^4+2800*C*b^5*c^2*x^4+26880*A*a^2*c^5*x^3+40320*A*a*b^2*c^4*x^3+
1680*A*b^4*c^3*x^3+4480*C*a^3*c^4*x^3+12320*C*a^2*b^2*c^3*x^3+8680*C*a*b^4*
c^2*x^3+350*C*b^6*c*x^3+40320*A*a^2*b*c^4*x^2+6720*A*a*b^3*c^3*x^2-168*A*b^
5*c^2*x^2+6720*C*a^3*b*c^3*x^2+9520*C*a^2*b^3*c^2*x^2+1372*C*a*b^5*c*x^2-35
*C*b^7*x^2+13440*A*a^3*c^4*x+10080*A*a^2*b^2*c^3*x-840*A*a*b^4*c^2*x+42*A*b
^6*c*x+6720*C*a^3*b^2*c^2*x+1120*C*a^2*b^4*c*x-28*C*a*b^6*x+6720*A*a^3*b*c^
3-1680*A*a^2*b^3*c^2+252*A*a*b^5*c-15*A*b^7+1920*C*a^4*b*c^2+320*C*a^3*b^3*
c-8*C*a^2*b^5)/(256*a^4*c^4-256*a^3*b^2*c^3+96*a^2*b^4*c^2-16*a*b^6*c+b^8)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+A)/(c*x^2+b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [**B**] time = 5.06, size = 1018, normalized size = 4.63

$$\frac{x\left(\frac{2\,c^2\,\left(160\,C\,b^2+768\,A\,c^2+96\,C\,a\,c\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}-\frac{64\,C\,a\,c^3}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{32\,C\,b^2\,c^2}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}\right)+\frac{b\,c\,\left(160\,C\,b^2+768\,A\,c^2+96\,C\,a\,c\right)}{105\,\left(4\,a\,c^2-b^2\,c\right)\,{\left(4\,a\,c-b^2\right)}^2}+\frac{}{105}}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)/(a + b*x + c*x^2)^(9/2),x)
```

```
[Out] (x*((2*c^2*(768*A*c^2 + 160*C*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*
c - b^2)^2) - (64*C*a*c^3)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*
b^2*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2)) + (b*c*(768*A*c^2 + 160*C
*b^2 + 96*C*a*c))/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (32*C*a*b*c^2)/
(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(3/2) - ((8*C*b)
/(105*(4*a*c - b^2)^2) - (16*C*c*x)/(105*(4*a*c - b^2)^2))/(a + b*x + c*x^2
)^(3/2) + ((8*C*b*c)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*c^2*x)/(
105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(3/2) - ((4*C*x)/(3
5*(4*a*c - b^2)) - (2*C*b)/(35*c*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) +
((b*c*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4*a*c^2 - b^2*c)*(4*
a*c - b^2)^3) + (2*c^2*x*(6144*A*c^3 + 896*C*a*c^2 + 1312*C*b^2*c))/(105*(4
*a*c^2 - b^2*c)*(4*a*c - b^2)^3))/(a + b*x + c*x^2)^(1/2) + (x*((4*A*c^2)/(
7*(4*a*c^2 - b^2*c)) + (2*C*b^2)/(7*(4*a*c^2 - b^2*c)) - (4*C*a*c)/(7*(4*a*
```

```
c^2 - b^2*c))) + (2*A*b*c)/(7*(4*a*c^2 - b^2*c)) + (2*C*a*b)/(7*(4*a*c^2 -
b^2*c)))/(a + b*x + c*x^2)^(7/2) + (x*((2*c*(48*A*c^2 + 12*C*b^2 + 8*C*a*c)
)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) + (16*C*a*c^2)/(35*(4*a*c^2 - b^2*c)
*(4*a*c - b^2)) - (8*C*b^2*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2))) + (b*(4
8*A*c^2 + 12*C*b^2 + 8*C*a*c))/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)) - (8*C*
a*b*c)/(35*(4*a*c^2 - b^2*c)*(4*a*c - b^2)))/(a + b*x + c*x^2)^(5/2) - ((32
*C*b*c^2)/(105*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (64*C*c^3*x)/(105*(4*a*
c^2 - b^2*c)*(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2) + ((64*C*b*c^2)/(105
*(4*a*c^2 - b^2*c)*(4*a*c - b^2)^2) + (128*C*c^3*x)/(105*(4*a*c^2 - b^2*c)*
(4*a*c - b^2)^2))/(a + b*x + c*x^2)^(1/2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+A)/(c*x**2+b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

# 3.186 $\int (g+hx)^3 \sqrt{a+bx+cx^2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=930

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c\right.}{}$$

```
[Out] 1/280*(33*b^2*f*h^2-2*c*h*(16*a*f*h+21*b*e*h+8*b*f*g)-4*c^2*(3*f*g^2-7*h*(2
*d*h+e*g)))*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^3/h-1/84*(11*b*f*h-14*c*e*h+6*c
*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2/h+1/7*f*(h*x+g)^4*(c*x^2+b*x+a)^(3/
2)/c/h+1/13440*(1155*b^4*f*h^4-128*c^4*g^2*(3*f*g^2-7*h*(12*d*h+e*g))-42*b^
2*c*h^3*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+343*a*b*h*(e*h
+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(15*f*g^2+7*h*(
d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*c*h*(231*b^3*f*h^3-6*b*c
*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*g*(3*f*g^2-7*h*(7*d*h+e*g))+8*c^2*
h*(a*h*(35*e*h+41*f*g)+b*(5*f*g^2+7*h*(7*d*h+9*e*g))))*x)*(c*x^2+b*x+a)^(3/
2)/c^5/h-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*
f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*
h^2+3*e*g*h+3*f*g^2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+
16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^
2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13
/2)+1/1024*(256*c^5*d*g^3-33*b^5*f*h^3+6*b^3*c*h^2*(20*a*f*h+7*b*(e*h+3*f*g
))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^2+3*e*g*h+3*f*g^
2))-64*c^4*g*(2*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+16*c^3*(2*a^2*h^2*
(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6
```

**Rubi [A]** time = 3.01, antiderivative size = 927, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{3/2}(g+hx)^4}{7ch}-\frac{(6cfg-14ceh+11bfh)\left(cx^2+bx+a\right)^{3/2}(g+hx)^3}{84c^2h}+\frac{\left(-4\left(3fg^2-7h(eg+2dh)\right)c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((256*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*
b*g*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h
*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)
) + 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(10
```

```
24*c^6) + ((33*b^2*f*h^2 - 2*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3
*f*g^2 - 7*h*(e*g + 2*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(280*c^3*
h) - ((6*c*f*g - 14*c*e*h + 11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/
(84*c^2*h) + (f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) + ((1155*b^4*f
*h^4 - 128*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h
+ 35*b*(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h)
+ b^2*(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h
*(3*e*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3
*f*h^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g
*h*(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*
g + 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(13440*c^5*h) - ((b^2 - 4*a*c)*(2
56*c^5*d*g^3 - 33*b^5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g
*(e*g + 3*d*h)) + 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(1
0*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*
b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2048*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch} + \frac{\int (g+hx)^3\left(-\frac{1}{2}h(3bfg-14cdh+\right.}{}$$

$$= -\frac{(6cfg-14ceh+11bfh)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2h} + \frac{f(g+hx)}{}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2a\right.\right.}{280c^3h}$$

$$= \frac{\left(33b^2fh^2-2ch(8bfg+21beh+16afh)-4c^2\left(3fg^2-7h(eg+2a\right.\right.}{280c^3h}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

$$= \frac{\left(256c^5dg^3-33b^5fh^3-64c^4g\left(afg^2+3ah(eg+dh)+2bg(eg+3\right.\right.}{}$$

**Mathematica [A]** time = 2.42, size = 1093, normalized size = 1.18

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(-3465fh^3b^6+210ch^2(63fg+21eh+11fhx)b^5-84ch\left(-260afh^2+35c(6eg+2dh+ehx)h\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g +
 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h + e*
h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*h^3
 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h*x +
 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*(20*g
^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*h*x +
189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*(35*g +
 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3) + 7*h*
(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 32*b*c^3*(
a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 609*g^2*h*x
+ 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(195*g^2 + 87
```

```
*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g*h^2*x^2 + h
^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + f*x*(35*
g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(128*a^3*f*h^3 -
a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315*g*h*x + 64*h^2
*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) + 7*e*(40*g^3 + 45
*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*g^2*h*x + 35*g*h^
2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x + 15*g*h^2*x^2 + 4
*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 + 10*h^3*x^3) + 3*f*
x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))))) + 105*(b^2 - 4*a*c)
*(-256*c^5*d*g^3 + 33*b^5*f*h^3 + 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2
*b*g*(e*g + 3*d*h)) - 6*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) + 8*b*c^2*
h*(10*a^2*f*h^2 + 14*a*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2
)) - 16*c^3*(2*a^2*h^2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) +
6*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])])/(215040*c^(13/2))
```

**fricas [A]** time = 2.45, size = 2817, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^
5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*
c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)
*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*
c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3
 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3
*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^
7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*
h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(
48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 -
 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^
4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b
*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 -
1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^
4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e +
 (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 +
 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b
*c^6*e - (9*b^2*c^5 - 20*a*c^6)*f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 2
```

0*a*c^6)*e + (99*b^3*c^4 - 316*a*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c
^6*f)*g^3 + 168*(80*c^7*d + 10*b*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h +
42*(40*b*c^6*d - 4*(7*b^2*c^5 - 16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*
g*h^2 - (56*(7*b^2*c^5 - 16*a*c^6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (23
1*b^4*c^3 - 972*a*b^2*c^4 + 512*a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8
*b*c^6*e - (5*b^2*c^5 - 12*a*c^6)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5
- 12*a*c^6)*e + (35*b^3*c^4 - 116*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 1
2*a*c^6)*d - 4*(35*b^3*c^4 - 116*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4
+ 240*a^2*c^5)*f)*g*h^2 + (56*(35*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^
3 - 448*a*b^2*c^4 + 240*a^2*c^5)*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*
a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^7, 1/215040*(105*(16*(16*(b^
2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e + (5*b^4*c^3 - 24*a*b^2*c^4
+ 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*c^5)*d - 2*(5*b^4*c^3 - 24*a
*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*f)*g^2
*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 4*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64
*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*d - 2*(21
*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*e + (33*b^7 - 252*a*
b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(15360*c^
7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(5
04*c^7*f*g^2*h + 42*(12*c^7*e + b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e -
(11*b^2*c^5 - 24*a*c^6)*f)*h^3)*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*
c^6)*e + (15*b^3*c^4 - 52*a*b*c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d
 - 10*(15*b^3*c^4 - 52*a*b*c^5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*
c^5)*f)*g^2*h + 42*(40*(15*b^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a
*b^2*c^4 + 256*a^2*c^5)*e + (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)
*f)*g*h^2 - (56*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5
*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 +
 34608*a^2*b^2*c^3 - 8192*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^
7*e + b*c^6*f)*g^2*h + 42*(120*c^7*d + 12*b*c^6*e - (9*b^2*c^5 - 20*a*c^6)*
f)*g*h^2 + (168*b*c^6*d - 14*(9*b^2*c^5 - 20*a*c^6)*e + (99*b^3*c^4 - 316*a
*b*c^5)*f)*h^3)*x^3 + 8*(560*(8*c^7*e + b*c^6*f)*g^3 + 168*(80*c^7*d + 10*b
*c^6*e - (7*b^2*c^5 - 16*a*c^6)*f)*g^2*h + 42*(40*b*c^6*d - 4*(7*b^2*c^5 -
16*a*c^6)*e + (21*b^3*c^4 - 68*a*b*c^5)*f)*g*h^2 - (56*(7*b^2*c^5 - 16*a*c^
6)*d - 14*(21*b^3*c^4 - 68*a*b*c^5)*e + (231*b^4*c^3 - 972*a*b^2*c^4 + 512*
a^2*c^5)*f)*h^3)*x^2 + 2*(560*(48*c^7*d + 8*b*c^6*e - (5*b^2*c^5 - 12*a*c^6
)*f)*g^3 + 168*(80*b*c^6*d - 10*(5*b^2*c^5 - 12*a*c^6)*e + (35*b^3*c^4 - 11
6*a*b*c^5)*f)*g^2*h - 42*(40*(5*b^2*c^5 - 12*a*c^6)*d - 4*(35*b^3*c^4 - 116
*a*b*c^5)*e + (105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)*f)*g*h^2 + (56*(3
5*b^3*c^4 - 116*a*b*c^5)*d - 14*(105*b^4*c^3 - 448*a*b^2*c^4 + 240*a^2*c^5)
*e + (1155*b^5*c^2 - 6048*a*b^3*c^3 + 6352*a^2*b*c^4)*f)*h^3)*x)*sqrt(c*x^2
 + b*x + a))/c^7]

**giac [A]** time = 0.31, size = 1702, normalized size = 1.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h^
2 + b*c^5*f*h^3 + 14*c^6*h^3*e)/c^6)*x + (504*c^6*f*g^2*h + 42*b*c^5*f*g*h^
2 + 168*c^6*d*h^3 - 11*b^2*c^4*f*h^3 + 24*a*c^5*f*h^3 + 504*c^6*g*h^2*e + 1
4*b*c^5*h^3*e)/c^6)*x + (1680*c^6*f*g^3 + 504*b*c^5*f*g^2*h + 5040*c^6*d*g*
h^2 - 378*b^2*c^4*f*g*h^2 + 840*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 + 99*b^3*c^
3*f*h^3 - 316*a*b*c^4*f*h^3 + 5040*c^6*g^2*h*e + 504*b*c^5*g*h^2*e - 126*b^
2*c^4*h^3*e + 280*a*c^5*h^3*e)/c^6)*x + (560*b*c^5*f*g^3 + 13440*c^6*d*g^2*
h - 1176*b^2*c^4*f*g^2*h + 2688*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 + 882*b^
3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 + 896*a*c^5*d*h^3
- 231*b^4*c^2*f*h^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3 + 4480*c^6*g^
3*e + 1680*b*c^5*g^2*h*e - 1176*b^2*c^4*g*h^2*e + 2688*a*c^5*g*h^2*e + 294*
b^3*c^3*h^3*e - 952*a*b*c^4*h^3*e)/c^6)*x + (26880*c^6*d*g^3 - 2800*b^2*c^4
*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g^2*h + 5880*b^3*c^3*f*g^2*h - 19
488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 - 4410*b^4
*c^2*f*g*h^2 + 18816*a*b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c
^3*d*h^3 - 6496*a*b*c^4*d*h^3 + 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6
352*a^2*b*c^3*f*h^3 + 4480*b*c^5*g^3*e - 8400*b^2*c^4*g^2*h*e + 20160*a*c^5
*g^2*h*e + 5880*b^3*c^3*g*h^2*e - 19488*a*b*c^4*g*h^2*e - 1470*b^4*c^2*h^3*
e + 6272*a*b^2*c^3*h^3*e - 3360*a^2*c^4*h^3*e)/c^6)*x + (26880*b*c^5*d*g^3
+ 8400*b^3*c^3*f*g^3 - 29120*a*b*c^4*f*g^3 - 40320*b^2*c^4*d*g^2*h + 107520
*a*c^5*d*g^2*h - 17640*b^4*c^2*f*g^2*h + 77280*a*b^2*c^3*f*g^2*h - 43008*a^
2*c^4*f*g^2*h + 25200*b^3*c^3*d*g*h^2 - 87360*a*b*c^4*d*g*h^2 + 13230*b^5*c
*f*g*h^2 - 70560*a*b^3*c^2*f*g*h^2 + 75936*a^2*b*c^3*f*g*h^2 - 5880*b^4*c^2
*d*h^3 + 25760*a*b^2*c^3*d*h^3 - 14336*a^2*c^4*d*h^3 - 3465*b^6*f*h^3 + 218
40*a*b^4*c*f*h^3 - 34608*a^2*b^2*c^2*f*h^3 + 8192*a^3*c^3*f*h^3 - 13440*b^2
*c^4*g^3*e + 35840*a*c^5*g^3*e + 25200*b^3*c^3*g^2*h*e - 87360*a*b*c^4*g^2*
h*e - 17640*b^4*c^2*g*h^2*e + 77280*a*b^2*c^3*g*h^2*e - 43008*a^2*c^4*g*h^2
*e + 4410*b^5*c*h^3*e - 23520*a*b^3*c^2*h^3*e + 25312*a^2*b*c^3*h^3*e)/c^6)
 + 1/2048*(256*b^2*c^5*d*g^3 - 1024*a*c^6*d*g^3 + 80*b^4*c^3*f*g^3 - 384*a*
b^2*c^4*f*g^3 + 256*a^2*c^5*f*g^3 - 384*b^3*c^4*d*g^2*h + 1536*a*b*c^5*d*g^
2*h - 168*b^5*c^2*f*g^2*h + 960*a*b^3*c^3*f*g^2*h - 1152*a^2*b*c^4*f*g^2*h
+ 240*b^4*c^3*d*g*h^2 - 1152*a*b^2*c^4*d*g*h^2 + 768*a^2*c^5*d*g*h^2 + 126*
b^6*c*f*g*h^2 - 840*a*b^4*c^2*f*g*h^2 + 1440*a^2*b^2*c^3*f*g*h^2 - 384*a^3*
c^4*f*g*h^2 - 56*b^5*c^2*d*h^3 + 320*a*b^3*c^3*d*h^3 - 384*a^2*b*c^4*d*h^3
- 33*b^7*f*h^3 + 252*a*b^5*c*f*h^3 - 560*a^2*b^3*c^2*f*h^3 + 320*a^3*b*c^3*
f*h^3 - 128*b^3*c^4*g^3*e + 512*a*b*c^5*g^3*e + 240*b^4*c^3*g^2*h*e - 1152*
a*b^2*c^4*g^2*h*e + 768*a^2*c^5*g^2*h*e - 168*b^5*c^2*g*h^2*e + 960*a*b^3*c
```

```
^3*g*h^2*e - 1152*a^2*b*c^4*g*h^2*e + 42*b^6*c*h^3*e - 280*a*b^4*c^2*h^3*e
+ 480*a^2*b^2*c^3*h^3*e - 128*a^3*c^4*h^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

**maple [B]** time = 0.02, size = 3543, normalized size = 3.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 3/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g^2*h-7/40/c^2*b*x*(c*x^2+b*x+a)^(3/2)*d*h^
3-3/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2*h+3/4*x
*(c*x^2+b*x+a)^(3/2)/c*d*g*h^2+3/4*x*(c*x^2+b*x+a)^(3/2)/c*e*g^2*h-5/8/c^2*
b*(c*x^2+b*x+a)^(3/2)*d*g*h^2+15/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*d*g*h^2+15/
64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*e*g^2*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^3-15/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*d*g*h^2-15/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e*g^2*h-1/8*a/c*x*(c*x^2+b*x+a)^(1/2)*f*g^3-1/16*a/c^2*(c*
x^2+b*x+a)^(1/2)*b*f*g^3-3/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*d*g*h^2-33/320*f*h^3/c^4*b^3*x*(c*x^2+b*x+a)^(3/2)-33/512*f*h^3/c
^5*b^5*x*(c*x^2+b*x+a)^(1/2)+15/128*f*h^3/c^5*b^4*a*(c*x^2+b*x+a)^(1/2)-39/
160*f*h^3/c^4*b^2*a*(c*x^2+b*x+a)^(3/2)-5/64*f*h^3/c^4*b^2*a^2*(c*x^2+b*x+a
)^(1/2)-63/512*f*h^3/c^(11/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*a+35/128*f*h^3/c^(9/2)*b^3*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))-5/32*f*h^3/c^(7/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3
5*f*h^3*a/c^2*x^2*(c*x^2+b*x+a)^(3/2)-11/84*f*h^3/c^2*b*x^3*(c*x^2+b*x+a)^(
3/2)+33/280*f*h^3/c^3*b^2*x^2*(c*x^2+b*x+a)^(3/2)+7/16/c^3*b^2*(c*x^2+b*x+a
)^(3/2)*f*g^2*h-7/64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*d*h^3-21/128/c^4*b^4*(c*
x^2+b*x+a)^(1/2)*e*g*h^2-21/128/c^4*b^4*(c*x^2+b*x+a)^(1/2)*f*g^2*h-2/5*a/c
^2*(c*x^2+b*x+a)^(3/2)*e*g*h^2-2/5*a/c^2*(c*x^2+b*x+a)^(3/2)*f*g^2*h-5/32/c
^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h^3+21/256/c^(9/
2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h^2-3/8*a/c*x*(c*x^2
+b*x+a)^(1/2)*e*g^2*h-3/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*g*h^2-3/16*a/c^2*(
c*x^2+b*x+a)^(1/2)*b*e*g^2*h-3/4/c*b*x*(c*x^2+b*x+a)^(1/2)*d*g^2*h-3/4/c^(3
/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g^2*h-3/8*a/c*x*(c*x^
2+b*x+a)^(1/2)*d*g*h^2+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g^3+1/2*d*g^3*x*(c*x^2+b
*x+a)^(1/2)+21/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
f*g^2*h+3/32/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*d*h^3+3/16/c^(5/2)*b*a^2*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^3+7/16/c^3*b^2*(c*x^2+b*x+a)^(3/2)
*e*g*h^2-5/8/c^2*b*(c*x^2+b*x+a)^(3/2)*e*g^2*h+5/32/c^2*b^2*x*(c*x^2+b*x+a)
^(1/2)*f*g^3-1/4/c*b*x*(c*x^2+b*x+a)^(1/2)*e*g^3-3/8/c^2*b^2*(c*x^2+b*x+a)^
(1/2)*d*g^2*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e
*g^3+3/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g^2*h+2
1/512/c^5*b^5*(c*x^2+b*x+a)^(1/2)*e*h^3+15/64*f*h^3/c^4*b^3*a*x*(c*x^2+b*x+
```

a)^(1/2)+111/560*f*h^3/c^3*b*a*x*(c*x^2+b*x+a)^(3/2)-15/32/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g^2*h+3/16/c^2*b*a*x*(c*x^2+b*x+a)^(1/2)*d*h^3+9/32/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*e*g*h^2+9/32/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*f*g^2*h+9/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h^2+9/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2*h-21/40/c^2*b*x*(c*x^2+b*x+a)^(3/2)*e*g*h^2-21/40/c^2*b*x*(c*x^2+b*x+a)^(3/2)*f*g^2*h-21/64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*e*g*h^2-21/64/c^3*b^3*x*(c*x^2+b*x+a)^(1/2)*f*g^2*h+7/48/c^3*b^2*(c*x^2+b*x+a)^(3/2)*d*h^3-7/128/c^4*b^4*(c*x^2+b*x+a)^(1/2)*d*h^3+7/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^3-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*d*h^3+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g^3-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g^3+5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g^3-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^3-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^3+(c*x^2+b*x+a)^(3/2)/c*d*g^2*h-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*e*g^3+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^3+1/4*d*g^3/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d*g^3/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d*g^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+3/32*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b*f*g*h^2-3/8*a/c^2*x*(c*x^2+b*x+a)^(3/2)*f*g*h^2+3/16*a^2/c^2*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2+105/256/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g*h^2-7/32/c^3*b^2*a*x*(c*x^2+b*x+a)^(1/2)*e*h^3-21/64/c^4*b^3*a*(c*x^2+b*x+a)^(1/2)*f*g*h^2-45/64/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2+49/80/c^3*b*a*(c*x^2+b*x+a)^(3/2)*f*g*h^2+3/5*x^2*(c*x^2+b*x+a)^(3/2)/c*e*g*h^2+63/160/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)*f*g*h^2+63/256/c^4*b^4*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2-5/32*f*h^3/c^3*b*a^2*x*(c*x^2+b*x+a)^(1/2)-15/32/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h^2+15/32/c^2*b^2*x*(c*x^2+b*x+a)^(1/2)*d*g*h^2+15/32/c^2*b^2*x*(c*x^2+b*x+a)^(1/2)*e*g^2*h+9/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h^2+9/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2*h-21/32/c^3*b^2*a*x*(c*x^2+b*x+a)^(1/2)*f*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^(1/2)*e*g*h^2+9/16/c^2*b*a*x*(c*x^2+b*x+a)^(1/2)*f*g^2*h-7/64/c^4*b^3*(c*x^2+b*x+a)^(3/2)*e*h^3-9/20/c^2*b*x^2*(c*x^2+b*x+a)^(3/2)*f*g*h^2-3/20/c^2*b*x^2*(c*x^2+b*x+a)^(3/2)*e*h^3+21/160/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)*e*h^3-21/64/c^4*b^3*(c*x^2+b*x+a)^(3/2)*f*g*h^2+21/256/c^4*b^4*x*(c*x^2+b*x+a)^(1/2)*e*h^3+63/512/c^5*b^5*(c*x^2+b*x+a)^(1/2)*f*g*h^2+35/256/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^3-63/1024/c^(11/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2-7/64/c^4*b^3*a*(c*x^2+b*x+a)^(1/2)*e*h^3-15/64/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+49/240/c^3*b*a*(c*x^2+b*x+a)^(3/2)*e*h^3-1/8*a/c^2*x*(c*x^2+b*x+a)^(3/2)*e*h^3+1/16*a^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e*h^3+1/32*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b*e*h^3+3/16*a^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h^2+1/2*x^3*(c*x^2+b*x+a)^(3/2)/c*f*g*h^2-21/1024/c^(11/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+1/16*a^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h^3+1/6*x^3*(c*x^2+b*x+a)^(3/2)/c*e*h^3+33/2048*f*h^3/c^(13/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/105*f*h^

```
3*a^2/c^3*(c*x^2+b*x+a)^(3/2)+11/128*f*h^3/c^5*b^4*(c*x^2+b*x+a)^(3/2)-33/1
024*f*h^3/c^6*b^6*(c*x^2+b*x+a)^(1/2)+1/7*f*h^3*x^4*(c*x^2+b*x+a)^(3/2)/c+1
/5*x^2*(c*x^2+b*x+a)^(3/2)/c*d*h^3
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 14.70, size = 3262, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*x
^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6) +
(d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)^(3
/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2 + b
/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x)/c^(1
/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*log((b +
 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
- (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1/2) + 2*(a
+ b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3
*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e*g^3*(8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (33*b^7*f*h
^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048*c^(13/2)) + (f
*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x
*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/
2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*
c^(3/2))))/(4*c)))/(2*c) + (7*b*d*h^3*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(
```

a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (3*b*e*h^3*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (3*d*g*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*e*g^2*h*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (3*a*f*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(2*c) + (21*b*e*g*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (21*b*f*g^2*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (9*b*f*g*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (35*a^2*b^3*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(128*c^(9/2)) + (13*a*b^4*f*h^3*(a + b*x + c*x^2)^(1/2))/(64*c^5) - (4*a*f*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(35*c^2) - (11*b*f*h^3*x^3*(a + b*x + c*x^2)^(3/2))/(84*c^2) - (33*b^3*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(320*c^4) + (11*b^5*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(512*c^5) + (3*e*g*h^2*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (3*f*g^2*h*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (f*g*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(2*c) - (3*a*d*g*h^2*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (3*a*e*g^2*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^

```
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/
(2*c^(3/2))))/(4*c) + (3*d*g^2*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (103*a^2*b^2*f*h^3*(a + b*x + c*
x^2)^(1/2))/(320*c^4) - (6*a*e*g*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*d*g*h^2*((log(
(b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/
2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2
)))/(8*c) - (6*a*f*g^2*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(
a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (15*b*e*g^2*h*((log((b + 2*c*x)/
c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) +
(8*a^2*f*h^3*x^2*(a + b*x + c*x^2)^(1/2))/(105*c^2) + (33*b^2*f*h^3*x^2*(a
+ b*x + c*x^2)^(3/2))/(280*c^3) + (11*b^4*f*h^3*x^2*(a + b*x + c*x^2)^(1/2)
)/(128*c^4) + (d*g^2*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2
)^(1/2))/(8*c^2) - (5*a^3*b*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2)
 + 2*c*x))/(32*c^(7/2)) - (63*a*b^5*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^
2)^(1/2) + 2*c*x))/(512*c^(11/2)) - (39*a*b^2*f*h^3*x^2*(a + b*x + c*x^2)^(
1/2))/(160*c^3) + (111*a*b*f*h^3*x*(a + b*x + c*x^2)^(3/2))/(560*c^3) - (26
9*a^2*b*f*h^3*x*(a + b*x + c*x^2)^(1/2))/(3360*c^3) - (3*a*b^3*f*h^3*x*(a +
 b*x + c*x^2)^(1/2))/(320*c^4)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g+hx\right)^3 \sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

# 3.187 $\int (g+hx)^2\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=584

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(dh^2+2egh+fg^2\right)\right)-28b^2ch(2afh+\right.}{1024c^{11/2}}$$

```
[Out] -1/20*(3*b*f*h-4*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2/h+1/6*f*(
h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^3+64*c^3*g*(f*g^2-2*h*(
5*d*h+e*g))-28*b*c*h^2*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*h*(16*a*h*(e*h+2*f*g
)+b*(7*f*g^2+25*h*(d*h+2*e*g)))-6*c*h*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*e*h+
2*b*f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4/h-1/1024
*(-4*a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g
)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+8*c^2*(2*a^2*f*h^2+6*a
*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2
)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/512*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*
h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^3*(2*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2)
)+8*c^2*(2*a^2*f*h^2+6*a*b*h*(e*h+2*f*g)+5*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

**Rubi** [A] time = 1.44, antiderivative size = 581, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\,\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bf\right.}{512c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 3
2*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*
h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)
*Sqrt[a + b*x + c*x^2])/(512*c^5) - ((2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x
)^2*(a + b*x + c*x^2)^(3/2))/(20*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^
(3/2))/(6*c*h) - ((105*b^3*f*h^3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 2
8*b*c*h^2*(7*a*f*h + 5*b*(2*f*g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*
g + d*h) + 16*a*h*(2*f*g + e*h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7
*b*e*h + 5*a*f*h) - 8*c^2*(f*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)
^(3/2))/(960*c^4*h) - ((b^2 - 4*a*c)*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 28*b^2
*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 2*
b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^2*(f*
```

```
g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(1024*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch} + \frac{\int (g+hx)^2\left(-\frac{3}{2}h(bfg-4cdh+2\right.}{}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx)^3}{}$$

$$= -\frac{(2cfg-4ceh+3bfh)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2h} + \frac{f(g+hx)^3}{}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg\right.\right.}{}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg\right.\right.}{}$$

$$= \frac{\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3\left(afg\right.\right.}{}$$

**Mathematica [A]** time = 0.97, size = 436, normalized size = 0.75

$$\frac{3h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c\right.}{512c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((-3*(2*c*f*g - 4*c*e*h + 3*b*f*h)*(g + h*x)^2*(a + x*(b + c*x))^(3/2))/(10
*c) + f*(g + h*x)^3*(a + x*(b + c*x))^(3/2) - ((a + x*(b + c*x))^(3/2)*(105
*b^3*f*h^3 - 14*b*c*h^2*(14*a*f*h + b*(20*f*g + 10*e*h + 9*f*h*x)) + 8*c^2*
```

```
h*(b*f*g*(7*g + 6*h*x) + b*h*(50*e*g + 25*d*h + 21*e*h*x) + a*h*(32*f*g + 1
6*e*h + 15*f*h*x)) + 16*c^3*(f*g^2*(4*g + 3*h*x) - h*(2*e*g*(4*g + 3*h*x) +
 5*d*h*(8*g + 3*h*x))))))/(160*c^3) + (3*h*(128*c^4*d*g^2 + 21*b^4*f*h^2 - 2
8*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a*f*g^2 + a*h*(2*e*g + d*h)
 + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6*a*b*h*(2*f*g + e*h) + 5*b^
2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]
- (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(5
12*c^(9/2)))/(6*c*h)
```

**fricas [A]** time = 1.75, size = 1791, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5
*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*
d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 +
 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7
*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b
^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*x^5 + 1
28*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(
10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*
f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52
*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*
c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c
^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315
*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f
)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^
5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 +
2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d
 - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5
*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 44
8*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6, 1/15360*(
15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (5*b^4*c^2 -
24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4)*d - 2*(5*b^
4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c
^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d - 4*(7*b^5*c - 40
*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64
*a^3*c^3)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(1280*c^6*f*h^2*x^5 + 128*(24*c^6*f*g*h
+ (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^2 + 24*(10*c^6*e + b*c^5*
```

```
f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 20*a*c^5)*f)*h^2)*x^3 + 40*
(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^3*c^3 - 52*a*b*c^4)*f)*g^2
- 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^3 - 52*a*b*c^4)*e + (105*b^4
*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (40*(15*b^3*c^3 - 52*a*b*c^4)*
d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*e + (315*b^5*c - 1680*a*b
^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c^6*e + b*c^5*f)*g^2 + 8*(80*c^6
*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)*g*h + (40*b*c^5*d - 4*(7*b^2*c^
4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4)*f)*h^2)*x^2 + 2*(40*(48*c^6*d +
 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4
- 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c
^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e + (105*b^4*c^2 - 448*a*b^2*c^3 + 240
*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

**giac [A]** time = 0.31, size = 1012, normalized size = 1.73

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,fh^2x+\frac{24\,c^5fgh+bc^4fh^2+12\,c^5h^2e}{c^5}\right)x+\frac{120\,c^5fg^2+24\,bc^4fgh+120\,c^5dh\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + b*c^
4*f*h^2 + 12*c^5*h^2*e)/c^5)*x + (120*c^5*f*g^2 + 24*b*c^4*f*g*h + 120*c^5*
d*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^2 + 240*c^5*g*h*e + 12*b*c^4*h^2*e)/
c^5)*x + (40*b*c^4*f*g^2 + 640*c^5*d*g*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g
*h + 40*b*c^4*d*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2 + 320*c^5*g^2*e +
 80*b*c^4*g*h*e - 28*b^2*c^3*h^2*e + 64*a*c^4*h^2*e)/c^5)*x + (1920*c^5*d*g
^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c^4*d*g*h + 280*b^3*c^2*f*
g*h - 928*a*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 - 105*b^4*c*f
*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h^2 + 320*b*c^4*g^2*e - 400*b^2*
c^3*g*h*e + 960*a*c^4*g*h*e + 140*b^3*c^2*h^2*e - 464*a*b*c^3*h^2*e)/c^5)*x
 + (1920*b*c^4*d*g^2 + 600*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^
3*d*g*h + 5120*a*c^4*d*g*h - 840*b^4*c*f*g*h + 3680*a*b^2*c^2*f*g*h - 2048*
a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2 + 315*b^5*f*h^2 - 16
80*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2 - 960*b^2*c^3*g^2*e + 2560*a*c^4*g^
2*e + 1200*b^3*c^2*g*h*e - 4160*a*b*c^3*g*h*e - 420*b^4*c*h^2*e + 1840*a*b^
2*c^2*h^2*e - 1024*a^2*c^3*h^2*e)/c^5) + 1/1024*(128*b^2*c^4*d*g^2 - 512*a*
c^5*d*g^2 + 40*b^4*c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 12
8*b^3*c^3*d*g*h + 512*a*b*c^4*d*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h
- 384*a^2*b*c^3*f*g*h + 40*b^4*c^2*d*h^2 - 192*a*b^2*c^3*d*h^2 + 128*a^2*c^
4*d*h^2 + 21*b^6*f*h^2 - 140*a*b^4*c*f*h^2 + 240*a^2*b^2*c^2*f*h^2 - 64*a^3
*c^3*f*h^2 - 64*b^3*c^3*g^2*e + 256*a*b*c^4*g^2*e + 80*b^4*c^2*g*h*e - 384*
a*b^2*c^3*g*h*e + 256*a^2*c^4*g*h*e - 28*b^5*c*h^2*e + 160*a*b^3*c^2*h^2*e
```

```
- 192*a^2*b*c^3*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(
c) - b))/c^(11/2)
```

**maple [B]** time = 0.02, size = 2179, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h-1/4*a/c*
(c*x^2+b*x+a)^(1/2)*x*e*g*h-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h-7/32/c^3*
b^3*(c*x^2+b*x+a)^(1/2)*x*f*g*h-5/16/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a*f*g*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g^2+1/2*d*g^2*(c*x^2+
b*x+a)^(1/2)*x-1/4*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
e*g*h+1/16*f*h^2*a^2/c^2*(c*x^2+b*x+a)^(1/2)*x+1/32*f*h^2*a^2/c^3*(c*x^2+b*
x+a)^(1/2)*b+21/256*f*h^2/c^4*b^4*(c*x^2+b*x+a)^(1/2)*x+35/256*f*h^2/c^(9/2
)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*f*h^2/c^4*b^3*a*(c
*x^2+b*x+a)^(1/2)-15/64*f*h^2/c^(7/2)*b^2*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))+49/240*f*h^2/c^3*b*a*(c*x^2+b*x+a)^(3/2)-1/8*f*h^2*a/c^2*x*(
c*x^2+b*x+a)^(3/2)+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*f*g^2+5/32/c^3*b^3*(c
*x^2+b*x+a)^(1/2)*e*g*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*a*d*h^2+2/5*x^2*(c*x^2+b*x+a)^(3/2)/c*f*g*h-7/40/c^2*b*x*(c*x^2+b
*x+a)^(3/2)*e*h^2+7/24/c^3*b^2*(c*x^2+b*x+a)^(3/2)*f*g*h-7/64/c^3*b^3*(c*x^
2+b*x+a)^(1/2)*x*e*h^2-7/64/c^4*b^4*(c*x^2+b*x+a)^(1/2)*f*g*h-5/32/c^(7/2)*
b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h^2+7/128/c^(9/2)*b^5*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+3/32/c^3*b^2*a*(c*x^2+b*x+
a)^(1/2)*e*h^2+3/16/c^(5/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*e*h^2-4/15*a/c^2*(c*x^2+b*x+a)^(3/2)*f*g*h-5/12/c^2*b*(c*x^2+b*x+a)^(3/2
)*e*g*h+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*d*h^2-3/20*f*h^2/c^2*b*x^2*(c*x^
2+b*x+a)^(3/2)+21/160*f*h^2/c^3*b^2*x*(c*x^2+b*x+a)^(3/2)-1/8*a/c*(c*x^2+b*
x+a)^(1/2)*x*d*h^2-1/8*a/c*(c*x^2+b*x+a)^(1/2)*x*f*g^2-1/16*a/c^2*(c*x^2+b*
x+a)^(1/2)*b*d*h^2-1/16*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g^2-1/8*d*g^2/c^(3/2)
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/5*x^2*(c*x^2+b*x+a)^(3/2
)/c*e*h^2+7/48/c^3*b^2*(c*x^2+b*x+a)^(3/2)*e*h^2-7/128/c^4*b^4*(c*x^2+b*x+a
)^(1/2)*e*h^2+7/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*e*h^2-2/15*a/c^2*(c*x^2+b*x+a)^(3/2)*e*h^2+1/6*f*h^2*x^3*(c*x^2+b*x+a)^(3/
2)/c-7/64*f*h^2/c^4*b^3*(c*x^2+b*x+a)^(3/2)+21/512*f*h^2/c^5*b^5*(c*x^2+b*x
+a)^(1/2)-21/1024*f*h^2/c^(11/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))+1/16*f*h^2*a^3/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/
24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g^2+3/16/c^2*b*a*(c*x^2+b*x+a)^(1/2)*x*e*h^2
+3/16/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*f*g*h+3/8/c^(5/2)*b*a^2*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h+5/16/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*e*g*h
+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h-1/2/c*
b*(c*x^2+b*x+a)^(1/2)*x*d*g*h-7/32*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^(1/2)*x-7/
```

```
20/c^2*b*x*(c*x^2+b*x+a)^(3/2)*f*g*h+1/2*d*g^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*a+1/4*d*g^2/c*(c*x^2+b*x+a)^(1/2)*b+5/64/c^3*b^3*(c
*x^2+b*x+a)^(1/2)*d*h^2+5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g^2-5/128/c^(7/2
)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-5/128/c^(7/2)*b^4*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/8*a^2/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*f*g^2+2/3*(c*x^2+b*x+a)^(3/2)/c*d*g*h-1/8/c^2*b^2*
(c*x^2+b*x+a)^(1/2)*e*g^2+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*e*g^2+1/4*x*(c*x^2+b*x+a)^(3/2)/c*d*h^2+1/4*x*(c*x^2+b*x+a)^(3/
2)/c*f*g^2-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*d*h^2+3/8/c^2*b*a*(c*x^2+b*x+a)^(
1/2)*x*f*g*h+3/16/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a
*f*g^2-5/64/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h+1
/2*x*(c*x^2+b*x+a)^(3/2)/c*e*g*h-1/4/c*b*(c*x^2+b*x+a)^(1/2)*x*e*g^2-1/4/c^
2*b^2*(c*x^2+b*x+a)^(1/2)*d*g*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a*e*g^2+1/8/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*d*g*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 7.91, size = 1881, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^2
)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/(4*c
))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(
1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(1/2) +
 (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b + 2*c*
x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (2*
```

```
a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b
*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^
(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g^2*((log((b
+ 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
 + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2)
)/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*g^2*x*(a + b*x +
c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x
 + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2
+ 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)
^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c
*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))
/(2*c) + (7*b*e*h^2*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*
(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*
c) + (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2)
 + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (
3*b*f*h^2*((7*b*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a +
 b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) +
 (a*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (
a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) - (2*a*
((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16
*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2)))/(5*c) + (x^2*(a + b*x + c*x^2)^(3/2))/(5*c)))/(4*c) + (2*f*g*h*x^
2*(a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*g*h*((x/2 + b/(4*c))*(a + b*x + c*x
^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/
4))/(2*c^(3/2))))/(2*c) + (d*g*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x
^2)^(1/2))*(b^3 - 4*a*b*c))/(8*c^(5/2)) - (4*a*f*g*h*((log((b + 2*c*x)/c^(1
/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a +
 c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b
*e*g*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(4*c) + (d*g*h*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(12*c^2) + (e*g*h*x*(a + b*x + c*x^2)^(3/2))/(2*c) + (7*b
*f*g*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*
x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x +
 c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(5*c)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^2 \sqrt{a+bx+cx^2}\,(d+ex+fx^2)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

# 3.188 $\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=322

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg\right)}{128c^4}+\frac{\left(a+bx\right.}{}$$

```
[Out] 1/5*f*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-16*c^2*(3*f*g^2
-5*h*(d*h+e*g))-2*c*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6*c
*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*f*h
-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+1/128*(32*c^3*d*g-7*b^3
*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*(2*
c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

**Rubi [A]** time = 0.50, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+25b(eh+fg))+35b^2fh^2-6chx(7bfh-10ceh+6cfg)+c^2\left(-\left(48fg^2-80h(dh\right.\right.\right.}{240c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2*b*
c*(6*a*f*h + 5*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^4)
 + (f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) + ((35*b^2*f*h^2 - c^2*(
48*f*g^2 - 80*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) - 6*c*h*
(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(240*c^3*h) - ((
b^2 - 4*a*c)*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a
*e*h) + 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(256*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(3bfg-10cdh+4a\ldots\right.}{\ldots}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch} + \frac{\left(35b^2fh^2-c^2\left(48fg^2-80h(eg+a\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5\ldots\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5\ldots\right.}{128c^4}$$

$$= \frac{\left(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5\ldots\right.}{128c^4}$$

**Mathematica [A]** time = 0.48, size = 258, normalized size = 0.80

$$\frac{\frac{(a+x(b+cx))^{3/2}\left(-2ch(16afh+b(25eh+25fg+21fhx))+35b^2fh^2+c^2(20h(4dh+4eg+3ehx)-12fg(4g+3hx))\right)}{48c^2} - \frac{5h\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\text{ta}\ldots\right.}{\ldots}}{5ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(3/2) + ((a + x*(b + c*x))^(3/2)*(35*b^2*f
*h^2 + c^2*(-12*f*g*(4*g + 3*h*x) + 20*h*(4*e*g + 4*d*h + 3*e*h*x)) - 2*c*h
*(16*a*f*h + b*(25*f*g + 25*e*h + 21*f*h*x))))/(48*c^2) - (5*h*(-32*c^3*d*g
 + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 5*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*
a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(256*c^(7/2))
)/(5*c*h)
```

**fricas [A]** time = 1.16, size = 1009, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5*
b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2*(
```

```
5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2
)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10*c^5*
e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*c^4*e
- (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 - 8*a*c^
4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d - 10*(1
5*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c^3)*f)*h
+ 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80*b*c^4*d -
 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*x)*sqrt(c*x
^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 -
4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 -
4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^
3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4 + 48*(10*c^5*f*
g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d +
 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*
c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^
4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^
2*c^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (
80*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

**giac [A]** time = 0.24, size = 495, normalized size = 1.54

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8fhx+\frac{10c^4fg+bc^3fh+10c^4he}{c^4}\right)x+\frac{10bc^3fg+80c^4dh-7b^2c^2fh+16ac^3fh}{c^4}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + b*c^3*f*h +
10*c^4*h*e)/c^4)*x + (10*b*c^3*f*g + 80*c^4*d*h - 7*b^2*c^2*f*h + 16*a*c^3*
f*h + 80*c^4*g*e + 10*b*c^3*h*e)/c^4)*x + (480*c^4*d*g - 50*b^2*c^2*f*g + 1
20*a*c^3*f*g + 80*b*c^3*d*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h + 80*b*c^3*g*e
 - 50*b^2*c^2*h*e + 120*a*c^3*h*e)/c^4)*x + (480*b*c^3*d*g + 150*b^3*c*f*g
- 520*a*b*c^2*f*g - 240*b^2*c^2*d*h + 640*a*c^3*d*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h - 240*b^2*c^2*g*e + 640*a*c^3*g*e + 150*b^3*c*h*
e - 520*a*b*c^2*h*e)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g + 10*b^4*
c*f*g - 48*a*b^2*c^2*f*g + 32*a^2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h
 - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h - 16*b^3*c^2*g*e + 64*a*b*
c^3*g*e + 10*b^4*c*h*e - 48*a*b^2*c^2*h*e + 32*a^2*c^3*h*e)*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 1117, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*(c*x^2+b*x+a)^(3/2)/c*d*h+1/3*(c*x^2+b*x+a)^(3/2)/c*e*g+1/2*d*g*(c*x^2+
b*x+a)^(1/2)*x+3/16*h*f/c^2*b*a*(c*x^2+b*x+a)^(1/2)*x-7/40*h*f/c^2*b*x*(c*x
^2+b*x+a)^(3/2)+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*e*h+3/16*h*f/c^(5/2)*b*a
^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/32*h*f/c^3*b^2*a*(c*x^2+b*
x+a)^(1/2)+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e*h+1/4*x*(c*x^2+b*x+a)^(3/2)/c*f*g-
5/32*h*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*h*f
/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x+5/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*f*g+3/16/
c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h+3/16/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/8*a/c*(c*x^2+b*x+a)
^(1/2)*x*e*h-1/8*a/c*(c*x^2+b*x+a)^(1/2)*x*f*g-1/16*a/c^2*(c*x^2+b*x+a)^(1/
2)*b*e*h-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h-5/
24/c^2*b*(c*x^2+b*x+a)^(3/2)*e*h-5/24/c^2*b*(c*x^2+b*x+a)^(3/2)*f*g+5/64/c^
3*b^3*(c*x^2+b*x+a)^(1/2)*e*h+1/2*d*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*a-1/8*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*b^2-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*d*h-1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*e
*g+1/16/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h+1/16/c^
(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/4*d*g/c*(c*x^2+
b*x+a)^(1/2)*b+1/5*h*f*x^2*(c*x^2+b*x+a)^(3/2)/c+7/48*h*f/c^3*b^2*(c*x^2+b*
x+a)^(3/2)-7/128*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)+7/256*h*f/c^(9/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/15*h*f*a/c^2*(c*x^2+b*x+a)^(3/2)+
5/64/c^3*b^3*(c*x^2+b*x+a)^(1/2)*f*g-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*e*h-5/128/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*f*g-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e*h-1/8*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/1
6*a/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g-1/4/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*a*e*g-1/4/c*b*(c*x^2+b*x+a)^(1/2)*x*d*h-1/4/c*b*(c*x^2+b*x
+a)^(1/2)*x*e*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 5.62, size = 877, normalized size = 2.72

$$dg\left(\frac{x}{2}+\frac{b}{4c}\right)\sqrt{cx^2+bx+a}-\frac{2afh\left(\frac{\ln\left(\frac{b+2cx}{\sqrt{c}}+2\sqrt{cx^2+bx+a}\right)\left(b^3-4abc\right)}{16c^{5/2}}+\frac{\left(-3b^2+2cxb+8c\left(cx^2+a\right)\right)\sqrt{cx^2+bx+a}}{24c^2}\right)}{5c}-\frac{5be}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5
*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(
1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x +
c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2
*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a + c*x^2) -
 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a + c*x^2)
 - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a + b*x + c
*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*b*f*h*((5*b
*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(1
6*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/
(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*
(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(a + b*x + c*x^2)
^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2
 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*
c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g*l
og((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)
) + (d*h*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*
c))/(16*c^(5/2)) + (e*g*log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (g + hx) \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

# 3.189 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=175

$$-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c^2d\right)}{128c^{7/2}}+\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-\right.}{64c^3}$$

```
[Out] 1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-1
/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*
d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
```

**Rubi [A]** time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(16c^2d-8bce+5b\right.}{}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+\right.}{24c^2}$$

**Mathematica [A]** time = 0.30, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)+15b^3f-2\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(7/2)
)
```

**fricas [A]** time = 0.80, size = 465, normalized size = 2.66

$$\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

**giac [A]** time = 0.24, size = 212, normalized size = 1.21

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{bc^2f+8\,c^3e}{c^3}\right)x+\frac{48\,c^3d-5\,b^2cf+12\,ac^2f+8\,bc^2e}{c^3}\right)x+\frac{48\,bc^2d+15\,b^3f-52}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
```

```
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

**maple [B]** time = 0.01, size = 453, normalized size = 2.59

$$-\frac{a^2 f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}+\frac{3a\,b^2 f \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{5}{2}}}-\frac{abe \ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{3}{2}}}+\frac{ad \ln\left(\frac{cx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f/c^2*b*(c*x^2+b*x+a)^(3/2)+5/32*f/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*x+5/64*f/c^3*b^3*(c*x^2+b*x+a)^(1/2)+3/16*f/c^(5/2)*
b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f/c^(7/2)*b^4*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*(c*x^2+b*x+a)^(1/2)*x-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e/c*b*(c*x^2+b*x+a)^(1/2)
*x-1/8*e/c^2*b^2*(c*x^2+b*x+a)^(1/2)-1/4*e/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*(c*x^2+b*x+a)^(1/2)*x+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [B]** time = 4.24, size = 320, normalized size = 1.83

$$d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}-\frac{a\,f\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}+\frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
+ c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c -
 b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^
2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b + 2*c*x
)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*
c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)
+ (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
+ (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

# 3.190 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

**Optimal**. Leaf size=321

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}-\sqrt{a}$$

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/16*(4*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f
*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-b^2*h^2-4*c*h*(-a*h+b*g)))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^4+(d*h^2-e*g*h+f*g^2)*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)*(a*h^2-b*g*h+c*g^2)^(1/2)/h^4-1/8*(4*c*h*(b*f*g-2*c*d*h)-(-b*h+4*c*g)*(b*
f*h-2*c*e*h+2*c*f*g)+2*c*h*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(1/2)/c
^2/h^3
```

**Rubi [A]** time = 0.78, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(bfh-2ceh+2cfg)\right)}{16c^{5/2}h^4}-\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] -((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*
c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(8*c^2*h^3) + (f*
(a + b*x + c*x^2)^(3/2))/(3*c*h) + ((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h)
- (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*h^4) + (S
qrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (
2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^4
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch} + \frac{\int \frac{\left(-\frac{3}{2}h(bfg-2cdh)-\frac{3}{2}h(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{g+hx}\,dx}{3ch^2}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

$$= -\frac{(4ch(bfg-2cdh)-(4cg-bh)(2cfg-2ceh+bfh)+2ch(2cfg-2ceh+}{8c^2h^3}$$

**Mathematica [A]** time = 0.79, size = 331, normalized size = 1.03

$$2\sqrt{c}\left(h\sqrt{a+x(b+cx)}\left(2ch(4afh+b(3eh-3fg+fhx))-3b^2fh^2+4c^2\left(3h(2dh-2eg+ehx)+f\left(6g^2-3ghx+\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (-3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2*a*f*h) + 16*c^3*g*(f*g^
2 + h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(e*g) + d*h) + a*h*(-(f*g)
 + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c
]*(h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e
*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x + 2
*h^2*x^2))) - 24*c^2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(f*g^2 + h*(-(e*g) + d*
h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) +
a*h)]*Sqrt[a + x*(b + c*x)])]))/(48*c^(5/2)*h^4)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 2549, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x)
```

```
[Out] 1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+1/2/h*e*(c*x^2+b*x+a)^(1/2)*x-1/h^2*((x+g/h)^
2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g+1/h^3*((x+g/h)
^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/h*((x+g/h
)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d+1/h^3*ln((1/2*
(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g^2*e-1/h^4*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*
c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))*c^(1/2)*g^3*f-1/h/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g
^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*
c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*d-1/8/h*
f/c^2*b^2*(c*x^2+b*x+a)^(1/2)+1/16/h*f/c^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))+1/4/h*e/c*(c*x^2+b*x+a)^(1/2)*b+1/2/h*e/c^(1/2)*ln((c*x
+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8/h*e/c^(3/2)*ln((c*x+1/2*b)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*b^2-1/2/h^2*f*g*(c*x^2+b*x+a)^(1/2)*x-1/2/h^2*ln((1
/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b*e*g+1/2/h^3*ln((1/2*(b*h-2*c*g)/h+(x+
g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)
^(1/2))/c^(1/2)*b*f*g^2+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-
b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((
x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a
*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b
*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c
```

```
*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*a*f*g^2+1/h^2/((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*g*d-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2))/(x+g/h))*b*g^2*e+1/2/h*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)*b
*d-1/h^2*ln((1/2*(b*h-2*c*g)/h+(x+g/h)*c)/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/
h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))*c^(1/2)*g*d+1/h^4/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2))/(x+g/h))*b*g^3*f-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*c*g^2*d+1/h^4/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*
h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/
h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3
*e-1/h^5/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h
-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^4*f-1/4/h*f/c*b*(c
*x^2+b*x+a)^(1/2)*x-1/4/h*f/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*a-1/4/h^2*f*g/c*(c*x^2+b*x+a)^(1/2)*b-1/2/h^2*f*g/c^(1/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/h^2*f*g/c^(3/2)*ln((c*x+1/2*b)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{g+hx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

# 3.191 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2} \, dx$

**Optimal**. Leaf size=459

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2ch^2x\left(\right.\right.}{}$$

```
[Out] -(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/8
*(b^2*f*h^2+4*c*h*(-a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*g*(3*
f*g^2-h*(-d*h+2*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(d*h^2-3*e*g*h+5*f*g^2)))*arc
tanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)
^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(1/2)-1/4*(b*f*h^2*(-a*h+b*g)+4*c^2*g*(3*f*
g^2-h*(-d*h+2*e*g))+c*h*(4*a*h*(-e*h+2*f*g)-b*(4*d*h^2-8*e*g*h+13*f*g^2))+2
*c*h^2*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/c/h
^3/(a*h^2-b*g*h+c*g^2)
```

**Rubi [A]** time = 1.10, antiderivative size = 453, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 814, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{8c^{3/2}h^4}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/h - d*h) + 4*a*c*h*(2*f*g
 - e*h) - b*c*(13*f*g^2 - 8*e*g*h + 4*d*h^2) + 2*c*h*(2*c*e*g + b*f*g - (3*
c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2*(c*g^2 - b
*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)) - ((b^2*f*h^2 + 4*c*h*(2*b*f*g - b*e*h - a*f*
h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2]])])/(8*c^(3/2)*h^4) - ((2*c*(3*f*g^3 - g*h*(2*e*g - d*h))
 - h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2
*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
]])])/(2*h^4*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+3beg+2afg-\frac{3bfg^2}{h}-bdh-2aeh\right.\right.}{}}{cg^2 -}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\right.\right.}{4ch^2\left(cg^2-\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\right.\right.}{4ch^2\left(cg^2-\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\right.\right.}{4ch^2\left(cg^2-\right.}$$

$$= -\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc\left(13fg^2-\right.\right.}{4ch^2\left(cg^2-\right.}$$

**Mathematica [A]** time = 1.56, size = 486, normalized size = 1.06

$$-\frac{\frac{\left(h(ah-bg)+cg^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4ch(afh+beh-2bfg)-b^2fh^2+8c^2\left(h(dh-2eg)+3fg^2\right)\right)}{\sqrt{c}}+2h\sqrt{a+x(b+cx)}\left(ch(2ah(2eh-4fg+fhx)+4bh(dh-2eg)+bfg(13g-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x) - ((3*c*f*g^2 + f*h*(-(b*g) + a*h) +
 2*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))
*(g + h*x)) - (2*h*Sqrt[a + x*(b + c*x)]*(b*f*h^2*(-(b*g) + a*h) + c*h*(4*b
```

```
*h*(-2*e*g + d*h) + b*f*g*(13*g - 2*h*x) + 2*a*h*(-4*f*g + 2*e*h + f*h*x))
+ c^2*(6*f*g^2*(-2*g + h*x) + 4*h*(e*g*(2*g - h*x) + d*h*(-g + h*x)))) + ((
c*g^2 + h*(-(b*g) + a*h))*(-(b^2*f*h^2) + 4*c*h*(-2*b*f*g + b*e*h + a*f*h)
+ 8*c^2*(3*f*g^2 + h*(-2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + x*(b + c*x)])]])/Sqrt[c] + 4*c*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*(2*c*(3*f*g
^3 + g*h*(-2*e*g + d*h)) - h*(5*b*f*g^2 + b*h*(-3*e*g + d*h) + 2*a*h*(-2*f*
g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(
b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]])/(4*h^3*(-(c*g^2) + h*(b*g - a*h))))/(
2*c*h)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 6218, normalized size = 13.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

## 3.192 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=448

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^2+1/8*(8*c^2*g^3*(-e*h+3*f*g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*e*
g*h+9*f*g^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h+3*
e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c
*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(3/2)-1/2*(-b*f*h-2*c*e*h+6*c*f*
g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/c^(1/2)+1/4*(4*c*
g^2*(-e*h+3*f*g)/h+4*a*h*(-e*h+3*f*g)-b*(-d*h^2-3*e*g*h+11*f*g^2)-2*h*(c*e*
g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/(a*h^2-b*g*
h+c*g^2)/(h*x+g)
```

**Rubi [A]** time = 0.87, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3\right.\right.}{8h^4\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f*g - e*h))/h - 4*a*h*(3*f*
g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h - c*d*h - 2*a*f*h)*x)*Sqrt[
a + b*x + c*x^2])/(4*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 - h*(
e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)
^2) - ((6*c*f*g - 2*c*e*h - b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[c]*h^4) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c*h*(b*g^2*
(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 - 4*
a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(b*g - 2*a
*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])
])/(8*h^4*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
```

```
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \int \frac{\left(\frac{1}{2}\left(-4cdg+3beg+4afg-\frac{3bfg^2}{h}+bdh-4a\right.\right.}{2\left(cg\right.}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bf\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bf\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bf\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx}$$

$$= -\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bf\right.\right.}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx}$$

**Mathematica [A]** time = 3.58, size = 645, normalized size = 1.44

$$-\frac{\frac{2c\sqrt{a+x(b+cx)}\left(h^2\left(-4a^2fh^2-4abh(eh-4fg)+b^2\left(dh^2+3egh-11fg^2\right)\right)+ch\left(b\left(h(dh(hx-g)+eg(3hx-7g))+fg^2(23g-7hx)\right)-2ah(h(dh-3eg+2ehx)+fg(9g-4hx))\right)-2c^2\left(gh\left(dh^2x+eg(hx-\right.\right.\right.}{h^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] ((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - ((3*c*f*g^2 + 2*f*h*(-(b*g) + a*
h) + c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(2*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^2) - ((-2*c*(6*c*f*g^3 - 2*c*g*h*(e*g + d*h) - 4*a*h^2*(-2*f
*g + e*h) + b*h*(-7*f*g^2 + h*(3*e*g + d*h)))*(a + x*(b + c*x))^(3/2))/(g +
```

```
 h*x) + (2*c*Sqrt[a + x*(b + c*x)]*(h^2*(-4*a^2*f*h^2 - 4*a*b*h*(-4*f*g + e
*h) + b^2*(-11*f*g^2 + 3*e*g*h + d*h^2)) - 2*c^2*(3*f*g^3*(2*g - h*x) + g*h
*(d*h^2*x + e*g*(-2*g + h*x))) + c*h*(-2*a*h*(f*g*(9*g - 4*h*x) + h*(-3*e*g
 + d*h + 2*e*h*x)) + b*(f*g^2*(23*g - 7*h*x) + h*(d*h*(-g + h*x) + e*g*(-7*
g + 3*h*x))))))/h^2 + (4*Sqrt[c]*(6*c*f*g - 2*c*e*h - b*f*h)*(c*g^2 + h*(-(
b*g) + a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*S
qrt[c*g^2 + h*(-(b*g) + a*h)]*(8*c^2*g^3*(3*f*g - e*h) + 4*c*h*(b*g^2*(-10*
f*g + 3*e*h) + a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*
h*(-6*f*g + e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTanh[(-(b*g) + 2*a
*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x
)])])/h^3)/(8*(c*g^2 + h*(-(b*g) + a*h))^2))/(c*h)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.81Unable to divide, perha
ps due to rounding error%%%{1,[6,0,0,7,0,0,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,0,0,6,1,0,0,0]%%%}+%%%{3,[4,1,0,6,1,0,0,0]%%%}+%%%{-3,[4,
0,0,7,0,0,1,0]%%%}+%%%{%%%{12,[1]%%%},[4,0,0,5,2,0,0,0]%%%}+%%%{%%{[-12,0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,5,2,0,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]
%%%}]%%},[3,0,0,6,1,0,1,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,0,0,4,3,0,0,0]%%%}+%%%{3,[2,2,0,5,2,0,0,0]%%%}+%%%{-6,[2,1,0,6,1,0
,1,0]%%%}+%%%{%%%{12,[1]%%%},[2,1,0,4,3,0,0,0]%%%}+%%%{3,[2,0,0,7,0,0,2,0]%
%%}+%%%{%%%{-12,[1]%%%},[2,0,0,5,2,0,1,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,2,0,4,3,0,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,0
,5,2,0,1,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,0,6,1,0,2,0]%%%}
+%%%{1,[0,3,0,4,3,0,0,0]%%%}+%%%{-3,[0,2,0,5,2,0,1,0]%%%}+%%%{3,[0,1,0,6,1,
0,2,0]%%%}+%%%{-1,[0,0,0,7,0,0,3,0]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[6,0,0,3,0,0,0,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0,0,2,1,0,
0,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,1,0,2,1,0,0,0]
```

```
%%%}+%%%{%%{poly1[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,3,0,0,1,
0]%%%}+%%%{%%{poly1[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,1,2,0,
0,0]%%%}+%%%{%%%{-12,[2]%%%},[3,1,0,1,2,0,0,0]%%%}+%%%{%%%{12,[2]%%%},[3,0,
0,2,1,0,1,0]%%%}+%%%{%%%{-8,[3]%%%},[3,0,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{3,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,1,2,0,0,0]%%%}+%%%{%%{[%%%{-6,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,2,1,0,1,0]%%%}+%%%{%%{[%%%{12,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,0,3,0,0,0]%%%}+%%%{%%{poly1[%%%{3,[1]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,3,0,0,2,0]%%%}+%%%{%%{poly1[%%%{-12,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,1,2,0,1,0]%%%}+%%%{%%%{-6,[2]%%%},[1,2
,0,0,3,0,0,0]%%%}+%%%{%%%{12,[2]%%%},[1,1,0,1,2,0,1,0]%%%}+%%%{%%%{-6,[2]%%
%},[1,0,0,2,1,0,2,0]%%%}+%%%{%%{[%%%{1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,3,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,0,1,2,0,1,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,0,2
,1,0,2,0]%%%}+%%%{%%{poly1[%%%{-1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0
,3,0,0,3,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 12139, normalized size = 27.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^2} \left(d + ex + fx^2\right)}{\left(g + hx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**3,x)
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

# 3.193 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

**Optimal**. Leaf size=603

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(h^2\left(8a^2fh^2-2abh(10fg-eh)+b^2\left(11fg^2-h(dh+eg)\right)\right)+2cgh\left(2ah(6fg-eh)-b\left(12fg\right.\right.\right.\right.}{}$$

```
[Out] -1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^3-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*a^
2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^2*(
-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*arctan
h(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1
/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*c^(1/2)/h^4-1/8*(8*c^2*f*g^5-2*c*g*h*(-2*a*d*h^3-6*a*f*g^2*h+b
*d*g*h^2+7*b*f*g^3)+h^2*(4*a^2*e*h^3+b^2*g*(d*h^2+e*g*h+5*f*g^2)-2*a*b*h*(d
*h^2+2*e*g*h+3*f*g^2))+h*(4*c^2*(-d*g^2*h^2+3*f*g^4)+h^2*(8*a^2*f*h^2-2*a*b
*h*(-e*h+10*f*g)+b^2*(11*f*g^2-h*(d*h+e*g)))+2*c*g*h*(2*a*h*(-e*h+6*f*g)-b*
(12*f*g^2-h*(2*d*h+e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^3/(a*h^2-b*g*h+c*g^2)^2
/(h*x+g)^2
```

**Rubi [A]** time = 1.45, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(hx\left(8a^2fh^3-2b\left(ah^2(10fg-eh)-cgh(2dh+eg)+12cfg^3\right)+4acgh(6fg-eh)+b^2h\left(11fg^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(3*f*g^2 + d*h^2) + b^2*g*h*(5
*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2 + 2*e*g*h + d*h^2) + c*(7*f*g
^4 + d*g^2*h^2)) + h*(8*a^2*f*h^3 + 4*a*c*g*h*(6*f*g - e*h) + c^2*((12*f*g^
4)/h - 4*d*g^2*h) + b^2*h*(11*f*g^2 - h*(e*g + d*h)) - 2*b*(12*c*f*g^3 - c*
g*h*(e*g + 2*d*h) + a*h^2*(10*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^2
*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x
 + c*x^2)^(3/2))/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + (Sqrt[c]*f*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^4 - ((16*c^3*f*g^5 -
 8*c^2*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b
```

```
*h*(6*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*f
*g - e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(16*h^4*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(-\frac{3}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2aeh\right.\right.}{(g+hx)}}{3\left(cg^2-bg\right.}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2\right.}{}$$

$$= -\frac{\left(\frac{8c^2fg^5}{h}+4a^2eh^4+4acgh\left(3fg^2+dh^2\right)+b^2gh\left(5fg^2+h(eg+dh)\right)-2\right.}{}$$

**Mathematica [A]** time = 1.93, size = 439, normalized size = 0.73

$$\frac{\frac{\left(\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(2ah^2(eh-2fg)-bh\left(h(dh+eg)-3fg^2\right)+c\left(2dgh^2-2fg^3\right)\right)}{2\left(h(ah-bg)+cg^2\right)}-\frac{h(a+x(b+cx))^{3/2}}{3(g+hx)^3\left(h(\right.}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] (-1/3*(h*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-
(b*g) + a*h))*(g + h*x)^3) + ((2*a*h^2*(-2*f*g + e*h) + c*(-2*f*g^3 + 2*d*g
*h^2) - b*h*(-3*f*g^2 + h*(e*g + d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2
*c*g*x + b*(g - h*x)))/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 -
 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/(
2*(c*g^2 + h*(-(b*g) + a*h))) + (f*(-((h*Sqrt[a + x*(b + c*x)])/(g + h*x))
+ Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + ((2*c*g
- b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g)
 + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)])))/h^2)
/h^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 42.01Unable to divide, perh
aps due to rounding error%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,0,0,0,0,0
```

,8,0]%%%}+%%%{%%%{8,[1]%%%},[7,0,0,0,0,0,7,1]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1
,[1]%%%}]%%},[6,0,1,0,0,0,7,1]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,
0,0,0,1,0,8,0]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,
0,0,0,0,6,2]%%%}+%%%{%%%{24,[1]%%%},[5,0,1,0,0,0,6,2]%%%}+%%%{%%%{-24,[1]%%
%},[5,0,0,0,1,0,7,1]%%%}+%%%{%%%{32,[2]%%%},[5,0,0,0,0,0,5,3]%%%}+%%%{%%{[-
6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,2,0,0,0,6,2]%%%}+%%%{%%{[12,0]:[1,0,%%%{-
1,[1]%%%}]%%},[4,0,1,0,1,0,7,1]%%%}+%%%{%%{[%%%{-48,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[4,0,1,0,0,0,5,3]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,
0,0,0,2,0,8,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0
,0,1,0,6,2]%%%}+%%%{%%{[%%%{-16,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,0
,0,0,4,4]%%%}+%%%{%%%{24,[1]%%%},[3,0,2,0,0,0,5,3]%%%}+%%%{%%%{-48,[1]%%%},
[3,0,1,0,1,0,6,2]%%%}+%%%{%%%{32,[2]%%%},[3,0,1,0,0,0,4,4]%%%}+%%%{%%%{24,[
1]%%%},[3,0,0,0,2,0,7,1]%%%}+%%%{%%%{-32,[2]%%%},[3,0,0,0,1,0,5,3]%%%}+%%%{
%%{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,3,0,0,0,5,3]%%%}+%%%{%%{[12,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,0,2,0,1,0,6,2]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[2,0,2,0,0,0,4,4]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]%%%}]%
%},[2,0,1,0,2,0,7,1]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[2,0,1,0,1,0,5,3]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,0,3,0,8,0
]%%%}+%%%{%%{[%%%{-24,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0,0,2,0,6,2]%
%%}+%%%{%%%{8,[1]%%%},[1,0,3,0,0,0,4,4]%%%}+%%%{%%%{-24,[1]%%%},[1,0,2,0,1,
0,5,3]%%%}+%%%{%%%{24,[1]%%%},[1,0,1,0,2,0,6,2]%%%}+%%%{%%%{-8,[1]%%%},[1,0
,0,0,3,0,7,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,4,0,0,0,4,4]%%
%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3,0,1,0,5,3]%%%}+%%%{%%{[-6,0]
:[1,0,%%%{-1,[1]%%%}]%%},[0,0,2,0,2,0,6,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]
%%%}]%%},[0,0,1,0,3,0,7,1]%%%}+%%%{%%{[-1,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0
,0,4,0,8,0]%%%} / %%%{%%%{1,[2]%%%},[8,0,0,0,0,0,4,0]%%%}+%%%{%%{poly1[%%%{
-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,0,0,0,0,3,1]%%%}+%%%{%%%{4,[2]%%
%},[6,0,1,0,0,0,3,1]%%%}+%%%{%%%{-4,[2]%%%},[6,0,0,0,1,0,4,0]%%%}+%%%{%%%{2
4,[3]%%%},[6,0,0,0,0,0,2,2]%%%}+%%%{%%{[%%%{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[5,0,1,0,0,0,2,2]%%%}+%%%{%%{poly1[%%%{24,[2]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,0,0,0,1,0,3,1]%%%}+%%%{%%{poly1[%%%{-32,[3]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[5,0,0,0,0,0,1,3]%%%}+%%%{%%%{6,[2]%%%},[4,0,2,0,0,0,2,2]%%%}+
%%%{%%%{-12,[2]%%%},[4,0,1,0,1,0,3,1]%%%}+%%%{%%%{48,[3]%%%},[4,0,1,0,0,0,1
,3]%%%}+%%%{%%%{6,[2]%%%},[4,0,0,0,2,0,4,0]%%%}+%%%{%%%{-48,[3]%%%},[4,0,0,
0,1,0,2,2]%%%}+%%%{%%%{16,[4]%%%},[4,0,0,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{-2
4,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,2,0,0,0,1,3]%%%}+%%%{%%{[%%%{48,[
2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,1,0,1,0,2,2]%%%}+%%%{%%{[%%%{-32,[3]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,1,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{-24,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0,0,2,0,3,1]%%%}+%%%{%%{poly1[%%%{3
2,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0,0,1,0,1,3]%%%}+%%%{%%%{4,[2]%%%
},[2,0,3,0,0,0,1,3]%%%}+%%%{%%%{-12,[2]%%%},[2,0,2,0,1,0,2,2]%%%}+%%%{%%%{2
4,[3]%%%},[2,0,2,0,0,0,0,4]%%%}+%%%{%%%{12,[2]%%%},[2,0,1,0,2,0,3,1]%%%}+%%
%{%%%{-48,[3]%%%},[2,0,1,0,1,0,1,3]%%%}+%%%{%%%{-4,[2]%%%},[2,0,0,0,3,0,4,0
]%%%}+%%%{%%%{24,[3]%%%},[2,0,0,0,2,0,2,2]%%%}+%%%{%%{[%%%{-8,[2]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,0,3,0,0,0,0,4]%%%}+%%%{%%{poly1[%%%{24,[2]%%%},0]:

```
[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,0,1,0,1,3]%%%}+%%%{%%{[%%%{-24,[2]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[1,0,1,0,2,0,2,2]%%%}+%%%{%%{poly1[%%%{8,[2]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[1,0,0,0,3,0,3,1]%%%}+%%%{%%%{1,[2]%%%},[0,0,4,0,0,0
,0,4]%%%}+%%%{%%%{-4,[2]%%%},[0,0,3,0,1,0,1,3]%%%}+%%%{%%%{6,[2]%%%},[0,0,2
,0,2,0,2,2]%%%}+%%%{%%%{-4,[2]%%%},[0,0,1,0,3,0,3,1]%%%}+%%%{%%%{1,[2]%%%},
[0,0,0,0,4,0,4,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 19321, normalized size = 32.04

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                               +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**4,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

# 3.194 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx$

**Optimal**. Leaf size=497

$$\frac{\sqrt{a+bx+cx^2}\,(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(a\left(dh^2-5egh+fg^2\right)+2bg(2dh+eg)\right)-8abh(eh+2fg\cdots}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] -1/4*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^4+1/24*(2*c*g*(3*f*g^2+h*(-5*d*h+e*g))+h*(8*a*h*(-e*h+2*f*g)-b*(-5*d*h^2-3
*e*g*h+11*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/
128*(-4*a*c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)+b^2*(5*d*h^
2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*h+f*g^2)))*arctanh
(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/
2))/(a*h^2-b*g*h+c*g^2)^(7/2)+1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+
2*f*g)+b^2*(5*d*h^2+3*e*g*h+5*f*g^2)-4*c*(2*b*g*(2*d*h+e*g)+a*(d*h^2-5*e*g*
h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^
2)^3/(h*x+g)^2
```

**Rubi [A]** time = 0.86, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\,(-2ah+x(2cg-bh)+bg)\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2f\cdots}{64(g+hx)^2\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] ((16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*
(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(
b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(64*(c*g^2 - b*g*h +
a*h^2)^3*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(
4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + ((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d
*h) + 8*a*h^2*(2*f*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x
+ c*x^2)^(3/2))/(24*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((b^2 - 4*a*
c)*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*
h*(5*e*g - d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))
*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(128*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+3beg+8afg-\frac{3bfg^2}{h}+5bdh-8ae\right.\right.}{}}{4\left(cg^2\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} + \frac{\left(6cfg^3+2cgh(eg-5dh)+8ah^2\right.}{24h}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg\right.\right.}{64\left(cg^2-bgh\right.}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg\right.\right.}{64\left(cg^2-bgh\right.}$$

$$= \frac{\left(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c\left(afg^2-ah(5eg-dh)+2bg(eg\right.\right.}{64\left(cg^2-bgh\right.}$$

**Mathematica [A]** time = 3.90, size = 447, normalized size = 0.90

$$\frac{\frac{3}{2}ch\left(\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{8\left(h(ah-bg)+cg^2\right)^{3/2}}+\frac{\sqrt{a+x(b+cx)}(-2ah+b(g-hx)+2cgx)}{4(g+hx)^2\left(h(ah-bg)+cg^2\right)}\right)\left(16a^2fh^2-4c\left(ah(dh-5eg)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(\right.\right.}{24\left(h(ah-bg)+cg^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (-((f*(a + x*(b + c*x))^(3/2))/(g + h*x)^4) + ((3*c*f*g^2 + 4*f*h*(-(b*g) +
 a*h) + c*h*(e*g - d*h))*(a + x*(b + c*x))^(3/2))/(4*(c*g^2 + h*(-(b*g) + a
*h))*(g + h*x)^4) + ((c*(6*c*f*g^3 + 2*c*g*h*(e*g - 5*d*h) - 8*a*h^2*(-2*f*
g + e*h) + b*h*(-11*f*g^2 + h*(3*e*g + 5*d*h)))*(a + x*(b + c*x))^(3/2))/(g
 + h*x)^3 + (3*c*h*(16*c^2*d*g^2 + 16*a^2*f*h^2 - 8*a*b*h*(2*f*g + e*h) - 4
*c*(a*f*g^2 + a*h*(-5*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + b^2*(5*f*g^2 + h*
(3*e*g + 5*d*h)))*((Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
```

```
/(4*(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(
b + c*x)])])/(8*(c*g^2 + h*(-(b*g) + a*h))^(3/2))))/2)/(24*(c*g^2 + h*(-(b*
g) + a*h))^2))/(c*h)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.03, size = 29161, normalized size = 58.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
```

```
more details)Is a*h^2-b*g*h                                    +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx+cx^2}\,\left(d+ex+fx^2\right)}{\left(g+hx\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**5,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

# 3.195 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx$

**Optimal**. Leaf size=824

$$\frac{\left(4c^2\left(3fg^2+h(2eg-27dh)\right)g^2-5h^2\left(\left(3fg^2+3ehg+7dh^2\right)b^2-2ah(6fg+5eh)b+16a^2fh^2\right)-2ch\left(bg\left(16fg^2\right.\right.\right.}{240h\left(cg^2-bhg+ah^2\right)^3(g+hx)^3}$$

```
[Out] -1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)
^5+1/40*(2*c*g*(3*f*g^2+h*(-7*d*h+2*e*g))+h*(10*a*h*(-e*h+2*f*g)-b*(-7*d*h^
2-3*e*g*h+13*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^4
+1/240*(4*c^2*g^2*(3*f*g^2+h*(-27*d*h+2*e*g))-5*h^2*(16*a^2*f*h^2-2*a*b*h*(
5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*c*h*(b*g*(-54*d*h^2-21*e*g*h+
16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-
b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3
*d*h+e*g)+a*(3*d*h^2-6*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g
)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^
2+3*e*g*h+3*f*g^2)+b^2*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*arctanh(1/2*(b*g-2*a*
h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g
*h+c*g^2)^(9/2)+1/128*(32*c^3*d*g^3-8*c^2*g*(2*b*g*(3*d*h+e*g)+a*(3*d*h^2-6
*e*g*h+f*g^2))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h
+3*f*g^2))+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(-d*h^2+3*e*g*h+3*f*g^2)+b^2
*g*(15*d*h^2+6*e*g*h+5*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1
/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)^2
```

**Rubi [A]** time = 2.33, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{\left(4\left(3fg^4+h(2eg-27dh)g^2\right)c^2-2h\left(bg\left(16fg^2-21ehg-54dh^2\right)-2ah\left(18fg^2-33ehg+8dh^2\right)\right)c-5h^2\left(\left(3fg\right.\right.\right.}{240h\left(cg^2-bhg+ah^2\right)^3(g+hx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] ((32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*
h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) +
b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g
 + 5*e*h) + b^2*(3*f*g^2 + h*(3*e*g + 7*d*h))))*(b*g - 2*a*h + (2*c*g - b*h
)*x)*Sqrt[a + b*x + c*x^2])/(128*(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) - (
```

```
(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^5) + ((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g -
 e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2))/(40*h*
(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^4) + ((4*c^2*(3*f*g^4 + g^2*h*(2*e*g -
27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + 3
*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) - 2*a*h*(1
8*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(240*h*(c*g^2 - b*
g*h + a*h^2)^3*(g + h*x)^3) - ((b^2 - 4*a*c)*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g
^2 - 3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e
*h) - 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5
*d*h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3
*e*g + 7*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(256*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(\frac{1}{2}\left(-10cdg+3beg+10afg-\frac{3bfg^2}{h}+7bdh-\cdots\right.\right.}{\cdots}}{5\left(c\cdots\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10a\cdots\right.}{40\cdots}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} + \frac{\left(6cfg^3+2cgh(2eg-7dh)+10a\cdots\right.}{40\cdots}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(32c^3dg^3-8c^2g\left(afg^2-3ah(2eg-dh)+2bg(eg+3dh)\right)+2c\left(4a^2h^2(6fg\cdots\right.\right.}{\cdots}$$

**Mathematica [A]** time = 6.33, size = 1128, normalized size = 1.37

$$\frac{\sqrt{a+x(b+cx)}\left(-\frac{\left(\frac{1}{2}h(3bfg+4cdh-10afh)-\frac{1}{2}g(6cfg+4ceh-7bfh)\right)\left(cx^2+bx+a\right)^{3/2}}{5\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{-\frac{\left(2cg\left(3cfg^2-5fh(bg-ah)+2ch(eg-dh)\right)-ch\left(3bfg^2-bh(3eg+7dh)+10h\cdots\right.\right.}{4\left(cg^2-bhg+ah^2\right)(g+hx)^4}}{\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -1/2*(f*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(c*h*(g + h*x)^5) + (Sqrt[
a + x*(b + c*x)]*(-1/5*(((h*(3*b*f*g + 4*c*d*h - 10*a*f*h))/2 - (g*(6*c*f*g
 + 4*c*e*h - 7*b*f*h))/2)*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)
*(g + h*x)^5) - (-1/4*((2*c*g*(3*c*f*g^2 - 5*f*h*(b*g - a*h) + 2*c*h*(e*g -
 d*h)) - c*h*(3*b*f*g^2 - b*h*(3*e*g + 7*d*h) + 10*h*(c*d*g - a*f*g + a*e*h
)))*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (((c^2
*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*
f*g^2 - h*(3*e*g + 7*d*h))) - (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) +
 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h
*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2)*(a +
b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-2*(a*c^2*h
*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(13*f*
g^2 - h*(3*e*g + 7*d*h))) + (c^2*g*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)) +
 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 16*h
*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2) + b*(
c^2*g*(6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(2*f*g - e*h) - b*h*(
13*f*g^2 - h*(3*e*g + 7*d*h))) + (c*h*(5*b^2*h*(3*f*g^2 + h*(3*e*g + 7*d*h)
) + 2*b*(3*c*f*g^3 - c*g*h*(18*e*g + 47*d*h) - 5*a*h^2*(6*f*g + 5*e*h)) + 1
6*h*(5*c^2*d*g^2 + 5*a^2*f*h^2 - a*c*(2*f*g^2 - h*(7*e*g - 2*d*h)))))/2))*(
((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*
h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*h + a*h^
2)))/(4*(c*g^2 - b*g*h + a*h^2)))/(5*(c*g^2 - b*g*h + a*h^2))))/(2*c*h*Sqrt
[a + b*x + c*x^2])
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [B] time = 0.04, size = 40336, normalized size = 48.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                             +c*g^2 zero or nonze
ro?
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

```
[Out] int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx + cx^{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2)/(h*x+g)**6,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

# 3.196 $\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=1169

$$\frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch}-\frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h}+\frac{\left(-12\left(5fg^2-3h(3eg+8dh\right.\right.}{}$$

```
[Out] 1/12288*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g
))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2
))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^2*
(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g))
))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h^2-2*c*h*(64*a*f*h+
99*b*e*h+24*b*f*g)-12*c^2*(5*f*g^2-3*h*(8*d*h+3*e*g)))*(h*x+g)^2*(c*x^2+b*x
+a)^(5/2)/c^3/h-1/144*(13*b*f*h-18*c*e*h+10*c*f*g)*(h*x+g)^3*(c*x^2+b*x+a)^
(5/2)/c^2/h+1/9*f*(h*x+g)^4*(c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b^4*f*h^4
-192*c^4*g^2*(5*f*g^2-3*h*(64*d*h+3*e*g))-198*b^2*c*h^3*(38*a*f*h+21*b*(e*h
+3*f*g))+8*c^2*h^2*(256*a^2*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553*f*g^2+756
*h*(d*h+3*e*g)))-16*c^3*h*(32*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g*(13*f*g^2+
9*h*(196*d*h+141*e*g)))-10*c*h*(429*b^3*f*h^3-22*b*c*h^2*(34*a*f*h+27*b*e*h
+29*b*f*g)+16*c^3*g*(5*f*g^2-9*h*(12*d*h+e*g))+8*c^2*h*(a*h*(63*e*h+61*f*g)
+3*b*(f*g^2+6*h*(6*d*h+7*e*g))))*x)*(c*x^2+b*x+a)^(5/2)/c^5/h+1/65536*(-4*a
*c+b^2)^2*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f
*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g
^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h^
2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g
))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(15/2)-1/32768*(-
4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*
f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*
g^2))-256*c^4*g*(3*b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+32*c^3*(3*a^2*h
^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*
g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7
```

**Rubi [A]** time = 3.70, antiderivative size = 1166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{f\left(cx^2+bx+a\right)^{5/2}(g+hx)^4}{9ch}-\frac{(10cfg-18ceh+13bfh)\left(cx^2+bx+a\right)^{5/2}(g+hx)^3}{144c^2h}+\frac{\left(-12\left(5fg^2-3h(3eg+8dh\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*
h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g
+ e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2
+ 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(32768*c^7) + ((1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g
*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f
*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) +
 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*
b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(b
 + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(12288*c^6) + ((143*b^2*f*h^2 - 2*c*h*(2
4*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8*d*h)))*(g
 + h*x)^2*(a + b*x + c*x^2)^(5/2))/(2016*c^3*h) - ((10*c*f*g - 18*c*e*h + 1
3*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(144*c^2*h) + (f*(g + h*x)^4*
(a + b*x + c*x^2)^(5/2))/(9*c*h) + ((3003*b^4*f*h^4 - 192*c^4*(5*f*g^4 - 3*
g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*h + 21*b*(3*f*g + e*h)) + 8
*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*h) + b^2*(1553*f*g^2 + 756*h
*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2 + 9*h*(3*e*g + d*h)) + b*g*(1
3*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*(429*b^3*f*h^3 - 22*b*c*h^2*(2
9*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f*g^3 - 9*g*h*(e*g + 12*d*h)) +
8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2 + 6*h*(7*e*g + 6*d*h))))*x)*(a
+ b*x + c*x^2)^(5/2))/(80640*c^5*h) + ((b^2 - 4*a*c)^2*(1536*c^5*d*g^3 - 14
3*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h))
 + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 +
9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*
h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2
+ h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(65536*c^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^3\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch} + \frac{\int (g+hx)^3\left(-\frac{1}{2}h(5bfg-18cdh\right.}{}$$

$$= -\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{144c^2h} + \frac{f(g+}{}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3h(3\right.\right.}{2016c^3h}$$

$$= \frac{\left(143b^2fh^2-2ch(24bfg+99beh+64afh)-12c^2\left(5fg^2-3h(3\right.\right.}{2016c^3h}$$

$$= \frac{\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg+dh)+3bg(\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg\right.\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(1536c^5dg^3-143b^5fh^3-256c^4g\left(afg^2+3ah(eg\right.\right.}{}$$

**Mathematica [A]** time = 2.71, size = 721, normalized size = 0.62

$$\frac{(a+x(b+cx))^{5/2}\left(4c^2h^2\left(512a^2fh^2+2abh(837eh+2511fg+935fhx)+b^2(27h(56dh+168eg+55ehx)+fg(3106g+1595hx))\right)-66b^2ch^3(114afh+b(63eh+189fg\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (((143*b^2*f*h^2 - 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2
 - 3*h*(3*e*g + 8*d*h)))*(g + h*x)^2*(a + x*(b + c*x))^(5/2))/(224*c^2) - (
(13*b*f*h + 2*c*(5*f*g - 9*e*h))*(g + h*x)^3*(a + x*(b + c*x))^(5/2))/(16*c
) + f*(g + h*x)^4*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(3003*
b^4*f*h^4 - 66*b^2*c*h^3*(114*a*f*h + b*(189*f*g + 63*e*h + 65*f*h*x)) - 32
*c^4*(5*f*g^3*(6*g + 5*h*x) - 9*g*h*(e*g*(6*g + 5*h*x) + 4*d*h*(32*g + 15*h
*x))) + 4*c^2*h^2*(512*a^2*f*h^2 + 2*a*b*h*(2511*f*g + 837*e*h + 935*f*h*x)
 + b^2*(f*g*(3106*g + 1595*h*x) + 27*h*(168*e*g + 56*d*h + 55*e*h*x))) - 16
```

```
*c^3*h*(a*h*(f*g*(544*g + 305*h*x) + 9*h*(96*e*g + 32*d*h + 35*e*h*x)) + b*
(f*g^2*(13*g + 15*h*x) + 9*h*(4*d*h*(49*g + 15*h*x) + e*g*(141*g + 70*h*x))
))))/(8960*c^4) + (3*h*(1536*c^5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2
 + 3*a*h*(e*g + d*h) + 3*b*g*(e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*
(3*f*g + e*h)) - 48*b*c^2*h*(5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3
*f*g^2 + 3*e*g*h + d*h^2)) + 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*
g^2 + 3*h*(e*g + d*h)) + 12*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(2*Sqrt[c]*
(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2))
+ 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])
)/(65536*c^(13/2)))/(9*c*h)
```

**fricas [B]** time = 10.43, size = 4751, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [-1/41287680*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*
c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b
^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)
*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*
b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 -
28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c
^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b
^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 33
6*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b
^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f
)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*
c*x + b)*sqrt(c) - 4*a*c) - 4*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^
2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e
 + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f
)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h + 18*
(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*c^8*d
+ 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 + 128*(
1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(b^2*c^
7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e - 3*(
11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*(11*b^
3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f)*h^3)
*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*b^2*c^
6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f)*g^3
+ 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5*c^4 -
 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488
```

*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 1383984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3) + 16*(1344*(120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8*d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 18*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 8536*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 + 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 22968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d - 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 - (288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/c^8, -1/20643840*(315*(64*(24*(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d - 12*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*e + (7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*f)*g^3 - 96*(24*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d - 2*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*e + 3*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*f)*g^2*h + 6*(32*(7*b^6*c^3 - 60*a*b^4*c^4 + 144*a^2*b^2*c^5 - 64*a^3*c^6)*d - 48*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*e + 3*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*f)*g*h^2 - (96*(3*b^7*c^2 - 28*a*b^5*c^3 + 80*a^2*b^3*c^4 - 64*a^3*b*c^5)*d - 6*(33*b^8*c - 336*a*b^6*c^2 + 1120*a^2*b^4*c^3 - 1280*a^3*b^2*c^4 + 256*a^4*c^5)*e + (143*b^9 - 1584*a*b^7*c + 6048*a^2*b^5*c^2 - 8960*a^3*b^3*c^3 + 3840*a^4*b*c^4)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1146880*c^9*f*h^3*x^8 + 71680*(54*c^9*f*g*h^2 + (18*c^9*e + 19*b*c^8*f)*h^3)*x^7 + 5120*(864*c^9*f*g^2*h + 54*(16*c^9*e + 17*b*c^8*f)*g*h^2 + (288*c^9*d + 306*b*c^8*e + (3*b^2*c^7 + 320*a*c^8)*f)*h^3)*x^6 + 1280*(1344*c^9*f*g^3 + 288*(14*c^9*e + 15*b*c^8*f)*g^2*h

```
 + 18*(224*c^9*d + 240*b*c^8*e + 3*(b^2*c^7 + 84*a*c^8)*f)*g*h^2 + (1440*b*
c^8*d + 18*(b^2*c^7 + 84*a*c^8)*e - (13*b^3*c^6 - 60*a*b*c^7)*f)*h^3)*x^5 +
 128*(1344*(12*c^9*e + 13*b*c^8*f)*g^3 + 288*(168*c^9*d + 182*b*c^8*e + 3*(
b^2*c^7 + 64*a*c^8)*f)*g^2*h + 18*(2912*b*c^8*d + 48*(b^2*c^7 + 64*a*c^8)*e
 - 3*(11*b^3*c^6 - 52*a*b*c^7)*f)*g*h^2 + (288*(b^2*c^7 + 64*a*c^8)*d - 18*
(11*b^3*c^6 - 52*a*b*c^7)*e + (143*b^4*c^5 - 804*a*b^2*c^6 + 768*a^2*c^7)*f
)*h^3)*x^4 - 1344*(120*(3*b^3*c^6 - 20*a*b*c^7)*d - 12*(15*b^4*c^5 - 100*a*
b^2*c^6 + 128*a^2*c^7)*e + (105*b^5*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*f
)*g^3 + 288*(168*(15*b^4*c^5 - 100*a*b^2*c^6 + 128*a^2*c^7)*d - 14*(105*b^5
*c^4 - 760*a*b^3*c^5 + 1296*a^2*b*c^6)*e + 3*(315*b^6*c^3 - 2520*a*b^4*c^4
+ 5488*a^2*b^2*c^5 - 2048*a^3*c^6)*f)*g^2*h - 18*(224*(105*b^5*c^4 - 760*a*
b^3*c^5 + 1296*a^2*b*c^6)*d - 48*(315*b^6*c^3 - 2520*a*b^4*c^4 + 5488*a^2*b
^2*c^5 - 2048*a^3*c^6)*e + 3*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 81648*a^2*b^
3*c^4 - 58816*a^3*b*c^5)*f)*g*h^2 + (288*(315*b^6*c^3 - 2520*a*b^4*c^4 + 54
88*a^2*b^2*c^5 - 2048*a^3*c^6)*d - 18*(3465*b^7*c^2 - 30660*a*b^5*c^3 + 816
48*a^2*b^3*c^4 - 58816*a^3*b*c^5)*e + (45045*b^8*c - 438900*a*b^6*c^2 + 138
3984*a^2*b^4*c^3 - 1467072*a^3*b^2*c^4 + 262144*a^4*c^5)*f)*h^3 + 16*(1344*
(120*c^9*d + 132*b*c^8*e + (3*b^2*c^7 + 140*a*c^8)*f)*g^3 + 288*(1848*b*c^8
*d + 14*(3*b^2*c^7 + 140*a*c^8)*e - 3*(9*b^3*c^6 - 44*a*b*c^7)*f)*g^2*h + 1
8*(224*(3*b^2*c^7 + 140*a*c^8)*d - 48*(9*b^3*c^6 - 44*a*b*c^7)*e + 3*(99*b^
4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*f)*g*h^2 - (288*(9*b^3*c^6 - 44*a*b*c^
7)*d - 18*(99*b^4*c^5 - 568*a*b^2*c^6 + 560*a^2*c^7)*e + (1287*b^5*c^4 - 85
36*a*b^3*c^5 + 12912*a^2*b*c^6)*f)*h^3)*x^3 + 8*(1344*(360*b*c^8*d + 12*(b^
2*c^7 + 32*a*c^8)*e - (7*b^3*c^6 - 36*a*b*c^7)*f)*g^3 + 288*(168*(b^2*c^7 +
 32*a*c^8)*d - 14*(7*b^3*c^6 - 36*a*b*c^7)*e + 3*(21*b^4*c^5 - 124*a*b^2*c^
6 + 128*a^2*c^7)*f)*g^2*h - 18*(224*(7*b^3*c^6 - 36*a*b*c^7)*d - 48*(21*b^4
*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*e + 3*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2
416*a^2*b*c^6)*f)*g*h^2 + (288*(21*b^4*c^5 - 124*a*b^2*c^6 + 128*a^2*c^7)*d
 - 18*(231*b^5*c^4 - 1560*a*b^3*c^5 + 2416*a^2*b*c^6)*e + (3003*b^6*c^3 - 2
2968*a*b^4*c^4 + 47280*a^2*b^2*c^5 - 16384*a^3*c^6)*f)*h^3)*x^2 + 2*(1344*(
120*(b^2*c^7 + 20*a*c^8)*d - 12*(5*b^3*c^6 - 28*a*b*c^7)*e + (35*b^4*c^5 -
216*a*b^2*c^6 + 240*a^2*c^7)*f)*g^3 - 288*(168*(5*b^3*c^6 - 28*a*b*c^7)*d -
 14*(35*b^4*c^5 - 216*a*b^2*c^6 + 240*a^2*c^7)*e + 3*(105*b^5*c^4 - 728*a*b
^3*c^5 + 1168*a^2*b*c^6)*f)*g^2*h + 18*(224*(35*b^4*c^5 - 216*a*b^2*c^6 + 2
40*a^2*c^7)*d - 48*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*e + 3*(11
55*b^6*c^3 - 8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*f)*g*h^2 -
(288*(105*b^5*c^4 - 728*a*b^3*c^5 + 1168*a^2*b*c^6)*d - 18*(1155*b^6*c^3 -
8988*a*b^4*c^4 + 18896*a^2*b^2*c^5 - 6720*a^3*c^6)*e + (15015*b^7*c^2 - 130
284*a*b^5*c^3 + 338832*a^2*b^3*c^4 - 236864*a^3*b*c^5)*f)*h^3)*x)*sqrt(c*x^
2 + b*x + a))/c^8]
```

**giac [B]** time = 0.44, size = 2977, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")

[Out] 1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (54*c^9*f*g*h^2 + 19*b*c^8*f*h^3 + 18*c^9*h^3*e)/c^8)*x + (864*c^9*f*g^2*h + 918*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 3*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3 + 864*c^9*g*h^2*e + 306*b*c^8*h^3*e)/c^8)*x + (1344*c^9*f*g^3 + 4320*b*c^8*f*g^2*h + 4032*c^9*d*g*h^2 + 54*b^2*c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8*d*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3 + 4032*c^9*g^2*h*e + 4320*b*c^8*g*h^2*e + 18*b^2*c^7*h^3*e + 1512*a*c^8*h^3*e)/c^8)*x + (17472*b*c^8*f*g^3 + 48384*c^9*d*g^2*h + 864*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b*c^8*d*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^2 + 288*b^2*c^7*d*h^3 + 18432*a*c^8*d*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*c^7*f*h^3 + 16128*c^9*g^3*e + 52416*b*c^8*g^2*h*e + 864*b^2*c^7*g*h^2*e + 55296*a*c^8*g*h^2*e - 198*b^3*c^6*h^3*e + 936*a*b*c^7*h^3*e)/c^8)*x + (161280*c^9*d*g^3 + 4032*b^2*c^7*f*g^3 + 188160*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*g*h^2 + 564480*a*c^8*d*g*h^2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 30240*a^2*c^7*f*g*h^2 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 - 1287*b^5*c^4*f*h^3 + 8536*a*b^3*c^5*f*h^3 - 12912*a^2*b*c^6*f*h^3 + 177408*b*c^8*g^3*e + 12096*b^2*c^7*g^2*h*e + 564480*a*c^8*g^2*h*e - 7776*b^3*c^6*g*h^2*e + 38016*a*b*c^7*g*h^2*e + 1782*b^4*c^5*h^3*e - 10224*a*b^2*c^6*h^3*e + 10080*a^2*c^7*h^3*e)/c^8)*x + (483840*b*c^8*d*g^3 - 9408*b^3*c^6*f*g^3 + 48384*a*b*c^7*f*g^3 + 48384*b^2*c^7*d*g^2*h + 1548288*a*c^8*d*g^2*h + 18144*b^4*c^5*f*g^2*h - 107136*a*b^2*c^6*f*g^2*h + 110592*a^2*c^7*f*g^2*h - 28224*b^3*c^6*d*g*h^2 + 145152*a*b*c^7*d*g*h^2 - 12474*b^5*c^4*f*g*h^2 + 84240*a*b^3*c^5*f*g*h^2 - 130464*a^2*b*c^6*f*g*h^2 + 6048*b^4*c^5*d*h^3 - 35712*a*b^2*c^6*d*h^3 + 36864*a^2*c^7*d*h^3 + 3003*b^6*c^3*f*h^3 - 22968*a*b^4*c^4*f*h^3 + 47280*a^2*b^2*c^5*f*h^3 - 16384*a^3*c^6*f*h^3 + 16128*b^2*c^7*g^3*e + 516096*a*c^8*g^3*e - 28224*b^3*c^6*g^2*h*e + 145152*a*b*c^7*g^2*h*e + 18144*b^4*c^5*g*h^2*e - 107136*a*b^2*c^6*g*h^2*e + 110592*a^2*c^7*g*h^2*e - 4158*b^5*c^4*h^3*e + 28080*a*b^3*c^5*h^3*e - 43488*a^2*b*c^6*h^3*e)/c^8)*x + (161280*b^2*c^7*d*g^3 + 3225600*a*c^8*d*g^3 + 47040*b^4*c^5*f*g^3 - 290304*a*b^2*c^6*f*g^3 + 322560*a^2*c^7*f*g^3 - 241920*b^3*c^6*d*g^2*h + 1354752*a*b*c^7*d*g^2*h - 90720*b^5*c^4*f*g^2*h + 628992*a*b^3*c^5*f*g^2*h - 1009152*a^2*b*c^6*f*g^2*h + 141120*b^4*c^5*d*g*h^2 - 870912*a*b^2*c^6*d*g*h^2 + 967680*a^2*c^7*d*g*h^2 + 62370*b^6*c^3*f*g*h^2 - 485352*a*b^4*c^4*f*g*h^2 + 1020384*a^2*b^2*c^5*f*g*h^2 - 362880*a^3*c^6*f*g*h^2 - 30240*b^5*c^4*d*h^3 + 209664*a*b^3*c^5*d*h^3 - 336384*a^2*b*c^6*d*h^3 - 15015*b^7*c^2*f*h^3 + 130284*a*b^5*c^3*f*h^3 - 338832*a^2*b^3*c^4*f*h^3 + 236864*a^3*b*c^5*f*h^3 - 80640*b^3*c^6*g^3*e + 451584*a*b*c^7*g^3*e + 141120*b^4*c^5*g^2*h*e - 870912*a*b^2*c^6*g^2*h*e + 967680*a^2*c^7*g^2*h*e - 90720*b^5*c^4*g*h^2*e + 628992*a*b^3*c^5*g*h^2*e - 1009152*a^2*b*c^6*g*h^2*e + 20790*b^6*c^3*h^3*e - 161784*a*b^4*c^4*h^3*e + 340128*a^2*b^2*c^5*h^3*e - 120960*a^3*c^6*h^3*e)/c^8)*x - (483840*b^3*c^6*d*g^3 - 3225600*a*b*c^7*d*g^3 + 141120*b^5*c^4*f*g^3 - 1021440*a*b^3*c^5*f*g^3 + 1741824*a^2*b*c^6*f*g^3 - 725760*b^4*c^5*d*g^2*h + 4838

```
400*a*b^2*c^6*d*g^2*h - 6193152*a^2*c^7*d*g^2*h - 272160*b^6*c^3*f*g^2*h +
2177280*a*b^4*c^4*f*g^2*h - 4741632*a^2*b^2*c^5*f*g^2*h + 1769472*a^3*c^6*f
*g^2*h + 423360*b^5*c^4*d*g*h^2 - 3064320*a*b^3*c^5*d*g*h^2 + 5225472*a^2*b
*c^6*d*g*h^2 + 187110*b^7*c^2*f*g*h^2 - 1655640*a*b^5*c^3*f*g*h^2 + 4408992
*a^2*b^3*c^4*f*g*h^2 - 3176064*a^3*b*c^5*f*g*h^2 - 90720*b^6*c^3*d*h^3 + 72
5760*a*b^4*c^4*d*h^3 - 1580544*a^2*b^2*c^5*d*h^3 + 589824*a^3*c^6*d*h^3 - 4
5045*b^8*c*f*h^3 + 438900*a*b^6*c^2*f*h^3 - 1383984*a^2*b^4*c^3*f*h^3 + 146
7072*a^3*b^2*c^4*f*h^3 - 262144*a^4*c^5*f*h^3 - 241920*b^4*c^5*g^3*e + 1612
800*a*b^2*c^6*g^3*e - 2064384*a^2*c^7*g^3*e + 423360*b^5*c^4*g^2*h*e - 3064
320*a*b^3*c^5*g^2*h*e + 5225472*a^2*b*c^6*g^2*h*e - 272160*b^6*c^3*g*h^2*e
+ 2177280*a*b^4*c^4*g*h^2*e - 4741632*a^2*b^2*c^5*g*h^2*e + 1769472*a^3*c^6
*g*h^2*e + 62370*b^7*c^2*h^3*e - 551880*a*b^5*c^3*h^3*e + 1469664*a^2*b^3*c
^4*h^3*e - 1058688*a^3*b*c^5*h^3*e)/c^8) - 1/65536*(1536*b^4*c^5*d*g^3 - 12
288*a*b^2*c^6*d*g^3 + 24576*a^2*c^7*d*g^3 + 448*b^6*c^3*f*g^3 - 3840*a*b^4*
c^4*f*g^3 + 9216*a^2*b^2*c^5*f*g^3 - 4096*a^3*c^6*f*g^3 - 2304*b^5*c^4*d*g^
2*h + 18432*a*b^3*c^5*d*g^2*h - 36864*a^2*b*c^6*d*g^2*h - 864*b^7*c^2*f*g^2
*h + 8064*a*b^5*c^3*f*g^2*h - 23040*a^2*b^3*c^4*f*g^2*h + 18432*a^3*b*c^5*f
*g^2*h + 1344*b^6*c^3*d*g*h^2 - 11520*a*b^4*c^4*d*g*h^2 + 27648*a^2*b^2*c^5
*d*g*h^2 - 12288*a^3*c^6*d*g*h^2 + 594*b^8*c*f*g*h^2 - 6048*a*b^6*c^2*f*g*h
^2 + 20160*a^2*b^4*c^3*f*g*h^2 - 23040*a^3*b^2*c^4*f*g*h^2 + 4608*a^4*c^5*f
*g*h^2 - 288*b^7*c^2*d*h^3 + 2688*a*b^5*c^3*d*h^3 - 7680*a^2*b^3*c^4*d*h^3
+ 6144*a^3*b*c^5*d*h^3 - 143*b^9*f*h^3 + 1584*a*b^7*c*f*h^3 - 6048*a^2*b^5*
c^2*f*h^3 + 8960*a^3*b^3*c^3*f*h^3 - 3840*a^4*b*c^4*f*h^3 - 768*b^5*c^4*g^3
*e + 6144*a*b^3*c^5*g^3*e - 12288*a^2*b*c^6*g^3*e + 1344*b^6*c^3*g^2*h*e -
11520*a*b^4*c^4*g^2*h*e + 27648*a^2*b^2*c^5*g^2*h*e - 12288*a^3*c^6*g^2*h*e
 - 864*b^7*c^2*g*h^2*e + 8064*a*b^5*c^3*g*h^2*e - 23040*a^2*b^3*c^4*g*h^2*e
 + 18432*a^3*b*c^5*g*h^2*e + 198*b^8*c*h^3*e - 2016*a*b^6*c^2*h^3*e + 6720*
a^2*b^4*c^3*h^3*e - 7680*a^3*b^2*c^4*h^3*e + 1536*a^4*c^5*h^3*e)*log(abs(-2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(15/2)
```

**maple [B]** time = 0.03, size = 5881, normalized size = 5.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+hx)^3 \left(cx^2+bx+a\right)^{3/2} \left(fx^2+ex+d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{\frac{3}{2}} \left(d+ex+fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**3*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

# 3.197 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=753

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(dh^2+2egh+fg^2\right)\right)-72b^2ch(3af\right.}{32768c^{13/2}}$$

```
[Out] 1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128
*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a*b
*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)
/c^5-1/112*(11*b*f*h-16*c*e*h+10*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c^2/h
+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(693*b^3*f*h^3+96*c^3*g*(5
*f*g^2-8*h*(7*d*h+e*g))-36*b*c*h^2*(31*a*f*h+28*b*(e*h+2*f*g))+8*c^2*h*(96*
a*h*(e*h+2*f*g)+b*(31*f*g^2+196*h*(d*h+2*e*g)))-10*c*h*(99*b^2*f*h^2-8*c^2*
(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e*h+f*g)))*x)*(c*x^2+b*x
+a)^(5/2)/c^4/h+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c
*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*
g^2))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(13/2)-1/16384*(-4*
a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-1
28*c^3*(3*b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+16*c^2*(3*a^2*f*h^2+12*a
*b*h*(e*h+2*f*g)+14*b^2*(d*h^2+2*e*g*h+f*g^2)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/
2)/c^6
```

**Rubi** [A] time = 2.10, antiderivative size = 749, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh\right.}{6144c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e
*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h))
 + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g
+ d*h))))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4*d*g^2
+ 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^
2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b
*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(b + 2*c*x)*(a + b*x
```

```
+ c*x^2)^(3/2))/(6144*c^5) - ((10*c*f*g - 16*c*e*h + 11*b*f*h)*(g + h*x)^2*
(a + b*x + c*x^2)^(5/2))/(112*c^2*h) + (f*(g + h*x)^3*(a + b*x + c*x^2)^(5/
2))/(8*c*h) - ((693*b^3*f*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36
*b*c*h^2*(31*a*f*h + 28*b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2
*e*g + d*h) + 96*a*h*(2*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2
 - 4*h*(2*e*g + 7*d*h)) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h))))*x)*(a + b*x
 + c*x^2)^(5/2))/(13440*c^4*h) + ((b^2 - 4*a*c)^2*(768*c^4*d*g^2 + 99*b^4*f
*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a*h*(2
*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g
+ e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
```

```
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^3 \left(a+bx+cx^2\right)^{5/2}}{8ch} + \frac{\int (g+hx)^2 \left(-\frac{1}{2}h(5bfg-16cd\right.}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g}{}$$

$$= -\frac{(10cfg-16ceh+11bfh)(g+hx)^2 \left(a+bx+cx^2\right)^{5/2}}{112c^2h} + \frac{f(g}{}$$

$$= \frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3a\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3a\right.}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3a\right.}{}$$

**Mathematica [A]** time = 1.60, size = 468, normalized size = 0.62

$$\frac{h\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)\right)\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-7\right.}{12288c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (-1/14*((11*b*f*h + 2*c*(5*f*g - 8*e*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2
))/c + f*(g + h*x)^3*(a + x*(b + c*x))^(5/2) - ((a + x*(b + c*x))^(5/2)*(69
3*b^3*f*h^3 + 8*c^2*h*(b*f*g*(31*g + 30*h*x) + 4*b*h*(98*e*g + 49*d*h + 45*
e*h*x) + 3*a*h*(64*f*g + 32*e*h + 35*f*h*x)) - 18*b*c*h^2*(62*a*f*h + b*(11
2*f*g + 56*e*h + 55*f*h*x)) + 16*c^3*(5*f*g^2*(6*g + 5*h*x) - 4*h*(2*e*g*(6
*g + 5*h*x) + 7*d*h*(12*g + 5*h*x)))))/(1680*c^3) + (h*(768*c^4*d*g^2 + 99*
b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a*f*h) - 128*c^3*(a*f*g^2 + a
*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c^2*(3*a^2*f*h^2 + 12*a*b*h*(2
*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h))))*(2*Sqrt[c]*(b + 2*c*x)*Sqr
t[a + x*(b + c*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*
c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(12288*c^(11/
2)))/(8*c*h)
```

**fricas [B]** time = 4.35, size = 3145, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas"
)
```

```
[Out] [1/6881280*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2
*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d
 - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*(3*b^7
*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^6*c^2
- 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*
b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
```

^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7, -1/3440640*(105*(32*(24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 12*(
b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e + (7*b^6*c^2 - 60*a*b^4*c^3 + 144*a
^2*b^2*c^4 - 64*a^3*c^5)*f)*g^2 - 32*(24*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*
c^5)*d - 2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e + 3*
(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*f)*g*h + (32*(7*b^
6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 48*(3*b^7*c - 28*a
*b^5*c^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e + 3*(33*b^8 - 336*a*b^6*c + 112
0*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f)*h^2)*sqrt(-c)*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2
15040*c^8*f*h^2*x^7 + 15360*(32*c^8*f*g*h + (16*c^8*e + 17*b*c^7*f)*h^2)*x^
6 + 1280*(224*c^8*f*g^2 + 32*(14*c^8*e + 15*b*c^7*f)*g*h + (224*c^8*d + 240
*b*c^7*e + 3*(b^2*c^6 + 84*a*c^7)*f)*h^2)*x^5 + 128*(224*(12*c^8*e + 13*b*c
^7*f)*g^2 + 32*(168*c^8*d + 182*b*c^7*e + 3*(b^2*c^6 + 64*a*c^7)*f)*g*h + (
2912*b*c^7*d + 48*(b^2*c^6 + 64*a*c^7)*e - 3*(11*b^3*c^5 - 52*a*b*c^6)*f)*h
^2)*x^4 + 16*(224*(120*c^8*d + 132*b*c^7*e + (3*b^2*c^6 + 140*a*c^7)*f)*g^2
 + 32*(1848*b*c^7*d + 14*(3*b^2*c^6 + 140*a*c^7)*e - 3*(9*b^3*c^5 - 44*a*b*
c^6)*f)*g*h + (224*(3*b^2*c^6 + 140*a*c^7)*d - 48*(9*b^3*c^5 - 44*a*b*c^6)*
e + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f)*h^2)*x^3 - 224*(120*(3*
b^3*c^5 - 20*a*b*c^6)*d - 12*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*e +
 (105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*f)*g^2 + 32*(168*(15*b^4*c^
4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d - 14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296
*a^2*b*c^5)*e + 3*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a
^3*c^5)*f)*g*h - (224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 48

```
*(315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e + 3*(34
65*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f)*h^2 +
8*(224*(360*b*c^7*d + 12*(b^2*c^6 + 32*a*c^7)*e - (7*b^3*c^5 - 36*a*b*c^6)*
f)*g^2 + 32*(168*(b^2*c^6 + 32*a*c^7)*d - 14*(7*b^3*c^5 - 36*a*b*c^6)*e + 3
*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*f)*g*h - (224*(7*b^3*c^5 - 36*a
*b*c^6)*d - 48*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e + 3*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f)*h^2)*x^2 + 2*(224*(120*(b^2*c^6 + 2
0*a*c^7)*d - 12*(5*b^3*c^5 - 28*a*b*c^6)*e + (35*b^4*c^4 - 216*a*b^2*c^5 +
240*a^2*c^6)*f)*g^2 - 32*(168*(5*b^3*c^5 - 28*a*b*c^6)*d - 14*(35*b^4*c^4 -
 216*a*b^2*c^5 + 240*a^2*c^6)*e + 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2
*b*c^5)*f)*g*h + (224*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*d - 48*(10
5*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e + 3*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))
/c^7]
```

**giac [B]** time = 0.37, size = 1852, normalized size = 2.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^8
*f*g*h + 17*b*c^7*f*h^2 + 16*c^8*h^2*e)/c^7)*x + (224*c^8*f*g^2 + 480*b*c^7
*f*g*h + 224*c^8*d*h^2 + 3*b^2*c^6*f*h^2 + 252*a*c^7*f*h^2 + 448*c^8*g*h*e
+ 240*b*c^7*h^2*e)/c^7)*x + (2912*b*c^7*f*g^2 + 5376*c^8*d*g*h + 96*b^2*c^6
*f*g*h + 6144*a*c^7*f*g*h + 2912*b*c^7*d*h^2 - 33*b^3*c^5*f*h^2 + 156*a*b*c
^6*f*h^2 + 2688*c^8*g^2*e + 5824*b*c^7*g*h*e + 48*b^2*c^6*h^2*e + 3072*a*c^
7*h^2*e)/c^7)*x + (26880*c^8*d*g^2 + 672*b^2*c^6*f*g^2 + 31360*a*c^7*f*g^2
+ 59136*b*c^7*d*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*
d*h^2 + 31360*a*c^7*d*h^2 + 297*b^4*c^4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680
*a^2*c^6*f*h^2 + 29568*b*c^7*g^2*e + 1344*b^2*c^6*g*h*e + 62720*a*c^7*g*h*e
 - 432*b^3*c^5*h^2*e + 2112*a*b*c^6*h^2*e)/c^7)*x + (80640*b*c^7*d*g^2 - 15
68*b^3*c^5*f*g^2 + 8064*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d
*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h + 12288*a^2*c^6*f*g*h - 1
568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 - 693*b^5*c^3*f*h^2 + 4680*a*b^3*c^4
*f*h^2 - 7248*a^2*b*c^5*f*h^2 + 2688*b^2*c^6*g^2*e + 86016*a*c^7*g^2*e - 31
36*b^3*c^5*g*h*e + 16128*a*b*c^6*g*h*e + 1008*b^4*c^4*h^2*e - 5952*a*b^2*c^
5*h^2*e + 6144*a^2*c^6*h^2*e)/c^7)*x + (26880*b^2*c^6*d*g^2 + 537600*a*c^7*
d*g^2 + 7840*b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 -
26880*b^3*c^5*d*g*h + 150528*a*b*c^6*d*g*h - 10080*b^5*c^3*f*g*h + 69888*a*
b^3*c^4*f*g*h - 112128*a^2*b*c^5*f*g*h + 7840*b^4*c^4*d*h^2 - 48384*a*b^2*c
^5*d*h^2 + 53760*a^2*c^6*d*h^2 + 3465*b^6*c^2*f*h^2 - 26964*a*b^4*c^3*f*h^2
 + 56688*a^2*b^2*c^4*f*h^2 - 20160*a^3*c^5*f*h^2 - 13440*b^3*c^5*g^2*e + 75
264*a*b*c^6*g^2*e + 15680*b^4*c^4*g*h*e - 96768*a*b^2*c^5*g*h*e + 107520*a^
```

```
2*c^6*g*h*e - 5040*b^5*c^3*h^2*e + 34944*a*b^3*c^4*h^2*e - 56064*a^2*b*c^5*
h^2*e)/c^7)*x - (80640*b^3*c^5*d*g^2 - 537600*a*b*c^6*d*g^2 + 23520*b^5*c^3
*f*g^2 - 170240*a*b^3*c^4*f*g^2 + 290304*a^2*b*c^5*f*g^2 - 80640*b^4*c^4*d*
g*h + 537600*a*b^2*c^5*d*g*h - 688128*a^2*c^6*d*g*h - 30240*b^6*c^2*f*g*h +
 241920*a*b^4*c^3*f*g*h - 526848*a^2*b^2*c^4*f*g*h + 196608*a^3*c^5*f*g*h +
 23520*b^5*c^3*d*h^2 - 170240*a*b^3*c^4*d*h^2 + 290304*a^2*b*c^5*d*h^2 + 10
395*b^7*c*f*h^2 - 91980*a*b^5*c^2*f*h^2 + 244944*a^2*b^3*c^3*f*h^2 - 176448
*a^3*b*c^4*f*h^2 - 40320*b^4*c^4*g^2*e + 268800*a*b^2*c^5*g^2*e - 344064*a^
2*c^6*g^2*e + 47040*b^5*c^3*g*h*e - 340480*a*b^3*c^4*g*h*e + 580608*a^2*b*c
^5*g*h*e - 15120*b^6*c^2*h^2*e + 120960*a*b^4*c^3*h^2*e - 263424*a^2*b^2*c^
4*h^2*e + 98304*a^3*c^5*h^2*e)/c^7) - 1/32768*(768*b^4*c^4*d*g^2 - 6144*a*b
^2*c^5*d*g^2 + 12288*a^2*c^6*d*g^2 + 224*b^6*c^2*f*g^2 - 1920*a*b^4*c^3*f*g
^2 + 4608*a^2*b^2*c^4*f*g^2 - 2048*a^3*c^5*f*g^2 - 768*b^5*c^3*d*g*h + 6144
*a*b^3*c^4*d*g*h - 12288*a^2*b*c^5*d*g*h - 288*b^7*c*f*g*h + 2688*a*b^5*c^2
*f*g*h - 7680*a^2*b^3*c^3*f*g*h + 6144*a^3*b*c^4*f*g*h + 224*b^6*c^2*d*h^2
- 1920*a*b^4*c^3*d*h^2 + 4608*a^2*b^2*c^4*d*h^2 - 2048*a^3*c^5*d*h^2 + 99*b
^8*f*h^2 - 1008*a*b^6*c*f*h^2 + 3360*a^2*b^4*c^2*f*h^2 - 3840*a^3*b^2*c^3*f
*h^2 + 768*a^4*c^4*f*h^2 - 384*b^5*c^3*g^2*e + 3072*a*b^3*c^4*g^2*e - 6144*
a^2*b*c^5*g^2*e + 448*b^6*c^2*g*h*e - 3840*a*b^4*c^3*g*h*e + 9216*a^2*b^2*c
^4*g*h*e - 4096*a^3*c^5*g*h*e - 144*b^7*c*h^2*e + 1344*a*b^5*c^2*h^2*e - 38
40*a^2*b^3*c^3*h^2*e + 3072*a^3*b*c^4*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

**maple [B]** time = 0.02, size = 3769, normalized size = 5.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/5*(c*x^2+b*x+a)^(5/2)/c*e*g^2+1/4*d*g^2*(c*x^2+b*x+a)^(3/2)*x+3/16/c^(5/2
)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g*h-15/128/c^(7/2)*b^
4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g*h-1/12*a/c*(c*x^2+b*x+a
)^(3/2)*x*e*g*h-1/24*a/c^2*(c*x^2+b*x+a)^(3/2)*b*e*g*h-1/8*a^2/c*(c*x^2+b*x
+a)^(1/2)*x*e*g*h-7/30/c^2*b*(c*x^2+b*x+a)^(5/2)*e*g*h+3/20/c^3*b^2*(c*x^2+
b*x+a)^(5/2)*f*g*h-3/64/c^4*b^4*(c*x^2+b*x+a)^(3/2)*f*g*h+9/512/c^5*b^6*(c*
x^2+b*x+a)^(1/2)*f*g*h-15/128/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*a^2*e*h^2+21/512/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*e*h^2+3/32/c^(5/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*h^2-3/28/c^2*b*x*(c*x^2+b*x+a)^(5/2)*e*h^2-3/64/c^3*b^3*(c*x^2+b*x+a
)^(3/2)*x*e*h^2-9/256*f*h^2/c^4*b^3*a*(c*x^2+b*x+a)^(3/2)-57/1024*f*h^2/c^4
*b^3*a^2*(c*x^2+b*x+a)^(1/2)-11/112*f*h^2/c^2*b*x^2*(c*x^2+b*x+a)^(5/2)+93/
1120*f*h^2/c^3*b*a*(c*x^2+b*x+a)^(5/2)+105/1024*f*h^2/c^(9/2)*b^4*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-63/2048*f*h^2/c^(11/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*e*g^
```

2-3/256/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g^2+1/8*d
*g^2/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*g^2*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d*g^2/c
^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d*g^2/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*a^2+7/96/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x*d*h^2+7/96/c^2*b^2*(c*
x^2+b*x+a)^(3/2)*x*f*g^2+7/96/c^3*b^3*(c*x^2+b*x+a)^(3/2)*e*g*h-7/256/c^3*b
^4*(c*x^2+b*x+a)^(1/2)*x*d*h^2-9/128*f*h^2/c^3*b^2*a*(c*x^2+b*x+a)^(3/2)*x-
57/512*f*h^2/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)*x-1/16/c^2*b^2*(c*x^2+b*x+a)^(
3/2)*e*g^2+2/5*(c*x^2+b*x+a)^(5/2)/c*d*g*h-3/32/c^3*b^3*(c*x^2+b*x+a)^(1/2)
*x*a*e*h^2-15/64/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^
2*f*g*h+21/256/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*
g*h+1/16/c^2*b*a*(c*x^2+b*x+a)^(3/2)*x*e*h^2+1/16/c^3*b^2*a*(c*x^2+b*x+a)^(
3/2)*f*g*h+3/32/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)*f*g*h+3/32/c^2*b*a^2*(c*x^2
+b*x+a)^(1/2)*x*e*h^2+9/32/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*a^2*e*g*h+1/8/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*e*g*h-1/4/c*b*(c*x^2+b*
x+a)^(3/2)*x*d*g*h-3/16/c*b*(c*x^2+b*x+a)^(1/2)*x*a*e*g^2+3/32/c^2*b^3*(c*x
^2+b*x+a)^(1/2)*x*d*g*h-3/16/c^2*b^2*(c*x^2+b*x+a)^(1/2)*a*d*g*h-3/8/c^(3/2
)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*g*h+3/16/c^(5/2)*b^3*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*g*h+9/256/c^4*b^5*(c*x^2+b*
x+a)^(1/2)*x*f*g*h-3/14/c^2*b*x*(c*x^2+b*x+a)^(5/2)*f*g*h-3/32/c^3*b^3*(c*x
^2+b*x+a)^(3/2)*x*f*g*h-3/32/c^4*b^4*(c*x^2+b*x+a)^(1/2)*a*f*g*h-3/32/c^2*b
^2*(c*x^2+b*x+a)^(1/2)*a*e*g^2+3/64/c^3*b^4*(c*x^2+b*x+a)^(1/2)*d*g*h-3/16/
c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g^2+3/32/c^(5/2
)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g^2-3/128/c^(7/2)*b^5
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*g*h-3/32*d*g^2/c*(c*x^2+b*x+
a)^(1/2)*x*b^2+2/7*x^2*(c*x^2+b*x+a)^(5/2)/c*f*g*h-4/35*a/c^2*(c*x^2+b*x+a)
^(5/2)*f*g*h-15/128*f*h^2/c^(7/2)*b^2*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))+1/128*f*h^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b+3/128*f*h^2*a^3/c^2*(c
*x^2+b*x+a)^(1/2)*x+3/256*f*h^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b-1/16*f*h^2*a/
c^2*x*(c*x^2+b*x+a)^(5/2)+1/64*f*h^2*a^2/c^2*(c*x^2+b*x+a)^(3/2)*x+33/448*f
*h^2/c^3*b^2*x*(c*x^2+b*x+a)^(5/2)+33/1024*f*h^2/c^4*b^4*(c*x^2+b*x+a)^(3/2
)*x-99/8192*f*h^2/c^5*b^6*(c*x^2+b*x+a)^(1/2)*x-1/8/c*b*(c*x^2+b*x+a)^(3/2)
*x*e*g^2-1/8/c^2*b^2*(c*x^2+b*x+a)^(3/2)*d*g*h+3/64/c^2*b^3*(c*x^2+b*x+a)^(
1/2)*x*e*g^2+9/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*x*e*h^2-3/64/c^4*b^4*(c*x^2+
b*x+a)^(1/2)*a*e*h^2-9/1024/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*f*g*h+1/32/c^3*b^2*a*(c*x^2+b*x+a)^(3/2)*e*h^2+3/64/c^3*b^2*a^2*(
c*x^2+b*x+a)^(1/2)*e*h^2-15/256/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*a*d*h^2-15/256/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*a*f*g^2+7/512/c^(9/2)*b^6*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*e*g*h-1/24*a/c*(c*x^2+b*x+a)^(3/2)*x*d*h^2-1/24*a/c*(c*x^2+b*x+a)^(3/2)
*x*f*g^2-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*d*h^2-1/48*a/c^2*(c*x^2+b*x+a)^(3
/2)*b*f*g^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*d*h^2-1/16*a^2/c*(c*x^2+b*x+a)
^(1/2)*x*f*g^2-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*d*h^2-1/32*a^2/c^2*(c*x^2
+b*x+a)^(1/2)*b*f*g^2-1/8*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*e*g*h+1/3*x*(c*x^2+b*x+a)^(5/2)/c*e*g*h+3/16*d*g^2/c*(c*x^2+b*x+a)^(
1/2)*b*a-3/16*d*g^2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2

```
*a-1/16*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*e*g*h+7/48/c^2*b^2*(c*x^2+b*x+a)^(3/2
)*x*e*g*h+1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a*d*h^2+1/8/c^2*b^2*(c*x^2+b*x+
a)^(1/2)*x*a*f*g^2-7/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*e*g*h+153/2048*f*h^2
/c^4*b^4*(c*x^2+b*x+a)^(1/2)*x*a-7/256/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*f*g^2+
1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*d*h^2+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a
*f*g^2-7/256/c^4*b^5*(c*x^2+b*x+a)^(1/2)*e*g*h+9/64/c^(5/2)*b^2*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h^2+9/64/c^(5/2)*b^2*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g^2+153/4096*f*h^2/c^5*b^5*(c*x^2+b*x+a)
^(1/2)*a+33/2048*f*h^2/c^5*b^5*(c*x^2+b*x+a)^(3/2)-99/16384*f*h^2/c^6*b^7*(
c*x^2+b*x+a)^(1/2)+3/128*d*g^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))*b^4-9/2048/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*e*h^2+1/7*x^2*(c*x^2+b*x+a)^(5/2)/c*e*h^2-2/35*a/c^2*(c*x^2+b*x+a)^(5/2)*e
*h^2+3/40/c^3*b^2*(c*x^2+b*x+a)^(5/2)*e*h^2-3/128/c^4*b^4*(c*x^2+b*x+a)^(3/
2)*e*h^2+9/1024/c^5*b^6*(c*x^2+b*x+a)^(1/2)*e*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)
/c*d*h^2+1/6*x*(c*x^2+b*x+a)^(5/2)/c*f*g^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*d
*h^2-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)
*d*h^2+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g^2-7/512/c^4*b^5*(c*x^2+b*x+a)^
(1/2)*d*h^2-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g^2+7/1024/c^(9/2)*b^6*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+7/1024/c^(9/2)*b^6*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*f*g^2+3/128*f*h^2*a^4/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+99/32768*f*h^2/c^(13/2)*b^8*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+1/8*f*h^2*x^3*(c*x^2+b*x+a)^(5/2)/c-33/640*f*h^2/c^4*b^3*(c*x
^2+b*x+a)^(5/2)-3/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*x*a*f*g*h+1/8/c^2*b*a*(c*x
^2+b*x+a)^(3/2)*x*f*g*h+3/16/c^2*b*a^2*(c*x^2+b*x+a)^(1/2)*x*f*g*h+1/4/c^2*
b^2*(c*x^2+b*x+a)^(1/2)*x*a*e*g*h-3/8/c*b*(c*x^2+b*x+a)^(1/2)*x*a*d*g*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(g+h\,x\right)^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g+hx\right)^{2}\left(a+bx+cx^{2}\right)^{\frac{3}{2}}\left(d+ex+fx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)**2*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

# 3.198 $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

**Optimal**. Leaf size=418

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)}{384c^4}+\frac{(a$$

```
[Out] 1/384*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*
f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*x^
2+b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h^2-24*c^2*(5*f*g^2-7*h*(d*h+e*g))-2*c*h
*(24*a*f*h+49*b*(e*h+f*g))-10*c*h*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2+b*x
+a)^(5/2)/c^3/h+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*
f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)-1/1024*(-4*a*c+b^2)*(48*c^3*d*g-9*b^3*
f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*(2*c
*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
```

**Rubi [A]** time = 0.65, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 621, 206}

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+fg))+63b^2fh^2-10chx(9bfh-14ceh+10cfg)-24c^2\left(5fg^2-7h(d\right.\right.}{840c^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h
+ a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*Sqrt[a + b*x + c*
x^2])/(1024*c^5) + ((48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*
d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(b + 2*c*x)*(a + b*x + c*
x^2)^(3/2))/(384*c^4) + (f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) + (
(63*b^2*f*h^2 - 24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b
*(f*g + e*h)) - 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)
^(5/2))/(840*c^3*h) + ((b^2 - 4*a*c)^2*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b
*e*g + a*f*g + 3*b*d*h + a*e*h) + 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{\int (g+hx)\left(-\frac{1}{2}h(5bfg-14cdh \cdots}{\cdots}$$

$$= \frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch} + \frac{\left(63b^2fh^2-24c^2\left(5fg^2-7h(eg \cdots}{\cdots}$$

$$= \frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh \cdots}{384c^4 \cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh) \cdots}{1024c^5 \cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh) \cdots}{1024c^5 \cdots}$$

$$= -\frac{\left(b^2-4ac\right)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh) \cdots}{1024c^5 \cdots}$$

**Mathematica [A]** time = 0.78, size = 285, normalized size = 0.68

$$\frac{\frac{(a+x(b+cx))^{5/2}\left(-2ch(24afh+b(49eh+49fg+45fhx))+63b^2fh^2-4c^2(5fg(6g+5hx)-7h(6dh+6eg+5ehx))\right)}{120c^2} - \frac{7h\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2 \cdots\right.\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + ((a + x*(b + c*x))^(5/2)*(63*b^2*f
*h^2 - 4*c^2*(5*f*g*(6*g + 5*h*x) - 7*h*(6*e*g + 6*d*h + 5*e*h*x)) - 2*c*h*
(24*a*f*h + b*(49*f*g + 49*e*h + 45*f*h*x))))/(120*c^2) - (7*h*(-48*c^3*d*g
 + 9*b^3*f*h + 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) - 2*b*c*(6*a*f*h +
 7*b*(f*g + e*h)))*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b^2 + 8
*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(6144*c^(9/2)))/(7*c*h)
```

**fricas [B]** time = 1.03, size = 1833, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3
 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b
^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5
*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x
- b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7
*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*
c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f
)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g +
 (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*
f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 3
6*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)
*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^
3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (
105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100
*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*
c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f
)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c
^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 -
728*a*b^3*c^3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6, -1/215
040*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2 - 8*a*
b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*
a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(7*b^6*c -
 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a*b^5*c + 8
0*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(15360*c^7*f*h*x^6 + 1
280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(14*(12*c^7*e + 13*b
*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*a*c^6)*f)*h)*x^4 + 1
6*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^6)*f)*g + (1848*b*c^6
*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44*a*b*c^5)*f)*h)*x^3 +
8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*b^3*c^4 - 36*a*b*c^5)*f
)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 - 36*a*b*c^5)*e + 3*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 - 14*(120*(3*b^3*c^4 - 20*a
*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*e + (105*b^5*c^2
- 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(15*b^4*c^3 - 100*a*b^2*c^4 +
 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*e + 3*(
315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4)*f)*h + 2*(14*
(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 - 28*a*b*c^5)*e + (35*b^4*c^3 -
 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5*b^3*c^4 - 28*a*b*c^5)*d - 14*(
35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*e + 3*(105*b^5*c^2 - 728*a*b^3*c^
3 + 1168*a^2*b*c^4)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

**giac [B]** time = 0.29, size = 955, normalized size = 2.28

$$\frac{1}{107520}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(12\,cfhx+\frac{14\,c^7fg+15\,bc^6fh+14\,c^7he}{c^6}\right)x+\frac{182\,bc^6fg+168\,c^7dh+3\,b^2}{}\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
15*b*c^6*f*h + 14*c^7*h*e)/c^6)*x + (182*b*c^6*f*g + 168*c^7*d*h + 3*b^2*c^
5*f*h + 192*a*c^6*f*h + 168*c^7*g*e + 182*b*c^6*h*e)/c^6)*x + (1680*c^7*d*g
 + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h - 27*b^3*c^4*f*h + 132*
a*b*c^5*f*h + 1848*b*c^6*g*e + 42*b^2*c^5*h*e + 1960*a*c^6*h*e)/c^6)*x + (5
040*b*c^6*d*g - 98*b^3*c^4*f*g + 504*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a
*c^6*d*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h + 168*b^2*c
^5*g*e + 5376*a*c^6*g*e - 98*b^3*c^4*h*e + 504*a*b*c^5*h*e)/c^6)*x + (1680*
b^2*c^5*d*g + 33600*a*c^6*d*g + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360
*a^2*c^5*f*g - 840*b^3*c^4*d*h + 4704*a*b*c^5*d*h - 315*b^5*c^2*f*h + 2184*
a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h - 840*b^3*c^4*g*e + 4704*a*b*c^5*g*e + 4
90*b^4*c^3*h*e - 3024*a*b^2*c^4*h*e + 3360*a^2*c^5*h*e)/c^6)*x - (5040*b^3*
c^4*d*g - 33600*a*b*c^5*d*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d*
h - 945*b^6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c
^4*f*h - 2520*b^4*c^3*g*e + 16800*a*b^2*c^4*g*e - 21504*a^2*c^5*g*e + 1470*
b^5*c^2*h*e - 10640*a*b^3*c^3*h*e + 18144*a^2*b*c^4*h*e)/c^6) - 1/2048*(48*
b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g + 14*b^6*c*f*g - 120*a*b^
4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 192*a*
b^3*c^3*d*h - 384*a^2*b*c^4*d*h - 9*b^7*f*h + 84*a*b^5*c*f*h - 240*a^2*b^3*
c^2*f*h + 192*a^3*b*c^3*f*h - 24*b^5*c^2*g*e + 192*a*b^3*c^3*g*e - 384*a^2*
b*c^4*g*e + 14*b^6*c*h*e - 120*a*b^4*c^2*h*e + 288*a^2*b^2*c^3*h*e - 128*a^
3*c^4*h*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(
11/2)
```

**maple [B]** time = 0.01, size = 2026, normalized size = 4.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/4*d*g*(c*x^2+b*x+a)^(3/2)*x+1/5*(c*x^2+b*x+a)^(5/2)/c*d*h+1/5*(c*x^2+b*x+
a)^(5/2)/c*e*g+9/1024*h*f/c^5*b^6*(c*x^2+b*x+a)^(1/2)-3/32*h*f/c^3*b^3*(c*x
^2+b*x+a)^(1/2)*x*a+3/32*h*f/c^2*b*a^2*(c*x^2+b*x+a)^(1/2)*x+1/16*h*f/c^2*b
```

```
*a*(c*x^2+b*x+a)^(3/2)*x-3/16/c*b*(c*x^2+b*x+a)^(1/2)*x*a*d*h-3/16/c*b*(c*x
^2+b*x+a)^(1/2)*x*a*e*g+1/8/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a*f*g+1/8/c^2*b^2
*(c*x^2+b*x+a)^(1/2)*x*a*e*h-3/128*h*f/c^4*b^4*(c*x^2+b*x+a)^(3/2)-15/128*h
*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-15/256/c^(7/
2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f*g-1/24*a/c*(c*x^2+b*
x+a)^(3/2)*x*e*h-1/48*a/c^2*(c*x^2+b*x+a)^(3/2)*b*f*g-1/48*a/c^2*(c*x^2+b*x
+a)^(3/2)*b*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c*e*h+1/6*x*(c*x^2+b*x+a)^(5/2)/c
*f*g-1/16/c^2*b^2*(c*x^2+b*x+a)^(3/2)*d*h-1/16/c^2*b^2*(c*x^2+b*x+a)^(3/2)*
e*g+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)*d*h+3/128/c^3*b^4*(c*x^2+b*x+a)^(1/2)
*e*g-3/256/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h-7/25
6/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*e*h-7/256/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x*f*g
+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*e*h+1/16/c^3*b^3*(c*x^2+b*x+a)^(1/2)*a*
f*g+9/64/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*h+9/
64/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*f*g-15/256/c
^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*h-1/16*a^2/c*(c*
x^2+b*x+a)^(1/2)*x*e*h-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*f*g-1/32*a^2/c^2*(c
*x^2+b*x+a)^(1/2)*b*e*h-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*f*g+7/96/c^2*b^2
*(c*x^2+b*x+a)^(3/2)*x*e*h+7/96/c^2*b^2*(c*x^2+b*x+a)^(3/2)*x*f*g-1/8/c*b*(
c*x^2+b*x+a)^(3/2)*x*d*h-1/8/c*b*(c*x^2+b*x+a)^(3/2)*x*e*g+3/64/c^2*b^3*(c*
x^2+b*x+a)^(1/2)*x*d*h+3/64/c^2*b^3*(c*x^2+b*x+a)^(1/2)*x*e*g-3/32/c^2*b^2*
(c*x^2+b*x+a)^(1/2)*a*d*h-3/32/c^2*b^2*(c*x^2+b*x+a)^(1/2)*a*e*g-3/16/c^(3/
2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*h-3/16/c^(3/2)*b*ln(
(c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*e*g+3/32/c^(5/2)*b^3*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*h+3/32/c^(5/2)*b^3*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*a*e*g-3/32*d*g/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/16*
d*g/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d*g/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*b^2*a-1/24*a/c*(c*x^2+b*x+a)^(3/2)*x*f*g+1/32*h*f/c^3*b^2*a
*(c*x^2+b*x+a)^(3/2)+3/64*h*f/c^3*b^2*a^2*(c*x^2+b*x+a)^(1/2)-3/28*h*f/c^2*
b*x*(c*x^2+b*x+a)^(5/2)+21/512*h*f/c^(9/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a+3/32*h*f/c^(5/2)*b*a^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))-3/64*h*f/c^3*b^3*(c*x^2+b*x+a)^(3/2)*x+9/512*h*f/c^4*b^5*(c*x^2+b
*x+a)^(1/2)*x-3/64*h*f/c^4*b^4*(c*x^2+b*x+a)^(1/2)*a-3/256/c^(7/2)*b^5*ln((
c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g+1/8*d*g/c*(c*x^2+b*x+a)^(3/2)*b
+3/8*d*g*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d*g/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d
*g/c^(1/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d*g/c^(5/2
)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-7/60/c^2*b*(c*x^2+b*x+a)^
(5/2)*e*h-7/60/c^2*b*(c*x^2+b*x+a)^(5/2)*f*g+7/192/c^3*b^3*(c*x^2+b*x+a)^(3
/2)*e*h+7/192/c^3*b^3*(c*x^2+b*x+a)^(3/2)*f*g-7/512/c^4*b^5*(c*x^2+b*x+a)^(
1/2)*e*h-7/512/c^4*b^5*(c*x^2+b*x+a)^(1/2)*f*g+7/1024/c^(9/2)*b^6*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h+7/1024/c^(9/2)*b^6*ln((c*x+1/2*b)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e*h-1/16*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*f*g-9/2048*h*f/c^(11/2)*b^7*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))+1/7*h*f*x^2*(c*x^2+b*x+a)^(5/2)/c-2/35*h*f*a/c^2*(c*x^2+b*x+a)^(5/2)+3/
40*h*f/c^3*b^2*(c*x^2+b*x+a)^(5/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((g + h*x)*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

# 3.199 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=236

$$\frac{\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+3be)+7b^2f+24c^2d\right)}{1024c^{9/2}}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af\right.}{512c^4}$$

```
[Out] 1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^
3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*x^2+b*x+a)^(5/2)/
c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b
^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4
```

**Rubi [A]** time = 0.24, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right)^{3}}{6c}$$

$$= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}+\frac{\left(2c(6cd-af)\right.}{}$$

$$= \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3}+\frac{(12ce-7}{}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4}+$$

**Mathematica [A]** time = 0.69, size = 392, normalized size = 1.66

$$\frac{360d\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}}-60be\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\,(b+2cx)\sqrt{\right.}\right.}{c^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

**fricas [A]** time = 0.51, size = 839, normalized size = 3.56

$$\left[-\frac{15\left(24\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)d-12\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)e+\left(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3\right)f\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
```

```
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

**giac [A]** time = 0.26, size = 417, normalized size = 1.77

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{13\,bc^5f+12\,c^6e}{c^5}\right)x+\frac{120\,c^6d+3\,b^2c^4f+140\,ac^5f+132\,bc^5e}{c^5}\right)x+\frac{36\cdots}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 862, normalized size = 3.65

$$-\frac{a^3f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{9a^2b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{64c^{\frac{5}{2}}}-\frac{3a^2be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{16c^{\frac{3}{2}}}+\frac{3a^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/8*f/c^2*b^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e/c*b*(c*x^2+b*x+a)^(1/2)*x*a+1/
6*f*x*(c*x^2+b*x+a)^(5/2)/c+1/8*d/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d*(c*x^2+b*x+
a)^(1/2)*x*a-3/64*d/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d/c^(1/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*b^4-1/16*e/c^2*b^2*(c*x^2+b*x+a)^(3/2)+3/128*e/c^3*b^4*
```

```
(c*x^2+b*x+a)^(1/2)-3/256*e/c^(7/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-7/60*f/c^2*b*(c*x^2+b*x+a)^(5/2)+7/192*f/c^3*b^3*(c*x^2+b*x+a)^(3/
2)-7/512*f/c^4*b^5*(c*x^2+b*x+a)^(1/2)+7/1024*f/c^(9/2)*b^6*ln((c*x+1/2*b)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/16*f*a^3/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3/16*d/c*(c*x^2+b*x+a)^(1/2)*b*a-3/16*d/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+1/4*d*(c*x^2+b*x+a)^(3/2)*x+1/5*e*(
c*x^2+b*x+a)^(5/2)/c-1/24*f*a/c*(c*x^2+b*x+a)^(3/2)*x-1/8*e/c*b*(c*x^2+b*x+
a)^(3/2)*x-1/32*f*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b-1/48*f*a/c^2*(c*x^2+b*x+a)^
(3/2)*b-1/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x-3/32*d/c*(c*x^2+b*x+a)^(1/2)*x*b
^2+3/64*e/c^2*b^3*(c*x^2+b*x+a)^(1/2)*x-3/32*e/c^2*b^2*(c*x^2+b*x+a)^(1/2)*
a-3/16*e/c^(3/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/32*e/c
^(5/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+7/96*f/c^2*b^2*(c*
x^2+b*x+a)^(3/2)*x-7/256*f/c^3*b^4*(c*x^2+b*x+a)^(1/2)*x+1/16*f/c^3*b^3*(c*
x^2+b*x+a)^(1/2)*a+9/64*f/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*a^2-15/256*f/c^(7/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*a
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

# 3.200 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx$

**Optimal**. Leaf size=660

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-4ach-3b^2h+8bcg\right)(bfh-2ceh+2c\cdots\right.\right.}{\cdots}$$

```
[Out] -1/48*(8*c*h*(b*f*g-2*c*d*h)-(-3*b*h+8*c*g)*(b*f*h-2*c*e*h+2*c*f*g)+6*c*h*(
b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/h^3+1/5*f*(c*x^2+b*x+a)^(
5/2)/c/h-1/256*(4*c*h*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b*g)*(b*f*g-2*c*d*h)-g*(-
4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))-2*(4*c^2*g^2-1/2*b^2*h^2-
2*c*h*(-a*h+b*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*
g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^6+(a*h^2-b*g*h+c*g^2)^(3/2)*(f*g^2-h*(-d
*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^6+1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b*e*h+b*f*
g)+128*c^4*g^2*(f*g^2-h*(-d*h+e*g))-32*c^3*h*(-4*a*h+5*b*g)*(f*g^2-h*(-d*h+
e*g))-8*b*c^2*h^2*(3*a*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c*h*(8*c*h*(
-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4
*c*h*(-3*a*h+2*b*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5
```

**Rubi [A]** time = 1.83, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2bg-3ah)-3b^2h^2+16c^2g^2\right)(bfh-2ceh+2cfg)\right)+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] ((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4
*a*h)*(f*g^2 - h*(e*g - d*h)) + 128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c
^2*h^2*(3*a*h*(f*g - e*h) - 2*b*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*
c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*
b^2*h^2 - 4*c*h*(2*b*g - 3*a*h)))*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*h^5) -
 ((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g - 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) +
6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(48*c^2*h^3)
+ (f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - ((4*c*h*(2*c*g - b*h)*(8*c*h*(b*g -
 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*
```

```
e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c*h*(2*c
*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b
^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(256*c^(7/2)*h^6) + ((c*g^2 - b*g*h + a*h^2)^(3/2)*(f*g^2 -
h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h^6
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{g+hx}\,dx = \frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch} + \frac{\int \frac{\left(-\frac{5}{2}h(bfg-2cdh)-\frac{5}{2}h(2cfg-2ceh+bfh)x\right)\left(a+bx+cx^2\right)^{3/2}}{g+hx}\,dx}{5ch^2}$$

$$= -\frac{(8ch(bfg-2cdh)-(8cg-3bh)(2cfg-2ceh+bfh)+6ch(2cfg-2ce}{48c^2h^3}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

$$= \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)\left(fg^2-h(eg-\right.\right.}{}$$

**Mathematica [A]** time = 2.21, size = 635, normalized size = 0.96

$$-\frac{(a+x(b+cx))^{3/2}\left(3b^2fh^2+6bch(f(g+hx)-eh)-4c^2(h(4dh-4eg+3ehx)+fg(4g-3hx))\right)}{48c^2h^3}+\frac{-\tanh^{-1}\left(\frac{\ }{2\sqrt{\ }}\right.}{\ }$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

```
[Out] (f*(a + x*(b + c*x))^(5/2))/(5*c*h) - ((a + x*(b + c*x))^(3/2)*(3*b^2*f*h^2
 + 6*b*c*h*(-(e*h) + f*(g + h*x)) - 4*c^2*(f*g*(4*g - 3*h*x) + h*(-4*e*g +
4*d*h + 3*e*h*x))))/(48*c^2*h^3) + (Sqrt[c]*h*Sqrt[a + x*(b + c*x)]*(3*b^4*
f*h^4 + 64*c^4*g*(f*g^2 + h*(-(e*g) + d*h))*(2*g - h*x) + 6*b^2*c*h^3*(-(b*
e*h) - 2*a*f*h + b*f*(g + h*x)) + 4*b*c^2*h^2*(6*a*e*h^2 - 6*a*f*h*(g + h*x
) + b*f*g*(4*g + 3*h*x) + b*h*(-4*e*g + 4*d*h - 3*e*h*x)) - 16*c^3*h*(2*b*(
f*g^2 + h*(-(e*g) + d*h))*(5*g - h*x) + a*h*(f*g*(-8*g + 3*h*x) + h*(8*e*g
- 8*d*h - 3*e*h*x)))) - (2*c*h*(2*c*g - b*h)*(8*c*h*(b*g - 2*a*h)*(b*f*g -
2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g - 2*c*e*h + b*f*h)) + (
-4*c^2*g^2 + (b^2*h^2)/2 + 2*c*h*(b*g - a*h))*(8*c*h*(2*c*g - b*h)*(b*f*g -
 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b^2*h^2 + 4*c*h*(-2
*b*g + 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 1
28*c^(7/2)*(c*g^2 + h*(-(b*g) + a*h))^(3/2)*(f*g^2 + h*(-(e*g) + d*h))*ArcT
anh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sq
rt[a + x*(b + c*x)])])/(128*c^(7/2)*h^6)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 6715, normalized size = 10.17

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{g + hx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

# 3.201 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

**Optimal**. Leaf size=754

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2b\right.}{128c^{5}}$$

```
[Out] -1/24*(3*b*f*h^2*(-a*h+b*g)+8*c^2*g*(5*f*g^2-h*(-3*d*h+4*e*g))+c*h*(8*a*h*(
-e*h+2*f*g)-b*(43*f*g^2-8*h*(-3*d*h+4*e*g)))+6*c*h^2*(4*c*e*g+b*f*g-5*c*f*g
^2/h-4*c*d*h-a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/c/h^3/(a*h^2-b*g*h+c*g^2)-(f*g^2
-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*(3*b
^4*f*h^4+8*b^2*c*h^3*(-3*a*f*h-b*e*h+2*b*f*g)+128*c^4*g^2*(5*f*g^2-h*(-3*d*
h+4*e*g))+48*c^2*h^2*(a^2*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g*h+3*f
*g^2))+192*c^3*h*(a*h*(d*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4*f*g^2)
))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1/2*(2*c*
g*(5*f*g^2-h*(-3*d*h+4*e*g))+h*(2*a*h*(-e*h+2*f*g)-b*(3*d*h^2-5*e*g*h+7*f*g
^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))*(a*h^2-b*g*h+c*g^2)^(1/2)/h^6-1/64*(3*b^3*f*h^3+4*b*c*h^2*
(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*g*(5*f*g^2-h*(-3*d*h+4*e*g))+16*c^2*h*(4*
a*h*(-e*h+2*f*g)-b*(9*d*h^2-14*e*g*h+19*f*g^2))+2*c*h*(3*b^2*f*h^2+4*c*h*(-
3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h*(-3*d*h+4*e*g)))*x)*(c*x^2+b*x+a
)^(1/2)/c^2/h^5
```

**Rubi [A]** time = 2.50, antiderivative size = 750, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 814, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2h^2\left(a^2fh^2-2abh(2fg-eh)+b^2\left(dh^2-2egh+3fg^2\right)\right)+8b^2ch^3(-3afh-beh+2b\right.}{128c^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -((3*b^3*f*h^3 + 4*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f*g^3
- g*h*(4*e*g - 3*d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) - 4*a*
h*(2*f*g - e*h)) + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h)
 - 16*c^2*(5*f*g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(64*c^2*
h^5) - ((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4*e*g - 3*d*h)))/h -
c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g - e*h)) + 6*c*h*(4*c*e
```

```
*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(
24*c*h^2*(c*g^2 - b*g*h + a*h^2)) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x
^2)^(5/2))/(h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) + ((3*b^4*f*h^4 + 8*b^2*c*
h^3*(2*b*f*g - b*e*h - 3*a*f*h) + 128*c^4*(5*f*g^4 - g^2*h*(4*e*g - 3*d*h))
 + 48*c^2*h^2*(a^2*f*h^2 - 2*a*b*h*(2*f*g - e*h) + b^2*(3*f*g^2 - 2*e*g*h +
 d*h^2)) + 192*c^3*h*(a*h*(3*f*g^2 - 2*e*g*h + d*h^2) - b*g*(4*f*g^2 - 3*e*
g*h + 2*d*h^2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1
28*c^(5/2)*h^6) - (Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g -
 3*d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTa
nh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(2*h^6)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\left(\frac{1}{2}\left(-2cdg+5beg+2afg-\frac{5bfg^2}{h}-3bdh-\right.\right.}{}}{cg}$$

$$= -\frac{\left(3bfh(bg-ah)+\frac{8c^2\left(5fg^3-gh(4eg-3dh)\right)}{h}-c\left(43bfg^2-8bh(4eg-3dh)-\right.\right.}{24ch^2\left(cg\right.}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

$$= -\frac{\left(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3\left(5fg^3-gh(4eg-3dh)\right)\right.}{}$$

**Mathematica** [A] time = 4.16, size = 756, normalized size = 1.00

$$-\frac{-2ch\sqrt{a+x(b+cx)}\left(h(ah-bg)+cg^2\right)\left(-4c^2h(ah(8eh-16fg+3fhx)+2b(h(9dh-14eg+ehx)+fg(19g-2hx)))+bch^2(b(-4eh+8fg+3fhx)-6afh)+\frac{3}{2}b^3fh^3+16c^3(2g-hx)\left(h(3dh-4eg)+5fg^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

```
[Out] -1/4*(-((f*(a + x*(b + c*x))^(5/2))/(g + h*x)) + ((5*c*f*g^2 + f*h*(-(b*g)
+ a*h) + 4*c*h*(-(e*g) + d*h))*(a + x*(b + c*x))^(5/2))/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x)) + (((a + x*(b + c*x))^(3/2)*(3*b*f*h^2*(-(b*g) + a*h) +
 c*h*(8*b*h*(-4*e*g + 3*d*h) + b*f*g*(43*g - 6*h*x) + 2*a*h*(-8*f*g + 4*e*h
 + 3*f*h*x)) + c^2*(10*f*g^2*(-4*g + 3*h*x) + 8*h*(e*g*(4*g - 3*h*x) + 3*d*
```

```
h*(-g + h*x)))))/(6*h^2) + (-2*c*h*(c*g^2 + h*(-(b*g) + a*h))*Sqrt[a + x*(b
 + c*x)]*((3*b^3*f*h^3)/2 + 16*c^3*(5*f*g^2 + h*(-4*e*g + 3*d*h))*(2*g - h*
x) + b*c*h^2*(-6*a*f*h + b*(8*f*g - 4*e*h + 3*f*h*x)) - 4*c^2*h*(a*h*(-16*f
*g + 8*e*h + 3*f*h*x) + 2*b*(f*g*(19*g - 2*h*x) + h*(-14*e*g + 9*d*h + e*h*
x)))) + Sqrt[c]*(c*g^2 + h*(-(b*g) + a*h))*(2*c*h*(2*c*g - b*h)*(3*b^2*f*g*
h + 4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 + h*(-4*e*g + 3*d*h))) + ((8*c
^2*g^2 - b^2*h^2 + 4*c*h*(-(b*g) + a*h))*(-3*b^2*f*h^2 + 4*c*h*(-4*b*f*g +
2*b*e*h + 3*a*f*h) + 16*c^2*(5*f*g^2 + h*(-4*e*g + 3*d*h))))/2)*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 32*c^3*(c*g^2 + h*(-(b*g) + a
*h))^(3/2)*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-7*b*f*g^2 + b*h*(5*e
*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*
h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(16*c^2*h^5
))/(-(c*g^2) + h*(b*g - a*h)))/(c*h)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 14734, normalized size = 19.54

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^{2}\right)^{\frac{3}{2}}\left(d+ex+fx^{2}\right)}{\left(g+hx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

# 3.202 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$

**Optimal**. Leaf size=824

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(4cg\left(-\frac{10fg^2}{h}+6eg-3dh\right)-4ah(7fg-3eh)+b\left(31fg^2-3h(5eg-dh)\right)\right.}{12h^2\left(cg^2-bhg+a\right.}$$

```
[Out] -1/12*(4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-4*a*h*(-3*e*h+7*f*g)+b*(31*f*g^2-3*h*
(-d*h+5*e*g))+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*x^2+b
*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2
+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*h^2*(-2
*a*f*h-b*e*h+3*b*f*g)+16*c^3*g*(10*f*g^2-3*h*(-d*h+2*e*g))+24*c^2*h*(a*h*(-
e*h+3*f*g)-b*(d*h^2-3*e*g*h+6*f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*g^2*(10*f*g^2-3*h*(-d*h+2*e*g))+4*c*h
*(a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-b*g*(6*d*h^2-15*e*g*h+28*f*g^2))+h^2*(8*a^
2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*g))))*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)-1/8*(b^2*f*h^3*(-a*h+b*g)-8*c^3*g^2*(10*f*g
^2-3*h*(-d*h+2*e*g))-2*c^2*h*(2*a*h*(3*d*h^2-9*e*g*h+19*f*g^2)-3*b*g*(5*d*h
^2-12*e*g*h+22*f*g^2))-c*h^2*(8*a^2*f*h^2-18*a*b*h*(-e*h+3*f*g)+b^2*(53*f*g
^2-6*h*(-d*h+4*e*g)))+2*c*h*(b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-3*h*(-d*h
+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3*b*(d*h^2-3*e*g*h+6*f*g^2)))*x)*(c*x^2+
b*x+a)^(1/2)/c/h^5/(a*h^2-b*g*h+c*g^2)
```

**Rubi** [A] time = 2.14, antiderivative size = 819, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 812, 814, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\left(31bfg^2+4c\left(-\frac{10fg^2}{h}+6eg-3dh\right)g-3bh(5eg-dh)-4ah(7fg-3eh)\right.}{12h^2\left(cg^2-bhg+a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] -((b^2*f*h^2*(b*g - a*h) + 8*c^3*g^2*(6*e*g - (10*f*g^2)/h - 3*d*h) - 2*c^2
*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2 - 12*e*g*h + 5*d*h
^2)) - c*h*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^2*(53*f*g^2 - 6*h*(4*e
*g - d*h))) + 2*c*(b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - 3*g*h*(2*e*g - d
*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2 - 3*e*g*h + d*h^2)))*x)*Sq
```

```
rt[a + b*x + c*x^2])/(8*c*h^4*(c*g^2 - b*g*h + a*h^2)) - ((31*b*f*g^2 + 4*c
*g*(6*e*g - (10*f*g^2)/h - 3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g - 3*
e*h) + 2*h*(3*c*e*g + 2*b*f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a +
b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)) - ((f*g^2 -
h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(2*h*(c*g^2 - b*g*h + a*h^2)*(g + h
*x)^2) - ((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10*f
*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) - a*h
*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1
6*c^(3/2)*h^6) + ((8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28*b*f
*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) + h^2*
(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g - d*h)
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(8*h^6*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\left(\frac{1}{2}\left(-4cdg+5beg+4afg-\frac{5bfg^2}{h}-bdh-4\cdots\right.\right.}{\cdots}}{2\left(\cdots\right.}$$

$$= -\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)\cdots\right.}{12h^2\left(cg^2-bgh+a\cdots\right.}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh\cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2\left(2ah\left(19fg^2-9egh\cdots\right.\right.\right.}{\cdots}$$

**Mathematica [B]**   time = 6.27, size = 4162, normalized size = 5.05

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

```
[Out] (f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(3*c*h*(g + h*x)^2) - ((a + x
*(b + c*x))^(3/2)*(-1/2*(((h*(5*b*f*g - 6*c*d*h - 4*a*f*h))/2 - (g*(10*c*f*
g - 6*c*e*h + b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*
(g + h*x)^2) - (((-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)
) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
)*(a + b*x + c*x^2)^(5/2))/((-(c*g^2) + b*g*h - a*h^2)*(g + h*x)) + (((4*c*
(4*c*g - (3*b*h)/2)*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d
*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2) + 4*c*h*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
```

```
 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2) - 1
2*c^2*h*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2)*x)*(a +
 b*x + c*x^2)^(3/2))/(12*c*h^2) - (((2*c*h*(-4*c*(2*a*c*g*h + b*g*(-4*c*g +
 (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) +
(3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) +
4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g -
 d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e
*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*
e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))
/2)) - (2*c*g - (b*h)/2)*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g +
6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h
*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)
) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))
/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*
c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))
 + c*h*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*
c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*
(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*
a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f
*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*
f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(
5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*
h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))*x)*Sqrt[a + b*x + c
*x^2])/(2*c*h^2) - (((2*c*h*(2*c*g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g +
(3*b*h)/2))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (
3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4
*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g -
d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*
h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e
*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) -
 (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/
2)) + (-4*c^2*g^2 + (b^2*h^2)/2 - c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 +
 (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*
h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*
g - a*f*g + a*e*h)))/2) + 4*c*h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*
(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h)
+ 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h)
 + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g -
a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*
```

```
d*g - a*f*g + a*e*h)))/2))/2)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[c]*h) - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(-(g*(2*c*h*(2*c*
g - b*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c*g*(5*c*f*g^2 -
2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2 - b*h*(5*e*g - d
*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h)*(-3*a*c*h*(5*c
*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*(5*b*f*g^2 - b*h
*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h*(5*b*f*g^2 - b*
h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3*c*g*(5*c*f*g^2
- 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^2 - b*h*(5*e*g -
 d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (-4*c^2*g^2 + (b^2*h^2)/2 -
 c*h*(-2*b*g + 2*a*h))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g + 6*
a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(
2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h))
- (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2
 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/
2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*
h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))))
+ h*(2*c*h*(b*g - 2*a*h)*(-4*c*(2*a*c*g*h + b*g*(-4*c*g + (3*b*h)/2))*(-3*c
*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c*h*(5*b*f*g^2
- b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*h*(b*g - 2*a*h
)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c^2*g*
(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (3*b*c*h
*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2 + (5*b*(3
*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3*c*h*(5*b*f*g^
2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2)) + (2*a*c*g*h
+ b*g*(-2*c*g + (b*h)/2))*(-4*c*(-8*c^2*g^2 + (3*b^2*h^2)/2 - c*h*(-4*b*g +
 6*a*h))*(-3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) + (3*c
*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2) + 4*c*
h*(2*c*g - b*h)*(-3*a*c*h*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h
)) - (3*c^2*g*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h))
)/2 + (3*b*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)
))/2 + (5*b*(3*c*g*(5*c*f*g^2 - 2*f*h*(b*g - a*h) - 3*c*h*(e*g - d*h)) - (3
*c*h*(5*b*f*g^2 - b*h*(5*e*g - d*h) + 4*h*(c*d*g - a*f*g + a*e*h)))/2))/2))
))*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2
]*Sqrt[a + b*x + c*x^2])])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4*c*h^2))/(8
*c*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2))))/(3*c*h*(
a + b*x + c*x^2)^(3/2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas"
```

```
)
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.47Unable to divide, perha
ps due to rounding error%%%{1,[6,0,0,0,9,0,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,0,0,0,8,1,0,0]%%%}+%%%{-3,[4,1,0,0,9,0,0,0]%%%}+%%%{3,[4,
0,1,0,8,1,0,0]%%%}+%%%{%%%{12,[1]%%%},[4,0,0,0,7,2,0,0]%%%}+%%%{%%{[12,0]:[
1,0,%%%{-1,[1]%%%}]%%},[3,1,0,0,8,1,0,0]%%%}+%%%{%%{[-12,0]:[1,0,%%%{-1,[1]
%%%}]%%},[3,0,1,0,7,2,0,0]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,0,0,0,6,3,0,0]%%%}+%%%{3,[2,2,0,0,9,0,0,0]%%%}+%%%{-6,[2,1,1,0,8,1
,0,0]%%%}+%%%{%%%{-12,[1]%%%},[2,1,0,0,7,2,0,0]%%%}+%%%{3,[2,0,2,0,7,2,0,0]
%%%}+%%%{%%%{12,[1]%%%},[2,0,1,0,6,3,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,2,0,0,8,1,0,0]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,1
,0,7,2,0,0]%%%}+%%%{%%{[-6,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2,0,6,3,0,0]%%%}
+%%%{-1,[0,3,0,0,9,0,0,0]%%%}+%%%{3,[0,2,1,0,8,1,0,0]%%%}+%%%{-3,[0,1,2,0,7
,2,0,0]%%%}+%%%{1,[0,0,3,0,6,3,0,0]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[6,0,0,0,3,0,0,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0,0,0,2,1,
0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,1,0,0,3,0,0,0
]%%%}+%%%{%%{poly1[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1,0,2,1,0,
0]%%%}+%%%{%%{poly1[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0,0,1,2,
0,0]%%%}+%%%{%%%{12,[2]%%%},[3,1,0,0,2,1,0,0]%%%}+%%%{%%%{-12,[2]%%%},[3,0,
1,0,1,2,0,0]%%%}+%%%{%%%{-8,[3]%%%},[3,0,0,0,0,3,0,0]%%%}+%%%{%%{[%%%{3,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{-6,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,1,0,2,1,0,0]%%%}+%%%{%%{[%%%{-12,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{3,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2,0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{12,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1,0,0,3,0,0]%%%}+%%%{%%%{-6,[2]%%%},[1,2
,0,0,2,1,0,0]%%%}+%%%{%%%{12,[2]%%%},[1,1,1,0,1,2,0,0]%%%}+%%%{%%%{-6,[2]%%
%},[1,0,2,0,0,3,0,0]%%%}+%%%{%%{[%%%{-1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[0,3,0,0,3,0,0,0]%%%}+%%%{%%{[%%%{3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2
,1,0,2,1,0,0]%%%}+%%%{%%{[%%%{-3,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,2,
0,1,2,0,0]%%%}+%%%{%%{poly1[%%%{1,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3
,0,0,3,0,0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 26596, normalized size = 32.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                              +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

# 3.203 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$

**Optimal**. Leaf size=833

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(2cg\left(-\frac{10fg^2}{h}+4eg-dh\right)-6ah(3fg-eh)+b\left(17fg^2-h(5eg+dh)\right)+2\right.}{12h^2\left(cg^2-bhg+ah^2\right.}$$

```
[Out] -1/12*(2*c*g*(4*e*g-10*f*g^2/h-d*h)-6*a*h*(-e*h+3*f*g)+b*(17*f*g^2-h*(d*h+5
*e*g))+2*h*(2*c*e*g+3*b*f*g-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)^(
3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+
a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/16*(16*c^3*g^3*(10*f*g^2-h*(-d*h
+4*e*g))-b*h^3*(24*a^2*f*h^2-6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h+35*f
*g^2))+6*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2)-2*a*
b*h*(d*h^2-7*e*g*h+25*f*g^2))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^2)-a*h*
(d*h^2-4*e*g*h+11*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*
g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2)+1/8*(3*
b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f*g^2-h*(-d*h+4*e*g)))*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/c^(1/2)-1/8*(8*c^2*g^2
*(10*f*g^2-h*(-d*h+4*e*g))-2*c*h*(3*b*g*(d*h^2-6*e*g*h+18*f*g^2)-2*a*h*(2*d
*h^2-8*e*g*h+23*f*g^2))+h^2*(12*a^2*f*h^2-6*a*b*h*(-e*h+7*f*g)+b^2*(29*f*g^
2-h*(d*h+5*e*g)))+2*h*(3*b*f*h^2*(-a*h+b*g)+2*c^2*g*(10*f*g^2-h*(-d*h+4*e*g
))+c*h*(6*a*h*(-e*h+3*f*g)-b*(d*h^2-7*e*g*h+22*f*g^2)))*x)*(c*x^2+b*x+a)^(1
/2)/h^5/(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

**Rubi [A]** time = 2.26, antiderivative size = 829, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}-\frac{\left(17bfg^2+2c\left(-\frac{10fg^2}{h}+4eg-dh\right)g-bh(5eg+dh)-6ah(3fg-eh)+2\right.}{12h^2\left(cg^2-bhg+ah^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] -((12*a^2*f*h^3 - 8*c^2*g^2*(4*e*g - (10*f*g^2)/h - d*h) - 6*a*b*h^2*(7*f*g
 - e*h) + 4*a*c*h*(23*f*g^2 - 2*h*(4*e*g - d*h)) - 6*b*c*g*(18*f*g^2 - h*(6
*e*g - d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + d*h)) + 2*(3*b*f*h^2*(b*g - a*h
) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(22*b*f*g^2 - b*h*(7*e*g - d
*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*x^2])/(8*h^4*(c*g^2 - b*g*h
```

```
 + a*h^2)*(g + h*x)) - ((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h - d*h) -
b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g - (5*c*f*g
^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(12*h^2*(c*g^2 - b*g
*h + a*h^2)*(g + h*x)^2) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2)
)/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((3*b^2*f*h^2 - 12*c*h*(4*b*f
*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*h^6) - ((16*c^3*(10*f*g^5
 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h) + b^
2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) + b^2*g*
(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2)) - 24*
c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*h + d*h^2
)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*
Sqrt[a + b*x + c*x^2])])/(16*h^6*(c*g^2 - b*g*h + a*h^2)^(3/2))
```

## Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

## Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

## Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

## Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3} - \frac{\int \frac{\left(\frac{1}{2}\left(-6cdg+5beg+6afg-\frac{5bfg^2}{h}+bdh-6\cdots\right.\right.}{\cdots}}{3\left(\cdots\right.}$$

$$= -\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2\cdots\right.}{12h^2\left(cg^2-bgh+ah^2\cdots\right.}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach\left(23fg^2\cdots\right.\right.}{\cdots}$$

**Mathematica [B]** time = 6.49, size = 7806, normalized size = 9.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

```
[Out] Result too large to show
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas"
)
```

[Out] Timed out

**giac [B]** time = 26.17, size = 7319, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 5*b*c*f*h^11 -
4*c^2*h^11*e)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 336*b*c^2*f*g^4*h + 16*c^3*d
*g^3*h^2 + 210*b^2*c*f*g^3*h^2 + 264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 -
 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a*c^2*d*g*h^
4 + 60*a*b^2*f*g*h^4 + 96*a^2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 24*a
^2*b*f*h^5 - 64*c^3*g^4*h*e + 120*b*c^2*g^3*h^2*e - 60*b^2*c*g^2*h^3*e - 96
*a*c^2*g^2*h^3*e + 5*b^3*g*h^4*e + 84*a*b*c*g*h^4*e - 6*a*b^2*h^5*e - 24*a^
2*c*h^5*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt
(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g^2 + b*g*
h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*f*g^5
*h^2 - 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*f*g^4*h^3 + 144*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*d*g^3*h^4 + 522*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^5*a*c^(5/2)*f*g^3*h^4 - 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*b*c^(5/2)*d*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqr
t(c)*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*f*g^
2*h^5 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c^(3/2)*d*g*h^6 + 120*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^(5/2)*d*g*h^6 + 108*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^5*a^2*c^(3/2)*f*g*h^6 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^3*sqrt(c)*d*h^7 - 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*
d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*sqrt(c)*f*h^7 - 288*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^(7/2)*g^4*h^3*e + 504*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b*c^(5/2)*g^3*h^4*e - 252*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^5*b^2*c^(3/2)*g^2*h^5*e - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^5*a*c^(5/2)*g^2*h^5*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*sqr
t(c)*g*h^6*e + 228*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c^(3/2)*g*h^6*
e - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*sqrt(c)*h^7*e - 24*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c^(3/2)*h^7*e + 1680*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*c^4*f*g^6*h - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^4*b*c^3*f*g^5*h^2 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*d*g^4*h^
3 + 1362*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*f*g^4*h^3 + 1464*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*f*g^4*h^3 - 504*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*b*c^3*d*g^3*h^4 - 147*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^4*b^3*c*f*g^3*h^4 - 876*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*f*
g^3*h^4 + 54*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*d*g^2*h^5 + 216*
```

(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*d*g^2*h^5 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*f*g^2*h^5 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*f*g^2*h^5 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*d*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*d*g*h^6 + 216*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*f*g*h^6 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*d*h^7 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*d*h^7 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*c*f*h^7 - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*c^4*g^5*h^2*e + 1464*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b*c^3*g^4*h^3*e - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^2*c^2*g^3*h^4*e - 672*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*c^3*g^3*h^4*e + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*b^3*c*g^2*h^5*e + 180*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b*c^2*g^2*h^5*e + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a*b^2*c*g*h^6*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*c^2*g*h^6*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^2*b*c*h^7*e + 1504*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*f*g^7 - 1072*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*f*g^6*h + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*d*g^5*h^2 - 1308*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*f*g^5*h^2 - 656*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*f*g^5*h^2 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*d*g^4*h^3 + 1042*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*f*g^4*h^3 + 4056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*f*g^4*h^3 - 420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*d*g^3*h^4 - 272*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*d*g^3*h^4 - 136*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*f*g^3*h^4 - 2712*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*f*g^3*h^4 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*f*g^3*h^4 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*d*g^2*h^5 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*d*g^2*h^5 + 328*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*f*g^2*h^5 + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*f*g^2*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*d*g*h^6 - 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*d*g*h^6 - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*d*g*h^6 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*sqrt(c)*f*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*c^(3/2)*f*g*h^6 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*d*h^7 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*sqrt(c)*f*h^7 - 832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^(9/2)*g^6*h*e + 400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c^(7/2)*g^5*h^2*e + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*c^(5/2)*g^4*h^3*e + 512*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c^(7/2)*g^4*h^3*e - 478*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^3*c^(3/2)*g^3*h^4*e - 2232*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*c^(5/2)*g^3*h^4*e + 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^4*sqrt(c)*g^2*h^5*e + 1092*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^2*c^(3/2)*g^2*h^5*e + 1104*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*c^(5/2)*g^2*h^5*e - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*sqrt(c)*g*h^6*e - 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c^(3/2)*g*h^6*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x

+ a))^3*a^2*b^2*sqrt(c)*h^7*e + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*b*c^4*f*g^7 - 3420*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*f*g^6*h -
2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*f*g^6*h + 528*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^2*b*c^4*d*g^5*h^2 + 1218*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^2*b^3*c^2*f*g^5*h^2 + 5976*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a*b*c^3*f*g^5*h^2 - 516*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*c^3*d*g^
4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^4*d*g^4*h^3 - 24*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*f*g^4*h^3 - 1944*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^2*a*b^2*c^2*f*g^4*h^3 - 2208*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^2*a^2*c^3*f*g^4*h^3 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c
^2*d*g^3*h^4 + 840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*d*g^3*h^4
- 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*f*g^3*h^4 + 192*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*f*g^3*h^4 + 24*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^2*b^4*c*d*g^2*h^5 + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^2*a*b^2*c^2*d*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*c^3
*d*g^2*h^5 + 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*f*g^2*h^5
+ 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*f*g^2*h^5 - 24*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*d*g*h^6 - 288*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^2*b*c^2*d*g*h^6 - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
2*a^3*b*c*f*g*h^6 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*d*h^7
+ 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*c*f*h^7 - 1248*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b*c^4*g^6*h*e + 1656*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*b^2*c^3*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c
^4*g^5*h^2*e - 414*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^3*c^2*g^4*h^3*e
- 2760*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*c^3*g^4*h^3*e - 24*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2*b^4*c*g^3*h^4*e + 420*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))^2*a*b^2*c^2*g^3*h^4*e + 912*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*c^3*g^3*h^4*e + 168*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b^3*c*
g^2*h^5*e + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b*c^2*g^2*h^5*e -
288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*b^2*c*g*h^6*e - 384*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^2*g*h^6*e + 144*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*a^3*b*c*h^7*e + 1128*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2
*c^(7/2)*f*g^7 - 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*f*g^6
*h - 2832*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*f*g^6*h + 264*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*d*g^5*h^2 + 720*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*b^4*c^(3/2)*f*g^5*h^2 + 5580*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a*b^2*c^(5/2)*f*g^5*h^2 + 1776*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*c^(7/2)*f*g^5*h^2 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*c^(
5/2)*d*g^4*h^3 - 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c^(7/2)*d*g^4*
h^3 - 57*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*f*g^4*h^3 - 2514*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*f*g^4*h^3 - 5688*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*f*g^4*h^3 + 36*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*b^4*c^(3/2)*d*g^3*h^4 + 852*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*b^2*c^(5/2)*d*g^3*h^4 + 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2
*c^(7/2)*d*g^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*

f*g^3*h^4 + 3078*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*f*g^3*
h^4 + 1848*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*f*g^3*h^4 + 3*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*d*g^2*h^5 - 90*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*d*g^2*h^5 - 864*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^2*b*c^(5/2)*d*g^2*h^5 - 249*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*f*g^2*h^5 - 1476*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^
3*b*c^(3/2)*f*g^2*h^5 - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^4*sqrt(c)
*d*g*h^6 + 90*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^2*c^(3/2)*d*g*h^6 +
 264*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*c^(5/2)*d*g*h^6 + 132*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*f*g*h^6 + 192*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a^4*c^(3/2)*f*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*a^2*b^3*sqrt(c)*d*h^7 - 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^
(3/2)*d*h^7 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*sqrt(c)*f*h^7 -
624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c^(7/2)*g^6*h*e + 876*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*b^3*c^(5/2)*g^5*h^2*e + 1536*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))*a*b*c^(7/2)*g^5*h^2*e - 282*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*b^4*c^(3/2)*g^4*h^3*e - 2664*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^
2*c^(5/2)*g^4*h^3*e - 960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c^(7/2)*g
^4*h^3*e + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^5*sqrt(c)*g^3*h^4*e + 8
94*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^3*c^(3/2)*g^3*h^4*e + 2640*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*c^(5/2)*g^3*h^4*e - 48*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^4*sqrt(c)*g^2*h^5*e - 936*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))*a^2*b^2*c^(3/2)*g^2*h^5*e - 816*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^3*c^(5/2)*g^2*h^5*e + 51*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3
*sqrt(c)*g*h^6*e + 300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c^(3/2)*g*
h^6*e - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*sqrt(c)*h^7*e + 24*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c^(3/2)*h^7*e + 188*b^3*c^3*f*g^7 -
272*b^4*c^2*f*g^6*h - 708*a*b^2*c^3*f*g^6*h + 44*b^3*c^3*d*g^5*h^2 + 87*b^5
*c*f*g^5*h^2 + 1214*a*b^3*c^2*f*g^5*h^2 + 888*a^2*b*c^3*f*g^5*h^2 - 44*b^4*
c^2*d*g^4*h^3 - 156*a*b^2*c^3*d*g^4*h^3 - 426*a*b^4*c*f*g^4*h^3 - 2010*a^2*
b^2*c^2*f*g^4*h^3 - 376*a^3*c^3*f*g^4*h^3 + 3*b^5*c*d*g^3*h^4 + 182*a*b^3*c
^2*d*g^3*h^4 + 192*a^2*b*c^3*d*g^3*h^4 + 807*a^2*b^3*c*f*g^3*h^4 + 1468*a^3
*b*c^2*f*g^3*h^4 - 6*a*b^4*c*d*g^2*h^5 - 294*a^2*b^2*c^2*d*g^2*h^5 - 88*a^3
*c^3*d*g^2*h^5 - 732*a^3*b^2*c*f*g^2*h^5 - 400*a^4*c^2*f*g^2*h^5 + 3*a^2*b^
3*c*d*g*h^6 + 220*a^3*b*c^2*d*g*h^6 + 312*a^4*b*c*f*g*h^6 - 64*a^4*c^2*d*h^
7 - 48*a^5*c*f*h^7 - 104*b^3*c^3*g^6*h*e + 134*b^4*c^2*g^5*h^2*e + 384*a*b^
2*c^3*g^5*h^2*e - 33*b^5*c*g^4*h^3*e - 578*a*b^3*c^2*g^4*h^3*e - 480*a^2*b*
c^3*g^4*h^3*e + 144*a*b^4*c*g^3*h^4*e + 936*a^2*b^2*c^2*g^3*h^4*e + 208*a^3
*c^3*g^3*h^4*e - 237*a^2*b^3*c*g^2*h^5*e - 676*a^3*b*c^2*g^2*h^5*e + 174*a^
3*b^2*c*g*h^6*e + 184*a^4*c^2*g*h^6*e - 48*a^4*b*c*h^7*e)/((c^(3/2)*g^2*h^6
 - b*sqrt(c)*g*h^7 + a*sqrt(c)*h^8)*((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^3) - 1/8*(
80*c^2*f*g^2 - 48*b*c*f*g*h + 8*c^2*d*h^2 + 3*b^2*f*h^2 + 12*a*c*f*h^2 - 32
*c^2*g*h*e + 12*b*c*h^2*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sq
rt(c) + b))/(sqrt(c)*h^6)

**maple [B]** time = 0.03, size = 40092, normalized size = 48.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                               +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^{2}\right)^{\frac{3}{2}} \left(d + ex + fx^{2}\right)}{\left(g + hx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

# 3.204 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5} \, dx$

**Optimal**. Leaf size=1097

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}-\frac{\left(16c^2(5fg-eh)g^4-4ch\left(bg\left(31fg^2-5ehg+3dh^2\right)-ah\left(25fg^2-5ehg+\right.\right.\right.}{}$$

```
[Out] -1/96*(16*c^2*g^4*(-e*h+5*f*g)-h^2*(16*a^2*h^2*(-2*e*h+f*g)-b^2*g*(3*d*h^2+
5*e*g*h+35*f*g^2)+4*a*b*h*(3*d*h^2+7*e*g*h+7*f*g^2))-4*c*g*h*(b*g*(3*d*h^2-
5*e*g*h+31*f*g^2)-a*h*(9*d*h^2-5*e*g*h+25*f*g^2))+3*h*(8*c^2*g^2*(5*f*g^2-h
*(d*h+e*g))+h^2*(16*a^2*f*h^2-8*a*b*h*(-e*h+6*f*g)+b^2*(-3*d*h^2-5*e*g*h+29
*f*g^2))-4*c*h*(2*b*g*(-d*h^2-2*e*g*h+9*f*g^2)-a*h*(d*h^2-5*e*g*h+17*f*g^2)
))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/4*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/128*(128*
c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*h+5*f*g))+
8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h+14*f*g)+3*
a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e*h+6*f*g)-5*
b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4*(48*a^2*f*h^
2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arctanh(1/2*(b*g-2*
a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*h
^2-b*g*h+c*g^2)^(5/2)-1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/h^6+1/64*(64*c^3*g^4*(-e*h+5*f*g)-16*
c^2*g^2*h*(b*g*(-7*e*h+41*f*g)-8*a*h*(-e*h+5*f*g))+4*c*h^2*(2*b^2*g^2*(-5*e
*h+46*f*g)+16*a^2*h^2*(-e*h+5*f*g)-a*b*h*(-3*d*h^2-25*e*g*h+173*f*g^2))-b*h
^3*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2))+2*c*h
*(16*c^2*g^3*(-e*h+5*f*g)-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*h*(35*f*g^2-h*(-3*d
*h+7*e*g)))+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+b^2*(61*f*g^2-h*(3*d*h+
5*e*g))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

**Rubi [A]** time = 3.12, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 810, 812, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}-\frac{\left(\frac{16c^2(5fg-eh)g^4}{h}-4c\left(bg\left(31fg^2-5ehg+3dh^2\right)-ah\left(25fg^2-5ehg+9dh^2\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

```
[Out] (((64*c^3*g^4*(5*f*g - e*h))/h - 16*c^2*g^2*(b*g*(41*f*g - 7*e*h) - 8*a*h*(
5*f*g - e*h)) + 4*c*h*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h^2*(5*f*g - e*h
) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^2*(48*a^2*f*h^2 - 8*a*b*h
*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*c*(16*c^2*g^3*(5*
f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g^2 - h*(7*e*g - 3*d*
h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^2*(61*f*g^2 - h*(5*e*
g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(64*h^4*(c*g^2 - b*g*h + a*h^2)^2*(
g + h*x)) - (((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h^2*(f*g - 2*e*h) -
b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2 + 7*e*g*h + 3*d*h^2
)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(25*f*g^2 - 5*e*g*h +
9*d*h^2)) + 3*h*((40*c^2*f*g^4)/h + 16*a^2*f*h^3 - 8*c^2*g^2*(e*g + d*h) -
8*a*b*h^2*(6*f*g - e*h) + 4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h)) - 8*b*c*g*(9
*f*g^2 - h*(2*e*g + d*h)) + b^2*h*(29*f*g^2 - h*(5*e*g + 3*d*h)))*x)*(a + b
*x + c*x^2)^(3/2))/(96*h^2*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^3) - ((f*g^2
 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(4*h*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^4) - (Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*h^6) + ((128*c^4*g^5*(5*f*g - e*h) - 64
*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)) + 8*c*h^3*(24*a^3*f
*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*g - e*h) + 3*a*b^2*h*(
55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b*g^2*h*(6*f*g - e*h) - 5*b
^2*g^3*(7*f*g - e*h) - a^2*h^2*(25*f*g^2 - 5*e*g*h + d*h^2)) + b^2*h^4*(48*
a^2*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)))*A
rcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(128*h^6*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^5}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{4h\left(cg^2-bgh+ah^2\right)(g+hx)^4} - \frac{\int \frac{\left(\frac{1}{2}\left(-8cdg+5beg+8afg-\frac{5bfg^2}{h}+3bdh\right.\right.}{}}{4}$$

$$= -\frac{\left(\frac{16c^2g^4(5fg-eh)}{h} - h\left(16a^2h^2(fg-2eh) - b^2g\left(35fg^2+5egh+3dh^2\right)+\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g^2\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g^2\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g^2\right.\right.}{}$$

$$= \frac{\left(\frac{64c^3g^4(5fg-eh)}{h} - 16c^2g^2(bg(41fg-7eh)-8ah(5fg-eh)) + 4ch\left(2b^2g^2\right.\right.}{}$$

**Mathematica [B]** time = 6.62, size = 46895, normalized size = 42.75

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
[Out] Result too large to show
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.06, size = 57957, normalized size = 52.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                                +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

```
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right)}{\left(g + hx\right)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

# 3.205 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

**Optimal**. Leaf size=1226

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{\left(16c^2fg^5-2ch\left(13bfg^3-10afhg^2+3bdh^2g-6adh^3\right)g-h^2\left(-g\left(7fg^2+\right.\right.\right.}{}$$

```
[Out] -1/48*(16*c^2*f*g^5-2*c*g*h*(-6*a*d*h^3-10*a*f*g^2*h+3*b*d*g*h^2+13*b*f*g^3
)-h^2*(4*a^2*h^2*(-3*e*h+2*f*g)-b^2*g*(7*f*g^2+3*h*(d*h+e*g))+2*a*b*h*(f*g^
2+3*h*(d*h+2*e*g)))+h*(4*c^2*(-3*d*g^2*h^2+7*f*g^4)+2*c*g*h*(2*a*h*(-3*e*h+
14*f*g)-b*(-6*d*h^2-3*e*g*h+28*f*g^2))+h^2*(16*a^2*f*h^2-2*a*b*h*(-3*e*h+22
*f*g)+b^2*(25*f*g^2-3*h*(d*h+e*g))))*x)*(c*x^2+b*x+a)^(3/2)/h^3/(a*h^2-b*g*
h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-
b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+32*c^3*g*h^2*(
35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*c^2*h^3*(35*b^3
*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*f*g^2)-3*a*b^2*g
*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*
g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*(e*h+8*f*g)-b^3*(-
3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))*arctanh(1/2*(b*g-2
*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6/(a*
h^2-b*g*h+c*g^2)^(7/2)-1/128*(128*c^4*f*g^7-32*c^3*f*g^5*h*(-10*a*h+11*b*g)
+8*c^2*g*h^2*(38*b^2*f*g^4+2*a^2*h^2*(3*d*h^2+13*f*g^2)-a*b*g*h*(3*d*h^2+65
*f*g^2))-2*c*h^3*(8*a^3*h^3*(-3*e*h+2*f*g)-2*a*b^2*g^2*h*(3*e*h+34*f*g)+4*a
^2*b*h^2*(3*d*h^2+6*e*g*h+5*f*g^2)+b^3*(-3*d*g^2*h^2+35*f*g^4))-b*h^4*(-2*a
*h+b*g)*(16*a^2*f*h^2-2*a*b*h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))+h
*(128*c*f*(c*g^2-h*(-a*h+b*g))^3+(-b*h+2*c*g)*(32*c^3*f*g^5-8*c^2*g*h*(3*a*
d*h^3-11*a*f*g^2*h+10*b*f*g^3)+2*c*h^2*(4*a^2*h^2*(-3*e*h+10*f*g)-6*a*b*h*(
-d*h^2-e*g*h+11*f*g^2)+b^2*(3*d*g*h^2+29*f*g^3))-b*h^3*(16*a^2*f*h^2-2*a*b*
h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))))*x)*(c*x^2+b*x+a)^(1/2)/h^5/
(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2
```

**Rubi [A]** time = 4.00, antiderivative size = 1223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 810, 843, 621, 206, 724}

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}-\frac{\left(16c^2fg^5-2ch\left(13bfg^3-10afhg^2+3bdh^2g-6adh^3\right)g-h^2\left(-g\left(7fg^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -((((128*c^4*f*g^7)/h - 32*c^3*f*g^5*(11*b*g - 10*a*h) + 8*c^2*g*h*(38*b^2*f
*g^4 + 2*a^2*h^2*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - b*h
^3*(b*g - 2*a*h)*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 +
3*h*(e*g + d*h))) - 2*c*h^2*(8*a^3*h^3*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*
f*g + 3*e*h) + b^3*(35*f*g^4 - 3*d*g^2*h^2) + 4*a^2*b*h^2*(5*f*g^2 + 3*h*(2
*e*g + d*h))) + (128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^3*
f*g^5 - 8*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^2*
h^2*(10*f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3 +
 3*d*g*h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^
2 + 3*h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(128*h^4*(c*g^2 - b*g*h +
 a*h^2)^3*(g + h*x)^2) - ((16*c^2*f*g^5 - 2*c*g*h*(13*b*f*g^3 - 10*a*f*g^2*
h + 3*b*d*g*h^2 - 6*a*d*h^3) - h^2*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*g*(7*f*
g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + h^2*(16*a^2
*f*h^3 + 4*a*c*g*h*(14*f*g - 3*e*h) + c^2*((28*f*g^4)/h - 12*d*g^2*h) + b^2
*h*(25*f*g^2 - 3*h*(e*g + d*h)) - b*(56*c*f*g^3 - 6*c*g*h*(e*g + 2*d*h) + 2
*a*h^2*(22*f*g - 3*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/(48*h^3*(c*g^2 - b*g*
h + a*h^2)^2*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2
))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (c^(3/2)*f*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h^6 - ((256*c^5*f*g^7 - 896*c^4*f*g^
5*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*(35
*f*g^2 - 3*d*h^2)) - 16*c^2*h^3*(35*b^3*f*g^4 - 6*a^3*h^3*(6*f*g - e*h) + 3
*a^2*b*h^2*(35*f*g^2 - e*g*h - d*h^2) - 3*a*b^2*g*h*(35*f*g^2 + d*h^2)) + b
^3*h^5*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g +
 d*h))) - 2*b*c*h^4*(96*a^3*f*h^3 - 24*a^2*b*h^2*(8*f*g + e*h) - b^3*(35*f*
g^3 - 3*d*g*h^2) + 4*a*b^2*h*(35*f*g^2 + 3*h*(e*g + d*h))))*ArcTanh[(b*g -
2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^
2])])/(256*h^6*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

<u>Rule 206</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 621</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 724</u>

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^6}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{5/2}}{5h\left(cg^2-bgh+ah^2\right)(g+hx)^5} - \frac{\int \frac{\left(-\frac{5}{2}\left(2cdg-beg-2afg+\frac{bfg^2}{h}-bdh+2a \cdots\right.\right.}{(g \cdots}}{5\left(cg^2- \cdots\right.}$$

$$= -\frac{\left(16c^2fg^5-2cgh\left(13bfg^3-10afg^2h+3bdgh^2-6adh^3\right)-h^2\left(4a^2h^2( \cdots\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2 \cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2 \cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2 \cdots\right.\right.\right.}{\cdots}$$

$$= -\frac{\left(\frac{128c^4fg^7}{h}-32c^3fg^5(11bg-10ah)+8c^2gh\left(38b^2fg^4+2a^2h^2\left(13fg^2 \cdots\right.\right.\right.}{\cdots}$$

**Mathematica** [A] time = 6.30, size = 1111, normalized size = 0.91

$$f(a+x(b+cx))^{3/2}\left(\frac{-\frac{(bh-2cg)\left(cx^2+bx+a\right)^{3/2}}{2\left(cg^2-bhg+ah^2\right)(g+hx)^2}-\frac{\frac{\left(\frac{1}{2}h\left(hb^2+2cgb-8ach\right)-cg(2cg-bh)\right)\left(cx^2+bx+a\right)^{3/2}}{\left(-cg^2+bhg-ah^2\right)(g+hx)}+\frac{\left(h\left(4c^2g^2-b^2h^2-4ch(bg-2ah)\right)xc^2-\left(2cg-\frac{bh}{2}\right)\left(4c^2g^2-b^2h^2-4ch(b\right.\right.}{}}{}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

```
[Out] -(((a + x*(b + c*x))^(3/2)*(((g*h*(2*f*g - e*h) - h*(f*g^2 - d*h^2))*(a + b
*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*h^2*(2
*f*g - e*h) + c*g*(f*g^2 - d*h^2)) + b*(g*h*(2*f*g - e*h) + h*(f*g^2 - d*h^
2)))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b
*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b
^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*
h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2
 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h +
 a*h^2))))/(h^2*(a + b*x + c*x^2)^(3/2))) + (f*(a + x*(b + c*x))^(3/2)*(-1/
3*(a + b*x + c*x^2)^(3/2)/(h*(g + h*x)^3) + (-1/2*((-2*c*g + b*h)*(a + b*x
+ c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g - b
*h)) + (h*(2*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*g^
2) + b*g*h - a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b^2*
h^2 - 4*c*h*(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h^2 +
 4*b*c*(2*c*g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g -
2*a*h))*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 - h*(b*g
 - a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h - (4*S
qrt[c*g^2 - b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c^2*g^2 - 6*
b^2*c*g*h - 8*a*c^2*g*h - b^3*h^2 + 12*a*b*c*h^2)) + 16*c^3*g*(c*g^2 - h*(b
```

```
*g - a*h))^2)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4
*c*h^2))/(-(c*g^2) + b*g*h - a*h^2))/(2*(c*g^2 - b*g*h + a*h^2)))/(2*h)))/(
h^2*(a + b*x + c*x^2)^(3/2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.07, size = 76693, normalized size = 62.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
```

```
more details)Is a*h^2-b*g*h                              +c*g^2 zero or nonze
ro?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

# 3.206 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^7}\,dx$

**Optimal**. Leaf size=657

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(a\left(dh^2-7egh+fg^2\right)+3bg(2dh+eg)\right)-12abh(eh\right.}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

```
[Out] 1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+
7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+
2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*(f*g^2-h*
(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1/60*(2*c*g
*(5*f*g^2+h*(-7*d*h+e*g))+h*(12*a*h*(-e*h+2*f*g)-b*(-7*d*h^2-5*e*g*h+17*f*g
^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/1024*(-4*a*c+
b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h
+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-7*e*g*h+f*g^2)))*arctanh(1/2*(b*g
-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^
2-b*g*h+c*g^2)^(9/2)-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h
*(e*h+2*f*g)+b^2*(7*d*h^2+5*e*g*h+7*f*g^2)-4*c*(3*b*g*(2*d*h+e*g)+a*(d*h^2-
7*e*g*h+f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*
h+c*g^2)^4/(h*x+g)^2
```

**Rubi [A]** time = 1.22, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 806, 720, 724, 206}

$$\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh\right.}{192(g+hx)^4\left(ah^2-bgh+cg^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d*g^2 + 24*a^2*f*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c
*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*
e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(512*
(c*g^2 - b*g*h + a*h^2)^4*(g + h*x)^2) + ((24*c^2*d*g^2 + 24*a^2*f*h^2 - 12
*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g + 2*d*
h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a
 + b*x + c*x^2)^(3/2))/(192*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^4) - ((f*g^
2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(6*h*(c*g^2 - b*g*h + a*h^2)*(g
```

```
 + h*x)^6) + ((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^2 - b*h*(5*e
*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/(60*h*(c*g^2
- b*g*h + a*h^2)^2*(g + h*x)^5) + ((b^2 - 4*a*c)^2*(24*c^2*d*g^2 + 24*a^2*f
*h^2 - 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e
*g + 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*ArcTanh[(b*g - 2*a*h + (2
*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])]])/(102
4*(c*g^2 - b*g*h + a*h^2)^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
```

```
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6} - \int \frac{(\frac{1}{2}(-12cdg+5beg+12afg-\frac{5bfg^2}{h}+7b}{}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6} + \frac{(2c(5fg^3+gh(eg-7dh))-}{}$$

$$= \frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3}{192(cg^2}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7}{512}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7}{512}$$

$$= -\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7}{512}$$

**Mathematica** [A] time = 6.24, size = 766, normalized size = 1.17

$$(a + x(b + cx))^{3/2} \left( -\frac{\frac{(a+bx+cx^2)^{5/2}\left(\frac{1}{2}ch\left(12h(aeh-afg+cdg)-bh(7dh+5eg)+5bfg^2\right)-cg\left(-6fh(bg-ah)+ch(eg-dh)+5cfg^2\right)\right)}{5(g+hx)^5\left(ah^2-bgh+cg^2\right)} - \left(\frac{(a+bx+cx^2)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)} - \cdots\right.}{\cdots} \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

```
[Out] -((f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^6)) + ((a +
x*(b + c*x))^(3/2)*(-1/6*(((h*(5*b*f*g + 2*c*d*h - 12*a*f*h))/2 - (g*(-7*b*
f*h + 2*c*(5*f*g + e*h)))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h
^2)*(g + h*x)^6) - (((-(c*g*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h
))) + (c*h*(5*b*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h))
)/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-
2*(-(a*c*h*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c^2*g*(5*b
*f*g^2 - b*h*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2) + b*(-(c*g
*(5*c*f*g^2 - 6*f*h*(b*g - a*h) + c*h*(e*g - d*h))) - (c*h*(5*b*f*g^2 - b*h
*(5*e*g + 7*d*h) + 12*h*(c*d*g - a*f*g + a*e*h)))/2))*(((b*g - 2*a*h + (2*c
*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^
2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x +
c*x^2])]])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(
16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(6*(c*g^2 - b*g*
h + a*h^2))))/(c*h*(a + b*x + c*x^2)^(3/2))
```

**fricas** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple** [**B**] time = 0.13, size = 100754, normalized size = 153.35

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

```
[Out] result too large to display
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                              +c*g^2 zero or nonze
ro?
```

**mupad** [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
[Out] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}{\left(g+hx\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

# 3.207 $\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}\,dx$

**Optimal**. Leaf size=1062

$$\frac{\left(4c^2\left(5fg^2+h(2eg-51dh)\right)g^2-7h^2\left(\left(5fg^2+5ehg+9dh^2\right)b^2-2ah(10fg+7eh)b+24a^2fh^2\right)-2ch\left(3bg\left(8fg\right.\right.\right.}{840h\left(cg^2-bhg+ah^2\right)^3(g+hx)^5}$$

```
[Out] 1/384*(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b
*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*
(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^
2+10*e*g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2
-b*g*h+c*g^2)^4/(h*x+g)^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a
*h^2-b*g*h+c*g^2)/(h*x+g)^7+1/84*(2*c*g*(5*f*g^2+h*(-9*d*h+2*e*g))+h*(14*a*
h*(-e*h+2*f*g)-b*(-9*d*h^2-5*e*g*h+19*f*g^2)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2
-b*g*h+c*g^2)^2/(h*x+g)^6+1/840*(4*c^2*g^2*(5*f*g^2+h*(-51*d*h+2*e*g))-7*h^
2*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))-2*c*h
*(3*b*g*(-34*d*h^2-15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*f*g^2)))*(
c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*a*c+b^2)^2*
(48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24
*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2
*h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e
*g*h+7*f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(11/2)-1/1024*(-4*a*c+b^2)*(
48*c^3*d*g^3-8*c^2*g*(3*b*g*(3*d*h+e*g)+a*(3*d*h^2-8*e*g*h+f*g^2))-b*h*(24*
a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))+2*c*(4*a^2*
h^2*(-e*h+8*f*g)-2*a*b*h*(-3*d*h^2+13*e*g*h+13*f*g^2)+b^2*g*(21*d*h^2+10*e*
g*h+7*f*g^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+
c*g^2)^5/(h*x+g)^2
```

**Rubi** [A] time = 3.00, antiderivative size = 1062, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{\left(4\left(5fg^4+h(2eg-51dh)g^2\right)c^2-2h\left(3bg\left(8fg^2-15ehg-34dh^2\right)-2ah\left(26fg^2-61ehg+12dh^2\right)\right)c-7h^2\left(\left(5f\right.\right.\right.}{840h\left(cg^2-bhg+ah^2\right)^3(g+hx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -((b^2 - 4*a*c)*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*
b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*
g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(1024*(c*g^2 - b*g*h + a*h^
2)^5*(g + h*x)^2) + ((48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h)
 + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^
2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(
13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*(b
*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(384*(c*g^2 - b*g*h
+ a*h^2)^4*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))
/(7*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^7) + ((2*c*(5*f*g^3 + g*h*(2*e*g -
9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a +
 b*x + c*x^2)^(5/2))/(84*h*(c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^6) + ((4*c^2
*(5*f*g^4 + g^2*h*(2*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g
 + 7*e*h) + b^2*(5*f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15
*e*g*h - 34*d*h^2) - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*
x^2)^(5/2))/(840*h*(c*g^2 - b*g*h + a*h^2)^3*(g + h*x)^5) + ((b^2 - 4*a*c)^
2*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3*d
*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5*e
*g + 9*d*h))) + 2*c*(4*a^2*h^2*(8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*e*
g - 3*d*h)) + b^2*(7*f*g^3 + g*h*(10*e*g + 21*d*h))))*ArcTanh[(b*g - 2*a*h
+ (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(2048*(c*g^2 - b*g*h + a*h^2)^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} - \frac{\int \frac{(\frac{1}{2}(-14cdg+5beg+14afg-\frac{5bfg^2}{h}+9bd}{}}{7}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} + \frac{(2c(5fg^3+gh(2eg-9dh))-h}{}$$

$$= -\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{7h(cg^2-bgh+ah^2)(g+hx)^7} + \frac{(2c(5fg^3+gh(2eg-9dh))-h}{}$$

$$= \frac{(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-bh(24a^2fh^2}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-}{}$$

$$= -\frac{(b^2-4ac)(48c^3dg^3-8c^2g(afg^2-ah(8eg-3dh)+3bg(eg+3dh))-}{}$$

**Mathematica** [A] time = 6.42, size = 1221, normalized size = 1.15

$$(a+x(b+cx))^{3/2}\left(-\frac{\left(\frac{1}{2}h(5bfg+4cdh-14afh)-\frac{1}{2}g(10cfg+4ceh-9bfh)\right)\left(cx^2+bx+a\right)^{5/2}}{7\left(cg^2-bhg+ah^2\right)(g+hx)^7}-\frac{-\frac{\left(2cg\left(5cfg^2-7fh(bg-ah)+2ch(eg-dh)\right)-ch\left(5bfg^2-bh(5eg+9d\ldots\right.\right.}{6\left(cg^2-bhg+ah^2\right)(g+hx\ldots}}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

```
[Out] -1/2*(f*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(c*h*(g + h*x)^7) + ((a
+ x*(b + c*x))^(3/2)*(-1/7*(((h*(5*b*f*g + 4*c*d*h - 14*a*f*h))/2 - (g*(10*
c*f*g + 4*c*e*h - 9*b*f*h))/2)*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a
*h^2)*(g + h*x)^7) - (-1/6*((2*c*g*(5*c*f*g^2 - 7*f*h*(b*g - a*h) + 2*c*h*(
e*g - d*h)) - c*h*(5*b*f*g^2 - b*h*(5*e*g + 9*d*h) + 14*h*(c*d*g - a*f*g +
a*e*h)))*(a + b*x + c*x^2)^(5/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) - (
((c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g +
 9*d*h) - 14*a*h*(2*f*g - e*h))) - (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d*
h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h))
+ 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2)
*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a
*c^2*h*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*e*g +
9*d*h) - 14*a*h*(2*f*g - e*h))) + (c^2*g*(7*b^2*h*(5*f*g^2 + h*(5*e*g + 9*d
*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e*h))
 + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))))/2
) + b*(c^2*g*(2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*g^2 - b*h*(5*
e*g + 9*d*h) - 14*a*h*(2*f*g - e*h))) + (c*h*(7*b^2*h*(5*f*g^2 + h*(5*e*g +
```

```
 9*d*h)) + 2*b*(5*c*f*g^3 - c*g*h*(40*e*g + 93*d*h) - 7*a*h^2*(10*f*g + 7*e
*h)) + 24*h*(7*c^2*d*g^2 + 7*a^2*f*h^2 - a*c*(2*f*g^2 - h*(9*e*g - 2*d*h)))
))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2
 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g -
 b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + (
(b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g
^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h
 + a*h^2)))/(6*(c*g^2 - b*g*h + a*h^2)))/(7*(c*g^2 - b*g*h + a*h^2))))/(2*c
*h*(a + b*x + c*x^2)^(3/2))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.18, size = 126612, normalized size = 119.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

```
[Out] result too large to display
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                              +c*g^2 zero or nonze
ro?
```

**mupad [F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

```
[Out] \text{Hanged}
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

```
[Out] Timed out
```

# 3.208 $\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx$

**Optimal**. Leaf size=143

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

```
[Out] 17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21*
(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*(75295+26982*x)*(3*x^2-x+2)^(3/2)+12403
9/93312*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+5393/15552*(1-6*x)*(3*x^2-x+
2)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4+\frac{67}{378}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{17}{105}\left(3x^2-x+2\right)^{3/2}(2x+1)^2-\frac{(26982x+75295)\left(3x^2\right.}{68040}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (5393*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/15552 + (17*(1 + 2*x)^2*(2 - x + 3*x^2
)^(3/2))/105 + (67*(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/378 + (2*(1 + 2*x)^4*
(2 - x + 3*x^2)^(3/2))/21 - ((75295 + 26982*x)*(2 - x + 3*x^2)^(3/2))/68040
 + (124039*ArcSinh[(1 - 6*x)/Sqrt[23]])/(31104*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} + \frac{1}{84}\int (1+2x)^3(-32+268x)\sqrt{2-x}$$

$$= \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2} + \frac{\int(}{}$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{2}$$

$$= \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2} + \frac{2}{2}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}$$

$$= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552} + \frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{67}{378}$$

**Mathematica [A]** time = 0.05, size = 70, normalized size = 0.49

$$\frac{6\sqrt{3x^2-x+2}\left(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069\right)-434}{3265920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 + 7
491456*x^4 + 6462720*x^5 + 2488320*x^6) - 4341365*Sqrt[3]*ArcSinh[(-1 + 6*x
)/Sqrt[23]])/3265920
```

**fricas [A]** time = 0.75, size = 83, normalized size = 0.58

$$\frac{1}{544320}\left(2488320\,x^6+6462720\,x^5+7491456\,x^4+5497776\,x^3+3280872\,x^2+1493894\,x-543069\right)\sqrt{3\,x^2-x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*x
^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(4*
sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac [A]** time = 0.26, size = 78, normalized size = 0.55

$$\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3x^2-x+2}+\frac{12}{93}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x +
 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sqrt
(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple [A]** time = 0.02, size = 115, normalized size = 0.80

$$\frac{32\left(3x^2-x+2\right)^{\frac{3}{2}}x^4}{21}+\frac{844\left(3x^2-x+2\right)^{\frac{3}{2}}x^3}{189}+\frac{1594\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{315}+\frac{7849\left(3x^2-x+2\right)^{\frac{3}{2}}x}{3780}-\frac{124039\sqrt{3}\ \mathrm{arcs}}{933}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/21*x^4*(3*x^2-x+2)^(3/2)+844/189*x^3*(3*x^2-x+2)^(3/2)+7849/3780*x*(3*x^
2-x+2)^(3/2)+1594/315*x^2*(3*x^2-x+2)^(3/2)-124039/93312*3^(1/2)*arcsinh(6/
23*23^(1/2)*(x-1/6))-5393/15552*(6*x-1)*(3*x^2-x+2)^(1/2)-45739/68040*(3*x^
2-x+2)^(3/2)
```

**maxima [A]** time = 0.95, size = 126, normalized size = 0.88

$$\frac{32}{21}\left(3x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{7849}{3780}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594/
315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739/6
8040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93312
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x + 2)
```

**mupad [B]** time = 5.54, size = 170, normalized size = 1.19

$$\frac{1594\,x^2\left(3\,x^2-x+2\right)^{3/2}}{315}+\frac{844\,x^3\left(3\,x^2-x+2\right)^{3/2}}{189}+\frac{32\,x^4\left(3\,x^2-x+2\right)^{3/2}}{21}-\frac{137057\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,(3}{3}\right.}{136080}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
+ (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)^(
1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x + 2
)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/1632960 +
 (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 - x +
2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3\sqrt{3x^2-x+2}\,\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

# 3.209 $\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx$

**Optimal**. Leaf size=118

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x-}}{1296}$$

```
[Out] 1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+3
06*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+23
5/1296*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3+\frac{1}{5}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{1}{810}(306x+25)\left(3x^2-x+2\right)^{3/2}+\frac{235(1-6x)\sqrt{3x^2-x-}}{1296}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (235*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/1296 + ((1 + 2*x)^2*(2 - x + 3*x^2)^(3/
2))/5 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((25 + 306*x)*(2 - x + 3*x^
2)^(3/2))/810 + (5405*ArcSinh[(1 - 6*x)/Sqrt[23]])/(2592*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{72}\int (1+2x)^2(-12+216x)\sqrt{2-x$$

$$= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{\int(1}{}$$

$$= \frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}($$

$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+$$

$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+$$

$$= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{9}(1+$$

**Mathematica [A]** time = 0.04, size = 65, normalized size = 0.55

$$\frac{6\sqrt{3x^2-x+2}\left(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607\right)-27025\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{38880}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
+ 17280*x^5) - 27025*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/38880
```

**fricas [A]** time = 0.95, size = 78, normalized size = 0.66

$$\frac{1}{6480}\left(17280\,x^5+35712\,x^4+33552\,x^3+22344\,x^2+14638\,x+5607\right)\sqrt{3\,x^2-x+2}+\frac{5405}{15552}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sqr
t(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

**giac** [A] time = 0.30, size = 73, normalized size = 0.62

$$\frac{1}{6480}\left(2\left(12\left(6\left(8\left(15x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3x^2-x+2}+\frac{5405}{7776}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt(
3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

**maple** [A] time = 0.01, size = 98, normalized size = 0.83

$$\frac{8\left(3x^2-x+2\right)^{\frac{3}{2}}x^3}{9}+\frac{32\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{15}+\frac{83\left(3x^2-x+2\right)^{\frac{3}{2}}x}{45}-\frac{5405\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}+\frac{277\left(3x^2-x+\right.}{810}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2-x+2)^(3/2)*x^3+32/15*(3*x^2-x+2)^(3/2)*x^2+83/45*(3*x^2-x+2)^(3/
2)*x+277/810*(3*x^2-x+2)^(3/2)-235/1296*(6*x-1)*(3*x^2-x+2)^(1/2)-5405/7776
*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima** [A] time = 0.96, size = 109, normalized size = 0.92

$$\frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{235}{216}\sqrt{3x^2-x+2}\,x-\frac{5405}{7776}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3*
x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2 -
 x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/1296*s
qrt(3*x^2 - x + 2)
```

**mupad** [B] time = 5.15, size = 153, normalized size = 1.30

$$\frac{32x^2\left(3x^2-x+2\right)^{3/2}}{15}+\frac{8x^3\left(3x^2-x+2\right)^{3/2}}{9}-\frac{2783\sqrt{3}\ \ln\left(\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{3240}-\frac{121\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3x^2-x}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (2783
*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (121*
(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*x^2
 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*log(2
*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \sqrt{3x^2-x+2} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

# 3.210 $\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx$

**Optimal**. Leaf size=93

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

```
[Out] 2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*(745+738*x)*(3*x^2-x+2)^(3/2)+437/1
5552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+19/2592*(1-6*x)*(3*x^2-x+2)^(1/
2)
```

**Rubi [A]** time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2+\frac{(738x+745)\left(3x^2-x+2\right)^{3/2}}{1620}+\frac{19(1-6x)\sqrt{3x^2-x+2}}{2592}+\frac{437\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (19*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/15 + ((745 + 738*x)*(2 - x + 3*x^2)^(3/2))/1620 + (437*ArcSinh[(1 - 6*
x)/Sqrt[23]])/(5184*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)dx = \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{1}{60}\int (1+2x)(8+164x)\sqrt{2-x+3$$

$$= \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620}-\frac{1}{21}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+}{}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+}{}$$

$$= \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592}+\frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2}+\frac{(745+}{}$$

**Mathematica [A]** time = 0.03, size = 60, normalized size = 0.65

$$\frac{6\sqrt{3x^2-x+2}\left(20736x^4+31536x^3+24072x^2+17374x+15471\right)-2185\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{77760}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^4
) - 2185*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/77760
```

**fricas [A]** time = 0.89, size = 73, normalized size = 0.78

$$\frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{31104}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72
*x^2 + 24*x - 25)
```

**giac [A]** time = 0.26, size = 68, normalized size = 0.73

$$\frac{1}{12960}\left(2\left(12\left(18\left(48\,x+73\right)x+1003\right)x+8687\right)x+15471\right)\sqrt{3\,x^2-x+2}+\frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
 + 1)
```

**maple [A]** time = 0.01, size = 81, normalized size = 0.87

$$\frac{8\left(3x^2-x+2\right)^{\frac{3}{2}}x^2}{15}+\frac{89\left(3x^2-x+2\right)^{\frac{3}{2}}x}{90}-\frac{437\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}+\frac{961\left(3x^2-x+2\right)^{\frac{3}{2}}}{1620}-\frac{19\left(6x-1\right)\sqrt{3x^2-}}{2592}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/15*(3*x^2-x+2)^(3/2)*x^2+89/90*(3*x^2-x+2)^(3/2)*x+961/1620*(3*x^2-x+2)^(
3/2)-19/2592*(6*x-1)*(3*x^2-x+2)^(1/2)-437/15552*3^(1/2)*arcsinh(6/23*23^(1
/2)*(x-1/6))
```

**maxima [A]** time = 0.96, size = 92, normalized size = 0.99

$$\frac{8}{15}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{961}{1620}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{19}{432}\sqrt{3\,x^2-x+2}\,x-\frac{437}{15552}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)*(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*(
3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*arc
sinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

**mupad [B]** time = 4.88, size = 136, normalized size = 1.46

$$\frac{8\,x^2\left(3\,x^2-x+2\right)^{3/2}}{15}-\frac{253\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\left(3\,x-\frac{1}{2}\right)}{3}\right)}{810}-\frac{44\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3\,x^2-x+2}}{45}+\frac{961\,\sqrt{3\,x^2-x+2}}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] (8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2) +
 (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45
 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2 - x
 + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*
x - 1))/3))/77760
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)*(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1), x)
```

# 3.211 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

**Optimal**. Leaf size=101

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\,\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

```
[Out] 2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/8*arc
tanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/72*(13+30*x)*(3*x^
2-x+2)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{9}\left(3x^2-x+2\right)^{3/2}+\frac{1}{72}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{8}\sqrt{13}\,\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-\frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((13 + 30*x)*Sqrt[2 - x + 3*x^2])/72 + (2*(2 - x + 3*x^2)^(3/2))/9 - (43*Ar
cSinh[(1 - 6*x)/Sqrt[23]])/(144*Sqrt[3]) - (Sqrt[13]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])])/8
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{9}\left(2-x+3x^2\right)^{3/2} + \frac{1}{36}\int \frac{(48+60x)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{\int \frac{-3324-1032x}{(1+2x)\sqrt{2-x+3x^2}}\,dx}{1728}$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} + \frac{43}{144}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{13}{4}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx$$

$$= \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{43\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}} - \frac{1}{8}\sqrt{1}$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 0.85

$$\frac{1}{432}\left(6\sqrt{3x^2-x+2}\left(48x^2+14x+45\right)-54\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+43\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 43*Sqrt[3]*ArcSinh[(-1 + 6*x)
/Sqrt[23]] - 54*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])
])/432
```

**fricas [A]** time = 0.91, size = 115, normalized size = 1.14

$$\frac{1}{72}\left(48\,x^2+14\,x+45\right)\sqrt{3\,x^2-x+2}+\frac{43}{864}\,\sqrt{3}\,\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+\frac{1}{16}\,\sqrt{13}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt(
3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1))
```

**giac [A]** time = 0.39, size = 126, normalized size = 1.25

$$\frac{1}{72}\left(2\left(24\,x+7\right)x+45\right)\sqrt{3\,x^2-x+2}-\frac{43}{432}\,\sqrt{3}\,\log\left(-6\,\sqrt{3}\,x+\sqrt{3}\,+6\,\sqrt{3\,x^2-x+2}\right)+\frac{1}{8}\,\sqrt{13}\,\log\left(-\frac{\left|-4\,\sqrt{3}\,x\right.}{2\left(2\,\sqrt{3}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqrt
(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*sqr
t(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqr
t(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.94

$$\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{432}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{8}+\frac{2\left(3x^2-x+2\right)^{\frac{3}{2}}}{9}+\frac{5\left(6x-1\right)\sqrt{3x^2-x+2}}{72}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1),x)
```

```
[Out] 2/9*(3*x^2-x+2)^(3/2)+5/72*(6*x-1)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh
(6/23*23^(1/2)*(x-1/6))+1/8*(12*(x+1/2)^2-16*x+5)^(1/2)-1/8*13^(1/2)*arctan
h(2/13*(9/2-4*x)*13^(1/2)/(12*(x+1/2)^2-16*x+5)^(1/2))
```

**maxima [A]** time = 0.96, size = 96, normalized size = 0.95

$$\frac{2}{9}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{5}{12}\,\sqrt{3\,x^2-x+2}\,x+\frac{43}{432}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{1}{8}\,\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\,\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2 - x + 2}\left(4x^2 + 3x + 1\right)}{2x + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x),x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

## 3.212 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

**Optimal**. Leaf size=108

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] -1/13*(3*x^2-x+2)^(3/2)/(1+2*x)-11/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+17/104*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/156*(6
7-96*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}-\frac{1}{156}(67-96x)\sqrt{3x^2-x+2}+\frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}}-\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -((67 - 96*x)*Sqrt[2 - x + 3*x^2])/156 - (2 - x + 3*x^2)^(3/2)/(13*(1 + 2*x
)) - (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(
2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(8*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{15}{2}-32x\right)\sqrt{2-x+3x^2}}{1+2x}\,dx$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{1}{624}\int \frac{-182+22}{(1+2x)\sqrt{2-}}$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{11}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,d$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} + \frac{17}{4}\text{Subst}\left(\int \frac{1}{52-x^2}\right.$$

$$= -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2} - \frac{\left(2-x+3x^2\right)^{3/2}}{13(1+2x)} - \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}} + \frac{17}{}$$

**Mathematica [A]** time = 0.09, size = 92, normalized size = 0.85

$$\frac{\sqrt{3x^2-x+2}\left(12x^2-2x-7\right)}{24x+12} + \frac{17\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8\sqrt{13}} + \frac{11\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) + (11*ArcSinh[(-1 + 6
*x)/Sqrt[23]])/(6*Sqrt[3]) + (17*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/(8*Sqrt[13])
```

**fricas [A]** time = 0.91, size = 133, normalized size = 1.23

$$\frac{572\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+153\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(}{4}\right.}{1872\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
```

```
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*x^2
- 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

**giac [B]** time = 0.72, size = 380, normalized size = 3.52

$$\frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}}\right.}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqrt(
3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sgn(1
/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x + 1))
+ 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3
*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) +
sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 13/(2*x +
 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sgn(1/(2*x
+ 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
3)^2
```

**maple [A]** time = 0.01, size = 123, normalized size = 1.14

$$\frac{11\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{17\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{104}+\frac{(6x-1)\sqrt{3x^2-x+2}}{12}-\frac{17\sqrt{-16x+12\left(x\right.}}{104}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^2,x)
```

```
[Out] 1/12*(6*x-1)*(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
-17/104*(-16*x+12*(x+1/2)^2+5)^(1/2)+17/104*13^(1/2)*arctanh(2/13*(-4*x+9/2
)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(3*(x+1/2)^2-4*x+5/4)
^(3/2)+1/52*(6*x-1)*(3*(x+1/2)^2-4*x+5/4)^(1/2)
```

**maxima [A]** time = 0.97, size = 103, normalized size = 0.95

$$\frac{1}{2}\sqrt{3x^2-x+2}\,x+\frac{11}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{17}{104}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)-\frac{1}{3}\sqrt{3x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sq
rt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(
23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2*x
+ 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)}{\left(2x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**2,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

# 3.213 $\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

**Optimal**. Leaf size=115

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

```
[Out] -1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1
/2)-803/2704*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+11/1
04*(7+10*x)*(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{3/2}}{26(2x+1)^2}+\frac{11(10x+7)\sqrt{3x^2-x+2}}{104(2x+1)}-\frac{803\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{208\sqrt{13}}+\frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] (11*(7 + 10*x)*Sqrt[2 - x + 3*x^2])/(104*(1 + 2*x)) - (2 - x + 3*x^2)^(3/2)
/(26*(1 + 2*x)^2) + (11*ArcSinh[(1 - 6*x)/Sqrt[23]])/(8*Sqrt[3]) - (803*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(208*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{33}{2}-55x\right)\sqrt{2-x+3x^2}}{(1+2x)^2}\,dx$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{517-572x}{(1+2x)\sqrt{2-x+3x^2}}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{11}{8}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx + \frac{8}{2}$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} - \frac{803}{104}\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx, x\right.$$

$$= \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{\left(2-x+3x^2\right)^{3/2}}{26(1+2x)^2} + \frac{11\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\tan}{}$$

**Mathematica [A]** time = 0.09, size = 93, normalized size = 0.81

$$\frac{\frac{78\sqrt{3x^2-x+2}\left(208x^2+268x+69\right)}{(2x+1)^2} - 2409\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) - 3718\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{8112}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((78*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 - 3718*Sqrt[3]
*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 2409*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]
*Sqrt[2 - x + 3*x^2])])/8112
```

**fricas [A]** time = 0.85, size = 149, normalized size = 1.30

$$\frac{3718\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+2409\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\right.}{16224\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sq
rt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x
+ 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-389344,[6]%%%}+%%%{%%{[1168032,0]:[1,0,-3]%%},[5]%%%}+%%%{-584016,[
4]%%%}+%%%{%%{[-4672128,0]:[1,0,-3]%%},[3]%%%}+%%%{1460040,[2]%%%}+%%%{%%{[
7300200,0]:[1,0,-3]%%},[1]%%%}+%%%{6083500,[0]%%%} / %%%{%%{[24,0]:[1,0,-3]
%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},[4]%%%}+%%%{864,[3]%
%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%}+%%%{%%{[-375,0]:[1
,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

**maple [A]** time = 0.01, size = 125, normalized size = 1.09

$$-\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{2704}+\frac{803\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}{2704}+11\left(-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(2*x+1)^3,x)
```

```
[Out] 803/2704*(-16*x+12*(x+1/2)^2+5)^(1/2)-11/24*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))-803/2704*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2
)^2+5)^(1/2))+11/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-11/676*(6*x-1)*(-
4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

**maxima [A]** time = 0.99, size = 114, normalized size = 0.99

$$-\frac{11}{24}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{55}{104}\sqrt{3\,x^{2}-x+2}-\frac{(3}{26(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)*(3*x^2-x+2)^(1/2)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] -11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13)
*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/10
4*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 11/5
2*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3\,x^2-x+2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x^{2} - x + 2} \left(4x^{2} + 3x + 1\right)}{\left(2x + 1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)*(3*x**2-x+2)**(1/2)/(1+2*x)**3,x)
```

```
[Out] Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

# 3.214 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

**Optimal**. Leaf size=158

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+\frac{54593(1-6x)\left(3x^2-x\right.}{559872}$$

```
[Out] 54593/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(5
/2)+913/486*x^2*(3*x^2-x+2)^(5/2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^
4*(3*x^2-x+2)^(5/2)+28879697/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)+1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi** [A] time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 12, 779, 612, 619, 215}

$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4+\frac{77}{81}x^3\left(3x^2-x+2\right)^{5/2}+\frac{913}{486}x^2\left(3x^2-x+2\right)^{5/2}-\frac{11(283-5850x)\left(3x^2-x+2\right)^{5/2}}{58320}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (1255639*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 + (54593*(1 - 6*x)*(2 - x +
 3*x^2)^(3/2))/559872 - (11*(283 - 5850*x)*(2 - x + 3*x^2)^(5/2))/58320 + (
913*x^2*(2 - x + 3*x^2)^(5/2))/486 + (77*x^3*(2 - x + 3*x^2)^(5/2))/81 + (2
*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (28879697*ArcSinh[(1 - 6*x)/Sqrt[2
3]])/(8957952*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{1}{108}\int 308x(1+2x)^3 \left(2-x+\right.$$

$$= \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{27}\int x(1+2x)^3 \left(2-x+3x^2\right)$$

$$= \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4 \left(2-x+3x^2\right)^{5/2} + \frac{77}{648}\int$$

$$= \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3 \left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4$$

$$= -\frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2 \left(2-x+3x^2\right)^{5/2} +$$

$$= \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\left(2-x+3x\right.}{58320}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)}{559872}$$

$$= \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)}{559872}$$

**Mathematica [A]** time = 0.05, size = 80, normalized size = 0.51

$$\frac{6\sqrt{3x^2-x+2}\left(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3\right.}{134369280}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*x
^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 238878
720*x^8) - 144398485*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/134369280
```

**fricas [A]** time = 0.91, size = 93, normalized size = 0.59

$$\frac{1}{22394880}\left(238878720\,x^8+510105600\,x^7+635765760\,x^6+711210240\,x^5+649452672\,x^4+421626672\,x^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5 +
 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*sqr
t(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x +
 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac [A]** time = 0.26, size = 88, normalized size = 0.56

$$\frac{1}{22394880}\left(2\left(12\left(6\left(8\left(30\left(36\left(2\left(96x+205\right)x+511\right)x+20579\right)x+563761\right)x+2927963\right)x+8387071\right)x+42007\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 563
761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x +
 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x
+ 2)) + 1)
```

**maple [A]** time = 0.02, size = 134, normalized size = 0.85

$$\frac{32\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{4}}{27}+\frac{269\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{3}}{81}+\frac{1777\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{2}}{486}+\frac{1099\left(3x^{2}-x+2\right)^{\frac{5}{2}}x}{648}-\frac{28879697\sqrt{3}\ \mathrm{arc}}{2687}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 32/27*x^4*(3*x^2-x+2)^(5/2)+269/81*x^3*(3*x^2-x+2)^(5/2)+1777/486*x^2*(3*x^
2-x+2)^(5/2)+1099/648*x*(3*x^2-x+2)^(5/2)-54593/559872*(6*x-1)*(3*x^2-x+2)^
(3/2)-28879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1255639/4478
976*(6*x-1)*(3*x^2-x+2)^(1/2)+1207/58320*(3*x^2-x+2)^(5/2)
```

**maxima [A]** time = 0.98, size = 155, normalized size = 0.98

$$\frac{32}{27}\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{4}+\frac{269}{81}\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{3}+\frac{1777}{486}\left(3x^{2}-x+2\right)^{\frac{5}{2}}x^{2}+\frac{1099}{648}\left(3x^{2}-x+2\right)^{\frac{5}{2}}x+\frac{1207}{58320}\left(3x^{2}-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

[Out] 32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/4
86*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/5832
0*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55987
2*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 28879697/2
6873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqrt(3*x
^2 - x + 2)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^3\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)

[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)

[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)

## 3.215 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

**Optimal**. Leaf size=141

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x+2\right.}{3456}$$

```
[Out] 91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/63*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1+
2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29+50*x)*(3*x^2-x+2)^(5/2)+48139/165888*a
rcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2093/27648*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3+\frac{8}{63}\left(3x^2-x+2\right)^{5/2}(2x+1)^2+\frac{13(50x+29)\left(3x^2-x+2\right)^{5/2}}{2520}+\frac{91(1-6x)\left(3x^2-x+2\right.}{3456}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2093*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/27648 + (91*(1 - 6*x)*(2 - x + 3*x^2)^
(3/2))/3456 + (8*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/63 + ((1 + 2*x)^3*(2 -
x + 3*x^2)^(5/2))/12 + (13*(29 + 50*x)*(2 - x + 3*x^2)^(5/2))/2520 + (48139
*ArcSinh[(1 - 6*x)/Sqrt[23]])/(55296*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{1}{96} \int (1+2x)^2(20+256x) \left(2-\right.$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \ldots$$

$$= \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3 \left(2-x+3x^2\right)^{5/2} + \frac{13}{\ldots}$$

$$= \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{12}$$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}($$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}($$

$$= \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}($$

**Mathematica [A]** time = 0.05, size = 75, normalized size = 0.53

$$\frac{6\sqrt{3x^2-x+2}\left(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+\right.}{5806080}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) - 1684865*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/5806080
```

**fricas [A]** time = 0.91, size = 88, normalized size = 0.62

$$\frac{1}{967680}\left(5806080\,x^7+9262080\,x^6+10656000\,x^5+12173952\,x^4+10119792\,x^3+5694024\,x^2+2735918\,x+15\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 1011979
2*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/3317
76*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25
)
```

**giac [A]** time = 0.27, size = 83, normalized size = 0.59

$$\frac{1}{967680}\,(2\,(12\,(2\,(8\,(30\,(12\,(42\,x+67)x+925)x+31703)x+210829)x+237251)x+1367959)x+1517367)\sqrt{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*x
 + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*sqr
t(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple [A]** time = 0.01, size = 117, normalized size = 0.83

$$\frac{2\left(3x^2-x+2\right)^{\frac{5}{2}}x^3}{3}+\frac{95\left(3x^2-x+2\right)^{\frac{5}{2}}x^2}{63}+\frac{319\left(3x^2-x+2\right)^{\frac{5}{2}}x}{252}-\frac{48139\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}-\frac{91\,(6x-1)\,(}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 2/3*(3*x^2-x+2)^(5/2)*x^3+95/63*(3*x^2-x+2)^(5/2)*x^2+319/252*(3*x^2-x+2)^(
5/2)*x-91/3456*(6*x-1)*(3*x^2-x+2)^(3/2)-48139/165888*3^(1/2)*arcsinh(6/23*
23^(1/2)*(x-1/6))-2093/27648*(6*x-1)*(3*x^2-x+2)^(1/2)+907/2520*(3*x^2-x+2)
^(5/2)
```

**maxima [A]** time = 0.93, size = 138, normalized size = 0.98

$$\frac{2}{3}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{95}{63}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{319}{252}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x+\frac{907}{2520}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{91}{576}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*(
3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 - x
 + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x +
2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/27648*s
qrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^2\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

# 3.216 $\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx$

**Optimal**. Leaf size=116

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-}}{20736}$$

```
[Out] -71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*(
109+102*x)*(3*x^2-x+2)^(5/2)-37559/124416*arcsinh(1/23*(1-6*x)*23^(1/2))*3^
(1/2)-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}+\frac{1}{378}(102x+109)\left(3x^2-x+2\right)^{5/2}-\frac{71(1-6x)\left(3x^2-x+2\right)^{3/2}}{2592}-\frac{1633(1-6x)\sqrt{3x^2-}}{20736}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1633*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/20736 - (71*(1 - 6*x)*(2 - x + 3*x^2)
^(3/2))/2592 + (2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + ((109 + 102*x)*(2
 - x + 3*x^2)^(5/2))/378 - (37559*ArcSinh[(1 - 6*x)/Sqrt[23]])/(41472*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{1}{84}\int (1+2x)(40+204x)\left(2-x\right.$$

$$= \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/}$$

$$= -\frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592} + \frac{2}{21}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{}{3}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736} - \frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592} + \frac{2}{21}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736} - \frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592} + \frac{2}{21}$$

$$= -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736} - \frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592} + \frac{2}{21}$$

**Mathematica [A]** time = 0.04, size = 70, normalized size = 0.60

$$\frac{6\sqrt{3x^2-x+2}\left(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337\right)+262913\sqrt{3}}{870912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 65318
4*x^4 + 518400*x^5 + 497664*x^6) + 262913*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[2
3]])/870912
```

**fricas [A]** time = 0.93, size = 83, normalized size = 0.72

$$\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{3\,x^2-x+2}+\frac{3}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt(3
)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac [A]** time = 0.21, size = 78, normalized size = 0.67

$$\frac{1}{145152}\left(2\left(12\left(18\left(24\left(2\left(24x+25\right)x+63\right)x+1723\right)x+22141\right)x+137705\right)x+203337\right)\sqrt{3x^2-x+2}-\frac{37559}{124416}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 137
705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3)*
(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple [A]** time = 0.00, size = 100, normalized size = 0.86

$$\frac{8\left(3x^2-x+2\right)^{\frac{5}{2}}x^2}{21}+\frac{41\left(3x^2-x+2\right)^{\frac{5}{2}}x}{63}+\frac{37559\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}+\frac{145\left(3x^2-x+2\right)^{\frac{5}{2}}}{378}+\frac{71\left(6x-1\right)\left(3x^2\right.}{2592}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/21*(3*x^2-x+2)^(5/2)*x^2+41/63*(3*x^2-x+2)^(5/2)*x+145/378*(3*x^2-x+2)^(5
/2)+71/2592*(6*x-1)*(3*x^2-x+2)^(3/2)+1633/20736*(6*x-1)*(3*x^2-x+2)^(1/2)+
37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima [A]** time = 0.97, size = 121, normalized size = 1.04

$$\frac{8}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{41}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{71}{432}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{71}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{16}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(3
*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x +
 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsinh(
1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1), x)
```

# 3.217 $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{1+2x}\,dx$

**Optimal**. Leaf size=124

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x}}\right.$$

```
[Out] 1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(1
/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+
2)^(1/2))*13^(1/2)+1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}+\frac{1}{144}(30x+7)\left(3x^2-x+2\right)^{3/2}+\frac{(402x+869)\sqrt{3x^2-x+2}}{1152}-\frac{13}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((869 + 402*x)*Sqrt[2 - x + 3*x^2])/1152 + ((7 + 30*x)*(2 - x + 3*x^2)^(3/2
))/144 + (2*(2 - x + 3*x^2)^(5/2))/15 + (2203*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(2304*Sqrt[3]) - (13*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*
x^2])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{1+2x}\,dx = \frac{2}{15}\left(2-x+3x^2\right)^{5/2} + \frac{1}{60}\int \frac{(80+100x)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)^{5/2} - \frac{\int \frac{(-13380-8040x)\sqrt{2-}}{1+2x}}{5760}$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x\right.$$

$$= \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x\right.$$

**Mathematica** [A] time = 0.06, size = 96, normalized size = 0.77

$$\frac{-14040\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + 6\sqrt{3x^2-x+2}\left(6912x^4-1008x^3+9624x^2+1058x+7977\right) - 11015\sqrt{3}\,\mathrm{s}}{34560}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) - 1
1015*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 14040*Sqrt[13]*ArcTanh[(9 - 8*x
)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/34560
```

**fricas** [A] time = 0.91, size = 125, normalized size = 1.01

$$\frac{1}{5760}\left(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977\right)\sqrt{3\,x^2-x+2}+\frac{2203}{13824}\sqrt{3}\,\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2
+ 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9
) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

**giac [A]** time = 0.28, size = 136, normalized size = 1.10

$$\frac{1}{5760}\,(2\,(12\,(6\,(48\,x-7)x+401)x+529)x+7977)\sqrt{3\,x^2-x+2}+\frac{2203}{6912}\,\sqrt{3}\,\log\Big(-6\,\sqrt{3}\,x+\sqrt{3}\,+6\,\sqrt{3\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2)
 + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) +
13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

**maple [A]** time = 0.01, size = 151, normalized size = 1.22

$$-\frac{2203\sqrt{3}\;\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{6912}-\frac{13\sqrt{13}\;\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{32}+\frac{2\left(3x^2-x+2\right)^{\frac{5}{2}}}{15}+\frac{5\,(6x-1)\left(3x^2-x\right.}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1),x)
```

```
[Out] 2/15*(3*x^2-x+2)^(5/2)+5/144*(6*x-1)*(3*x^2-x+2)^(3/2)+115/1152*(6*x-1)*(3*
x^2-x+2)^(1/2)-2203/6912*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1/12*(-4*x+
3*(x+1/2)^2+5/4)^(3/2)-1/24*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)+13/32*(-16
*x+12*(x+1/2)^2+5)^(1/2)-13/32*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-
16*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.99, size = 125, normalized size = 1.01

$$\frac{2}{15}\left(3\,x^2-x+2\right)^{\frac{5}{2}}+\frac{5}{24}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{7}{144}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{67}{192}\,\sqrt{3\,x^2-x+2}\,x-\frac{2203}{6912}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6/2
3*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(
2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}-x+2\right)^{\frac{3}{2}}\left(4x^{2}+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

## 3.218 $\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

**Optimal**. Leaf size=131

$$-\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-}\right.$$

```
[Out] -1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-1/13*(3*x^2-x+2)^(5/2)/(1+2*x)-2327/1152
*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+25/32*arctanh(1/26*(9-8*x)*13^(1/2)
/(3*x^2-x+2)^(1/2))*13^(1/2)-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}-\frac{1}{104}(23-38x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{192}(349-294x)\sqrt{3x^2-x+2}+\frac{25}{32}\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] -((349 - 294*x)*Sqrt[2 - x + 3*x^2])/192 - ((23 - 38*x)*(2 - x + 3*x^2)^(3/
2))/104 - (2 - x + 3*x^2)^(5/2)/(13*(1 + 2*x)) - (2327*ArcSinh[(1 - 6*x)/Sq
rt[23]])/(384*Sqrt[3]) + (25*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
- x + 3*x^2])])/32
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{13}{2}-38x\right)\left(2-x+3x^2\right)^{3/2}}{1+2x}\,dx$$

$$= -\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{\left(2-x+3x^2\right)^{5/2}}{13(1+2x)} + \frac{\int \frac{(-78+7644x)\sqrt{2-x}}{1+2x}}{1248}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2}{}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2}{}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2}{}$$

$$= -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2} - \frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} - \frac{(2}{}$$

**Mathematica [A]** time = 0.09, size = 103, normalized size = 0.79

$$\frac{900\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}\left(288x^4-96x^3+564x^2-332x-493\right)}{2x+1} + 2327\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1152}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 + 2
*x) + 2327*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] + 900*Sqrt[13]*ArcTanh[(9 -
 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/1152
```

**fricas [A]** time = 0.88, size = 143, normalized size = 1.09

$$\frac{2327\sqrt{3}\,(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+900\sqrt{13}\,(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+}}{}\right.}{2304\,(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288*x^
4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

**giac** [B] time = 0.84, size = 570, normalized size = 4.35

$$\frac{25}{32}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{2327}{1152}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2x+1}\cdots}\right.}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2x+1}\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt
(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*sq
rt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*sg
n(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(2*x + 1)
 + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 48*sqrt(1
3)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(
2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
 + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1
)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 24624*sqrt
(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)
/(2*x + 1))^2 - 3)^4
```

**maple** [A] time = 0.01, size = 179, normalized size = 1.37

$$\frac{2327\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1152}+\frac{25\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{32}+\frac{(6x-1)\left(3x^2-x+2\right)^{\frac{3}{2}}}{24}+\frac{23\,(6x-1)\sqrt{3x\cdots}}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^2,x)
```

```
[Out] 1/24*(6*x-1)*(3*x^2-x+2)^(3/2)+23/192*(6*x-1)*(3*x^2-x+2)^(1/2)+2327/1152*3
^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-25/156*(-4*x+3*(x+1/2)^2+5/4)^(3/2)+1
3/96*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-25/32*(-16*x+12*(x+1/2)^2+5)^(1/2
)+25/32*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1
/2))-1/26/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+1/52*(6*x-1)*(-4*x+3*(x+1/2)
^2+5/4)^(3/2)
```

**maxima** [A] time = 0.98, size = 132, normalized size = 1.01

$$\frac{1}{4}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x-\frac{1}{8}\left(3\,x^2-x+2\right)^{\frac{3}{2}}+\frac{49}{32}\,\sqrt{3\,x^2-x+2}\,x+\frac{2327}{1152}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)-\frac{25}{32}\,\sqrt{13}\,\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
- x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 2
5/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
+ 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x + 1)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(3\,x^2-x+2\right)}^{3/2}\,\left(4\,x^2+3\,x+1\right)}{{\left(2\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2} - x + 2\right)^{\frac{3}{2}} \left(4x^{2} + 3x + 1\right)}{\left(2x + 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

# 3.219 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3} dx$

**Optimal**. Leaf size=138

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+$$

```
[Out] 1/312*(151+122*x)*(3*x^2-x+2)^(3/2)/(1+2*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)^
2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1153/832*arctanh(1/26*(9-
8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/624*(1858-771*x)*(3*x^2-x+2)^(1
/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}+\frac{(122x+151)(3x^2-x+2)^{3/2}}{312(2x+1)}+\frac{1}{624}(1858-771x)\sqrt{3x^2-x+2}-\frac{1153\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{64\sqrt{13}}+$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/624 + ((151 + 122*x)*(2 - x + 3*x^2)^(
3/2))/(312*(1 + 2*x)) - (2 - x + 3*x^2)^(5/2)/(26*(1 + 2*x)^2) + (1519*ArcS
inh[(1 - 6*x)/Sqrt[23]])/(192*Sqrt[3]) - (1153*ArcTanh[(9 - 8*x)/(2*Sqrt[13
]*Sqrt[2 - x + 3*x^2])])/(64*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{31}{2}-61x\right)\left(2-x+3x^2\right)^{3/2}}{(1+2x)^2}\,dx$$

$$= \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{\left(2-x+3x^2\right)^{5/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(639-1028x)}{1+}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{(2-x}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{(2-x}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{(2-x}{26(}$$

$$= \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)\left(2-x+3x^2\right)^{3/2}}{312(1+2x)} - \frac{(2-x}{26(}$$

**Mathematica [A]** time = 0.11, size = 103, normalized size = 0.75

$$\frac{-10377\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{156\sqrt{3x^2-x+2}\left(96x^4-68x^3+390x^2+627x+182\right)}{(2x+1)^2}-19747\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{7488}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 + 2
*x)^2 - 19747*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]] - 10377*Sqrt[13]*ArcTanh
[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/7488
```

**fricas [A]** time = 0.89, size = 159, normalized size = 1.15

$$\frac{19747\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+10377\sqrt{13}\left(4x^2+4x+1\right)\mathrm{lo}}{14976\left(4x^2+4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*
x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x + 2)
)/(4*x^2 + 4*x + 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-18688512,[6]%%%}+%%%{%%{[56065536,0]:[1,0,-3]%%},[5]%%%}+%%%{-28032
768,[4]%%%}+%%%{%%{[-224262144,0]:[1,0,-3]%%},[3]%%%}+%%%{70081920,[2]%%%}+
%%%{%%{[350409600,0]:[1,0,-3]%%},[1]%%%}+%%%{292008000,[0]%%%} / %%%{%%{[24
,0]:[1,0,-3]%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},[4]%%%}+
%%%{864,[3]%%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%}+%%%{%%
{[-375,0]:[1,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

**maple [A]** time = 0.01, size = 162, normalized size = 1.17

$$-\frac{1519\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576}-\frac{1153\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{832}+\frac{1153\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}{4056}-\frac{257}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(2*x+1)^3,x)
```

```
[Out] 1153/4056*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-257/1248*(6*x-1)*(-4*x+3*(x+1/2)^2+5
/4)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+1153/832*(-16*x+1
2*(x+1/2)^2+5)^(1/2)-1153/832*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-1
6*x+12*(x+1/2)^2+5)^(1/2))+15/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-15/6
76*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+5
/4)^(5/2)
```

**maxima [A]** time = 0.98, size = 143, normalized size = 1.04

$$\frac{61}{312}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{26\left(4x^2+4x+1\right)}-\frac{257}{208}\sqrt{3x^2-x+2}\,x-\frac{1519}{576}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1153}{832}\sqrt{1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1)
 - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)*x
 - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1)
- 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*x^2
- x + 2)^(3/2)/(2*x + 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2-x+2\right)^{3/2}\,\left(4x^2+3x+1\right)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

```
[Out] int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2-x+2\right)^{\frac{3}{2}}\left(4x^2+3x+1\right)}{(2x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

```
[Out] Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

# 3.220 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

**Optimal**. Leaf size=189

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3\right.}{498960}$$

```
[Out] 117047/1492992*(1-6*x)*(3*x^2-x+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(5
/2)-1/498960*(26353-21350*x)*(3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-x+
2)^(7/2)+29/330*(1+2*x)^3*(3*x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(7/2
)+61917863/71663616*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2692081/11943936
*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4+\frac{29}{330}\left(3x^2-x+2\right)^{7/2}(2x+1)^3+\frac{133\left(3x^2-x+2\right)^{7/2}(2x+1)^2}{1485}-\frac{(26353-21350x)\left(3\right.}{498960}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (2692081*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/11943936 + (117047*(1 - 6*x)*(2 - x
 + 3*x^2)^(3/2))/1492992 + (5089*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/155520 -
((26353 - 21350*x)*(2 - x + 3*x^2)^(7/2))/498960 + (133*(1 + 2*x)^2*(2 - x
+ 3*x^2)^(7/2))/1485 + (29*(1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/330 + (2*(1 +
 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + (61917863*ArcSinh[(1 - 6*x)/Sqrt[23]])/
(23887872*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{2}{33}(1+2x)^4 \left(2-x+3x^2\right)^{7/2} + \frac{1}{132} \int (1+2x)^3(32+348x) \left(2\right.$$

$$= \frac{29}{330}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+2x)^4 \left(2-x+3x^2\right)^{7/2}$$

$$= \frac{133(1+2x)^2 \left(2-x+3x^2\right)^{7/2}}{1485} + \frac{29}{330}(1+2x)^3 \left(2-x+3x^2\right)^{7/2}$$

$$= -\frac{(26353-21350x) \left(2-x+3x^2\right)^{7/2}}{498960} + \frac{133(1+2x)^2 \left(2-x+3\right.}{1485}$$

$$= \frac{5089(1-6x) \left(2-x+3x^2\right)^{5/2}}{155520} - \frac{(26353-21350x) \left(2-x+3x\right.}{498960}$$

$$= \frac{117047(1-6x) \left(2-x+3x^2\right)^{3/2}}{1492992} + \frac{5089(1-6x) \left(2-x+3x^2\right)^{5}}{155520}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)}{1492992}$$

$$= \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x) \left(2-x+3x^2\right)}{1492992}$$

**Mathematica [A]** time = 0.06, size = 90, normalized size = 0.48

$$\frac{6\sqrt{3x^2-x+2} \left(120394874880x^{10} + 207681159168x^9 + 308846297088x^8 + 419978151936x^7 + 415908006912\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 1612
69204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 419
978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10) -
23838377255*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/27590492160
```

**fricas [A]** time = 0.89, size = 103, normalized size = 0.54

$$\frac{1}{4598415360} \left(120394874880\, x^{10} + 207681159168\, x^9 + 308846297088\, x^8 + 419978151936\, x^7 + 415908006912\, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 419
978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 + 16
1269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^2 -
 x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25)
```

**giac** [A] time = 0.20, size = 98, normalized size = 0.52

$$\frac{1}{4598415360}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(48\left(18\left(40\,x+69\right)x+1847\right)x+120557\right)x+1671441\right)x+50238389\right)x+22885\right.\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 1205
57)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 300369106
1)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663616
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple** [A] time = 0.02, size = 153, normalized size = 0.81

$$\frac{32\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{4}}{33}+\frac{436\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{3}}{165}+\frac{4258\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{2}}{1485}+\frac{10073\left(3x^{2}-x+2\right)^{\frac{7}{2}}x}{7128}-\frac{61917863\sqrt{3}\ \mathrm{ar}}{716}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 32/33*x^4*(3*x^2-x+2)^(7/2)+436/165*x^3*(3*x^2-x+2)^(7/2)+4258/1485*x^2*(3*
x^2-x+2)^(7/2)+10073/7128*x*(3*x^2-x+2)^(7/2)-5089/155520*(6*x-1)*(3*x^2-x+
2)^(5/2)-117047/1492992*(6*x-1)*(3*x^2-x+2)^(3/2)-61917863/71663616*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))-2692081/11943936*(6*x-1)*(3*x^2-x+2)^(1/2)+
92423/498960*(3*x^2-x+2)^(7/2)
```

**maxima** [A] time = 0.98, size = 184, normalized size = 0.97

$$\frac{32}{33}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{4}+\frac{436}{165}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{3}+\frac{4258}{1485}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{2}+\frac{10073}{7128}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x+\frac{92423}{498960}\left(3\,x^{2}-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258/
1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 92423
/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 5089/1
55520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 11704
7/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*x - 6
1917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1194393
6*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2\,x+1)^3\,{\left(3\,x^2-x+2\right)}^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x+1\right)^{3} \left(3x^{2}-x+2\right)^{\frac{5}{2}} \left(4x^{2}+3x+1\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

# 3.221 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

**Optimal**. Leaf size=164

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2-x\right.}{58320}$$

```
[Out] -6739/559872*(1-6*x)*(3*x^2-x+2)^(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)+
37/405*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/17010
*(2731+3430*x)*(3*x^2-x+2)^(7/2)-3564931/26873856*arcsinh(1/23*(1-6*x)*23^(
1/2))*3^(1/2)-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 832, 779, 612, 619, 215}

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}+\frac{37}{405}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{(3430x+2731)\left(3x^2-x+2\right)^{7/2}}{17010}-\frac{293(1-6x)\left(3x^2-x\right.}{58320}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-154997*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/4478976 - (6739*(1 - 6*x)*(2 - x +
3*x^2)^(3/2))/559872 - (293*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/58320 + (37*(1
 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/405 + ((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/
15 + ((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/17010 - (3564931*ArcSinh[(1 -
6*x)/Sqrt[23]])/(8957952*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + \frac{1}{120} \int (1+2x)^2(52+296x) \left(2-\right.$$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + $$

$$= \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3 \left(2-x+3x^2\right)^{7/2} + $$

$$= -\frac{293(1-6x) \left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2 \left(2-x+3x^2\right)^{7/2} + $$

$$= -\frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(1-6x) \left(2-x+3x^2\right)^{5/2}}{58320} + $$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872}$$

$$= -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x) \left(2-x+3x^2\right)^{3/2}}{559872}$$

**Mathematica [A]** time = 0.06, size = 85, normalized size = 0.52

$$\frac{6\sqrt{3x^2-x+2} \left(2257403904x^9 + 2675441664x^8 + 4427716608x^7 + 5671627776x^6 + 4996802304x^5 + 41715797\right.}{940584960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 30961049
76*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^7
+ 2675441664*x^8 + 2257403904*x^9) + 124772585*Sqrt[3]*ArcSinh[(-1 + 6*x)/S
qrt[23]])/940584960
```

**fricas [A]** time = 0.95, size = 98, normalized size = 0.60

$$\frac{1}{156764160} \left(2257403904\, x^9 + 2675441664\, x^8 + 4427716608\, x^7 + 5671627776\, x^6 + 4996802304\, x^5 + 417157977\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776*
x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 + 6
92659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*log(
-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac** [A] time = 0.21, size = 93, normalized size = 0.57

$$\frac{1}{156764160}\left(2\left(12\left(6\left(8\left(6\left(36\left(14\left(24\left(27\,x+32\right)x+1271\right)x+22793\right)x+722917\right)x+3621163\right)x+21500729\right)x+\right.\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
+ 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 38718
2961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3
)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple** [A] time = 0.01, size = 136, normalized size = 0.83

$$\frac{8\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{3}}{15}+\frac{472\left(3x^{2}-x+2\right)^{\frac{7}{2}}x^{2}}{405}+\frac{235\left(3x^{2}-x+2\right)^{\frac{7}{2}}x}{243}+\frac{3564931\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+\frac{293\,(6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/15*(3*x^2-x+2)^(7/2)*x^3+472/405*(3*x^2-x+2)^(7/2)*x^2+235/243*(3*x^2-x+2
)^(7/2)*x+293/58320*(6*x-1)*(3*x^2-x+2)^(5/2)+6739/559872*(6*x-1)*(3*x^2-x+
2)^(3/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+154997/447
8976*(6*x-1)*(3*x^2-x+2)^(1/2)+5419/17010*(3*x^2-x+2)^(7/2)
```

**maxima** [A] time = 0.97, size = 167, normalized size = 1.02

$$\frac{8}{15}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{3}+\frac{472}{405}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x^{2}+\frac{235}{243}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}x+\frac{5419}{17010}\left(3\,x^{2}-x+2\right)^{\frac{7}{2}}+\frac{293}{9720}\left(3\,x^{2}-x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/24
3*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(3*
x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x^2
- x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sqrt(3
*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 154997/4478976*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

```
[Out] int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{\frac{5}{2}} \left(4x^2+3x+1\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

```
[Out] Integral((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

# 3.222 $\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

**Optimal**. Leaf size=139

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x^2\right.}{746496}$$

```
[Out] -51175/746496*(1-6*x)*(3*x^2-x+2)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2)
+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-27071
575/35831808*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1177025/5971968*(1-6*x)
*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1653, 779, 612, 619, 215}

$$\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}+\frac{1}{648}(122x+137)\left(3x^2-x+2\right)^{7/2}-\frac{445(1-6x)\left(3x^2-x+2\right)^{5/2}}{15552}-\frac{51175(1-6x)\left(3x^2\right.}{746496}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (-1177025*(1 - 6*x)*Sqrt[2 - x + 3*x^2])/5971968 - (51175*(1 - 6*x)*(2 - x
+ 3*x^2)^(3/2))/746496 - (445*(1 - 6*x)*(2 - x + 3*x^2)^(5/2))/15552 + (2*(
1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + ((137 + 122*x)*(2 - x + 3*x^2)^(7/2)
)/648 - (27071575*ArcSinh[(1 - 6*x)/Sqrt[23]])/(11943936*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx = \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{108}\int (1+2x)(72+244x)\left(2-\right.$$

$$= \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/}$$

$$= -\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552} + \frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2} +$$

$$= -\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496} - \frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3}}{746496}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3}}{746496}$$

$$= -\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968} - \frac{51175(1-6x)\left(2-x+3x^2\right)^{3}}{746496}$$

**Mathematica [A]** time = 0.05, size = 80, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+4103\right.}{35831808}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^3
 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x^8
) + 27071575*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/35831808
```

**fricas [A]** time = 0.90, size = 93, normalized size = 0.67

$$\frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+58946544\,x^3+4103104\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

```
[Out] 1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 6696
9216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^2
- x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25)
```

**giac** [A] time = 0.19, size = 88, normalized size = 0.63

$$\frac{1}{5971968}\left(2\left(12\left(6\left(8\left(6\left(36\left(2\left(96x+61\right)x+319\right)x+11579\right)x+58133\right)x+409351\right)x+1709627\right)x+9930031\right)x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

```
[Out] 1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133)
*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2) -
27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

**maple** [A] time = 0.01, size = 119, normalized size = 0.86

$$\frac{8\left(3x^2-x+2\right)^{\frac{7}{2}}x^2}{27}+\frac{157\left(3x^2-x+2\right)^{\frac{7}{2}}x}{324}+\frac{27071575\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}+\frac{185\left(3x^2-x+2\right)^{\frac{7}{2}}}{648}+\frac{445\left(6x-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

```
[Out] 8/27*(3*x^2-x+2)^(7/2)*x^2+157/324*(3*x^2-x+2)^(7/2)*x+185/648*(3*x^2-x+2)^
(7/2)+445/15552*(6*x-1)*(3*x^2-x+2)^(5/2)+51175/746496*(6*x-1)*(3*x^2-x+2)^
(3/2)+1177025/5971968*(6*x-1)*(3*x^2-x+2)^(1/2)+27071575/35831808*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima** [A] time = 0.97, size = 150, normalized size = 1.08

$$\frac{8}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{157}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{445}{2592}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{445}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

```
[Out] 8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648*
(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x^2
 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3*x^
2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/35831808
```

```
*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2 - x
+ 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+1)\left(3x^2-x+2\right)^{5/2}\left(4x^2+3x+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
[Out] int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}\left(4x^2+3x+1\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
[Out] Integral((2*x + 1)*(3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1), x)
```

# 3.223 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx$

**Optimal**. Leaf size=147

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x}}{82944}$$

```
[Out] 1/10368*(2449+2154*x)*(3*x^2-x+2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2)
+2/21*(3*x^2-x+2)^(7/2)+944521/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)-169/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+1/82944
*(221999-17850*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}+\frac{(150x+29)\left(3x^2-x+2\right)^{5/2}}{1080}+\frac{(2154x+2449)\left(3x^2-x+2\right)^{3/2}}{10368}+\frac{(221999-17850x)\sqrt{3x^2-x}}{82944}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] ((221999 - 17850*x)*Sqrt[2 - x + 3*x^2])/82944 + ((2449 + 2154*x)*(2 - x +
3*x^2)^(3/2))/10368 + ((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/1080 + (2*(2 - x
 + 3*x^2)^(7/2))/21 + (944521*ArcSinh[(1 - 6*x)/Sqrt[23]])/(165888*Sqrt[3])
 - (169*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{1+2x}\,dx=\frac{2}{21}\left(2-x+3x^2\right)^{7/2}+\frac{1}{84}\int \frac{(112+140x)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$=\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2-x+3x^2\right)^{7/2}-\frac{\int \frac{(-29708-20104x)\left(2-x\right.}{1+2x}}{12096}$$

$$=\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(29+150x)\left(2-x+3x^2\right)^{5/2}}{1080}+\frac{2}{21}\left(2\right.$$

$$=\frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(2}{}$$

$$=\frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(2}{}$$

$$=\frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(2}{}$$

$$=\frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944}+\frac{(2449+2154x)\left(2-x+3x^2\right)^{3/2}}{10368}+\frac{(2}{}$$

**Mathematica [A]** time = 0.08, size = 106, normalized size = 0.72

$$\frac{-22997520\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+6\sqrt{3x^2-x+2}\left(7464960x^6-3836160x^5+15700608x^4-3646512x^3\right.}{17418240}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3 +
 15700608*x^4 - 3836160*x^5 + 7464960*x^6) - 33058235*Sqrt[3]*ArcSinh[(-1 +
 6*x)/Sqrt[23]] - 22997520*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 -
x + 3*x^2])])/17418240
```

**fricas [A]** time = 0.88, size = 135, normalized size = 0.92

$$\frac{1}{2903040}\left(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+12466776\,x^2-2120998\,x+11665053\right)\sqrt{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 1246677
6*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)*l
og(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/256*
sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x +
 185)/(4*x^2 + 4*x + 1))
```

**giac [A]** time = 0.31, size = 146, normalized size = 0.99

$$\frac{1}{2903040}\left(2\left(12\left(18\left(8\left(30\left(72x-37\right)x+4543\right)x-8441\right)x+519449\right)x-1060499\right)x+11665053\right)\sqrt{3x^2-x+2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x -
 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-6*
sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2*ab
s(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)
*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

**maple [A]** time = 0.01, size = 207, normalized size = 1.41

$$-\frac{944521\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{497664}-\frac{169\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{128}+\frac{2\left(3x^2-x+2\right)^{\frac{7}{2}}}{21}+\frac{5\left(6x-1\right)\left(3x^2\right.}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1),x)
```

```
[Out] 2/21*(3*x^2-x+2)^(7/2)+5/216*(6*x-1)*(3*x^2-x+2)^(5/2)+575/10368*(6*x-1)*(3
*x^2-x+2)^(3/2)+13225/82944*(6*x-1)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))+1/20*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-1/48*(6*x
-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-25/128*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2
)+13/48*(-4*x+3*(x+1/2)^2+5/4)^(3/2)+169/128*(-16*x+12*(x+1/2)^2+5)^(1/2)-1
69/128*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/
2))
```

**maxima [A]** time = 0.97, size = 154, normalized size = 1.05

$$\frac{2}{21}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{5}{36}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{29}{1080}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{359}{1728}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{2449}{10368}\left(3x^2-x+2\right)^{\frac{3}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

```
[Out] 2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
- x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x +
 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arcsin
h(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt(23)
*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^2 - x
 + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)}{2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^{2}-x+2\right)^{\frac{5}{2}}\left(4x^{2}+3x+1\right)}{2x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

```
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

# 3.224 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx$

**Optimal**. Leaf size=154

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}-\frac{11(37-60x)\left(3x^2-x+2\right)^{5/2}}{2340}-\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+}}{6912}$$

```
[Out] -11/864*(67-78*x)*(3*x^2-x+2)^(3/2)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-1/1
3*(3*x^2-x+2)^(7/2)/(1+2*x)-315623/41472*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)+429/128*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-11/6
912*(4727-3090*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}-\frac{11(37-60x)\left(3x^2-x+2\right)^{5/2}}{2340}-\frac{11}{864}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{11(4727-3090x)\sqrt{3x^2-x+}}{6912}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] (-11*(4727 - 3090*x)*Sqrt[2 - x + 3*x^2])/6912 - (11*(67 - 78*x)*(2 - x + 3
*x^2)^(3/2))/864 - (11*(37 - 60*x)*(2 - x + 3*x^2)^(5/2))/2340 - (2 - x + 3
*x^2)^(7/2)/(13*(1 + 2*x)) - (315623*ArcSinh[(1 - 6*x)/Sqrt[23]])/(13824*Sq
rt[3]) + (429*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])
/128
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^2}\,dx = -\frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} - \frac{1}{13}\int \frac{\left(-\frac{11}{2}-44x\right)\left(2-x+3x^2\right)^{5/2}}{1+2x}\,dx$$

$$= -\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} + \frac{\int \frac{(-286+14872x)\left(2-x\right.}{1+2x}}{1872}$$

$$= -\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340} - \frac{(2-}{1}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{1}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{1}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{1}{}$$

$$= -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912} - \frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} - \frac{1}{}$$

**Mathematica [A]** time = 0.12, size = 113, normalized size = 0.73

$$\frac{694980\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right) + \frac{6\sqrt{3x^2-x+2}\left(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257\right)}{2x+1} + 1578}{207360}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 251
424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) + 1578115*Sqrt[3]*ArcSinh[(-1
+ 6*x)/Sqrt[23]] + 694980*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x
 + 3*x^2])])/207360
```

**fricas [A]** time = 0.96, size = 153, normalized size = 0.99

$$\frac{1578115\,\sqrt{3}\,(2\,x+1)\log\left(-4\,\sqrt{3}\,\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-25\right)+694980\,\sqrt{13}\,(2\,x+1)\log\left(\frac{4\,\sqrt{13}\,\sqrt{3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqr
t(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12
*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 322972*x
- 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

**giac [B]** time = 1.10, size = 760, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

```
[Out] 429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*abs(
-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x +
1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1
/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) + 3946881
5*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9*sgn(1/(2
*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x + 1) + 13/(2*
x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084680*sqrt(13)
*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*
x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x
 + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 1397032245*(sqrt(-8
/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) -
 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*
x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538588*sqrt(13)*sgn(1/(2*x
 + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2 -
 3)^6
```

**maple [A]** time = 0.01, size = 235, normalized size = 1.53

$$\frac{315623\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{41472}+\frac{429\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{128}+\frac{(6x-1)\left(3x^{2}-x+2\right)^{\frac{5}{2}}}{36}+\frac{115\,(6x-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^2,x)
```

```
[Out] 1/36*(6*x-1)*(3*x^2-x+2)^(5/2)+115/1728*(6*x-1)*(3*x^2-x+2)^(3/2)+2645/1382
4*(6*x-1)*(3*x^2-x+2)^(1/2)+315623/41472*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1
/6))-33/260*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+19/192*(6*x-1)*(-4*x+3*(x+1/2)^2+5
/4)^(3/2)+965/1536*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-11/16*(-4*x+3*(x+1/
2)^2+5/4)^(3/2)-429/128*(-16*x+12*(x+1/2)^2+5)^(1/2)+429/128*13^(1/2)*arcta
nh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(-4*
x+3*(x+1/2)^2+5/4)^(7/2)+1/52*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)
```

**maxima [A]** time = 1.00, size = 161, normalized size = 1.05

$$\frac{1}{6}\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}x-\frac{7}{90}\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}+\frac{143}{144}\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}x-\frac{737}{864}\left(3\,x^{2}-x+2\right)^{\frac{3}{2}}-\frac{\left(3\,x^{2}-x+2\right)^{\frac{5}{2}}}{4\,(2\,x+1)}+\frac{5665}{1152}\sqrt{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2 -
 x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/2)
/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcsinh
(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(23)*
x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 - x +
2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^{2}-x+2\right)^{5/2}\,\left(4\,x^{2}+3\,x+1\right)}{\left(2\,x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

```
[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2 - x + 2\right)^{\frac{5}{2}} \left(4x^2 + 3x + 1\right)}{(2x + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

# 3.225 $\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx$

**Optimal**. Leaf size=161

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)\left(3x^2-x+2\right)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)\left(3x^2-x+2\right)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-}}{1536}$$

```
[Out] 1/832*(1227-838*x)*(3*x^2-x+2)^(3/2)+1/520*(257+134*x)*(3*x^2-x+2)^(5/2)/(1
+2*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*23^
(1/2))*3^(1/2)-1631/256*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13
^(1/2)+1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}+\frac{(134x+257)\left(3x^2-x+2\right)^{5/2}}{520(2x+1)}+\frac{1}{832}(1227-838x)\left(3x^2-x+2\right)^{3/2}+\frac{(21317-10470x)\sqrt{3x^2-}}{1536}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/1536 + ((1227 - 838*x)*(2 - x + 3*x
^2)^(3/2))/832 + ((257 + 134*x)*(2 - x + 3*x^2)^(5/2))/(520*(1 + 2*x)) - (2
 - x + 3*x^2)^(7/2)/(26*(1 + 2*x)^2) + (118423*ArcSinh[(1 - 6*x)/Sqrt[23]])
/(3072*Sqrt[3]) - (1631*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/256
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)}{(1+2x)^3}\,dx = -\frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} - \frac{1}{26}\int \frac{\left(-\frac{29}{2}-67x\right)\left(2-x+3x^2\right)^{5/2}}{(1+2x)^2}\,dx$$

$$= \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} + \frac{1}{208}\int \frac{(793-1676\ldots}{\ldots}$$

$$= \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \ldots$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \ldots$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \ldots$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \ldots$$

$$= \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \ldots$$

**Mathematica [A]** time = 0.11, size = 113, normalized size = 0.70

$$\frac{-293580\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)+\frac{6\sqrt{3x^2-x+2}\left(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057\right)}{(2x+1)^2}-592115}{46080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

```
[Out] ((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 83616
*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 - 592115*Sqrt[3]*ArcSinh[(-1 + 6
*x)/Sqrt[23]] - 293580*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
3*x^2])])/46080
```

**fricas [A]** time = 0.94, size = 169, normalized size = 1.05

$$592115\sqrt{3}\left(4x^2+4x+1\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+293580\sqrt{13}\left(4x^2+4x+1\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] 1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*x^2
+ 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-299016192,[6]%%%}+%%%{%%{[897048576,0]:[1,0,-3]%%},[5]%%%}+%%%{-448
524288,[4]%%%}+%%%{%%{[-3588194304,0]:[1,0,-3]%%},[3]%%%}+%%%{1121310720,[2
]%%%}+%%%{%%{[5606553600,0]:[1,0,-3]%%},[1]%%%}+%%%{4672128000,[0]%%%} / %%
%{%%{[24,0]:[1,0,-3]%%},[6]%%%}+%%%{-216,[5]%%%}+%%%{%%{[36,0]:[1,0,-3]%%},
[4]%%%}+%%%{864,[3]%%%}+%%%{%%{[-90,0]:[1,0,-3]%%},[2]%%%}+%%%{-1350,[1]%%%
}+%%%{%%{[-375,0]:[1,0,-3]%%},[0]%%%} Error: Bad Argument Value
```

**maple [A]** time = 0.02, size = 199, normalized size = 1.24

$$-\frac{118423\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9216}-\frac{1631\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{256}+\frac{1631\left(-4x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}\right)^{\frac{5}{2}}}{6760}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(2*x+1)^3,x)
```

```
[Out] 1631/6760*(-4*x+3*(x+1/2)^2+5/4)^(5/2)+1631/1248*(-4*x+3*(x+1/2)^2+5/4)^(3/
2)+1631/256*(-16*x+12*(x+1/2)^2+5)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(x+1/2)^2+
5/4)^(7/2)-1745/1536*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)+19/338/(x+1/2)*(-
4*x+3*(x+1/2)^2+5/4)^(7/2)-19/676*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(5/2)-419/
2496*(6*x-1)*(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1631/256*13^(1/2)*arctanh(2/13*(-
4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-118423/9216*3^(1/2)*arcsinh
(6/23*23^(1/2)*(x-1/6))
```

**maxima [A]** time = 0.99, size = 172, normalized size = 1.07

$$\frac{67}{520}\left(3x^{2}-x+2\right)^{\frac{5}{2}}-\frac{\left(3x^{2}-x+2\right)^{\frac{7}{2}}}{26\left(4x^{2}+4x+1\right)}-\frac{419}{416}\left(3x^{2}-x+2\right)^{\frac{3}{2}}x+\frac{1227}{832}\left(3x^{2}-x+2\right)^{\frac{3}{2}}+\frac{19\left(3x^{2}-x+2\right)^{\frac{5}{2}}}{52\left(2x+1\right)}-\frac{1745}{256}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1)
 - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/52
*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 118423/
9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(13)*a
rcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21317/1
536*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^{2}-x+2\right)^{5/2}\left(4x^{2}+3x+1\right)}{\left(2x+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

[Out] int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(3x^2 - x + 2\right)^{\frac{5}{2}} \left(4x^2 + 3x + 1\right)}{(2x + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
[Out] Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

# 3.226 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=693

$$\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(96c^3\left(a^2h^2(eh+3fg)+2abh\left(h(dh+3eg)+3fg^2\right)+b^2g\left(3h(dh+eg)+fg^2\right)\right)-80bc^2h\right.$$

```
[Out] 1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-80*b
*c^2*h*(3*a^2*f*h^2+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))-128*c^
4*g*(b*g*(3*d*h+e*g)+a*(f*g^2+3*h*(d*h+e*g)))+96*c^3*(a^2*h^2*(e*h+3*f*g)+b
^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(63*b^2*f*h^2-2*c*h*(32
*a*f*h+35*b*e*h+24*b*f*g)-c^2*(12*f*g^2-20*h*(4*d*h+3*e*g)))*(h*x+g)^2*(c*x
^2+b*x+a)^(1/2)/c^3/h-1/40*(9*b*f*h+2*c*(-5*e*h+f*g))*(h*x+g)^3*(c*x^2+b*x+
a)^(1/2)/c^2/h+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/1920*(945*b^4*f*h^
4-64*c^4*g^2*(3*f*g^2-5*h*(16*d*h+3*e*g))-210*b^2*c*h^3*(14*a*f*h+5*b*(e*h+
3*f*g))+8*c^2*h^2*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b^2*(129*f*g^2+50*
h*(d*h+3*e*g)))-16*c^3*h*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))+b*g*(39*f*g^2+5
*h*(54*d*h+47*e*g)))-2*c*h*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f*h+25*b*e*h+39*
b*f*g)+16*c^3*g*(3*f*g^2-5*h*(10*d*h+3*e*g))+8*c^2*h*(a*h*(45*e*h+71*f*g)+b
*(50*d*h^2+80*e*g*h+21*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c^5/h
```

**Rubi** [A] time = 2.10, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 621, 206}

$$\sqrt{a+bx+cx^2}\left(8c^2h^2\left(128a^2fh^2+275abh(eh+3fg)+3b^2\left(50h(dh+3eg)+129fg^2\right)\right)-2chx\left(8c^2h\left(ah(45eh\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((63*b^2*f*h^2 - 2*c*h*(24*b*f*g + 35*b*e*h + 32*a*f*h) - c^2*(12*f*g^2 - 2
0*h*(3*e*g + 4*d*h)))*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(240*c^3*h) - ((9*
b*f*h + 2*c*(f*g - 5*e*h))*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(40*c^2*h) +
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) + ((945*b^4*f*h^4 - 64*c^4*(3
*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*g +
 e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(129*f*
g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g + d*h)
) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 - 14*b*c
*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + 16*c^3*(3*f*g^3 - 5*g*h*(3*e*g + 10
```

```
*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g + 45*e*
h)))*x)*Sqrt[a + b*x + c*x^2])/(1920*c^5*h) + ((256*c^5*d*g^3 - 63*b^5*f*h^
3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(
e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g
+ e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) +
 b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
```

```
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}dx = \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch} + \frac{\int \frac{(g+hx)^3\left(-\frac{1}{2}h(bfg-10cdh+8afh)-\frac{1}{2}h(2cfg-10ceh+9bfh)x\right)}{\sqrt{a+bx+cx^2}} d}{5ch^2}$$

$$= -\frac{(9bfh+2c(fg-5eh))(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2h} + \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+}{240c^3h}$$

$$= \frac{\left(63b^2fh^2-2ch(24bfg+35beh+32afh)-c^2\left(12fg^2-20h(3eg+4dh)\right)\right)(g+}{240c^3h}$$

**Mathematica [A]** time = 1.25, size = 588, normalized size = 0.85

$$\frac{\sqrt{a+x(b+cx)}\left(4c^2h\left(256a^2fh^2+2abh(275eh+825fg+161fhx)+b^2\left(25h(12dh+36eg+7ehx)+3f\left(300g^2\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h + 14*a*f*h +
 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*x^2) + 15*e*
(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 + 20*g^2*h*x +
```

```
15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h*(825*f*g + 27
5*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) + 3*f*(300*g^2 +
 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*d*h + 9*e*h*x) +
 f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 + 50*g^2*h*x + 35*g
*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54*g^2 + 30*g*h*x +
7*h^2*x^2))))))/(1920*c^5) + ((256*c^5*d*g^3 - 63*b^5*f*h^3 + 70*b^3*c*h^2*
(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + b*g*
(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h) + b^2*(3*f
*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2*g*(f*g^2 + 3
*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

**fricas [A]** time = 1.73, size = 1435, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*(
8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h +
 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*c -
 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d - 2*
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 240*a^2
*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c^5*e -
3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*f)*g^
2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 - 44*a*
b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*a*b*c
^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^5*f*g
*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^5*e -
 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f)*h^3
)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^5*d -
 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35*b^2*
c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x
 + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)
*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)
*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e +
(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*
c^3)*d - 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*
c + 240*a^2*b*c^2)*f)*h^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*f*h^3*x^4 + 480*(4*c^5
*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^4)*
f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^2 -
```

```
44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2 - 44*
a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*(30*c^
5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 30*(8*c^
5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*a*c^4)*f
)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h + 30*(48*c^
5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*d - 10*(35
*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*sqrt(c*x^2
+ b*x + a))/c^6]
```

**giac** [A] time = 0.32, size = 822, normalized size = 1.19

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,fh^3x}{c}+\frac{30\,c^4fgh^2-9\,bc^3fh^3+10\,c^4h^3e}{c^5}\right)x+\frac{240\,c^4fg^2h-210\,bc^3fgh^2+80\,c^4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 - 9*b*
c^3*f*h^3 + 10*c^4*h^3*e)/c^5)*x + (240*c^4*f*g^2*h - 210*b*c^3*f*g*h^2 + 8
0*c^4*d*h^3 + 63*b^2*c^2*f*h^3 - 64*a*c^3*f*h^3 + 240*c^4*g*h^2*e - 70*b*c^
3*h^3*e)/c^5)*x + (480*c^4*f*g^3 - 1200*b*c^3*f*g^2*h + 1440*c^4*d*g*h^2 +
1050*b^2*c^2*f*g*h^2 - 1080*a*c^3*f*g*h^2 - 400*b*c^3*d*h^3 - 315*b^3*c*f*h
^3 + 644*a*b*c^2*f*h^3 + 1440*c^4*g^2*h*e - 1200*b*c^3*g*h^2*e + 350*b^2*c^
2*h^3*e - 360*a*c^3*h^3*e)/c^5)*x - (1440*b*c^3*f*g^3 - 5760*c^4*d*g^2*h -
3600*b^2*c^2*f*g^2*h + 3840*a*c^3*f*g^2*h + 4320*b*c^3*d*g*h^2 + 3150*b^3*c
*f*g*h^2 - 6600*a*b*c^2*f*g*h^2 - 1200*b^2*c^2*d*h^3 + 1280*a*c^3*d*h^3 - 9
45*b^4*f*h^3 + 2940*a*b^2*c*f*h^3 - 1024*a^2*c^2*f*h^3 - 1920*c^4*g^3*e + 4
320*b*c^3*g^2*h*e - 3600*b^2*c^2*g*h^2*e + 3840*a*c^3*g*h^2*e + 1050*b^3*c*
h^3*e - 2200*a*b*c^2*h^3*e)/c^5) - 1/256*(256*c^5*d*g^3 + 96*b^2*c^3*f*g^3
- 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f
*g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 + 210*b^4*c*f*g*h^2 - 720*
a*b^2*c^2*f*g*h^2 + 288*a^2*c^3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*
h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*a^2*b*c^2*f*h^3 - 128*b*c^4*g^
3*e + 288*b^2*c^3*g^2*h*e - 384*a*c^4*g^2*h*e - 240*b^3*c^2*g*h^2*e + 576*a
*b*c^3*g*h^2*e + 70*b^4*c*h^3*e - 240*a*b^2*c^2*h^3*e + 96*a^2*c^3*h^3*e)*l
og(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

**maple** [B] time = 0.02, size = 1869, normalized size = 2.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

[Out] g^3*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/c*(c*x^2+b*x+a)^(1/2)*g^3*e-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*g^3*f+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f+161/240*h^3*f/c^3*b*a*x*(c*x^2+b*x+a)^(1/2)-7/8/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)*g*h^2*f+35/32/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)*g*h^2*f-45/16/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f+55/16/c^3*b*a*(c*x^2+b*x+a)^(1/2)*g*h^2*f-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*g*h^2*f-5/4/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g*h^2*e-5/4/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g^2*h*f+9/4/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e+9/4/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+3/4*x^3/c*(c*x^2+b*x+a)^(1/2)*g*h^2*f-7/24/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)*h^3*e-21/64*h^3*f/c^4*b^3*x*(c*x^2+b*x+a)^(1/2)+35/32*h^3*f/c^(9/2)*b^3*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*h^3*f/c^4*b^2*a*(c*x^2+b*x+a)^(1/2)-15/16*h^3*f/c^(7/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-9/40*h^3*f/c^2*b*x^3*(c*x^2+b*x+a)^(1/2)+21/80*h^3*f/c^3*b^2*x^2*(c*x^2+b*x+a)^(1/2)+15/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g^2*h*f-15/16/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e-15/16/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+3/4/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d-2*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h^2*e-2*a/c^2*(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2*x/c*(c*x^2+b*x+a)^(1/2)*g*h^2*d+3/2*x/c*(c*x^2+b*x+a)^(1/2)*g^2*h*e-9/4/c^2*b*(c*x^2+b*x+a)^(1/2)*g*h^2*d-9/4/c^2*b*(c*x^2+b*x+a)^(1/2)*g^2*h*e+9/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d+9/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*e-3/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d-3/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*e-3/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*d+1/5*h^3*f*x^4/c*(c*x^2+b*x+a)^(1/2)+63/128*h^3*f/c^5*b^4*(c*x^2+b*x+a)^(1/2)-63/256*h^3*f/c^(11/2)*b^5*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/96/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)*h^3*e+8/15*h^3*f*a^2/c^3*(c*x^2+b*x+a)^(1/2)+1/4*x^3/c*(c*x^2+b*x+a)^(1/2)*h^3*e-35/64/c^4*b^3*(c*x^2+b*x+a)^(1/2)*h^3*e+35/128/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+3/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+1/3*x^2/c*(c*x^2+b*x+a)^(1/2)*h^3*d+5/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*h^3*d-5/16/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*h^3*d+1/2*x/c*(c*x^2+b*x+a)^(1/2)*g^3*f+3/c*(c*x^2+b*x+a)^(1/2)*g^2*h*d-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*e-105/64/c^4*b^3*(c*x^2+b*x+a)^(1/2)*g*h^2*f+105/128/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-15/16/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+55/48/c^3*b*a*(c*x^2+b*x+a)^(1/2)*h^3*e-3/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*h^3*e+9/8*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f+x^2/c*(c*x^2+b*x+a)^(1/2)*g*h^2*e+x^2/c*(c*x^2+b*x+a)^(1/2)*g^2*h*f-5/12/c^2*b*x*(c*x^2+b*x+a)^(1/2)*h^3*d+15/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g*h^2*e-4/15*h^3*f*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^3\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)^{3} \left(d + ex + fx^{2}\right)}{\sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.227 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=420

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(dh^2+2egh+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-\right.}{128c^{9/2}}$$

```
[Out] 1/128*(128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^3
*(b*g*(2*d*h+e*g)+a*(d*h^2+2*e*g*h+f*g^2))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h+2
*f*g)+b^2*(d*h^2+2*e*g*h+f*g^2)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+
a)^(1/2))/c^(9/2)-1/24*(7*b*f*h-8*c*e*h+2*c*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(1
/2)/c^2/h+1/4*f*(h*x+g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^3+32*c
^3*g*(f*g^2-4*h*(3*d*h+e*g))-20*b*c*h^2*(11*a*f*h+6*b*(e*h+2*f*g))+8*c^2*h*
(16*a*h*(e*h+2*f*g)+b*(11*f*g^2+18*h*(d*h+2*e*g)))-2*c*h*(35*b^2*f*h^2-4*c*
h*(9*a*f*h+10*b*e*h+6*b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g)))*x)*(c*x^2+b*x
+a)^(1/2)/c^4/h
```

**Rubi [A]** time = 1.01, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] -((2*c*f*g - 8*c*e*h + 7*b*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(24*c^2*
h) + (f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - ((105*b^3*f*h^3 + 32*c
^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g + e*h)
) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h)) - 2*
c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g^2 - 2
*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(192*c^4*h) + ((128*c^4*d*g^
2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2
 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*
f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} + \frac{\int \frac{(g+hx)^2\left(-\frac{1}{2}h(bfg-8cdh+6afh)-\frac{1}{2}h(2cfg-8ceh+7bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{4ch^2}$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} +$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

$$= -\frac{(2cfg-8ceh+7bfh)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2h} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} -$$

**Mathematica [A]** time = 0.65, size = 343, normalized size = 0.82

$$3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*(
24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*g*
h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e*g
+ 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20*g*h
*x + 7*h^2*x^2))) + 3*(128*c^4*d*g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g +
 b*e*h + 3*a*f*h) - 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)
) + 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h
))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(9/2))
```

**fricas [A]** time = 1.33, size = 861, normalized size = 2.05

$$\left[\frac{3\left(16\left(8c^4d-4bc^3e+\left(3b^2c^2-4ac^3\right)f\right)g^2-16\left(8bc^3d-2\left(3b^2c^2-4ac^3\right)e+\left(5b^3c-12abc^2\right)f\right)gh+\left(16\left(3b^2c\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b
*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3
*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4*c^4*
e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)
*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^
2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*
g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 -
 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(8*c^4*d -
4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*b^2*c^2 - 4
*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*a*c^3)*d - 8
*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f)*h^2)*sqr
t(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*
x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4
*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2
*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (
8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*
g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^
2 + b*x + a))/c^5]
```

**giac [A]** time = 0.30, size = 457, normalized size = 1.09

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,fh^2x}{c}+\frac{16\,c^3fgh-7\,bc^2fh^2+8\,c^3h^2e}{c^4}\right)x+\frac{48\,c^3fg^2-80\,bc^2fgh+48\,c^3dh^2+35\,b^2}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h - 7*b*c^2*f*
h^2 + 8*c^3*h^2*e)/c^4)*x + (48*c^3*f*g^2 - 80*b*c^2*f*g*h + 48*c^3*d*h^2 +
 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2 + 96*c^3*g*h*e - 40*b*c^2*h^2*e)/c^4)*x -
(144*b*c^2*f*g^2 - 384*c^3*d*g*h - 240*b^2*c*f*g*h + 256*a*c^2*f*g*h + 144*
b*c^2*d*h^2 + 105*b^3*f*h^2 - 220*a*b*c*f*h^2 - 192*c^3*g^2*e + 288*b*c^2*g
*h*e - 120*b^2*c*h^2*e + 128*a*c^2*h^2*e)/c^4) - 1/128*(128*c^4*d*g^2 + 48*
b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h - 80*b^3*c*f*g*h + 192*a*b
*c^2*f*g*h + 48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 + 35*b^4*f*h^2 - 120*a*b^2*c
*f*h^2 + 48*a^2*c^2*f*h^2 - 64*b*c^3*g^2*e + 96*b^2*c^2*g*h*e - 128*a*c^3*g
*h*e - 40*b^3*c*h^2*e + 96*a*b*c^2*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 1069, normalized size = 2.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/c*(c*x^2+b*x+a)^(1/2)*g^2*e+g^2*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)-5/6/c^2*b*x*(c*x^2+b*x+a)^(1/2)*g*h*f+3/2/c^(5/2)*b*a*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h*f+5/4/c^3*b^2*(c*x^2+b*x+a)^(1/2)*g
*h*f-4/3*a/c^2*(c*x^2+b*x+a)^(1/2)*g*h*f+x/c*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/
c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2/3*x^2/c*(c*
x^2+b*x+a)^(1/2)*g*h*f-5/12/c^2*b*x*(c*x^2+b*x+a)^(1/2)*h^2*e+2/c*(c*x^2+b*
x+a)^(1/2)*g*h*d-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*
g^2*e+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+1/3
*x^2/c*(c*x^2+b*x+a)^(1/2)*h^2*e-5/8/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*g*h*f+35/96*h^2*f/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*h^2
*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*h^2*f/c^
3*b*a*(c*x^2+b*x+a)^(1/2)-3/8*h^2*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-7/24*h^2*f/
c^2*b*x^2*(c*x^2+b*x+a)^(1/2)-3/2/c^2*b*(c*x^2+b*x+a)^(1/2)*e*g*h+3/4/c^(5/
2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-a/c^(3/2)*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*g*h-b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))*g*h*d-2/3*a/c^2*(c*x^2+b*x+a)^(1/2)*h^2*e+1/2*x/c*(c*x
^2+b*x+a)^(1/2)*d*h^2+1/4*h^2*f*x^3/c*(c*x^2+b*x+a)^(1/2)-35/64*h^2*f/c^4*b
^3*(c*x^2+b*x+a)^(1/2)+35/128*h^2*f/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3/8*h^2*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g^2-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*d*h
^2-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2-1/2*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g^2-3/4/c^2*b*(c*x^2+b*
x+a)^(1/2)*f*g^2+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)*f*g^2+5/8/c^3*b^2*(c*x^2+b*x+a)^(1/2)*h^2*e-5/16/c^(7/2)*b^3*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g+hx\right)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.228 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=223

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+c}}{}$$

```
[Out] 1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*h+b*e*g)+6*b*c*(2*a*f*h+b
*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/3
*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h^2-8*c^2*(f*g^2-3*h*(d
*h+e*g))-2*c*h*(8*a*f*h+9*b*(e*h+f*g))-2*c*h*(5*b*f*h-6*c*e*h+2*c*f*g)*x)*(
c*x^2+b*x+a)^(1/2)/c^3/h
```

**Rubi [A]** time = 0.30, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2-2chx(5bfh-6ceh+2cfg)-8c^2\left(fg^2-3h(dh+eg)\right)\right)}{24c^3h}+\frac{\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) + ((15*b^2*f*h^2 - 8*c^2*(f*g
^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(2*c*f*g
- 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*h) + ((16*c^3*d*g -
5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b*e*h +
2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/
2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\int \frac{(g+hx)\left(-\frac{1}{2}h(bfg-6cdh+4afh)-\frac{1}{2}h(2cfg-6ceh+5bfh)x\right)}{\sqrt{a+bx+cx^2}}\,dx}{3ch^2}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+\right.}{}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+\right.}{}$$

$$= \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh^2-8c^2\left(fg^2-3h(eg+dh)\right)-2ch(8afh+\right.}{}$$

**Mathematica [A]** time = 0.23, size = 215, normalized size = 0.96

$$\frac{-\frac{3h\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(8c^2(aeh+afg+bdh+beg)-6bc(2afh+beh+bfg)+5b^3fh-16c^3dg\right)}{16c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\left(-2ch(8afh+b(9eh+9fg+5fhx))+15b\right.}{8c^2}}{3ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (f*(g + h*x)^2*Sqrt[a + x*(b + c*x)] + (Sqrt[a + x*(b + c*x)]*(15*b^2*f*h^2
 - 4*c^2*(f*g*(2*g + h*x) - 3*h*(2*e*g + 2*d*h + e*h*x)) - 2*c*h*(8*a*f*h +
 b*(9*f*g + 9*e*h + 5*f*h*x))))/(8*c^2) - (3*h*(-16*c^3*d*g + 5*b^3*f*h + 8
*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b*c*(b*f*g + b*e*h + 2*a*f*h))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(16*c^(5/2)))/(3*c*h)
```

**fricas [A]** time = 1.11, size = 461, normalized size = 2.07

$$\left[\frac{3\left(2\left(8\,c^3d-4\,bc^2e+\left(3\,b^2c-4\,ac^2\right)f\right)g-\left(8\,bc^2d-2\left(3\,b^2c-4\,ac^2\right)e+\left(5\,b^3-12\,abc\right)f\right)h\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2 -
 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(
8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (15*b^
2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt(c*x^
2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f
)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(
-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
+ a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c
^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)
*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

**giac [A]** time = 0.27, size = 210, normalized size = 0.94

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,fhx}{c}+\frac{6\,c^2fg-5\,bcfh+6\,c^2he}{c^3}\right)x-\frac{18\,bcfg-24\,c^2dh-15\,b^2fh+16\,acfh-24\,c^2ge+1}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g - 5*b*c*f*h + 6*c^2*h
*e)/c^3)*x - (18*b*c*f*g - 24*c^2*d*h - 15*b^2*f*h + 16*a*c*f*h - 24*c^2*g*
e + 18*b*c*h*e)/c^3) - 1/16*(16*c^3*d*g + 6*b^2*c*f*g - 8*a*c^2*f*g - 8*b*c
```

```
^2*d*h - 5*b^3*f*h + 12*a*b*c*f*h - 8*b*c^2*g*e + 6*b^2*c*h*e - 8*a*c^2*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

**maple [B]** time = 0.01, size = 505, normalized size = 2.26

$$\frac{\sqrt{c\,x^2+bx+a}\;fh\,x^2}{3c}+\frac{3abfh\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{5}{2}}}-\frac{aeh\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}-\frac{afg\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\right.}}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*h*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*h*f/c^2*b*x*(c*x^2+b*x+a)^(1/2)+5/8*
h*f/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*h*f/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))+3/4*h*f/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))-2/3*h*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e*
h+1/2*x/c*(c*x^2+b*x+a)^(1/2)*f*g-3/4/c^2*b*(c*x^2+b*x+a)^(1/2)*e*h-3/4/c^2
*b*(c*x^2+b*x+a)^(1/2)*f*g+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*e*h+3/8/c^(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*f*g-1/2*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e*h-1/2*a/c^
(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g+1/c*(c*x^2+b*x+a)^(1/
2)*d*h+1/c*(c*x^2+b*x+a)^(1/2)*e*g-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*d*h-1/2*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*g+d*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)\left(d + ex + fx^2\right)}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.229 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] 1/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)+1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x
^2+b*x+a)^(1/2)/c
```

**Rubi [A]** time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{\quad}{\sqrt{a+b}}}{4c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\text{Subst}}{2c^2}$$

$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{\quad}{2}\right.}{8c^{5/2}}$$

**Mathematica [A]** time = 0.15, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(8*c^(5/2))
```

**fricas** [A] time = 0.86, size = 227, normalized size = 1.96

$$\left[-\frac{\left(8\,c^2d-4\,bce+\left(3\,b^2-4\,ac\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)-4\left(2\,c^2f\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

**giac** [A] time = 0.25, size = 98, normalized size = 0.84

$$\frac{1}{4}\,\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^2}\right)-\frac{\left(8\,c^2d+3\,b^2f-4\,acf-4\,bce\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}\,-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

**maple** [A] time = 0.01, size = 185, normalized size = 1.59

$$-\frac{af\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3b^2f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{be\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{d\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*f/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
```

```
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.230 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=179

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx}}{ch}$$

```
[Out] -1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x
)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(1
/2)+f*(c*x^2+b*x+a)^(1/2)/c/h
```

**Rubi [A]** time = 0.29, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{2c^{3/2}h^2}+\frac{\left(fg^2-h(eg-dh)\right)\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h^2\sqrt{ah^2-bgh+cg^2}}+\frac{f\sqrt{a+bx}}{ch}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x + c*x^2])/(c*h) - ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*h^2) + ((f*g^2 - h*(
e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
 a*h^2]*Sqrt[a + b*x + c*x^2])])/(h^2*Sqrt[c*g^2 - b*g*h + a*h^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} + \frac{\int \frac{-\frac{1}{2}h(bfg-2cdh)-\frac{1}{2}h(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{a+bx+cx^2}}\,dx}{ch^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2ch^2} + \frac{\left(fg^2-egh+dh^2\right)\int -}{h^2}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{ch^2} - \frac{\left(2\left(fg\right.\right.}{}$$

$$= \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} + \frac{\left(fg^2-h(eg-\right.}{}$$

**Mathematica [A]** time = 0.28, size = 172, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)(bfh-2ceh+2cfg)}{c^{3/2}}+\frac{2\left(h(dh-eg)+fg^2\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)}{\sqrt{h(ah-bg)+cg^2}}-\frac{2fh\sqrt{a+x(b+cx)}}{c}}{2h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/2*((-2*f*h*Sqrt[a + x*(b + c*x)])/c + ((2*c*f*g - 2*c*e*h + b*f*h)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (2*(f*g^2 + h*
(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 +
h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)])/
h^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 599, normalized size = 3.35

$$-\frac{d\ln\left(\frac{\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{2a h^2-2bgh+2c g^2}{h^2}+2\sqrt{\frac{a h^2-bgh+c g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2 c+\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{a h^2-bgh+c g^2}{h^2}}}{x+\frac{g}{h}}\right)}{\sqrt{\frac{a h^2-bgh+c g^2}{h^2}}\, h}+\frac{eg\ln\left(\frac{\frac{(hb-2cg)\left(x+\frac{g}{h}\right)}{h}+\frac{2a h^2-2bgh+2c g^2}{h^2}+2\right.}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h*f*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))+1/h*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^
2*f*g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*d+1/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((
(b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(
x+g/h))*e*g-1/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h
+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(
b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,\sqrt{c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\left(g + hx\right) \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

# 3.231 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=241

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h\left(2ah(2fg-eh)-b\left(-dh^2-egh+3fg^2\right.\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

```
[Out] -1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-e*g*h+3*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/h^2/c^(1/2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/
(a*h^2-b*g*h+c*g^2)/(h*x+g)
```

**Rubi [A]** time = 0.37, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}-\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-b\right.\right.}{2h^2\left(ah^2-bgh+cg^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^2
)*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) -
2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*h^2*(c*g^2 - b*g*h + a*h^2)^(
3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} - \frac{\int \frac{\frac{1}{2}\left(-2cdg+beg+2afg-\frac{bfg^2}{h}+bdh-2aeh\right)+f\left(bg-\frac{cg^2}{h}\right.}{(g+hx)\sqrt{a+bx+cx^2}}}{cg^2-bgh+ah^2}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{h^2} - \frac{\left(2c\left(fg^3-dgh^2\right)-\right.}{}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{(2f)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{h^2} +$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)} + \frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,h^2} - \frac{\left(2c\left(fg^3-\right.\right.}{}$$

**Mathematica [A]** time = 0.35, size = 227, normalized size = 0.94

$$\frac{-\frac{h\sqrt{a+x(b+cx)}\left(h(dh-eg)+fg^2\right)}{(g+hx)\left(h(ah-bg)+cg^2\right)} + \frac{\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h\left(-2ah(eh-2fg)+bh(dh+eg)-3bfg^2\right)+2c\left(fg^3-dgh^2\right)\right)}{2\left(h(ah-bg)+cg^2\right)^{3/2}} + \frac{f\tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a}}\right.}{\sqrt{c}}}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g)
 + a*h))*(g + h*x))) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])])/Sqrt[c] + ((2*c*(f*g^3 - d*g*h^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h)
- 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt
[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*(c*g^2 + h*(-(b*g) +
 a*h))^(3/2)))/h^2
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.71Error: Bad Argument Type
```

**maple [B]** time = 0.02, size = 1671, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] f/h^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/(a*h^2-b*g*h+c*
g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+1/2/(a*h^2-
b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d-1/2/h/(a*h^2-b
*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*
h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*
c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g+1/2/h^2/(a*h^
2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*
(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h
-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*f*g^2-1/h/(a*h
^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2
*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*
h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g*d+1/h^2/(a*
h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+
2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b
*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e-1/h^3/
(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)
/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c
+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^3*f-1/h
```

```
^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h
+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g
/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*e+2/h^3/((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^
2)/h^2)^(1/2))/(x+g/h))*f*g
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\left(g + hx\right)^{2} \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

# 3.232 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=336

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(a\left(dh^2-3egh+fg^2\right)+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(3dh\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

```
[Out] 1/8*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)+b^2*(3*d*h^2+e*g*h+3*f*g^2
)-4*c*(b*g*(2*d*h+e*g)+a*(d*h^2-3*e*g*h+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b
*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*
g^2)^(5/2)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^
2)/(h*x+g)^2+1/4*(2*c*g*(f*g^2+h*(-3*d*h+e*g))+h*(4*a*h*(-e*h+2*f*g)-b*(-3*
d*h^2-e*g*h+5*f*g^2)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

**Rubi [A]** time = 0.66, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c\left(-ah(3eg-dh)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) + ((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b*h*(e
*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(4*h*(c*g^2 - b*
g*h + a*h^2)^2*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g +
e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f*g^2
 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2
- b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} - \frac{\int \frac{\frac{1}{2}\left(-4cdg+beg+4afg-\frac{bfg^2}{h}+3bdh-4aeh\right)-\left(ceg-2bfg \cdots\right.}{(g+hx)^2\sqrt{a+bx+cx^2}}}{2\left(cg^2-bgh+ah^2\right)}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh \cdots\right.\right.}{4h\left(cg^2-bgh \cdots\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh \cdots\right.\right.}{4h\left(cg^2-bgh \cdots\right.}$$

$$= -\frac{\left(fg^2-h(eg-dh)\right)\sqrt{a+bx+cx^2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2} + \frac{\left(2c\left(fg^3+gh(eg-3dh)\right)-h\left(5bfg^2-bh \cdots\right.\right.}{4h\left(cg^2-bgh \cdots\right.}$$

**Mathematica [A]** time = 1.10, size = 367, normalized size = 1.09

$$\frac{-\frac{ch\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(8a^2fh^2-4c\left(ah(dh-3eg)+afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)}{8\left(h(ah-bg)+cg^2\right)^{5/2}}+\frac{\sqrt{a+x(b+cx)}\,(\cdots}{2(g \cdots}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-((f*Sqrt[a + x*(b + c*x)])/(g + h*x)^2) + ((c*f*g^2 + 2*f*h*(-(b*g) + a*h
) + c*h*(e*g - d*h))*Sqrt[a + x*(b + c*x)])/(2*(c*g^2 + h*(-(b*g) + a*h))*(
g + h*x)^2) + (c*(2*c*(f*g^3 + g*h*(e*g - 3*d*h)) + h*(-5*b*f*g^2 + b*h*(e*
g + 3*d*h) - 4*a*h*(-2*f*g + e*h)))*Sqrt[a + x*(b + c*x)])/(4*(c*g^2 + h*(-
(b*g) + a*h))^2*(g + h*x)) - (c*h*(8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f
*g + e*h) - 4*c*(a*f*g^2 + a*h*(-3*e*g + d*h) + b*g*(e*g + 2*d*h)) + b^2*(3
*f*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sq
rt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(8*(c*g^2 + h*(-(b*g)
 + a*h))^(5/2)))/(c*h)
```

**fricas [B]** time = 174.73, size = 2034, normalized size = 6.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)

[Out] [1/16*(((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f -
(b^2 + 12*a*c)*e)*g^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 +
 ((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b
^2 + 12*a*c)*e)*g*h^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 +
2*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^
2 + 12*a*c)*e)*g^2*h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*
sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2
- (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h +
a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2
 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*(2*a^
2*d*h^5 - (4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)
*g^4*h - (13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f
- 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (
2*c^2*e - 7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 +
(9*b*c*d - 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^
2 + 2*a*c)*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c
^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*
g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^
2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 -
 (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h -
3*b*c^2*g^6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 -
 (b^3 + 6*a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x), 1/8*(((8*c^2*d -
4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g
^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2 + ((8*c^2*d - 4*b*c*
e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g*h
^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^2 + 2*((8*c^2*d - 4*b*c
*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f - (b^2 + 12*a*c)*e)*g^2*
h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^3)*x)*sqrt(-c*g^2 + b*g*h
 - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b
*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c
*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*(2*a^2*d*h^5 -
(4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2 + 2*a*c)*f)*g^4*h - (
13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*e + 6*a^2*f - 5*(b^2 +
 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*c^2*f*g^5 + (2*c^2*e -
7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)*f)*g^3*h^2 + (9*b*c*d -
 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e + 8*a^2*f - 3*(b^2 + 2*a*c)
*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/(c^3*g^8 - 3
*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3*(b^2*c + a*c^2)*g^6*h^2 -
(b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4*h^4 + (c^3*g^6*h^2 - 3*b*c^
2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c + a*c^2)*g^4*h^4 - (b^3 + 6*
a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6)*x^2 + 2*(c^3*g^7*h - 3*b*c^2*g^
6*h^2 - 3*a^2*b*g^2*h^6 + a^3*g*h^7 + 3*(b^2*c + a*c^2)*g^5*h^3 - (b^3 + 6*
```

```
a*b*c)*g^4*h^4 + 3*(a*b^2 + a^2*c)*g^3*h^5)*x)]
```

**giac [B]** time = 0.54, size = 2307, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(8*c^2*d*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h - 8*a*b*f*g*h +
3*b^2*d*h^2 - 4*a*c*d*h^2 + 8*a^2*f*h^2 - 4*b*c*g^2*e + b^2*g*h*e + 12*a*c*
g*h*e - 4*a*b*h^2*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(
c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h^2 +
 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2)) + 1/4
*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^3*c^2*d*g^2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 +
20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*b*c*g^2*h^3*e - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e
- 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*g*h^4*e + 4*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*a*b*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c
^(5/2)*f*g^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h -
 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - (sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
^2*b*c^(3/2)*d*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c
)*f*g^2*h^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 +
 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 - 16*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*c^(5/2)*g^4*h*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*
c^(3/2)*g^3*h^2*e + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2
*h^3*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*g*h^4*e + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*h^5*e + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))*b*c^2*f*g^5 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h -
 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*f*g^4*h - 24*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*b*c^2*d*g^3*h^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
b^3*f*g^3*h^2 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 + 20
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*d*g^2*h^3 + 40*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3 - 11*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a*b^2*f*g^2*h^3 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 -
 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c
```

```
*x^2 + b*x + a))*a*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*
b*f*g*h^4 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d*h^5 + 4*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))*b*c^2*g^4*h*e - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*g^3*h^2*e
+ (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e - 16*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
*b^2*g*h^4*e + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*g*h^4*e - 4*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 2*b^2*c^(3/2)*f*g^5 - 5*b^3*
sqrt(c)*f*g^4*h - 4*a*b*c^(3/2)*f*g^4*h - 6*b^2*c^(3/2)*d*g^3*h^2 + 21*a*b^
2*sqrt(c)*f*g^3*h^2 + 4*a^2*c^(3/2)*f*g^3*h^2 + 3*b^3*sqrt(c)*d*g^2*h^3 + 2
0*a*b*c^(3/2)*d*g^2*h^3 - 32*a^2*b*sqrt(c)*f*g^2*h^3 - 11*a*b^2*sqrt(c)*d*g
*h^4 - 12*a^2*c^(3/2)*d*g*h^4 + 16*a^3*sqrt(c)*f*g*h^4 + 8*a^2*b*sqrt(c)*d*
h^5 + 2*b^2*c^(3/2)*g^4*h*e + b^3*sqrt(c)*g^3*h^2*e - 8*a*b*c^(3/2)*g^3*h^2
*e - 5*a*b^2*sqrt(c)*g^2*h^3*e + 4*a^2*c^(3/2)*g^2*h^3*e + 12*a^2*b*sqrt(c)
*g*h^4*e - 8*a^3*sqrt(c)*h^5*e)/((c^2*g^4*h^2 - 2*b*c*g^3*h^3 + b^2*g^2*h^4
 + 2*a*c*g^2*h^4 - 2*a*b*g*h^5 + a^2*h^6)*((sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c)*g + b*g - a*h)^2)
```

**maple [B]** time = 0.02, size = 3615, normalized size = 10.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b
*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))-3/2/h/(a*h^2-b*g*h+c*g^2
)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*
h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+
g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g^2*e+3/2/h^2/(a*h^2-b*
g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a
*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g^3*f-3/2/h^2/
(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*c*g^3*f-3/8/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2))/(x+g/h))*b^2*f*g^2+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*c*g*d-3/2/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2
+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^2*d+3/2/h^2/(a*h^2-b*g*h+c*g^2)^2
```

```
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c
*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h
)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^3*e-3/2/h^3/(a*h^2-b*g*h
+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^
2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c^2*g^4*f-1/2/h/(a*h^
2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*d-1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h
)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g+1/2
/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g
/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^
2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e+1/2/
h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*e*g-1/2/h^3/(a*h^2-b*g*h+c*g^2)/(x+g/h)^2*((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+3/4*h/(a*h^
2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*b*d-3/4/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c
*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*e*g-3/2/(a*h^2-b*g*h+c*g^2)^
2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*c*g*d-3/8*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h
-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/
h))*b^2*d+3/8/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*
h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g
/h))*b^2*e*g+1/2/h*c/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+g/h))*d-1/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((
b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x
+g/h))*b*f*g-3/2/h^2/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln
(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
/(x+g/h))*c*g*e+5/2/h^3/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*c*g^2*f+3/4/h/(a*h^2-b*g*h+c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-
2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^2+3/2/h/(a*h^2-b*g*h+
c*g^2)^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)*c*g^2*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                            +c*g^2    positive,
negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(g+hx\right)^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

## 3.233 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

**Optimal**. Leaf size=504

$$\frac{h\sqrt{a+bx+cx^2}\left(8c^2\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2chx\left(-8c^2(9aeh+11afg+3b\right.\right.}{}$$

```
[Out] -1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*g*(f*g^2+3*h*
(d*h+e*g))+24*c^2*h*(a*h*(e*h+3*f*g)+b*(d*h^2+3*e*g*h+3*f*g^2)))*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)+2*(c*(2*a*e-b*(d+a*f/c))-(
-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^3/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/
2)+1/3*(-16*a*c*f+7*b^2*f-6*b*c*e+12*c^2*d)*h*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)
/c^2/(-4*a*c+b^2)+1/24*h*(192*c^4*d*g^2+105*b^4*f*h^2-10*b^2*c*h*(46*a*f*h+
9*b*(e*h+3*f*g))-16*c^3*(3*b*g*(3*d*h+2*e*g)+4*a*(3*d*h^2+9*e*g*h+7*f*g^2))
+8*c^2*(32*a^2*f*h^2+39*a*b*h*(e*h+3*f*g)+b^2*(20*f*g^2+9*h*(d*h+3*e*g)))+2
*c*h*(48*c^3*d*g-35*b^3*f*h-8*c^2*(9*a*e*h+11*a*f*g+3*b*d*h+3*b*e*g)+2*b*c*
(58*a*f*h+15*b*e*h+17*b*f*g))*x)*(c*x^2+b*x+a)^(1/2)/c^4/(-4*a*c+b^2)
```

**Rubi [A]** time = 1.18, antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1644, 832, 779, 621, 206}

$$\frac{h\sqrt{a+bx+cx^2}\left(8c\left(32a^2fh^2+39abh(eh+3fg)+b^2\left(9h(dh+3eg)+20fg^2\right)\right)+2hx\left(-8c^2(9aeh+11afg+3bd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((12*c^2*d - 6*b*c*e +
 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c^2*(b^2 - 4*a
*c)) + (h*(192*c^3*d*g^2 + (105*b^4*f*h^2)/c - 10*b^2*h*(46*a*f*h + 9*b*(3*
f*g + e*h)) - 16*c^2*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*
h^2)) + 8*c*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h*(3
*e*g + d*h))) + 2*h*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*a*f*g +
3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sqrt[a + b*
x + c*x^2])/(24*c^3*(b^2 - 4*a*c)) - ((35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*g +
 b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(3*b*f*g^
```

```
2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(16*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
```

```
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)^2\left(-\right.}{}}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2\right.}{}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^3}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(12c^2d+7b^2\right.}{}$$

**Mathematica [A]** time = 1.59, size = 715, normalized size = 1.42

$$\frac{3\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+x(b+cx)}}\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2af\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*(105*b^5*f*h^3*x + 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h
+ 7*f*h*x)) - 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*
e*g - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) - 16*c^2*(-
16*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) -
3*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
```

```
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
3*h*(4*d*h + 3*e*(4*g + h*x)))) - 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3*d
*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h*x
- 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*g*h*
x - 5*h^2*x^2)))) + 4*b^2*c*(-115*a^2*f*h^3 + a*c*h*(3*h*(18*e*g + 6*d*h +
31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) + c^2*x*(f*(-12*g^3 + 18*g
^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*g^2 +
6*g*h*x + h^2*x^2))))) + 3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*b*f*
g + b*e*h + 2*a*f*h) - 16*c^3*g*(f*g^2 + 3*h*(e*g + d*h)) + 24*c^2*h*(3*b*f
*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*Sqrt[a + x*(b + c*x)]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]])/(48*c^(9/2)*(-b^2 + 4*a*c
)*Sqrt[a + x*(b + c*x)])
```

**fricas [B]** time = 28.67, size = 2937, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*
e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d -
 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^
3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c
^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16
*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a
*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e
+ 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b
*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b
^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(
2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*
c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c
^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c -
24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)
*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^
5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c
^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e
+ (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d
- 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*f)*h^3 + 2*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5
)*e - 7*(b^3*c^3 - 4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h
 + 18*(4*(b^2*c^4 - 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^
2*c^4 - 4*a*c^5)*d - 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c
```

```
^3 + 128*a^2*c^4)*f)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5
)*f)*g^3 - 72*(2*b*c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^
4)*f)*g^2*h + 18*(8*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e +
(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b
*c^4)*d - 6*(15*b^4*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a
*b^3*c^2 + 488*a^2*b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*
a^2*c^6 + (b^2*c^6 - 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x), -1/48*(3*(16*
(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)*e - 3*(a*b^3*
c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d - 12*(a*b^3*c^
2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c^3)*f)*g*h^2 -
 (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3*c
^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3 + (16*(b^2*c^4 - 4
*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3 - 4*a*b*c^4)*f)*g^
2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*c^4)*e + 3*(5*b^4*c^
2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3 - 4*a*b*c^4)*d - 6*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c - 40*a*b^3*c^2 + 48*a
^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*g^3 + 24*(2*(b^3*c^3 -
4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h + 6*(8*(b^3*c^3 - 4*a*b*c
^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b
*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d - 6*(5*b^5*c - 24*a*b^3*c^2
+ 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*f)*h^3)*x)*sqrt
(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) - 2*(8*(b^2*c^4 - 4*a*c^5)*f*h^3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a
*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*
g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 5
2*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 -
 52*a^2*b*c^3)*e + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*f)*h^3 + 2
*(18*(b^2*c^4 - 4*a*c^5)*f*g*h^2 + (6*(b^2*c^4 - 4*a*c^5)*e - 7*(b^3*c^3 -
4*a*b*c^4)*f)*h^3)*x^3 + (72*(b^2*c^4 - 4*a*c^5)*f*g^2*h + 18*(4*(b^2*c^4 -
 4*a*c^5)*e - 5*(b^3*c^3 - 4*a*b*c^4)*f)*g*h^2 + (24*(b^2*c^4 - 4*a*c^5)*d
- 30*(b^3*c^3 - 4*a*b*c^4)*e + (35*b^4*c^2 - 172*a*b^2*c^3 + 128*a^2*c^4)*f
)*h^3)*x^2 - (48*(2*c^6*d - b*c^5*e + (b^2*c^4 - 2*a*c^5)*f)*g^3 - 72*(2*b*
c^5*d - 2*(b^2*c^4 - 2*a*c^5)*e + (3*b^3*c^3 - 10*a*b*c^4)*f)*g^2*h + 18*(8
*(b^2*c^4 - 2*a*c^5)*d - 4*(3*b^3*c^3 - 10*a*b*c^4)*e + (15*b^4*c^2 - 62*a*
b^2*c^3 + 24*a^2*c^4)*f)*g*h^2 - (24*(3*b^3*c^3 - 10*a*b*c^4)*d - 6*(15*b^4
*c^2 - 62*a*b^2*c^3 + 24*a^2*c^4)*e + (105*b^5*c - 530*a*b^3*c^2 + 488*a^2*
b*c^3)*f)*h^3)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^5 - 4*a^2*c^6 + (b^2*c^6
- 4*a*c^7)*x^2 + (b^3*c^5 - 4*a*b*c^6)*x)]
```

**giac [B]** time = 0.35, size = 1054, normalized size = 2.09

$$\left(\left(\left(2\left(\frac{4\left(b^2c^3fh^3-4\,ac^4fh^3\right)x}{b^2c^4-4\,ac^5}+\frac{18\,b^2c^3fgh^2-72\,ac^4fgh^2-7\,b^3c^2fh^3+28\,abc^3fh^3+6\,b^2c^3h^3e-24\,ac^4h^3e}{b^2c^4-4\,ac^5}\right)x+\frac{72\,b^2c^3fg^2h-288\,ac^4fg^2h-90\,b^3c^2fgh^2}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b^
2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 - 7*b^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3 + 6*b
^2*c^3*h^3*e - 24*a*c^4*h^3*e)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^2*h
 - 288*a*c^4*f*g^2*h - 90*b^3*c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^
3*d*h^3 - 96*a*c^4*d*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3 + 128*a^2*c
^3*f*h^3 + 72*b^2*c^3*g*h^2*e - 288*a*c^4*g*h^2*e - 30*b^3*c^2*h^3*e + 120*
a*b*c^3*h^3*e)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 + 48*b^2*c^3*f*g^3 -
96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^
2*h + 144*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*
b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^
3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*c^2*f*h^3 - 48*b*c^4*g^3*
e + 144*b^2*c^3*g^2*h*e - 288*a*c^4*g^2*h*e - 216*b^3*c^2*g*h^2*e + 720*a*b
*c^3*g*h^2*e + 90*b^4*c*h^3*e - 372*a*b^2*c^2*h^3*e + 144*a^2*c^3*h^3*e)/(b
^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 + 48*a*b*c^3*f*g^3 - 288*a*c^4*d*g^2
*h - 216*a*b^2*c^2*f*g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 + 27
0*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a*b^2*c^2*d*h^3 + 192*a^2*c^
3*d*h^3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3 - 96*a*
c^4*g^3*e + 144*a*b*c^3*g^2*h*e - 216*a*b^2*c^2*g*h^2*e + 576*a^2*c^3*g*h^2
*e + 90*a*b^3*c*h^3*e - 312*a^2*b*c^2*h^3*e)/(b^2*c^4 - 4*a*c^5))/sqrt(c*x^
2 + b*x + a) - 1/16*(16*c^3*f*g^3 - 72*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 90*
b^2*c*f*g*h^2 - 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 35*b^3*f*h^3 + 60*a*b*c
*f*h^3 + 48*c^3*g^2*h*e - 72*b*c^2*g*h^2*e + 30*b^2*c*h^3*e - 24*a*c^2*h^3*
e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.02, size = 2780, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^3*f-1/c/(c*x^2+b*x+
a)^(1/2)*g^3*e-3/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*h^3*d-3/2/c^(5/2)*b*ln((c*x+
1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*d+2*a/c^2/(c*x^2+b*x+a)^(1/2)*h^3*d
-35/32*h^3*f/c^5*b^4/(c*x^2+b*x+a)^(1/2)-35/16*h^3*f/c^(9/2)*b^3*ln((c*x+1/
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-8/3*h^3*f*a^2/c^3/(c*x^2+b*x+a)^(1/2)+1/2
*x^3/c/(c*x^2+b*x+a)^(1/2)*h^3*e+15/16/c^4*b^3/(c*x^2+b*x+a)^(1/2)*h^3*e+15
/8/c^(7/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e-3/2*a/c^(5
/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^3*e+1/3*h^3*f*x^4/c/(c*x^
2+b*x+a)^(1/2)+2*g^3*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/c^(3/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*d+3/c^(3/2)*ln((c*x+1/2*b
```

)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*e-3/c/(c*x^2+b*x+a)^(1/2)*g^2*h*d+x^2/
c/(c*x^2+b*x+a)^(1/2)*h^3*d+45/16/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g
*h^2*f-39/4/c^3*b*a/(c*x^2+b*x+a)^(1/2)*g*h^2*f-13/4/c^3*b^3*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)*h^3*e-9/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2
*e-9/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f+2*a/c^2*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*h^3*d+9/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*g*h^2*e+12*a/
c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*f+3/2/c^2*b^3/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*g*h^2*d+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*f-3/2/
c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d-3*b^2/c/(4*a*c-b^2)/(c*x^2+
b*x+a)^(1/2)*g^2*h*d+115/24*h^3*f/c^4*b^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
-15/4*h^3*f/c^3*b*a*x/(c*x^2+b*x+a)^(1/2)-8/3*h^3*f*a^2/c^3*b^2/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)-35/16*h^3*f/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+
9/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*g^2*h*f-15/4/c^2*b*x^2/(c*x^2+b*x+a)^(1/2)*
g*h^2*f-45/8/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*f+15/8/c^3*b^4/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*h^3*e-9/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*g*h^2*e-9/4/c^
3*b^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f-3/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)*h^3*d-9/2/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*e+
3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*g*h^2*f-5/4/c^2*b*x^2/(c*x^2+b*x+a)^(1/2)*h^3
*e-15/8/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)*h^3*e+45/16/c^4*b^3/(c*x^2+b*x+a)^(1/
2)*g*h^2*f+15/16/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^3*e+45/8/c^(7/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-13/4/c^3*b*a/(c*x^
2+b*x+a)^(1/2)*h^3*e+3/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*h^3*e-9/2*a/c^(5/2)*ln
((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h^2*f-9/2/c^(5/2)*b*ln((c*x+1/2
*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g^2*h*f+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g*h^2*
e+6*a/c^2/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g*h^2*e+9
/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*g*h^2*f+3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g^2*h*e-6*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*d+45/8/c^3*b^4/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-13/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*h^3*e-39/4/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h^2*
f+115/12*h^3*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-16/3*h^3*f*a^2/c
^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*g*h^2*e+6*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*h*f+4*a/c*b
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^3*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*g*h^2*d+3/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*h*e-9/2/c^2*b
^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*e-9/2/c^2*b^3/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*g^2*h*f+3*x^2/c/(c*x^2+b*x+a)^(1/2)*g^2*h*f+3/2/c^2*b*x/(c*
x^2+b*x+a)^(1/2)*h^3*d-3*x/c/(c*x^2+b*x+a)^(1/2)*g*h^2*d-3*x/c/(c*x^2+b*x+a
)^(1/2)*g^2*h*e+3/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g*h^2*d+3/2/c^2*b/(c*x^2+b*x+
a)^(1/2)*g^2*h*e+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g^3*f-2*b/(4*a
*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^3*e-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g
^3*e-7/12*h^3*f/c^2*b*x^3/(c*x^2+b*x+a)^(1/2)+35/24*h^3*f/c^3*b^2*x^2/(c*x^
2+b*x+a)^(1/2)+35/16*h^3*f/c^4*b^3*x/(c*x^2+b*x+a)^(1/2)-35/32*h^3*f/c^5*b^
6/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+115/24*h^3*f/c^4*b^2*a/(c*x^2+b*x+a)^(1/2
)+15/4*h^3*f/c^(7/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/3*h^
3*f*a/c^2*x^2/(c*x^2+b*x+a)^(1/2)+12*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*

```
x*g*h^2*e-39/2/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h^2*f-x/c/(c*x
^2+b*x+a)^(1/2)*g^3*f+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*g^3*f
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g+h\,x\right)}^3\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)^{3} \left(d + ex + fx^{2}\right)}{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

# 3.234 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=289

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2\ldots\right.}{8c^{7/\ldots}}$$

```
[Out] 1/8*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g)))
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+2*(c*(2*a*e-b*(
d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*x+g)^2/c/(-4*a*c+b^2)/(c*x^2
+b*x+a)^(1/2)+1/4*h*(32*c^3*d*g-15*b^3*f*h-8*c^2*(4*a*e*h+8*a*f*g+b*d*h+2*b
*e*g)+4*b*c*(13*a*f*h+3*b*e*h+6*b*f*g)+2*c*(-12*a*c*f+5*b^2*f-4*b*c*e+8*c^2
*d)*h*x)*(c*x^2+b*x+a)^(1/2)/c^3/(-4*a*c+b^2)
```

**Rubi [A]** time = 0.39, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1644, 779, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{7/2}}+\frac{2(g+hx)^2\left(c\left(2ae-b\ldots\right.\right.}{c\left(\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (h*(32*c^2*d*g - (15*b
^3*f*h)/c - 8*c*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*(6*b*f*g + 3*b*
e*h + 13*a*f*h) + 2*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sqrt[a +
b*x + c*x^2])/(4*c^2*(b^2 - 4*a*c)) + ((15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*
e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{(g+hx)\left(-\frac{b^{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)^2}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{h\left(32c^2dg-\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.83, size = 412, normalized size = 1.43

$$\frac{2\sqrt{c}\left(4bc\left(-13a^2fh^2+ac\left(2h(dh+2eg+5ehx)+f\left(2g^2+20ghx-5h^2x^2\right)\right)+2c^2g(d(g-2hx)-egx)\right)+8c^2\left(a^2h\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*f*
h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*(2*
h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*c*(a
*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f*(-4*
g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*e*h +
3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) + f*x*(-
2*g^2 + 4*g*h*x + h^2*x^2)))) - (b^2 - 4*a*c)*(15*b^2*f*h^2 - 12*c*h*(2*b*f
*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*Sqrt[a + x*(b + c*x)
]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(7/2)*(-b^2
+ 4*a*c)*Sqrt[a + x*(b + c*x)])
```

**fricas [B]** time = 20.32, size = 1769, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e -
 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^
3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 + (8*
(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b
*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*
f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*h + (8*
(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*
c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*
f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4*d - 2*a*b*
c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*a*b^2*c^2 -
8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*
f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)*h^2)*x^2 - (8
*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^4*d - 2*(b^2*c^
3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 2*a*c^4)*d
 - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f)
*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6
)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 +
8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b
^2*c^2 - 4*a^2*c^3)*d - 12*(a*b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*
b^2*c + 16*a^3*c^2)*f)*h^2 + (8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 -
 4*a*c^4)*e - 3*(b^3*c^2 - 4*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 1
2*(b^3*c^2 - 4*a*b*c^3)*e + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)
*x^2 + (8*(b^3*c^2 - 4*a*b*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b
^4*c - 4*a*b^2*c^2)*f)*g*h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b
^2*c^2)*e + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(-c)*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) -
 2*(2*(b^2*c^3 - 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g
^2 + 8*(4*a*c^4*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b
*c^3*d - 4*(3*a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2
 + (8*(b^2*c^3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4
*a*b*c^3)*f)*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2
- 8*(2*b*c^4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h
+ (8*(b^2*c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*
a*b^2*c^2 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^
2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x)]
```

**giac [B]** time = 0.32, size = 580, normalized size = 2.01

$$\left(\left(\frac{2\left(b^2c^2fh^2-4\,ac^3fh^2\right)x}{b^2c^3-4\,ac^4}+\frac{8\,b^2c^2fgh-32\,ac^3fgh-5\,b^3cfh^2+20\,abc^2fh^2+4\,b^2c^2h^2e-16\,ac^3h^2e}{b^2c^3-4\,ac^4}\right)x-\frac{16\,c^4dg^2+8\,b^2c^2fg^2-16\,ac^3fg^2-16\,bc^3dgh-2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^2
*f*g*h - 32*a*c^3*f*g*h - 5*b^3*c*f*h^2 + 20*a*b*c^2*f*h^2 + 4*b^2*c^2*h^2*
e - 16*a*c^3*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 + 8*b^2*c^2*f*g^
2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8
*b^2*c^2*d*h^2 - 16*a*c^3*d*h^2 + 15*b^4*f*h^2 - 62*a*b^2*c*f*h^2 + 24*a^2*
c^2*f*h^2 - 8*b*c^3*g^2*e + 16*b^2*c^2*g*h*e - 32*a*c^3*g*h*e - 12*b^3*c*h^
2*e + 40*a*b*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 + 8*a*b*c^2
*f*g^2 - 32*a*c^3*d*g*h - 24*a*b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d
*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2 - 16*a*c^3*g^2*e + 16*a*b*c^2*g*h*
e - 12*a*b^2*c*h^2*e + 32*a^2*c^2*h^2*e)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 +
b*x + a) - 1/8*(8*c^2*f*g^2 - 24*b*c*f*g*h + 8*c^2*d*h^2 + 15*b^2*f*h^2 - 1
2*a*c*f*h^2 + 16*c^2*g*h*e - 12*b*c*h^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

**maple [B]** time = 0.01, size = 1557, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -13/2*h^2*f/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/c^(3/2)*ln((c*x+1
/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*h^2+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*f*g^2-1/c/(c*x^2+b*x+a)^(1/2)*g^2*e+4*a/c*b/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*h^2*e+4*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*g*
h*f+2/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g*h-3/c^2*b^3/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*x*g*h*f+8*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h*f+
2*g^2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+15/16*h^2*f/c^4*b^3/(c*x^
2+b*x+a)^(1/2)+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f*g^2-5/4*h^2*f/c^
2*b*x^2/(c*x^2+b*x+a)^(1/2)-15/8*h^2*f/c^3*b^2*x/(c*x^2+b*x+a)^(1/2)+15/16*
h^2*f/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-13/4*h^2*f/c^3*b*a/(c*x^2+b*x
+a)^(1/2)+3/2*h^2*f*a/c^2*x/(c*x^2+b*x+a)^(1/2)+2*x^2/c/(c*x^2+b*x+a)^(1/2)
*g*h*f+3/2/c^2*b*x/(c*x^2+b*x+a)^(1/2)*h^2*e-3/2/c^3*b^2/(c*x^2+b*x+a)^(1/2
)*g*h*f-3/4/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*h^2*e-3/c^(5/2)*b*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*g*h*f+4*a/c^2/(c*x^2+b*x+a)^(1/2)*g*
h*f-2*x/c/(c*x^2+b*x+a)^(1/2)*e*g*h+1/c^2*b/(c*x^2+b*x+a)^(1/2)*e*g*h+1/2/c
^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*h^2+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*f*g^2-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g^2*e-b^2/c/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*g^2*e+1/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*
e*g*h-3/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*h^2*e-3/2/c^3*b^4/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*f+2*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
```

```
)*h^2*e+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*h^2+3/c^2*b*x/(c*x^2+b*
x+a)^(1/2)*g*h*f-4*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*g*h*d-2*b^2/c/(4*a*c
-b^2)/(c*x^2+b*x+a)^(1/2)*g*h*d+15/8*h^2*f/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*x-13/4*h^2*f/c^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+15/8*h^2*f/c
^(7/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*h^2*f*a/c^(5/2)*
ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-x/c/(c*x^2+b*x+a)^(1/2)*d*h^2-x
/c/(c*x^2+b*x+a)^(1/2)*f*g^2+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*d*h^2+1/2/c^2*b/
(c*x^2+b*x+a)^(1/2)*f*g^2+2/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*e*g*h-2/c/(c*x^2+b*x+a)^(1/2)*g*h*d+1/2*h^2*f*x^3/c/(c*x^2+b*x+a)^(1/2
)+x^2/c/(c*x^2+b*x+a)^(1/2)*h^2*e-3/4/c^3*b^2/(c*x^2+b*x+a)^(1/2)*h^2*e-3/2
/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*h^2*e+2*a/c^2/(c*x^2
+b*x+a)^(1/2)*h^2*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g+h\,x\right)^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(g + hx\right)^{2} \left(d + ex + fx^{2}\right)}{\left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
[Out] Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

# 3.235 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=186

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}$$

```
[Out] -1/2*(3*b*f*h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(5/2)+2*(c*(2*a*e-b*(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)*(h*
x+g)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*h
*(c*x^2+b*x+a)^(1/2)/c^2/(-4*a*c+b^2)
```

**Rubi [A]** time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1644, 640, 621, 206}

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{h\sqrt{a+bx+cx^2}\left(-8acf+3b^2f-2bce+4c^2d\right)}{c^2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(g
 + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((4*c^2*d - 2*b*c*e + 3*
b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/(c^2*(b^2 - 4*a*c)) - ((3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{b^2fg+2b(cd+a\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\cdots\right.}{\cdots}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)(g+hx)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}} + \frac{\left(4c^2d+3b^2f\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.73, size = 205, normalized size = 1.10

$$\frac{\frac{2\sqrt{c}\left(4c\left(2a^2fh-ac(dh+e(g+hx)+fx(g-hx))+c^2dgx\right)+b^2(cx(2eh+2fg-fhx)-3afh)+2bc(aeh+af(g+5hx)+cd(g-hx)-cegx)-3b^3fhx\right)}{\sqrt{a+x(b+cx)}}+\left(b^2-4ac\right)}{2c^{5/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g
 + 5*h*x)) + b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h
+ c^2*d*g*x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x)))))/Sqrt[a + x*(b + c*
x)] + (b^2 - 4*a*c)*(3*b*f*h - 2*c*(f*g + e*h))*Log[b + 2*c*x + 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]])/(2*c^(5/2)*(-b^2 + 4*a*c))
```

**fricas [B]** time = 14.45, size = 905, normalized size = 4.87

$$\left[-\frac{\left(2\left(ab^2c-4a^2c^2\right)fg+\left(2\left(b^2c^2-4ac^3\right)fg+\left(2\left(b^2c^2-4ac^3\right)e-3\left(b^3c-4abc^2\right)f\right)h\right)x^2+\left(2\left(ab^2c-4a^2c^2\right)e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*
c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^
2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c
- 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*
c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((b^2*
c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d
 - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^
2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 1
0*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^
4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c - 4*a^2*c^2)
*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4
*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)
*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b
^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-
c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d -
 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c
^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d -
2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x
```

```
+ a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
^4)*x)]
```

**giac [A]** time = 0.28, size = 271, normalized size = 1.46

$$\frac{\left(\frac{\left(b^2cfh-4\,ac^2fh\right)x}{b^2c^2-4\,ac^3}-\frac{4\,c^3dg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh-3\,b^3fh+10\,abcfh-2\,bc^2ge+2\,b^2che-4\,ac^2he}{b^2c^2-4\,ac^3}\right)x-\frac{2\,bc^2dg+2\,abcfg-4\,ac^2dh-3\,ab^2fh+8\,a^2cf}{b^2c^2-4\,ac^3}}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] (((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g + 2*b^2*c*f*
g - 4*a*c^2*f*g - 2*b*c^2*d*h - 3*b^3*f*h + 10*a*b*c*f*h - 2*b*c^2*g*e + 2*
b^2*c*h*e - 4*a*c^2*h*e)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g + 2*a*b*c*f*
g - 4*a*c^2*d*h - 3*a*b^2*f*h + 8*a^2*c*f*h - 4*a*c^2*g*e + 2*a*b*c*h*e)/(b
^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g - 3*b*f*h + 2*c*h*e
)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

**maple [B]** time = 0.01, size = 735, normalized size = 3.95

$$\frac{4abfhx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{3b^3fhx}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}+\frac{b^2ehx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}+\frac{b^2fgx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*g-1/c/(c*x^2+b*x+a)
^(1/2)*d*h-1/c/(c*x^2+b*x+a)^(1/2)*e*g+1/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*e*h-3/2*f*h/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2*f
*h*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f*h*a/c*b/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x+1/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*h+1/c*b^2/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f*g-x/c/(c*x^2+b*x+a)^(1/2)*e*h+2*f*h*a/c^2/(c
*x^2+b*x+a)^(1/2)-3/2*f*h/c^(5/2)*b*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))+f*h*x^2/c/(c*x^2+b*x+a)^(1/2)-3/4*f*h/c^3*b^2/(c*x^2+b*x+a)^(1/2)+2*d*
g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*f
*g-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e*g-2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*d*h+1/2/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*g+1/2/c^2*b^3/(4*
a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*h+3/2*f*h/c^2*b*x/(c*x^2+b*x+a)^(1/2)-3/4*f*
h/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*d*h-b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e*g-x/c/(c*x^2+b*x+a)^(1/2)*
f*g+1/2/c^2*b/(c*x^2+b*x+a)^(1/2)*e*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g + h\,x)\,\left(f\,x^2 + e\,x + d\right)}{\left(c\,x^2 + b\,x + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g + hx)\left(d + ex + fx^2\right)}{\left(a + bx + cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

# 3.236 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)+2*(c*(2*a*e-b*
(d+a*f/c))-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c*x^2+b*x+a)^(
1/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \right.}{c}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

**Mathematica [A]** time = 0.30, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

**fricas [B]** time = 1.37, size = 429, normalized size = 3.86

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)fx^2+\left(b^3-4\,abc\right)fx+\left(ab^2-4\,a^2c\right)f\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{\right.}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

**giac [A]** time = 0.27, size = 122, normalized size = 1.10

$$-\frac{2\left(\frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

**maple [B]** time = 0.01, size = 249, normalized size = 2.24

$$\frac{b^2fx}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c}-\frac{2bex}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3f}{2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}\;c^2}-\frac{b^2e}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f/c^2*b/(c*x^2+b*x+a)^(1/2)+f/c*b^2/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [B]** time = 4.54, size = 143, normalized size = 1.29

$$\frac{f\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}}-\frac{e\,\left(4\,a+2\,b\,x\right)}{\left(4\,a\,c-b^2\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}+\frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

```
[Out] (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c - b
^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c -
b^2/4)*(a + b*x + c*x^2)^(1/2))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

## 3.237 $\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=225

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2d\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

```
[Out] (f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*
g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)+2*(b^2*d*h-b*(a*e
*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)-(2*c^2*d*g+b*f*(-a*h+b*g)-c*(-2*a*e
*h+2*a*f*g+b*d*h+b*e*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^2+b*x+a)^
(1/2)
```

**Rubi [A]** time = 0.27, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+b^2d\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2*
c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((b^
2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*(e*
g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a
*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

$$= \frac{2\left(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)-\left(2c^2dg+bf(bg-ah)\right.\right.}{\left(b^2-4ac\right)\left(cg^2-bgh+ah^2\right)\sqrt{a+bx+cx^2}}$$

**Mathematica [A]** time = 0.49, size = 271, normalized size = 1.20

$$\frac{\frac{-b^2\left(afh^2+2cdh^2+cfg(g-2hx)\right)-2bch(-aeh+af(g+hx)+c(-dg+dhx+egx))+4c^2(ah(dh-eg+ehx)+afg(g-hx)+cdghx)+b^3fgh}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(h(bg-ah)-cg^2\right)}-\frac{ch\left(h(dh-eg)+fg^2\right)\tanh}{\left(h(ah-\right.}}{ch}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/Sqrt[a + x*(b + c*x)]) + (b^3*f*g*h - b^2*(2*c*d*h^2 + a*f*h^2 + c*f*g
*(g - 2*h*x)) - 2*b*c*h*(-(a*e*h) + a*f*(g + h*x) + c*(-(d*g) + e*g*x + d*h
*x)) + 4*c^2*(c*d*g*h*x + a*f*g*(g - h*x) + a*h*(-(e*g) + d*h + e*h*x)))/((
b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (c*h*(f*g^
2 + h*(-(e*g) + d*h))*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*
g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))
^(3/2))/(c*h)
```

**fricas [B]** time = 34.40, size = 1905, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c
)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a
*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 -
4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 -
 (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sq
rt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h
)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*
x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c
- a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a
*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 +
((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*
c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b
^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 +
 b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h +
 (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3
+ (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c
^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^
4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2
```

```
 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x), (((a*b^2
- 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*c)*d*h^2 + ((b^
2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4*a*c^2)*d*h^2)*x
^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3 - 4*a*b*c)*d*h^2
)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*arctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*
sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h +
a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)
*x)) - 2*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2*(b^2*c - a*c
^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*c)*d - (a*b^2
+ 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)*d)*h^3 + ((2*c
^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b^2*c + 2*a*c^2)*
e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2)*d - 2*(a*b^2 -
a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)*sqrt(c*x^2 + b*x
+ a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*g^3*h + (a*b
^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*a^3*b*c)*g*h^3 + (a^
3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b^3*c^2 - 4*a*b*c^3)*g
^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*(a*b^3*c - 4*a^2*b*c^2
)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c^2 - 4*a*b*c^3)*g^4 - 2
*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*g^2*h^2 - 2*
(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*b*c)*h^4)*x)]
```

**giac [B]** time = 0.29, size = 719, normalized size = 3.20

$$-\frac{2\left(\frac{\left(2\,c^3dg^3+b^2cfg^3-2\,ac^2fg^3-3\,bc^2dg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2\,ac^2dgh^2+2\,ab^2fgh^2-2\,a^2cfgh^2-abcdh^3-a^2bfh^3-bc^2g^3e+b^2cg^2he+2\,ac^2g^2}{b^2c^2g^4-4\,ac^3g^4-2\,b^3cg^3h+8\,abc^2g^3h+b^4g^2h^2-2\,ab^2cg^2h^2-8\,a^2c^2g^2h^2-2\,ab^3gh^3+8\,a^2bcgh^3+a^2b^2h^4-4\,a^3ch^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^3*d*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g^2*h - b^3*f*g^
2*h + a*b*c*f*g^2*h + b^2*c*d*g*h^2 + 2*a*c^2*d*g*h^2 + 2*a*b^2*f*g*h^2 - 2
*a^2*c*f*g*h^2 - a*b*c*d*h^3 - a^2*b*f*h^3 - b*c^2*g^3*e + b^2*c*g^2*h*e +
2*a*c^2*g^2*h*e - 3*a*b*c*g*h^2*e + 2*a^2*c*h^3*e)*x/(b^2*c^2*g^4 - 4*a*c^3
*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2 -
8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3*c
*h^4) + (b*c^2*d*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h + 2*a*c^2*d*g^2*h - a*
b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b^3*d*g*h^2 - a*b*c*d*g*h^2 + 3*a^2*b*f*g*h
^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 - 2*a^3*f*h^3 - 2*a*c^2*g^3*e + 3*a*b*c*g^
2*h*e - a*b^2*g*h^2*e - 2*a^2*c*g*h^2*e + a^2*b*h^3*e)/(b^2*c^2*g^4 - 4*a*c
^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h^2
- 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4*a^3
*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 + d*h^2 - g*h*e)*arctan(-((sqrt(c
```

```
)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/(
(c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

**maple [B]** time = 0.01, size = 2079, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -4/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g^2*e+4/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^
2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*
g^3*f-2*h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/
h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*d+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/
((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*e*g
-2/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g^2*e+2/h^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g^3*
f-1/h*f/c/(c*x^2+b*x+a)^(1/2)+h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g
)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*d-1/(a*h^2-b*g*h+c*g^2)/((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-h/(a*h^2-b*g*
h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*b^2*d+1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g
)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*e*g-1/h/(a*h^2-b*g*h+c*g^2)/
((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*
g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)
/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g^2-2/h*f*b/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*x-1/h*f*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+4/h*e/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x*c-2/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-2/h
/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*f*g^2+2/h*e/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b
+1/h/(a*h^2-b*g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*f*g^2-h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*d+1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b
*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+
g/h))*e*g+2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h
)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c*g*d-4/h^2*f*g/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*x*c+4/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x
+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^2*g*d-1/h/(a*h^2-b*g*h+c*g^2)/(4
*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*b^2*f*g^2
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{\left(g+h\,x\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex + fx^{2}}{\left(g + hx\right) \left(a + bx + cx^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

# 3.238 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=421

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2f\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\ (}$$

```
[Out] 1/2*(2*c*g*(f*g^2-h*(-3*d*h+2*e*g))-h*(2*a*h*(-e*h+2*f*g)-b*(-3*d*h^2+e*g*h
+f*g^2)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/
(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)-2*(b^3*d*h^2-b^2*h*(a*e*h+2*
c*d*g)-2*a*c*(c*g*(-2*d*h+e*g)+a*h*(-e*h+2*f*g))+b*(c^2*d*g^2+a^2*f*h^2+a*c
*(-3*d*h^2+2*e*g*h+f*g^2))+c*(2*c^2*d*g^2+2*a^2*f*h^2-a*b*h*(e*h+2*f*g)+b^2
*(d*h^2+f*g^2)-c*(b*g*(2*d*h+e*g)+2*a*(d*h^2-2*e*g*h+f*g^2)))*x)/(-4*a*c+b^
2)/(a*h^2-b*g*h+c*g^2)^2/(c*x^2+b*x+a)^(1/2)-h*(f*g^2-h*(-d*h+e*g))*(c*x^2+
b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)
```

**Rubi [A]** time = 0.80, antiderivative size = 418, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2f\right.\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\ (}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*(
2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2))
 + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h^2)
 - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4*a*c
)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) - (h*(f*g^2 - h*(e*g - d
*h))*Sqrt[a + b*x + c*x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)) + ((2*c*(
f*g^3 - g*h*(2*e*g - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*
g - e*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2]])])/(2*(c*g^2 - b*g*h + a*h^2)^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\right.\right.}{}$$

$$= -\frac{2\left(b^3dh^2-b^2h(2cdg+aeh)-2ac(cg(eg-2dh)+ah(2fg-eh))+b\left(c^2dg^2+\right.\right.}{}$$

**Mathematica [A]** time = 2.46, size = 487, normalized size = 1.16

$$\frac{ch\left(\frac{\left(4ac-b^2\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h\left(2ah(eh-2fg)+bh(eg-3dh)+bfg^2\right)+2c\left(gh(3dh-2eg)+fg^3\right)\right)}{\left(h(ah-bg)+cg^2\right)^{5/2}}-\frac{2h\sqrt{a+x(b+cx)}\left(4a^2fh^2-2c\left(2ah(2dh-3eg)+4afg^2+bg(2dh+eg\right.\right.}{(g+hx)\left(h(ah-bg)+\right.}\right.}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/((g + h*x)*Sqrt[a + x*(b + c*x)])) + (b^3*f*g*h - b^2*(4*c*d*h^2 + a*f
*h^2 + c*f*g*(g - 4*h*x)) - 4*b*c*h*(a*h*(-e + f*x) + c*(-(d*g) + e*g*x + d
*h*x)) + 4*c*(-(a^2*f*h^2) + 2*c^2*d*g*h*x + a*c*(f*g*(g - 2*h*x) + 2*h*(-(
e*g) + d*h + e*h*x))))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*
Sqrt[a + x*(b + c*x)]) + (c*h*((-2*h*(4*c^2*d*g^2 + 4*a^2*f*h^2 - 2*a*b*h*(
2*f*g + e*h) - 2*c*(4*a*f*g^2 + 2*a*h*(-3*e*g + 2*d*h) + b*g*(e*g + 2*d*h))
 + b^2*(3*f*g^2 + h*(-(e*g) + 3*d*h)))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(
-(b*g) + a*h))^2*(g + h*x)) + ((-b^2 + 4*a*c)*(2*c*(f*g^3 + g*h*(-2*e*g + 3
*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) + 2*a*h*(-2*f*g + e*h)))*ArcTanh[(-
(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a +
 x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(5/2)))/(b^2 - 4*a*c))/(2*c*h)
```

**fricas [B]** time = 107.70, size = 5098, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g^4 - (4*(a*b^2*c - 4*a^2*c^2)*e - (a*b^3
- 4*a^2*b*c)*f)*g^3*h + (6*(a*b^2*c - 4*a^2*c^2)*d + (a*b^3 - 4*a^2*b*c)*e
- 4*(a^2*b^2 - 4*a^3*c)*f)*g^2*h^2 - (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2
- 4*a^3*c)*e)*g*h^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h - (4*(b^2*c^2 - 4*a*c^
3)*e - (b^3*c - 4*a*b*c^2)*f)*g^2*h^2 + (6*(b^2*c^2 - 4*a*c^3)*d + (b^3*c -
 4*a*b*c^2)*e - 4*(a*b^2*c - 4*a^2*c^2)*f)*g*h^3 - (3*(b^3*c - 4*a*b*c^2)*d
 - 2*(a*b^2*c - 4*a^2*c^2)*e)*h^4)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 - (4*
(b^2*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*g^3*h + (6*(b^2*c^2 - 4*a*
c^3)*d - 3*(b^3*c - 4*a*b*c^2)*e + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*f)*g^2*h^
2 + (3*(b^3*c - 4*a*b*c^2)*d + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*e - 4*(a*b^3 -
 4*a^2*b*c)*f)*g*h^3 - (3*(b^4 - 4*a*b^2*c)*d - 2*(a*b^3 - 4*a^2*b*c)*e)*h^
4)*x^2 + (2*(b^3*c - 4*a*b*c^2)*f*g^4 - (4*(b^3*c - 4*a*b*c^2)*e - (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*f)*g^3*h + (6*(b^3*c - 4*a*b*c^2)*d + (b^4 - 8*a*b^2*
c + 16*a^2*c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*d - 3*(a*b^3 - 4*a^2*b*c)*e + 4*(a^2*b^2 - 4*a^3*c)*f)*g*h^3 -
 (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*h^4)*x)*sqrt(c*g^2 - b
*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 -
 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*
x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 -
 (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2)) - 4*((a^2*b^2 - 4*a^3*c
)*d*h^5 + 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g^5 + (8*a*b*c^2*e - 2*(3*b^2
*c^2 - 4*a*c^3)*d - (a*b^2*c + 12*a^2*c^2)*f)*g^4*h + (6*(b^3*c - 2*a*b*c^2
)*d - (7*a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 16*a^2*b*c)*f)*g^3*h^2 - ((2*b^4
 - 3*a*b^2*c - 4*a^2*c^2)*d - (3*a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 + 12*a^3*c
)*f)*g^2*h^3 + (2*a^3*b*f + (a*b^3 - 2*a^2*b*c)*d - (3*a^2*b^2 - 8*a^3*c)*e
)*g*h^4 + ((4*c^4*d - 2*b*c^3*e + (3*b^2*c^2 - 8*a*c^3)*f)*g^4*h - (8*b*c^3
*d - (b^2*c^2 + 12*a*c^3)*e + (3*b^3*c - 4*a*b*c^2)*f)*g^3*h^2 + ((7*b^2*c^
2 - 4*a*c^3)*d + (b^3*c - 16*a*b*c^2)*e + (7*a*b^2*c - 4*a^2*c^2)*f)*g^2*h^
3 - (8*a^2*b*c*f + (3*b^3*c - 4*a*b*c^2)*d - (a*b^2*c + 12*a^2*c^2)*e)*g*h^
4 - (2*a^2*b*c*e - 4*a^3*c*f - (3*a*b^2*c - 8*a^2*c^2)*d)*h^5)*x^2 + (2*(2*
c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g^5 - (6*b*c^3*d - 2*(b^2*c^2 + 2*
a*c^3)*e + (b^3*c + 2*a*b*c^2)*f)*g^4*h + (8*a*c^3*d - b^3*c*e - (b^4 - 8*a
*b^2*c + 8*a^2*c^2)*f)*g^3*h^2 + (a*b^3*f + (5*b^3*c - 16*a*b*c^2)*d + (b^4
 - 8*a*b^2*c + 8*a^2*c^2)*e)*g^2*h^3 - ((3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d -
 (a*b^3 + 2*a^2*b*c)*e + 2*(a^2*b^2 + 2*a^3*c)*f)*g*h^4 + (2*a^3*b*f + (3*a
*b^3 - 10*a^2*b*c)*d - 2*(a^2*b^2 - 2*a^3*c)*e)*h^5)*x)*sqrt(c*x^2 + b*x +
```

a))/((a*b^2*c^3 - 4*a^2*c^4)*g^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*g^6*h + 3*(a
*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*g^5*h^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3
*b*c^2)*g^4*h^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*g^3*h^4 - 3*(a^3*b^
3 - 4*a^4*b*c)*g^2*h^5 + (a^4*b^2 - 4*a^5*c)*g*h^6 + ((b^2*c^4 - 4*a*c^5)*g
^6*h - 3*(b^3*c^3 - 4*a*b*c^4)*g^5*h^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c
^4)*g^4*h^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*g^3*h^4 + 3*(a*b^4*c - 3
*a^2*b^2*c^2 - 4*a^3*c^3)*g^2*h^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*g*h^6 + (a^
3*b^2*c - 4*a^4*c^2)*h^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*g^7 - 2*(b^3*c^3 - 4*a
*b*c^4)*g^6*h + 3*(a*b^2*c^3 - 4*a^2*c^4)*g^5*h^2 + (2*b^5*c - 11*a*b^3*c^2
 + 12*a^2*b*c^3)*g^4*h^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^
3*h^4 + 3*(a*b^5 - 4*a^2*b^3*c)*g^2*h^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4
*c^2)*g*h^6 + (a^3*b^3 - 4*a^4*b*c)*h^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*g^7 -
 (3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*g^6*h + 3*(b^5*c - 4*a*b^3*c^2)*g^5
*h^2 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^4*h^3 + (2*a*b^5 - 1
1*a^2*b^3*c + 12*a^3*b*c^2)*g^3*h^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*g^2*h^5 - 2
*(a^3*b^3 - 4*a^4*b*c)*g*h^6 + (a^4*b^2 - 4*a^5*c)*h^7)*x), 1/2*((2*(a*b^2*
c - 4*a^2*c^2)*f*g^4 - (4*(a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 4*a^2*b*c)*f)*
g^3*h + (6*(a*b^2*c - 4*a^2*c^2)*d + (a*b^3 - 4*a^2*b*c)*e - 4*(a^2*b^2 - 4
*a^3*c)*f)*g^2*h^2 - (3*(a*b^3 - 4*a^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*g*
h^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^3*h - (4*(b^2*c^2 - 4*a*c^3)*e - (b^3*c -
4*a*b*c^2)*f)*g^2*h^2 + (6*(b^2*c^2 - 4*a*c^3)*d + (b^3*c - 4*a*b*c^2)*e -
4*(a*b^2*c - 4*a^2*c^2)*f)*g*h^3 - (3*(b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c -
4*a^2*c^2)*e)*h^4)*x^3 + (2*(b^2*c^2 - 4*a*c^3)*f*g^4 - (4*(b^2*c^2 - 4*a*c
^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*g^3*h + (6*(b^2*c^2 - 4*a*c^3)*d - 3*(b^3*
c - 4*a*b*c^2)*e + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*f)*g^2*h^2 + (3*(b^3*c -
4*a*b*c^2)*d + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*e - 4*(a*b^3 - 4*a^2*b*c)*f)*g
*h^3 - (3*(b^4 - 4*a*b^2*c)*d - 2*(a*b^3 - 4*a^2*b*c)*e)*h^4)*x^2 + (2*(b^3
*c - 4*a*b*c^2)*f*g^4 - (4*(b^3*c - 4*a*b*c^2)*e - (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*f)*g^3*h + (6*(b^3*c - 4*a*b*c^2)*d + (b^4 - 8*a*b^2*c + 16*a^2*c^2)*
e - 3*(a*b^3 - 4*a^2*b*c)*f)*g^2*h^2 - (3*(b^4 - 6*a*b^2*c + 8*a^2*c^2)*d -
 3*(a*b^3 - 4*a^2*b*c)*e + 4*(a^2*b^2 - 4*a^3*c)*f)*g*h^3 - (3*(a*b^3 - 4*a
^2*b*c)*d - 2*(a^2*b^2 - 4*a^3*c)*e)*h^4)*x)*sqrt(-c*g^2 + b*g*h - a*h^2)*a
rctan(-1/2*sqrt(-c*g^2 + b*g*h - a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h
 + (2*c*g - b*h)*x)/(a*c*g^2 - a*b*g*h + a^2*h^2 + (c^2*g^2 - b*c*g*h + a*c*
h^2)*x^2 + (b*c*g^2 - b^2*g*h + a*b*h^2)*x)) - 2*((a^2*b^2 - 4*a^3*c)*d*h^5
 + 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g^5 + (8*a*b*c^2*e - 2*(3*b^2*c^2 -
4*a*c^3)*d - (a*b^2*c + 12*a^2*c^2)*f)*g^4*h + (6*(b^3*c - 2*a*b*c^2)*d - (
7*a*b^2*c - 4*a^2*c^2)*e - (a*b^3 - 16*a^2*b*c)*f)*g^3*h^2 - ((2*b^4 - 3*a*
b^2*c - 4*a^2*c^2)*d - (3*a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 + 12*a^3*c)*f)*g^
2*h^3 + (2*a^3*b*f + (a*b^3 - 2*a^2*b*c)*d - (3*a^2*b^2 - 8*a^3*c)*e)*g*h^4
 + ((4*c^4*d - 2*b*c^3*e + (3*b^2*c^2 - 8*a*c^3)*f)*g^4*h - (8*b*c^3*d - (b
^2*c^2 + 12*a*c^3)*e + (3*b^3*c - 4*a*b*c^2)*f)*g^3*h^2 + ((7*b^2*c^2 - 4*a
*c^3)*d + (b^3*c - 16*a*b*c^2)*e + (7*a*b^2*c - 4*a^2*c^2)*f)*g^2*h^3 - (8*
a^2*b*c*f + (3*b^3*c - 4*a*b*c^2)*d - (a*b^2*c + 12*a^2*c^2)*e)*g*h^4 - (2*
a^2*b*c*e - 4*a^3*c*f - (3*a*b^2*c - 8*a^2*c^2)*d)*h^5)*x^2 + (2*(2*c^4*d -

```
 b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*g^5 - (6*b*c^3*d - 2*(b^2*c^2 + 2*a*c^3)*
e + (b^3*c + 2*a*b*c^2)*f)*g^4*h + (8*a*c^3*d - b^3*c*e - (b^4 - 8*a*b^2*c
+ 8*a^2*c^2)*f)*g^3*h^2 + (a*b^3*f + (5*b^3*c - 16*a*b*c^2)*d + (b^4 - 8*a*
b^2*c + 8*a^2*c^2)*e)*g^2*h^3 - ((3*b^4 - 8*a*b^2*c - 4*a^2*c^2)*d - (a*b^3
 + 2*a^2*b*c)*e + 2*(a^2*b^2 + 2*a^3*c)*f)*g*h^4 + (2*a^3*b*f + (3*a*b^3 -
10*a^2*b*c)*d - 2*(a^2*b^2 - 2*a^3*c)*e)*h^5)*x)*sqrt(c*x^2 + b*x + a))/((a
*b^2*c^3 - 4*a^2*c^4)*g^7 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*g^6*h + 3*(a*b^4*c
- 3*a^2*b^2*c^2 - 4*a^3*c^3)*g^5*h^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)
*g^4*h^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*g^3*h^4 - 3*(a^3*b^3 - 4*a
^4*b*c)*g^2*h^5 + (a^4*b^2 - 4*a^5*c)*g*h^6 + ((b^2*c^4 - 4*a*c^5)*g^6*h -
3*(b^3*c^3 - 4*a*b*c^4)*g^5*h^2 + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*g^4
*h^3 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*g^3*h^4 + 3*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*g^2*h^5 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*g*h^6 + (a^3*b^2*c
 - 4*a^4*c^2)*h^7)*x^3 + ((b^2*c^4 - 4*a*c^5)*g^7 - 2*(b^3*c^3 - 4*a*b*c^4)
*g^6*h + 3*(a*b^2*c^3 - 4*a^2*c^4)*g^5*h^2 + (2*b^5*c - 11*a*b^3*c^2 + 12*a
^2*b*c^3)*g^4*h^3 - (b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^3*h^4 +
 3*(a*b^5 - 4*a^2*b^3*c)*g^2*h^5 - (3*a^2*b^4 - 13*a^3*b^2*c + 4*a^4*c^2)*g
*h^6 + (a^3*b^3 - 4*a^4*b*c)*h^7)*x^2 + ((b^3*c^3 - 4*a*b*c^4)*g^7 - (3*b^4
*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*g^6*h + 3*(b^5*c - 4*a*b^3*c^2)*g^5*h^2 -
(b^6 - a*b^4*c - 15*a^2*b^2*c^2 + 12*a^3*c^3)*g^4*h^3 + (2*a*b^5 - 11*a^2*b
^3*c + 12*a^3*b*c^2)*g^3*h^4 + 3*(a^3*b^2*c - 4*a^4*c^2)*g^2*h^5 - 2*(a^3*b
^3 - 4*a^4*b*c)*g*h^6 + (a^4*b^2 - 4*a^5*c)*h^7)*x)]
```

**giac [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

**maple [B]** time = 0.02, size = 4930, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -1/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*d-1/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2))/(x+g/h))*e+3/2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
```

```
ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*b*f*g^2+3*h/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g
/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g*d+3/h/(a*h^2-b*g*h+c*g^2)^2/((x+g/
h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^3*f+3/2*h^2
/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*d+3/2*h^2/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*d+1/h/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h)
^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e*g-2/h/(a*h^2-b*
g*h+c*g^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*f*g-1/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*e+2/h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2))/(x+g/h))*f*g-1/h^2/(a*h^2-b*g*h+c*g^2)/(x+g/h)/((x+g/h
)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*f*g^2+3/2*h/(a*h
^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^
2)^(1/2)*b*e*g+3/2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*f*g^2+1/(a*h^2-b*g*h+c*g^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*e-12/h/(
a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*x*b*c^2*g^3*f-3*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)
/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*
e*g+4/h/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*f*g-12*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-
b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*
c^2*g*d-3/2/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*b*f*g^2-3/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-
2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*c*g^2*e+3/(a*h^2-b*g*h+c*g^
2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g
*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x
+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*c*g^2*e-8*c^2/(a*h^2-b*g*h
+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*x*d-4*c/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)
*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d+2*f/h^2*(2*c*x+b)/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)-3/2*h^2/(a*h^2-b*g*h+c*g^2)^2/((x+g/h)^2*c+(b*h-2*c*g)
*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*d+12/h^2/(a*h^2-b*g*h+c*g^2)^2/
(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)*x*c^3*g^4*f-6*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g*d+6/h^2/(a*h^2-b*g*h+c*
g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h
^2)^(1/2)*b*c^2*g^4*f-6/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b
*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^2*c*g^3*f-16/h^2*c^2/(
```

```
a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)*x*f*g^2+6/h*c/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h
)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*e*g-8/h^2*c/(a
*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*b*f*g^2+12/h*c^2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g
/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*e*g-6/h/(a*h
^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)*b*c^2*g^3*e+3*h^2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((
x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c*d+1
2/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c^2*g^2*e+3/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2
)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b^2*c
*f*g^2-3/2*h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+
g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b^3*e*g-3/2*h/(a*h^2-b*g*h+c*g^2)^2/(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g
^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/
h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))*b*e*g+12/(a*h^2-b*g*h+c*g^2)^2/(
4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)*x*c^3*g^2*d+6/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*
(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*b*c^2*g^2*d+6/(a*h^2-b*g*h+c*g^2)^
2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*b^2*c*g^2*e-3*h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*
ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)/
h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
))/(x+g/h))*c*g*d-3/h/(a*h^2-b*g*h+c*g^2)^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)
*ln(((b*h-2*c*g)*(x+g/h)/h+2*(a*h^2-b*g*h+c*g^2)/h^2+2*((a*h^2-b*g*h+c*g^2)
/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2))/(x+g/h))*c*g^3*f-2/(a*h^2-b*g*h+c*g^2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*
c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*b*c*e+2/h/(a*h^2-b*g*h+c*g^
2)/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^
(1/2)*b^2*f*g-12/h/(a*h^2-b*g*h+c*g^2)^2/(4*a*c-b^2)/((x+g/h)^2*c+(b*h-2*c*
g)*(x+g/h)/h+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*x*c^3*g^3*e
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `assum
```

```
e?` for more details)Is (b/h-(2*c*g)/h^2)^2    -(4*c       *((-(b*g)/h)
    +(c*g^2)/h^2+a))     /h^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,{\left(c\,x^2+b\,x+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

# 3.239 $\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=713

$$\frac{\tanh^{-1}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)-4ch\left(ah\left(3dh^2-\right.\right.\right.}{8\left(ah^2-bgh+cg^2\right)^{7/2}}$$

```
[Out] 1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+h^2*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*f
*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g)))-4*c*h*(a*h*(3*d*h^2-9*e*g*h+11*f*g^2)-b*g
*(2*f*g^2+3*h*(-4*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2
-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(7/2)+2*(b^4*d
*h^3-b^3*h^2*(a*e*h+3*c*d*g)+b^2*h*(3*c^2*d*g^2+a^2*f*h^2+a*c*h*(-4*d*h+3*e
*g))-b*c*(c^2*d*g^3+3*a^2*h^2*(-e*h+f*g)+a*c*g*(-9*d*h^2+3*e*g*h+f*g^2))-2*
a*c*(a^2*f*h^3-c^2*g^2*(-3*d*h+e*g)-a*c*h*(d*h^2-3*e*g*h+3*f*g^2))-c*(2*c^3
*d*g^3-b*(a^2*f-a*b*e+b^2*d)*h^3-c^2*g*(b*g*(3*d*h+e*g)+2*a*(3*d*h^2-3*e*g*
h+f*g^2))+c*(2*a^2*h^2*(-e*h+3*f*g)-3*a*b*h*(-d*h^2+e*g*h+f*g^2)+b^2*(3*d*g
*h^2+f*g^3)))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^3/(c*x^2+b*x+a)^(1/2)-1/2
*h*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2
-1/4*h*(2*c*g*(3*f*g^2-h*(-7*d*h+5*e*g))-h*(4*a*h*(-e*h+2*f*g)-b*(-7*d*h^2+
3*e*g*h+f*g^2)))*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)
```

**Rubi [A]** time = 2.67, antiderivative size = 707, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2\left(-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg^3\right)\right)-bh^3\left(a^2f-abe+b^2d\right)-c^2g\left(-6ah(\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
+ a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g*(
f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) - a*
c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e + a^2*
f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c*(2*a
^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(e*g -
d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x + c*x^2])
 - (h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(2*(c*g^2 - b*g*h + a*
```

```
h^2)^2*(g + h*x)^2) - (h*(6*c*f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*
f*g - e*h) + b*h*(f*g^2 + h*(3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(4*(c*
g^2 - b*g*h + a*h^2)^3*(g + h*x)) + ((8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2)
 + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*g^2 - 9*e*g*h + 3*d
*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b^2*(f*g^2 + 3*h*(e*g
 - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
+ a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\,(\ldots}{\ldots}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\,(\ldots}{\ldots}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\,(\ldots}{\ldots}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\,(\ldots}{\ldots}$$

$$= \frac{2\left(b^4dh^3-b^3h^2(3cdg+aeh)+b^2h\left(3c^2dg^2+a^2fh^2+ach(3eg-4dh)\right)-bc\,(\ldots}{\ldots}$$

**Mathematica [A]** time = 5.29, size = 762, normalized size = 1.07

$$3ch\left(-\frac{4h\sqrt{a+x(b+cx)}\left(8a^2fh^2-4c\left(ah(3dh-5eg)+3afg^2+bg(2dh+eg)\right)-4abh(eh+2fg)+b^2\left(h(5dh-eg)+5fg^2\right)+8c^2dg^2\right)}{(g+hx)^2}+\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2ah-bg+bhx-2cgx}{2\sqrt{a+x(b+cx)}\sqrt{h(ah-bg)+cg^2}}\right)\left(h^2\left(-\ldots\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

```
[Out] (-(f/((g + h*x)^2*Sqrt[a + x*(b + c*x)])) + (b^3*f*g*h - b^2*(6*c*d*h^2 + a
*f*h^2 + c*f*g*(g - 6*h*x)) + 2*b*c*h*(3*a*e*h + a*f*(g - 3*h*x) - 3*c*(-(d
*g) + e*g*x + d*h*x)) + 4*c*(-2*a^2*f*h^2 + 3*c^2*d*g*h*x + a*c*(f*g*(g - 3
*h*x) + 3*h*(-(e*g) + d*h + e*h*x))))/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a
*h))*(g + h*x)^2*Sqrt[a + x*(b + c*x)]) + (3*c*h*((-4*h*(8*c^2*d*g^2 + 8*a^
2*f*h^2 - 4*a*b*h*(2*f*g + e*h) - 4*c*(3*a*f*g^2 + b*g*(e*g + 2*d*h) + a*h*
(-5*e*g + 3*d*h)) + b^2*(5*f*g^2 + h*(-(e*g) + 5*d*h)))*Sqrt[a + x*(b + c*x
)])/(g + h*x)^2 - (2*h*(16*c^3*d*g^3 - 8*c^2*g*(5*a*f*g^2 + b*g*(e*g + 3*d*
h) + a*h*(-11*e*g + 13*d*h)) + b*h*(-8*a^2*f*h^2 + 4*a*b*h*(-2*f*g + 3*e*h)
 + b^2*(f*g^2 + 3*h*(e*g - 5*d*h))) + 2*c*(8*a^2*h^2*(5*f*g - 2*e*h) + 2*a*
b*h*(-7*f*g^2 + h*(-9*e*g + 13*d*h)) + b^2*(7*f*g^3 + g*h*(-5*e*g + 19*d*h)
)))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + ((b^2 -
 4*a*c)*(-8*c^2*g^2*(f*g^2 - 3*e*g*h + 6*d*h^2) - 4*c*h*(2*b*f*g^3 + 3*b*g*
h*(e*g - 4*d*h) + a*h*(-11*f*g^2 + 9*e*g*h - 3*d*h^2)) + h^2*(-8*a^2*f*h^2
+ 4*a*b*h*(-2*f*g + 3*e*h) + b^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*ArcTanh[(-(b
*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x
*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(8*(b^2 - 4*a*c)*(c*g^2 +
 h*(-(b*g) + a*h))^2))/(3*c*h)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

**giac [B]** time = 0.88, size = 5637, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^7*d*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d*g^8*h - 3*b^3*
c^4*f*g^8*h + 3*a*b*c^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 + 3*b^4*c^3*f*g^7*h^
2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3 - b^5*c^2*f*g^6*h^3 - 13*a
*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c^3*d*g^5*h^4 + 6*a*b^
2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2
```

*c^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*d*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*g^3*h^6 - 16*a^3*c^4*d*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7 + 24*a^3*b*c^3*d*g^2*h^7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 + 3*a^2*b^4*c*d*g*h^8 - 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 + 3*a^4*b^2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 + 3*a^4*b*c^2*d*h^9 - a^5*b*c*f*h^9 - b*c^6*g^9*e + 3*b^2*c^5*g^8*h*e + 6*a*c^6*g^8*h*e - 3*b^3*c^4*g^7*h^2*e - 24*a*b*c^5*g^7*h^2*e + b^4*c^3*g^6*h^3*e + 34*a*b^2*c^4*g^6*h^3*e + 16*a^2*c^5*g^6*h^3*e - 21*a*b^3*c^3*g^5*h^4*e - 42*a^2*b*c^4*g^5*h^4*e + 6*a*b^4*c^2*g^4*h^5*e + 36*a^2*b^2*c^3*g^4*h^5*e + 12*a^3*c^4*g^4*h^5*e - a*b^5*c*g^3*h^6*e - 13*a^2*b^3*c^2*g^3*h^6*e - 16*a^3*b*c^3*g^3*h^6*e + 3*a^2*b^4*c*g^2*h^7*e + 6*a^3*b^2*c^2*g^2*h^7*e - 3*a^3*b^3*c*g*h^8*e + 3*a^4*b*c^2*g*h^8*e + a^4*b^2*c*h^9*e - 2*a^5*c^2*h^9*e)*x/(b^2*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12) + (b*c^6*d*g^9 + a*b*c^5*f*g^9 - 6*b^2*c^5*d*g^8*h + 6*a*c^6*d*g^8*h - 3*a*b^2*c^4*f*g^8*h - 6*a^2*c^5*f*g^8*h + 15*b^3*c^4*d*g^7*h^2 - 24*a*b*c^5*d*g^7*h^2 + 3*a*b^3*c^3*f*g^7*h^2 + 24*a^2*b*c^4*f*g^7*h^2 - 20*b^4*c^3*d*g^6*h^3 + 34*a*b^2*c^4*d*g^6*h^3 + 16*a^2*c^5*d*g^6*h^3 - a*b^4*c^2*f*g^6*h^3 - 34*a^2*b^2*c^3*f*g^6*h^3 - 16*a^3*c^4*f*g^6*h^3 + 15*b^5*c^2*d*g^5*h^4 - 15*a*b^3*c^3*d*g^5*h^4 - 54*a^2*b*c^4*d*g^5*h^4 + 21*a^2*b^3*c^2*f*g^5*h^4 + 42*a^3*b*c^3*f*g^5*h^4 - 6*b^6*c*d*g^4*h^5 - 9*a*b^4*c^2*d*g^4*h^5 + 66*a^2*b^2*c^3*d*g^4*h^5 + 12*a^3*c^4*d*g^4*h^5 - 6*a^2*b^4*c*f*g^4*h^5 - 36*a^3*b^2*c^2*f*g^4*h^5 - 12*a^4*c^3*f*g^4*h^5 + b^7*d*g^3*h^6 + 11*a*b^5*c*d*g^3*h^6 - 31*a^2*b^3*c^2*d*g^3*h^6 - 32*a^3*b*c^3*d*g^3*h^6 + a^2*b^5*f*g^3*h^6 + 13*a^3*b^3*c*f*g^3*h^6 + 16*a^4*b*c^2*f*g^3*h^6 - 3*a*b^6*d*g^2*h^7 + 30*a^3*b^2*c^2*d*g^2*h^7 - 3*a^3*b^4*f*g^2*h^7 - 6*a^4*b^2*c*f*g^2*h^7 + 3*a^2*b^5*d*g*h^8 - 9*a^3*b^3*c*d*g*h^8 - 3*a^4*b*c^2*d*g*h^8 + 3*a^4*b^3*f*g*h^8 - 3*a^5*b*c*f*g*h^8 - a^3*b^4*d*h^9 + 4*a^4*b^2*c*d*h^9 - 2*a^5*c^2*d*h^9 - a^5*b^2*f*h^9 + 2*a^6*c*f*h^9 - 2*a*c^6*g^9*e + 9*a*b*c^5*g^8*h*e - 18*a*b^2*c^4*g^7*h^2*e + 21*a*b^3*c^3*g^6*h^3*e - 15*a*b^4*c^2*g^5*h^4*e - 6*a^2*b^2*c^3*g^5*h^4*e + 12*a^3*c^4*g^5*h^4*e + 6*a*b^5*c*g^4*h^5*e + 15*a^2*b^3*c^2*g^4*h^5*e - 30*a^3*b*c^3*g^4*h^5*e - a*b^6*g^3*h^6*e - 12*a^2*b^4*c*g^3*h^6*e + 18*a^3*b^2*c^2*g^3*h^6*e + 16*a^4*c^3*g^3*h^6*e + 3*a^2*b^5*g^2*h^7*e

+ 3*a^3*b^3*c*g^2*h^7*e - 24*a^4*b*c^2*g^2*h^7*e - 3*a^3*b^4*g*h^8*e + 6*a^4*b^2*c*g*h^8*e + 6*a^5*c^2*g*h^8*e + a^4*b^3*h^9*e - 3*a^5*b*c*h^9*e)/(b^2*c^6*g^12 - 4*a*c^7*g^12 - 6*b^3*c^5*g^11*h + 24*a*b*c^6*g^11*h + 15*b^4*c^4*g^10*h^2 - 54*a*b^2*c^5*g^10*h^2 - 24*a^2*c^6*g^10*h^2 - 20*b^5*c^3*g^9*h^3 + 50*a*b^3*c^4*g^9*h^3 + 120*a^2*b*c^5*g^9*h^3 + 15*b^6*c^2*g^8*h^4 - 225*a^2*b^2*c^4*g^8*h^4 - 60*a^3*c^5*g^8*h^4 - 6*b^7*c*g^7*h^5 - 36*a*b^5*c^2*g^7*h^5 + 180*a^2*b^3*c^3*g^7*h^5 + 240*a^3*b*c^4*g^7*h^5 + b^8*g^6*h^6 + 26*a*b^6*c*g^6*h^6 - 30*a^2*b^4*c^2*g^6*h^6 - 340*a^3*b^2*c^3*g^6*h^6 - 80*a^4*c^4*g^6*h^6 - 6*a*b^7*g^5*h^7 - 36*a^2*b^5*c*g^5*h^7 + 180*a^3*b^3*c^2*g^5*h^7 + 240*a^4*b*c^3*g^5*h^7 + 15*a^2*b^6*g^4*h^8 - 225*a^4*b^2*c^2*g^4*h^8 - 60*a^5*c^3*g^4*h^8 - 20*a^3*b^5*g^3*h^9 + 50*a^4*b^3*c*g^3*h^9 + 120*a^5*b*c^2*g^3*h^9 + 15*a^4*b^4*g^2*h^10 - 54*a^5*b^2*c*g^2*h^10 - 24*a^6*c^2*g^2*h^10 - 6*a^5*b^3*g*h^11 + 24*a^6*b*c*g*h^11 + a^6*b^2*h^12 - 4*a^7*c*h^12))/sqrt(c*x^2 + b*x + a) + 1/4*(8*c^2*f*g^4 + 8*b*c*f*g^3*h + 48*c^2*d*g^2*h^2 - b^2*f*g^2*h^2 - 44*a*c*f*g^2*h^2 - 48*b*c*d*g*h^3 + 8*a*b*f*g*h^3 + 15*b^2*d*h^4 - 12*a*c*d*h^4 + 8*a^2*f*h^4 - 24*c^2*g^3*h*e + 12*b*c*g^2*h^2*e - 3*b^2*g*h^3*e + 36*a*c*g*h^3*e - 12*a*b*h^4*e)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^3*g^6 - 3*b*c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*a*b*c*g^3*h^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6)*sqrt(-c*g^2 + b*g*h - a*h^2)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*g^3*h^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*g^2*h^3*e - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*g*h^4*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*g*h^4*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*h^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*f*g^3*h^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*d*g^2*h^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*d*g*h^4 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*d*g*h^4 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*f*g*h^4 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*d*h^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*g^4*h*e + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*g^3*h^2*e - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b^2*sqrt(c)*g^2*h^3*e + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*c^(3/2)*g^2*h^3*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a*b*sqrt(c)*g*h^4*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^2*sqrt(c)*h^5*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*f*g^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^2*c*f*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x

```
+ a))*a*c^2*f*g^4*h + 56*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*d*g^3*h^
2 + (sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*f*g^3*h^2 - 28*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*a*b*c*f*g^3*h^2 - 44*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*b^2*c*d*g^2*h^3 - 88*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*c^2*d*g^2*h^3
+ 7*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*f*g^2*h^3 + 44*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^2*c*f*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*b^3*d*g*h^4 + 60*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*d*g*h^4 - 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*f*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*a*b^2*d*h^5 - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d*h
^5 - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b*c^2*g^4*h*e + 24*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*b^2*c*g^3*h^2*e + 64*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*a*c^2*g^3*h^2*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*b^3*g^2*h^3*e
 - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b*c*g^2*h^3*e + (sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*a*b^2*g*h^4*e - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*a^2*c*g*h^4*e + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b*h^5*e + 6*b^2*
c^(3/2)*f*g^5 + b^3*sqrt(c)*f*g^4*h - 20*a*b*c^(3/2)*f*g^4*h + 14*b^2*c^(3/
2)*d*g^3*h^2 - 9*a*b^2*sqrt(c)*f*g^3*h^2 + 12*a^2*c^(3/2)*f*g^3*h^2 - 7*b^3
*sqrt(c)*d*g^2*h^3 - 44*a*b*c^(3/2)*d*g^2*h^3 + 24*a^2*b*sqrt(c)*f*g^2*h^3
+ 23*a*b^2*sqrt(c)*d*g*h^4 + 28*a^2*c^(3/2)*d*g*h^4 - 16*a^3*sqrt(c)*f*g*h^
4 - 16*a^2*b*sqrt(c)*d*h^5 - 10*b^2*c^(3/2)*g^4*h*e + 3*b^3*sqrt(c)*g^3*h^2
*e + 32*a*b*c^(3/2)*g^3*h^2*e - 7*a*b^2*sqrt(c)*g^2*h^3*e - 20*a^2*c^(3/2)*
g^2*h^3*e - 4*a^2*b*sqrt(c)*g*h^4*e + 8*a^3*sqrt(c)*h^5*e)/((c^3*g^6 - 3*b*
c^2*g^5*h + 3*b^2*c*g^4*h^2 + 3*a*c^2*g^4*h^2 - b^3*g^3*h^3 - 6*a*b*c*g^3*h
^3 + 3*a*b^2*g^2*h^4 + 3*a^2*c*g^2*h^4 - 3*a^2*b*g*h^5 + a^3*h^6)*((sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*h + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqr
t(c)*g + b*g - a*h)^2)
```

**maple** [B] time = 0.02, size = 9126, normalized size = 12.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x)
[Out] result too large to display
```

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` for
more details)Is a*h^2-b*g*h                              +c*g^2    positive,
negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f\,x^2 + e\,x + d}{\left(g + h\,x\right)^3\,\left(c\,x^2 + b\,x + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

# 3.240 $\int \frac{(1+2x)^3 (1+3x+4x^2)}{\sqrt{2-x+3x^2}} dx$

**Optimal**. Leaf size=120

$$\frac{2}{15}\sqrt{3x^2-x+2}\,(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}$$

```
[Out] 9211/3888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+44/135*(1+2*x)^2*(3*x^2-x+
2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^4*(3*x^2-x+2)^(1/2)
-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{2}{15}\sqrt{3x^2-x+2}\,(2x+1)^4+\frac{19}{60}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{44}{135}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{(6298x+24897)\sqrt{3x^2-x+2}}{3240}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (44*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/135 + (19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/60 + (2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 - ((24897 + 6298*x)*Sqrt[2
 - x + 3*x^2])/3240 + (9211*ArcSinh[(1 - 6*x)/Sqrt[23]])/(1296*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{60}\int \frac{(1+2x)^3(-64+228x)}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} + \frac{1}{720}\int \frac{(1+2x)^2(-3\ldots}{\sqrt{2-x\ldots}}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x\ldots}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x\ldots}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x\ldots}$$

$$= \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2} + \frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x\ldots}$$

**Mathematica [A]** time = 0.04, size = 60, normalized size = 0.50

$$\frac{6\sqrt{3x^2-x+2}\left(6912x^4+22032x^3+26904x^2+7538x-22383\right)-46055\sqrt{3}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{19440}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4)
 - 46055*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/19440
```

**fricas [A]** time = 0.84, size = 73, normalized size = 0.61

$$\frac{1}{3240}\left(6912\,x^4+22032\,x^3+26904\,x^2+7538\,x-22383\right)\sqrt{3\,x^2-x+2}+\frac{9211}{7776}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x +
 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25)
```

**giac [A]** time = 0.24, size = 68, normalized size = 0.57

$$\frac{1}{3240}\left(2\left(12\left(18\left(16x+51\right)x+1121\right)x+3769\right)x-22383\right)\sqrt{3x^2-x+2}+\frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 - x
 + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
+ 1)
```

**maple [A]** time = 0.02, size = 96, normalized size = 0.80

$$\frac{32\sqrt{3x^2-x+2}\,x^4}{15}+\frac{34\sqrt{3x^2-x+2}\,x^3}{5}+\frac{1121\sqrt{3x^2-x+2}\,x^2}{135}+\frac{3769\sqrt{3x^2-x+2}\,x}{1620}-\frac{9211\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x}{23}\right.}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 32/15*x^4*(3*x^2-x+2)^(1/2)+34/5*x^3*(3*x^2-x+2)^(1/2)+1121/135*x^2*(3*x^2-
x+2)^(1/2)+3769/1620*x*(3*x^2-x+2)^(1/2)-829/120*(3*x^2-x+2)^(1/2)-9211/388
8*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima [A]** time = 0.95, size = 97, normalized size = 0.81

$$\frac{32}{15}\sqrt{3x^2-x+2}\,x^4+\frac{34}{5}\sqrt{3x^2-x+2}\,x^3+\frac{1121}{135}\sqrt{3x^2-x+2}\,x^2+\frac{3769}{1620}\sqrt{3x^2-x+2}\,x-\frac{9211}{3888}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sqr
t(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3)*
arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

# 3.241 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx$

**Optimal**. Leaf size=95

$$\frac{1}{6}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

```
[Out] 4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-143/324*(3-2*x)*(3*x^2-x+2
)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(1+2*x)^3*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1653, 832, 779, 619, 215}

$$\frac{1}{6}\sqrt{3x^2-x+2}\,(2x+1)^3+\frac{11}{27}\sqrt{3x^2-x+2}\,(2x+1)^2-\frac{143}{324}(3-2x)\sqrt{3x^2-x+2}+\frac{4147\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (-143*(3 - 2*x)*Sqrt[2 - x + 3*x^2])/324 + (11*(1 + 2*x)^2*Sqrt[2 - x + 3*x
^2])/27 + ((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 + (4147*ArcSinh[(1 - 6*x)/Sqr
t[23]])/(648*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{48}\int \frac{(1+2x)^2(-44+176x)}{\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2} + \frac{1}{432}\int \frac{(1+2x)(-1716}{\sqrt{2-x+3}}$$
$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+}$$

$$= -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2} + \frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+}$$

**Mathematica [A]** time = 0.03, size = 55, normalized size = 0.58

$$\frac{6\sqrt{3x^2-x+2}\left(432x^3+1176x^2+1138x-243\right)-4147\sqrt{3}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{1944}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) - 4147*Sqrt[3]*
ArcSinh[(-1 + 6*x)/Sqrt[23]])/1944
```

**fricas [A]** time = 0.76, size = 68, normalized size = 0.72

$$\frac{1}{324}\left(432\,x^3+1176\,x^2+1138\,x-243\right)\sqrt{3\,x^2-x+2}+\frac{4147}{3888}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}\,(6\,x-1)-72\,x^2+24\,x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*s
qrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac [A]** time = 0.22, size = 63, normalized size = 0.66

$$\frac{1}{324}\left(2\left(12\left(18\,x+49\right)x+569\right)x-243\right)\sqrt{3\,x^2-x+2}+\frac{4147}{1944}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944*
sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple [A]** time = 0.01, size = 79, normalized size = 0.83

$$\frac{4\sqrt{3x^2-x+2}\,x^3}{3}+\frac{98\sqrt{3x^2-x+2}\,x^2}{27}+\frac{569\sqrt{3x^2-x+2}\,x}{162}-\frac{4147\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}-\frac{3\sqrt{3x^2-x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 4/3*(3*x^2-x+2)^(1/2)*x^3+98/27*(3*x^2-x+2)^(1/2)*x^2+569/162*(3*x^2-x+2)^(
1/2)*x-3/4*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6
))
```

**maxima [A]** time = 0.97, size = 80, normalized size = 0.84

$$\frac{4}{3}\sqrt{3\,x^2-x+2}\,x^3+\frac{98}{27}\sqrt{3\,x^2-x+2}\,x^2+\frac{569}{162}\sqrt{3\,x^2-x+2}\,x-\frac{4147}{1944}\sqrt{3}\ \operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\,(6\,x-1)\right)-\frac{3}{4}\sqrt{3\,x^2-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt(
3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3/4
*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^2\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

# 3.242 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx$

**Optimal**. Leaf size=70

$$\frac{2}{9}\sqrt{3x^2-x+2}\,(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

```
[Out] 251/324*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/9*(1+2*x)^2*(3*x^2-x+2)^(1
/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 779, 619, 215}

$$\frac{2}{9}\sqrt{3x^2-x+2}\,(2x+1)^2+\frac{1}{54}(62x+69)\sqrt{3x^2-x+2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + ((69 + 62*x)*Sqrt[2 - x + 3*x^2])/5
4 + (251*ArcSinh[(1 - 6*x)/Sqrt[23]])/(108*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx &= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{36}\int \frac{(1+2x)(-24+124x)}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251}{108}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}-\frac{251\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx\right.}{108\sqrt{69}} \\
&= \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2}+\frac{1}{54}(69+62x)\sqrt{2-x+3x^2}+\frac{251\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 0.71

$$\frac{1}{324}\left(6\sqrt{3x^2-x+2}\,\left(48x^2+110x+81\right)-251\sqrt{3}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

```
[Out] (6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) - 251*Sqrt[3]*ArcSinh[(-1 + 6*
x)/Sqrt[23]])/324
```

**fricas** [A] time = 0.76, size = 63, normalized size = 0.90

$$\frac{1}{54}\left(48\,x^{2}+110\,x+81\right)\sqrt{3\,x^{2}-x+2}+\frac{251}{648}\,\sqrt{3}\,\log\left(4\,\sqrt{3}\,\sqrt{3\,x^{2}-x+2}\,(6\,x-1)-72\,x^{2}+24\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

**giac** [A] time = 0.26, size = 58, normalized size = 0.83

$$\frac{1}{54}\,(2\,(24\,x+55)x+81)\sqrt{3\,x^{2}-x+2}\,+\,\frac{251}{324}\,\sqrt{3}\,\log\left(-2\,\sqrt{3}\left(\sqrt{3}\,x-\sqrt{3\,x^{2}-x+2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

**maple** [A] time = 0.01, size = 62, normalized size = 0.89

$$\frac{8\sqrt{3x^{2}-x+2}\;x^{2}}{9}+\frac{55\sqrt{3x^{2}-x+2}\;x}{27}-\frac{251\sqrt{3}\;\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{324}+\frac{3\sqrt{3x^{2}-x+2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 8/9*(3*x^2-x+2)^(1/2)*x^2+55/27*(3*x^2-x+2)^(1/2)*x+3/2*(3*x^2-x+2)^(1/2)-2
51/324*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima** [A] time = 0.96, size = 63, normalized size = 0.90

$$\frac{8}{9}\,\sqrt{3\,x^{2}-x+2}\,x^{2}+\frac{55}{27}\,\sqrt{3\,x^{2}-x+2}\,x-\frac{251}{324}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)+\frac{3}{2}\,\sqrt{3\,x^{2}-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

```
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/sqrt(3*x**2 - x + 2), x)
```

# 3.243 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

**Optimal**. Leaf size=78

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

```
[Out] -5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/3*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 - (5*ArcSinh[(1 - 6*x)/Sqrt[23]])/(6*Sqrt[3]) - A
rcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx &= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{12}\int \frac{16+20x}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{1}{2}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\frac{5}{6}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx \\
&= \frac{2}{3}\sqrt{2-x+3x^2}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{6\sqrt{69}}-\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\sqrt{\right.} \\
&= \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}
\end{aligned}$$

**Mathematica** [A]   time = 0.05, size = 78, normalized size = 1.00

$$\frac{2}{3}\sqrt{3x^2-x+2}-\frac{\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}+\frac{5\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] (2*Sqrt[2 - x + 3*x^2])/3 + (5*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]) -
ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[13])
```

**fricas** [A]   time = 0.77, size = 105, normalized size = 1.35

$$\frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+\frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+2}{4x^2+4x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x
^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

**giac** [A]   time = 0.57, size = 116, normalized size = 1.49

$$-\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)+\frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*sq
rt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 -
x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/3*s
qrt(3*x^2 - x + 2)
```

**maple** [A]   time = 0.01, size = 60, normalized size = 0.77

$$\frac{5\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}-\frac{\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{26}+\frac{2\sqrt{3x^2-x+2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 2/3*(3*x^2-x+2)^(1/2)+5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26*13^(
1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.97, size = 67, normalized size = 0.86

$$\frac{5}{18}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1}{26}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{2}{3}\sqrt{3\,x^2-x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(3*
x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

# 3.244 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

**Optimal**. Leaf size=83

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+9/338*arctanh(1/26*(9-8*x)*13^(
1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(13*(1 + 2*x)) - ArcSinh[(1 - 6*x)/Sqrt[23]]/Sqrt[3] +
 (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx &= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{1}{13}\int \frac{-\frac{17}{2}-26x}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{9}{26}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx+\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}+\frac{9}{13}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\right.}{\sqrt{6}}\\
&= -\frac{\sqrt{2-x+3x^2}}{13(1+2x)}-\frac{\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 82, normalized size = 0.99

$$-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}+\frac{9\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{26\sqrt{13}}+\frac{\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) + ArcSinh[(-1 + 6*x)/Sqrt[23]]/Sqrt[3]
+ (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(26*Sqrt[13])
```

**fricas** [A] time = 0.87, size = 123, normalized size = 1.48

$$\frac{338\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8}{4x^2}\right.}{2028(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
- 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(3*x
^2 - x + 2))/(2*x + 1)
```

**giac** [A] time = 0.28, size = 48, normalized size = 0.58

$$\frac{1}{26}\sqrt{3}\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)-\frac{\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}{26\,\mathrm{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/26*sqrt(3)*sgn(1/(2*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3
)/sgn(1/(2*x + 1))
```

**maple** [A] time = 0.01, size = 67, normalized size = 0.81

$$\frac{\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\ \mathrm{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{338}-\frac{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{26\left(x+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+9/338*13^(1/2)*arctanh(2/13*(-4*
x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)*(-4*x+3*(x+1/2)^
2+5/4)^(1/2)
```

**maxima [A]** time = 0.98, size = 74, normalized size = 0.89

$$\frac{1}{3}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{9}{338}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{13\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcsi
nh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt(3
*x^2 - x + 2)/(2*x + 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)^2\,\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\,\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

# 3.245 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

**Optimal**. Leaf size=89

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

```
[Out] -581/8788*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-1/26*(3
*x^2-x+2)^(1/2)/(1+2*x)^2+7/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 724, 206}

$$\frac{7\sqrt{3x^2-x+2}}{169(2x+1)}-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{676\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] -Sqrt[2 - x + 3*x^2]/(26*(1 + 2*x)^2) + (7*Sqrt[2 - x + 3*x^2])/(169*(1 + 2
*x)) - (581*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(676*Sqrt[
13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
```

```
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx &= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}-\frac{1}{26}\int \frac{-\frac{35}{2}-49x}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{581}{676}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581}{338}\operatorname{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{581\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 69, normalized size = 0.78

$$\frac{\frac{26(28x+1)\sqrt{3x^2-x+2}}{(2x+1)^2}-581\sqrt{13}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{8788}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

```
[Out] ((26*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 - 581*Sqrt[13]*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/8788
```

**fricas [A]** time = 0.68, size = 96, normalized size = 1.08

$$\frac{581\sqrt{13}\left(4x^2+4x+1\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+52\sqrt{3x^2-x+2}(28x+1)}{17576\left(4x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2
)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 - x
 + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

**giac [B]** time = 0.33, size = 204, normalized size = 2.29

$$\frac{581}{8788}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{190\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-53\sqrt{3}\left(\sqrt{3}x-\right.}{338\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(3)*
x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x^2 -
 x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sq
rt(3*x^2 - x + 2)) - 5)^2
```

**maple [A]** time = 0.04, size = 74, normalized size = 0.83

$$-\frac{581\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{8788}+\frac{7\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{338\left(x+\frac{1}{2}\right)}-\frac{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}{104\left(x+\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(1/2),x)
```

```
[Out] -581/8788*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^
(1/2))+7/338/(x+1/2)*(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^2*(-4*x+3*(
x+1/2)^2+5/4)^(1/2)
```

**maxima [A]** time = 0.98, size = 82, normalized size = 0.92

$$\frac{581}{8788}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)-\frac{\sqrt{3\,x^2-x+2}}{26\left(4\,x^2+4\,x+1\right)}+\frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2 -
 x + 2)/(2*x + 1)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)^3\,\sqrt{3\,x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\,\sqrt{3x^2-x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

# 3.246 $\int \frac{(1+2x)^3 \left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}} \, dx$

**Optimal**. Leaf size=103

$$\frac{32}{27}\sqrt{3x^2-x+2}\,x^2+\frac{412}{81}\sqrt{3x^2-x+2}\,x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

```
[Out] 353/243*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/1863*(12839-3871*x)/(3*x^2
-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*(3*x^2-x+2)^(1/2)+32/27*x^2*(
3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{32}{27}\sqrt{3x^2-x+2}\,x^2+\frac{412}{81}\sqrt{3x^2-x+2}\,x+\frac{746}{81}\sqrt{3x^2-x+2}+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}+\frac{353\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (746*Sqrt[2 - x + 3*x^2])
/81 + (412*x*Sqrt[2 - x + 3*x^2])/81 + (32*x^2*Sqrt[2 - x + 3*x^2])/27 + (3
53*ArcSinh[(1 - 6*x)/Sqrt[23]])/(81*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{1127}{81}+\frac{7682x}{27}+\frac{2852x^2}{9}+\frac{368x^3}{3}}{\sqrt{2-x+3x^2}}dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{2}{207}\int \frac{\frac{1127}{9}+2070x+\frac{9476x^2}{3}}{\sqrt{2-x+3x^2}}dx$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x+3x^2} + \frac{1}{621}\int \frac{-556}{\sqrt{2}}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x}$$

$$= \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}} + \frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2} + \frac{32}{27}x^2\sqrt{2-x}$$

**Mathematica [A]** time = 0.04, size = 69, normalized size = 0.67

$$\frac{6\left(3312x^4+13110x^3+23207x^2-2974x+29997\right)-8119\sqrt{9x^2-3x+6}\,\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{5589\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4) - 8119*Sqrt[6 - 3*x
+ 9*x^2]*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(5589*Sqrt[2 - x + 3*x^2])
```

**fricas [A]** time = 0.71, size = 97, normalized size = 0.94

$$\frac{8119\sqrt{3}\left(3x^2-x+2\right)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+12\left(3312x^4+13110x^3+23207\right.}{11178\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

[Out] 1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 2974*x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)

**giac** [A] time = 0.21, size = 67, normalized size = 0.65

$$\frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(23\left(6\left(24x+95\right)x+1009\right)x-2974\right)x+29997\right)}{1863\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")

[Out] 353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2)

**maple** [A] time = 0.01, size = 115, normalized size = 1.12

$$\frac{32x^4}{9\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}-\frac{521\,(6x}{414\sqrt{3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)

[Out] 32/9*x^4/(3*x^2-x+2)^(1/2)+380/27*x^3/(3*x^2-x+2)^(1/2)+2018/81*x^2/(3*x^2-x+2)^(1/2)+353/81*x/(3*x^2-x+2)^(1/2)-521/414*(6*x-1)/(3*x^2-x+2)^(1/2)-353/243*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+557/18/(3*x^2-x+2)^(1/2)

**maxima** [A] time = 0.96, size = 97, normalized size = 0.94

$$\frac{32x^4}{9\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}-\frac{353}{243}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}\left(6x-1\right)\right)-\frac{5948x}{1863\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")

[Out] 32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^2/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 5948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

## 3.247 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=82

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] -64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+2/621*(1249-2273*x)/(3*x^2-x+
2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}+\frac{8}{9}x\sqrt{3x^2-x+2}+\frac{112}{27}\sqrt{3x^2-x+2}-\frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (112*Sqrt[2 - x + 3*x^2])/2
7 + (8*x*Sqrt[2 - x + 3*x^2])/9 - (64*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[
3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{2116}{27}+\frac{1150x}{9}+\frac{184x^2}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{1}{69}\int \frac{\frac{3128}{9}+\frac{2576x}{3}}{\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{64}{9}\int \frac{1}{\sqrt{2-x+3\ldots}}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} + \frac{64\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\ldots}}\ldots\right)}{9\sqrt{\ldots}}$$

$$= \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} - \frac{64\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

**Mathematica [A]** time = 0.03, size = 61, normalized size = 0.74

$$\frac{2\left(828x^3+3588x^2+736\sqrt{9x^2-3x+6}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-3009x+3825\right)}{621\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(3825 - 3009*x + 3588*x^2 + 828*x^3 + 736*Sqrt[6 - 3*x + 9*x^2]*ArcSinh[
(-1 + 6*x)/Sqrt[23]]))/(621*Sqrt[2 - x + 3*x^2])
```

**fricas [A]** time = 0.60, size = 92, normalized size = 1.12

$$\frac{2\left(368\sqrt{3}\left(3x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(276x^3+1196x^2-1003x+\right.\right.}{621\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(3*
x^2 - x + 2))/(3*x^2 - x + 2)
```

**giac [A]** time = 0.27, size = 62, normalized size = 0.76

$$-\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left((92(3x+13)x-1003)x+1275\right)}{207\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/20
7*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

**maple [A]** time = 0.01, size = 98, normalized size = 1.20

$$\frac{8x^3}{3\sqrt{3x^2-x+2}}+\frac{104x^2}{9\sqrt{3x^2-x+2}}-\frac{64x}{9\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}+\frac{107}{9\sqrt{3x^2-x+2}}-\frac{89(6x-1)}{207\sqrt{3x^2-x+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2-x+2)^(1/2)*x^3+104/9/(3*x^2-x+2)^(1/2)*x^2-64/9/(3*x^2-x+2)^(1/2
)*x+107/9/(3*x^2-x+2)^(1/2)-89/207*(6*x-1)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*
arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima** [A] time = 0.97, size = 80, normalized size = 0.98

$$\frac{8\,x^{3}}{3\,\sqrt{3\,x^{2}-x+2}}+\frac{104\,x^{2}}{9\,\sqrt{3\,x^{2}-x+2}}+\frac{64}{27}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(6\,x-1)\right)-\frac{2006\,x}{207\,\sqrt{3\,x^{2}-x+2}}+\frac{850}{69\,\sqrt{3\,x^{2}-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3)
*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/69
/sqrt(3*x^2 - x + 2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^2\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x+1\right)^{2} \left(4x^{2}+3x+1\right)}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

## 3.248 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=63

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

```
[Out] -14/9*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-2/207*(73+367*x)/(3*x^2-x+2)^(
1/2)+8/9*(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1660, 640, 619, 215}

$$-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}+\frac{8}{9}\sqrt{3x^2-x+2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (8*Sqrt[2 - x + 3*x^2])/9 - (
14*ArcSinh[(1 - 6*x)/Sqrt[23]])/(3*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{437}{9}+\frac{92x}{3}}{\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14}{3}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}+\frac{14\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6x\right)}{3\sqrt{69}}\\ &= -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 50, normalized size = 0.79

$$\frac{2\left(92x^2-153x+37\right)}{69\sqrt{3x^2-x+2}}+\frac{14\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

```
[Out] (2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) + (14*ArcSinh[(-1 + 6*x)
/Sqrt[23]])/(3*Sqrt[3])
```

**fricas [A]** time = 0.65, size = 87, normalized size = 1.38

$$\frac{161\sqrt{3}\left(3x^2-x+2\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}\,(6x-1)-72x^2+24x-25\right)+6\left(92x^2-153x+37\right)\sqrt{3x^2-x}}{207\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
- 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))/(
3*x^2 - x + 2)
```

**giac [A]** time = 0.27, size = 57, normalized size = 0.90

$$-\frac{14}{9}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2((92x-153)x+37)}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69*
((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

**maple [A]** time = 0.01, size = 81, normalized size = 1.29

$$\frac{8x^2}{3\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\ \operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{9}+\frac{10}{9\sqrt{3x^2-x+2}}+\frac{\frac{16x}{69}-\frac{8}{207}}{\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 8/3/(3*x^2-x+2)^(1/2)*x^2-14/3/(3*x^2-x+2)^(1/2)*x+10/9/(3*x^2-x+2)^(1/2)+8
/207*(6*x-1)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima [A]** time = 0.94, size = 63, normalized size = 1.00

$$\frac{8x^2}{3\sqrt{3x^2-x+2}}+\frac{14}{9}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{102x}{23\sqrt{3x^2-x+2}}+\frac{74}{69\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1))
 - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2\,x+1)\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
[Out] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

# 3.249 $\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=62

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

```
[Out] -2/169*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/299*(101
-77*x)/(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 12, 724, 206}

$$-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt[
13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{23}{13(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}+\frac{2}{13}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{4}{13}\,\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-8x}{\sqrt{2-x+3x^2}}\right)\\
&= -\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 73, normalized size = 1.18

$$-\frac{2\left(23\sqrt{13}\sqrt{3x^2-x+2}\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)-1001x+1313\right)}{3887\sqrt{3x^2-x+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
[Out] (-2*(1313 - 1001*x + 23*Sqrt[13]*Sqrt[2 - x + 3*x^2]*ArcTanh[(9 - 8*x)/(2*S
qrt[13]*Sqrt[2 - x + 3*x^2])]))/(3887*Sqrt[2 - x + 3*x^2])
```

**fricas [A]** time = 0.61, size = 96, normalized size = 1.55

$$\frac{23\sqrt{13}\left(3x^2-x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}\,(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+26\sqrt{3x^2-x+2}\,(77x-101)}{3887\left(3x^2-x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8
*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x + 2
)*(77*x - 101))/(3*x^2 - x + 2)
```

**giac [A]** time = 0.58, size = 91, normalized size = 1.47

$$\frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(77x-101)}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
 + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

**maple [B]** time = 0.01, size = 102, normalized size = 1.65

$$-\frac{2\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{169}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{\frac{10x}{23}-\frac{5}{69}}{\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{\frac{24x}{299}}{\sqrt{-4x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(3/2),x)
```

```
[Out] -2/3/(3*x^2-x+2)^(1/2)+5/69*(6*x-1)/(3*x^2-x+2)^(1/2)+1/13/(-4*x+3*(x+1/2)^
2+5/4)^(1/2)+4/299*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-2/169*13^(1/2)*arct
anh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.97, size = 64, normalized size = 1.03

$$\frac{2}{169}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{154x}{299\sqrt{3x^2-x+2}}-\frac{202}{299\sqrt{3x^2-x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x
 + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

# 3.250 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] 2/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)-2/3887*(19
7-837*x)/(3*x^2-x+2)^(1/2)-4/169*(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi** [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{169(2x+1)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) - (4*Sqrt[2 - x + 3*x^2])/(16
9*(1 + 2*x)) + (2*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169
*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx &= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}+\frac{2}{23}\int \frac{\frac{184}{169}-\frac{230x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}-\frac{2}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}\,dx\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{4}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{9-}{\sqrt{2-}}\right.\\
&= -\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 74, normalized size = 0.85

$$\frac{2\left(1536x^2+489x-289\right)}{3887(2x+1)\sqrt{3x^2-x+2}}+\frac{2\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(-289 + 489*x + 1536*x^2))/(3887*(1 + 2*x)*Sqrt[2 - x + 3*x^2]) + (2*Arc
Tanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

**fricas [A]** time = 0.69, size = 106, normalized size = 1.22

$$\frac{23\sqrt{13}\left(6x^3+x^2+3x+2\right)\log\left(\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)-220x^2+196x-185}{4x^2+4x+1}\right)+26\left(1536x^2+489x-289\right)\sqrt{3x^2-x+2}}{50531\left(6x^3+x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 - x
 + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^2
+ 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

**giac [B]** time = 0.30, size = 168, normalized size = 1.93

$$-\frac{2}{50531}\sqrt{13}\left(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{2\left(\frac{\frac{1047}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}+\frac{2\sqrt{13}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sgn
(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*x
+ 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x + 1)^
2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

**maple [A]** time = 0.01, size = 109, normalized size = 1.25

$$\frac{2\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2197}+\frac{\frac{12x}{23}-\frac{2}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{169\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}-\frac{82(6x-1)}{3887\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 2/23*(6*x-1)/(3*x^2-x+2)^(1/2)-1/169/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-82/3887*(
6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+2/2197*13^(1/2)*arctanh(2/13*(-4*x+9/2)
*13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)
^(1/2)
```

**maxima [A]** time = 0.96, size = 96, normalized size = 1.10

$$-\frac{2}{2197}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1536\,x}{3887\,\sqrt{3\,x^2-x+2}}-\frac{279}{3887\,\sqrt{3\,x^2-x+2}}-\frac{1}{13\left(2\,\sqrt{3\,x^2-x+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2) -
 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)^2\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

# 3.251 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=112

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] -487/28561*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+2/5053
1*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-4/2197*
(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi [A]** time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2\sqrt{3x^2-x+2}}{169(2x+1)^2}-\frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) - (2*Sqrt[2 - x + 3*x^2])/(
169*(1 + 2*x)^2) - (4*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (487*ArcTanh[
(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
```

```
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} + \frac{2}{23}\int \frac{\frac{8349}{2197}+\frac{20838x}{2197}+\frac{23828x^2}{2197}}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{1}{299}\int \frac{-\frac{11615}{169}-\frac{22034x}{169}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} + \frac{487\int \frac{1}{(1+2x)\sqrt{2-x+3x}}}{2197}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{974\,\mathrm{Subst}\left(\int \frac{1}{52-x^2}\right.}{219}$$

$$= \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{487\tanh^{-1}\left(\frac{9}{2\sqrt{13}\sqrt{}}\right.}{2197\sqrt{13}}$$

**Mathematica** [A] time = 0.06, size = 79, normalized size = 0.71

$$\frac{2\left(14496x^3+23281x^2+13306x+1673\right)}{50531(2x+1)^2\sqrt{3x^2-x+2}} - \frac{487\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

```
[Out] (2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
+ 3*x^2]) - (487*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197
*Sqrt[13])
```

**fricas** [A] time = 0.64, size = 126, normalized size = 1.12

$$\frac{11201\sqrt{13}\left(12x^4+8x^3+7x^2+7x+2\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+52\left(14496x^3+23281x^2+\right.}{1313806\left(12x^4+8x^3+7x^2+7x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)
```

```
) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12*x^
4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

**giac [B]** time = 0.31, size = 223, normalized size = 1.99

$$\frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}+\frac{2\left(62\left(\sqrt{3}x-\sqrt{3x^2-\right.\right.}{219}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)*x
- sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 +
 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt
(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

**maple [A]** time = 0.01, size = 111, normalized size = 0.99

$$-\frac{487\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{28561}+\frac{487}{4394\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{\frac{7248x}{50531}-\frac{1208}{50531}}{\sqrt{-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}}}+\frac{}{338\left(x+\frac{1}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(3/2),x)
```

```
[Out] 487/4394/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+1208/50531*(6*x-1)/(-4*x+3*(x+1/2)^2+
5/4)^(1/2)-487/28561*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x
+1/2)^2+5)^(1/2))+3/338/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-1/104/(x+1/2)^
2/(-4*x+3*(x+1/2)^2+5/4)^(1/2)
```

**maxima [A]** time = 0.97, size = 145, normalized size = 1.29

$$\frac{487}{28561}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|}-\frac{9\sqrt{23}}{23|2x+1|}\right)+\frac{7248x}{50531\sqrt{3x^2-x+2}}+\frac{8785}{101062\sqrt{3x^2-x+2}}-\frac{}{26\left(4\sqrt{3x^2-x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 - x
+ 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(3*x
^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)^3\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

# 3.252 $\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=86

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

```
[Out] 2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/2
))*3^(1/2)-28/128547*(35809+42240*x)/(3*x^2-x+2)^(1/2)+32/27*(3*x^2-x+2)^(1
/2)
```

**Rubi [A]** time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 640, 619, 215}

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}+\frac{32}{27}\sqrt{3x^2-x+2}-\frac{28(42240x+35809)}{128547\sqrt{3x^2-x+2}}-\frac{296\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (28*(35809 + 42240*x))/
(128547*Sqrt[2 - x + 3*x^2]) + (32*Sqrt[2 - x + 3*x^2])/27 - (296*ArcSinh[(
1 - 6*x)/Sqrt[23]])/(27*Sqrt[3])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{4361}{81}+\frac{7682x}{9}+\frac{2852x^2}{3}+368x^3}{\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{37030}{9}+\frac{4232x}{3}}{\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296}{27}\int \frac{}{\sqrt{2}}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}+\frac{296\,\text{Subst}\Big(}{}$$

$$= \frac{2(12839-3871x)}{5589\left(2-x+3x^2\right)^{3/2}}-\frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}}+\frac{32}{27}\sqrt{2-x+3x^2}-\frac{296\sinh^{-1}}{27\sqrt{3}}$$

**Mathematica [A]** time = 0.06, size = 71, normalized size = 0.83

$$\frac{2\left(228528x^4-743712x^3+25890x^2+78292\sqrt{3}\left(3x^2-x+2\right)^{3/2}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)-358377x-134217\right)}{42849\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-134217 - 358377*x + 25890*x^2 - 743712*x^3 + 228528*x^4 + 78292*Sqrt[3
]*(2 - x + 3*x^2)^(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(42849*(2 - x + 3*x^
2)^(3/2))
```

**fricas [A]** time = 0.59, size = 117, normalized size = 1.36

$$\frac{2\left(39146\sqrt{3}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(76176x\right.\right.}{42849\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904*x
^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*
x^2 - 4*x + 4)
```

**giac [A]** time = 0.21, size = 67, normalized size = 0.78

$$-\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(2\left(8\left(4761x-15494\right)x+4315\right)x-119459\right)x-44739\right)}{14283\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/1
4283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x +
2)^(3/2)
```

**maple [B]** time = 0.01, size = 163, normalized size = 1.90

$$\frac{32x^4}{3\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{296x^3}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{8x^2}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{296x}{27\sqrt{3x^2-x+2}}-\frac{461x}{81\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{296\sqrt{3}\ \text{arcsinh}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 13763/33534*(6*x-1)/(3*x^2-x+2)^(3/2)-296/27/(3*x^2-x+2)^(1/2)*x+65264/1285
47*(6*x-1)/(3*x^2-x+2)^(1/2)+296/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-
148/81/(3*x^2-x+2)^(1/2)-1727/1458/(3*x^2-x+2)^(3/2)+32/3*x^4/(3*x^2-x+2)^(
3/2)-296/27*x^3/(3*x^2-x+2)^(3/2)+8/27*x^2/(3*x^2-x+2)^(3/2)-461/81*x/(3*x^
2-x+2)^(3/2)
```

**maxima [B]** time = 0.96, size = 202, normalized size = 2.35

$$\frac{32\,x^4}{3\left(3\,x^2-x+2\right)^{\frac{3}{2}}}+\frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}}-\frac{4761\,x^2}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3\,x^2-x+2}}+\frac{805\,x}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3\,x^2-x+2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) - 4
761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x +
 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sqrt(
23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3*x^2
 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 - x + 2
) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^3\,\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

# 3.253 $\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=68

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

```
[Out] 2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2))
*3^(1/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 12, 619, 215}

$$\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{3x^2-x+2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) - (8*(23257 - 1473*x))/(42
849*Sqrt[2 - x + 3*x^2]) - (16*ArcSinh[(1 - 6*x)/Sqrt[23]])/(9*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx &= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{1802}{27}+\frac{1150x}{3}+184x^2}{\left(2-x+3x^2\right)^{3/2}}\,dx\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{4\int \frac{2116}{3\sqrt{2-x+3x^2}}\,dx}{1587}\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16}{9}\int \frac{1}{\sqrt{2-x+3x^2}}\,dx\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}+\frac{16\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+6
\right.}{9\sqrt{69}}\\
&= \frac{2(1249-2273x)}{1863\left(2-x+3x^2\right)^{3/2}}-\frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}}-\frac{16\sinh^{-1}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.13, size = 66, normalized size = 0.97

$$
\frac{2\left(5892x^3-94992x^2+4232\sqrt{3}\left(3x^2-x+2\right)^{3/2}\sinh^{-1}\left(\frac{6x-1}{\sqrt{23}}\right)+17511x-52443\right)}{14283\left(3x^2-x+2\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (2*(-52443 + 17511*x - 94992*x^2 + 5892*x^3 + 4232*Sqrt[3]*(2 - x + 3*x^2)^
(3/2)*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/(14283*(2 - x + 3*x^2)^(3/2))
```

**fricas [B]** time = 0.86, size = 112, normalized size = 1.65

$$\frac{2\left(2116\sqrt{3}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+3\left(1964x^3-\right.\right.}{14283\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sqr
t(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^2
+ 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

**giac [A]** time = 0.24, size = 62, normalized size = 0.91

$$-\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)+\frac{2\left(\left(4\left(491x-7916\right)x+5837\right)x-17481\right)}{4761\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/47
61*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

**maple [B]** time = 0.01, size = 146, normalized size = 2.15

$$-\frac{16x^3}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{92x^2}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{67x}{27\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{16x}{9\sqrt{3x^2-x+2}}+\frac{16\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}-\frac{}{486\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -16/9/(3*x^2-x+2)^(3/2)*x^3-92/9/(3*x^2-x+2)^(3/2)*x^2-67/27/(3*x^2-x+2)^(3
/2)*x-2653/486/(3*x^2-x+2)^(3/2)+4585/11178*(6*x-1)/(3*x^2-x+2)^(3/2)+18892
/42849*(6*x-1)/(3*x^2-x+2)^(1/2)-16/9/(3*x^2-x+2)^(1/2)*x-8/27/(3*x^2-x+2)^
(1/2)+16/27*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

**maxima [B]** time = 0.97, size = 185, normalized size = 2.72

$$\frac{16}{14283}x\left(\frac{426x}{\sqrt{3x^2-x+2}}-\frac{4761x^2}{\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{3x^2-x+2}}+\frac{805x}{\left(3x^2-x+2\right)^{\frac{3}{2}}}-\frac{2162}{\left(3x^2-x+2\right)^{\frac{3}{2}}}\right)+\frac{16}{27}\sqrt{3}\,\operatorname{arsin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 71
/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(
3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt(3*
x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x + 2)
^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/2) -
3394/621/(3*x^2 - x + 2)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+1)^2\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

## 3.254 $\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

```
[Out] -2/621*(73+367*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1660, 636}

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

```
[Out] (-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283*
Sqrt[2 - x + 3*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{577}{9}+92x}{\left(2-x+3x^2\right)^{3/2}}\,dx$$
$$= -\frac{2(73+367x)}{621\left(2-x+3x^2\right)^{3/2}} - \frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

**Mathematica [A]** time = 0.06, size = 33, normalized size = 0.70

$$\frac{2\left(2860x^3-3546x^2+1833x-1915\right)}{1587\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
[Out] (2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

**fricas [A]** time = 0.80, size = 51, normalized size = 1.09

$$\frac{2\left(2860\,x^3-3546\,x^2+1833\,x-1915\right)\sqrt{3\,x^2-x+2}}{1587\left(9\,x^4-6\,x^3+13\,x^2-4\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
[Out] 2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 - 6
*x^3 + 13*x^2 - 4*x + 4)
```

**giac [A]** time = 0.20, size = 28, normalized size = 0.60

$$\frac{2\left(\left(2\left(1430\,x-1773\right)x+1833\right)x-1915\right)}{1587\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
[Out] 2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

**maple** [A] time = 0.00, size = 30, normalized size = 0.64

$$\frac{\frac{5720}{1587}x^3-\frac{2364}{529}x^2+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

**maxima** [A] time = 0.44, size = 76, normalized size = 1.62

$$\frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}}-\frac{8\,x^2}{3\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{2860}{14283\,\sqrt{3\,x^2-x+2}}-\frac{182\,x}{621\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}-\frac{1250}{621\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/1428
3/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2 -
 x + 2)^(3/2)
```

**mupad** [B] time = 4.20, size = 49, normalized size = 1.04

$$-\frac{442\,x-5720\,x\,\left(3\,x^2-x+2\right)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,\left(14283\,x^2-4761\,x+9522\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

```
[Out] -(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/2
)*(14283*x^2 - 4761*x + 9522))
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

## 3.255 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=85

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

```
[Out] -2/897*(101-77*x)/(3*x^2-x+2)^(3/2)-8/2197*arctanh(1/26*(9-8*x)*13^(1/2)/(3
*x^2-x+2)^(1/2))*13^(1/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 822, 12, 724, 206}

$$-\frac{4(691-13668x)}{268203\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}-\frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) - (4*(691 - 13668*x))/(268203
*Sqrt[2 - x + 3*x^2]) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2
])])/(169*Sqrt[13])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{223}{13}+\frac{308x}{13}}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{4\int \frac{3174}{13(1+2x)\sqrt{2-x+3x^2}}\,dx}{20631}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} + \frac{8}{169}\int \frac{1}{(1+2x)\sqrt{2-x+3x^2}}$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{16}{169}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, -$$

$$= -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.85

$$\frac{2\left(82008x^3-31482x^2+79077x-32963\right)}{268203\left(3x^2-x+2\right)^{3/2}} - \frac{8\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{169\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/2
)) - (8*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(169*Sqrt[13])
```

**fricas [A]** time = 0.68, size = 126, normalized size = 1.48

$$\frac{2\left(3174\sqrt{13}\left(9x^4-6x^3+13x^2-4x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\left(82008x^3-31482x^2\right.\right.}{3486639\left(9x^4-6x^3+13x^2-4x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(13
)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

```
+ 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*x^4
- 6*x^3 + 13*x^2 - 4*x + 4)
```

**giac [A]** time = 0.43, size = 101, normalized size = 1.19

$$\frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(4556x-1749\right)x+26359\right)x-32963\right)}{268203\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)^(3
/2)
```

**maple [B]** time = 0.01, size = 158, normalized size = 1.86

$$-\frac{8\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{2197}-\frac{2}{9\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{\frac{10x}{69}-\frac{5}{207}}{\left(3x^2-x+2\right)^{\frac{3}{2}}}+\frac{\frac{80x}{529}-\frac{40}{1587}}{\sqrt{3x^2-x+2}}+\frac{1}{39\left(-4x+3\left(x+\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)/(3*x^2-x+2)^(5/2),x)
```

```
[Out] -2/9/(3*x^2-x+2)^(3/2)+5/207*(6*x-1)/(3*x^2-x+2)^(3/2)+40/1587*(6*x-1)/(3*x
^2-x+2)^(1/2)+1/39/(-4*x+3*(x+1/2)^2+5/4)^(3/2)+4/897*(6*x-1)/(-4*x+3*(x+1/
2)^2+5/4)^(3/2)+784/89401*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+4/169/(-4*x+
3*(x+1/2)^2+5/4)^(1/2)-8/2197*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-1
6*x+12*(x+1/2)^2+5)^(1/2))
```

**maxima [A]** time = 0.96, size = 93, normalized size = 1.09

$$\frac{8}{2197}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\left|2x+1\right|}-\frac{9\sqrt{23}}{23\left|2x+1\right|}\right)+\frac{18224x}{89401\sqrt{3x^2-x+2}}-\frac{2764}{268203\sqrt{3x^2-x+2}}+\frac{154x}{897\left(3x^2-x+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x +
2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

## 3.256 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=110

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

```
[Out] -2/11661*(197-837*x)/(3*x^2-x+2)^(3/2)-56/28561*arctanh(1/26*(9-8*x)*13^(1/
2)/(3*x^2-x+2)^(1/2))*13^(1/2)-24/1162213*(841-6633*x)/(3*x^2-x+2)^(1/2)-16
/2197*(3*x^2-x+2)^(1/2)/(1+2*x)
```

**Rubi** [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 806, 724, 206}

$$-\frac{24(841-6633x)}{1162213\sqrt{3x^2-x+2}}-\frac{16\sqrt{3x^2-x+2}}{2197(2x+1)}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}-\frac{56\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2197\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) - (24*(841 - 6633*x))/(116
2213*Sqrt[2 - x + 3*x^2]) - (16*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)) - (56
*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2197*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{2226}{169}+\frac{462x}{13}+\frac{6696x^2}{169}}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}+\frac{4\int \frac{\frac{50784}{2197}+\frac{19044x}{2197}}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx}{1587}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}+\frac{56\int \frac{}{(1}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{112\,\mathrm{Sul}}{}$$

$$= -\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{56\tanh}{}$$

**Mathematica [A]** time = 0.06, size = 111, normalized size = 1.01

$$\frac{26\left(1318464x^4+133308x^3+1021566x^2+569989x-170239\right)-88872\sqrt{13}\sqrt{3x^2-x+2}\left(6x^3+x^2+3x+2\right)}{45326307(2x+1)\left(3x^2-x+2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (26*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4) - 88872*S
qrt[13]*Sqrt[2 - x + 3*x^2]*(2 + 3*x + x^2 + 6*x^3)*ArcTanh[(9 - 8*x)/(2*Sq
rt[13]*Sqrt[2 - x + 3*x^2])])/(45326307*(1 + 2*x)*(2 - x + 3*x^2)^(3/2))
```

**fricas [A]** time = 0.74, size = 141, normalized size = 1.28

$$\frac{2\left(22218\sqrt{13}\left(18x^5-3x^4+20x^3+5x^2+4x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\left(1318464x\right.\right.}{45326307\left(18x^5-3x^4+20x^3+5x^2+4x+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
+ 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17023
9)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

**giac [B]** time = 0.36, size = 233, normalized size = 2.12

$$-\frac{56}{15108769}\sqrt{13}\left(872\sqrt{13}\sqrt{3}-529\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right)-\frac{56\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+}\right.\right.}{28561\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4)
)*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13
/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/3486639*(
((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x + 1
) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*x + 1
) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt(-8/(2
*x + 1) + 13/(2*x + 1)^2 + 3))
```

**maple** [A] time = 0.01, size = 165, normalized size = 1.50

$$-\frac{56\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^{2}+5}}\right)}{28561}+\frac{\frac{4x}{23}-\frac{2}{69}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}+\frac{\frac{96x}{529}-\frac{16}{529}}{\sqrt{3x^{2}-x+2}}+\frac{7}{507\left(-4x+3\left(x+\frac{1}{2}\right)^{2}+\frac{5}{4}\right)^{\frac{3}{2}}}-\frac{}{11661\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^2/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 2/69*(6*x-1)/(3*x^2-x+2)^(3/2)+16/529*(6*x-1)/(3*x^2-x+2)^(1/2)+7/507/(-4*x
+3*(x+1/2)^2+5/4)^(3/2)-128/11661*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(3/2)-1073
6/1162213*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+28/2197/(-4*x+3*(x+1/2)^2+5/
4)^(1/2)-56/28561*13^(1/2)*arctanh(2/13*(-4*x+9/2)*13^(1/2)/(-16*x+12*(x+1/
2)^2+5)^(1/2))-1/26/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

**maxima** [A] time = 0.97, size = 125, normalized size = 1.14

$$\frac{56}{28561}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{146496\,x}{1162213\sqrt{3\,x^{2}-x+2}}-\frac{9604}{1162213\sqrt{3\,x^{2}-x+2}}+\frac{420\,x}{3887\left(3\,x^{2}-x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^2
- x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3/2)
*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2 + 3x + 1}{(2x + 1)^2 \left(3x^2 - x + 2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

## 3.257 $\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=135

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

```
[Out] 2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)-2084/371293*arctanh(1/26*(9-8*x)*1
3^(1/2)/(3*x^2-x+2)^(1/2))*13^(1/2)+12/15108769*(25771+103526*x)/(3*x^2-x+2
)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144/28561*(3*x^2-x+2)^(1/2)/(1+2
*x)
```

**Rubi [A]** time = 0.21, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}-\frac{144\sqrt{3x^2-x+2}}{28561(2x+1)}-\frac{8\sqrt{3x^2-x+2}}{2197(2x+1)^2}+\frac{12(103526x+25771)}{15108769\sqrt{3x^2-x+2}}-\frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (12*(25771 + 103526*x)
)/(15108769*Sqrt[2 - x + 3*x^2]) - (8*Sqrt[2 - x + 3*x^2])/(2197*(1 + 2*x)^
2) - (144*Sqrt[2 - x + 3*x^2])/(28561*(1 + 2*x)) - (2084*ArcTanh[(9 - 8*x)/
(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{32433}{2197}+\frac{106830x}{2197}+\frac{160116x^2}{2197}+\frac{59088x^3}{2197}}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} + \frac{4\int \frac{\frac{1434648}{28561}+\frac{3345396x}{28561}+\frac{3097824x}{28561}}{(1+2x)^3\sqrt{2-x+3x^2}}}{1587}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{2\int \frac{-2}{(1+}}{}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2}}{2856}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2}}{2856}$$

$$= \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2}}{2856}$$

**Mathematica [A]** time = 0.07, size = 89, normalized size = 0.66

$$\frac{2\left(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141\right)}{45326307(2x+1)^2\left(3x^2-x+2\right)^{3/2}} - \frac{2084\tanh^{-1}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+}}\right.}{28561\sqrt{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

```
[Out] (2*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3 + 20074356*x^4 + 2030
4864*x^5))/(45326307*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)) - (2084*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(28561*Sqrt[13])
```

**fricas [A]** time = 0.83, size = 156, normalized size = 1.16

$$\frac{2\left(826827\sqrt{13}\left(36x^6+12x^5+37x^4+30x^3+13x^2+12x+4\right)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right)+13\right.}{589241991\left(36x^6+12x^5+37x^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x
+ 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x^3
+ 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12*x^5
 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

**giac [B]** time = 0.35, size = 233, normalized size = 1.73

$$\frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)+\frac{2\left(3\left(6\left(310578x-26213\right)x+1455755\right)x+\right.}{45326307\left(3x^2-x+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4
*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(3*x
^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 21*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(3) +
1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

**maple [A]** time = 0.01, size = 148, normalized size = 1.10

$$-\frac{2084\sqrt{13}\ \operatorname{arctanh}\left(\frac{2\left(-4x+\frac{9}{2}\right)\sqrt{13}}{13\sqrt{-16x+12\left(x+\frac{1}{2}\right)^2+5}}\right)}{371293}+\frac{521}{13182\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{1772x}{50531}-\frac{886}{151593}}{\left(-4x+3\left(x+\frac{1}{2}\right)^2+\frac{5}{4}\right)^{\frac{3}{2}}}+\frac{\frac{1128}{1510}}{\sqrt{-4x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+3*x+1)/(2*x+1)^3/(3*x^2-x+2)^(5/2),x)
```

```
[Out] 521/13182/(-4*x+3*(x+1/2)^2+5/4)^(3/2)+886/151593*(6*x-1)/(-4*x+3*(x+1/2)^2
+5/4)^(3/2)+188008/15108769*(6*x-1)/(-4*x+3*(x+1/2)^2+5/4)^(1/2)+1042/28561
/(-4*x+3*(x+1/2)^2+5/4)^(1/2)-2084/371293*13^(1/2)*arctanh(2/13*(-4*x+9/2)*
13^(1/2)/(-16*x+12*(x+1/2)^2+5)^(1/2))-1/338/(x+1/2)/(-4*x+3*(x+1/2)^2+5/4)
^(3/2)-1/104/(x+1/2)^2/(-4*x+3*(x+1/2)^2+5/4)^(3/2)
```

**maxima [A]** time = 0.99, size = 174, normalized size = 1.29

$$\frac{2084}{371293}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}\,x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}}+\frac{363210}{15108769\sqrt{3\,x^2-x+2}}+\frac{17}{50531\,(3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/a
bs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/sqr
t(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 - x
+ 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1/169
/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3*x^2
- x + 2)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x^2+3\,x+1}{(2\,x+1)^3\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

```
[Out] int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

# 3.258 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=208

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-$$

```
[Out] -f/c/h^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)+1/3*(6*b*c*e*h^2-3*b^2*f*h
^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*(2*c*x+b)/c/h^2/(-b*h+2*c*g)^3/(-g*(-b*h+c*
g)+b*h^2*x+c*h^2*x^2)^(1/2)+2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*x+g
)/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

**Rubi** [A] time = 0.42, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.064, Rules used = {1638, 792, 613}

$$\frac{(b+2cx)\left(-3b^2fh^2+6bceh^2+4c^2\left(fg^2-h(2dh+eg)\right)\right)}{3ch^2(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}+\frac{2\left(dh^2-egh+fg^2\right)}{3h^3(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^(
3/2)),x]
```

```
[Out] -(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) + ((6*b*c*e*h^2 -
 3*b^2*f*h^2 + 4*c^2*(f*g^2 - h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*c*h^2*(2*c*
g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) + (2*(f*g^2 - e*g*
h + d*h^2))/(3*h^3*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) + b*h^2*x
+ c*h^2*x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + b*x + c*x
^2)^(p + 1))/((2*c*d - b*e)*(m + p + 1)), x] + Dist[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1)), Int[(d + e*x)
^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
```

```
x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1]
&&  !IGtQ[m + p + 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0
]) && NeQ[m + p + 1, 0]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{\int \frac{\frac{1}{2}h^3(bfg-2cdh)+\frac{1}{2}h^3(2cfg-2ce\cdots}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2\cdots\right.}}{ch^4}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{2\left(fg^2-e\cdots\right.}{3h^3(2cg-bh)(g+hx)\sqrt{-\cdots}}$$

$$= -\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} + \frac{\left(6bceh^2-3b^2fh^2+4c^2\left(\cdots\right.\right.}{3ch^2(2cg-bh)^3\sqrt{-g(\cdots}}$$

**Mathematica [A]** time = 0.51, size = 219, normalized size = 1.05

$$\frac{2b^2h^2\left(f\left(8g^2+12ghx+3h^2x^2\right)-h(dh+2eg+3ehx)\right)-4bch\left(h\left(e\left(g^2+2ghx+3h^2x^2\right)-2dh(2g+hx)\right)+2fg^2\cdots\right.}{3h^3(g+hx)(bh-2cg)^3\sqrt{(g+h\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*
x^2)^(3/2)),x]
```

```
[Out] (2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2)
) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*x^
```

```
2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x) +
h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g + b
*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

**fricas [B]** time = 33.08, size = 465, normalized size = 2.24

$$\frac{2\left(8\,c^2fg^4 - b^2dh^4 + 4\left(c^2e - 4\,bcf\right)g^3h - 2\left(2\,c^2d + bce - 4\,b^2f\right)g^2h^2 + 2\left(4\,bcd - b^2e\right)gh^3 - \left(4\,c^2fg^2h^2 - 4\,c^2eg\right.}{3\left(8\,c^4g^6h^3 - 20\,bc^3g^5h^4 + 18\,b^2c^2g^4h^5 - 7\,b^3cg^3h^6 + b^4g^2h^7 - \left(8\,c^4g^3h^6 - 12\,bc^3g^2h^7 + 6\,b^2c^2gh^8 -\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="fricas")
```

```
[Out] 2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*c
*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*c^
2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*(c^
2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d - 3
*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h^3 -
20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7 - (8*
c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 - (8*c^4*
g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4*h^9)*x^
2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^3*c*g^2*h
^7 + 2*b^4*g*h^8)*x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx^2 + ex + d}{\left(ch^2x^2 + bh^2x - cg^2 + bgh\right)^{\frac{3}{2}}\left(hx + g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="giac")
```

```
[Out] integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(h
*x + g)), x)
```

**maple [A]** time = 0.01, size = 324, normalized size = 1.56

$$-\frac{2\left(chx + hb - cg\right)\left(-3b^2f\,h^4x^2 + 6bce\,h^4x^2 - 8c^2d\,h^4x^2 - 4c^2eg\,h^3x^2 + 4c^2f\,g^2h^2x^2 + 3b^2e\,h^4x - 12b^2fg\,h^3x - \right.}{3\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x)
```

```
[Out] -2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c^
2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*h^
4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x-8*
c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b*c*e
*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^3*h^3
-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2
)^(3/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, al
gorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for mo
re details)Is b*h-2*c*g zero or nonzero?
```

**mupad [B]** time = 5.75, size = 1089, normalized size = 5.24

$$\frac{16\,c^2\,f\,g^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-2\,b^2\,d\,h^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-8\,c^2\,d\,g^2\,h^2\,\sqrt{-c\,g^2+b\,g\,h}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/2
)),x)
```

```
[Out] (16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b*
h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2
 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*b^2*e
*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2 + c*h^
2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4
*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b*c*e*h^4*x
^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*
```

```
g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2)
+ 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b*c*d*
g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*f*g^3*h*(b*h^2*x
 - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h^2*x - c*g^2 + c*
h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^9*x^2 - 60*b*c^3*g^
5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*g^5*h^4*x + 54*b^2*c^2*
g^4*h^5 - 24*c^4*g^4*h^5*x^2 - 24*c^4*g^3*h^6*x^3 + 6*b^4*g*h^8*x + 18*b^2*
c^2*g^2*h^7*x^2 - 84*b*c^3*g^4*h^5*x - 39*b^3*c*g^2*h^7*x - 15*b^3*c*g*h^8*
x^2 + 90*b^2*c^2*g^3*h^6*x + 12*b*c^3*g^3*h^6*x^2 + 36*b*c^3*g^2*h^7*x^3 -
18*b^2*c^2*g*h^8*x^3)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2}{\left(\left(g+hx\right)\left(bh-cg+chx\right)\right)^{\frac{3}{2}}\left(g+hx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2)
,x)
```

```
[Out] Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h*
x)), x)
```

# 3.259 $\int \sqrt{d+ex}\,\sqrt{a+bx+cx^2}\,\left(A+Bx+Cx^2\right)dx$

**Optimal**. Leaf size=906

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\,\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\,\left(d\left(8Cd^2-3e(4Bd\right.\right.}{}$$

```
[Out] 2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-2/21*(-3*B*c*e+2*C*b*e+2*C*c*d)
*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c^2/e+2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*B*
b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e+C*
d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b
*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3
/e^3+1/315*(2*(4*c^2*d^2-b^2*e^2-3/2*c*e*(-2*a*e+b*d))*(8*b^2*C*e^2-c*e*(12
*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))-5*c*e*(-b*e+2*c*d)*(6*
b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C*e^2-c*d*(9*B*e+C*d))))*El
lipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*
(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^4/
e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-
2/315*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e^2*(-7*A*b*e-10*B*a*e+B*b*d+2
*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B
*d)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a
)^(1/2)
```

**Rubi [A]** time = 2.70, antiderivative size = 905, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1653, 832, 814, 843, 718, 424, 419}

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\,\left(cx^2+bx+a\right)^{3/2}}{21c^2e}+\frac{2\sqrt{d+ex}\,\left(\left(8Cd^3-3de(4Bd\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] (2*Sqrt[d + e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(
8*C*d^3 - 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) -
 a*e*(C*d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e)
 - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(315*c^3*e^
3) - (2*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2)
)/(21*c^2*e) + (2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - (Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 - b*
c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 - b^
2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a
*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*
d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e -
10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d*e*
(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*c^4*e^4*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
```

```
b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
```

```
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)dx = \frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce} + \frac{2\int\sqrt{d+ex}\left(-\frac{3}{2}e(bCd-3A\right.}{}$$

$$= -\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{21c^2e} + \frac{2C(d+}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3\right.\right.}{}$$

$$= \frac{2\sqrt{d+ex}\left(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3\left(8Cd^3-3\right.\right.}{}$$

**Mathematica [C]** time = 14.99, size = 15669, normalized size = 17.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

**maple [B]** time = 0.19, size = 19955, normalized size = 22.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
[Out] int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex}\left(A+Bx+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

# 3.260 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$

**Optimal**. Leaf size=668

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/7*C*(c*x^2+b*x+a)^(3/2)*(e*x+d)^(1/2)/c/e-2/105*(5*c*e*(-7*A*c*e+C*a*e+3*
C*b*d)-(-b*e+4*c*d)*(-7*B*c*e+4*C*b*e+6*C*c*d)+3*c*e*(-7*B*c*e+4*C*b*e+6*C*
c*d)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+1/105*(5*c*e*(-b*e+2*c*d)
*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e+6*C*c*d)*(8*c^2*d^2-2*b^2*e^2-3
*c*e*(-2*a*e+b*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^3/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)+2/105*(a*e^2-b*d*e+c*d^2)*(4*b^2*C*e^2+c*e*(-7*B*b*e-
10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d)))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.19, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce(-10aCe-7bBe+8bCd)+c^2\left(48Cd^2-14\right.\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[d + e*x]*(5*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C
*d - 7*B*c*e + 4*b*C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b
```

```
*x + c*x^2])/(105*c^2*e^3) + (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7
*c*e) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*e
+ a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*(b*
d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^
3*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*A
*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^3*e^4*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2
])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx = \frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce} + \frac{2\int \frac{\left(-\frac{1}{2}e(3bCd-7Ace+aCe)-\frac{1}{2}e(6cCd-7Bce+4bC}{\sqrt{d+ex}}}{7ce^2}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe}{105c^2e^3}$$

$$= -\frac{2\sqrt{d+ex}\,(5ce(3bCd-7Ace+aCe)-(4cd-be)(6cCd-7Bce+4bCe}{105c^2e^3}$$

**Mathematica [C]** time = 14.41, size = 9965, normalized size = 14.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

**maple [B]** time = 0.07, size = 12761, normalized size = 19.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

**sympy** [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

# 3.261 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}} dx$

**Optimal**. Leaf size=749

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce\left(2ae(9Cd-5Be)-b\left(32Cd^2-5e(5Bd-3Ae)\right)\right)+bCe^2(bd}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-2/15*(b*C*e^2*(-a*e+b*d)+c^2*d*(24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(a*e*
(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+b*C*d-6*c*C*
d^2/e-5*A*c*e-a*C*e)*x)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-b*d*
e+c*d^2)-1/15*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*C*a*e+8*C*b*d)-c^2*(48*C*d^2-10*
e*(-3*A*e+4*B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)/c^2/e^4/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)+2/15*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(24*C*d^2-5*e*(-3*A*e
+4*B*d))-c*e*(2*a*e*(-5*B*e+9*C*d)-b*(32*C*d^2-5*e*(-3*A*e+5*B*d))))*Ellipt
icF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2
*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))^(1/2)/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.52, antiderivative size = 746, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 814, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-}{15c^2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C*d^3 - 5*d*e*(4*B*d - 3*A
*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 - 4*B*d*e + 3*A*e^2)) + 3*c*
e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e - a*C*e)*x)*Sqrt[a + b*x + c
*x^2])/(15*c*e^3*(c*d^2 - b*d*e + a*e^2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (Sqrt[2]*S
qrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*e) - c^2*(48
*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/
(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[
b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(15*c^2*e^4*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*e^2*(b*d -
a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d^2 - 5*b*e*(
5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
(-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^2*e^4*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
```

```
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd-2Ae)+e(Acd-aCd\cdots}{2e}\right.}{\cdots}}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9C\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9C\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9C\cdots\right.\right.}{\cdots}$$

$$= -\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2\left(24Cd^3-5de(4Bd-3Ae)\right)+ce\left(ae(9C\cdots\right.\right.}{\cdots}$$

**Mathematica [C]** time = 14.04, size = 13240, normalized size = 17.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
[Out] Result too large to show
```

**fricas [F]** time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^2x^2+2\,dex+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^2*x^2 + 2
*d*e*x + d^2), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

**maple [B]** time = 0.08, size = 8221, normalized size = 10.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{{\left(d + e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(3/2),x)
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

## 3.262 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

**Optimal**. Leaf size=712

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)F\left(\sin^{-1}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)-2/3*(e*(-a*e+b*d)*(-3*B*e+7*C*d)-c*d*(8*C*d^2-e*(-A*e+4*B*d))+e^2*(B
*c*d+b*C*d-2*c*C*d^2/e-A*c*e-a*C*e)*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e
+c*d^2)/(e*x+d)^(1/2)+1/3*(2*(4*c*d-1/2*b*e)*(B*c*d+b*C*d-2*c*C*d^2/e-A*c*e
-a*C*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d)))*EllipticE(1/2*((b+2*c*x
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e
*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^3/(a*e^2-b*d*e+c*d^2)
/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3
*(e*(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))*EllipticF(1/2*
((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a
*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))^(1/2)/c/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.27, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 812, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(e(-2aCe-3bBe+8bCd)-2c\left(8Cd^2-e(4Bd-Ae)\right)\right)F\left(\sin^{-1}\right.}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((8*c*C*d^3)/e - c*d*(4*B*d - A*e) - (b*d - a*e)*(7*C*d - 3*B*e) - e*(B*
c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(3*e
```

```
^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (2*(C*d^2 - e*(B*d - A*e))*(a +
 b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + (Sqrt[
2]*Sqrt[b^2 - 4*a*c]*(2*(4*c*d - (b*e)/2)*(B*c*d + b*C*d - (2*c*C*d^2)/e -
A*c*e - a*C*e) + 6*c*(b*d*(C*d - B*e) + e*(A*c*d - a*C*d + a*B*e)))*Sqrt[d
+ e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^3*(c*d^2 - b*d*e + a
*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
+ c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) -
 2*c*(8*C*d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^4*Sqrt[d + e
*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
```

```
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{\left(-\frac{3(bd(Cd-Be)+e(Acd-aCd+aBe))}{2e}+\right.}{}}{3\left(c\right.}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd + bCd - \frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd + bCd - \frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd + bCd - \frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$= \frac{2\left(\frac{8cCd^3}{e} - cd(4Bd-Ae) - (bd-ae)(7Cd-3Be) - e\left(Bcd + bCd - \frac{2cCd^2}{e}\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

**Mathematica [C]** time = 14.27, size = 8456, normalized size = 11.88

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
[Out] Result too large to show
```

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^3x^3+3\,de^2x^2+3\,d^2ex+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^3*x^3 + 3
*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

**maple [B]** time = 0.13, size = 21038, normalized size = 29.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^2\right)\sqrt{a + bx + cx^2}}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(5/2),x)
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

# 3.263 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx$

**Optimal**. Leaf size=992

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(c^2\left(24Cd^2-e(4Bd+Ae)\right)d^3-ce\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37\right.\right.\right.}{}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)-2/15*(c^2*d^3*(24*C*d^2-e*(A*e+4*B*d))+e^2*(15*b^2*C*d^3+5*a^2*e^2*(
B*e+C*d)-a*b*e*(2*A*e^2+3*B*d*e+22*C*d^2))-c*d*e*(b*d*(A*e^2-6*B*d*e+41*C*d
^2)-a*e*(7*A*e^2-7*B*d*e+37*C*d^2))+e*(5*c^2*d^2*(6*C*d^2-e*(A*e+B*d))+e^2*
(15*a^2*C*e^2-5*a*b*e*(-B*e+8*C*d)+b^2*(-2*A*e^2-3*B*d*e+23*C*d^2))-c*e*(5*
b*d*(-A*e^2-2*B*d*e+11*C*d^2)-a*e*(3*A*e^2-13*B*d*e+53*C*d^2)))*x)*(c*x^2+b
*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/2)+1/15*(2*c^2*d^2*(24*C*d
^2-e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A*e^2-3*B
*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e^2-8*B*d*
e+43*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))
^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+
b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/15*(15*b*C*e^2*(-a*e+b*d)+2*c^2*d*(24*C
*d^2-e*(A*e+4*B*d))+c*e*(10*a*e*(-B*e+5*C*d)-b*(-A*e^2-9*B*d*e+64*C*d^2)))*
EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^4/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.92, antiderivative size = 989, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 810, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{2\left(\left(24Cd^5-d^3e(4Bd+Ae)\right)c^2-de\left(bd\left(41Cd^2-6Bed+Ae^2\right)-ae\left(37\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] (-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^2*(15*b^2*C*d^3 + 5*a^2*e^2*(
C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*e^2)) - c*d*e*(b*d*(41*C*d^2 -
 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e + 7*A*e^2)) + e^2*((30*c^2*C*d^
4)/e + 15*a^2*C*e^3 - 5*c^2*d^2*(B*d + A*e) - 5*a*b*e^2*(8*C*d - B*e) + a*c
*e*(53*C*d^2 - e*(13*B*d - 3*A*e)) - 5*b*c*d*(11*C*d^2 - e*(2*B*d + A*e)) +
 b^2*e*(23*C*d^2 - e*(3*B*d + 2*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*e^3*(c
*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a +
b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(5/2)) + (Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C
*e^2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(
b*d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2))
)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^4*(c*d^2 - b
*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(15*b*C*e^2*(b*d - a*e)
+ c^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e
) - 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c*e^4*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
```

```
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+2Ae)+5e(Acd-aC\ldots}{2e}\right.}{\ldots}}{5\ldots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\ldots\right.\right.}{\ldots}$$

$$= -\frac{2\left(c^2\left(24Cd^5-d^3e(4Bd+Ae)\right)+e^2\left(15b^2Cd^3+5a^2e^2(Cd+Be)-abe\ldots\right.\right.}{\ldots}$$

**Mathematica [C]** time = 14.67, size = 12997, normalized size = 13.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^4x^4+4\,de^3x^3+6\,d^2e^2x^2+4\,d^3ex+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^4*x^4 + 4
*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="gia
c")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(d*exp(1)+x*exp(1
)^2)]integrate()  Bad Argument Typeintegrate()  Bad Argument TypeEvaluation
 time: 23.8Unable to transpose Error: Bad Argument Value
```

**maple [B]** time = 0.25, size = 48427, normalized size = 48.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(7/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2 + B\,x + A\right)\,\sqrt{c\,x^2 + b\,x + a}}{{(d + e\,x)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^{2}\right)\sqrt{a + bx + cx^{2}}}{\left(d + ex\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(7/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

# 3.264 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx$

**Optimal**. Leaf size=1363

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}-\frac{2\left(c^2\left(24Cd^2+e(4Bd+3Ae)\right)d^3-ce\left(bd\left(43Cd^2+6Bed+15Ae^2\right)-a\right.\right.}{}$$

```
[Out] -2/7*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(7/2)-2/105*(c^2*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-e^2*(7*a^2*e^2*(-3*B*e+C*d
)-b^2*d*(8*A*e^2+6*B*d*e+15*C*d^2)+a*b*e*(12*A*e^2+23*B*d*e+12*C*d^2))-c*d*
e*(b*d*(15*A*e^2+6*B*d*e+43*C*d^2)-a*e*(19*A*e^2+9*B*d*e+33*C*d^2))+e*(7*c^
2*d^2*(6*C*d^2+e*(-3*A*e+B*d))+e^2*(35*a^2*C*e^2-7*a*b*e*(-B*e+12*C*d)+b^2*
(-4*A*e^2-3*B*d*e+45*C*d^2))+c*e*(a*e*(-5*A*e^2-9*B*d*e+93*C*d^2)-b*(-21*A*
d*e^2+91*C*d^3)))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(5/2)+2/105*(2*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*
b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*
(-29*A*e+15*B*d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B
*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B
*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)-1/105*(2
*c^3*d^3*(24*C*d^2+e*(3*A*e+4*B*d))-b*e^3*(35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d
)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*
d))-b*d*(128*C*d^2+e*(9*A*e+19*B*d)))+c*e^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b
*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*d*(103*C*d^2+e*(19*A*e+9*B*d))))*Ellipti
cE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*
e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^
2-b*d*e+c*d^2)^3/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2/105*(2*c^2*d^2*(24*C*d^2+e*(3*A*e+4*B*d))+c*e*(2*a*e*(51*C*d
^2+e*(-5*A*e+12*B*d))-b*d*(104*C*d^2+3*e*(2*A*e+5*B*d)))+e^2*(70*a^2*C*e^2-
7*a*b*e*(B*e+18*C*d)+b^2*(60*C*d^2+e*(4*A*e+3*B*d))))*EllipticF(1/2*((b+2*c
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 4.20, antiderivative size = 1363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}-\frac{2\left(\left(24Cd^5+e(4Bd+3Ae)d^3\right)c^2-de\left(bd\left(43Cd^2+6Bed+15Ae^2\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

```
[Out] (2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 14*a*b
*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*e*(69
*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e))) + c*e
^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e)) + b^
2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d
^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]) - (2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3
*A*e)) - e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2
) + a*b*e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*
e + 15*A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(7*c^2*(6*C*d^4 +
d^2*e*(B*d - 3*A*e)) + e^2*(35*a^2*C*e^2 - 7*a*b*e*(12*C*d - B*e) + b^2*(45
*C*d^2 - 3*B*d*e - 4*A*e^2)) + c*e*(a*e*(93*C*d^2 - 9*B*d*e - 5*A*e^2) - b*
(91*C*d^3 - 21*A*d*e^2)))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e
 + a*e^2)^2*(d + e*x)^(5/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)
^(3/2))/(7*e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(7/2)) - (Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(2*c^3*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*C*e^2 - 1
4*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2*d*e*(2*a*
e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d + 9*A*e)))
+ c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*d - 29*A*e))
 + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]
*Sqrt[b^2 - 4*a*c]*(c^2*(48*C*d^4 + 2*d^2*e*(4*B*d + 3*A*e)) + c*e*(2*a*e*(
51*C*d^2 + e*(12*B*d - 5*A*e)) - b*(104*C*d^3 + 3*d*e*(5*B*d + 2*A*e))) + e
^2*(70*a^2*C*e^2 - 7*a*b*e*(18*C*d + B*e) + b^2*(60*C*d^2 + e*(3*B*d + 4*A*
e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c
] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(105*e^4*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x
]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{7e\left(cd^2-bde+ae^2\right)(d+ex)^{7/2}} - \frac{2\int \frac{\left(-\frac{3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+}{2e}\right.}{}}{7(}$$

$$= -\frac{2\left(c^2\left(24Cd^5+d^3e(4Bd+3Ae)\right)-e^2\left(7a^2e^2(Cd-3Be)-b^2d\left(15Cd^2+\right.\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b\right.\right.}{}$$

$$= \frac{2\left(c^3\left(48Cd^5+2d^3e(4Bd+3Ae)\right)-be^3\left(35a^2Ce^2-14abe(3Cd+Be)+b\right.\right.}{}$$

**Mathematica [C]** time = 16.02, size = 19853, normalized size = 14.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
[Out] Result too large to show
```

**fricas [F]** time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{e^5x^5+5\,de^4x^4+10\,d^2e^3x^3+10\,d^3e^2x^2+5\,d^4ex+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^5*x^5 + 5
*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

**maple [B]** time = 0.43, size = 88790, normalized size = 65.14

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(9/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}}{{\left(d+e\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + Bx + Cx^{2}\right) \sqrt{a + bx + cx^{2}}}{\left(d + ex\right)^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(9/2),x)
[Out] Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

# 3.265 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx$

**Optimal**. Leaf size=1904

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{9e\left(cd^2-bed+ae^2\right)(d+ex)^{9/2}}-\frac{2\left(c^2\left(8Cd^2+e(4Bd+5Ae)\right)d^3-ce\left(3bd\left(5Cd^2+2Bed+5Ae^2\right)-ae\right.\right.}{}$$

```
[Out] -2/9*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(9/2)+2/315*(2*c^3*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^
2+7*B*d*e+9*C*d^2)-b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B
*e+17*C*d)-a*b*e*(-9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^
2))-b*e^3*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)
))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(3/2)-2/105*(c^2*d
^3*(8*C*d^2+e*(5*A*e+4*B*d))-e^2*(3*a^2*e^2*(-5*B*e+3*C*d)-a*b*e*(-10*A*e^2
-17*B*d*e+2*C*d^2)-b^2*d*(8*A*e^2+4*B*d*e+5*C*d^2))-c*d*e*(3*b*d*(5*A*e^2+2
*B*d*e+5*C*d^2)-a*e*(13*A*e^2+11*B*d*e+7*C*d^2))+e*(3*c^2*d^2*(6*C*d^2+e*(-
5*A*e+3*B*d))+c*e*(a*e*(-7*A*e^2+B*d*e+47*C*d^2)-3*b*d*(-5*A*e^2+2*B*d*e+15
*C*d^2))+e^2*(21*a^2*C*e^2-3*a*b*e*(-B*e+16*C*d)+b^2*(25*C*d^2-e*(2*A*e+B*d
))))*x)*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(7/2)+2/315*(
2*c^4*d^4*(8*C*d^2+e*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+
3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e
+30*C*d^2)-a^2*e^2*(7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11
*C*d^2))-c*e^3*(126*a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*
e^2+7*B*d*e+5*C*d^2)+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(
-34*A*e^2+8*B*d*e+11*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*(c*x^2+b*x+a
)^(1/2)/e^3/(a*e^2-b*d*e+c*d^2)^4/(e*x+d)^(1/2)-1/315*(2*c^4*d^4*(8*C*d^2+e
*(5*A*e+4*B*d))+2*b^2*e^4*(21*a^2*C*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+
4*B*d*e+5*C*d^2))-6*c^2*e^2*(a*b*d*e*(-34*A*e^2-5*B*d*e+30*C*d^2)-a^2*e^2*(
7*A*e^2-36*B*d*e+30*C*d^2)-b^2*d^2*(11*A*e^2+3*B*d*e+11*C*d^2))-c*e^3*(126*
a^3*C*e^3-3*a^2*b*e^2*(29*B*e+12*C*d)-6*a*b^2*e*(-12*A*e^2+7*B*d*e+5*C*d^2)
+b^3*d*(56*A*e^2+25*B*d*e+20*C*d^2))+c^3*d^2*e*(6*a*e*(-34*A*e^2+8*B*d*e+11
*C*d^2)-b*d*(56*C*d^2+5*e*(4*A*e+5*B*d))))*EllipticE(1/2*((b+2*c*x+(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c
*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^4/(c*x^2+b
*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(2*c^3
*d^3*(8*C*d^2+e*(5*A*e+4*B*d))+3*c^2*d*e*(2*a*e*(-9*A*e^2+7*B*d*e+9*C*d^2)-
b*d*(5*A*e^2+7*B*d*e+16*C*d^2))+3*c*e^2*(2*a^2*e^2*(-5*B*e+17*C*d)-a*b*e*(-
```

```
9*A*e^2+5*B*d*e+41*C*d^2)+b^2*d*(7*A*e^2+3*B*d*e+15*C*d^2))-b*e^3*(21*a^2*C
*e^2-6*a*b*e*(2*B*e+3*C*d)+b^2*(8*A*e^2+4*B*d*e+5*C*d^2)))*EllipticF(1/2*((
b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c
+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2)/e^4/(a*e^2-b*d*e+c*d^2)^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1
/2)
```

**Rubi [A]** time = 6.24, antiderivative size = 1904, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1650, 810, 834, 843, 718, 424, 419}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]

[Out] (2*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a*e*(9*C*d^2 + 7
*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*c*e^2*(2*a^2*e^
2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) + b^2*d*(15*C*d^2
 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) + b^
2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 -
b*d*e + a*e^2)^3*(d + e*x)^(3/2)) + (2*(2*c^4*(8*C*d^6 + d^4*e*(4*B*d + 5*A
*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d + 4*A*e) - 6*a*e*(11*C*d^2 +
8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2 - 6*a*b*e*(3*C*d + 2*B*e) +
b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2*(a*b*d*e*(30*C*d^2 - 5*B*d*e
 - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e + 7*A*e^2) - b^2*d^2*(11*C*d^2
+ 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3 - 3*a^2*b*e^2*(12*C*d + 29*B*
e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2) + b^3*d*(20*C*d^2 + 25*B*d*e
+ 56*A*e^2)))*Sqrt[a + b*x + c*x^2])/(315*e^3*(c*d^2 - b*d*e + a*e^2)^4*Sqr
t[d + e*x]) - (2*(c^2*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) - e^2*(3*a^2*e^2*(3
*C*d - 5*B*e) - a*b*e*(2*C*d^2 - 17*B*d*e - 10*A*e^2) - b^2*d*(5*C*d^2 + 4*
B*d*e + 8*A*e^2)) - c*d*e*(3*b*d*(5*C*d^2 + 2*B*d*e + 5*A*e^2) - a*e*(7*C*d
^2 + 11*B*d*e + 13*A*e^2)) + e^2*((3*c^2*(6*C*d^4 + d^2*e*(3*B*d - 5*A*e)))
/e + c*(a*e*(47*C*d^2 + e*(B*d - 7*A*e)) - 3*b*(15*C*d^3 + d*e*(2*B*d - 5*A
*e))) + e*(21*a^2*C*e^2 - 3*a*b*e*(16*C*d - B*e) + b^2*(25*C*d^2 - e*(B*d +
 2*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(105*e^3*(c*d^2 - b*d*e + a*e^2)^2*(d
+ e*x)^(7/2)) - (2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(9*e*(c
*d^2 - b*d*e + a*e^2)*(d + e*x)^(9/2)) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c^4*
(8*C*d^6 + d^4*e*(4*B*d + 5*A*e)) - c^3*d^2*e*(56*b*C*d^3 + 5*b*d*e*(5*B*d
+ 4*A*e) - 6*a*e*(11*C*d^2 + 8*B*d*e - 34*A*e^2)) + 2*b^2*e^4*(21*a^2*C*e^2
 - 6*a*b*e*(3*C*d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)) - 6*c^2*e^2
*(a*b*d*e*(30*C*d^2 - 5*B*d*e - 34*A*e^2) - a^2*e^2*(30*C*d^2 - 36*B*d*e +
7*A*e^2) - b^2*d^2*(11*C*d^2 + 3*B*d*e + 11*A*e^2)) - c*e^3*(126*a^3*C*e^3
- 3*a^2*b*e^2*(12*C*d + 29*B*e) - 6*a*b^2*e*(5*C*d^2 + 7*B*d*e - 12*A*e^2)
```

```
+ b^3*d*(20*C*d^2 + 25*B*d*e + 56*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x
 + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(315*e^4*(c*d^2 - b*d*e + a*e^2)^4*Sqrt[(c*(d + e*x)
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*(2*c^3*(8*C*d^5 + d^3*e*(4*B*d + 5*A*e)) + 3*c^2*d*e*(2*a
*e*(9*C*d^2 + 7*B*d*e - 9*A*e^2) - b*d*(16*C*d^2 + 7*B*d*e + 5*A*e^2)) + 3*
c*e^2*(2*a^2*e^2*(17*C*d - 5*B*e) - a*b*e*(41*C*d^2 + 5*B*d*e - 9*A*e^2) +
b^2*d*(15*C*d^2 + 3*B*d*e + 7*A*e^2)) - b*e^3*(21*a^2*C*e^2 - 6*a*b*e*(3*C*
d + 2*B*e) + b^2*(5*C*d^2 + 4*B*d*e + 8*A*e^2)))*Sqrt[(c*(d + e*x))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*
EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sq
rt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315
*e^4*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
```

```
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{11/2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(a+bx+cx^2\right)^{3/2}}{9e\left(cd^2-bde+ae^2\right)(d+ex)^{9/2}} - \frac{2\int \frac{\left(-\frac{3\left(bCd^2-be(Bd+2Ae)+3e(Acd-aC\cdots\right.}{2e}\right.}{\cdots}}{9\cdots}$$

$$= -\frac{2\left(c^2\left(8Cd^5+d^3e(4Bd+5Ae)\right)-e^2\left(3a^2e^2(3Cd-5Be)-abe\left(2Cd^2-\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(2c^3\left(8Cd^5+d^3e(4Bd+5Ae)\right)+3c^2de\left(2ae\left(9Cd^2+7Bde-9Ae^2\right)-\cdots\right.\right.}{\cdots}$$

**Mathematica [C]** time = 18.39, size = 29140, normalized size = 15.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(11/2),x]
```

```
[Out] Result too large to show
```

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}}{e^6x^6 + 6\,de^5x^5 + 15\,d^2e^4x^4 + 20\,d^3e^3x^3 + 15\,d^4e^2x^2 + 6\,d^5ex + d^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="fr
icas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(e^6*x^6 + 6
*d*e^5*x^5 + 15*d^2*e^4*x^4 + 20*d^3*e^3*x^3 + 15*d^4*e^2*x^2 + 6*d^5*e*x +
 d^6), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="gi
ac")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

**maple [B]** time = 0.80, size = 153623, normalized size = 80.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}}{(ex + d)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^(11/2),x, algorithm="ma
xima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(11/2), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(C x^2 + B x + A\right) \sqrt{c x^2 + b x + a}}{(d + e x)^{11/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2),x)
```

```
[Out] int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(11/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**(11/2),x)
```

```
[Out] Timed out
```

# 3.266 $\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=724

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-\right.\right.\right.}{105c^4e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] -2/35*(-7*B*c*e+6*C*b*e+2*C*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+2/
7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/2)/c/e+2/105*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(e*x+d)^(1/2)*(c*x^2+b
*x+a)^(1/2)/c^3/e-1/105*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+13*C*a*e+9*C*b*d)+
c^3*d*(6*C*d^2-7*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*C*d)+b*(70*A*e^2+9
1*B*d*e+12*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)/c^4/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))^(1/2)-2/105*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e*(28*B*b*e
+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*EllipticF(1/2*((b+2*c*
x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2))))^(1/2)/c^4/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.78, antiderivative size = 724, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)+c^2\left(-\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{105c^3e}-\frac{2\sqrt{2}\sqrt{b^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*e) - (2*(
2*c*C*d - 7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(35*c^2
*e) + (2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - (Sqrt[2]*Sqrt[b
^2 - 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*
(6*C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C
*d^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(105*c^4*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2
- 7*e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(105*c^4*e^2*Sqrt[d + e*x]*Sq
rt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
```

```
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce} + \frac{2\int \frac{(d+ex)^{3/2}\left(-\frac{1}{2}e(bCd-7Ace+5aCe)-\frac{1}{2}e(2cCd-7Bce+6b}{\sqrt{a+bx+cx^2}}}{7ce^2}$$

$$= -\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e} + \frac{2C(d+ex)^{5/2}\sqrt{a+bx}}{7ce}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{}}{105c^3e}$$

$$= \frac{2\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2\left(6Cd^2-7e(3Bd+5Ae)\right)\right)\sqrt{}}{105c^3e}$$

**Mathematica [C]** time = 14.55, size = 9972, normalized size = 13.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
[Out] Result too large to show
```

**fricas [F]** time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Cex^3+(Cd+Be)x^2+Ad+(Bd+Ae)x\right)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*e*x^3 + (C*d + B*e)*x^2 + A*d + (B*d + A*e)*x)*sqrt(e*x + d)/sq
rt(c*x^2 + b*x + a), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)(ex + d)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

**maple [B]** time = 0.08, size = 14084, normalized size = 19.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)(ex + d)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^{3/2}\,\left(C\,x^2 + B\,x + A\right)}{\sqrt{c\,x^2 + b\,x + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.267 $\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=557

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\text{si}\right.}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] 2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/15*(-5*B*c*e+4*C*b*e+2*C*c*d)
*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e+1/15*(8*b^2*C*e^2-c*e*(10*B*b*e+9*
C*a*e+3*C*b*d)-c^2*(2*C*d^2-5*e*(3*A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(
2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^3/e^2/(c*x^2+b*x+a)^(1/2)/(c*
(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(-5*B*c*e+4*C*b*e+2*C*
c*d)*(a*e^2-b*d*e+c*d^2)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2
))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c^3/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.89, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1653, 832, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)+c^2\left(-\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E\left(\right.}{15c^3e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (-2*(2*c*C*d - 5*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(15*
c^2*e) + (2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) + (Sqrt[2]*Sqr
t[b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C
```

```
*d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)])/(15*c^3*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e +
4*b*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*c^3*e^2*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}dx = \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} + \frac{2\int \frac{\sqrt{d+ex}\left(-\frac{1}{2}e(bCd-5Ace+3aCe)-\frac{1}{2}e(2cCd-5Bce+4bCe)x\right.}{\sqrt{a+bx+cx^2}}}{5ce^2}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+c}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+c}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+c}}{5ce}$$

$$= -\frac{2(2cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+c}}{5ce}$$

**Mathematica [C]** time = 11.59, size = 992, normalized size = 1.78

$$\frac{\left(\frac{2(cCd+5Bce-4bCe)}{15c^2e}+\frac{2Cx}{5c}\right)\sqrt{d+ex}\left(cx^2+bx+a\right)}{\sqrt{a+x(b+cx)}} - \frac{2(d+ex)^{3/2}\sqrt{cx^2+bx+a}\left(\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2+e(3b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (((2*(c*C*d + 5*B*c*e - 4*b*C*e))/(15*c^2*e) + (2*C*x)/(5*c))*Sqrt[d + e*x]
*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(d + e*x)^(3/2)*Sqrt[a + b*x
 + c*x^2]*((-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(2*C*d^
2 - 5*e*(B*d + 3*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) +
 (a*e)/(d + e*x)))/(d + e*x)) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e
)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2
 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*
b*B*e + 9*a*C*e) + c^2*(-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I*ArcSinh[
(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b
*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*C*d + 10*B*c
*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d -
 5*B*e) - 15*A*c*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^2*(14*c*C*d +
10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2 - 17*a*C*e^2 +
 3*C*d*Sqrt[(b^2 - 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b^2 - 4*a*c)*e^2]
)))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*
d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d +
e*x])))/(15*c^3*e^3*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
 e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

**fricas [F]** time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

**maple [B]** time = 0.06, size = 8161, normalized size = 14.65

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)\sqrt{ex + d}}{\sqrt{cx^2 + bx + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d + e\,x}\,\left(C\,x^2 + B\,x + A\right)}{\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex}\,\left(A + Bx + Cx^2\right)}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

# 3.268 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=471

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)+c\left(2Cd^2-3e(Bd-Ae)\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

```
[Out] 2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)*
EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c^
2/e^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2
)+2/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*EllipticF(1/2*((b+2*c*x+(
-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
)))^(1/2)/c^2/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.48, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1653, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - (Sqrt[2]*Sqrt[b^2 - 4*a
*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*c^2*e^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
```

```
)*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C*
e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c^2*e^2*Sqrt[d
+ e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
```

```
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} + \frac{2\int \frac{-\frac{1}{2}e(bCd-3Ace+aCe)-\frac{1}{2}e(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{3ce^2}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{(2cCd-3Bce+2bCe)\int \frac{\sqrt{d+ex}}{\sqrt{a+bx+cx^2}}\,dx}{3ce^2} + \frac{\left(2cCd^2+\right.}{}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\cdots}\right.}{3c^2e^2\sqrt{\frac{c(d\cdots}{2cd-be-\cdots}}}$$

$$= \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}\,(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(\cdots}{\cdots}}}{3c^2e^2\sqrt{\frac{c(d+ex\cdots}{2cd-\left(b+\sqrt{b^2\cdots}\right.}}}$$

**Mathematica [C]** time = 12.50, size = 1080, normalized size = 2.29

$$\frac{\sqrt{cx^2+bx+a}\left(-4(2cCd-3Bce+2bCe)\sqrt{\frac{cd^2+e(ae-bd)}{-2cd+be+\sqrt{(b^2-4ac)e^2}}}\left(c\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ae}{d+ex}\right)}{d+ex}\right)+\frac{i\sqrt{2}\,(2cCd-3Bce+2b\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d +
 e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[(c
*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 +
 d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) +
(I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] -
(I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*
(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*B*e)))*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b
*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(S
qrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b
*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipti
cF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2
])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*c^2*e^3*Sq
rt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqr
t[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d
)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

**fricas [F]** time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{cex^3+(cd+be)x^2+ad+(bd+ae)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e*x^3 + (
c*d + b*e)*x^2 + a*d + (b*d + a*e)*x), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

**maple [B]** time = 0.05, size = 4251, normalized size = 9.03

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/3/c^2*(-2*C*x^3*c^2*e^3-2*C*x*b*c*d*e^2-2*C*x*a*c*e^3-2*C*x^2*b*c*e^3-2*
C*x^2*c^2*d*e^2+C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*El
lipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*
c+b^2)^(1/2)*b*d*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)
)^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(
1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))
*b*c*d*e^2-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*
((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((
b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellip
ticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*c*d*e^2
-3*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(
-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-
4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*
c*d*e^2+3*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-
2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2
*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Elliptic
F(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b*c*d*e^2+2*
C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*
a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a
*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*
(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*c*d
```

```
^2*e+6*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c
*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*
x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2
^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2
)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*c*d*e^2-2*C*a
*c*d*e^2-4*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((
-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipti
cE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*c^2*d^3+6*B
*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a
*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*
c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+
b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*c^2*d^2*e-4*C*2^(1/2)*(
-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1
/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/
2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c
/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)
/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*b*e^3+4*C*2^(1/2)*(-(e*x+d)*c/(
e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*
(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+
b^2)^(1/2)-b*e+2*c*d))^(1/2))*b^2*d*e^2+3*A*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^
2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)
-b*e+2*c*d))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3-6*A*2^(1/2)*(-(e*x+d)*c/(e*(-4
*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a
*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+
b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^
(1/2)-b*e+2*c*d))^(1/2))*c^2*d*e^2-6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^
(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1
/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-
2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+
2*c*d))^(1/2))*a*c*e^3+6*B*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*
c*d))^(1/2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*
d))^(1/2)*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))
^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2
),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2
))*a*c*e^3-C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((
```

```
-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+
2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*Ellipti
cF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*(-4*a*c+b^2
)^(1/2)*a*e^3+3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/
2)*((-2*c*x+(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)
*((b+2*c*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*El
lipticF(2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-
4*a*c+b^2)^(1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*a*b*e^
3-3*C*2^(1/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+
(-4*a*c+b^2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(
-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(
1/2)+b*e-2*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b^2*d*e^2+3*A*2^(1
/2)*(-(e*x+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*((-2*c*x+(-4*a*c+b^
2)^(1/2)-b)*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2)*((b+2*c*x+(-4*a*c+b^2
)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2)*EllipticF(2^(1/2)*(-(e*x
+d)*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)+b*e-2
*c*d)/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d))^(1/2))*b*c*e^3)*(e*x+d)^(1/2)*(c*x^
2+b*x+a)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)/e^3
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{\sqrt{d + e\,x}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

# 3.269 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=508

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(Ce(bd-ae)-c\left(2Cd^2-e(Bd-Ae)\right)\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b}}{2cd-(b+}\right.}{ce^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] -2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(
1/2)-(C*e*(-a*e+b*d)-c*(2*C*d^2-e*(-A*e+B*d)))*EllipticE(1/2*((b+2*c*x+(-4*
a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*x
^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2*(-B*e+
2*C*d)*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))
*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d
)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)
^(1/2)
```

**Rubi [A]** time = 0.65, antiderivative size = 506, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1650, 843, 718, 424, 419}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(-Ce(bd-ae)-ce(Bd-Ae)+2cCd^2\right)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2}}{2cd-(b+}\right.}{ce^2\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e^
2)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - C*e*(b*d - a*e)
 - c*e*(B*d - A*e))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c
))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
```

```
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(c*e^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d -
B*e)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a +
 b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
```

```
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{2\int \frac{-\frac{bd(Cd-Be)+e(Acd-aCd+aBe)}{2e}+\frac{1}{2}\left(Bcd+bCd-\frac{2cC\cdots}{}\right.}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}}{cd^2-bde+ae^2}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx}{e^2} - \frac{\left(B\cdots\right.}{}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\left(\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-\cdots\right.\right.}{ce\cdots}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\left(Bcd+bCd-\frac{2cCd^2}{e}-\cdots\right.}{ce\left(cd\cdots\right.}$$

**Mathematica** [C] time = 7.28, size = 772, normalized size = 1.52

$$\frac{2\left(-\frac{i(d+ex)^{3/2}\sqrt{1-\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)}}\sqrt{\frac{2\left(e(ae-bd)+cd^2\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+be-2cd\right)}+1}\left(\left(\sqrt{e^2\left(b^2-4ac\right)}-be+2cd\right)\left(Ce(ae-bd)+ce(Ae-Bd)+2cCd^2\right)E\left(i\sinh^{-1}\left(\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2 +
 C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(d
+ e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(
b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2
*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*Ellip
ticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(
b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) + 2*a*c*C*
d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e*Sqrt[(b^2
 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*d + Sqrt[(b
^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sqrt[(b^2 - 4*
a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*
c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)])
```

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{ce^2x^4+\left(2\,cde+be^2\right)x^3+ad^2+\left(cd^2+2\,bde+ae^2\right)x^2+\left(bd^2+2\,ade\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^2*x^4 +
 (2*c*d*e + b*e^2)*x^3 + a*d^2 + (c*d^2 + 2*b*d*e + a*e^2)*x^2 + (b*d^2 + 2
*a*d*e)*x), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

**maple [B]** time = 0.07, size = 6053, normalized size = 11.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{(d + e\,x)^{3/2}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{\frac{3}{2}}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

# 3.270 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=684

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(3Ce(bd-ae)-c\left(e(Bd-Ae)+2Cd^2\right)\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-\ldots}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(3/2)+2/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-
B*d*e+4*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)-
1/3*(c*d*(2*C*d^2+e*(-4*A*e+B*d))+e*(3*a*e*(-B*e+2*C*d)-b*(-2*A*e^2-B*d*e+4
*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/
2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*(3*C*e*(-a*e+b*d)-c*(2*C*d^2+e*(-A*e+B*d)
))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)/(2
*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/
2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 1.19, antiderivative size = 680, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 834, 843, 718, 424, 419}

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4a\ldots}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{3ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(3/2)) + (2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*
```

```
d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d
^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d +
2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]],
 (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*e^2*(c*d
^2 - b*d*e + a*e^2)^2*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 - 3*C*e
*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*c*e^2*(c*d^2 - b*d
*e + a*e^2)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(Bcd-3\right.}{(d+ex)^{3/2}\sqrt{a+bx+cx}}}{3\left(cd^2-bde+ae\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}} + \frac{2\left(2cCd^3+cde(Bd-4Ae)+3ae^2(2C\right.}{3e\left(cd^2-\right.}$$

**Mathematica** [C] time = 12.16, size = 1194, normalized size = 1.75

$$\frac{2\sqrt{cx^2+bx+a}\left(\frac{i\sqrt{1-\frac{2\left(cd^2+e(ae-bd)\right)}{\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}}\sqrt{\frac{2\left(cd^2+e(ae-bd)\right)}{\left(-2cd+be+\sqrt{\left(b^2-4ac\right)e^2}\right)(d+ex)}+1}\left(\left(2cd-be+\sqrt{\left(b^2-4ac\right)e^2}\right)\left(cd\left(2Cd^2+e(Bd-4Ae)\right)+e\left(-4bCd^2\right.\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
- b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e -
b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*d^2
 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(3/2)
*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(-2*C*
d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(
b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1 - (2*(c
*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x
))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(c*d*(2*C*d
^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*e*(-2*C*d +
 B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - (2*a*c*C*d^2*
e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*a*c)*e^2] + B*c
*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - 3*a*
B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^2 - 2*d*Sqrt[(b^2
 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b*e*(2*A*e^2*(3*c*d
 + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2])
 + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e)
)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[d + e*x])))/(3*e^3*(c*d^2
- b*d*e + a*e^2)^2*Sqrt[a + x*(b + c*x)]*Sqrt[((d + e*x)^2*(c*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2])
```

**fricas [F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2 + Bx + A\right)\sqrt{cx^2 + bx + a}\,\sqrt{ex + d}}{ce^3x^5 + \left(3\,cde^2 + be^3\right)x^4 + ad^3 + \left(3\,cd^2e + 3\,bde^2 + ae^3\right)x^3 + \left(cd^3 + 3\,bd^2e + 3\,ade^2\right)x^2 + \left(bd^3 + 3\,ad^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^3*x^5 +
 (3*c*d*e^2 + b*e^3)*x^4 + a*d^3 + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^3 + (c
*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^2 + (b*d^3 + 3*a*d^2*e)*x), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

**maple [B]** time = 0.13, size = 20481, normalized size = 29.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\,(ex + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2 + B\,x + A}{(d + e\,x)^{5/2}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{\frac{5}{2}}\,\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

# 3.271 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=944

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(c^2\left(2Cd^2+e(3Bd-23Ae)\right)d^2-e^2\left(\left(3Cd^2+2Bed+8Ae\right.\right.\right.}{}$$

```
[Out] -2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)
^(5/2)+2/15*(c*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e
^2-B*d*e+6*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(3/
2)+2/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*(B*
e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d^2)-
a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)
^3/(e*x+d)^(1/2)-1/15*(c^2*d^2*(2*C*d^2+e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^
2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B
*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*EllipticE(1/2*((b+2*c*x+(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^3/(c*
x^2+b*x+a)^(1/2)/(c*(e*x+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/15*(c
*d*(2*C*d^2+e*(-8*A*e+3*B*d))+e*(5*a*e*(-B*e+2*C*d)-b*(-4*A*e^2-B*d*e+6*C*d
^2)))*EllipticF(1/2*((b+2*c*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*
2^(1/2)*(-4*a*c+b^2)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(c*(e*x+d)
/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^
(1/2)/(c*x^2+b*x+a)^(1/2)
```

**Rubi** [A] time = 2.26, antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1650, 834, 843, 718, 424, 419}

$$-\frac{2\sqrt{cx^2+bx+a}\left(Cd^2-e(Bd-Ae)\right)}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}-\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed-23\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + (2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*
C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(15*e*
(c*d^2 - b*d*e + a*e^2)^2*(d + e*x)^(3/2)) + (2*(c^2*(2*C*d^4 + d^2*e*(3*B*
d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2*
B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^2
 - 29*B*d*e + 9*A*e^2))))*Sqrt[a + b*x + c*x^2])/(15*e*(c*d^2 - b*d*e + a*e^
2)^3*Sqrt[d + e*x]) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 + 2
*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*d^
2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^3*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sq
rt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6*C*d^2 -
 e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*
e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*e^2*(c*d^2 - b*d*e + a*e^2)^2*
Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} - \frac{2\int \frac{-\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)}{2e}-\frac{1}{2}\left(3Bcd-\right.}{(d+ex)^{5/2}\sqrt{a+bx+c}}}{5\left(cd^2-bde+a\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2C\right.}{15e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2C\right.}{15e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2C\right.}{15e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2C\right.}{15e\left(cd^2-\right.}$$

$$= -\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}} + \frac{2\left(2cCd^3+cde(3Bd-8Ae)+5ae^2(2C\right.}{15e\left(cd^2-\right.}$$

<u>**Mathematica**</u> [C]   time = 15.04, size = 12295, normalized size = 13.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
[Out] Result too large to show
```

**fricas [F]** time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)\sqrt{cx^2+bx+a}\,\sqrt{ex+d}}{ce^4x^6+\left(4\,cde^3+be^4\right)x^5+ad^4+\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^4+2\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^3+\left(cd^4+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)/(c*e^4*x^6 +
 (4*c*d*e^3 + b*e^4)*x^5 + a*d^4 + (6*c*d^2*e^2 + 4*b*d*e^3 + a*e^4)*x^4 +
2*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^3 + (c*d^4 + 4*b*d^3*e + 6*a*d^2*
e^2)*x^2 + (b*d^4 + 4*a*d^3*e)*x), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\,(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

**maple [B]** time = 0.26, size = 46697, normalized size = 49.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\,(ex+d)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{7/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^{2}}{\left(d + ex\right)^{\frac{7}{2}} \sqrt{a + bx + cx^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

# 3.272 $\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=510

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g+h}{2cg-\left(b-\sqrt{b^2}\right.}\right.}{ch^3(m+1)(m+2p+3}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(1+p)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+c*(
2*f*g^2*(1+p)-h*(-d*h+e*g)*(3+m+2*p)))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appell
F1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(
2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/(2*c
*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(
1/2))))^p)-(b*f*h*(2+m+p)+c*(2*f*g*(1+p)-e*h*(3+m+2*p)))*(h*x+g)^(2+m)*(c*x
^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/
2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))/c/h^3/(2+m)/(3+m+2*p)/((
1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h
*(b+(-4*a*c+b^2)^(1/2))))^p)
```

**Rubi [A]** time = 0.83, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1653, 843, 759, 133}

$$\frac{(g+hx)^{m+1}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(m+1;-p,-p;m+2;\frac{2c(g+h}{2cg-\left(b-\sqrt{b^2}\right.}\right.}{ch^3(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) + ((f*h
*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*p))*(
g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (2*c*(g
 + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b +
 Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(1 + m)*(3 + m + 2*p)*(1 - (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + S
qrt[b^2 - 4*a*c])*h))^p) - ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3
 + m + 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p,
3 + m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))
/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(c*h^3*(2 + m)*(3 + m + 2*p)*(1 - (2
*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(
2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\int (g+hx)^m(-h(afh(1+m)}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{(2cfg(1+p)+bfh(2+m}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} - \frac{\left((2cfg(1+p)+bfh(2+m\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{1+p}}{ch(3+m+2p)} + \frac{\left(fh(bg-ah)(1+m)+2cf\right.}{}$$

**Mathematica [F]** time = 2.28, size = 0, normalized size = 0.00

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

**fricas [F]** time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

**maple [F]** time = 0.16, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+ex+d\right)\left(hx+g\right)^{m}\left(c\,x^{2}+bx+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
[Out] int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^{2}+ex+d\right)\left(cx^{2}+bx+a\right)^{p}\left(hx+g\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,{\left(c\,x^2+b\,x+a\right)}^p\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
[Out] Timed out
```

# 3.273 $\int (g+hx)^m \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=496

$$\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)+c\,(\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

```
[Out] f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(f*h*(-a*h+b*g)*(1+m)+c*(3*f*
g^2-h*(-d*h+e*g)*(4+m)))*(h*x+g)^(1+m)*AppellF1(1+m,-1/2,-1/2,2+m,2*c*(h*x+
g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1
/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-h*(b-(-4
*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^(
1/2)-1/2*(b*f*h*(5+2*m)+c*(6*f*g-2*e*h*(4+m)))*(h*x+g)^(2+m)*AppellF1(2+m,-
1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*
g-h*(b+(-4*a*c+b^2)^(1/2))))*(c*x^2+b*x+a)^(1/2)/c/h^3/(2+m)/(4+m)/(1-2*c*(
h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^(1/2)
```

**Rubi [A]** time = 0.67, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1653, 843, 759, 133}

$$\frac{\sqrt{a+bx+cx^2}\,(g+hx)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)\left(fh(m+1)(bg-ah)-ch\right.}{ch^3(m+1)(m+4)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) + ((3*c*f*g^2 +
 f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)*Sqrt[
a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/(2*c*g
- (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*
c])*h)])/(c*h^3*(1 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt
[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) - ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*S
qrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(2*
c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
```

```
4*a*c])*h)])/(2*c*h^3*(2 + m)*(4 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h)])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\int (g+hx)^m\left(-\frac{1}{2}h(3bfg+2a\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2+fh(bg-ah)(1+m)\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(\left(3cfg^2+fh(bg-ah)(1+m\right.\right.}{}$$

$$= \frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)} + \frac{\left(3cfg^2+fh(bg-ah)(1+m)\right.}{}$$

**Mathematica [F]** time = 1.46, size = 0, normalized size = 0.00

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

**fricas [F]** time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)\left(hx+g\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \left(f\,x^{2}+e x +d \right) \sqrt{c\,x^{2}+b x +a}\, \left(h x +g \right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] int((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx^{2}+bx+a}\left(fx^{2}+ex+d\right)\left(hx+g\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^m*(f*x^2+e*x+d)*(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g+h\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

```
[Out] int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(g + hx\right)^{m} \sqrt{a + bx + cx^{2}} \left(d + ex + fx^{2}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**m*(f*x**2+e*x+d)*(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

# 3.274 $\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$

**Optimal**. Leaf size=590

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g+}{2cg-\left(b-\sqrt{b}\right.}\right.}{2h^3p}$$

```
[Out] -1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1+p)/h/(a*h^2-b*g*h+c*g^2)/(1+p)/(
(h*x+g)^(2+2*p))-1/2*(2*c*(-d*g*h^2+f*g^3)+h*(2*a*h*(-e*h+2*f*g)-b*(-d*h^2-
e*g*h+3*f*g^2)))*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p],[-
2*p],-4*c*(h*x+g)*(-4*a*c+b^2)^(1/2)/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*
(b+(-4*a*c+b^2)^(1/2))))*(b+2*c*x-(-4*a*c+b^2)^(1/2))/h^2/(a*h^2-b*g*h+c*g^
2)/(1+2*p)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2)))/(((2*c*g-h*(b-(-4*a*c+b^2)^(1/2
)))*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(b+2*c*x-(-4*a*c+b^2)^(1/2))/(2*c*g-h*(b+(
-4*a*c+b^2)^(1/2))))^p)-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,2*c
*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)^(1/2))),2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b
^2)^(1/2))))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-h*(b-(-4*a*c+b^2)
^(1/2))))^p)/((1-2*c*(h*x+g)/(2*c*g-h*(b+(-4*a*c+b^2)^(1/2))))^p)
```

**Rubi [A]** time = 0.76, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1655, 759, 133, 806, 726}

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}F_1\left(-2p;-p,-p;1-2p;\frac{2c(g+}{2cg-\left(b-\sqrt{b}\right.}\right.}{2h^3p}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

```
[Out] -((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2*h*(c*g^2 - b*g*h +
a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) - (f*(a + b*x + c*x^2)^p*AppellF1[-2*
p, -p, -p, 1 - 2*p, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2
*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(2*h^3*p*(g + h*x)^(2*p
)*(1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g
+ h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])*h))^p) - ((2*c*(f*g^3 - d*g*h^2) -
 h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeometric2F1[-1
- 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (b + Sqrt[b^2
```

```
 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*h^2*(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b - Sqrt[b^2
 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sqrt[b^2 - 4*
a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 726

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((b - Rt[b^2 - 4*a*c, 2] + 2*c*x)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^p*Hypergeometric2F1[m + 1, -p, m + 2, (-4*c*Rt[b^2 - 4*a*c, 2]*(
d + e*x))/((2*c*d - b*e - e*Rt[b^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2
*c*x))])/((m + 1)*(2*c*d - b*e + e*Rt[b^2 - 4*a*c, 2])*(((2*c*d - b*e + e*R
t[b^2 - 4*a*c, 2])*(b + Rt[b^2 - 4*a*c, 2] + 2*c*x))/((2*c*d - b*e - e*Rt[b
^2 - 4*a*c, 2])*(b - Rt[b^2 - 4*a*c, 2] + 2*c*x)))^p), x] /; FreeQ[{a, b, c
, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 759

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - (e*(b - q))/(2*c)))^p*(1 - (d + e*x)/(d - (e*(b + q))/(2*c)))
^p), Subst[Int[x^m*Simp[1 - x/(d - (e*(b - q))/(2*c)), x]^p*Simp[1 - x/(d -
 (e*(b + q))/(2*c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*
d - b*e, 0] &&  !IntegerQ[p]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1655

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Dist[Coeff[Pq, x, q]/e^q, Int[(d
 + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Dist[1/e^q, Int[(d + e*x)^m*(
a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x], x]
, x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b, c
, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx=\frac{\int (g+hx)^{-3-2p}\left(-fg^2+dh^2-h(2fg-eh)x\right)\left(a+bx+cx^2\right.}{h^2}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{(2}{}$$

$$=-\frac{\left(fg^2-h(eg-dh)\right)(g+hx)^{-2(1+p)}\left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}-\frac{f(}{}$$

**Mathematica [F]** time = 3.46, size = 0, normalized size = 0.00

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
[Out] Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

**fricas [F]** time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fric
as")
```

```
[Out] integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac
")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \left(f\,x^2+ex+d\right)\left(hx+g\right)^{-2p-3}\left(c\,x^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)^(-2*p-3)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

```
[Out] int((h*x+g)^(-2*p-3)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p\left(hx+g\right)^{-2\,p-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

```
[Out] int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

## 3.275 $\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right)$

**Optimal**. Leaf size=41

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

```
[Out] b*f*(3+2*p)*(f*x^2+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+d)^(1+p)
```

**Rubi [A]** time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1815, 12, 261}

$$\frac{bf(2p+3)\left(d+fx^2\right)^{p+1}}{p+1}+2cfx\left(d+fx^2\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x
]
```

```
[Out] (b*f*(3 + 2*p)*(d + f*x^2)^(1 + p))/(1 + p) + 2*c*f*x*(d + f*x^2)^(1 + p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+fx^2\right)^p \left(2cdf+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = 2cfx\left(d+fx^2\right)^{1+p} + \frac{\int 2bf^3(3+2p)^2x\left(d+fx^2\right)^{\cdots}}{f(3+2p)}$$
$$= 2cfx\left(d+fx^2\right)^{1+p} + \left(2bf^2(3+2p)\right)\int x\left(d+fx^{\cdots}\right.$$
$$= \frac{bf(3+2p)\left(d+fx^2\right)^{1+p}}{1+p} + 2cfx\left(d+fx^2\right)^{1+p}$$

**Mathematica [C]** time = 0.11, size = 119, normalized size = 2.90

$$\frac{f\left(d+fx^2\right)^p\left(\frac{fx^2}{d}+1\right)^{-p}\left((2p+3)\left(3b\left(d+fx^2\right)\left(\frac{fx^2}{d}+1\right)^p+2cf(p+1)x^3\,{}_2F_1\left(\frac{3}{2},-p;\frac{5}{2};-\frac{fx^2}{d}\right)\right)+6cd(p+1)x\,{}_2\cdots\right.}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + f*x^2)^p*(2*c*d*f + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*
x^2),x]
```

```
[Out] (f*(d + f*x^2)^p*(6*c*d*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -((f*x^2)
/d)] + (3 + 2*p)*(3*b*(d + f*x^2)*(1 + (f*x^2)/d)^p + 2*c*f*(1 + p)*x^3*Hyp
ergeometric2F1[3/2, -p, 5/2, -((f*x^2)/d)])))/(3*(1 + p)*(1 + (f*x^2)/d)^p)
```

**fricas [A]** time = 0.58, size = 75, normalized size = 1.83

$$\frac{\left(2\,bdfp+2\left(cf^2p+cf^2\right)x^3+3\,bdf+\left(2\,bf^2p+3\,bf^2\right)x^2+2\left(cdfp+cdf\right)x\right)\left(fx^2+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="fricas")
```

```
[Out] (2*b*d*f*p + 2*(c*f^2*p + c*f^2)*x^3 + 3*b*d*f + (2*b*f^2*p + 3*b*f^2)*x^2
+ 2*(c*d*f*p + c*d*f)*x)*(f*x^2 + d)^p/(p + 1)
```

**giac [B]** time = 0.18, size = 141, normalized size = 3.44

$$\frac{2\left(fx^2+d\right)^p cf^2px^3+2\left(fx^2+d\right)^p bf^2px^2+2\left(fx^2+d\right)^p cf^2x^3+2\left(fx^2+d\right)^p cdfpx+3\left(fx^2+d\right)^p bf^2x^2+2\left(f\cdots\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="giac")
```

```
[Out] (2*(f*x^2 + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + d)^p*b*f^2*p*x^2 + 2*(f*x^2 + d)^
p*c*f^2*x^3 + 2*(f*x^2 + d)^p*c*d*f*p*x + 3*(f*x^2 + d)^p*b*f^2*x^2 + 2*(f*
x^2 + d)^p*b*d*f*p + 2*(f*x^2 + d)^p*c*d*f*x + 3*(f*x^2 + d)^p*b*d*f)/(p +
1)
```

**maple [A]** time = 0.00, size = 36, normalized size = 0.88

$$\frac{\left(2pcx+2pb+2cx+3b\right)f\left(f\,x^{2}+d\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] f*(f*x^2+d)^(1+p)*(2*c*p*x+2*b*p+2*c*x+3*b)/(1+p)
```

**maxima [A]** time = 0.58, size = 59, normalized size = 1.44

$$\frac{\left(2\,cf^{2}\left(p+1\right)x^{3}+bf^{2}\left(2\,p+3\right)x^{2}+2\,cdf\left(p+1\right)x+bdf\left(2\,p+3\right)\right)\left(fx^{2}+d\right)^{p}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+d)^p*(2*c*d*f+2*b*f^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, al
gorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*f^2*(2*p + 3)*x^2 + 2*c*d*f*(p + 1)*x + b*d*f*(2*p
 + 3))*(f*x^2 + d)^p/(p + 1)
```

**mupad [B]** time = 4.25, size = 58, normalized size = 1.41

$$\left(f\,x^2+d\right)^p\,\left(2\,c\,f^2\,x^3+2\,c\,d\,f\,x+\frac{b\,f^2\,x^2\,\left(2\,p+3\right)}{p+1}+\frac{b\,d\,f\,\left(2\,p+3\right)}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*x^2)^p*(2*c*d*f + 2*b*f^2*x*(2*p + 3) + 2*c*f^2*x^2*(2*p + 3)),x
)
```

```
[Out] (d + f*x^2)^p*(2*c*f^2*x^3 + 2*c*d*f*x + (b*f^2*x^2*(2*p + 3))/(p + 1) + (b
*d*f*(2*p + 3))/(p + 1))
```

**sympy [B]** time = 13.25, size = 221, normalized size = 5.39

$$\begin{cases} \frac{2bdfp(d+fx^2)^p}{p+1} + \frac{3bdf(d+fx^2)^p}{p+1} + \frac{2bf^2px^2(d+fx^2)^p}{p+1} + \frac{3bf^2x^2(d+fx^2)^p}{p+1} + \frac{2cdfpx(d+fx^2)^p}{p+1} + \frac{2cdfx(d+fx^2)^p}{p+1} + \frac{2cf^2px^3(d+fx^2)^p}{p+1} + \\ bf\log\left(-i\sqrt{d}\sqrt{\frac{1}{f}}+x\right) + bf\log\left(i\sqrt{d}\sqrt{\frac{1}{f}}+x\right) + 2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+d)**p*(2*c*d*f+2*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),
x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + f*x**2)**p/(p + 1) + 3*b*d*f*(d + f*x**2)**p/(p +
 1) + 2*b*f**2*p*x**2*(d + f*x**2)**p/(p + 1) + 3*b*f**2*x**2*(d + f*x**2)*
*p/(p + 1) + 2*c*d*f*p*x*(d + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + f*x**2)**
p/(p + 1) + 2*c*f**2*p*x**3*(d + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d + f*
x**2)**p/(p + 1), Ne(p, -1)), (b*f*log(-I*sqrt(d)*sqrt(1/f) + x) + b*f*log(
I*sqrt(d)*sqrt(1/f) + x) + 2*c*f*x, True))
```

# 3.276 $\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)\right.$

**Optimal**. Leaf size=46

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -c*e*(2+p)*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1+p)
```

**Rubi [A]** time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{ce(p+2)\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 + 2*p)*x
^2),x]
```

```
[Out] -((c*e*(2 + p)*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*f*x*(d + e*x + f*x
^2)^(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf-ce^2p+2cf^2(3+2p)x^2\right)\,dx = 2cfx\left(d+ex+fx^2\right)^{1+p} + \frac{\int \left(-ce^2f(2+p)(3 \right.}{}$$

$$= -\frac{ce(2+p)\left(d+ex+fx^2\right)^{1+p}}{1+p} + 2cfx\left(d+ex\right.$$

**Mathematica** [A] time = 0.13, size = 34, normalized size = 0.74

$$\frac{c(2f(p+1)x-e(p+2))(d+x(e+fx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f - c*e^2*p + 2*c*f^2*(3 +
2*p)*x^2),x]
```

```
[Out] (c*(-(e*(2 + p)) + 2*f*(1 + p)*x)*(d + x*(e + f*x))^(1 + p))/(1 + p)
```

**fricas** [A] time = 0.63, size = 83, normalized size = 1.80

$$\frac{\left(cefpx^2-cdep+2\left(cf^2p+cf^2\right)x^3-2\,cde-\left(2\,ce^2-2\,cdf+\left(ce^2-2\,cdf\right)p\right)x\right)\left(fx^2+ex+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="fricas")
```

```
[Out] (c*e*f*p*x^2 - c*d*e*p + 2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e - (2*c*e^2 - 2*c
*d*f + (c*e^2 - 2*c*d*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

**giac** [B] time = 0.24, size = 191, normalized size = 4.15

$$\frac{2\left(fx^2+xe+d\right)^p cf^2px^3+2\left(fx^2+xe+d\right)^p cf^2x^3+\left(fx^2+xe+d\right)^p cfpx^2e+2\left(fx^2+xe+d\right)^p cdfpx+2\left(fx^2\right.}{p+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x
^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2 + x*e + d)^p*c*d*f*p*x + 2*(f*x^2 +
```

```
x*e + d)^p*c*d*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*
d*p*e - 2*(f*x^2 + x*e + d)^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1
)
```

**maple [A]** time = 0.00, size = 39, normalized size = 0.85

$$-\frac{\left(-2fpx+ep-2fx+2e\right)c\left(fx^2+ex+d\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] -c*(f*x^2+e*x+d)^(p+1)*(-2*f*p*x+e*p-2*f*x+2*e)/(p+1)
```

**maxima [A]** time = 0.58, size = 66, normalized size = 1.43

$$\frac{\left(2cf^2\left(p+1\right)x^3+cefpx^2-cde\left(p+2\right)-\left(e^2\left(p+2\right)-2df\left(p+1\right)\right)cx\right)\left(fx^2+ex+d\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f-c*e^2*p+2*c*f^2*(3+2*p)*x^2),x,
 algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + c*e*f*p*x^2 - c*d*e*(p + 2) - (e^2*(p + 2) - 2*d*f*(
p + 1))*c*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

**mupad [B]** time = 4.39, size = 78, normalized size = 1.70

$$\left(fx^2+ex+d\right)^p\left(2cf^2x^3+\frac{cx\left(2df-e^2p-2e^2+2dfp\right)}{p+1}-\frac{cde\left(p+2\right)}{p+1}+\frac{cefpx^2}{p+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(d + e*x + f*x^2)^p*(2*c*e^2 - 2*c*d*f + c*e^2*p - 2*c*f^2*x^2*(2*p +
3)),x)
```

```
[Out] (d + e*x + f*x^2)^p*(2*c*f^2*x^3 + (c*x*(2*d*f - e^2*p - 2*e^2 + 2*d*f*p))/
(p + 1) - (c*d*e*(p + 2))/(p + 1) + (c*e*f*p*x^2)/(p + 1))
```

**sympy [A]** time = 173.95, size = 280, normalized size = 6.09

$$\begin{cases} -\frac{cdep\left(d+ex+fx^2\right)^p}{p+1}-\frac{2cde\left(d+ex+fx^2\right)^p}{p+1}+\frac{2cdfpx\left(d+ex+fx^2\right)^p}{p+1}+\frac{2cdfx\left(d+ex+fx^2\right)^p}{p+1}-\frac{ce^2px\left(d+ex+fx^2\right)^p}{p+1}-\frac{2ce^2x\left(d+ex+fx^2\right)^p}{p+1}+\frac{cefpx^2(\ldots}{} \\ -ce\log\left(\frac{e}{2f}+x-\frac{\sqrt{-4df+e^2}}{2f}\right)-ce\log\left(\frac{e}{2f}+x+\frac{\sqrt{-4df+e^2}}{2f}\right)+2cfx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f-c*e**2*p+2*c*f**2*(3+2*p)*x*
*2),x)
```

```
[Out] Piecewise((-c*d*e*p*(d + e*x + f*x**2)**p/(p + 1) - 2*c*d*e*(d + e*x + f*x*
*2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*x*(d +
 e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*(d + e*x + f*x**2)**p/(p + 1) - 2*c*
e**2*x*(d + e*x + f*x**2)**p/(p + 1) + c*e*f*p*x**2*(d + e*x + f*x**2)**p/(
p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*x**3*(d +
 e*x + f*x**2)**p/(p + 1), Ne(p, -1)), (-c*e*log(e/(2*f) + x - sqrt(-4*d*f
+ e**2)/(2*f)) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x
, True))
```

## 3.277 $\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2bef\right.$

**Optimal**. Leaf size=57

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

```
[Out] -(c*e*(2+p)-b*f*(3+2*p))*(f*x^2+e*x+d)^(1+p)/(1+p)+2*c*f*x*(f*x^2+e*x+d)^(1
+p)
```

**Rubi** [A] time = 0.12, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1661, 629}

$$2cfx\left(d+ex+fx^2\right)^{p+1}-\frac{(ce(p+2)-bf(2p+3))\left(d+ex+fx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b*e*f*p
 + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] -(((c*e*(2 + p) - b*f*(3 + 2*p))*(d + e*x + f*x^2)^(1 + p))/(1 + p)) + 2*c*
f*x*(d + e*x + f*x^2)^(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(d+ex+fx^2\right)^p \left(-2ce^2+2cdf+3bef-ce^2p+2befp+2bf^2(3+2p)x+2cf^2(3+2p)x^2\right) dx = 2cfx\left(d+ex\right.$$

$$= -\frac{(ce(2+p)}{}$$

**Mathematica [A]** time = 0.31, size = 43, normalized size = 0.75

$$\frac{(d+x(e+fx))^{p+1}(bf(2p+3)-ce(p+2)+2cf(p+1)x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^p*(-2*c*e^2 + 2*c*d*f + 3*b*e*f - c*e^2*p + 2*b
*e*f*p + 2*b*f^2*(3 + 2*p)*x + 2*c*f^2*(3 + 2*p)*x^2),x]
```

```
[Out] ((-(c*e*(2 + p)) + b*f*(3 + 2*p) + 2*c*f*(1 + p)*x)*(d + x*(e + f*x))^(1 +
p))/(1 + p)
```

**fricas [B]** time = 0.52, size = 123, normalized size = 2.16

$$\frac{\left(2\left(cf^2p+cf^2\right)x^3-2\,cde+3\,bdf+\left(3\,bf^2+\left(cef+2\,bf^2\right)p\right)x^2-\left(cde-2\,bdf\right)p-\left(2\,ce^2-(2\,cd+3\,be)f+\left(ce\right.\right.\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="fricas")
```

```
[Out] (2*(c*f^2*p + c*f^2)*x^3 - 2*c*d*e + 3*b*d*f + (3*b*f^2 + (c*e*f + 2*b*f^2)
*p)*x^2 - (c*d*e - 2*b*d*f)*p - (2*c*e^2 - (2*c*d + 3*b*e)*f + (c*e^2 - 2*(
c*d + b*e)*f)*p)*x)*(f*x^2 + e*x + d)^p/(p + 1)
```

**giac [B]** time = 0.31, size = 314, normalized size = 5.51

$$\frac{2\left(fx^2+xe+d\right)^p cf^2px^3+2\left(fx^2+xe+d\right)^p bf^2px^2+2\left(fx^2+xe+d\right)^p cf^2x^3+\left(fx^2+xe+d\right)^p cfpx^2e+2\left(fx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="giac")
```

```
[Out] (2*(f*x^2 + x*e + d)^p*c*f^2*p*x^3 + 2*(f*x^2 + x*e + d)^p*b*f^2*p*x^2 + 2*
(f*x^2 + x*e + d)^p*c*f^2*x^3 + (f*x^2 + x*e + d)^p*c*f*p*x^2*e + 2*(f*x^2
+ x*e + d)^p*c*d*f*p*x + 3*(f*x^2 + x*e + d)^p*b*f^2*x^2 + 2*(f*x^2 + x*e +
 d)^p*b*f*p*x*e + 2*(f*x^2 + x*e + d)^p*b*d*f*p + 2*(f*x^2 + x*e + d)^p*c*d
*f*x - (f*x^2 + x*e + d)^p*c*p*x*e^2 - (f*x^2 + x*e + d)^p*c*d*p*e + 3*(f*x
^2 + x*e + d)^p*b*f*x*e + 3*(f*x^2 + x*e + d)^p*b*d*f - 2*(f*x^2 + x*e + d)
^p*c*x*e^2 - 2*(f*x^2 + x*e + d)^p*c*d*e)/(p + 1)
```

**maple [A]** time = 0.01, size = 51, normalized size = 0.89

$$\frac{\left(2cfxp+2bfp-cep+2cfx+3bf-2ce\right)\left(f\,x^{2}+ex+d\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f^2*(3+
2*p)*x+2*c*f^2*(3+2*p)*x^2),x)
```

```
[Out] (f*x^2+e*x+d)^(p+1)*(2*c*f*p*x+2*b*f*p-c*e*p+2*c*f*x+3*b*f-2*c*e)/(p+1)
```

**maxima [A]** time = 0.60, size = 98, normalized size = 1.72

$$\frac{\left(2\,cf^{2}\left(p+1\right)x^{3}+bdf\left(2\,p+3\right)-cde\left(p+2\right)+\left(bf^{2}\left(2\,p+3\right)+cefp\right)x^{2}+\left(bef\left(2\,p+3\right)-\left(e^{2}\left(p+2\right)-2\,df\left(p+1\right.\right.\right.\right.}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^p*(-2*c*e^2+2*c*d*f+3*b*e*f-c*e^2*p+2*b*e*f*p+2*b*f
^2*(3+2*p)*x+2*c*f^2*(3+2*p)*x^2),x, algorithm="maxima")
```

```
[Out] (2*c*f^2*(p + 1)*x^3 + b*d*f*(2*p + 3) - c*d*e*(p + 2) + (b*f^2*(2*p + 3) +
 c*e*f*p)*x^2 + (b*e*f*(2*p + 3) - (e^2*(p + 2) - 2*d*f*(p + 1))*c)*x)*(f*x
^2 + e*x + d)^p/(p + 1)
```

**mupad [B]** time = 4.46, size = 120, normalized size = 2.11

$$\left(f\,x^2+e\,x+d\right)^p\,\left(\frac{x^2\,\left(3\,b\,f^2+2\,b\,f^2\,p+c\,e\,f\,p\right)}{p+1}+2\,c\,f^2\,x^3+\frac{d\,\left(3\,b\,f-2\,c\,e+2\,b\,f\,p-c\,e\,p\right)}{p+1}+\frac{x\,\left(3\,b\,e\,f-2\,c\,e\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2)^p*(3*b*e*f - 2*c*e^2 + 2*c*d*f - c*e^2*p + 2*b*f^2*x*
(2*p + 3) + 2*c*f^2*x^2*(2*p + 3) + 2*b*e*f*p),x)
```

```
[Out] (d + e*x + f*x^2)^p*((x^2*(3*b*f^2 + 2*b*f^2*p + c*e*f*p))/(p + 1) + 2*c*f^
2*x^3 + (d*(3*b*f - 2*c*e + 2*b*f*p - c*e*p))/(p + 1) + (x*(3*b*e*f - 2*c*e
^2 + 2*c*d*f - c*e^2*p + 2*b*e*f*p + 2*c*d*f*p))/(p + 1))
```

**sympy [B]** time = 171.17, size = 483, normalized size = 8.47

$$\begin{cases} \frac{2bdfp\left(d+ex+fx^2\right)^p}{p+1} + \frac{3bdf\left(d+ex+fx^2\right)^p}{p+1} + \frac{2befpx\left(d+ex+fx^2\right)^p}{p+1} + \frac{3befx\left(d+ex+fx^2\right)^p}{p+1} + \frac{2bf^2px^2\left(d+ex+fx^2\right)^p}{p+1} + \frac{3bf^2x^2\left(d+ex+fx^2\right)^p}{p+1} - \\ bf\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) + bf\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x - \frac{\sqrt{-4df+e^2}}{2f}\right) - ce\log\left(\frac{e}{2f} + x + \frac{\sqrt{-4d}}{2}\right. \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**p*(-2*c*e**2+2*c*d*f+3*b*e*f-c*e**2*p+2*b*e*f*p+2
*b*f**2*(3+2*p)*x+2*c*f**2*(3+2*p)*x**2),x)
```

```
[Out] Piecewise((2*b*d*f*p*(d + e*x + f*x**2)**p/(p + 1) + 3*b*d*f*(d + e*x + f*x
**2)**p/(p + 1) + 2*b*e*f*p*x*(d + e*x + f*x**2)**p/(p + 1) + 3*b*e*f*x*(d
+ e*x + f*x**2)**p/(p + 1) + 2*b*f**2*p*x**2*(d + e*x + f*x**2)**p/(p + 1)
+ 3*b*f**2*x**2*(d + e*x + f*x**2)**p/(p + 1) - c*d*e*p*(d + e*x + f*x**2)*
*p/(p + 1) - 2*c*d*e*(d + e*x + f*x**2)**p/(p + 1) + 2*c*d*f*p*x*(d + e*x +
 f*x**2)**p/(p + 1) + 2*c*d*f*x*(d + e*x + f*x**2)**p/(p + 1) - c*e**2*p*x*
(d + e*x + f*x**2)**p/(p + 1) - 2*c*e**2*x*(d + e*x + f*x**2)**p/(p + 1) +
c*e*f*p*x**2*(d + e*x + f*x**2)**p/(p + 1) + 2*c*f**2*p*x**3*(d + e*x + f*x
**2)**p/(p + 1) + 2*c*f**2*x**3*(d + e*x + f*x**2)**p/(p + 1), Ne(p, -1)),
(b*f*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)) + b*f*log(e/(2*f) + x + s
qrt(-4*d*f + e**2)/(2*f)) - c*e*log(e/(2*f) + x - sqrt(-4*d*f + e**2)/(2*f)
) - c*e*log(e/(2*f) + x + sqrt(-4*d*f + e**2)/(2*f)) + 2*c*f*x, True))
```

# 3.278 $\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17b\right.\right.$

**Optimal**. Leaf size=20

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

```
[Out] (e*x+d)^5*(c*x^2+b*x+a)^6
```

**Rubi [A]** time = 0.42, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 75, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {1624, 1590}

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*d
*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] (d + e*x)^5*(a + b*x + c*x^2)^6
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rule 1624

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Rubi steps

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right)\,dx=\int (d+$$

$$=(d+ex$$

**Mathematica [B]** time = 0.45, size = 167, normalized size = 8.35

$$x\left(a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)+6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

```
[Out] x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3*
x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^4*
(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 10*d
^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

**fricas [B]** time = 0.44, size = 2467, normalized size = 123.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

```
[Out] x^17*e^5*c^6 + 5*x^16*e^4*d*c^6 + 6*x^16*e^5*c^5*b + 10*x^15*e^3*d^2*c^6 +
30*x^15*e^4*d*c^5*b + 15*x^15*e^5*c^4*b^2 + 6*x^15*e^5*c^5*a + 10*x^14*e^2*
d^3*c^6 + 60*x^14*e^3*d^2*c^5*b + 75*x^14*e^4*d*c^4*b^2 + 20*x^14*e^5*c^3*b
^3 + 30*x^14*e^4*d*c^5*a + 30*x^14*e^5*c^4*b*a + 5*x^13*e*d^4*c^6 + 60*x^13
*e^2*d^3*c^5*b + 150*x^13*e^3*d^2*c^4*b^2 + 100*x^13*e^4*d*c^3*b^3 + 15*x^1
3*e^5*c^2*b^4 + 60*x^13*e^3*d^2*c^5*a + 150*x^13*e^4*d*c^4*b*a + 60*x^13*e^
5*c^3*b^2*a + 15*x^13*e^5*c^4*a^2 + x^12*d^5*c^6 + 30*x^12*e*d^4*c^5*b + 15
0*x^12*e^2*d^3*c^4*b^2 + 200*x^12*e^3*d^2*c^3*b^3 + 75*x^12*e^4*d*c^2*b^4 +
 6*x^12*e^5*c*b^5 + 60*x^12*e^2*d^3*c^5*a + 300*x^12*e^3*d^2*c^4*b*a + 300*
x^12*e^4*d*c^3*b^2*a + 60*x^12*e^5*c^2*b^3*a + 75*x^12*e^4*d*c^4*a^2 + 60*x
^12*e^5*c^3*b*a^2 + 6*x^11*d^5*c^5*b + 75*x^11*e*d^4*c^4*b^2 + 200*x^11*e^2
*d^3*c^3*b^3 + 150*x^11*e^3*d^2*c^2*b^4 + 30*x^11*e^4*d*c*b^5 + x^11*e^5*b^
6 + 30*x^11*e*d^4*c^5*a + 300*x^11*e^2*d^3*c^4*b*a + 600*x^11*e^3*d^2*c^3*b
^2*a + 300*x^11*e^4*d*c^2*b^3*a + 30*x^11*e^5*c*b^4*a + 150*x^11*e^3*d^2*c^
4*a^2 + 300*x^11*e^4*d*c^3*b*a^2 + 90*x^11*e^5*c^2*b^2*a^2 + 20*x^11*e^5*c^
3*a^3 + 15*x^10*d^5*c^4*b^2 + 100*x^10*e*d^4*c^3*b^3 + 150*x^10*e^2*d^3*c^2
*b^4 + 60*x^10*e^3*d^2*c*b^5 + 5*x^10*e^4*d*b^6 + 6*x^10*d^5*c^5*a + 150*x^
10*e*d^4*c^4*b*a + 600*x^10*e^2*d^3*c^3*b^2*a + 600*x^10*e^3*d^2*c^2*b^3*a
+ 150*x^10*e^4*d*c*b^4*a + 6*x^10*e^5*b^5*a + 150*x^10*e^2*d^3*c^4*a^2 + 60
0*x^10*e^3*d^2*c^3*b*a^2 + 450*x^10*e^4*d*c^2*b^2*a^2 + 60*x^10*e^5*c*b^3*a
^2 + 100*x^10*e^4*d*c^3*a^3 + 60*x^10*e^5*c^2*b*a^3 + 20*x^9*d^5*c^3*b^3 +
75*x^9*e*d^4*c^2*b^4 + 60*x^9*e^2*d^3*c*b^5 + 10*x^9*e^3*d^2*b^6 + 30*x^9*d
^5*c^4*b*a + 300*x^9*e*d^4*c^3*b^2*a + 600*x^9*e^2*d^3*c^2*b^3*a + 300*x^9*
```

```
e^3*d^2*c*b^4*a + 30*x^9*e^4*d*b^5*a + 75*x^9*e*d^4*c^4*a^2 + 600*x^9*e^2*d
^3*c^3*b*a^2 + 900*x^9*e^3*d^2*c^2*b^2*a^2 + 300*x^9*e^4*d*c*b^3*a^2 + 15*x
^9*e^5*b^4*a^2 + 200*x^9*e^3*d^2*c^3*a^3 + 300*x^9*e^4*d*c^2*b*a^3 + 60*x^9
*e^5*c*b^2*a^3 + 15*x^9*e^5*c^2*a^4 + 15*x^8*d^5*c^2*b^4 + 30*x^8*e*d^4*c*b
^5 + 10*x^8*e^2*d^3*b^6 + 60*x^8*d^5*c^3*b^2*a + 300*x^8*e*d^4*c^2*b^3*a +
300*x^8*e^2*d^3*c*b^4*a + 60*x^8*e^3*d^2*b^5*a + 15*x^8*d^5*c^4*a^2 + 300*x
^8*e*d^4*c^3*b*a^2 + 900*x^8*e^2*d^3*c^2*b^2*a^2 + 600*x^8*e^3*d^2*c*b^3*a^
2 + 75*x^8*e^4*d*b^4*a^2 + 200*x^8*e^2*d^3*c^3*a^3 + 600*x^8*e^3*d^2*c^2*b*
a^3 + 300*x^8*e^4*d*c*b^2*a^3 + 20*x^8*e^5*b^3*a^3 + 75*x^8*e^4*d*c^2*a^4 +
 30*x^8*e^5*c*b*a^4 + 6*x^7*d^5*c*b^5 + 5*x^7*e*d^4*b^6 + 60*x^7*d^5*c^2*b^
3*a + 150*x^7*e*d^4*c*b^4*a + 60*x^7*e^2*d^3*b^5*a + 60*x^7*d^5*c^3*b*a^2 +
 450*x^7*e*d^4*c^2*b^2*a^2 + 600*x^7*e^2*d^3*c*b^3*a^2 + 150*x^7*e^3*d^2*b^
4*a^2 + 100*x^7*e*d^4*c^3*a^3 + 600*x^7*e^2*d^3*c^2*b*a^3 + 600*x^7*e^3*d^2
*c*b^2*a^3 + 100*x^7*e^4*d*b^3*a^3 + 150*x^7*e^3*d^2*c^2*a^4 + 150*x^7*e^4*
d*c*b*a^4 + 15*x^7*e^5*b^2*a^4 + 6*x^7*e^5*c*a^5 + x^6*d^5*b^6 + 30*x^6*d^5
*c*b^4*a + 30*x^6*e*d^4*b^5*a + 90*x^6*d^5*c^2*b^2*a^2 + 300*x^6*e*d^4*c*b^
3*a^2 + 150*x^6*e^2*d^3*b^4*a^2 + 20*x^6*d^5*c^3*a^3 + 300*x^6*e*d^4*c^2*b*
a^3 + 600*x^6*e^2*d^3*c*b^2*a^3 + 200*x^6*e^3*d^2*b^3*a^3 + 150*x^6*e^2*d^3
*c^2*a^4 + 300*x^6*e^3*d^2*c*b*a^4 + 75*x^6*e^4*d*b^2*a^4 + 30*x^6*e^4*d*c*
a^5 + 6*x^6*e^5*b*a^5 + 6*x^5*d^5*b^5*a + 60*x^5*d^5*c*b^3*a^2 + 75*x^5*e*d
^4*b^4*a^2 + 60*x^5*d^5*c^2*b*a^3 + 300*x^5*e*d^4*c*b^2*a^3 + 200*x^5*e^2*d
^3*b^3*a^3 + 75*x^5*e*d^4*c^2*a^4 + 300*x^5*e^2*d^3*c*b*a^4 + 150*x^5*e^3*d
^2*b^2*a^4 + 60*x^5*e^3*d^2*c*a^5 + 30*x^5*e^4*d*b*a^5 + x^5*e^5*a^6 + 15*x
^4*d^5*b^4*a^2 + 60*x^4*d^5*c*b^2*a^3 + 100*x^4*e*d^4*b^3*a^3 + 15*x^4*d^5*
c^2*a^4 + 150*x^4*e*d^4*c*b*a^4 + 150*x^4*e^2*d^3*b^2*a^4 + 60*x^4*e^2*d^3*
c*a^5 + 60*x^4*e^3*d^2*b*a^5 + 5*x^4*e^4*d*a^6 + 20*x^3*d^5*b^3*a^3 + 30*x^
3*d^5*c*b*a^4 + 75*x^3*e*d^4*b^2*a^4 + 30*x^3*e*d^4*c*a^5 + 60*x^3*e^2*d^3*
b*a^5 + 10*x^3*e^3*d^2*a^6 + 15*x^2*d^5*b^2*a^4 + 6*x^2*d^5*c*a^5 + 30*x^2*
e*d^4*b*a^5 + 10*x^2*e^2*d^3*a^6 + 6*x*d^5*b*a^5 + 5*x*e*d^4*a^6
```

**giac [B]** time = 0.23, size = 2383, normalized size = 119.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

```
[Out] c^6*x^17*e^5 + 5*c^6*d*x^16*e^4 + 10*c^6*d^2*x^15*e^3 + 10*c^6*d^3*x^14*e^2
 + 5*c^6*d^4*x^13*e + c^6*d^5*x^12 + 6*b*c^5*x^16*e^5 + 30*b*c^5*d*x^15*e^4
 + 60*b*c^5*d^2*x^14*e^3 + 60*b*c^5*d^3*x^13*e^2 + 30*b*c^5*d^4*x^12*e + 6*
b*c^5*d^5*x^11 + 15*b^2*c^4*x^15*e^5 + 6*a*c^5*x^15*e^5 + 75*b^2*c^4*d*x^14
*e^4 + 30*a*c^5*d*x^14*e^4 + 150*b^2*c^4*d^2*x^13*e^3 + 60*a*c^5*d^2*x^13*e
^3 + 150*b^2*c^4*d^3*x^12*e^2 + 60*a*c^5*d^3*x^12*e^2 + 75*b^2*c^4*d^4*x^11
*e + 30*a*c^5*d^4*x^11*e + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 20*b^3*
```

c^3*x^14*e^5 + 30*a*b*c^4*x^14*e^5 + 100*b^3*c^3*d*x^13*e^4 + 150*a*b*c^4*d*x^13*e^4 + 200*b^3*c^3*d^2*x^12*e^3 + 300*a*b*c^4*d^2*x^12*e^3 + 200*b^3*c^3*d^3*x^11*e^2 + 300*a*b*c^4*d^3*x^11*e^2 + 100*b^3*c^3*d^4*x^10*e + 150*a*b*c^4*d^4*x^10*e + 20*b^3*c^3*d^5*x^9 + 30*a*b*c^4*d^5*x^9 + 15*b^4*c^2*x^13*e^5 + 60*a*b^2*c^3*x^13*e^5 + 15*a^2*c^4*x^13*e^5 + 75*b^4*c^2*d*x^12*e^4 + 300*a*b^2*c^3*d*x^12*e^4 + 75*a^2*c^4*d*x^12*e^4 + 150*b^4*c^2*d^2*x^11*e^3 + 600*a*b^2*c^3*d^2*x^11*e^3 + 150*a^2*c^4*d^2*x^11*e^3 + 150*b^4*c^2*d^3*x^10*e^2 + 600*a*b^2*c^3*d^3*x^10*e^2 + 150*a^2*c^4*d^3*x^10*e^2 + 75*b^4*c^2*d^4*x^9*e + 300*a*b^2*c^3*d^4*x^9*e + 75*a^2*c^4*d^4*x^9*e + 15*b^4*c^2*d^5*x^8 + 60*a*b^2*c^3*d^5*x^8 + 15*a^2*c^4*d^5*x^8 + 6*b^5*c*x^12*e^5 + 60*a*b^3*c^2*x^12*e^5 + 60*a^2*b*c^3*x^12*e^5 + 30*b^5*c*d*x^11*e^4 + 300*a*b^3*c^2*d*x^11*e^4 + 300*a^2*b*c^3*d*x^11*e^4 + 60*b^5*c*d^2*x^10*e^3 + 600*a*b^3*c^2*d^2*x^10*e^3 + 600*a^2*b*c^3*d^2*x^10*e^3 + 60*b^5*c*d^3*x^9*e^2 + 600*a*b^3*c^2*d^3*x^9*e^2 + 600*a^2*b*c^3*d^3*x^9*e^2 + 30*b^5*c*d^4*x^8*e + 300*a*b^3*c^2*d^4*x^8*e + 300*a^2*b*c^3*d^4*x^8*e + 6*b^5*c*d^5*x^7 + 60*a*b^3*c^2*d^5*x^7 + 60*a^2*b*c^3*d^5*x^7 + b^6*x^11*e^5 + 30*a*b^4*c*x^11*e^5 + 90*a^2*b^2*c^2*x^11*e^5 + 20*a^3*c^3*x^11*e^5 + 5*b^6*d*x^10*e^4 + 150*a*b^4*c*d*x^10*e^4 + 450*a^2*b^2*c^2*d*x^10*e^4 + 100*a^3*c^3*d*x^10*e^4 + 10*b^6*d^2*x^9*e^3 + 300*a*b^4*c*d^2*x^9*e^3 + 900*a^2*b^2*c^2*d^2*x^9*e^3 + 200*a^3*c^3*d^2*x^9*e^3 + 10*b^6*d^3*x^8*e^2 + 300*a*b^4*c*d^3*x^8*e^2 + 900*a^2*b^2*c^2*d^3*x^8*e^2 + 200*a^3*c^3*d^3*x^8*e^2 + 5*b^6*d^4*x^7*e + 150*a*b^4*c*d^4*x^7*e + 450*a^2*b^2*c^2*d^4*x^7*e + 100*a^3*c^3*d^4*x^7*e + b^6*d^5*x^6 + 30*a*b^4*c*d^5*x^6 + 90*a^2*b^2*c^2*d^5*x^6 + 20*a^3*c^3*d^5*x^6 + 6*a*b^5*x^10*e^5 + 60*a^2*b^3*c*x^10*e^5 + 60*a^3*b*c^2*x^10*e^5 + 30*a*b^5*d*x^9*e^4 + 300*a^2*b^3*c*d*x^9*e^4 + 300*a^3*b*c^2*d*x^9*e^4 + 60*a*b^5*d^2*x^8*e^3 + 600*a^2*b^3*c*d^2*x^8*e^3 + 600*a^3*b*c^2*d^2*x^8*e^3 + 60*a*b^5*d^3*x^7*e^2 + 600*a^2*b^3*c*d^3*x^7*e^2 + 600*a^3*b*c^2*d^3*x^7*e^2 + 30*a*b^5*d^4*x^6*e + 300*a^2*b^3*c*d^4*x^6*e + 300*a^3*b*c^2*d^4*x^6*e + 6*a*b^5*d^5*x^5 + 60*a^2*b^3*c*d^5*x^5 + 60*a^3*b*c^2*d^5*x^5 + 15*a^2*b^4*x^9*e^5 + 60*a^3*b^2*c*x^9*e^5 + 15*a^4*c^2*x^9*e^5 + 75*a^2*b^4*d*x^8*e^4 + 300*a^3*b^2*c*d*x^8*e^4 + 75*a^4*c^2*d*x^8*e^4 + 150*a^2*b^4*d^2*x^7*e^3 + 600*a^3*b^2*c*d^2*x^7*e^3 + 150*a^4*c^2*d^2*x^7*e^3 + 150*a^2*b^4*d^3*x^6*e^2 + 600*a^3*b^2*c*d^3*x^6*e^2 + 150*a^4*c^2*d^3*x^6*e^2 + 75*a^2*b^4*d^4*x^5*e + 300*a^3*b^2*c*d^4*x^5*e + 75*a^4*c^2*d^4*x^5*e + 15*a^2*b^4*d^5*x^4 + 60*a^3*b^2*c*d^5*x^4 + 15*a^4*c^2*d^5*x^4 + 20*a^3*b^3*x^8*e^5 + 30*a^4*b*c*x^8*e^5 + 100*a^3*b^3*d*x^7*e^4 + 150*a^4*b*c*d*x^7*e^4 + 200*a^3*b^3*d^2*x^6*e^3 + 300*a^4*b*c*d^2*x^6*e^3 + 200*a^3*b^3*d^3*x^5*e^2 + 300*a^4*b*c*d^3*x^5*e^2 + 100*a^3*b^3*d^4*x^4*e + 150*a^4*b*c*d^4*x^4*e + 20*a^3*b^3*d^5*x^3 + 30*a^4*b*c*d^5*x^3 + 15*a^4*b^2*x^7*e^5 + 6*a^5*c*x^7*e^5 + 75*a^4*b^2*d*x^6*e^4 + 30*a^5*c*d*x^6*e^4 + 150*a^4*b^2*d^2*x^5*e^3 + 60*a^5*c*d^2*x^5*e^3 + 150*a^4*b^2*d^3*x^4*e^2 + 60*a^5*c*d^3*x^4*e^2 + 75*a^4*b^2*d^4*x^3*e + 30*a^5*c*d^4*x^3*e + 15*a^4*b^2*d^5*x^2 + 6*a^5*c*d^5*x^2 + 6*a^5*b*x^6*e^5 + 30*a^5*b*d*x^5*e^4 + 60*a^5*b*d^2*x^4*e^3 + 60*a^5*b*d^3*x^3*e^2 + 30*a^5*b*d^4*x^2*e + 6*a^5*b*d^5*x + a^6*x^5*e^5 + 5*a^6*d*x^4*e^4 + 10*a^6*d^2*x^3*e^3 + 10*a^6*d^3*x^2*e^2 + 5*a^6*d^4*x*e

**maple [B]** time = 0.00, size = 8419, normalized size = 420.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)*
x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 0.50, size = 1779, normalized size = 88.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c
*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

```
[Out] c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c^
5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5*d
^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*x^1
4 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 + 10*
(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^5)*x^
13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 100*(2*
b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d*e^4 +
 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 + 15*(5*b
^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 150*(b^4*c^
2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^
3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)*x^11 + (3*
(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e + 150*(b^4*c^2
 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3
)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d*e^4 + 6*(a
*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*c^3 + 3*a*b*c^4
)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^5*c + 10*a*b^3*c
^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*
c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*e^4 + 3*(a^2*b^4 +
 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d
^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e + 2*(b^6 + 30*a*b^4*c +
90*a^2*b^2*c^2 + 20*a^3*c^3)*d^3*e^2 + 12*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*
c^2)*d^2*e^3 + 15*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d*e^4 + 2*(2*a^3*b^3 +
3*a^4*b*c)*e^5)*x^8 + (6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^5 + 5*(b^6
 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^4*e + 60*(a*b^5 + 10*a^2*b^3
*c + 10*a^3*b*c^2)*d^3*e^2 + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^3
+ 50*(2*a^3*b^3 + 3*a^4*b*c)*d*e^4 + 3*(5*a^4*b^2 + 2*a^5*c)*e^5)*x^7 + (6*
```

```
a^5*b*e^5 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*d^5 + 30*(a*b^
5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d^4*e + 150*(a^2*b^4 + 4*a^3*b^2*c + a^4*c
^2)*d^3*e^2 + 100*(2*a^3*b^3 + 3*a^4*b*c)*d^2*e^3 + 15*(5*a^4*b^2 + 2*a^5*c
)*d*e^4)*x^6 + (30*a^5*b*d*e^4 + a^6*e^5 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3
*b*c^2)*d^5 + 75*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^4*e + 100*(2*a^3*b^3 +
 3*a^4*b*c)*d^3*e^2 + 30*(5*a^4*b^2 + 2*a^5*c)*d^2*e^3)*x^5 + 5*(12*a^5*b*d
^2*e^3 + a^6*d*e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^5 + 10*(2*a^3*b^
3 + 3*a^4*b*c)*d^4*e + 6*(5*a^4*b^2 + 2*a^5*c)*d^3*e^2)*x^4 + 5*(12*a^5*b*d
^3*e^2 + 2*a^6*d^2*e^3 + 2*(2*a^3*b^3 + 3*a^4*b*c)*d^5 + 3*(5*a^4*b^2 + 2*a
^5*c)*d^4*e)*x^3 + (30*a^5*b*d^4*e + 10*a^6*d^3*e^2 + 3*(5*a^4*b^2 + 2*a^5*
c)*d^5)*x^2 + (6*a^5*b*d^5 + 5*a^6*d^4*e)*x
```

**mupad** [B] time = 4.87, size = 2026, normalized size = 101.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c*
d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

```
[Out] x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^2
*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^2
+ 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e +
300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^11*(
b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2*c^2*
e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3 + 30*
a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2 + 300*a
*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5*(a^6*e^
5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4*d^4*e +
75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a^4*b^2*d^2
*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^2) + x^3*(2
0*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^3*e^2 + 30*a
^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 60*a*b^3*c^2*e
^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 + 75*b^4*c^2*d*
e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d^4*e + 300*a*b*
c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b^5*c*d^5 + 5*b^6
*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*d^5 + 60*a*b^5*d^
3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b^4*d^2*e^3 + 150*a
^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 + 450*a^2*b^2*c^2*d^
4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 + 600*a^3*b^2*c*d^2*e^3
) + x^10*(6*a*b^5*e^5 + 6*a*c^5*d^5 + 5*b^6*d*e^4 + 15*b^2*c^4*d^5 + 60*a^2
*b^3*c*e^5 + 60*a^3*b*c^2*e^5 + 100*a^3*c^3*d*e^4 + 100*b^3*c^3*d^4*e + 60*
b^5*c*d^2*e^3 + 150*a^2*c^4*d^3*e^2 + 150*b^4*c^2*d^3*e^2 + 150*a*b*c^4*d^4
*e + 150*a*b^4*c*d*e^4 + 600*a*b^2*c^3*d^3*e^2 + 600*a*b^3*c^2*d^2*e^3 + 60
0*a^2*b*c^3*d^2*e^3 + 450*a^2*b^2*c^2*d*e^4) + x^8*(15*a^2*c^4*d^5 + 20*a^3
```

```
*b^3*e^5 + 15*b^4*c^2*d^5 + 10*b^6*d^3*e^2 + 60*a*b^2*c^3*d^5 + 60*a*b^5*d^
2*e^3 + 75*a^2*b^4*d*e^4 + 75*a^4*c^2*d*e^4 + 200*a^3*c^3*d^3*e^2 + 30*a^4*
b*c*e^5 + 30*b^5*c*d^4*e + 900*a^2*b^2*c^2*d^3*e^2 + 300*a*b^3*c^2*d^4*e +
300*a*b^4*c*d^3*e^2 + 300*a^2*b*c^3*d^4*e + 300*a^3*b^2*c*d*e^4 + 600*a^2*b
^3*c*d^2*e^3 + 600*a^3*b*c^2*d^2*e^3) + x^9*(15*a^2*b^4*e^5 + 15*a^4*c^2*e^
5 + 20*b^3*c^3*d^5 + 10*b^6*d^2*e^3 + 60*a^3*b^2*c*e^5 + 75*a^2*c^4*d^4*e +
 75*b^4*c^2*d^4*e + 60*b^5*c*d^3*e^2 + 200*a^3*c^3*d^2*e^3 + 30*a*b*c^4*d^5
 + 30*a*b^5*d*e^4 + 900*a^2*b^2*c^2*d^2*e^3 + 300*a*b^2*c^3*d^4*e + 300*a*b
^4*c*d^2*e^3 + 300*a^2*b^3*c*d*e^4 + 300*a^3*b*c^2*d*e^4 + 600*a*b^3*c^2*d^
3*e^2 + 600*a^2*b*c^3*d^3*e^2) + x^4*(5*a^6*d*e^4 + 15*a^2*b^4*d^5 + 15*a^4
*c^2*d^5 + 60*a^3*b^2*c*d^5 + 100*a^3*b^3*d^4*e + 60*a^5*b*d^2*e^3 + 60*a^5
*c*d^3*e^2 + 150*a^4*b^2*d^3*e^2 + 150*a^4*b*c*d^4*e) + x^13*(5*c^6*d^4*e +
 15*a^2*c^4*e^5 + 15*b^4*c^2*e^5 + 60*a*b^2*c^3*e^5 + 60*a*c^5*d^2*e^3 + 60
*b*c^5*d^3*e^2 + 100*b^3*c^3*d*e^4 + 150*b^2*c^4*d^2*e^3 + 150*a*b*c^4*d*e^
4) + c^6*e^5*x^17 + a^5*d^4*x*(5*a*e + 6*b*d) + 5*c^3*e^2*x^14*(4*b^3*e^3 +
 2*c^3*d^3 + 6*a*b*c*e^3 + 6*a*c^2*d*e^2 + 12*b*c^2*d^2*e + 15*b^2*c*d*e^2)
 + c^5*e^4*x^16*(6*b*e + 5*c*d) + a^4*d^3*x^2*(10*a^2*e^2 + 15*b^2*d^2 + 6*
a*c*d^2 + 30*a*b*d*e) + c^4*e^3*x^15*(15*b^2*e^2 + 10*c^2*d^2 + 6*a*c*e^2 +
 30*b*c*d*e)
```

**sympy [B]** time = 1.53, size = 2281, normalized size = 114.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e+
12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

```
[Out] c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*e
**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(30
*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e**4
 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5 + 60
*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**4*c**
2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d**3*e*
*2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e**4 + 60
*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3 + 60*a*
c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c**3*d**2*e
**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x**11*(20*a
**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4 + 150*a**2*
c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 600*a*b**2*c**
3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6*e**5 + 30*b*
*5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e**2 + 75*b**2*c
**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 100*a**3*c**3*d*
e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 600*a**2*b*c**3*d*
*2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*b**4*c*d*e**4 + 6
```

```
00*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 150*a*b*c**4*d**4*e
+ 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 + 150*b**4*c**2*d**3*
e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x**9*(15*a**4*c**2*e**5
+ 60*a**3*b**2*c*e**5 + 300*a**3*b*c**2*d*e**4 + 200*a**3*c**3*d**2*e**3 +
15*a**2*b**4*e**5 + 300*a**2*b**3*c*d*e**4 + 900*a**2*b**2*c**2*d**2*e**3 +
 600*a**2*b*c**3*d**3*e**2 + 75*a**2*c**4*d**4*e + 30*a*b**5*d*e**4 + 300*a
*b**4*c*d**2*e**3 + 600*a*b**3*c**2*d**3*e**2 + 300*a*b**2*c**3*d**4*e + 30
*a*b*c**4*d**5 + 10*b**6*d**2*e**3 + 60*b**5*c*d**3*e**2 + 75*b**4*c**2*d**
4*e + 20*b**3*c**3*d**5) + x**8*(30*a**4*b*c*e**5 + 75*a**4*c**2*d*e**4 + 2
0*a**3*b**3*e**5 + 300*a**3*b**2*c*d*e**4 + 600*a**3*b*c**2*d**2*e**3 + 200
*a**3*c**3*d**3*e**2 + 75*a**2*b**4*d*e**4 + 600*a**2*b**3*c*d**2*e**3 + 90
0*a**2*b**2*c**2*d**3*e**2 + 300*a**2*b*c**3*d**4*e + 15*a**2*c**4*d**5 + 6
0*a*b**5*d**2*e**3 + 300*a*b**4*c*d**3*e**2 + 300*a*b**3*c**2*d**4*e + 60*a
*b**2*c**3*d**5 + 10*b**6*d**3*e**2 + 30*b**5*c*d**4*e + 15*b**4*c**2*d**5)
 + x**7*(6*a**5*c*e**5 + 15*a**4*b**2*e**5 + 150*a**4*b*c*d*e**4 + 150*a**4
*c**2*d**2*e**3 + 100*a**3*b**3*d*e**4 + 600*a**3*b**2*c*d**2*e**3 + 600*a*
*3*b*c**2*d**3*e**2 + 100*a**3*c**3*d**4*e + 150*a**2*b**4*d**2*e**3 + 600*
a**2*b**3*c*d**3*e**2 + 450*a**2*b**2*c**2*d**4*e + 60*a**2*b*c**3*d**5 + 6
0*a*b**5*d**3*e**2 + 150*a*b**4*c*d**4*e + 60*a*b**3*c**2*d**5 + 5*b**6*d**
4*e + 6*b**5*c*d**5) + x**6*(6*a**5*b*e**5 + 30*a**5*c*d*e**4 + 75*a**4*b**
2*d*e**4 + 300*a**4*b*c*d**2*e**3 + 150*a**4*c**2*d**3*e**2 + 200*a**3*b**3
*d**2*e**3 + 600*a**3*b**2*c*d**3*e**2 + 300*a**3*b*c**2*d**4*e + 20*a**3*c
**3*d**5 + 150*a**2*b**4*d**3*e**2 + 300*a**2*b**3*c*d**4*e + 90*a**2*b**2*
c**2*d**5 + 30*a*b**5*d**4*e + 30*a*b**4*c*d**5 + b**6*d**5) + x**5*(a**6*e
**5 + 30*a**5*b*d*e**4 + 60*a**5*c*d**2*e**3 + 150*a**4*b**2*d**2*e**3 + 30
0*a**4*b*c*d**3*e**2 + 75*a**4*c**2*d**4*e + 200*a**3*b**3*d**3*e**2 + 300*
a**3*b**2*c*d**4*e + 60*a**3*b*c**2*d**5 + 75*a**2*b**4*d**4*e + 60*a**2*b*
*3*c*d**5 + 6*a*b**5*d**5) + x**4*(5*a**6*d*e**4 + 60*a**5*b*d**2*e**3 + 60
*a**5*c*d**3*e**2 + 150*a**4*b**2*d**3*e**2 + 150*a**4*b*c*d**4*e + 15*a**4
*c**2*d**5 + 100*a**3*b**3*d**4*e + 60*a**3*b**2*c*d**5 + 15*a**2*b**4*d**5
) + x**3*(10*a**6*d**2*e**3 + 60*a**5*b*d**3*e**2 + 30*a**5*c*d**4*e + 75*a
**4*b**2*d**4*e + 30*a**4*b*c*d**5 + 20*a**3*b**3*d**5) + x**2*(10*a**6*d**
3*e**2 + 30*a**5*b*d**4*e + 6*a**5*c*d**5 + 15*a**4*b**2*d**5) + x*(5*a**6*
d**4*e + 6*a**5*b*d**5)
```

# 3.279 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

**Optimal**. Leaf size=26

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(x+2)$$

```
[Out] 1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1593, 800, 632, 31}

$$\frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

```
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2+x^3}{-2+x+x^2}\,dx &= \int \frac{x^2(1+x)}{-2+x+x^2}\,dx\\ &= \int \left(x+\frac{2x}{-2+x+x^2}\right) dx\\ &= \frac{x^2}{2}+2\int \frac{x}{-2+x+x^2}\,dx\\ &= \frac{x^2}{2}+\frac{2}{3}\int \frac{1}{-1+x}\,dx+\frac{4}{3}\int \frac{1}{2+x}\,dx\\ &= \frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(2+x)\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
[Out] x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

**fricas [A]** time = 0.54, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2+\frac{4}{3}\log(x+2)+\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

**giac [A]** time = 0.16, size = 20, normalized size = 0.77

$$\frac{1}{2}x^2+\frac{4}{3}\log(|x+2|)+\frac{2}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
[Out] 1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

**maple [A]** time = 0.00, size = 19, normalized size = 0.73

$$\frac{x^2}{2} + \frac{4\ln(x+2)}{3} + \frac{2\ln(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)/(x^2+x-2),x)
[Out] 1/2*x^2+4/3*ln(x+2)+2/3*ln(x-1)
```

**maxima [A]** time = 0.43, size = 18, normalized size = 0.69

$$\frac{1}{2}x^2 + \frac{4}{3}\log(x+2) + \frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
[Out] 1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

**mupad [B]** time = 0.05, size = 18, normalized size = 0.69

$$\frac{2\ln(x-1)}{3} + \frac{4\ln(x+2)}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)/(x + x^2 - 2),x)
[Out] (2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

**sympy [A]** time = 0.24, size = 20, normalized size = 0.77

$$\frac{x^2}{2} + \frac{2\log(x-1)}{3} + \frac{4\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)/(x**2+x-2),x)
[Out] x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

# 3.280 $\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=346

$$\frac{x^2\sqrt{a+bx+cx^2}\,\left(-64acg+63b^2g-70bcf+80c^2e\right)}{240c^3}-\frac{\sqrt{a+bx+cx^2}\,\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+\right.\right.}{}$$

```
[Out] 1/256*(70*b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5*
g-40*b^3*c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)+1/240*(-64*a*c*g+63*b^2*g-70*b*c*f+80
*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x+a)^
(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-1/1920*(1050*b^3*c*f+40*b*c^2*(-5
5*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5*c*e)-
2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*e)-315*b^3*g+14*b*c*(46*a*g+25*b*f))*x)*(
c*x^2+b*x+a)^(1/2)/c^5
```

**Rubi [A]** time = 0.81, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1653, 832, 779, 621, 206}

$$-\frac{\sqrt{a+bx+cx^2}\,\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+4\right.}{1920c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a + b*x + c*x^2])/(24
0*c^3) + ((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x + c*x^2])/(40*c^2) + (g*x^4*Sqr
t[a + b*x + c*x^2])/(5*c) - ((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 9
45*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*x
)*Sqrt[a + b*x + c*x^2])/(1920*c^5) + ((70*b^4*c*f + 48*b^2*c^2*(2*c*d - 5*
a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) + 48*
a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(256*c^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(5cd+(5ce-4ag)x+\frac{1}{2}(10cf-9bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{5c}$$

$$= \frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{gx^4\sqrt{a+bx+cx^2}}{5c} + \frac{\int \frac{x^2\left(\frac{1}{2}\left(40c^2d-30acf+27abg\right)\cdots}{\sqrt{a\cdots}}}{}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx\cdots}}{40c^2}$$

$$= \frac{\left(80c^2e-70bcf+63b^2g-64acg\right)x^2\sqrt{a+bx+cx^2}}{240c^3} + \frac{(10cf-9bg)x^3\sqrt{a+bx\cdots}}{40c^2}$$

**Mathematica [A]** time = 0.74, size = 282, normalized size = 0.82

$$\frac{\sqrt{a+x(b+cx)}\left(16c^2\left(64a^2g-ac(80e+x(45f+32gx))+2c^2x(30d+x(20e+3x(5f+4gx)))\right)+4b^2c(-735ag\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*b^2*c*(300*
c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f + 161*g*x)) +
2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g - a*c*(80*e +
 x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*g*x))))))/(192
0*c^5) - ((-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d - 3*a
*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(256*c^(11/2))
```

**fricas [A]** time = 1.05, size = 701, normalized size = 2.03

$$\left[-\frac{15\left(32\left(3b^2c^3-4ac^4\right)d-16\left(5b^3c^2-12abc^3\right)e+2\left(35b^4c-120ab^2c^2+48a^2c^3\right)f-\left(63b^5-280ab^3c+240\cdots\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240*a
^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*(10*
c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a*c^4)
*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)*f + (
945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*c^4*e +
 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*sqrt(c*x^
2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2
- 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 2
80*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4 - 1440*b*c^4
*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3
 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*
b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 4
00*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*
x)*sqrt(c*x^2 + b*x + a))/c^6]
```

**giac [A]** time = 0.31, size = 330, normalized size = 0.95

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,gx}{c}+\frac{10\,c^4f-9\,bc^3g}{c^5}\right)x-\frac{70\,bc^3f-63\,b^2c^2g+64\,ac^3g-80\,c^4e}{c^5}\right)x+\frac{480\,c^4d+35\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^5
)*x - (70*b*c^3*f - 63*b^2*c^2*g + 64*a*c^3*g - 80*c^4*e)/c^5)*x + (480*c^4
*d + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c^2*g - 400*b*c^3*
e)/c^5)*x - (1440*b*c^3*d + 1050*b^3*c*f - 2200*a*b*c^2*f - 945*b^4*g + 294
0*a*b^2*c*g - 1024*a^2*c^2*g - 1200*b^2*c^2*e + 1280*a*c^3*e)/c^5) - 1/256*
(96*b^2*c^3*d - 128*a*c^4*d + 70*b^4*c*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f -
 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b*c^2*g - 80*b^3*c^2*e + 192*a*b*c^3*e)
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

**maple [B]** time = 0.01, size = 783, normalized size = 2.26

$$\frac{\sqrt{c\,x^2+bx+a}\;g\,x^4}{5c}-\frac{9\sqrt{c\,x^2+bx+a}\;bg\,x^3}{40c^2}+\frac{\sqrt{c\,x^2+bx+a}\;f\,x^3}{4c}-\frac{4\sqrt{c\,x^2+bx+a}\;ag\,x^2}{15c^2}+\frac{21\sqrt{c\,x^2+bx+a}\;b^2g}{80c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 161/240*g/c^3*b*a*x*(c*x^2+b*x+a)^(1/2)+35/128*f/c^(9/2)*b^4*ln((c*x+1/2*b)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*f*a^2/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+63/128*g/c^5*b^4*(c*x^2+b*x+a)^(1/2)-63/256*g/c^(11/2)*b^5
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+8/15*g*a^2/c^3*(c*x^2+b*x+a)^(
1/2)+1/2*d*x/c*(c*x^2+b*x+a)^(1/2)-3/4*d/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*d/c^
(5/2)*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*d*a/c^(3/2)*ln((c
*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*e*x^2/c*(c*x^2+b*x+a)^(1/2)+5/8*
e/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*e/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))-2/3*e*a/c^2*(c*x^2+b*x+a)^(1/2)+1/4*f*x^3/c*(c*x^2+b*x+a)
^(1/2)-35/64*f/c^4*b^3*(c*x^2+b*x+a)^(1/2)+1/5*g*x^4*(c*x^2+b*x+a)^(1/2)/c-
3/8*f*a/c^2*x*(c*x^2+b*x+a)^(1/2)-9/40*g/c^2*b*x^3*(c*x^2+b*x+a)^(1/2)+21/8
0*g/c^3*b^2*x^2*(c*x^2+b*x+a)^(1/2)-21/64*g/c^4*b^3*x*(c*x^2+b*x+a)^(1/2)+3
5/32*g/c^(9/2)*b^3*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-49/32*g/c^
4*b^2*a*(c*x^2+b*x+a)^(1/2)-7/24*f/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+35/96*f/c^
3*b^2*x*(c*x^2+b*x+a)^(1/2)-15/16*f/c^(7/2)*b^2*a*ln((c*x+1/2*b)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))-4/15*g*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)+55/48*f/c^3*b*a*(c*
x^2+b*x+a)^(1/2)-15/16*g/c^(7/2)*b*a^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)
^(1/2))+3/4*e/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/12*
e/c^2*b*x*(c*x^2+b*x+a)^(1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(d + ex + fx^2 + gx^3\right)}{\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

## 3.281 $\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=245

$$\frac{\sqrt{a+bx+cx^2}\,\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c\right.}{192c^4}$$

```
[Out] -1/128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e)
+16*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2)
)/c^(9/2)+1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x^2+
b*x+a)^(1/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(11*a
*g+6*b*f)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^(1/2)
/c^4
```

**Rubi [A]** time = 0.44, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1653, 779, 621, 206}

$$\frac{\sqrt{a+bx+cx^2}\,\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c\right.}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (g*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + ((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*c
*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sqrt
[a + b*x + c*x^2])/(192*c^4) - ((40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) - 35
*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(4cd+(4ce-3ag)x+\frac{1}{2}(8cf-7bg)x^2\right)}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{x\left(12c^2d-8acf+7abg+\frac{1}{4}\left(48c^2e-4\cdots\right.\right.}{\sqrt{a+bx+cx^2}}}{12c^2}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\cdots\right.}{\cdots}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\cdots\right.}{\cdots}$$

$$= \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{\left(192c^3d-16c^2(9be+8af)\cdots\right.}{\cdots}$$

**Mathematica [A]** time = 0.44, size = 199, normalized size = 0.81

$$\frac{\sqrt{a+x(b+cx)}\left(-8c^2\left(16af+9agx+18be+10bfx+7bgx^2\right)+10bc(22ag+12bf+7bgx)-105b^3g+16c^3\left(12\right.\right.}{192c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7*b*g*x) - 8
*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^3*(12*d + x*
(6*e + 4*f*x + 3*g*x^2))))/(192*c^4) + ((-40*b^3*c*f + 32*b*c^2*(-2*c*d + 3
*a*f) + 35*b^4*g + 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(-4*c*e + 3*a*g))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

**fricas [A]** time = 0.71, size = 499, normalized size = 2.04

$$\left[-\frac{3\left(64\,bc^3d-16\left(3\,b^2c^2-4\,ac^3\right)e+8\left(5\,b^3c-12\,abc^2\right)f-\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)g\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c
^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(48*c
^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^
2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*
f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(3*(64*
b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f - (35*b^4
 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 192*c^4*d
- 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*a*c^3)*f -
 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^2*c^2 - 36*
a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

**giac [A]** time = 0.39, size = 228, normalized size = 0.93

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x-\frac{40\,bc^2f-35\,b^2cg+36\,ac^2g-48\,c^3e}{c^4}\right)x+\frac{192\,c^3d+120\,b^2c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x -
 (40*b*c^2*f - 35*b^2*c*g + 36*a*c^2*g - 48*c^3*e)/c^4)*x + (192*c^3*d + 12
0*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g - 144*b*c^2*e)/c^4) + 1/1
28*(64*b*c^3*d + 40*b^3*c*f - 96*a*b*c^2*f - 35*b^4*g + 120*a*b^2*c*g - 48*
a^2*c^2*g - 48*b^2*c^2*e + 64*a*c^3*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*sqrt(c) - b))/c^(9/2)
```

**maple [B]** time = 0.01, size = 532, normalized size = 2.17

$$\frac{\sqrt{c\,x^2+bx+a}\;g\,x^3}{4c}-\frac{7\sqrt{c\,x^2+bx+a}\;bg\,x^2}{24c^2}+\frac{\sqrt{c\,x^2+bx+a}\;f\,x^2}{3c}+\frac{3a^2g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{15a\,b^2g\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/4*g*x^3*(c*x^2+b*x+a)^(1/2)/c-7/24*g/c^2*b*x^2*(c*x^2+b*x+a)^(1/2)+35/96*
g/c^3*b^2*x*(c*x^2+b*x+a)^(1/2)-35/64*g/c^4*b^3*(c*x^2+b*x+a)^(1/2)+35/128*
g/c^(9/2)*b^4*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/16*g/c^(7/2)*b
^2*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*g/c^3*b*a*(c*x^2+b*x
+a)^(1/2)-3/8*g*a/c^2*x*(c*x^2+b*x+a)^(1/2)+3/8*g*a^2/c^(5/2)*ln((c*x+1/2*b
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/3*f*x^2/c*(c*x^2+b*x+a)^(1/2)-5/12*f/c^2*b
*x*(c*x^2+b*x+a)^(1/2)+5/8*f/c^3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*f/c^(7/2)*b^3
*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/4*f/c^(5/2)*b*a*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/3*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*e*x/c*(
c*x^2+b*x+a)^(1/2)-3/4*e/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*e/c^(5/2)*b^2*ln((c*
x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*e*a/c^(3/2)*ln((c*x+1/2*b)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))+d/c*(c*x^2+b*x+a)^(1/2)-1/2*d*b/c^(3/2)*ln((c*x+1/2*
b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
[Out] int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral(x*(d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

# 3.282 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=177

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bc\right.}{24c^3}$$

```
[Out] 1/16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)+1/24*(-16*a*c*g+15*b^2*g-18*b*c*f
+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(1/2
)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c
```

**Rubi [A]** time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{16c^{7/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bc\right.}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x + c*x^2])/(24*c^3
) + ((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c*x^2])/(12*c^2) + (g*x^2*Sqrt[a + b*
x + c*x^2])/(3*c) + ((16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f +
 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{3cd+(3ce-2ag)x+\frac{1}{2}(6cf-5bg)x^2}{\sqrt{a+bx+cx^2}}\,dx}{3c}$$

$$= \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} + \frac{\int \frac{\frac{1}{2}\left(12c^2d-6acf+5abg\right)+\frac{1}{4}\left(24c^2e-18bc\right.}{\sqrt{a+bx+cx^2}}}{6c^2}$$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + $$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + $$

$$= \frac{\left(24c^2e-18bcf+15b^2g-16acg\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + $$

**Mathematica [A]** time = 0.26, size = 141, normalized size = 0.80

$$\frac{3\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(-2c(8ag+9b\right.}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
+ 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*
g + 6*b*c*(b*f + 2*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])/(48*c^(7/2))
```

**fricas [A]** time = 0.76, size = 341, normalized size = 1.93

$$\left[\frac{3\left(16\,c^3d-8\,bc^2e+2\left(3\,b^2c-4\,ac^2\right)f-\left(5\,b^3-12\,abc\right)g\right)\sqrt{c}\,\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+\right.}{96\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*
c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4, -
1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b*c
)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^
2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2*c - 16
*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

**giac [A]** time = 0.27, size = 149, normalized size = 0.84

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,gx}{c}+\frac{6\,c^2f-5\,bcg}{c^3}\right)x-\frac{18\,bcf-15\,b^2g+16\,acg-24\,c^2e}{c^3}\right)-\frac{\left(16\,c^3d+6\,b^2cf-8\,ac^2f-5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x - (18*b
*c*f - 15*b^2*g + 16*a*c*g - 24*c^2*e)/c^3) - 1/16*(16*c^3*d + 6*b^2*c*f -
8*a*c^2*f - 5*b^3*g + 12*a*b*c*g - 8*b*c^2*e)*log(abs(-2*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

**maple [B]** time = 0.01, size = 333, normalized size = 1.88

$$\frac{\sqrt{c\,x^2+bx+a}\;g\,x^2}{3c}+\frac{3abg\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{4c^{\frac{5}{2}}}-\frac{af\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}-\frac{5b^3g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c-5/12*g/c^2*b*x*(c*x^2+b*x+a)^(1/2)+5/8*g/c^
3*b^2*(c*x^2+b*x+a)^(1/2)-5/16*g/c^(7/2)*b^3*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+3/4*g/c^(5/2)*b*a*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
-2/3*g*a/c^2*(c*x^2+b*x+a)^(1/2)+1/2*(c*x^2+b*x+a)^(1/2)/c*f*x-3/4*(c*x^2+b
*x+a)^(1/2)*b/c^2*f+3/8*b^2/c^(5/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))-1/2*a/c^(3/2)*f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+(c*x^2+b
*x+a)^(1/2)/c*e-1/2*b/c^(3/2)*e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
+1/c^(1/2)*d*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/sqrt(a + b*x + c*x**2), x)
```

# 3.283 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=155

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+gx$$

```
[Out] 1/8*(8*c^2*e+3*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)-d*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^
(1/2)+1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(1/2
)/c
```

**Rubi** [A] time = 0.26, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1653, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(4cf-3bg)}{4c^2}-\frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+gx$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (g*x*Sqrt[a + b*x + c*x^2
])/(2*c) - (d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[
a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd+(2ce-ag)x+\frac{1}{2}(4cf-3bg)x^2}{x\sqrt{a+bx+cx^2}}\,dx}{2c}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2c^2d+\frac{1}{4}\left(8c^2e+3b^2g-4c(bf+ag)\right)x}{x\sqrt{a+bx+cx^2}}\,dx}{2c^2}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} + d\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx + \frac{\left(8c^2e+\right.}{}$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - (2d)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a}{\sqrt{a+}}\right.$$

$$= \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2} + \frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{\left(8c^2e\right.}{}$$

**Mathematica [A]** time = 0.37, size = 134, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}\,(-3bg+4cf+2cgx)}{4c^2} - \frac{d\tanh^{-1}\left(\frac{2a+b}{2\sqrt{a}\sqrt{a+x}}\right.}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) - (d*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + ((8*c^2*e + 3*b^2*g -
 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(
8*c^(5/2))
```

**fricas [A]** time = 5.78, size = 733, normalized size = 4.73

$$\left[\frac{8\sqrt{a}\,c^3d\log\left(-\frac{8abx+\left(b^2+4ac\right)x^2-4\sqrt{cx^2+bx+a}\,(bx+2a)\sqrt{a}+8a^2}{x^2}\right) - \left(8ac^2e-4abcf+\left(3ab^2-4a^2c\right)g\right)\sqrt{c}\log\left(-8c^2x^2-\right.}{16ac^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b
^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*
g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^
2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2)
- (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2
*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16*(16*sqrt
(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 +
 a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(c)*lo
g(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
- 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(
a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a
^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3)]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitu
tion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subs
titution variable should perhaps be purged.index.cc index_m operator + Erro
r: Bad Argument Value
```

**maple [A]** time = 0.01, size = 220, normalized size = 1.42

$$-\frac{ag\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}-\frac{d\ln\left(\frac{bx+2a+2\sqrt{c\,x^{2}+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{3b^{2}g\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{5}{2}}}-\frac{bf\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*g*x*(c*x^2+b*x+a)^(1/2)/c-3/4*g/c^2*b*(c*x^2+b*x+a)^(1/2)+3/8*g/c^(5/2)
*b^2*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*g*a/c^(3/2)*ln((c*x+1/
```

```
2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+f/c*(c*x^2+b*x+a)^(1/2)-1/2*f*b/c^(3/2)*l
n((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)-d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

# 3.284 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=139

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

```
[Out] 1/2*(-2*a*e+b*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
+1/2*(-b*g+2*c*f)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2
)+g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x
```

**Rubi [A]** time = 0.24, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1650, 1653, 843, 621, 206, 724}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}-\frac{d\sqrt{a+bx+cx^2}}{ax}+\frac{g\sqrt{a+bx+cx^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (g*Sqrt[a + b*x + c*x^2])/c - (d*Sqrt[a + b*x + c*x^2])/(a*x) + ((b*d - 2*a
*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)) + (
(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}(bd-2ae)-afx-agx^2}{x\sqrt{a+bx+cx^2}}\,dx}{a}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{\int \frac{\frac{1}{2}c(bd-2ae)-\frac{1}{2}a(2cf-bg)x}{x\sqrt{a+bx+cx^2}}\,dx}{ac}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} - \frac{(bd-2ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a} + \frac{(2cf-bg)\int \frac{}{\sqrt{}}}{2c}$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a} +$$

$$= \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf-}{}$$

**Mathematica [A]** time = 0.40, size = 127, normalized size = 0.91

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{2c^{3/2}} + \frac{\sqrt{a+x(b+cx)}\,(agx-cd)}{acx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((b*d - 2*a*e)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(2*a^(3/2)) + ((2*c*f - b*g)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*c^(3/2))
```

**fricas [A]** time = 3.72, size = 703, normalized size = 5.06

$$\left[-\frac{\left(2\,a^2cf-a^2bg\right)\sqrt{c}\,x\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)+\left(bc^2d-2\,ac^2e\right)\sqrt{a}\,x\,\mathrm{l}}{4\,a^2c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)*s
qrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x +
 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x + a)
)/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*d - 2*
a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2
+ b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^
2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + (2*
a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*
sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*
x + a))/(a^2*c^2*x)]
```

**giac [A]** time = 0.37, size = 171, normalized size = 1.23

$$\frac{\sqrt{cx^2+bx+a}\,g}{c}-\frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\left(2\,cf-bg\right)\log\left(\left|2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a
)*a)
```

**maple [A]** time = 0.01, size = 173, normalized size = 1.24

$$-\frac{e\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bd\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{bg\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{f\ln\left(\frac{cx+\frac{b}{2}}{\sqrt{c}}+\sqrt{c\,x^2+\right.}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*(c*x^2+b*x+a)^(1/2)/c-1/2*g*b/c^(3/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))+f*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-d*(c*x^2+b*x
+a)^(1/2)/a/x+1/2*d*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
-e/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima** [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad** [**B**] time = 4.46, size = 166, normalized size = 1.19

$$\frac{g\,\sqrt{c\,x^2+b\,x+a}}{c}-\frac{e\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}+\frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}}-\frac{b\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+\right.}{2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] (g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^
2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2
)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(2*c
^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/(a^(
1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

**sympy** [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

# 3.285 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=159

$$\frac{\sqrt{a+bx+cx^2}\,(3bd-4ae)}{4a^2x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\frac{\ }{2\ }\right.}{\sqrt{\ }}$$

```
[Out] -1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a)^(
1/2)/a^2/x
```

**Rubi [A]** time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 843, 621, 206, 724}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2f-4abe-4acd+3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+bx+cx^2}\,(3bd-4ae)}{4a^2x}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{g\tanh^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(2*a*x^2) + ((3*b*d - 4*a*e)*Sqrt[a + b*x + c*x^
2])/(4*a^2*x) - ((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*a + b*x
)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} - \frac{\int \frac{\frac{1}{2}(3bd-4ae)+(cd-2af)x-2agx^2}{x^2\sqrt{a+bx+cx^2}}\,dx}{2a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\int \frac{\frac{1}{4}\left(3b^2d-4abe-4a(cd-2af)\right)+2a^2gx}{x\sqrt{a+bx+cx^2}}\,dx}{2a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} + \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\int -}{8a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\mathrm{Sub}}{4a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{2ax^2} + \frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{\left(3b^2d-4acd-4abe+8a^2f\right)\tan}{8a^{5/2}}$$

**Mathematica [A]** time = 0.36, size = 137, normalized size = 0.86

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(4abe+4a(cd-2af)-3b^2d\right)}{8a^{5/2}}+\frac{\sqrt{a+x(b+cx)}\,(3bdx-2a(d+2ex))}{4a^2x^2}+\frac{g\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/(4*a^2*x^2) + ((-3*b^2*
d + 4*a*b*e + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])])/(8*a^(5/2)) + (g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)])])/Sqrt[c]
```

**fricas [A]** time = 4.81, size = 783, normalized size = 4.92

$$\left[\frac{8\,a^3\sqrt{c}\,gx^2\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{c}\,-4\,ac\right)-\left(4\,abce-8\,a^2cf-\left(3\,b^2c-4\,ac^2\right)d\right.}{16\,a^3cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c -
 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
 b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d - 4
*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(
8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x +
 a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*
c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d - (3*a*b*c*d -
 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^3*sqrt(-c)*g*x
^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
 a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arcta
n(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) +
 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^
2)]
```

**giac [B]** time = 0.36, size = 352, normalized size = 2.21

$$-\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)c-b\sqrt{c}\right|\right)}{\sqrt{c}}+\frac{\left(3b^2d-4acd+8a^2f-4abe\right)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2}-\frac{3\left(\sqrt{\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*c - b*sqrt(c)))/sqrt(c) +
 1/4*(3*b^2*d - 4*a*c*d + 8*a^2*f - 4*a*b*e)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^2 - a)^2*a^2)
```

**maple [A]** time = 0.01, size = 241, normalized size = 1.52

$$-\frac{f\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{be\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{cd\ln\left(\frac{bx+2a+2\sqrt{cx^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2d\ln\left(\frac{bx+2a+2\sqrt{cx^2+b}}{x}\right.}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] g*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d*(c*x^2+b*x+a)^(
1/2)/a/x^2+3/4*d*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*d*b^2/a^(5/2)*ln((b*x+2*a+
2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*d*c/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+
a)^(1/2)*a^(1/2))/x)-e/a/x*(c*x^2+b*x+a)^(1/2)+1/2*e*b/a^(3/2)*ln((b*x+2*a+
2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a^(1/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/
2)*a^(1/2))/x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

# 3.286 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=186

$$\frac{\sqrt{a+bx+cx^2}\,(5bd-6ae)}{12a^2x^2}+\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx}}{}$$

```
[Out] 1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh(1/
2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)-1/3*d*(c*x^2+b*x+a)^(1/2)/
a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^2-1/24*(24*a^2*f-18*a*b
*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x
```

**Rubi [A]** time = 0.32, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1650, 806, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{16a^{7/2}}-\frac{\sqrt{a+bx+cx^2}\,\left(24a^2f-18abe-16a\right.}{24a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(3*a*x^3) + ((5*b*d - 6*a*e)*Sqrt[a + b*x + c*x^
2])/(12*a^2*x^2) - ((15*b^2*d - 16*a*c*d - 18*a*b*e + 24*a^2*f)*Sqrt[a + b*
x + c*x^2])/(24*a^3*x) + ((5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*f) + 8*
a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(16*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} - \frac{\int \frac{\frac{1}{2}(5bd-6ae)+(2cd-3af)x-3agx^2}{x^3\sqrt{a+bx+cx^2}}\,dx}{3a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} + \frac{\int \frac{\frac{1}{4}\left(15b^2d-16acd-18abe+24a^2f\right)+\frac{1}{2}\left(5bcd-6\right.}{x^2\sqrt{a+bx+cx^2}}}{6a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)}{24a^3x}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3} + \frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2} - \frac{\left(15b^2d-16acd-18abe+24a^2f\right)}{24a^3x}$$

**Mathematica [A]** time = 0.31, size = 150, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(8a^2(ce-2ag)-6ab^2e+4ab(2af-3cd)+5b^3d\right)}{16a^{7/2}} - \frac{\sqrt{a+x(b+cx)}\left(4a^2(2d+3x(e+2fx\right.}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/24*(Sqrt[a + x*(b + c*x)]*(15*b^2*d*x^2 - 2*a*x*(5*b*d + 8*c*d*x + 9*b*e
*x) + 4*a^2*(2*d + 3*x*(e + 2*f*x))))/(a^3*x^3) + ((5*b^3*d - 6*a*b^2*e + 4
*a*b*(-3*c*d + 2*a*f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + x*(b + c*x)])])/(16*a^(7/2))
```

**fricas [A]** time = 5.55, size = 365, normalized size = 1.96

$$\left[-\frac{3\left(8\,a^2bf-16\,a^3g+\left(5\,b^3-12\,abc\right)d-2\left(3\,ab^2-4\,a^2c\right)e\right)\sqrt{a}\,x^3\log\left(-\frac{8\,abx+\left(b^2+4\,ac\right)x^2-4\,\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right.}{96\,a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^2
*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^3*f
 - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d -
 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*e - 24*
a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^4*x^3)]
```

**giac [B]** time = 0.29, size = 689, normalized size = 3.70

$$-\frac{\left(5\,b^3d-12\,abcd+8\,a^2bf-16\,a^3g-6\,ab^2e+8\,a^2ce\right)\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^3}+\frac{15\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)^5b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(5*b^3*d - 12*a*b*c*d + 8*a^2*b*f - 16*a^3*g - 6*a*b^2*e + 8*a^2*c*e)*
arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/24
*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))^5*a*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b*f
```

```
 - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*e + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a^2*c*e + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^3*
sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*a^3*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^2*e + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d - 96*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^2*a^4*sqrt(c)*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*
a^3*b*sqrt(c)*e + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^3*d + 36*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d + 24*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))*a^4*b*f - 30*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*c*e + 48*a^3*b^2*sqrt(c)*d - 32*a^4*c
^(3/2)*d + 48*a^5*sqrt(c)*f - 48*a^4*b*sqrt(c)*e)/(((sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2 - a)^3*a^3)
```

**maple [B]** time = 0.01, size = 375, normalized size = 2.02

$$-\frac{g\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{bf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{ce\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2*e/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*e*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*e*b^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*e*c/a^(3/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-f/a/x*(c*x^2+b*x+a)^(1/2)+1/2*f*
b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-g/a^(1/2)*ln((b*x+2
*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/3*d*(c*x^2+b*x+a)^(1/2)/a/x^3+5/12*d
*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*d*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/16*d*b
^3/a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-3/4*d*b/a^(5/2)*c*
ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*d*c/a^2/x*(c*x^2+b*x+a)^(
1/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

# 3.287 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=270

$$\frac{\sqrt{a+bx+cx^2}\,(7bd-8ae)}{24a^2x^3}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd\right.}{128a^{9/2}}$$

```
[Out] -1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f+3*c*d)-24*a*b^2*(-2*a*f+5*c*d)
+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2)
)/a^(9/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x+a)
^(1/2)/a^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)^(1/
2)/a^3/x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*(-3*a
*g+2*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x
```

**Rubi [A]** time = 0.49, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 834, 806, 724, 206}

$$\frac{\sqrt{a+bx+cx^2}\,\left(64a^2(2ce-3ag)-120ab^2e-4ab(55cd-36af)+105b^3d\right)}{192a^4x}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(4*a*x^4) + ((7*b*d - 8*a*e)*Sqrt[a + b*x + c*x^
2])/(24*a^2*x^3) - ((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*Sqrt[a + b*
x + c*x^2])/(96*a^3*x^2) + ((105*b^3*d - 120*a*b^2*e - 4*a*b*(55*c*d - 36*a
*f) + 64*a^2*(2*c*e - 3*a*g))*Sqrt[a + b*x + c*x^2])/(192*a^4*x) - ((35*b^4
*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d - 2*a*f) + 32*
a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(128*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} - \frac{\int \frac{\frac{1}{2}(7bd-8ae)+(3cd-4af)x-4agx^2}{x^4\sqrt{a+bx+cx^2}}\,dx}{4a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} + \frac{\int \frac{\frac{1}{4}\left(35b^2d-40abe-12a(3cd-4af)\right)+\left(7bcd-8a\right.}{x^3\sqrt{a+bx+cx^2}}}{12a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)}{96a^3x^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{4ax^4} + \frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{\left(35b^2d-36acd-40abe+48a^2f\right)}{96a^3x^2}$$

**Mathematica** [A] time = 0.52, size = 212, normalized size = 0.79

$$\frac{\sqrt{a+x(b+cx)}\,\left(-16a^3\left(3d+4ex+6x^2(f+2gx)\right)+8a^2x(7bd+2bx(5e+9fx)+cx(9d+16ex))-10abx^2(7bd+\right.}{192a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*d*x + 12*b
*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x)) - 16*a^3
*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/(192*a^4*x^4) - ((35*b^4*d - 40*a*b^3*
e + 16*a^2*c*(3*c*d - 4*a*f) + 24*a*b^2*(-5*c*d + 2*a*f) + 32*a^2*b*(3*c*e
- 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(128*a^(9
/2))
```

**fricas** [A] time = 10.83, size = 525, normalized size = 1.94

$$\left[\frac{3\left(64\,a^3bg-\left(35\,b^4-120\,ab^2c+48\,a^2c^2\right)d+8\left(5\,ab^3-12\,a^2bc\right)e-16\left(3\,a^2b^2-4\,a^3c\right)f\right)\sqrt{a}\,x^4\log\left(-\frac{8\,abx+\left(b^2+4\,ac\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
- 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2
) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d -
8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 -
 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*
x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*
a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*arctan(1/2
*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(4
8*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d - 8*(15*a^
2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^2 - 36*a^3*
c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(a^5*x^4)]
```

**giac [B]** time = 0.30, size = 1448, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d + 48*a^2*b^2*f - 64*a^3*c*f -
 64*a^3*b*g - 40*a*b^3*e + 96*a^2*b*c*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d + 14
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d + 144*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^7*a^2*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^
3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*g - 120*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)^7*a^2*b*c*e - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt(c)*g - 3
85*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*
a^3*c^2*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a^4*b*g + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e - 1
056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 384*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*sqrt(c)*g - 768*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2
)*e + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d - 1752*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^3*a^4*c^2*d + 624*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^2*f +
```

```
192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*c*f - 576*(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))^3*a^5*b*g - 584*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*
b^3*e + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b*c*e - 2048*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^2*a^4*b*c^(3/2)*d + 768*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^2*a^5*b*sqrt(c)*f - 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
*a^6*sqrt(c)*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^4*b^2*sqrt(c)*
e + 1024*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*c^(3/2)*e - 279*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*a^3*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*a^4*b^2*c*d + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*c^2*d - 240*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^2*f - 192*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))*a^6*c*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*g + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^3*e + 288*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*a^5*b*c*e - 384*a^4*b^3*sqrt(c)*d + 512*a^5*b*c^(3/2)*d - 38
4*a^6*b*sqrt(c)*f + 384*a^7*sqrt(c)*g + 384*a^5*b^2*sqrt(c)*e - 256*a^6*c^(
3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^4*a^4)
```

**maple [B]** time = 0.01, size = 591, normalized size = 2.19

$$\frac{bg\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}+\frac{cf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2f\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}-\frac{3bce\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+}}{x}\right.}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2*f/a/x^2*(c*x^2+b*x+a)^(1/2)+3/4*f*b/a^2/x*(c*x^2+b*x+a)^(1/2)-3/8*f*b^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*f*c/a^(3/2)*ln(
(b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/3*e/a/x^3*(c*x^2+b*x+a)^(1/2)+
5/12*e*b/a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*e*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/
16*e*b^3/a^(7/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-3/4*e*b/a^(5
/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*e*c/a^2/x*(c*x^2+b*
x+a)^(1/2)-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+7/24*d*b/a^2/x^3*(c*x^2+b*x+a)^(
1/2)-35/96*d*b^2/a^3/x^2*(c*x^2+b*x+a)^(1/2)+35/64*d*b^3/a^4/x*(c*x^2+b*x+a
)^(1/2)-35/128*d*b^4/a^(9/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+
15/16*d*b^2/a^(7/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-55/48*d
*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+3/8*d*c/a^2/x^2*(c*x^2+b*x+a)^(1/2)-3/8*d*c^
2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-g/a/x*(c*x^2+b*x+a)
^(1/2)+1/2*g*b/a^(3/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

# 3.288 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=371

$$\frac{\sqrt{a+bx+cx^2}\,(9bd-10ae)}{40a^2x^4}-\frac{\sqrt{a+bx+cx^2}\,\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-1050ab^3e-60ab^2(49cd\right.}{1920a^5x}$$

```
[Out] 1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c*d)-40*a*b^3*(-2*a*f+7*c*d
)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c*e))*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/40*
(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64*a*c
*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/x^3+1/960*(315*b^3*d-350*a*b^2*e-4*a*b
*(-100*a*f+161*c*d)+120*a^2*(-4*a*g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x^2-1/1
920*(945*b^4*d-1050*a*b^3*e-60*a*b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*a*f+4*c
*d)+40*a^2*b*(-36*a*g+55*c*e))*(c*x^2+b*x+a)^(1/2)/a^5/x
```

**Rubi [A]** time = 0.82, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1650, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx+cx^2}\,\left(40a^2b(55ce-36ag)+256a^2c(4cd-5af)-60ab^2(49cd-20af)-1050ab^3e+945b^4d\right)}{1920a^5x}+\frac{\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(d*Sqrt[a + b*x + c*x^2])/(5*a*x^5) + ((9*b*d - 10*a*e)*Sqrt[a + b*x + c*x
^2])/(40*a^2*x^4) - ((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)*Sqrt[a + b
*x + c*x^2])/(240*a^3*x^3) + ((315*b^3*d - 350*a*b^2*e - 4*a*b*(161*c*d - 1
00*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(960*a^4*x^2) - (
(945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2*c*(4*c*d -
 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(1920*a^5*x) +
 ((63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*(7*c*d - 2
*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(256*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} - \frac{\int \frac{\frac{1}{2}(9bd-10ae)+(4cd-5af)x-5agx^2}{x^5\sqrt{a+bx+cx^2}}\,dx}{5a}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} + \frac{\int \frac{\frac{1}{4}\left(63b^2d-64acd-70abe+80a^2f\right)+\frac{1}{2}(27bc}{x^4\sqrt{a+bx+cx^2}}}{20a^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)}{240a^3x^3}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{5ax^5} + \frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4} - \frac{\left(63b^2d-64acd-70abe+80a^2f\right)}{240a^3x^3}$$

**Mathematica [A]** time = 0.73, size = 299, normalized size = 0.81

$$\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)\left(32a^3c(4ag-3ce)-48a^2b^2(2ag-5ce)-48a^2bc(4af-5cd)-70ab^4e+40ab^3(2af-7cd\right.}{256a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -1/1920*(Sqrt[a + x*(b + c*x)]*(945*b^4*d*x^4 - 210*a*b^2*x^3*(3*b*d + 14*c
*d*x + 5*b*e*x) + 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) + 4*a^2*x^2*(
256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 12*f*x
))) - 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e + 2*x*
(5*f + 9*g*x))))))/(a^5*x^5) + ((63*b^5*d - 70*a*b^4*e + 40*a*b^3*(-7*c*d +
 2*a*f) - 48*a^2*b*c*(-5*c*d + 4*a*f) - 48*a^2*b^2*(-5*c*e + 2*a*g) + 32*a^
3*c*(-3*c*e + 4*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])
])/(256*a^(11/2))
```

**fricas [A]** time = 42.47, size = 727, normalized size = 1.96

$$\left[\frac{15\left(\left(63\,b^5-280\,ab^3c+240\,a^2bc^2\right)d-2\left(35\,ab^4-120\,a^2b^2c+48\,a^3c^2\right)e+16\left(5\,a^2b^3-12\,a^3bc\right)f-32\left(3\,a^3b^2-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a
^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 -
4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b*g -
 (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*
c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(
45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*
b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)
*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a*b^3*c +
240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*
b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(384
*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50
*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a
^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^
4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 4
8*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5)]
```

**giac [B]** time = 0.51, size = 2177, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d + 80*a^2*b^3*f - 192*a^3
*b*c*f - 96*a^3*b^2*g + 128*a^4*c*g - 70*a*b^4*e + 240*a^2*b^2*c*e - 96*a^3
*c^2*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^5
) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^9*a^2*b*c^2*d + 1200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^3*
f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b*c*f - 1440*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x +
```

a))^9*a^4*c*g - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^4*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^2*e - 4410*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c*d - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d - 5600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b^2*g - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*c*g + 4900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*c^2*e + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^5*d - 35840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^3*c*d + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*c^2*d + 10240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^3*f - 15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b*c*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^5*b^2*g - 8960*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b^4*e + 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b^2*c*e + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*c^(5/2)*d + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b^2*sqrt(c)*f - 17920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*c^(3/2)*f - 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^6*b*sqrt(c)*g + 20480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^5*b*c^(3/2)*e - 7110*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^5*d + 31600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^3*c*d + 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b*c^2*d - 8480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^3*f + 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b*c*f + 8640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*b^2*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^7*c*g + 7900*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^4*b^4*e - 13920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^5*b^2*c*e - 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^6*c^2*e + 38400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^2*c^(3/2)*d - 10240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*c^(5/2)*d - 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b^2*sqrt(c)*f + 12800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*c^(3/2)*f + 11520*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^7*b*sqrt(c)*g + 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^5*b^3*sqrt(c)*e - 25600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^6*b*c^(3/2)*e + 2895*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b^5*d + 4200*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^3*c*d - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b*c^2*d + 2640*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^3*f + 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b*c*f - 2400*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*b^2*g - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^8*c*g - 2790*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^5*b^4*e - 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^6*b^2*c*e + 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^7*c^2*e + 3840*a^5*b^4*sqrt(c)*d - 7680*a^6*b^2*c^(3/2)*d + 2048*a^7*c^(5/2)*d + 3840*a^7*b^2*sqrt(c)*f - 2560*a^8*c^(3/2)*f - 3840*a^8*b*sqrt(c)*g - 3840*a^6*b^3*sqrt(c)*e + 5120*a^7*b*c^(3/2)*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^5 *a^5)

**maple [B]** time = 0.02, size = 859, normalized size = 2.32

$$\frac{cg\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2g\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{8a^{\frac{5}{2}}}-\frac{3bcf\ln\left(\frac{bx+2a+2\sqrt{c\,x^2+bx+a}\,\sqrt{a}}{x}\right)}{4a^{\frac{5}{2}}}-\frac{3c^2e\ln\left(\frac{bx+2a+2\sqrt{c}}{}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/4*e/a/x^4*(c*x^2+b*x+a)^(1/2)-35/128*e*b^4/a^(9/2)*ln((b*x+2*a+2*(c*x^2+
b*x+a)^(1/2)*a^(1/2))/x)-3/8*e*c^2/a^(5/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2
)*a^(1/2))/x)-1/3*f/a/x^3*(c*x^2+b*x+a)^(1/2)+5/16*f*b^3/a^(7/2)*ln((b*x+2*
a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+63/256*d*b^5/a^(11/2)*ln((b*x+2*a+2*(c*
x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/2*g/a/x^2*(c*x^2+b*x+a)^(1/2)-3/8*g*b^2/a^(5
/2)*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1/2*g*c/a^(3/2)*ln((b*x+2
*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5-161/24
0*d*b/a^3*c/x^2*(c*x^2+b*x+a)^(1/2)-55/48*e*b/a^3*c/x*(c*x^2+b*x+a)^(1/2)+4
9/32*d*b^2/a^4*c/x*(c*x^2+b*x+a)^(1/2)+3/8*e*c/a^2/x^2*(c*x^2+b*x+a)^(1/2)+
9/40*d*b/a^2/x^4*(c*x^2+b*x+a)^(1/2)-21/80*d*b^2/a^3/x^3*(c*x^2+b*x+a)^(1/2
)+21/64*d*b^3/a^4/x^2*(c*x^2+b*x+a)^(1/2)-63/128*d*b^4/a^5/x*(c*x^2+b*x+a)^
(1/2)-35/32*d*b^3/a^(9/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+1
5/16*d*b/a^(7/2)*c^2*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+4/15*d*c
/a^2/x^3*(c*x^2+b*x+a)^(1/2)-8/15*d*c^2/a^3/x*(c*x^2+b*x+a)^(1/2)+5/12*f*b/
a^2/x^2*(c*x^2+b*x+a)^(1/2)-5/8*f*b^2/a^3/x*(c*x^2+b*x+a)^(1/2)-3/4*f*b/a^(
5/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))/x)+2/3*f*c/a^2/x*(c*x^2+b
*x+a)^(1/2)+7/24*e*b/a^2/x^3*(c*x^2+b*x+a)^(1/2)-35/96*e*b^2/a^3/x^2*(c*x^2
+b*x+a)^(1/2)+3/4*g*b/a^2/x*(c*x^2+b*x+a)^(1/2)+35/64*e*b^3/a^4/x*(c*x^2+b*
x+a)^(1/2)+15/16*e*b^2/a^(7/2)*c*ln((b*x+2*a+2*(c*x^2+b*x+a)^(1/2)*a^(1/2))
/x)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive, negative or zero?
```

**mupad** **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
[Out] Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

# 3.289 $\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=258

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(300d^4+170d^3e+102d^2e^2+12d\right.}{6e^7}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e^7
-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^5/e^7+
1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^6/e^7-2/7*(200*
d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+85*d*e+17*e^2)*(e*x
+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^7
```

**Rubi [A]** time = 0.26, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^8}{8e^7}-\frac{2\left(85d^2e+200d^3+34de^2+2e^3\right)(d+ex)^7}{7e^7}+\frac{\left(102d^2e^2+170d^3e+300d^4+12d\right.}{6e^7}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^
4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*
d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 +
2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8)/(8*
e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)dx=\int\left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5\right.}{e^6}\right.$$

$$=\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(}{4e^7}$$

**Mathematica** [A] time = 0.04, size = 212, normalized size = 0.82

$$6d^3x+\frac{1}{8}ex^8\left(60d^2-51de+17e^2\right)+dx^3\left(7d^2+7de+6e^2\right)+\frac{1}{2}d^2x^2(7d+18e)+\frac{1}{7}x^7\left(20d^3-51d^2e+51de^2-4e^3\right)+\frac{1}{6}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*d
^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2 +
 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20*d^
3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)*x^8
)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

**fricas** [A] time = 0.74, size = 237, normalized size = 0.92

$$2x^{10}e^3-\frac{17}{9}x^9e^3+\frac{20}{3}x^9e^2d+\frac{17}{8}x^8e^3-\frac{51}{8}x^8e^2d+\frac{15}{2}x^8ed^2-\frac{4}{7}x^7e^3+\frac{51}{7}x^7e^2d-\frac{51}{7}x^7ed^2+\frac{20}{7}x^7d^3+\frac{7}{2}x^6e^3-2x^6e^2d+\frac{17}{2}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 2*x^10*e^3 - 17/9*x^9*e^3 + 20/3*x^9*e^2*d + 17/8*x^8*e^3 - 51/8*x^8*e^2*d
+ 15/2*x^8*e*d^2 - 4/7*x^7*e^3 + 51/7*x^7*e^2*d - 51/7*x^7*e*d^2 + 20/7*x^7
*d^3 + 7/2*x^6*e^3 - 2*x^6*e^2*d + 17/2*x^6*e*d^2 - 17/6*x^6*d^3 + 7/5*x^5*
e^3 + 63/5*x^5*e^2*d - 12/5*x^5*e*d^2 + 17/5*x^5*d^3 + 3/2*x^4*e^3 + 21/4*x
^4*e^2*d + 63/4*x^4*e*d^2 - x^4*d^3 + 6*x^3*e^2*d + 7*x^3*e*d^2 + 7*x^3*d^3
 + 9*x^2*e*d^2 + 7/2*x^2*d^3 + 6*x*d^3
```

**giac** [A] time = 0.17, size = 230, normalized size = 0.89

$$2\,x^{10}e^3+\frac{20}{3}\,dx^9e^2+\frac{15}{2}\,d^2x^8e+\frac{20}{7}\,d^3x^7-\frac{17}{9}\,x^9e^3-\frac{51}{8}\,dx^8e^2-\frac{51}{7}\,d^2x^7e-\frac{17}{6}\,d^3x^6+\frac{17}{8}\,x^8e^3+\frac{51}{7}\,dx^7e^2+\frac{17}{2}\,d^2x^6e+\frac{17}{5}\,d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 2*x^10*e^3 + 20/3*d*x^9*e^2 + 15/2*d^2*x^8*e + 20/7*d^3*x^7 - 17/9*x^9*e^3
- 51/8*d*x^8*e^2 - 51/7*d^2*x^7*e - 17/6*d^3*x^6 + 17/8*x^8*e^3 + 51/7*d*x^
7*e^2 + 17/2*d^2*x^6*e + 17/5*d^3*x^5 - 4/7*x^7*e^3 - 2*d*x^6*e^2 - 12/5*d^
2*x^5*e - d^3*x^4 + 7/2*x^6*e^3 + 63/5*d*x^5*e^2 + 63/4*d^2*x^4*e + 7*d^3*x
^3 + 7/5*x^5*e^3 + 21/4*d*x^4*e^2 + 7*d^2*x^3*e + 7/2*d^3*x^2 + 3/2*x^4*e^3
 + 6*d*x^3*e^2 + 9*d^2*x^2*e + 6*d^3*x
```

**maple [A]** time = 0.00, size = 208, normalized size = 0.81

$$2e^{3}x^{10}+\frac{\left(60d\,e^{2}-17e^{3}\right)x^{9}}{9}+\frac{\left(60d^{2}e-51d\,e^{2}+17e^{3}\right)x^{8}}{8}+\frac{\left(20d^{3}-51d^{2}e+51d\,e^{2}-4e^{3}\right)x^{7}}{7}+\frac{\left(-17d^{3}+51d^{2}e-12\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 2*e^3*x^10+1/9*(60*d*e^2-17*e^3)*x^9+1/8*(60*d^2*e-51*d*e^2+17*e^3)*x^8+1/7
*(20*d^3-51*d^2*e+51*d*e^2-4*e^3)*x^7+1/6*(-17*d^3+51*d^2*e-12*d*e^2+21*e^3
)*x^6+1/5*(17*d^3-12*d^2*e+63*d*e^2+7*e^3)*x^5+1/4*(-4*d^3+63*d^2*e+21*d*e^
2+6*e^3)*x^4+1/3*(21*d^3+21*d^2*e+18*d*e^2)*x^3+1/2*(7*d^3+18*d^2*e)*x^2+6*
d^3*x
```

**maxima [A]** time = 0.43, size = 206, normalized size = 0.80

$$2\,e^{3}x^{10}+\frac{1}{9}\left(60\,de^{2}-17\,e^{3}\right)x^{9}+\frac{1}{8}\left(60\,d^{2}e-51\,de^{2}+17\,e^{3}\right)x^{8}+\frac{1}{7}\left(20\,d^{3}-51\,d^{2}e+51\,de^{2}-4\,e^{3}\right)x^{7}-\frac{1}{6}\left(17\,d^{3}-51\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e^
3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 51*
d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e^3)
*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3 + 7
*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

**mupad [B]** time = 4.20, size = 196, normalized size = 0.76

$$6\,d^{3}\,x+x^{8}\,\left(\frac{15\,d^{2}\,e}{2}-\frac{51\,d\,e^{2}}{8}+\frac{17\,e^{3}}{8}\right)-x^{6}\,\left(\frac{17\,d^{3}}{6}-\frac{17\,d^{2}\,e}{2}+2\,d\,e^{2}-\frac{7\,e^{3}}{2}\right)+x^{4}\,\left(-d^{3}+\frac{63\,d^{2}\,e}{4}+\frac{21\,d\,e^{2}}{4}+\frac{3\,e^{3}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 - (
17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4 -
d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^3)/
5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3*x^1
0 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^9*(60
*d - 17*e))/9
```

**sympy [A]** time = 0.52, size = 230, normalized size = 0.89

$$6d^3x+2e^3x^{10}+x^9\left(\frac{20de^2}{3}-\frac{17e^3}{9}\right)+x^8\left(\frac{15d^2e}{2}-\frac{51de^2}{8}+\frac{17e^3}{8}\right)+x^7\left(\frac{20d^3}{7}-\frac{51d^2e}{7}+\frac{51de^2}{7}-\frac{4e^3}{7}\right)+x^6\left(-\frac{17d^3}{6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e/
2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/7
- 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x**5*
(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d**2*
e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) + x**2*
(7*d**3/2 + 9*d**2*e)
```

# 3.290 $\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=157

$$\frac{1}{7}x^7\left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6\left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5\left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4\left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3\left(21\right.$$

```
[Out] 6*d^2*x+1/2*d*(7*d+12*e)*x^2+1/3*(21*d^2+14*d*e+6*e^2)*x^3-1/4*(4*d^2-42*d*
e-7*e^2)*x^4+1/5*(17*d^2-8*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+1/
7*(20*d^2-34*d*e+17*e^2)*x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9
```

**Rubi** [A] time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{1}{7}x^7\left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6\left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5\left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4\left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3\left(21\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4*
d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d^2
 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d - 1
7*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)^2\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)dx &= \int \left(6d^2+d(7d+12e)x+\left(21d^2+14de+6e^2\right)x^2-\left(\right.\right.\\ &= 6d^2x+\frac{1}{2}d(7d+12e)x^2+\frac{1}{3}\left(21d^2+14de+6e^2\right)x^3-\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 136, normalized size = 0.87

$$d^2\left(\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x\right)+de\left(5x^8-\frac{34x^7}{7}+\frac{17x^6}{3}-\frac{8x^5}{5}+\frac{21x^4}{2}+\frac{14x^3}{3}+6x^2\right)+\frac{e^2\left(56\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] (e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 5600
*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6
+ (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x^6)
/3 - (34*x^7)/7 + 5*x^8)
```

**fricas [A]** time = 0.67, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2 - \frac{17}{8}x^8e^2 + 5x^8ed + \frac{17}{7}x^7e^2 - \frac{34}{7}x^7ed + \frac{20}{7}x^7d^2 - \frac{2}{3}x^6e^2 + \frac{17}{3}x^6ed - \frac{17}{6}x^6d^2 + \frac{21}{5}x^5e^2 - \frac{8}{5}x^5ed + \frac{17}{5}x^5d^2 + \frac{7}{4}x^4e^2 + \frac{21}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 20/9*x^9*e^2 - 17/8*x^8*e^2 + 5*x^8*e*d + 17/7*x^7*e^2 - 34/7*x^7*e*d + 20/
7*x^7*d^2 - 2/3*x^6*e^2 + 17/3*x^6*e*d - 17/6*x^6*d^2 + 21/5*x^5*e^2 - 8/5*
x^5*e*d + 17/5*x^5*d^2 + 7/4*x^4*e^2 + 21/2*x^4*e*d - x^4*d^2 + 2*x^3*e^2 +
 14/3*x^3*e*d + 7*x^3*d^2 + 6*x^2*e*d + 7/2*x^2*d^2 + 6*x*d^2
```

**giac [A]** time = 0.16, size = 160, normalized size = 1.02

$$\frac{20}{9}x^9e^2 + 5\,dx^8e + \frac{20}{7}d^2x^7 - \frac{17}{8}x^8e^2 - \frac{34}{7}dx^7e - \frac{17}{6}d^2x^6 + \frac{17}{7}x^7e^2 + \frac{17}{3}dx^6e + \frac{17}{5}d^2x^5 - \frac{2}{3}x^6e^2 - \frac{8}{5}dx^5e - d^2x^4 + \frac{21}{5}x^5e^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 20/9*x^9*e^2 + 5*d*x^8*e + 20/7*d^2*x^7 - 17/8*x^8*e^2 - 34/7*d*x^7*e - 17/
6*d^2*x^6 + 17/7*x^7*e^2 + 17/3*d*x^6*e + 17/5*d^2*x^5 - 2/3*x^6*e^2 - 8/5*
d*x^5*e - d^2*x^4 + 21/5*x^5*e^2 + 21/2*d*x^4*e + 7*d^2*x^3 + 7/4*x^4*e^2 +
 14/3*d*x^3*e + 7/2*d^2*x^2 + 2*x^3*e^2 + 6*d*x^2*e + 6*d^2*x
```

**maple [A]** time = 0.00, size = 146, normalized size = 0.93

$$\frac{20e^2x^9}{9} + \frac{(40de - 17e^2)\,x^8}{8} + \frac{(20d^2 - 34de + 17e^2)\,x^7}{7} + \frac{(-17d^2 + 34de - 4e^2)\,x^6}{6} + \frac{(17d^2 - 8de + 21e^2)\,x^5}{5} + \frac{(-4d^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 20/9*e^2*x^9+1/8*(40*d*e-17*e^2)*x^8+1/7*(20*d^2-34*d*e+17*e^2)*x^7+1/6*(-1
7*d^2+34*d*e-4*e^2)*x^6+1/5*(17*d^2-8*d*e+21*e^2)*x^5+1/4*(-4*d^2+42*d*e+7*
e^2)*x^4+1/3*(21*d^2+14*d*e+6*e^2)*x^3+1/2*(7*d^2+12*d*e)*x^2+6*d^2*x
```

**maxima [A]** time = 0.43, size = 145, normalized size = 0.92

$$\frac{20}{9}e^2x^9+\frac{1}{8}\left(40\,de-17\,e^2\right)x^8+\frac{1}{7}\left(20\,d^2-34\,de+17\,e^2\right)x^7-\frac{1}{6}\left(17\,d^2-34\,de+4\,e^2\right)x^6+\frac{1}{5}\left(17\,d^2-8\,de+21\,e^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*x
^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^5
- 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 + 6*
d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

**mupad [B]** time = 4.11, size = 137, normalized size = 0.87

$$x^3\left(7\,d^2+\frac{14\,d\,e}{3}+2\,e^2\right)+x^4\left(-d^2+\frac{21\,d\,e}{2}+\frac{7\,e^2}{4}\right)-x^6\left(\frac{17\,d^2}{6}-\frac{17\,d\,e}{3}+\frac{2\,e^2}{3}\right)+x^5\left(\frac{17\,d^2}{5}-\frac{8\,d\,e}{5}+\frac{21\,e^2}{5}\right)+x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^6
*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (21*
e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2*x^
9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

**sympy [A]** time = 0.15, size = 158, normalized size = 1.01

$$6d^2x+\frac{20e^2x^9}{9}+x^8\left(5de-\frac{17e^2}{8}\right)+x^7\left(\frac{20d^2}{7}-\frac{34de}{7}+\frac{17e^2}{7}\right)+x^6\left(-\frac{17d^2}{6}+\frac{17de}{3}-\frac{2e^2}{3}\right)+x^5\left(\frac{17d^2}{5}-\frac{8de}{5}+\frac{21e^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 34
*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*d*
*2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*(7*
d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

# 3.291 $\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

**Optimal**. Leaf size=93

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

```
[Out] 6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*x
^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

**Rubi [A]** time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {1628}

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x^
8)/2
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx=\int \left(6d+(7d+6e)x+7(3d+e)x^2-(4d-21e)x^3+(17$$

$$=6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4+$$

**Mathematica [A]** time = 0.01, size = 93, normalized size = 1.00

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)+\frac{1}{4}x^4(21e-4d)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*x
^8)/2
```

**fricas [A]** time = 0.61, size = 83, normalized size = 0.89

$$\frac{5}{2}x^8e-\frac{17}{7}x^7e+\frac{20}{7}x^7d+\frac{17}{6}x^6e-\frac{17}{6}x^6d-\frac{4}{5}x^5e+\frac{17}{5}x^5d+\frac{21}{4}x^4e-x^4d+\frac{7}{3}x^3e+7x^3d+3x^2e+\frac{7}{2}x^2d+6xd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="frica
s")
```

```
[Out] 5/2*x^8*e - 17/7*x^7*e + 20/7*x^7*d + 17/6*x^6*e - 17/6*x^6*d - 4/5*x^5*e +
 17/5*x^5*d + 21/4*x^4*e - x^4*d + 7/3*x^3*e + 7*x^3*d + 3*x^2*e + 7/2*x^2*
d + 6*x*d
```

**giac [A]** time = 0.15, size = 90, normalized size = 0.97

$$\frac{5}{2}x^8e+\frac{20}{7}dx^7-\frac{17}{7}x^7e-\frac{17}{6}dx^6+\frac{17}{6}x^6e+\frac{17}{5}dx^5-\frac{4}{5}x^5e-dx^4+\frac{21}{4}x^4e+7dx^3+\frac{7}{3}x^3e+\frac{7}{2}dx^2+3x^2e+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac"
)
```

```
[Out] 5/2*x^8*e + 20/7*d*x^7 - 17/7*x^7*e - 17/6*d*x^6 + 17/6*x^6*e + 17/5*d*x^5
- 4/5*x^5*e - d*x^4 + 21/4*x^4*e + 7*d*x^3 + 7/3*x^3*e + 7/2*d*x^2 + 3*x^2*
e + 6*d*x
```

**maple [A]** time = 0.00, size = 84, normalized size = 0.90

$$\frac{5e\,x^8}{2}+\frac{(20d-17e)\,x^7}{7}+\frac{(-17d+17e)\,x^6}{6}+\frac{(17d-4e)\,x^5}{5}+\frac{(-4d+21e)\,x^4}{4}+\frac{(21d+7e)\,x^3}{3}+6dx+\frac{(7d+6e)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 5/2*e*x^8+1/7*(20*d-17*e)*x^7+1/6*(-17*d+17*e)*x^6+1/5*(17*d-4*e)*x^5+1/4*(
-4*d+21*e)*x^4+1/3*(21*d+7*e)*x^3+1/2*(7*d+6*e)*x^2+6*d*x
```

**maxima [A]** time = 0.43, size = 79, normalized size = 0.85

$$\frac{5}{2}ex^8+\frac{1}{7}(20d-17e)x^7-\frac{17}{6}(d-e)x^6+\frac{1}{5}(17d-4e)x^5-\frac{1}{4}(4d-21e)x^4+\frac{7}{3}(3d+e)x^3+\frac{1}{2}(7d+6e)x^2+6dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxim
a")
```

```
[Out] 5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^5
 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

**mupad [B]** time = 0.05, size = 77, normalized size = 0.83

$$\frac{5\,e\,x^8}{2}+\left(\frac{20\,d}{7}-\frac{17\,e}{7}\right)\,x^7+\left(\frac{17\,e}{6}-\frac{17\,d}{6}\right)\,x^6+\left(\frac{17\,d}{5}-\frac{4\,e}{5}\right)\,x^5+\left(\frac{21\,e}{4}-d\right)\,x^4+\left(7\,d+\frac{7\,e}{3}\right)\,x^3+\left(\frac{7\,d}{2}+3\,e\right)\,x^2+6\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6*
((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 - x
^4*(d - (21*e)/4)
```

**sympy [A]** time = 1.91, size = 87, normalized size = 0.94

$$6dx+\frac{5ex^8}{2}+x^7\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\left(7d+\frac{7e}{3}\right)+x^2\left(\frac{7d}{2}+3e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x**
5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/2
+ 3*e)
```

# 3.292 $\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

**Optimal**. Leaf size=42

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

```
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

**Rubi [A]** time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(6+7x+21x^2-4x^3+17x^4-17x^5+20x^6\right)\,dx\\ &= 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.00, size = 42, normalized size = 1.00

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

**fricas [A]** time = 0.64, size = 34, normalized size = 0.81

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

**giac [A]** time = 0.15, size = 34, normalized size = 0.81

$$\frac{20}{7}\,x^7 - \frac{17}{6}\,x^6 + \frac{17}{5}\,x^5 - x^4 + 7\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

**maple [A]** time = 0.00, size = 35, normalized size = 0.83

$$\frac{20}{7}x^7 - \frac{17}{6}x^6 + \frac{17}{5}x^5 - x^4 + 7x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
[Out] 6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

**maxima [A]** time = 0.42, size = 34, normalized size = 0.81

$$\frac{20}{7}\,x^7 - \frac{17}{6}\,x^6 + \frac{17}{5}\,x^5 - x^4 + 7\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
[Out] 20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

**mupad [B]** time = 0.03, size = 34, normalized size = 0.81

$$\frac{20\,x^7}{7} - \frac{17\,x^6}{6} + \frac{17\,x^5}{5} - x^4 + 7\,x^3 + \frac{7\,x^2}{2} + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

**sympy** **[A]** time = 0.29, size = 37, normalized size = 0.88

$$\frac{20x^7}{7} - \frac{17x^6}{6} + \frac{17x^5}{5} - x^4 + 7x^3 + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

# 3.293 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$

**Optimal**. Leaf size=228

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,\mathrm{lo}}{e^7}$$

```
[Out] -(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+17
*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4*e
^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+10/3
*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/
e^7
```

**Rubi** [A] time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(17d^2e+20d^3+17de^2+4e^3)}{3e^4}+\frac{x^2(17d^2e^2+17d^3e+20d^4+4de^3+21e^4)}{2e^5}-\frac{x(17d^3e^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
+ ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((20*
d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17*e^
2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^2 -
2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/
e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx=\int\left(\frac{-20d^5-17d^4e-17d^3e^2-4d^2e^3-21de^4+7e^5}{e^6}+\frac{(20d^4+}{}\right.$$

$$=-\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)x}{e^6}+\frac{(20d^4+}{}$$

**Mathematica [A]** time = 0.06, size = 179, normalized size = 0.79

$$ex\left(-1200d^5+60d^4e(10x-17)-10d^3e^2\left(40x^2-51x+102\right)+10d^2e^3\left(30x^3-34x^2+51x-24\right)-5de^4\left(48x^4-\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] (e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
+ 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^2
- 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200*x^
5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5
 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

**fricas [A]** time = 0.84, size = 230, normalized size = 1.01

$$200\,e^6x^6-12\left(20\,de^5+17\,e^6\right)x^5+15\left(20\,d^2e^4+17\,de^5+17\,e^6\right)x^4-20\left(20\,d^3e^3+17\,d^2e^4+17\,de^5+4\,e^6\right)x^3+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="frica
s")
```

```
[Out] 1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
+ 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*(2
0*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5*e
+ 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x +
 d))/e^7
```

**giac [A]** time = 0.16, size = 228, normalized size = 1.00

$$\left(20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{60}\left(200\,x^6e^5-240\,dx^5e^4+300\,d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac"
)
```

```
[Out] (20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)
*e^(-7)*log(abs(x*e + d)) + 1/60*(200*x^6*e^5 - 240*d*x^5*e^4 + 300*d^2*x^4
*e^3 - 400*d^3*x^3*e^2 + 600*d^4*x^2*e - 1200*d^5*x - 204*x^5*e^5 + 255*d*x
^4*e^4 - 340*d^2*x^3*e^3 + 510*d^3*x^2*e^2 - 1020*d^4*x*e + 255*x^4*e^5 - 3
```

```
40*d*x^3*e^4 + 510*d^2*x^2*e^3 - 1020*d^3*x*e^2 - 80*x^3*e^5 + 120*d*x^2*e^
4 - 240*d^2*x*e^3 + 630*x^2*e^5 - 1260*d*x*e^4 + 420*x*e^5)*e^(-6)
```

**maple [A]** time = 0.01, size = 286, normalized size = 1.25

$$\frac{10x^6}{3e}-\frac{4d\,x^5}{e^2}-\frac{17x^5}{5e}+\frac{5d^2x^4}{e^3}+\frac{17d\,x^4}{4e^2}+\frac{17x^4}{4e}-\frac{20d^3x^3}{3e^4}-\frac{17d^2x^3}{3e^3}-\frac{17d\,x^3}{3e^2}-\frac{4x^3}{3e}+\frac{10d^4x^2}{e^5}+\frac{17d^3x^2}{2e^4}+\frac{17d^2x^2}{2e^3}+\frac{2d\,x^2}{e^2}+\frac{21}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] 21/2*x^2/e+10/3*x^6/e+6/e*ln(e*x+d)+17/4/e*x^4-4/3/e*x^3+7/e*x-17/5/e*x^5+1
7/4/e^2*x^4*d-20/3/e^4*x^3*d^3-17/3/e^3*x^3*d^2-17/3/e^2*x^3*d+10/e^5*x^2*d
^4+17/2/e^4*x^2*d^3+17/2/e^3*x^2*d^2-4/e^2*x^5*d+5/e^3*x^4*d^2+4/e^4*ln(e*x
+d)*d^3-17/e^4*x*d^3-4/e^3*x*d^2-21/e^2*x*d+2/e^2*x^2*d-20/e^6*x*d^5-17/e^5
*x*d^4+21/e^3*ln(e*x+d)*d^2-7/e^2*ln(e*x+d)*d+20/e^7*ln(e*x+d)*d^6+17/e^6*l
n(e*x+d)*d^5+17/e^5*ln(e*x+d)*d^4
```

**maxima [A]** time = 0.43, size = 228, normalized size = 1.00

$$\frac{200\,e^5x^6-12\left(20\,de^4+17\,e^5\right)x^5+15\left(20\,d^2e^3+17\,de^4+17\,e^5\right)x^4-20\left(20\,d^3e^2+17\,d^2e^3+17\,de^4+4\,e^5\right)x^3+30}{60\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxim
a")
```

```
[Out] 1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
+ 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*(2
0*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 + 17
*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 + 17*d
^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x + d)/
e^7
```

**mupad [B]** time = 4.14, size = 260, normalized size = 1.14

$$x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e)
)/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e))
/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) + x^
2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e)) +
 (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d
^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

**sympy [A]** time = 1.17, size = 235, normalized size = 1.03

$$x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e))
 + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e)) +
```

```
x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21/(2
*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 21*d/
e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d**3*e
+ 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

# 3.294 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

**Optimal**. Leaf size=228

$$\frac{x^3\left(60d^2+34de+17e^2\right)}{3e^4}-\frac{x^2\left(80d^3+51d^2e+34de^2+4e^3\right)}{2e^5}-\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{e^7(d+ex)}$$

```
[Out] (100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34*
d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x^4
/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^
7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*x+d)
/e^7
```

**Rubi** [A] time = 0.19, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^3\left(60d^2+34de+17e^2\right)}{3e^4}-\frac{x^2\left(51d^2e+80d^3+34de^2+4e^3\right)}{2e^5}+\frac{x\left(51d^2e^2+68d^3e+100d^4+8de^3+21e^4\right)}{e^6}-\frac{\left(5d^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 + 5
1*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^3)
/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e + 3*
e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - ((120
*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])
/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \int \left(\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6} - \frac{(80d^3+51d^2e+\ldots}{e^5}\right.$$

$$= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6} - \frac{(80d^3+51d^2e+\ldots}{2e^5\ldots}$$

**Mathematica** [A] time = 0.09, size = 223, normalized size = 0.98

$$\frac{4e^3x^3(60d^2+34de+17e^2) - 6e^2x^2(80d^3+51d^2e+34de^2+4e^3) + 12ex(100d^4+68d^3e+51d^2e^2+8de^3+21e^4) \ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x
]
```

```
[Out] (12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d^
3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x^3
 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^4*e
^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5 + 8
5*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(12*e^7
)
```

**fricas** [A] time = 0.78, size = 319, normalized size = 1.40

$$\frac{48\,e^6x^6 - 240\,d^6 - 204\,d^5e - 204\,d^4e^2 - 48\,d^3e^3 - 252\,d^2e^4 + 84\,de^5 - 72\,e^6 - 3\left(24\,de^5 + 17\,e^6\right)x^5 + \left(120\,d^2e^4 + 8\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fri
cas")
```

```
[Out] 1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^2
*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*d*
e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3 +
6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*(100
*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d^6 +
85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*e + 85
*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x + d))/(e^
8*x + d*e^7)
```

**giac [A]** time = 0.17, size = 308, normalized size = 1.35

$$-\frac{1}{12}\,(xe+d)^5\left(\frac{3\left(120\,de+17\,e^2\right)e^{(-1)}}{xe+d}-\frac{4\left(300\,d^2e^2+85\,de^3+17\,e^4\right)e^{(-2)}}{(xe+d)^2}+\frac{12\left(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\right.}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gia
c")
```

```
[Out] -1/12*(x*e + d)^5*(3*(120*d*e + 17*e^2)*e^(-1)/(x*e + d) - 4*(300*d^2*e^2 +
 85*d*e^3 + 17*e^4)*e^(-2)/(x*e + d)^2 + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*
d*e^5 + 2*e^6)*e^(-3)/(x*e + d)^3 - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2
*e^6 + 12*d*e^7 + 21*e^8)*e^(-4)/(x*e + d)^4 - 48)*e^(-7) + (120*d^5 + 85*d
^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*e^(-7)*log(abs(x*e + d)*
e^(-1)/(x*e + d)^2) - (20*d^6*e^5/(x*e + d) + 17*d^5*e^6/(x*e + d) + 17*d^4
*e^7/(x*e + d) + 4*d^3*e^8/(x*e + d) + 21*d^2*e^9/(x*e + d) - 7*d*e^10/(x*e
 + d) + 6*e^11/(x*e + d))*e^(-12)
```

**maple [A]** time = 0.01, size = 313, normalized size = 1.37

$$\frac{4x^5}{e^2}-\frac{10d\,x^4}{e^3}-\frac{17x^4}{4e^2}+\frac{20d^2x^3}{e^4}+\frac{34d\,x^3}{3e^3}+\frac{17x^3}{3e^2}-\frac{40d^3x^2}{e^5}-\frac{51d^2x^2}{2e^4}-\frac{17d\,x^2}{e^3}-\frac{2x^2}{e^2}-\frac{20d^6}{(ex+d)\,e^7}-\frac{17d^5}{(ex+d)\,e^6}-\frac{120d^5\ln(}{e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] 21*x/e^2+4*x^5/e^2-2/e^2*x^2-6/e/(e*x+d)+7/e^2*ln(e*x+d)+17/3/e^2*x^3-17/4/
e^2*x^4-42/e^3*ln(e*x+d)*d-120/e^7*ln(e*x+d)*d^5-85/e^6*ln(e*x+d)*d^4-68/e^
5*ln(e*x+d)*d^3-12/e^4*ln(e*x+d)*d^2-21/e^3/(e*x+d)*d^2+7/e^2/(e*x+d)*d-20/
e^7/(e*x+d)*d^6-17/e^6/(e*x+d)*d^5-17/e^5/(e*x+d)*d^4-4/e^4/(e*x+d)*d^3+8/e
^3*x*d+34/3/e^3*x^3*d-40/e^5*x^2*d^3-51/2/e^4*x^2*d^2-17/e^3*x^2*d+100/e^6*
d^4*x+68/e^5*x*d^3+51/e^4*x*d^2-10/e^3*x^4*d+20/e^4*x^3*d^2
```

**maxima [A]** time = 0.43, size = 234, normalized size = 1.03

$$-\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}+\frac{48\,e^4x^5-3\left(40\,de^3+17\,e^4\right)x^4+4\left(60\,d^2e^2+34\,de^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="max
ima")
```

```
[Out] -(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6
)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*d^
```

```
2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4*e^
4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - (
120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e*x +
d)/e^7
```

**mupad** [B] time = 4.18, size = 363, normalized size = 1.59

$$x^3\left(\frac{17}{3\,e^2}-\frac{20\,d^2}{3\,e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{3\,e}\right)-x^2\left(\frac{2}{e^2}+\frac{d\,\left(\frac{17}{e^2}-\frac{20\,d^2}{e^4}+\frac{2\,d\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{40\,d}{e^3}+\frac{17}{e^2}\right)}{2\,e^2}\right)-x^4\,\left(\frac{10\,d}{e^3}+\frac{17}{4\,e^2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) - x
^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e -
 (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) + x*(
21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/
e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20*d^2)
/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d + e*x)*(
42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^7 - (17
*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2)/(e
*(d*e^6 + e^7*x))
```

**sympy** [A] time = 1.13, size = 238, normalized size = 1.04

$$x^4\left(-\frac{10d}{e^3}-\frac{17}{4e^2}\right)+x^3\left(\frac{20d^2}{e^4}+\frac{34d}{3e^3}+\frac{17}{3e^2}\right)+x^2\left(-\frac{40d^3}{e^5}-\frac{51d^2}{2e^4}-\frac{17d}{e^3}-\frac{2}{e^2}\right)+x\left(\frac{100d^4}{e^6}+\frac{68d^3}{e^5}+\frac{51d^2}{e^4}+\frac{8d}{e^3}+\frac{21}{e^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/(
3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) + x
*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-20*
d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5 - 6
```

```
*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d**3*e*
*2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

## 3.295 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$

**Optimal**. Leaf size=231

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^7(d+ex)^2}$$

```
[Out] -(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/e
^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e
+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e
^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e
^4)*ln(e*x+d)/e^7
```

**Rubi [A]** time = 0.20, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(102d^2e+200d^3+51de^2+4e^3)}{e^6}+\frac{68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7e^5}{e^7(d+ex)}-$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^2
- 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(d +
 e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5
)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4
)*Log[d + e*x])/e^7
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx = \int\left(\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{(120d^2+51de+17e^2)}{e^5}\right.$$
$$= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)}{2e^5}$$

**Mathematica [A]** time = 0.07, size = 204, normalized size = 0.88

$$\frac{660d^6+d^5e(459-480x)-51d^4e^2\left(40x^2+2x-7\right)-3d^3e^3\left(200x^3+357x^2-34x-20\right)+d^2e^4\left(150x^4-340x^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x
]
```

```
[Out] (660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^3
*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x^3
 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) + e^6
*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^3*e
+ 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d + e*
x)^2)
```

**fricas [A]** time = 0.57, size = 360, normalized size = 1.56

$$\frac{30\,e^6x^6+660\,d^6+459\,d^5e+357\,d^4e^2+60\,d^3e^3+189\,d^2e^4-21\,de^5-18\,e^6-2\left(30\,de^5+17\,e^6\right)x^5+\left(150\,d^2e^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fri
cas")
```

```
[Out] 1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2*
e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d*e
^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 - 3*
(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e + 17
*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 + 170*
d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^3*e^3
+ 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2 + 102*d
^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2
*e^7)
```

**giac [A]** time = 0.18, size = 216, normalized size = 0.94

$$\left(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4\right)e^{(-7)}\log\left(|xe+d|\right)+\frac{1}{6}\left(30\,x^4e^9-120\,dx^3e^8+360\,d^2x^2e^7-1200\,d^3xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gia
c")
```

```
[Out] (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*e^(-7)*log(abs(x*e
+ d)) + 1/6*(30*x^4*e^9 - 120*d*x^3*e^8 + 360*d^2*x^2*e^7 - 1200*d^3*x*e^6
- 34*x^3*e^9 + 153*d*x^2*e^8 - 612*d^2*x*e^7 + 51*x^2*e^9 - 306*d*x*e^8 - 2
4*x*e^9)*e^(-12) + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63
*d^2*e^4 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
 7*e^6)*x - 7*d*e^5 - 6*e^6)*e^(-7)/(x*e + d)^2
```

**maple [A]** time = 0.01, size = 336, normalized size = 1.45

$$\frac{5x^4}{e^3}-\frac{20d\,x^3}{e^4}-\frac{17x^3}{3e^3}-\frac{10d^6}{\left(ex+d\right)^2e^7}-\frac{17d^5}{2\left(ex+d\right)^2e^6}-\frac{17d^4}{2\left(ex+d\right)^2e^5}-\frac{2d^3}{\left(ex+d\right)^2e^4}-\frac{21d^2}{2\left(ex+d\right)^2e^3}+\frac{60d^2x^2}{e^5}+\frac{7d}{2\left(ex+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] -200/e^6*d^3*x-102/e^5*x*d^2-51/e^4*x*d+5*x^4/e^3-3/e/(e*x+d)^2+21/e^3*ln(e
*x+d)+17/2/e^3*x^2-4/e^3*x-7/e^2/(e*x+d)-17/3/e^3*x^3+120/e^7/(e*x+d)*d^5+8
5/e^6/(e*x+d)*d^4+68/e^5/(e*x+d)*d^3+12/e^4/(e*x+d)*d^2+42/e^3/(e*x+d)*d-20
/e^4*x^3*d+60/e^5*x^2*d^2+51/2/e^4*x^2*d-10/e^7/(e*x+d)^2*d^6-17/2/e^6/(e*x
+d)^2*d^5-17/2/e^5/(e*x+d)^2*d^4-2/e^4/(e*x+d)^2*d^3-21/2/e^3/(e*x+d)^2*d^2
+7/2/e^2/(e*x+d)^2*d+300/e^7*ln(e*x+d)*d^4+170/e^6*ln(e*x+d)*d^3+102/e^5*ln
(e*x+d)*d^2+12/e^4*ln(e*x+d)*d
```

**maxima [A]** time = 0.44, size = 240, normalized size = 1.04

$$\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\left(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4+42\,de^5\right.}{2\left(e^9x^2+2\,de^8x+d^2e^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="max
ima")
```

```
[Out] 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
- 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 -
```

```
7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^2 +
 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102*d^2
*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e
^3 + 21*e^4)*log(e*x + d)/e^7
```

**mupad [B]** time = 0.09, size = 297, normalized size = 1.29

$$x^2\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\,\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\,\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right)+\frac{x\,\left(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5\right)}{d^2\,e^6+2\,d\,e^7\,x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*(
(20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d
^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e^4
+ 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*(4/e
^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3
))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d + e*x)
*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

**sympy [A]** time = 2.63, size = 248, normalized size = 1.07

$$x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)+\frac{220d^6+153d^5e+119d^4e^2+20d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/(
2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220*d
**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**5 -
 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4 + 84
*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/e**3
+ (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d + e*x
)/e**7
```

# 3.296 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=391

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(7000d^4+1575d^3e+1665d\right.}{}$$

```
[Out] 1/4*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^4/e
^9-1/5*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4
-11*e^5)*(e*x+d)^5/e^9+1/6*(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3*e^3+888
*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^6/e^9-1/7*(5600*d^5+1575*d^4*e+2220*d^3
*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^7/e^9+1/8*(7000*d^4+1575*d^3*e+1
665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^8/e^9-1/9*(5600*d^3+945*d^2*e+666*d*
e^2+37*e^3)*(e*x+d)^9/e^9+1/10*(2800*d^2+315*d*e+111*e^2)*(e*x+d)^10/e^9-5/
11*(160*d+9*e)*(e*x+d)^11/e^9+25/3*(e*x+d)^12/e^9
```

**Rubi [A]** time = 0.39, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{10}}{10e^9}-\frac{\left(945d^2e+5600d^3+666de^2+37e^3\right)(d+ex)^9}{9e^9}+\frac{\left(1665d^2e^2+1575d^3e+700\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3
*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6 + 9
45*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*
(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e + 166
5*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 + 945*d^
2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d*e + 111
*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*e^9) + (2
5*(d + e*x)^12)/(3*e^9)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left( \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+\right.}{e^8} \right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right.}{4e^9}$$

**Mathematica [A]** time = 0.04, size = 277, normalized size = 0.71

$$18d^3x+\frac{3}{10}ex^{10}\left(100d^2-45de+37e^2\right)+\frac{1}{3}dx^3\left(107d^2+99de+54e^2\right)+\frac{3}{2}d^2x^2(11d+18e)+\frac{1}{9}x^9\left(100d^3-135d^2e+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^3
)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*d^
2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 107*
e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-45*d^
3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e + 333*d
*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (15*(20
*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

**fricas [A]** time = 0.70, size = 305, normalized size = 0.78

$$\frac{25}{3}x^{12}e^3-\frac{45}{11}x^{11}e^3+\frac{300}{11}x^{11}e^2d+\frac{111}{10}x^{10}e^3-\frac{27}{2}x^{10}e^2d+30x^{10}ed^2-\frac{37}{9}x^9e^3+37x^9e^2d-15x^9ed^2+\frac{100}{9}x^9d^3+\frac{37}{2}x^8e^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 25/3*x^12*e^3 - 45/11*x^11*e^3 + 300/11*x^11*e^2*d + 111/10*x^10*e^3 - 27/2
*x^10*e^2*d + 30*x^10*e*d^2 - 37/9*x^9*e^3 + 37*x^9*e^2*d - 15*x^9*e*d^2 +
100/9*x^9*d^3 + 37/2*x^8*e^3 - 111/8*x^8*e^2*d + 333/8*x^8*e*d^2 - 45/8*x^8
*d^3 + 65/7*x^7*e^3 + 444/7*x^7*e^2*d - 111/7*x^7*e*d^2 + 111/7*x^7*d^3 + 1
07/6*x^6*e^3 + 65/2*x^6*e^2*d + 74*x^6*e*d^2 - 37/6*x^6*d^3 + 33/5*x^5*e^3
+ 321/5*x^5*e^2*d + 39*x^5*e*d^2 + 148/5*x^5*d^3 + 9/2*x^4*e^3 + 99/4*x^4*e
^2*d + 321/4*x^4*e*d^2 + 65/4*x^4*d^3 + 18*x^3*e^2*d + 33*x^3*e*d^2 + 107/3
*x^3*d^3 + 27*x^2*e*d^2 + 33/2*x^2*d^3 + 18*x*d^3
```

**giac [A]** time = 0.16, size = 296, normalized size = 0.76

$$\frac{25}{3}\,x^{12}e^{3}+\frac{300}{11}\,dx^{11}e^{2}+30\,d^{2}x^{10}e+\frac{100}{9}\,d^{3}x^{9}-\frac{45}{11}\,x^{11}e^{3}-\frac{27}{2}\,dx^{10}e^{2}-15\,d^{2}x^{9}e-\frac{45}{8}\,d^{3}x^{8}+\frac{111}{10}\,x^{10}e^{3}+37\,dx^{9}e^{2}+\frac{333}{8}\,d^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 25/3*x^12*e^3 + 300/11*d*x^11*e^2 + 30*d^2*x^10*e + 100/9*d^3*x^9 - 45/11*x
^11*e^3 - 27/2*d*x^10*e^2 - 15*d^2*x^9*e - 45/8*d^3*x^8 + 111/10*x^10*e^3 +
 37*d*x^9*e^2 + 333/8*d^2*x^8*e + 111/7*d^3*x^7 - 37/9*x^9*e^3 - 111/8*d*x^
8*e^2 - 111/7*d^2*x^7*e - 37/6*d^3*x^6 + 37/2*x^8*e^3 + 444/7*d*x^7*e^2 + 7
4*d^2*x^6*e + 148/5*d^3*x^5 + 65/7*x^7*e^3 + 65/2*d*x^6*e^2 + 39*d^2*x^5*e
+ 65/4*d^3*x^4 + 107/6*x^6*e^3 + 321/5*d*x^5*e^2 + 321/4*d^2*x^4*e + 107/3*
d^3*x^3 + 33/5*x^5*e^3 + 99/4*d*x^4*e^2 + 33*d^2*x^3*e + 33/2*d^3*x^2 + 9/2
*x^4*e^3 + 18*d*x^3*e^2 + 27*d^2*x^2*e + 18*d^3*x
```

**maple [A]** time = 0.00, size = 264, normalized size = 0.68

$$\frac{25e^{3}x^{12}}{3}+\frac{\left(300d\,e^{2}-45e^{3}\right)x^{11}}{11}+\frac{\left(300d^{2}e-135d\,e^{2}+111e^{3}\right)x^{10}}{10}+\frac{\left(100d^{3}-135d^{2}e+333d\,e^{2}-37e^{3}\right)x^{9}}{9}+\frac{\left(-45d^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 25/3*e^3*x^12+1/11*(300*d*e^2-45*e^3)*x^11+1/10*(300*d^2*e-135*d*e^2+111*e^
3)*x^10+1/9*(100*d^3-135*d^2*e+333*d*e^2-37*e^3)*x^9+1/8*(-45*d^3+333*d^2*e
-111*d*e^2+148*e^3)*x^8+1/7*(111*d^3-111*d^2*e+444*d*e^2+65*e^3)*x^7+1/6*(-
37*d^3+444*d^2*e+195*d*e^2+107*e^3)*x^6+1/5*(148*d^3+195*d^2*e+321*d*e^2+33
*e^3)*x^5+1/4*(65*d^3+321*d^2*e+99*d*e^2+18*e^3)*x^4+1/3*(107*d^3+99*d^2*e+
54*d*e^2)*x^3+1/2*(33*d^3+54*d^2*e)*x^2+18*d^3*x
```

**maxima [A]** time = 0.43, size = 263, normalized size = 0.67

$$\frac{25}{3}\,e^{3}x^{12}+\frac{15}{11}\left(20\,de^{2}-3\,e^{3}\right)x^{11}+\frac{3}{10}\left(100\,d^{2}e-45\,de^{2}+37\,e^{3}\right)x^{10}+\frac{1}{9}\left(100\,d^{3}-135\,d^{2}e+333\,de^{2}-37\,e^{3}\right)x^{9}-\frac{1}{8}\left(45\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
+ 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8*(
```

```
45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e +
444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3)*x^
6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 + 321*
d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e + 54*d*
e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

**mupad [B]** time = 4.25, size = 251, normalized size = 0.64

$$18\,d^3\,x+x^3\,\left(\frac{107\,d^3}{3}+33\,d^2\,e+18\,d\,e^2\right)+x^9\,\left(\frac{100\,d^3}{9}-15\,d^2\,e+37\,d\,e^2-\frac{37\,e^3}{9}\right)+x^6\,\left(-\frac{37\,d^3}{6}+74\,d^2\,e+\frac{65\,d\,e^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^2
*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/6
+ (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3)/2
) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5*((3
21*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/7 - (
111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^10*(100
*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*x^11*(2
0*d - 3*e))/11
```

**sympy [A]** time = 0.20, size = 298, normalized size = 0.76

$$18d^3x+\frac{25e^3x^{12}}{3}+x^{11}\left(\frac{300de^2}{11}-\frac{45e^3}{11}\right)+x^{10}\left(30d^2e-\frac{27de^2}{2}+\frac{111e^3}{10}\right)+x^9\left(\frac{100d^3}{9}-15d^2e+37de^2-\frac{37e^3}{9}\right)+x^8\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(3
0*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37*d
*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37*e*
*3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x**6*
(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5 + 39
*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4 + 99*d
*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x**2*(33*
d**3/2 + 27*d**2*e)
```

# 3.297 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=201

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}$$

```
[Out] 18*d^2*x+3/2*d*(11*d+12*e)*x^2+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/4*(65*d^2+
214*d*e+33*e^2)*x^4+1/5*(148*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e-6
5*e^2)*x^6+37/7*(3*d^2-2*d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+1/9
*(100*d^2-90*d*e+111*e^2)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11
```

**Rubi [A]** time = 0.24, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7)
/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*x^
9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)dx=\int\left(18d^2+3d(11d+12e)x+\left(107d^2+66de+18e^2\right)x\right.$$

$$=18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)$$

**Mathematica [A]** time = 0.03, size = 201, normalized size = 1.00

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)+\frac{1}{8}x^8\left(-45d^2+222de-37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)+\frac{1}{6}x^6\left(-37d^2+296de+65e^2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] 18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
+ ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^5)
/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x^7
)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2)*
x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

**fricas [A]** time = 0.70, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}e^2-\frac{9}{2}x^{10}e^2+20x^{10}ed+\frac{37}{3}x^9e^2-10x^9ed+\frac{100}{9}x^9d^2-\frac{37}{8}x^8e^2+\frac{111}{4}x^8ed-\frac{45}{8}x^8d^2+\frac{148}{7}x^7e^2-\frac{74}{7}x^7ed+\frac{111}{7}x^7d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] 100/11*x^11*e^2 - 9/2*x^10*e^2 + 20*x^10*e*d + 37/3*x^9*e^2 - 10*x^9*e*d +
100/9*x^9*d^2 - 37/8*x^8*e^2 + 111/4*x^8*e*d - 45/8*x^8*d^2 + 148/7*x^7*e^2
 - 74/7*x^7*e*d + 111/7*x^7*d^2 + 65/6*x^6*e^2 + 148/3*x^6*e*d - 37/6*x^6*d
^2 + 107/5*x^5*e^2 + 26*x^5*e*d + 148/5*x^5*d^2 + 33/4*x^4*e^2 + 107/2*x^4*
e*d + 65/4*x^4*d^2 + 6*x^3*e^2 + 22*x^3*e*d + 107/3*x^3*d^2 + 18*x^2*e*d +
33/2*x^2*d^2 + 18*x*d^2
```

**giac [A]** time = 0.16, size = 206, normalized size = 1.02

$$\frac{100}{11}x^{11}e^2+20\,dx^{10}e+\frac{100}{9}\,d^2x^9-\frac{9}{2}\,x^{10}e^2-10\,dx^9e-\frac{45}{8}\,d^2x^8+\frac{37}{3}\,x^9e^2+\frac{111}{4}\,dx^8e+\frac{111}{7}\,d^2x^7-\frac{37}{8}\,x^8e^2-\frac{74}{7}\,dx^7e-\frac{37}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] 100/11*x^11*e^2 + 20*d*x^10*e + 100/9*d^2*x^9 - 9/2*x^10*e^2 - 10*d*x^9*e -
 45/8*d^2*x^8 + 37/3*x^9*e^2 + 111/4*d*x^8*e + 111/7*d^2*x^7 - 37/8*x^8*e^2
 - 74/7*d*x^7*e - 37/6*d^2*x^6 + 148/7*x^7*e^2 + 148/3*d*x^6*e + 148/5*d^2*
x^5 + 65/6*x^6*e^2 + 26*d*x^5*e + 65/4*d^2*x^4 + 107/5*x^5*e^2 + 107/2*d*x^
```

```
4*e + 107/3*d^2*x^3 + 33/4*x^4*e^2 + 22*d*x^3*e + 33/2*d^2*x^2 + 6*x^3*e^2
+ 18*d*x^2*e + 18*d^2*x
```

**maple [A]** time = 0.00, size = 186, normalized size = 0.93

$$\frac{100e^2x^{11}}{11}+\frac{(200de-45e^2)\,x^{10}}{10}+\frac{(100d^2-90de+111e^2)\,x^9}{9}+\frac{(-45d^2+222de-37e^2)\,x^8}{8}+\frac{(111d^2-74de+148e^2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 100/11*e^2*x^11+1/10*(200*d*e-45*e^2)*x^10+1/9*(100*d^2-90*d*e+111*e^2)*x^9
+1/8*(-45*d^2+222*d*e-37*e^2)*x^8+1/7*(111*d^2-74*d*e+148*e^2)*x^7+1/6*(-37
*d^2+296*d*e+65*e^2)*x^6+1/5*(148*d^2+130*d*e+107*e^2)*x^5+1/4*(65*d^2+214*
d*e+33*e^2)*x^4+1/3*(107*d^2+66*d*e+18*e^2)*x^3+1/2*(33*d^2+36*d*e)*x^2+18*
d^2*x
```

**maxima [A]** time = 0.43, size = 185, normalized size = 0.92

$$\frac{100}{11}\,e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9-\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+4\,e^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*e
^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e^2
)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 107*
e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e + 18
*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

**mupad [B]** time = 0.11, size = 175, normalized size = 0.87

$$x^3\left(\frac{107\,d^2}{3}+22\,d\,e+6\,e^2\right)+x^9\left(\frac{100\,d^2}{9}-10\,d\,e+\frac{37\,e^2}{3}\right)+x^4\left(\frac{65\,d^2}{4}+\frac{107\,d\,e}{2}+\frac{33\,e^2}{4}\right)-x^8\left(\frac{45\,d^2}{8}-\frac{111\,d\,e}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/3
) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*d*
e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(26*
d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148*e^2
```

```
)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x^10*(
40*d - 9*e))/2
```

**sympy [A]** time = 0.15, size = 206, normalized size = 1.02

$$18d^2x+\frac{100e^2x^{11}}{11}+x^{10}\left(20de-\frac{9e^2}{2}\right)+x^9\left(\frac{100d^2}{9}-10de+\frac{37e^2}{3}\right)+x^8\left(-\frac{45d^2}{8}+\frac{111de}{4}-\frac{37e^2}{8}\right)+x^7\left(\frac{111d^2}{7}-\frac{74de}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2/
9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**7*
(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65*e*
*2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107*d*e
/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/2 + 1
8*d*e)
```

# 3.298 $\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$

**Optimal**. Leaf size=121

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2$$

```
[Out] 18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(14
8*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(20
*d-9*e)*x^9+10*e*x^10
```

**Rubi [A]** time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \int \left(18d+3(11d+6e)x+(107d+33e)x^2+(65d+10\right.\\ &= 18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 121, normalized size = 1.00

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)*
x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^7)
/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

**fricas [A]** time = 0.67, size = 107, normalized size = 0.88

$$10x^{10}e-5x^9e+\frac{100}{9}x^9d+\frac{111}{8}x^8e-\frac{45}{8}x^8d-\frac{37}{7}x^7e+\frac{111}{7}x^7d+\frac{74}{3}x^6e-\frac{37}{6}x^6d+13x^5e+\frac{148}{5}x^5d+\frac{107}{4}x^4e+\frac{65}{4}x^4d+11x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 10*x^10*e - 5*x^9*e + 100/9*x^9*d + 111/8*x^8*e - 45/8*x^8*d - 37/7*x^7*e +
 111/7*x^7*d + 74/3*x^6*e - 37/6*x^6*d + 13*x^5*e + 148/5*x^5*d + 107/4*x^4
*e + 65/4*x^4*d + 11*x^3*e + 107/3*x^3*d + 9*x^2*e + 33/2*x^2*d + 18*x*d
```

**giac [A]** time = 0.16, size = 116, normalized size = 0.96

$$10\,x^{10}e+\frac{100}{9}\,dx^9-5\,x^9e-\frac{45}{8}\,dx^8+\frac{111}{8}\,x^8e+\frac{111}{7}\,dx^7-\frac{37}{7}\,x^7e-\frac{37}{6}\,dx^6+\frac{74}{3}\,x^6e+\frac{148}{5}\,dx^5+13\,x^5e+\frac{65}{4}\,dx^4+\frac{107}{4}\,x^4e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 10*x^10*e + 100/9*d*x^9 - 5*x^9*e - 45/8*d*x^8 + 111/8*x^8*e + 111/7*d*x^7
- 37/7*x^7*e - 37/6*d*x^6 + 74/3*x^6*e + 148/5*d*x^5 + 13*x^5*e + 65/4*d*x^
4 + 107/4*x^4*e + 107/3*d*x^3 + 11*x^3*e + 33/2*d*x^2 + 9*x^2*e + 18*d*x
```

**maple [A]** time = 0.00, size = 108, normalized size = 0.89

$$10e\,x^{10}+\frac{(100d-45e)\,x^9}{9}+\frac{(-45d+111e)\,x^8}{8}+\frac{(111d-37e)\,x^7}{7}+\frac{(-37d+148e)\,x^6}{6}+\frac{(148d+65e)\,x^5}{5}+\frac{(65d+107e)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 10*e*x^10+1/9*(100*d-45*e)*x^9+1/8*(-45*d+111*e)*x^8+1/7*(111*d-37*e)*x^7+1
/6*(-37*d+148*e)*x^6+1/5*(148*d+65*e)*x^5+1/4*(65*d+107*e)*x^4+1/3*(107*d+3
3*e)*x^3+1/2*(33*d+18*e)*x^2+18*d*x
```

**maxima [A]** time = 0.43, size = 105, normalized size = 0.87

$$10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*x
^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4 +
 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

**mupad [B]** time = 4.17, size = 101, normalized size = 0.83

$$10\,e\,x^{10}+\left(\frac{100\,d}{9}-5\,e\right)\,x^9+\left(\frac{111\,e}{8}-\frac{45\,d}{8}\right)\,x^8+\left(\frac{111\,d}{7}-\frac{37\,e}{7}\right)\,x^7+\left(\frac{74\,e}{3}-\frac{37\,d}{6}\right)\,x^6+\left(\frac{148\,d}{5}+13\,e\right)\,x^5+\left(\frac{65\,d}{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^6
*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13*e
) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 10*e
*x^10
```

**sympy [A]** time = 0.14, size = 112, normalized size = 0.93

$$18dx+10ex^{10}+x^9\left(\frac{100d}{9}-5e\right)+x^8\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^7\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^6\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^5\left(\frac{148d}{5}+13e\right)+x^4\left(\frac{65}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x**
7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) + x*
*4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

## 3.299 $\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$

**Optimal**. Leaf size=60

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

**Rubi [A]** time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {1657}

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \int \left(18+33x+107x^2+65x^3+148x^4-37x^5+111x^6-45x\right.$$
$$=18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45}{8}$$

**Mathematica [A]** time = 0.00, size = 60, normalized size = 1.00

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

**fricas [A]** time = 0.72, size = 44, normalized size = 0.73

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

**giac [A]** time = 0.15, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^9 - \frac{45}{8}\,x^8 + \frac{111}{7}\,x^7 - \frac{37}{6}\,x^6 + \frac{148}{5}\,x^5 + \frac{65}{4}\,x^4 + \frac{107}{3}\,x^3 + \frac{33}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

**maple [A]** time = 0.00, size = 45, normalized size = 0.75

$$\frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] 18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100/
9*x^9
```

**maxima [A]** time = 0.42, size = 44, normalized size = 0.73

$$\frac{100}{9}\,x^9 - \frac{45}{8}\,x^8 + \frac{111}{7}\,x^7 - \frac{37}{6}\,x^6 + \frac{148}{5}\,x^5 + \frac{65}{4}\,x^4 + \frac{107}{3}\,x^3 + \frac{33}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3*
x^3 + 33/2*x^2 + 18*x
```

**mupad [B]** time = 0.03, size = 44, normalized size = 0.73

$$\frac{100\,x^9}{9}-\frac{45\,x^8}{8}+\frac{111\,x^7}{7}-\frac{37\,x^6}{6}+\frac{148\,x^5}{5}+\frac{65\,x^4}{4}+\frac{107\,x^3}{3}+\frac{33\,x^2}{2}+18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] 18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 + (
111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

**sympy [A]** time = 0.15, size = 56, normalized size = 0.93

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] 100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
107*x**3/3 + 33*x**2/2 + 18*x
```

## 3.300 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

**Optimal**. Leaf size=352

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+}{e^9}$$

```
[Out] -(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-
33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65*d
*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d*e^
4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x^4/e
^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e+111*
e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*(4*d^4
+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

**Rubi [A]** time = 0.32, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3}-\frac{x^5(45d^2e+100d^3+111de^2+37e^3)}{5e^4}+\frac{x^4(111d^2e^2+45d^3e+100d^4+37de^3+148e^4)}{4e^5}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

```
[Out] -(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^
5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37*d
^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45*d^
4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((100*d
^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100*d^3
+ 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e + 111*e^
2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((5*d^2 -
 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x
])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{d+ex}\,dx = \int \left(\frac{-100d^7-45d^6e-111d^5e^2-37d^4e^3-148d^3e^4+65d^2e^5}{e^8}\right.$$

$$= -\frac{\left(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+\right.}{e^8}$$

**Mathematica [A]** time = 0.12, size = 262, normalized size = 0.74

$$\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\left(5d^2-2de+3e^2\right)^2\log(d+ex)}{e^9}+\frac{x\left(-42000d^7+2100d^6e(10x-9)-70d^5e^2\left(20\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x
]
```

```
[Out] (x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^2
) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x +
 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^3 -
 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9324*
x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x^3 -
1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*d*e +
3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/e^9
```

**fricas [A]** time = 0.87, size = 368, normalized size = 1.05

$$\frac{5250\,e^8x^8-300\left(20\,de^7+9\,e^8\right)x^7+70\left(100\,d^2e^6+45\,de^7+111\,e^8\right)x^6-84\left(100\,d^3e^5+45\,d^2e^6+111\,de^7+37\,e^8\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fri
cas")
```

```
[Out] 1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*e
^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^5
+ 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4 - 1
40*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 65*e^
8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 148*d^2
*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^5*e^3
+ 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 420*(100*
```

```
d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*
d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

**giac [A]** time = 0.17, size = 378, normalized size = 1.07

$$\left(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{420}\left(52\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gia
c")
```

```
[Out] (100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 +
 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*e^(-9)*log(abs(x*e + d)) + 1/420*(5250*x^
8*e^7 - 6000*d*x^7*e^6 + 7000*d^2*x^6*e^5 - 8400*d^3*x^5*e^4 + 10500*d^4*x^
4*e^3 - 14000*d^5*x^3*e^2 + 21000*d^6*x^2*e - 42000*d^7*x - 2700*x^7*e^7 +
3150*d*x^6*e^6 - 3780*d^2*x^5*e^5 + 4725*d^3*x^4*e^4 - 6300*d^4*x^3*e^3 + 9
450*d^5*x^2*e^2 - 18900*d^6*x*e + 7770*x^6*e^7 - 9324*d*x^5*e^6 + 11655*d^2
*x^4*e^5 - 15540*d^3*x^3*e^4 + 23310*d^4*x^2*e^3 - 46620*d^5*x*e^2 - 3108*x
^5*e^7 + 3885*d*x^4*e^6 - 5180*d^2*x^3*e^5 + 7770*d^3*x^2*e^4 - 15540*d^4*x
*e^3 + 15540*x^4*e^7 - 20720*d*x^3*e^6 + 31080*d^2*x^2*e^5 - 62160*d^3*x*e^
4 + 9100*x^3*e^7 - 13650*d*x^2*e^6 + 27300*d^2*x*e^5 + 22470*x^2*e^7 - 4494
0*d*x*e^6 + 13860*x*e^7)*e^(-8)
```

**maple [A]** time = 0.01, size = 465, normalized size = 1.32

$$\frac{25x^8}{2e}-\frac{100d\,x^7}{7e^2}-\frac{45x^7}{7e}+\frac{50d^2x^6}{3e^3}+\frac{15d\,x^6}{2e^2}+\frac{37x^6}{2e}-\frac{20d^3x^5}{e^4}-\frac{9d^2x^5}{e^3}-\frac{111d\,x^5}{5e^2}-\frac{37x^5}{5e}+\frac{25d^4x^4}{e^5}+\frac{45d^3x^4}{4e^4}+\frac{111d^2x^4}{4e^3}+\frac{37d}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

```
[Out] 107/2/e*x^2+25/2*x^8/e+37/2/e*x^6-45/7/e*x^7+18/e*ln(e*x+d)+37/e*x^4+65/3/e
*x^3+33/e*x-37/5/e*x^5-15/e^5*x^3*d^4+25/e^5*x^4*d^4-20/e^4*x^5*d^3+45/4/e^
4*x^4*d^3+50/3/e^3*x^6*d^2-9/e^3*x^5*d^2-100/7/e^2*x^7*d+15/2/e^2*x^6*d-100
/e^8*x*d^7-45/e^7*x*d^6+45/e^8*ln(e*x+d)*d^7-100/3/e^6*x^3*d^5+45/2/e^6*x^2
*d^5+50/e^7*x^2*d^6+100/e^9*ln(e*x+d)*d^8+37/4*d/e^2*x^4-37*d^3/e^4*x^3-37/
3*d^2/e^3*x^3-148/3*d/e^2*x^3+111/2*d^4/e^5*x^2+37/2*d^3/e^4*x^2+74*d^2/e^3
*x^2-111/5*d/e^2*x^5+111/4*d^2/e^3*x^4-65*d^3/e^4*ln(e*x+d)-148*d^3/e^4*x+6
5*d^2/e^3*x-107*d/e^2*x-65/2*d/e^2*x^2-111*d^5/e^6*x-37*d^4/e^5*x+107*d^2/e
^3*ln(e*x+d)-33*d/e^2*ln(e*x+d)+111*d^6/e^7*ln(e*x+d)+37*d^5/e^6*ln(e*x+d)+
148*d^4/e^5*ln(e*x+d)
```

**maxima [A]** time = 0.44, size = 366, normalized size = 1.04

$$\frac{5250\,e^7x^8-300\left(20\,de^6+9\,e^7\right)x^7+70\left(100\,d^2e^5+45\,de^6+111\,e^7\right)x^6-84\left(100\,d^3e^4+45\,d^2e^5+111\,de^6+37\,e^7\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="max
ima")
```

```
[Out] 1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*e
^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^5
+ 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4 - 1
40*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 65*e^
7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*d^2*e
^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d
^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (100*d^8 +
 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e
^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

**mupad [B]** time = 0.08, size = 434, normalized size = 1.23

$$x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^7\left(\frac{100\,d}{7\,e^2}+\frac{45}{7\,e}\right)+x^6\left(\frac{37}{2\,e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{6\,e}\right)-x^5\left(\frac{37}{5\,e}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

```
[Out] x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((10
0*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e))
 + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*(11
1/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(111/
e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/e + (
d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x^2*(10
7/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 4
5/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d^7*e - 3
3*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^4 + 37*d^
5*e^3 + 111*d^6*e^2))/e^9
```

**sympy [A]** time = 1.00, size = 372, normalized size = 1.06

$$x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

```
[Out] x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
+ 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*e)
) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*e**
2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 37*d*
*2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**5/(2*
e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2*e**2)
 + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 37*d**4/
e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/(2*e) +
(5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*e**3 + 2*
e**4)*log(d + e*x)/e**9
```

# 3.301 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

**Optimal**. Leaf size=353

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3}{e^9(d+ex)}$$

```
[Out] (700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*x
/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2/e
^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400*d^
3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e^4-5
/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*
d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

**Rubi [A]** time = 0.33, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(135d^2e+400d^3+222de^2+37e^3)}{4e^5}+\frac{x^3(333d^2e^2+180d^3e+500d^4+74de^3+148}{3e^6}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

```
[Out] ((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5
 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 + 29
6*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 74*d
*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e^3)*
x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d + 9*e
)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*
d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*e + 3*e
^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*Log[
d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \int \left(\frac{700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130d}{e^8}\right.$$

$$= \frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5}{e^8}$$

**Mathematica** [A] time = 0.14, size = 342, normalized size = 0.97

$$\frac{252e^5x^5(100d^2+30de+37e^2)-105e^4x^4(400d^3+135d^2e+222de^2+37e^3)+140e^3x^3(500d^4+180d^3e+333}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2
,x]
```

```
[Out] (420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130
*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*
e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e^2
+ 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*e)*x^
6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5*e^2 +
185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d + e*x])
/(420*e^9)
```

**fricas** [A] time = 1.00, size = 490, normalized size = 1.39

$$\frac{6000\,e^8x^8-42000\,d^8-18900\,d^7e-46620\,d^6e^2-15540\,d^5e^3-62160\,d^4e^4+27300\,d^3e^5-44940\,d^2e^6+13860\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="f
ricas")
```

```
[Out] 1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*e
^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e^8
 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*x^6
 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d^4*e
^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^5*e^3
```

```
 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3 + 210
*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^6 - 195
*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3 + 148*d^
4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 + 315*d^7*e
 + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^2*e^6 - 33
*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5
 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

**giac [A]** time = 0.18, size = 459, normalized size = 1.30

$$-\frac{1}{420}\,(xe+d)^7\left(\frac{350\left(160\,de+9\,e^2\right)e^{(-1)}}{xe+d}-\frac{84\left(2800\,d^2e^2+315\,de^3+111\,e^4\right)e^{(-2)}}{(xe+d)^2}+\frac{105\left(5600\,d^3e^3+945\,d^2e^4+6\right.}{(xe+d)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="g
iac")
```

```
[Out] -1/420*(x*e + d)^7*(350*(160*d*e + 9*e^2)*e^(-1)/(x*e + d) - 84*(2800*d^2*e
^2 + 315*d*e^3 + 111*e^4)*e^(-2)/(x*e + d)^2 + 105*(5600*d^3*e^3 + 945*d^2*
e^4 + 666*d*e^5 + 37*e^6)*e^(-3)/(x*e + d)^3 - 140*(7000*d^4*e^4 + 1575*d^3
*e^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)*e^(-4)/(x*e + d)^4 + 210*(5600*d
^5*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)*e
^(-5)/(x*e + d)^5 - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^
3*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)*e^(-6)/(x*e + d)^6 - 6000)*e^
(-9) + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195
*d^2*e^5 + 214*d*e^6 - 33*e^7)*e^(-9)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)
- (100*d^8*e^7/(x*e + d) + 45*d^7*e^8/(x*e + d) + 111*d^6*e^9/(x*e + d) + 3
7*d^5*e^10/(x*e + d) + 148*d^4*e^11/(x*e + d) - 65*d^3*e^12/(x*e + d) + 107
*d^2*e^13/(x*e + d) - 33*d*e^14/(x*e + d) + 18*e^15/(x*e + d))*e^(-16)
```

**maple [A]** time = 0.01, size = 500, normalized size = 1.42

$$\frac{100x^7}{7e^2}-\frac{100d\,x^6}{3e^3}-\frac{15x^6}{2e^2}+\frac{60d^2x^5}{e^4}+\frac{18d\,x^5}{e^3}+\frac{111x^5}{5e^2}-\frac{100d^3x^4}{e^5}-\frac{135d^2x^4}{4e^4}-\frac{111d\,x^4}{2e^3}-\frac{37x^4}{4e^2}+\frac{500d^4x^3}{3e^6}+\frac{60d^3x^3}{e^5}+\frac{111d^2x^3}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

```
[Out] 700/e^8*d^6*x-300/e^7*x^2*d^5-225/2/e^6*x^2*d^4+500/3/e^6*x^3*d^4+60/e^5*x^
3*d^3+270/e^7*x*d^5-100/e^5*x^4*d^3-135/4/e^4*x^4*d^2+60/e^4*x^5*d^2-100/3/
e^3*x^6*d+18/e^3*x^5*d-100/e^9/(e*x+d)*d^8-45/e^8/(e*x+d)*d^7-800/e^9*ln(e*
x+d)*d^7-315/e^8*ln(e*x+d)*d^6+107/e^2*x+100/7*x^7/e^2+111/5/e^2*x^5-15/2/e
^2*x^6+65/2/e^2*x^2-18/(e*x+d)/e+33/e^2*ln(e*x+d)+148/3/e^2*x^3-37/4/e^2*x^
4-214*d/e^3*ln(e*x+d)-666*d^5/e^7*ln(e*x+d)-185*d^4/e^6*ln(e*x+d)-592*d^3/e
```

```
^5*ln(e*x+d)+195*d^2/e^4*ln(e*x+d)-107/(e*x+d)*d^2/e^3+33/(e*x+d)*d/e^2-111
/(e*x+d)*d^6/e^7-37/(e*x+d)*d^5/e^6-148/(e*x+d)*d^4/e^5+65/(e*x+d)*d^3/e^4-
130*d/e^3*x+74/3*d/e^3*x^3-222*d^3/e^5*x^2-111/2*d^2/e^4*x^2-148*d/e^3*x^2+
555*d^4/e^6*x+148*d^3/e^5*x+444*d^2/e^4*x-111/2*d/e^3*x^4+111*d^2/e^4*x^3
```

**maxima [A]** time = 0.44, size = 372, normalized size = 1.05

$$-\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}+\frac{6000\,e^6x^7-350\left(40\,de^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="m
axima")
```

```
[Out] -(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
+ 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7 -
 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5 - 1
05*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*e^2
+ 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e + 22
5*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420*(700*
d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^5 + 107
*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e
^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

**mupad [B]** time = 4.22, size = 939, normalized size = 2.66

$$x^2 \left( \frac{65}{2e^2} - \frac{d\left(\frac{148}{e^2} + \frac{2d\left(\frac{37}{e^2} + \frac{2d\left(\frac{111}{e^2} - \frac{100d^2}{e^4} + \frac{2d\left(\frac{200d}{e^3} + \frac{45}{e^2}\right)}{e}\right)}{e} - \frac{d^2\left(\frac{200d}{e^3} + \frac{45}{e^2}\right)}{e^2}\right)}{e} - \frac{d^2\left(\frac{111}{e^2} - \frac{100d^2}{e^4} + \frac{2d\left(\frac{200d}{e^3} + \frac{45}{e^2}\right)}{e}\right)}{e^2}\right)}{e} + \frac{d^2\left(\frac{37}{e^2} + \frac{2d\left(\frac{111}{e^2} - \frac{100}{e}\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

```
[Out] x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^
4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))
/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))
/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/
e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(3*e^2
) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e
^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4*e^2) +
(d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*e) - (d^2
*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e^4 + (2*d*
((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^2)) - x*((2
*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 +
 (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e
- (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e
+ (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2
))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107/e^2 + (d^2*(148
/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 4
5/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d^2*(111/e^2 - (100
*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2) + (100*x^7)/(7*e^2)
 - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148
*d^4*e^4 + 37*d^5*e^3 + 111*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (log(d + e*x)*(2
14*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d
^4*e^3 + 666*d^5*e^2))/e^9
```

**sympy [A]** time = 2.31, size = 393, normalized size = 1.11

$$x^6\left(-\frac{100d}{3e^3}-\frac{15}{2e^2}\right)+x^5\left(\frac{60d^2}{e^4}+\frac{18d}{e^3}+\frac{111}{5e^2}\right)+x^4\left(-\frac{100d^3}{e^5}-\frac{135d^2}{4e^4}-\frac{111d}{2e^3}-\frac{37}{4e^2}\right)+x^3\left(\frac{500d^4}{3e^6}+\frac{60d^3}{e^5}+\frac{111d^2}{e^4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

```
[Out] x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 111
/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) - 37
/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74*d/
(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 222*d
**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6/e**8
 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 130*d/e*
*3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e**3 - 14
8*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(d*e**9 +
```

```
e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**5 + 127*d
**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d + e*x)/e**9
```

## 3.302 $\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$

**Optimal**. Leaf size=354

$$\frac{3x^4 (200d^2 + 45de + 37e^2)}{4e^5} - \frac{x^3 (1000d^3 + 270d^2e + 333de^2 + 37e^3)}{3e^6} - \frac{(5d^2 - 2de + 3e^2)^2 (4d^4 + 5d^3e + 3d^2e^2 -}{2e^9(d + ex)^2}$$

```
[Out] -(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*(
1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+270
*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(20*
d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*
e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^
3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665*d^4*e
^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

**Rubi** [A] time = 0.34, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{3x^4 (200d^2 + 45de + 37e^2)}{4e^5} - \frac{x^3 (270d^2e + 1000d^3 + 333de^2 + 37e^3)}{3e^6} + \frac{x^2 (666d^2e^2 + 450d^3e + 1500d^4 + 111de^3 -}{2e^7}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

```
[Out] -(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)
*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^2)
/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (3*(
200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (50*x^
6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^
3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d
^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x)) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
107*e^6)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^3}\,dx = \int \left(\frac{-2100d^5-675d^4e-1110d^3e^2-222d^2e^3-444de^4+65e^5}{e^8}\right.$$

$$= -\frac{\left(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5\right)x}{e^8}$$

**Mathematica [A]** time = 0.10, size = 311, normalized size = 0.88

$$\frac{9000d^8-390d^7e(40x-9)-18d^6e^2\left(2300x^2+240x-407\right)-2d^5e^3\left(5600x^3+6750x^2+2664x-999\right)+4d^4e^4\left(7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3
,x]
```

```
[Out] (9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x -
 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 6660*x
^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3 + 1
665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*x^3 +
 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3 + 888
*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^2
*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

**fricas [A]** time = 0.81, size = 545, normalized size = 1.54

$$\frac{200\,e^8x^8+9000\,d^8+3510\,d^7e+7326\,d^6e^2+1998\,d^5e^3+6216\,d^4e^4-1950\,d^3e^5+1926\,d^2e^6-198\,de^7-108\,e^8-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="f
ricas")
```

```
[Out] 1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 + 6
216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d*e
^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*e^5
 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5 + 16
65*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4 + 166
```

```
5*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2 + 2250
*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^2 - 12*
(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5 + 130*d^
2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^6*e^2 + 3
70*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*e^2 + 945*
d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x
^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 + 888*d^3*e^5
 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e^10*x + d^2*e
^9)
```

**giac [A]** time = 0.16, size = 354, normalized size = 1.00

$$\left(2800\,d^6+945\,d^5e+1665\,d^4e^2+370\,d^3e^3+888\,d^2e^4-195\,de^5+107\,e^6\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{12}\left(200\,x^6e^{15}-720\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="g
iac")
```

```
[Out] (2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^
5 + 107*e^6)*e^(-9)*log(abs(x*e + d)) + 1/12*(200*x^6*e^15 - 720*d*x^5*e^14
 + 1800*d^2*x^4*e^13 - 4000*d^3*x^3*e^12 + 9000*d^4*x^2*e^11 - 25200*d^5*x*
e^10 - 108*x^5*e^15 + 405*d*x^4*e^14 - 1080*d^2*x^3*e^13 + 2700*d^3*x^2*e^1
2 - 8100*d^4*x*e^11 + 333*x^4*e^15 - 1332*d*x^3*e^14 + 3996*d^2*x^2*e^13 -
13320*d^3*x*e^12 - 148*x^3*e^15 + 666*d*x^2*e^14 - 2664*d^2*x*e^13 + 888*x^
2*e^15 - 5328*d*x*e^14 + 780*x*e^15)*e^(-18) + 1/2*(1500*d^8 + 585*d^7*e +
1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 + 2*(
800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2
*e^6 + 214*d*e^7 - 33*e^8)*x - 33*d*e^7 - 18*e^8)*e^(-9)/(x*e + d)^2
```

**maple [A]** time = 0.01, size = 531, normalized size = 1.50

$$\frac{50x^6}{3e^3}-\frac{60d\,x^5}{e^4}-\frac{9x^5}{e^3}+\frac{150d^2x^4}{e^5}+\frac{135d\,x^4}{4e^4}+\frac{111x^4}{4e^3}-\frac{1000d^3x^3}{3e^6}-\frac{90d^2x^3}{e^5}-\frac{111d\,x^3}{e^4}-\frac{37x^3}{3e^3}-\frac{50d^8}{(ex+d)^2\,e^9}-\frac{45d^7}{2\,(ex+d)^2\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

```
[Out] -1110*d^3/e^6*x-222*d^2/e^5*x-444*d/e^4*x+50/3*x^6/e^3+111/4/e^3*x^4-9/e^3*
x^5-9/(e*x+d)^2/e+107/e^3*ln(e*x+d)+74/e^3*x^2+65/e^3*x-33/(e*x+d)/e^2-37/3
/e^3*x^3-50/e^9/(e*x+d)^2*d^8-45/2/e^8/(e*x+d)^2*d^7+2800/e^9*ln(e*x+d)*d^6
+945/e^8*ln(e*x+d)*d^5-60/e^4*x^5*d+150/e^5*x^4*d^2+135/4/e^4*x^4*d-1000/3/
e^6*x^3*d^3-90/e^5*x^3*d^2+750/e^7*x^2*d^4+225/e^6*x^2*d^3-2100/e^8*x*d^5-6
75/e^7*x*d^4+800/e^9/(e*x+d)*d^7+315/e^8/(e*x+d)*d^6+666/(e*x+d)*d^5/e^7+18
```

```
5/(e*x+d)*d^4/e^6+592/(e*x+d)*d^3/e^5-195/(e*x+d)*d^2/e^4+214/(e*x+d)*d/e^3
-111*d/e^4*x^3+333*d^2/e^5*x^2+111/2*d/e^4*x^2-111/2/(e*x+d)^2*d^6/e^7-37/2
/(e*x+d)^2*d^5/e^6-74/(e*x+d)^2*d^4/e^5+65/2/(e*x+d)^2*d^3/e^4-107/2/(e*x+d
)^2*d^2/e^3+33/2/(e*x+d)^2*d/e^2+1665*d^4/e^7*ln(e*x+d)+370*d^3/e^6*ln(e*x+
d)+888*d^2/e^5*ln(e*x+d)-195*d/e^4*ln(e*x+d)
```

**maxima [A]** time = 0.45, size = 378, normalized size = 1.07

$$\frac{1500\,d^8 + 585\,d^7e + 1221\,d^6e^2 + 333\,d^5e^3 + 1036\,d^4e^4 - 325\,d^3e^5 + 321\,d^2e^6 - 33\,de^7 - 18\,e^8 + 2\left(800\,d^7e + 315\,d^{6}\right.}{2\left(e^{11}x^2 + 2\,de^{10}x + d^2e^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="m
axima")
```

```
[Out] 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325
*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 6
66*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*
x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4 + 3
*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2 + 270
*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 666*d^2*
e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3*e^2 +
222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e + 1665*d^4
*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x + d)/e^9
```

**mupad [B]** time = 0.13, size = 771, normalized size = 2.18

$$x^4\left(\frac{111}{4e^3}-\frac{75d^2}{e^5}+\frac{3d\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{4e}\right)-x^3\left(\frac{37}{3e^3}+\frac{100d^3}{3e^6}+\frac{d\left(\frac{111}{e^3}-\frac{300d^2}{e^5}+\frac{3d\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{300d}{e^4}+\frac{45}{e^3}\right)}{e^2}\right)-x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

```
[Out] x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^3
*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300
*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*d)/
e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6 + (
3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*
((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (3*d*((
300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e + (3*d
^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/
e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 - (d^3*(111
/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x*(214*d*e^
6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 185*d^4*e^3
+ 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 321*d^2*e^6 -
325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e))/(d^2*e^8 +
e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d*(37/e^3 + (10
0*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e
))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*(111/e^3 - (300*
d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3*((300*d)/e^4 + 4
```

```
5/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 + 2800*d^6 + 107*e^
6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

**sympy [A]** time = 4.87, size = 394, normalized size = 1.11

$$x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right)+x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d^2}{e^5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

```
[Out] x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4*
e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e**3
)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4) +
 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d**2/
e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**2 + 3
33*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*e**7 -
 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**4*e**4
+ 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e**9 + 4*
d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d**5*e + 16
65*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e**6)*log(d
 + e*x)/e**9
```

# 3.303 $\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^4}\,dx$

**Optimal**. Leaf size=360

$$\frac{x^3\left(1000d^2+180de+111e^2\right)}{3e^6}-\frac{x^2\left(2000d^3+450d^2e+444de^2+37e^3\right)}{2e^7}-\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2\right.}{3e^9(d+ex)^3}$$

```
[Out] 2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450*
d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/4*
(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^
2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e
+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2+(-2800*d^6-945*d^5*e-1
665*d^4*e^2-370*d^3*e^3-888*d^2*e^4+195*d*e^5-107*e^6)/e^9/(e*x+d)-(5600*d^
5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

**Rubi** [A] time = 0.36, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{x^3\left(1000d^2+180de+111e^2\right)}{3e^6}-\frac{x^2\left(450d^2e+2000d^3+444de^2+37e^3\right)}{2e^7}+\frac{2x\left(555d^2e^2+450d^3e+1750d^4+74de^3\right.}{e^8}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

```
[Out] (2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((2000
*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*e
+ 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 - ((5
*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(3*e
^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^
2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)/(e^9
*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*
e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)}{(d+ex)^4}\,dx = \int\left(\frac{2\left(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4\right)}{e^8} - \frac{\left(2000d\right.}{}\right.$$

$$= \frac{2\left(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4\right)x}{e^8} - \frac{\left(2000d^3\right.}{}$$

**Mathematica** [A] time = 0.12, size = 344, normalized size = 0.96

$$\frac{4e^3x^3\left(1000d^2+180de+111e^2\right)-6e^2x^2\left(2000d^3+450d^2e+444de^2+37e^3\right)+24ex\left(1750d^4+450d^3e+555d^2e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4
,x]
```

```
[Out] (24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(2
000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e +
 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*e +
 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 + (6*(
800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5
 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 1665*d^4*
e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) - 12*(560
0*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*Log[d
 + e*x])/(12*e^9)
```

**fricas** [A] time = 0.65, size = 587, normalized size = 1.63

$$\frac{240\,e^8x^8-29200\,d^8-9630\,d^7e-16428\,d^6e^2-3478\,d^5e^3-7696\,d^4e^4+1430\,d^3e^5-428\,d^2e^6-66\,de^7-72\,e^8-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="f
ricas")
```

```
[Out] 1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3 -
 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d*e
^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3*e^
5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*e^5
+ 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d^4*e^
```

```
4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2 + 306
0*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 214*e^8)
*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 + 2664*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d^7*e + 22
20*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e^3 + 1575*
d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 + 3*(5600*d^
6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6 - 65*d*e^7)
*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e^4 + 592*d^3*
e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^10*x
+ d^3*e^9)
```

**giac** [A] time = 0.16, size = 345, normalized size = 0.96

$$-\left(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5\right)e^{(-9)}\log\left(|xe+d|\right)+\frac{1}{12}\left(240\,x^5e^{16}-1200\,dx^4e^{15}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="g
iac")
```

```
[Out] -(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*
e^(-9)*log(abs(x*e + d)) + 1/12*(240*x^5*e^16 - 1200*d*x^4*e^15 + 4000*d^2*
x^3*e^14 - 12000*d^3*x^2*e^13 + 42000*d^4*x*e^12 - 135*x^4*e^16 + 720*d*x^3
*e^15 - 2700*d^2*x^2*e^14 + 10800*d^3*x*e^13 + 444*x^3*e^16 - 2664*d*x^2*e^
15 + 13320*d^2*x*e^14 - 222*x^2*e^16 + 1776*d*x*e^15 + 1776*x*e^16)*e^(-20)
 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
 - 715*d^3*e^5 + 6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5
 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 + 214*d^2*e^6 + 3*(10400*d^7*e +
3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 2
14*d*e^7 + 33*e^8)*x + 33*d*e^7 + 36*e^8)*e^(-9)/(x*e + d)^3
```

**maple** [A] time = 0.01, size = 558, normalized size = 1.55

$$\frac{20x^5}{e^4}-\frac{100d\,x^4}{e^5}-\frac{45x^4}{4e^4}-\frac{100d^8}{3\left(ex+d\right)^3e^9}-\frac{15d^7}{\left(ex+d\right)^3e^8}-\frac{37d^6}{\left(ex+d\right)^3e^7}-\frac{37d^5}{3\left(ex+d\right)^3e^6}-\frac{148d^4}{3\left(ex+d\right)^3e^5}+\frac{65d^3}{3\left(ex+d\right)^3e^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x)
```

```
[Out] 20*x^5/e^4-107/e^3/(e*x+d)-6/e/(e*x+d)^3+65/e^4*ln(e*x+d)-33/2/e^2/(e*x+d)^
2+37/e^4*x^3-37/2/e^4*x^2+148/e^4*x-45/4/e^4*x^4-37/3/e^6/(e*x+d)^3*d^5-148
/3/e^5/(e*x+d)^3*d^4+65/3/e^4/(e*x+d)^3*d^3-107/3/e^3/(e*x+d)^3*d^2+11/e^2/
(e*x+d)^3*d-100/e^5*x^4*d+1000/3/e^6*x^3*d^2-5600/e^9*ln(e*x+d)*d^5-1575/e^
8*ln(e*x+d)*d^4-2220/e^7*ln(e*x+d)*d^3-370/e^6*ln(e*x+d)*d^2-592/e^5*ln(e*x
```

```
+d)*d+400/e^9/(e*x+d)^2*d^7+315/2/e^8/(e*x+d)^2*d^6+333/e^7/(e*x+d)^2*d^5+1
85/2/e^6/(e*x+d)^2*d^4+296/e^5/(e*x+d)^2*d^3-195/2/e^4/(e*x+d)^2*d^2+107/e^
3/(e*x+d)^2*d+60/e^5*x^3*d-1000/e^7*x^2*d^3-225/e^6*x^2*d^2-222/e^5*x^2*d+3
500/e^8*d^4*x+900/e^7*x*d^3+1110/e^6*x*d^2+148/e^5*x*d-2800/e^9/(e*x+d)*d^6
-945/e^8/(e*x+d)*d^5-1665/e^7/(e*x+d)*d^4-370/e^6/(e*x+d)*d^3-888/e^5/(e*x+
d)*d^2+195/e^4/(e*x+d)*d-100/3/e^9/(e*x+d)^3*d^8-15/e^8/(e*x+d)^3*d^7-37/e^
7/(e*x+d)^3*d^6
```

**maxima [A]** time = 0.46, size = 390, normalized size = 1.08

$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\left(2800\,d^6e^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="m
axima")
```

```
[Out] -1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4 -
 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^5*
e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^2 +
 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3*e^5
 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d^2*e^
10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(1000*d
^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 444*d*e^3
 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4
)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4
- 65*e^5)*log(e*x + d)/e^9
```

**mupad [B]** time = 4.28, size = 560, normalized size = 1.56

$$x^3\left(\frac{37}{e^4}-\frac{200\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{3\,e}\right)-x^2\left(\frac{37}{2\,e^4}+\frac{200\,d^3}{e^7}+\frac{2\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

```
[Out] x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(37
/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6 +
(3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4 + (
1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 + 36*e
^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d^6*e^2
)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*d^3*e^4
 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x + 3*d*e^1
0*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e^8 + (4*d
*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^
5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6*d^2*(111/e
^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*d^3*((400*d)
/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4 + 1575*d^4*e
 + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

**sympy [A]** time = 8.09, size = 401, normalized size = 1.11

$$x^4\left(-\frac{100d}{e^5}-\frac{45}{4e^4}\right)+x^3\left(\frac{1000d^2}{3e^6}+\frac{60d}{e^5}+\frac{37}{e^4}\right)+x^2\left(-\frac{1000d^3}{e^7}-\frac{225d^2}{e^6}-\frac{222d}{e^5}-\frac{37}{2e^4}\right)+x\left(\frac{3500d^4}{e^8}+\frac{900d^3}{e^7}+\frac{111}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

```
[Out] x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 + 3
7/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4))
 + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148/e*
*4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 3848*d
**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*(-168
00*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 5328*d**2
*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2 - 1798
2*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 642*d*e**7
 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*e**12*x**
3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 370*d**2*e*
*3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

# 3.304 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

**Optimal**. Leaf size=221

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(500d^3-2475d^2e+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(4125d^3-6075d^2e-6870de^2+8\right.}{6250}$$

```
[Out] 1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-607
5*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458*e
^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2/15
*e^3*x^6+1/156250*(57250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*
x+3)-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arctan(1/14*
(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(-2475d^2e+500d^3+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(-6075d^2e+4125d^3-6870de^2\right.}{6250}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3 -
 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e + 12
15*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/500
+ (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175*d^2
*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt[14])
 + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x + 5*x^2
])/156250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int\left(\frac{10125d^3+34350d^2e-13215de^2-5108e^3}{15625}-\frac{\left(4125d^3-6075d^2\right.}{}\right.$$

$$=\frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625}-\frac{\left(4125d^3-6075d^2\right.}{}$$

$$=\frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625}-\frac{\left(4125d^3-6075d^2\right.}{}$$

$$=\frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625}-\frac{\left(4125d^3-6075d^2\right.}{}$$

$$=\frac{\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)x}{15625}-\frac{\left(4125d^3-6075d^2\right.}{}$$

**Mathematica [A]** time = 0.12, size = 178, normalized size = 0.81

$$\frac{42\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(5x^2+2x+3\right)+35x\left(250d^3\left(200x^2-495x+486\right)+450d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2 +
 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) + e^
3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) - 6*S
qrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5
*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[
3 + 2*x + 5*x^2])/6562500
```

**fricas [A]** time = 0.84, size = 206, normalized size = 0.93

$$\frac{2}{15}e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

**giac [A]** time = 0.17, size = 212, normalized size = 0.96

$$\frac{2}{15}x^6e^3+\frac{12}{25}dx^5e^2+\frac{3}{5}d^2x^4e+\frac{4}{15}d^3x^3-\frac{33}{125}x^5e^3-\frac{99}{100}dx^4e^2-\frac{33}{25}d^2x^3e-\frac{33}{50}d^3x^2+\frac{81}{500}x^4e^3+\frac{81}{125}dx^3e^2+\frac{243}{250}d^2x^2e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 2/15*x^6*e^3 + 12/25*d*x^5*e^2 + 3/5*d^2*x^4*e + 4/15*d^3*x^3 - 33/125*x^5*
e^3 - 99/100*d*x^4*e^2 - 33/25*d^2*x^3*e - 33/50*d^3*x^2 + 81/500*x^4*e^3 +
 81/125*d*x^3*e^2 + 243/250*d^2*x^2*e + 81/125*d^3*x + 458/1875*x^3*e^3 + 6
87/625*d*x^2*e^2 + 1374/625*d^2*x*e - 881/6250*x^2*e^3 - 2643/3125*d*x*e^2
- 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*
arctan(1/14*sqrt(14)*(5*x + 1)) - 5108/15625*x*e^3 + 1/156250*(57250*d^3 -
66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

**maple [A]** time = 0.01, size = 291, normalized size = 1.32

$$\frac{2e^3x^6}{15}+\frac{12d\,e^2x^5}{25}-\frac{33e^3x^5}{125}+\frac{3d^2e\,x^4}{5}-\frac{99d\,e^2x^4}{100}+\frac{81e^3x^4}{500}+\frac{4d^3x^3}{15}-\frac{33d^2e\,x^3}{25}+\frac{81d\,e^2x^3}{125}+\frac{458e^3x^3}{1875}-\frac{33d^3x^2}{50}+\frac{243d^2e\,x}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] -881/6250*e^3*x^2+2/15*e^3*x^6+81/500*e^3*x^4-17967/43750*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d^2*e+54969/218750*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d*e^2+4/15*x^3*d^3-5108/15625*x*e^3-33/125*x^5*e^3-33/50*x^2*d^3+458
/1875*x^3*e^3+229/625*ln(5*x^2+2*x+3)*d^3+23431/156250*ln(5*x^2+2*x+3)*e^3+
81/125*d^3*x-2643/3125*x*d*e^2-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14^(1
/2))*d^3+53189/1093750*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^3+3/5*x^4*
d^2*e-99/100*x^4*d*e^2-33/25*x^3*d^2*e+687/625*x^2*d*e^2+1374/625*x*d^2*e+8
1/125*x^3*d*e^2+243/250*x^2*d^2*e-7662/15625*ln(5*x^2+2*x+3)*d*e^2+12/25*x^
5*d*e^2-2643/6250*ln(5*x^2+2*x+3)*d^2*e
```

**maxima [A]** time = 0.96, size = 206, normalized size = 0.93

$$\frac{2}{15}\,e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5+\frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^2
 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3 -
 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*sqrt
(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt
(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3
)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^
2 + 2*x + 3)
```

**mupad [B]** time = 4.18, size = 397, normalized size = 1.80

$$x^2\left(\frac{26\,e^2\,\left(12\,d-5\,e\right)}{625}-\frac{33\,e\,\left(4\,d^2-5\,d\,e+e^2\right)}{250}-\frac{3\,d\,e^2}{50}+\frac{3\,d^2\,e}{2}-\frac{33\,d^3}{50}+\frac{622\,e^3}{3125}\right)-x^3\left(\frac{11\,e^2\,\left(12\,d-5\,e\right)}{375}+\frac{2\,e}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e^
2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d -
5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^3)/
15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2*x +
 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 - (2343
1*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e^2))/2
0 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 + (3*d*(
d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*e^2)/125
 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(((14^(1/2
)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750 + (14^(1/2
)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/218750)/((54969*
d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)/78125))*(27
4845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

**sympy [C]** time = 2.58, size = 450, normalized size = 2.04

$$\frac{2e^3x^6}{15}+x^5\left(\frac{12de^2}{25}-\frac{33e^3}{125}\right)+x^4\left(\frac{3d^2e}{5}-\frac{99de^2}{100}+\frac{81e^3}{500}\right)+x^3\left(\frac{4d^3}{15}-\frac{33d^2e}{25}+\frac{81de^2}{125}+\frac{458e^3}{1875}\right)+x^2\left(-\frac{33d^3}{50}+\frac{243d^2}{250}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99*
d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/125
+ 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 - 88
1*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 5108*e
**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e
**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189
*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e
**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3
)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) + (229*d**3
/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 + sqrt(14)*
I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187500)*log(x
+ (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sqrt(14)*I*(52
875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(52875*d**3 + 449
175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

## 3.305 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

**Optimal**. Leaf size=156

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}+$$

```
[Out] 1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+1
/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5+1/15
625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)-1/218750*(10575*d^2+59890*
d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{375}x^3\left(100d^2-330de+81e^2\right)-\frac{x^2\left(825d^2-810de-458e^2\right)}{1250}+\frac{\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)}{15625}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*x
^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)/1
00 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)
/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*
x + 5*x^2])/15625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{2025d^2+4580de-881e^2}{3125}-\frac{1}{625}\left(825d^2-810de-458e^2\right)x+\frac{1}{12}\right.$$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\Big($$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\Big($$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\Big($$

$$=\frac{\left(2025d^2+4580de-881e^2\right)x}{3125}-\frac{\left(825d^2-810de-458e^2\right)x^2}{1250}+\frac{1}{375}\Big($$

**Mathematica [A]** time = 0.08, size = 130, normalized size = 0.83

$$\frac{84\left(5725d^2-4405de-2554e^2\right)\log\left(5x^2+2x+3\right)+35x\left(50d^2\left(200x^2-495x+486\right)+60de\left(250x^3-550x^2+\right.\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] (35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 250
*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt[1
4]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(572
5*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

**fricas [A]** time = 0.79, size = 141, normalized size = 0.90

$$\frac{4}{25}\,e^2x^5+\frac{1}{100}\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\left(100\,d^2-330\,de+81\,e^2\right)x^3-\frac{1}{1250}\left(825\,d^2-810\,de-458\,e^2\right)x^2-\frac{1}{218750}\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.16, size = 145, normalized size = 0.93

$$\frac{4}{25}\,x^5e^2+\frac{2}{5}\,dx^4e+\frac{4}{15}\,d^2x^3-\frac{33}{100}\,x^4e^2-\frac{22}{25}\,dx^3e-\frac{33}{50}\,d^2x^2+\frac{27}{125}\,x^3e^2+\frac{81}{125}\,dx^2e+\frac{81}{125}\,d^2x+\frac{229}{625}\,x^2e^2+\frac{916}{625}\,dxe-\frac{1}{218}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 4/25*x^5*e^2 + 2/5*d*x^4*e + 4/15*d^2*x^3 - 33/100*x^4*e^2 - 22/25*d*x^3*e
- 33/50*d^2*x^2 + 27/125*x^3*e^2 + 81/125*d*x^2*e + 81/125*d^2*x + 229/625*
x^2*e^2 + 916/625*d*x*e - 1/218750*sqrt(14)*(10575*d^2 + 59890*d*e - 18323*
e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) - 881/3125*x*e^2 + 1/15625*(5725*d^2 -
 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

**maple [A]** time = 0.01, size = 191, normalized size = 1.22

$$\frac{4e^2x^5}{25}+\frac{2de\,x^4}{5}-\frac{33e^2x^4}{100}+\frac{4d^2x^3}{15}-\frac{22de\,x^3}{25}+\frac{27e^2x^3}{125}-\frac{33d^2x^2}{50}+\frac{81de\,x^2}{125}+\frac{229e^2x^2}{625}+\frac{81d^2x}{125}-\frac{423\sqrt{14}\;d^2\arctan\left(\frac{(10x}{}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*e
^2*x^3-33/50*x^2*d^2+81/125*x^2*d*e+229/625*x^2*e^2+81/125*d^2*x+916/625*x*
d*e-881/3125*e^2*x+229/625*ln(5*x^2+2*x+3)*d^2-881/3125*ln(5*x^2+2*x+3)*d*e
-2554/15625*ln(5*x^2+2*x+3)*e^2-423/8750*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d^2-5989/21875*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e+18323/2187
50*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^2
```

**maxima [A]** time = 0.96, size = 141, normalized size = 0.90

$$\frac{4}{25}\,e^2x^5+\frac{1}{100}\left(40\,de-33\,e^2\right)x^4+\frac{1}{375}\left(100\,d^2-330\,de+81\,e^2\right)x^3-\frac{1}{1250}\left(825\,d^2-810\,de-458\,e^2\right)x^2-\frac{1}{218750}\,\sqrt{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] 4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81*
e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(10
575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(
2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*e^2)
*log(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 0.10, size = 223, normalized size = 1.43

$$x\left(\frac{4\,d\,e}{5}+\frac{52\,e\,\left(8\,d-5\,e\right)}{625}+\frac{81\,d^2}{125}+\frac{419\,e^2}{3125}\right)-\ln\left(5\,x^2+2\,x+3\right)\left(-\frac{229\,d^2}{625}+\frac{881\,d\,e}{3125}+\frac{2554\,e^2}{15625}\right)+x^4\left(\frac{e\,\left(8\,d-5\,e\right)}{20}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - lo
g(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) + x^
4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))/75
 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33*d^2
)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(59890*
d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575*d^2 -
 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/15625))
*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

**sympy [C]** time = 1.72, size = 303, normalized size = 1.94

$$\frac{4e^2x^5}{25}+x^4\left(\frac{2de}{5}-\frac{33e^2}{100}\right)+x^3\left(\frac{4d^2}{15}-\frac{22de}{25}+\frac{27e^2}{125}\right)+x^2\left(-\frac{33d^2}{50}+\frac{81de}{125}+\frac{229e^2}{625}\right)+x\left(\frac{81d^2}{125}+\frac{916de}{625}-\frac{881e^2}{3125}\right)+\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/25
 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*d*
*2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2554
*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*log(
x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 59890*
d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d**2/625
 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d*e - 18
323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - sqrt(14)*
I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*
e**2))
```

## 3.306 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

**Optimal**. Leaf size=99

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\tan}{3125\sqrt{14}}$$

```
[Out] 1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4+
1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)-1/43750*(2115*d+5989*e)*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi** [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1628, 634, 618, 204, 628}

$$\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)-\frac{(2115d+5989e)\tan}{3125\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/7
5 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt[1
4]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{1}{625}(405d+458e)-\frac{3}{125}(55d-27e)x+\frac{1}{25}(20d-33e)x^2+\frac{4ex}{5}\right.$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}+$$

$$=\frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2+\frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5}-$$

**Mathematica [A]** time = 0.05, size = 86, normalized size = 0.87

$$\frac{21(2290d-881e)\log\left(5x^2+2x+3\right)+35x\left(5d\left(200x^2-495x+486\right)+3e\left(250x^3-550x^2+405x+916\right)\right)-3}{131250}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3))
 - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d - 8
81*e)*Log[3 + 2*x + 5*x^2])/131250
```

**fricas** [A]   time = 0.78, size = 84, normalized size = 0.85

$$\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,(405$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

**giac** [A]   time = 0.16, size = 88, normalized size = 0.89

$$\frac{1}{5}\,x^4e+\frac{4}{15}\,dx^3-\frac{11}{25}\,x^3e-\frac{33}{50}\,dx^2+\frac{81}{250}\,x^2e-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,dx+\frac{458}{625}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/5*x^4*e + 4/15*d*x^3 - 11/25*x^3*e - 33/50*d*x^2 + 81/250*x^2*e - 1/43750
*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x +
458/625*x*e + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

**maple** [A]   time = 0.00, size = 102, normalized size = 1.03

$$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}-\frac{423\sqrt{14}\,\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}+\frac{229d\ln\left(5x^2+2x+3\right)}{625}+\frac{458ex}{625}-\frac{598}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 1/5*e*x^4+4/15*x^3*d-11/25*x^3*e-33/50*x^2*d+81/250*e*x^2+81/125*d*x+458/62
5*e*x+229/625*ln(5*x^2+2*x+3)*d-881/6250*e*ln(5*x^2+2*x+3)-423/8750*14^(1/2
)*arctan(1/28*(10*x+2)*14^(1/2))*d-5989/43750*14^(1/2)*arctan(1/28*(10*x+2)
*14^(1/2))*e
```

**maxima** [A]   time = 0.96, size = 84, normalized size = 0.85

$$\frac{1}{5}\,ex^4+\frac{1}{75}\,(20\,d-33\,e)x^3-\frac{3}{250}\,(55\,d-27\,e)x^2-\frac{1}{43750}\,\sqrt{14}\,(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,(405$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] 1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqrt
(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 458
*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 0.07, size = 107, normalized size = 1.08

$$x^3\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5}+x\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\sqrt{1}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^2
 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/62
5) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2115
*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/43750
```

**sympy [C]** time = 0.85, size = 163, normalized size = 1.65

$$\frac{ex^4}{5}+x^3\left(\frac{4d}{15}-\frac{11e}{25}\right)+x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right)+x\left(\frac{81d}{125}+\frac{458e}{625}\right)+\left(\frac{229d}{625}-\frac{881e}{6250}-\frac{\sqrt{14}\,i\,(2115d+5989e)}{87500}\right)\log\left(x+\frac{42}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/1
25 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/87
500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d +
5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500)*lo
g(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 5989*e)
)
```

# 3.307 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2} \, dx$

**Optimal**. Leaf size=56

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}-\frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

```
[Out] 81/125*x-33/50*x^2+4/15*x^3+229/625*ln(5*x^2+2*x+3)-423/8750*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

**Rubi** [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}-\frac{423\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(6
25*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx &= \int \left(\frac{81}{125}-\frac{33x}{25}+\frac{4x^2}{5}+\frac{7+458x}{125\left(3+2x+5x^2\right)}\right)dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{1}{125}\int \frac{7+458x}{3+2x+5x^2}\,dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\int \frac{2+10x}{3+2x+5x^2}\,dx-\frac{423}{625}\int \frac{1}{3+2x+5x^2}\,dx\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}+\frac{229}{625}\log\left(3+2x+5x^2\right)+\frac{846}{625}\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx,x,\right.\\
&= \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 50, normalized size = 0.89

$$\frac{35x\left(200x^2-495x+486\right)+9618\log\left(5x^2+2x+3\right)-1269\sqrt{14}\,\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{26250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

```
[Out] (35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
9618*Log[3 + 2*x + 5*x^2])/26250
```

**fricas [A]** time = 0.65, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

**giac** [A] time = 0.16, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

**maple** [A] time = 0.00, size = 44, normalized size = 0.79

$$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}+\frac{229\ln\left(5x^2+2x+3\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] 4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

**maxima** [A] time = 0.97, size = 43, normalized size = 0.77

$$\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

**mupad** [B] time = 0.04, size = 45, normalized size = 0.80

$$\frac{81\,x}{125}+\frac{229\,\ln\left(5\,x^2+2\,x+3\right)}{625}-\frac{423\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}-\frac{33\,x^2}{50}+\frac{4\,x^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

```
[Out] (81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/2
)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

**sympy [A]** time = 0.23, size = 61, normalized size = 1.09

$$\frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} + \frac{229\log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14} + \frac{\sqrt{14}}{14}\right)}{8750}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] 4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*s
qrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

# 3.308 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$

**Optimal**. Leaf size=168

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{x(2$$

```
[Out] -1/25*(20*d+33*e)*x/e^2+2/5*x^2/e+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(
e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+1/250*(458*d-7*e)*ln(5*x^2+2*x+3)/(5*d^2-2*d
*e+3*e^2)-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+
3*e^2)*14^(1/2)
```

**Rubi [A]** time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\left(5d^2-2de+3e^2\right)}+\frac{\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d-1367e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}-\frac{x(2$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -((20*d + 33*e)*x)/(25*e^2) + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1 +
 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3*e
 + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)) +
 ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{-20d-33e}{25e^2}+\frac{4x}{5e}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)}+\frac{7d+272e+(4}{25\left(5d^2-2de+3e^2\right.}\right.$$

$$=-\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{\int \frac{7d+}{}}{25\left(5\right.}$$

$$=-\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}-\frac{(423d}{12}$$

$$=-\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)}+\frac{(458d}{2}$$

$$=-\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}\left(5d^2-2de+3e^2\right)}+\frac{\left(4d^4+5d^3e+3d^2e^2-\right.}{e^3\left(5d^2-\right.}$$

**Mathematica [A]** time = 0.11, size = 146, normalized size = 0.87

$$\frac{70ex\left(5d^2-2de+3e^2\right)(e(10x-33)-20d)+1750\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)+7e^3(458d-7}{1750e^3\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
- 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^2])
/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

**fricas [A]** time = 1.00, size = 171, normalized size = 1.02

$$\frac{700\left(5\,d^2e^2-2\,de^3+3\,e^4\right)x^2-\sqrt{14}\left(423\,de^3-1367\,e^4\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-70\left(100\,d^3e+125\,d^2e^2-6\,de^3\right.}{1750\left(5\,d^2e^3-2\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="frica
s")
```

```
[Out] 1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367*
e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*e^
3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x
+ d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4 + 3
*e^5)
```

**giac [A]** time = 0.22, size = 158, normalized size = 0.94

$$\frac{1}{25}\left(10\,x^2e-20\,dx-33\,xe\right)e^{(-2)}-\frac{\sqrt{14}\,(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{1750\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{(458\,d-7\,e)\log\left(5\,x^2+2\,x+3\right.}{250\left(5\,d^2-2\,de+3\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac"
)
```

```
[Out] 1/25*(10*x^2*e - 20*d*x - 33*x*e)*e^(-2) - 1/1750*sqrt(14)*(423*d - 1367*e)
*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 - 2*d*e + 3*e^2) + 1/250*(458*d - 7
*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2
*e^2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(5*d^2*e^3 - 2*d*e^4 + 3*e^5)
```

**maple [A]** time = 0.01, size = 298, normalized size = 1.77

$$\frac{4d^4\ln(ex+d)}{\left(5d^2-2de+3e^2\right)e^3}+\frac{5d^3\ln(ex+d)}{\left(5d^2-2de+3e^2\right)e^2}+\frac{3d^2\ln(ex+d)}{\left(5d^2-2de+3e^2\right)e}-\frac{423\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70\left(125d^2-50de+75e^2\right)}-\frac{d\ln(ex+d)}{5d^2-2de+3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x)
```

```
[Out] 2/5/e*x^2-4/5*d/e^2*x-33/25/e*x+229/5/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+
3)*d-7/10/(125*d^2-50*d*e+75*e^2)*ln(5*x^2+2*x+3)*e-423/70/(125*d^2-50*d*e+
75*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d+1367/70/(125*d^2-50*d*e+7
5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e+4/e^3/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)*d^4+5/e^2/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d^3+3/e/(5*d^2-2*d*e+3*e^
2)*ln(e*x+d)*d^2-1/(5*d^2-2*d*e+3*e^2)*ln(e*x+d)*d+2*e/(5*d^2-2*d*e+3*e^2)*
ln(e*x+d)
```

**maxima [A]** time = 0.96, size = 160, normalized size = 0.95

$$-\frac{\sqrt{14}\,(423\,d-1367\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{1750\left(5\,d^2-2\,de+3\,e^2\right)}+\frac{\left(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4\right)\log\left(ex+d\right)}{5\,d^2e^3-2\,de^4+3\,e^5}+\frac{(458\,d-7\,e)}{250\left(5\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxim
a")
```

```
[Out] -1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(
5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

**mupad [B]** time = 6.39, size = 713, normalized size = 4.24

$$\frac{2\,x^2}{5\,e}-\ln\left(d+e\,x\right)\left(\frac{\frac{458\,d}{125}-\frac{7\,e}{125}}{5\,d^2-2\,d\,e+3\,e^2}-\frac{100\,d^2+165\,d\,e+81\,e^2}{125\,e^3}\right)-x\left(\frac{4\,\left(5\,d+2\,e\right)}{25\,e^2}+\frac{1}{e}\right)-\frac{\ln\left(\frac{-28\,d^3+1053\,d^2\,e+1791\,d}{25\,e^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3*
e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e^2
) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) - (x*
(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14^(1/
2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3 + 43
```

```
50*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e^3 - 6
250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e^5 - 14
500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6250*d^2*
e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/35
00 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^2*3i))*(d*
((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250)))/(d^2
*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/
(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) - ((
d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 + 7i/250))*((4
751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(
8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) + (
((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e
^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((136
7*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i
+ e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500 +
7i/250)))/(d^2*5i - d*e*2i + e^2*3i)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

# 3.309 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

**Optimal**. Leaf size=233

$$\frac{\left(229d^2-7de-136e^2\right)\log\left(5x^2+2x+3\right)}{25\left(5d^2-2de+3e^2\right)^2}-\frac{\left(423d^2-2734de+293e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}\left(5d^2-2de+3e^2\right)^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+}{e^3\left(5d^2-2de+3e^2\right)(d+}$$

```
[Out] 4/5*x/e^2+(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e
*x+d)-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)*ln(e*x+d)/e^3/(5*d^2
-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3
*e^2)^2-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d
^2-2*d*e+3*e^2)^2*14^(1/2)
```

**Rubi [A]** time = 0.25, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{\left(229d^2-7de-136e^2\right)\log\left(5x^2+2x+3\right)}{25\left(5d^2-2de+3e^2\right)^2}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(28d^3e^2+44d^2e^3+d^4e+40d^5-2\right.}{e^3\left(5d^2-2de+3e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

```
[Out] (4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 -
 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 + 5*
x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e
+ 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*d^2
 - 2*d*e + 3*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{4}{5e^2}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2}{e^2\left(5d^2-2de+3e^2\right)^2(d+e}\right.$$

$$=\frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4\right.}{e^3\left(5d^2-2de+3e^2\right)^2}$$

$$=\frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4\right.}{e^3\left(5d^2-2de+3e^2\right)^2}$$

$$=\frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4\right.}{e^3\left(5d^2-2de+3e^2\right)^2}$$

$$=\frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(d+ex)}-\frac{\left(423d^2-2734de+293e^2\right)\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right.}{25\sqrt{14}\left(5d^2-2de+3e^2\right)^2}$$

**Mathematica** [A] time = 0.16, size = 233, normalized size = 1.00

$$\frac{\left(229d^2-7de-136e^2\right)\log\left(5x^2+2x+3\right)}{25\left(5d^2-2de+3e^2\right)^2}+\frac{\left(-423d^2+2734de-293e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{14}\left(5d^2-2de+3e^2\right)^2}+\frac{-4d^4-5d^3e-3d^2e^2+d\cdots}{e^3\left(5d^2-2de+3e^2\right)(d\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] (4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^4*e
 - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*d
*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

**fricas** [A] time = 1.05, size = 416, normalized size = 1.79

$$-\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\left(25\,d^4e^2-20\,d^3e^3+34\,d^2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] -1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
- 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*
e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (423*
d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 280*(2
5*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*d^6 +
 d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^4*e^2
+ 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229*d^3*e^
3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*log(5*x^2
+ 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9*d*e^7 +
(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

**giac** [A] time = 0.18, size = 355, normalized size = 1.52

$$\frac{1}{25}\left(40\,d+33\,e\right)e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)-\frac{\sqrt{14}\left(423\,d^2e^2-2734\,de^3+293\,e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\cdots\right.\right.}{350\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] 1/25*(40*d + 33*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2) - 1/350*sqrt
(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2
/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)/(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + 4/5*(x*e + d)*e^(-3) + 1/25*(
229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x
*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(25*d^4 - 20*d^3*e + 3
4*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(x*e + d) + 5*d^3*e^4/(x*e + d)
+ 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^7/(x*e + d))/(5*d^2*e^6 - 2*d
*e^7 + 3*e^8)
```

**maple [B]** time = 0.02, size = 538, normalized size = 2.31

$$-\frac{40d^5 \ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2 e^3}-\frac{d^4 \ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2 e^2}-\frac{28d^3 \ln(ex+d)}{\left(5d^2-2de+3e^2\right)^2 e}-\frac{423\sqrt{14}\, d^2 \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{350\left(5d^2-2de+3e^2\right)^2}-\frac{44d^2 \ln(\cdots}{\left(5d^2-2d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x)
```

```
[Out] 4/5/e^2*x+229/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2-7/25/(5*d^2-2*d*
e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e-136/25/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)
*e^2-423/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d^2+1367/175/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*
d*e-293/350/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e
^2-4/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^4-5/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)*
d^3-3/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d^2+1/(5*d^2-2*d*e+3*e^2)/(e*x+d)*d-2*e
/(5*d^2-2*d*e+3*e^2)/(e*x+d)-40/e^3/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^5-1/e
^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^4-28/e/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
*d^3-44/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^2+2*e/(5*d^2-2*d*e+3*e^2)^2*ln(e*
x+d)*d-e^2/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)
```

**maxima [A]** time = 0.98, size = 294, normalized size = 1.26

$$-\frac{\sqrt{14}\left(423\,d^2-2734\,de+293\,e^2\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{350\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}-\frac{\left(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5\right)\log\left(e\cdots\right.}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e +
 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*d^3
*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2)*lo
g(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4
*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 + 3*d*e^
5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

**mupad [B]** time = 4.67, size = 312, normalized size = 1.34

$$\frac{4\,x}{5\,e^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}-\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}+\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}+\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}}+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

```
[Out] (4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350 -
7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log(x
+ (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((293*14
^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i - d^3
*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*d^4 +
2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2)) - (log(
d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^3*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

# 3.310 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)} dx$

**Optimal**. Leaf size=317

$$\frac{(458d^3-21d^2e-816de^2+113e^3)\log(5x^2+2x+3)}{10(5d^2-2de+3e^2)^3}-\frac{(423d^3-4101d^2e+879de^2+703e^3)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}(5d^2-2de+3e^2)^3}-\frac{4d^4+}{2e^3(5}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^
2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2)^
2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-242*d^3*e^3+141*d^2*e^4+120*d*e^5-
e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3+1/10*(458*d^3-21*d^2*e-816*d*e^2+1
13*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3-1/70*(423*d^3-4101*d^2*e+879*
d*e^2+703*e^3)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

**Rubi [A]** time = 0.29, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1628, 634, 618, 204, 628}

$$\frac{(-21d^2e+458d^3-816de^2+113e^3)\log(5x^2+2x+3)}{10(5d^2-2de+3e^2)^3}-\frac{3d^2e^2+5d^3e+4d^4-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{28d^3e^2+44d^2e^3+d^4e}{e^3(5d^2-2de+3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e^3*(5*d^2 - 2*d*e + 3*e^
2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e + 879*
d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3
*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d^2*e^4
+ 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) + ((458*d^
3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d^2 - 2*d*
e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)}\,dx = \int \left(\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\left(5d^2-2de+3e^2\right)(d+ex)^3}+\frac{-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-}{e^2\left(5d^2-2de+3e^2\right)^2(d+ex)^2}\right.$$

$$=-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}$$

$$=-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}$$

$$=-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}$$

$$=-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}$$

**Mathematica [A]** time = 0.43, size = 278, normalized size = 0.88

$$-\frac{-7\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)+\sqrt{14}\left(423d^3-4101d^2e+879de^2+703e^3\right)\tan^{-1}\left(\frac{5x+}{\sqrt{1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x
]
```

```
[Out] -1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e +
28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(423*d
^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70*(-1
00*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e^5 +
e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3
 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

**fricas [B]** time = 1.36, size = 698, normalized size = 2.20

$$\frac{10500\,d^8-6825\,d^7e+14175\,d^6e^2+10395\,d^5e^3-6160\,d^4e^4+12145\,d^3e^5-4305\,d^2e^6+1365\,de^7-630\,e^8-\sqrt{14}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] 1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^4
 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^5*
e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2*e^
6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*e^6
+ 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d^6*e^
2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 + 3*e^8)
*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e^4 + 12
0*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 242*d^3*e^
5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^2 + 228*d
^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(e*x + d) +
 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*d^3*e^5 - 2
1*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*e^5 - 816*d^
2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150*d^7*e^4 + 28
5*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e^9 + (125*d^6*
e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9 - 54*d*e^10 + 2
```

```
7*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 - 188*d^4*e^7 + 17
1*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

**giac [A]** time = 0.18, size = 406, normalized size = 1.28

$$-\frac{\sqrt{14}\left(423\,d^{3}-4101\,d^{2}e+879\,de^{2}+703\,e^{3}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{70\left(125\,d^{6}-150\,d^{5}e+285\,d^{4}e^{2}-188\,d^{3}e^{3}+171\,d^{2}e^{4}-54\,de^{5}+27\,e^{6}\right)}+\frac{\left(458\,d^{3}-21\,d^{2}e-816\,de^{2}+1\right.}{10\left(125\,d^{6}-150\,d^{5}e+285\,d^{4}e^{2}-188\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 17
1*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d
^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(x*e + d))/(125*d^6*e^3 - 15
0*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) +
1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4 + 141*d
^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 + 297*d^5*
e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8)*e^(-1))*
e^(-2)/((5*d^2 - 2*d*e + 3*e^2)^3*(x*e + d)^2)
```

**maple [B]** time = 0.02, size = 819, normalized size = 2.58

$$\frac{100d^{6}\ln\left(ex+d\right)}{\left(5d^{2}-2de+3e^{2}\right)^{3}e^{3}}-\frac{120d^{5}\ln\left(ex+d\right)}{\left(5d^{2}-2de+3e^{2}\right)^{3}e^{2}}+\frac{228d^{4}\ln\left(ex+d\right)}{\left(5d^{2}-2de+3e^{2}\right)^{3}e}-\frac{423\sqrt{14}\,d^{3}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{70\left(5d^{2}-2de+3e^{2}\right)^{3}}-\frac{242d^{3}\ln}{\left(5d^{2}-2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x)
```

```
[Out] -423/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3-7
03/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^3-2/e
^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^4-5/2/e^2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*
d^3-3/2/e/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d^2+40/e^3/(5*d^2-2*d*e+3*e^2)^2/(e
*x+d)*d^5+1/e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^4-2*e/(5*d^2-2*d*e+3*e^2)^2
/(e*x+d)*d-21/10/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e-408/5/(5*d^2-2
*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^2+120/(5*d^2-2*d*e+3*e^2)^3*e^2*ln(e*x+d)
*d+100/(5*d^2-2*d*e+3*e^2)^3/e^3*ln(e*x+d)*d^6-120/(5*d^2-2*d*e+3*e^2)^3/e^
2*ln(e*x+d)*d^5+228/(5*d^2-2*d*e+3*e^2)^3/e*ln(e*x+d)*d^4+141/(5*d^2-2*d*e+
```

```
3*e^2)^3*e*ln(e*x+d)*d^2+28/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^3+44/(5*d^2-2
*d*e+3*e^2)^2/(e*x+d)*d^2+1/2/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2*d-e/(5*d^2-2*d*
e+3*e^2)/(e*x+d)^2+e^2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)-242/(5*d^2-2*d*e+3*e^2
)^3*ln(e*x+d)*d^3-1/(5*d^2-2*d*e+3*e^2)^3*e^3*ln(e*x+d)+229/5/(5*d^2-2*d*e+
3*e^2)^3*ln(5*x^2+2*x+3)*d^3+113/10/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e
^3+4101/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^
2*e-879/70/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*
e^2
```

**maxima [A]** time = 1.00, size = 498, normalized size = 1.57

$$-\frac{\sqrt{14}\left(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{70\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{\left(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e\right.}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] -1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sqr
t(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2
*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^
3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*e^4
+ 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*(458*
d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + 1/2*(
60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5 - 6*e^6
+ 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)/(25*d
^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d^4*e^5 - 2
0*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 - 20*d^4*e^5
 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

**mupad [B]** time = 4.76, size = 493, normalized size = 1.56

$$\frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}+\frac{x\,\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^2\,\left(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4\right)}}{d^2+2\,d\,e\,x+e^2\,x^2}-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{423\,\sqrt{14}}{140}-\frac{229}{5}\mathrm{i}\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

```
[Out] ((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e^
2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*e
- 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d^3*
```

```
e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x - (
14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*14^(1/
2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((4101*14^
(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e
^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)*(d
^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/10) + d*e
^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 21i/10)))/(
d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i +
 d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6 - e^6 + 141
*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

# 3.311 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

**Optimal**. Leaf size=189

$$\frac{ex^2(840d^2-1722de+373e^2)}{3500}-\frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^3-17220d^2}{17}$$

```
[Out] 1/17500*(2800*d^3-17220*d^2*e+9921*d*e^2+6053*e^3)*x+1/3500*e*(840*d^2-1722
*d*e+373*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-1/3500*(1367+423*x
)*(e*x+d)^3/(5*x^2+2*x+3)-1/6250*(1025*d^3-1545*d^2*e-2601*d*e^2+832*e^3)*l
n(5*x^2+2*x+3)+1/1225000*(32825*d^3+317565*d^2*e-221643*d*e^2-67499*e^3)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.26, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{ex^2(840d^2-1722de+373e^2)}{3500}-\frac{(-1545d^2e+1025d^3-2601de^2+832e^3)\log(5x^2+2x+3)}{6250}+\frac{x(-17220d^2e+28}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^3 - 17220*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/17500 + (e*(840*d^2 -
1722*d*e + 373*e^2)*x^2)/3500 + ((60*d - 41*e)*e^2*x^3)/375 + (e^3*x^4)/25
- ((1367 + 423*x)*(d + e*x)^3)/(3500*(3 + 2*x + 5*x^2)) + ((32825*d^3 + 317
565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sq
rt[14]) - ((1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x
^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{(d+ex)^2\left(\frac{6}{125}(615d+1367e) - \frac{12}{125}\right.}{3}$$

$$= -\frac{(1367+423x)(d+ex)^3}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int\left(\frac{2}{625}\left(2800d^3-17220d^2e+9921d\right.\right.$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500} + \frac{e\left(840d^2-1722de+3\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500} + \frac{e\left(840d^2-1722de+3\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500} + \frac{e\left(840d^2-1722de+3\right.}{3500}$$

$$= \frac{\left(2800d^3-17220d^2e+9921de^2+6053e^3\right)x}{17500} + \frac{e\left(840d^2-1722de+3\right.}{3500}$$

**Mathematica [A]** time = 0.16, size = 209, normalized size = 1.11

$$\frac{14700ex^2\left(300d^2-615de+103e^2\right) - \frac{42\left(125d^3(423x+1367)+75d^2e(5989x-1269)-15de^2(18323x+17967)+e^3(54969-53189x)\right)}{5x^2+2x+3} + 2940\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (42
*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x
) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825*d^3
 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 29
40*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^2])/18
375000
```

**fricas [B]** time = 0.87, size = 350, normalized size = 1.85

$$\frac{3675000\,e^3x^6 + 1225000\left(12\,de^2-7\,e^3\right)x^5 + 122500\left(180\,d^2e-321\,de^2+47\,e^3\right)x^4 + 147000\left(100\,d^3-555\,d^2e+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180*
d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2 +
 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e^3
+ 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14)*(9
8475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 317565
*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(157125*d^3 -
 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 4635*d^2*e
 - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)
*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*x^2 + 2*x
+ 3))/(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.16, size = 206, normalized size = 1.09

$$\frac{1}{25}\,x^4e^3+\frac{4}{25}\,dx^3e^2+\frac{6}{25}\,d^2x^2e+\frac{4}{25}\,d^3x-\frac{41}{375}\,x^3e^3-\frac{123}{250}\,dx^2e^2-\frac{123}{125}\,d^2xe+\frac{103}{1250}\,x^2e^3+\frac{309}{625}\,dxe^2+\frac{1}{1225000}\,\sqrt{14}\,(32$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/25*x^4*e^3 + 4/25*d*x^3*e^2 + 6/25*d^2*x^2*e + 4/25*d^3*x - 41/375*x^3*e^
3 - 123/250*d*x^2*e^2 - 123/125*d^2*x*e + 103/1250*x^2*e^3 + 309/625*d*x*e^
2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3
)*arctan(1/14*sqrt(14)*(5*x + 1)) + 867/3125*x*e^3 - 1/6250*(1025*d^3 - 154
5*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3
 - 95175*d^2*e + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*x -
269505*d*e^2 + 54969*e^3)/(5*x^2 + 2*x + 3)
```

**maple [A]** time = 0.02, size = 283, normalized size = 1.50

$$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6d^2e\,x^2}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4d^3x}{25}+\frac{1313\sqrt{14}\;d^3\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d^3\ln\left(5x^2+\right.}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*d^2*e*x^2-123/250*d*e^2*x^2
+103/1250*e^3*x^2+4/25*d^3*x-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3*x
-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+6835/
28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-41/250*
d^3*ln(5*x^2+2*x+3)+309/1250*d^2*e*ln(5*x^2+2*x+3)+2601/6250*d*e^2*ln(5*x^2
+2*x+3)-416/3125*e^3*ln(5*x^2+2*x+3)+1313/49000*14^(1/2)*d^3*arctan(1/28*(1
0*x+2)*14^(1/2))+63513/245000*14^(1/2)*d^2*e*arctan(1/28*(10*x+2)*14^(1/2))
-221643/1225000*14^(1/2)*d*e^2*arctan(1/28*(10*x+2)*14^(1/2))-67499/1225000
*14^(1/2)*e^3*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima [A]** time = 0.96, size = 212, normalized size = 1.12

$$\frac{1}{25}e^3x^4+\frac{1}{375}\left(60\,de^2-41\,e^3\right)x^3+\frac{1}{1250}\left(300\,d^2e-615\,de^2+103\,e^3\right)x^2+\frac{1}{1225000}\sqrt{14}\left(32825\,d^3+317565\,d^2e-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e^
2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*d*
e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 3075*d
^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d*e^2
 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e - 269
505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^
3)*x)/(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 0.15, size = 333, normalized size = 1.76

$$\frac{\frac{53901\,d\,e^2}{28}+\frac{19035\,d^2\,e}{28}+x\left(-\frac{10575\,d^3}{28}-\frac{89835\,d^2\,e}{28}+\frac{54969\,d\,e^2}{28}+\frac{53189\,e^3}{140}\right)-\frac{34175\,d^3}{28}-\frac{54969\,e^3}{140}}{15625\,x^2+6250\,x+9375}+x^3\left(\frac{e^2\,\left(12\,d-5\,e\right)}{75}-\frac{16\,e^3}{375}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] ((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)/
28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140)/
(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375) -
x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*e^2)
/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3)*((2
601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) - x^2*(
(2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)/625) +
```

```
(e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^2*e - 32
825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565*d^2*e -
32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^2*e)/17500
 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*d^2*e - 328
25*d^3 + 67499*e^3))/1225000
```

**sympy [C]** time = 2.77, size = 444, normalized size = 2.35

$$\frac{e^3x^4}{25}+x^3\left(\frac{4de^2}{25}-\frac{41e^3}{375}\right)+x^2\left(\frac{6d^2e}{25}-\frac{123de^2}{250}+\frac{103e^3}{1250}\right)+x\left(\frac{4d^3}{25}-\frac{123d^2e}{125}+\frac{309de^2}{625}+\frac{867e^3}{3125}\right)+\left(-\frac{41d^3}{250}+\frac{309d^2e}{1250}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*d
*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/625
 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 41
6*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6749
9*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 6749
9*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e
**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-41*d
**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt(14)*I*(
32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*log(x + (
6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(14)*I*(328
25*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*d**3 + 3175
65*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 95175*d**2*e + 2
69505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 274845*d*e**2
+ 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

## 3.312 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

**Optimal**. Leaf size=140

$$-\frac{(1025d^2-1030de-867e^2)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^2-11480de+3307e^2)}{17500}+\frac{(32825d^2+211710de-7388}{87500\sqrt{14}}$$

```
[Out] 1/17500*(2800*d^2-11480*d*e+3307*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x^
3-1/3500*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)-1/6250*(1025*d^2-1030*d*e-867
*e^2)*ln(5*x^2+2*x+3)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(1/1
4*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$-\frac{(1025d^2-1030de-867e^2)\log(5x^2+2x+3)}{6250}+\frac{x(2800d^2-11480de+3307e^2)}{17500}+\frac{(32825d^2+211710de-7388}{87500\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((2800*d^2 - 11480*d*e + 3307*e^2)*x)/17500 + ((40*d - 41*e)*e*x^2)/250 + (
4*e^2*x^3)/75 - ((1367 + 423*x)*(d + e*x)^2)/(3500*(3 + 2*x + 5*x^2)) + ((3
2825*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(87500*Sqrt[
14]) - ((1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{(d+ex)\left(\frac{2}{125}(1845d+2734e)-\frac{6}{12}\right.}{3}$$

$$= -\frac{(1367+423x)(d+ex)^2}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{2}{625}\left(2800d^2-11480de+3307e^2\right.\right.$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

$$= \frac{\left(2800d^2-11480de+3307e^2\right)x}{17500}+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75}-\frac{(13}{35}$$

**Mathematica [A]** time = 0.11, size = 150, normalized size = 1.07

$$\frac{-\frac{42\left(25d^2(423x+1367)+10de(5989x-1269)-e^2(18323x+17967)\right)}{5x^2+2x+3}+588\left(-1025d^2+1030de+867e^2\right)\log\left(5x^2+2x+3\right)+5880x}{36}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]
```

```
[Out] (5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000*
e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(17967
 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7388
1*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2)*Lo
g[3 + 2*x + 5*x^2])/3675000
```

**fricas [A]** time = 0.80, size = 245, normalized size = 1.75

$$\frac{980000\,e^2x^5+24500\left(120\,de-107\,e^2\right)x^4+58800\left(50\,d^2-185\,de+41\,e^2\right)x^3+2940\left(400\,d^2-1040\,de-203\,e^2\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2 -
 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt(1
4)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e - 2
21643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(14)*
(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 232090
*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 3075*d^2
 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*x^2 + 2
*x + 3))/(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.16, size = 145, normalized size = 1.04

$$\frac{4}{75}\,x^{3}e^{2}+\frac{4}{25}\,dx^{2}e+\frac{4}{25}\,d^{2}x-\frac{41}{250}\,x^{2}e^{2}-\frac{82}{125}\,dxe+\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^{2}+211710\,de-73881\,e^{2}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 4/75*x^3*e^2 + 4/25*d*x^2*e + 4/25*d^2*x - 41/250*x^2*e^2 - 82/125*d*x*e +
1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14
)*(5*x + 1)) + 103/625*x*e^2 - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5
*x^2 + 2*x + 3) - 1/87500*(34175*d^2 + (10575*d^2 + 59890*d*e - 18323*e^2)*
x - 12690*d*e - 17967*e^2)/(5*x^2 + 2*x + 3)
```

**maple [A]** time = 0.01, size = 189, normalized size = 1.35

$$\frac{4e^{2}x^{3}}{75}+\frac{4de\,x^{2}}{25}-\frac{41e^{2}x^{2}}{250}+\frac{4d^{2}x}{25}+\frac{1313\sqrt{14}\;d^{2}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d^{2}\ln\left(5x^{2}+2x+3\right)}{250}-\frac{82dex}{125}+\frac{21171\sqrt{14}\;de}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/75*e^2*x^3+4/25*d*e*x^2-41/250*e^2*x^2+4/25*d^2*x-82/125*d*e*x+103/625*e^
2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d*e
-17967/700*e^2)/(x^2+2/5*x+3/5)-41/250*d^2*ln(5*x^2+2*x+3)+103/625*d*e*ln(5
*x^2+2*x+3)+867/6250*e^2*ln(5*x^2+2*x+3)+1313/49000*14^(1/2)*d^2*arctan(1/2
8*(10*x+2)*14^(1/2))+21171/122500*14^(1/2)*d*e*arctan(1/28*(10*x+2)*14^(1/2
))-73881/1225000*14^(1/2)*e^2*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima [A]** time = 0.96, size = 147, normalized size = 1.05

$$\frac{4}{75}\,e^{2}x^{3}+\frac{1}{250}\left(40\,de-41\,e^{2}\right)x^{2}+\frac{1}{1225000}\,\sqrt{14}\left(32825\,d^{2}+211710\,de-73881\,e^{2}\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
+ 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^2
- 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^2 +
 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 59890
*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 0.11, size = 211, normalized size = 1.51

$$\ln\left(5\,x^{2}+2\,x+3\right)\left(-\frac{41\,d^{2}}{250}+\frac{103\,d\,e}{625}+\frac{867\,e^{2}}{6250}\right)-x\left(\frac{2\,d\,e}{5}+\frac{4\,e\,(8\,d-5\,e)}{125}-\frac{4\,d^{2}}{25}-\frac{3\,e^{2}}{625}\right)+x^{2}\left(\frac{e\,(8\,d-5\,e)}{50}-\frac{8\,e^{2}}{125}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*((
2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8*d
 - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d^2)
/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 3125*x^
2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 32825*
d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 73881*e^2
))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))*(21171
0*d*e + 32825*d^2 - 73881*e^2))/1225000
```

**sympy [C]** time = 1.96, size = 298, normalized size = 2.13

$$\frac{4e^{2}x^{3}}{75}+x^{2}\left(\frac{4de}{25}-\frac{41e^{2}}{250}\right)+x\left(\frac{4d^{2}}{25}-\frac{82de}{125}+\frac{103e^{2}}{625}\right)+\left(-\frac{41d^{2}}{250}+\frac{103de}{625}+\frac{867e^{2}}{6250}-\frac{\sqrt{14}\,i\left(32825d^{2}+211710de-73\right.}{2450000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
+ 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*I*
(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 42342*
d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/5)/(
32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 + 867*
e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(
x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 + 211710
*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-34175*d**
2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e**2))/(437
500*x**2 + 175000*x + 262500)
```

# 3.313 $\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2} dx$

**Optimal**. Leaf size=97

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+\frac{2e}{2}$$

```
[Out] 1/125*(20*d-41*e)*x+2/25*e*x^2-1/3500*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)-1/
1250*(205*d-103*e)*ln(5*x^2+2*x+3)+1/245000*(6565*d+21171*e)*arctan(1/14*(1
+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.19, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1657, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}-\frac{(205d-103e)\log\left(5x^2+2x+3\right)}{1250}+\frac{1}{125}x(20d-41e)+\frac{(6565d+21171e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{17500\sqrt{14}}+\frac{2e}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] ((20*d - 41*e)*x)/125 + (2*e*x^2)/25 - ((1367 + 423*x)*(d + e*x))/(3500*(3
+ 2*x + 5*x^2)) + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(17500*Sq
rt[14]) - ((205*d - 103*e)*Log[3 + 2*x + 5*x^2])/1250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{\frac{2}{125}(1845d+1367e)-\frac{168}{125}(55d-27e\ldots}{3+2x+\ldots}$$

$$= -\frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{1}{56}\int\left(\frac{56}{125}(20d-41e)+\frac{224ex}{25}+\frac{2(165d\ldots}{\ldots}\right.$$

$$= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{\int \frac{165d+4811e-28(20\ldots}{3+2x+5x\ldots}}{3500\ldots}$$

$$= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{(-205d+103e)\ldots}{1250\ldots}$$

$$= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} - \frac{(205d-103e)\log\ldots}{12\ldots}$$

$$= \frac{1}{125}(20d-41e)x + \frac{2ex^2}{25} - \frac{(1367+423x)(d+ex)}{3500\left(3+2x+5x^2\right)} + \frac{(6565d+21171e\ldots}{17500\ldots}$$

**Mathematica [A]** time = 0.07, size = 96, normalized size = 0.99

$$\frac{-\frac{14(5d(423x+1367)+e(5989x-1269))}{5x^2+2x+3}+196(103e-205d)\log\left(5x^2+2x+3\right)+1960x(20d-41e)+\sqrt{14}\,(6565d+21171e\ldots}{245000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x
]
```

```
[Out] (1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x)/
Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

**fricas [A]** time = 0.82, size = 147, normalized size = 1.52

$$\frac{98000\,ex^4+9800\,(20\,d-37\,e)x^3+7840\,(10\,d-13\,e)x^2+\sqrt{14}\left(5\,(6565\,d+21171\,e)x^2+2\,(6565\,d+21171\,e)x+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] 1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 + s
qrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 6351
3*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*(20
5*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x + 3
) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

**giac** [A] time = 0.16, size = 94, normalized size = 0.97

$$\frac{2}{25}\,x^2e+\frac{1}{245000}\,\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{4}{25}\,dx-\frac{41}{125}\,xe-\frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 2/25*x^2*e + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 4/25*d*x - 41/125*x*e - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

**maple** [A] time = 0.01, size = 106, normalized size = 1.09

$$\frac{2e\,x^2}{25}+\frac{4dx}{25}+\frac{1313\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41d\ln\left(5x^2+2x+3\right)}{250}-\frac{41ex}{125}+\frac{21171\sqrt{14}\,e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{245000}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-1
269/700*e)/(x^2+2/5*x+3/5)-41/250*d*ln(5*x^2+2*x+3)+103/1250*e*ln(5*x^2+2*x
+3)+1313/49000*14^(1/2)*d*arctan(1/28*(10*x+2)*14^(1/2))+21171/245000*14^(1
/2)*e*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima** [A] time = 0.96, size = 90, normalized size = 0.93

$$\frac{2}{25}\,ex^2+\frac{1}{245000}\,\sqrt{14}\,(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{125}\,(20\,d-41\,e)x-\frac{1}{1250}\,(205\,d-103\,e)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x + 3
) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 4.15, size = 115, normalized size = 1.19

$$\frac{2\,e\,x^2}{25}-\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right)+x\,\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right)-\frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\,\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375}+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(6565\,}{245\mathrm{0}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)/
25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)/1
40))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171*e)
)/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (21171*e
)/17500))*(6565*d + 21171*e))/245000
```

**sympy [C]** time = 1.02, size = 165, normalized size = 1.70

$$\frac{2ex^2}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x\,(-2115d-5989e)}{87500x^2+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}\,i\,(6565d+21171e)}{490000}\right)\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 5989
*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*I*
(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(6565*d
 + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)*I*(
6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(6565*d
+ 21171*e)/5)/(6565*d + 21171*e))
```

# 3.314 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx$

**Optimal**. Leaf size=63

$$-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log(5x^2+2x+3)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

```
[Out] 4/25*x+1/3500*(-1367-423*x)/(5*x^2+2*x+3)-41/250*ln(5*x^2+2*x+3)+1313/49000
*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1660, 1657, 634, 618, 204, 628}

$$-\frac{423x+1367}{3500(5x^2+2x+3)}-\frac{41}{250}\log(5x^2+2x+3)+\frac{4x}{25}+\frac{1313\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{3500\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 - (1367 + 423*x)/(3500*(3 + 2*x + 5*x^2)) + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(3500*Sqrt[14]) - (41*Log[3 + 2*x + 5*x^2])/250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{\frac{738}{25}-\frac{1848x}{25}+\frac{224x^2}{5}}{3+2x+5x^2}\,dx$$

$$= -\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{56}\int \left(\frac{224}{25}+\frac{2(33-1148x)}{25\left(3+2x+5x^2\right)}\right)dx$$

$$= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1}{700}\int \frac{33-1148x}{3+2x+5x^2}\,dx$$

$$= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\int \frac{2+10x}{3+2x+5x^2}\,dx+\frac{1313\int \frac{1}{3+2x+5x^2}\,dx}{3500}$$

$$= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}-\frac{41}{250}\log\left(3+2x+5x^2\right)-\frac{1313\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,d\right.}{1750}$$

$$= \frac{4x}{25}-\frac{1367+423x}{3500\left(3+2x+5x^2\right)}+\frac{1313\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}}-\frac{41}{250}\log\left(3+2x+5x^2\right)$$

**Mathematica [A]** time = 0.04, size = 59, normalized size = 0.94

$$\frac{-\frac{14(423x+1367)}{5x^2+2x+3}-8036\log\left(5x^2+2x+3\right)+7840x+1313\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{49000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1 +
 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

**fricas [A]** time = 0.77, size = 78, normalized size = 1.24

$$\frac{39200\,x^3+1313\,\sqrt{14}\left(5\,x^2+2\,x+3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+15680\,x^2-8036\left(5\,x^2+2\,x+3\right)\log\left(5\,x^2+2\right.}{49000\left(5\,x^2+2\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 17598
*x - 19138)/(5*x^2 + 2*x + 3)
```

**giac** [A] time = 0.15, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

**maple** [A] time = 0.01, size = 51, normalized size = 0.81

$$\frac{4x}{25}+\frac{1313\sqrt{14}\;\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}-\frac{41\ln\left(5x^2+2x+3\right)}{250}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] 4/25*x-1/25*(423/700*x+1367/700)/(x^2+2/5*x+3/5)-41/250*ln(5*x^2+2*x+3)+131
3/49000*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima** [A] time = 0.95, size = 52, normalized size = 0.83

$$\frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x-\frac{423x+1367}{3500\left(5x^2+2x+3\right)}-\frac{41}{250}\log\left(5x^2+2x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

```
[Out] 1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423*
x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

**mupad** [B] time = 4.15, size = 52, normalized size = 0.83

$$\frac{4x}{25}-\frac{41\ln\left(5x^2+2x+3\right)}{250}-\frac{\frac{423x}{17500}+\frac{1367}{17500}}{x^2+\frac{2x}{5}+\frac{3}{5}}+\frac{1313\sqrt{14}\;\mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] (4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((2
*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/
49000
```

**sympy** **[A]** time = 0.19, size = 65, normalized size = 1.03

$$\frac{4x}{25} + \frac{-423x - 1367}{17500x^2 + 7000x + 10500} - \frac{41\log\left(x^2 + \frac{2x}{5} + \frac{3}{5}\right)}{250} + \frac{1313\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14} + \frac{\sqrt{14}}{14}\right)}{49000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] 4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x/
5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

## 3.315 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx$

**Optimal**. Leaf size=224

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(6565d^3-26423d^2\right.}{700\sqrt{}}$$

```
[Out] 1/700*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+(4
*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/50*
(205*d^3-61*d^2*e+23*d*e^2+14*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2+1/
9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)
```

**Rubi [A]** time = 0.34, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}-\frac{\left(-61d^2e+205d^3+23de^2+14e^3\right)\log\left(5x^2+2x+3\right)}{50\left(5d^2-2de+3e^2\right)^2}+\frac{\left(3d^2e^2+5d^3e+4\right.}{e\left(5d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*
x + 5*x^2)) + ((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(700*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((4*d^4 + 5*d^
3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e + 3*e^2)^2
) - ((205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*x^2])/(50*(5*
d^2 - 2*d*e + 3*e^2)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{1}{56}\int \frac{\frac{2\left(369d^2-421de+280e^2\right)}{5\left(5d^2-2de+3e^2\right)} - \frac{2\left(924d^2-285\right.}{5\left(5d^2-2\right.}}{(d+ex)\left(3+2x+\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{1}{56}\int \left(\frac{56\left(4d^4+5d^3e+3d^2e^2-de^3\right.}{\left(5d^2-2de+3e^2\right)^2(d+ex}\right.$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(}{e\left(5d^2-2de+3e^2\right)^2}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\left(6565d^3-26423d^2e+11089de^2-66\right.}{700\sqrt{14}\left(5d^2-2de+3\right.}$$

**Mathematica [A]** time = 0.16, size = 186, normalized size = 0.83

$$\frac{\frac{14\left(5d^2-2de+3e^2\right)(e(1367x+293)-d(423x+1367))}{5x^2+2x+3} - 196\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(5x^2+2x+3\right) + \sqrt{14}\left(6565d^3-\right.}{9800\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x
]
```

```
[Out] ((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3 +
 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*
ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2
*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3
+ 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

**fricas [B]** time = 0.94, size = 479, normalized size = 2.14

$$-\frac{95690\,d^3e-58786\,d^2e^2+65618\,de^3-12306\,e^4-\sqrt{14}\left(19695\,d^3e-79269\,d^2e^2+33267\,de^3-19869\,e^4+5\left(656\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fri
cas")
```

```
[Out] -1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*(
19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 264
23*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2 +
11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d^3*e
 - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e + 9*d^
2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x
^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d) + 196*
(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^2*e^2 +
23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14*e^4)*x)*
log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d*e^4 + 27*
e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x^2 + 2*(25
*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

**giac [A]** time = 0.17, size = 284, normalized size = 1.27

$$\frac{\sqrt{14}\left(6565\,d^{3}-26423\,d^{2}e+11089\,de^{2}-6623\,e^{3}\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{9800\left(25\,d^{4}-20\,d^{3}e+34\,d^{2}e^{2}-12\,de^{3}+9\,e^{4}\right)}-\frac{\left(205\,d^{3}-61\,d^{2}e+23\,de^{2}+14\,e^{3}\right)}{50\left(25\,d^{4}-20\,d^{3}e+34\,d^{2}e^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gia
c")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
- 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^
4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4)*log(abs(x*e + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 -
12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + (2115*d^3 - 7681*d^2*e +
 4003*d*e^2 - 4101*e^3)*x + 4687*d*e^2 - 879*e^3)/((5*d^2 - 2*d*e + 3*e^2)^
2*(5*x^2 + 2*x + 3))
```

**maple [B]** time = 0.02, size = 691, normalized size = 3.08

$$\frac{4d^{4}\ln\left(ex+d\right)}{\left(5d^{2}-2de+3e^{2}\right)^{2}e}-\frac{423d^{3}x}{700\left(5d^{2}-2de+3e^{2}\right)^{2}\left(x^{2}+\frac{2}{5}x+\frac{3}{5}\right)}+\frac{1313\sqrt{14}\;d^{3}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{1960\left(5d^{2}-2de+3e^{2}\right)^{2}}+\frac{5d^{3}\ln\left(ex+d\right.}{\left(5d^{2}-2de+3e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x)
```

```
[Out] -423/700/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3*x+7681/3500/(5*d^2-2*d*e
+3*e^2)^2/(x^2+2/5*x+3/5)*x*d^2*e-4003/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*
x+3/5)*x*d*e^2+4101/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*x*e^3-1367/7
00/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d^3+4199/3500/(5*d^2-2*d*e+3*e^2)^
2/(x^2+2/5*x+3/5)*d^2*e-4687/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*d*e
^2+879/3500/(5*d^2-2*d*e+3*e^2)^2/(x^2+2/5*x+3/5)*e^3-41/10/(5*d^2-2*d*e+3*
e^2)^2*ln(5*x^2+2*x+3)*d^3+61/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d^2*
e-23/50/(5*d^2-2*d*e+3*e^2)^2*ln(5*x^2+2*x+3)*d*e^2-7/25/(5*d^2-2*d*e+3*e^2
)^2*ln(5*x^2+2*x+3)*e^3+1313/1960/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1/2
8*(10*x+2)*14^(1/2))*d^3-26423/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arctan(1
/28*(10*x+2)*14^(1/2))*d^2*e+11089/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))*d*e^2-6623/9800/(5*d^2-2*d*e+3*e^2)^2*14^(1/2)*a
rctan(1/28*(10*x+2)*14^(1/2))*e^3+4/(5*d^2-2*d*e+3*e^2)^2/e*ln(e*x+d)*d^4+5
/(5*d^2-2*d*e+3*e^2)^2*ln(e*x+d)*d^3+3/(5*d^2-2*d*e+3*e^2)^2*e*ln(e*x+d)*d^
2-1/(5*d^2-2*d*e+3*e^2)^2*e^2*ln(e*x+d)*d+2/(5*d^2-2*d*e+3*e^2)^2*e^3*ln(e*
x+d)
```

**maxima [A]** time = 0.98, size = 289, normalized size = 1.29

$$\frac{\sqrt{14}\left(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{9800\left(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4\right)}+\frac{\left(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4\right)l}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="max
ima")
```

```
[Out] 1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1/
14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
+ (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e - 20
*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e + 23*d
*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*
e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2 - 2*d*
e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*x)
```

**mupad [B]** time = 4.61, size = 330, normalized size = 1.47

$$\frac{\ln\left(d+e\,x\right)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}+\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}\mathrm{i}\right)\,d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}\mathrm{i}\right)\,d}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3*
((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50) -
 d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
 + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^2))
+ (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3) - (
log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) - e^3*(
(6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i/50) -
d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*12i
+ e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 + 2*e^4 +
3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

# 3.316 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$

**Optimal**. Leaf size=313

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(41d^4-8d^3e-60d^2e^2+24de^3-5e^4\right)\log\left(5x^2+2x+\right.}{2\left(5d^2-2de+3e^2\right)^3}$$

```
[Out] (-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+1/14
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)+(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5
*d^2-2*d*e+3*e^2)^3-1/2*(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5*x^2
+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2+156*
d*e^3-271*e^4)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

**Rubi [A]** time = 0.50, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{140\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}-\frac{\left(-60d^2e^2-8d^3e+41d^4+24de^3-5e^4\right)\log\left(5x^2+2x\right.}{2\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -((4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e*(5*d^2 - 2*d*e + 3*e^2)^
2*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e
^2)*x)/(140*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((1313*d^4 - 100
44*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(28*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + ((41*d^4 - 8*d^3*e - 60*d^2*e^2 +
 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - ((41*d^4 - 8*d
^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d
*e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int \frac{2\left(369d^4-842d^3 \right.}{\left(5d\right.}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}+\frac{1}{56}\int\left(\frac{56\left(4d^4+5\right.}{\left(5d^2-\right.}\right.$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734d\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734d\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734d\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5\right.}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e\left(5d^2-2de+3e^2\right)^2(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2734d\right.}{140\left(5d^2-2de+3e^2\right)^2\left(3+2x+5\right.}$$

**Mathematica [A]** time = 0.26, size = 270, normalized size = 0.86

$$\frac{-\frac{14\left(5d^2-2de+3e^2\right)\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{5x^2+2x+3}+980\left(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4\right)\log\left(5x^2+2x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^
4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2*(
1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/S
qrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d +
e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3 + 2*x
+ 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

**fricas [B]** time = 1.12, size = 910, normalized size = 2.91

$$-\frac{117600\,d^6+195650\,d^5e+20664\,d^4e^2+48132\,d^3e^3+118552\,d^2e^4-70686\,de^5+35280\,e^6+14\left(14000\,d^6+11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 118552
*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*d^
4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*sqr
t(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d*e^5
 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*x^
3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*d*e^5
- 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182*d^2*e^
4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(5600*d^6 +
 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*e^5 - 429
*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^
5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d
^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*
d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(e*x + d) +
980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d*e^5 + 5*(41*d
^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (205*d^5*e + 42*d
^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e + 107*d^4*e^2 - 1
44*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2 + 2*x + 3))/(375
*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^3*e^5 - 162*d^2*e^
6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^4 - 188*d^3*e^5 + 1
71*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 500*d^6*e^2 + 1125*d^5*e
^3 - 370*d^4*e^4 + 479*d^3*e^5 + 72*d^2*e^6 + 27*d*e^7 + 54*e^8)*x^2 + (250
*d^7*e + 75*d^6*e^2 + 120*d^5*e^3 + 479*d^4*e^4 - 222*d^3*e^5 + 405*d^2*e^6
 - 108*d*e^7 + 81*e^8)*x)
```

**giac [A]** time = 0.20, size = 571, normalized size = 1.82

$$\frac{\sqrt{14}\left(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\left(5\,d-\frac{5\,d^2}{xe+d}+\frac{2\,de}{xe+d}-\frac{3\,e^2}{xe+d}-e\right)e^{(-1)}\right)}{392\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 2
71*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e
^2/(x*e + d) - e)*e^(-1))*e^(-2)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d
^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(x*e + d) + 5*d^2/(x*e + d)^2 + 2*e/(x*e
+ d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(125*d^6 - 150*d^5*e + 28
5*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - (4*d^4*e^3/(x*
e + d) + 5*d^3*e^4/(x*e + d) + 3*d^2*e^5/(x*e + d) - d*e^6/(x*e + d) + 2*e^
7/(x*e + d))/(25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/
28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e + 3*e^2
) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6)*e^(-
1)/((5*d^2 - 2*d*e + 3*e^2)*(x*e + d)))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/(x
*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e + d)^2 - 3*e^2/(x*
e + d)^2 - 5))
```

**maple [B]** time = 0.02, size = 986, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x)
```

```
[Out] 1/(5*d^2-2*d*e+3*e^2)^2*e^2/(e*x+d)*d-8/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3
*e-60/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^2*e^2+24/(5*d^2-2*d*e+3*e^2)^3*ln(e
*x+d)*d*e^3-423/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4*x-879/700/(5*
d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*e^4+1416/175/(5*d^2-2*d*e+3*e^2)^3/(x^
2+2/5*x+3/5)*d^3*e-879/350/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^2*e^2+88
/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d*e^3+4/(5*d^2-2*d*e+3*e^2)^3*ln
(5*x^2+2*x+3)*d^3*e+30/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e^2-12/(5*
d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^3+1313/392/(5*d^2-2*d*e+3*e^2)^3*14^
(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4-271/392/(5*d^2-2*d*e+3*e^2)^3*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^4-4/(5*d^2-2*d*e+3*e^2)^2/e/(e*x+d)*d
^4-3/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)*d^2-5/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)*d^
3-1367/140/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*d^4+2109/700/(5*d^2-2*d*e+
3*e^2)^3/(x^2+2/5*x+3/5)*e^4-41/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^4
+5/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*e^4-2/(5*d^2-2*d*e+3*e^2)^2*e^3/
(e*x+d)+41/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^4-5/(5*d^2-2*d*e+3*e^2)^3*ln(e
*x+d)*e^4+3629/175/(5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*d^3*e-4101/350/(
5*d^2-2*d*e+3*e^2)^3/(x^2+2/5*x+3/5)*x*d^2*e^2+2197/175/(5*d^2-2*d*e+3*e^2)
^3/(x^2+2/5*x+3/5)*x*d*e^3-2511/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))*d^3*e+2145/196/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(
1/28*(10*x+2)*14^(1/2))*d^2*e^2+39/98/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan
(1/28*(10*x+2)*14^(1/2))*d*e^3
```

**maxima [A]** time = 1.02, size = 548, normalized size = 1.75

$$\frac{\sqrt{14}\left(1313\,d^4 - 10044\,d^3e + 4290\,d^2e^2 + 156\,de^3 - 271\,e^4\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\left(125\,d^6 - 150\,d^5e + 285\,d^4e^2 - 188\,d^3e^3 + 171\,d^2e^4 - 54\,de^5 + 27\,e^6\right)} + \frac{\left(41\,d^4 - 8\,d^3e - 60\right.}{125\,d^6 - 150\,d^5e + 285\,d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4
)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*e^2
 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*
d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 60*d^2
*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^
4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680*d^4 +
3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*d^3*e +
2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e - 527*d^
2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*e^3 - 36*d
^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*d*e^5 + 9*
e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5 + 1
8*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 - 18*d*e^5 + 2
7*e^6)*x)
```

**mupad [B]** time = 4.84, size = 601, normalized size = 1.92

$$\ln(d+e\,x)\left(\frac{41}{25\,\left(5\,d^2-2\,d\,e+3\,e^2\right)} - \frac{4\,e^3\,\left(423\,d-1367\,e\right)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^3} + \frac{2\,e\,\left(310\,d-1323\,e\right)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}\right) - \frac{\frac{1680\,d^4+3467\,d^3\,e+674}{140\,e\left(25\,d^4-20\,d^3\,e+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(1
25*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*e
+ 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*e^2
)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573*d*e
^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 - 20*d^
3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3 + 280
0*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*
e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e)) + (lo
g(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) - e^4*((271
```

```
*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*e^3*((39*1
4^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*125i - d^5*e*
150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) -
(log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41i/2) - e^4*((
271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30i) + d*e^3*((3
9*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/(d^6*125i - d^5
*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

## 3.317 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$

**Optimal**. Leaf size=412

$$-\frac{1367d^3-879d^2e+x\left(423d^3-4101d^2e+879de^2+703e^3\right)-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{41d^4-8d^3e-60d^2e^2+24de^3}{\left(5d^2-2de+3e^2\right)^3(d+ex}$$

```
[Out] 1/2*(-4*d^4-5*d^3*e-3*d^2*e^2+d*e^3-2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^
2+(-41*d^4+8*d^3*e+60*d^2*e^2-24*d*e^3+5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)
+1/28*(-1367*d^3+879*d^2*e+2109*d*e^2-457*e^3-(423*d^3-4101*d^2*e+879*d*e^2
+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+(205*d^5-19*d^4*e-846*d^3*
e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(205*d
^5-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/(5*d^2
-2*d*e+3*e^2)^4+1/392*(6565*d^5-74017*d^4*e+35022*d^3*e^2+42858*d^2*e^3-172
47*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^4*14^(1
/2)
```

**Rubi** [A] time = 0.71, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{x\left(-4101d^2e+423d^3+879de^2+703e^3\right)-879d^2e+1367d^3-2109de^2+457e^3}{28\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}-\frac{\left(-846d^3e^2+396d^2e^3-19d^4\right.}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

```
[Out] -(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(2*e*(5*d^2 - 2*d*e + 3*e^2)
^2*(d + e*x)^2) - (41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)/((5*d^
2 - 2*d*e + 3*e^2)^3*(d + e*x)) - (1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*
e^3 + (423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*x)/(28*(5*d^2 - 2*d*e +
3*e^2)^3*(3 + 2*x + 5*x^2)) + ((6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42
858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(28*Sqrt[1
4]*(5*d^2 - 2*d*e + 3*e^2)^4) + ((205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^
2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - ((205*
d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[3 + 2*x
 + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx = -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{1367d^3-879d^2e-2109de^2+457e^3+\left(423d^3-4101d^2e+879de^2+703e^3\right)}{28\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{13}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{13}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{13}{}$$

$$= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e\left(5d^2-2de+3e^2\right)^2(d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{\left(5d^2-2de+3e^2\right)^3(d+ex)} - \frac{13}{}$$

**Mathematica [A]** time = 0.40, size = 363, normalized size = 0.88

$$\frac{-\frac{14\left(5d^2-2de+3e^2\right)\left(d^3(423x+1367)-3d^2e(1367x+293)+3de^2(293x-703)+e^3(703x+457)\right)}{5x^2+2x+3} - \frac{196\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\left(5d^2-2de+3e^2\right)^2}{e(d+ex)^2} + \frac{39}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2)
,x]
```

```
[Out] ((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e + 60
*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3*d*
e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(293
+ 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 35022*d^
3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*L
og[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3 - 57*d*e
^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

**fricas [B]** time = 1.46, size = 1499, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] -1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 395164
*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14*(
28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e
^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 -
 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7
 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^5*e^3
+ 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 - 74017*d
^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4 + 2*(32
825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 43377*d^2*
e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2 - 101263*d
^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425*d*e^7 + 17
37*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 + 147924*d^4*e
^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/14*sqrt(14)*(
5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 172
02*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 392*
(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^
2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7
- 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5
 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*
d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^
2 + 2*(205*d^7*e + 596*d^6*e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5
+ 150*d^2*e^6 - 63*d*e^7)*x)*log(e*x + d) + 196*(615*d^7*e - 57*d^6*e^2 - 2
538*d^5*e^3 + 1188*d^4*e^4 + 171*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19
*d^4*e^4 - 846*d^3*e^5 + 396*d^2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6
*e^2 + 110*d^5*e^3 - 4249*d^4*e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 -
 21*e^8)*x^3 + (1025*d^7*e + 725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 66
9*d^3*e^5 + 1311*d^2*e^6 + 87*d*e^7 - 63*e^8)*x^2 + 2*(205*d^7*e + 596*d^6*
e^2 - 903*d^5*e^3 - 2142*d^4*e^4 + 1245*d^3*e^5 + 150*d^2*e^6 - 63*d*e^7)*x
)*log(5*x^2 + 2*x + 3))/(1875*d^10*e - 3000*d^9*e^2 + 6300*d^8*e^3 - 5880*d
^7*e^4 + 6258*d^6*e^5 - 3528*d^5*e^6 + 2268*d^4*e^7 - 648*d^3*e^8 + 243*d^2
*e^9 + 5*(625*d^8*e^3 - 1000*d^7*e^4 + 2100*d^6*e^5 - 1960*d^5*e^6 + 2086*d
^4*e^7 - 1176*d^3*e^8 + 756*d^2*e^9 - 216*d*e^10 + 81*e^11)*x^4 + 2*(3125*d
^9*e^2 - 4375*d^8*e^3 + 9500*d^7*e^4 - 7700*d^6*e^5 + 8470*d^5*e^6 - 3794*d
^4*e^7 + 2604*d^3*e^8 - 324*d^2*e^9 + 189*d*e^10 + 81*e^11)*x^3 + (3125*d^1
0*e - 2500*d^9*e^2 + 8375*d^8*e^3 - 4400*d^7*e^4 + 8890*d^6*e^5 - 3416*d^5*
e^6 + 5334*d^4*e^7 - 1584*d^3*e^8 + 1809*d^2*e^9 - 324*d*e^10 + 243*e^11)*x
```

```
^2 + 2*(625*d^10*e + 875*d^9*e^2 - 900*d^8*e^3 + 4340*d^7*e^4 - 3794*d^6*e^
5 + 5082*d^5*e^6 - 2772*d^4*e^7 + 2052*d^3*e^8 - 567*d^2*e^9 + 243*d*e^10)*
x)
```

**giac [A]** time = 0.20, size = 595, normalized size = 1.44

$$\frac{\sqrt{14}\left(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)}{392\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}-\frac{}{2\left(625\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*
e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
+ 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57*d*e^5
- 21*e^6)*log(abs(x*e + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^3 - 1960
*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 + 81*e^9)
- 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 + 28226*d^4*
e^4 + (28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 4596
6*d^2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 - 31223*d^3*e^5 + (7000*d^8 + 31850*d^
7*e + 6400*d^6*e^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*
d^2*e^6 - 2841*d*e^7 + 1791*e^8)*x^2 + 12753*d^2*e^6 + (2800*d^8 + 14855*d^
7*e + 5815*d^6*e^2 - 18620*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*
d^2*e^6 + 7866*d*e^7 - 756*e^8)*x - 2646*d*e^7 + 756*e^8)*e^(-1)/((5*d^2 -
2*d*e + 3*e^2)^4*(5*x^2 + 2*x + 3)*(x*e + d)^2)
```

**maple [B]** time = 0.03, size = 1314, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x)
```

```
[Out] -3/2/(5*d^2-2*d*e+3*e^2)^2*e/(e*x+d)^2*d^2+1/2/(5*d^2-2*d*e+3*e^2)^2*e^2/(e
*x+d)^2*d+8/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^3*e+60/(5*d^2-2*d*e+3*e^2)^3/(e
*x+d)*d^2*e^2-24/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d*e^3-19/(5*d^2-2*d*e+3*e^2)
^4*ln(e*x+d)*d^4*e-846/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^3*e^2+396/(5*d^2-2
*d*e+3*e^2)^4*ln(e*x+d)*d^2*e^3+57/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d*e^4-42
```

```
3/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^5-2109/140/(5*d^2-2*d*e+3*e^
2)^4/(x^2+2/5*x+3/5)*x*e^5+7129/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d
^4*e+2343/70/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^3*e^2-1933/70/(5*d^2-2
*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^2*e^3+7241/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2
/5*x+3/5)*d*e^4+19/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^4*e+423/(5*d^2
-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^3*e^2-198/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+
2*x+3)*d^2*e^3-57/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d*e^4+6565/392/(5
*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5+579/392/(5*
d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5-2/(5*d^2-2*d
*e+3*e^2)^2/e/(e*x+d)^2*d^4-5/2/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^2*d^3+205/(5*
d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^5-21/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*e^5-136
7/28/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*d^5-1371/140/(5*d^2-2*d*e+3*e^2)
^4/(x^2+2/5*x+3/5)*e^5-205/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^5+21/2
/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*e^5-1/(5*d^2-2*d*e+3*e^2)^2*e^3/(e*x
+d)^2-41/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4+5/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*
e^4+21429/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
*d^2*e^3-17247/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(
1/2))*d*e^4+21351/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^4*e-6933/70
/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5)*x*d^3*e^2+5273/70/(5*d^2-2*d*e+3*e^2
)^4/(x^2+2/5*x+3/5)*x*d^2*e^3-1231/140/(5*d^2-2*d*e+3*e^2)^4/(x^2+2/5*x+3/5
)*x*d*e^4-74017/392/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^
(1/2))*d^4*e+17511/196/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*
14^(1/2))*d^3*e^2
```

**maxima [B]** time = 1.09, size = 851, normalized size = 2.07

$$\frac{\sqrt{14}\left(6565\,d^5-74017\,d^4e+35022\,d^3e^2+42858\,d^2e^3-17247\,de^4+579\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{392\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right)}+\frac{\cdots}{625\,d^8-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] 1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 17
247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*e
+ 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57
*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d
^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) -
1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*lo
g(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 20
86*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/28*(840*d
^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714*d*e^5 + 2
52*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^5 + 3*e^6)*
```

```
x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2349*d^2*e^4
- 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*e^2 - 4799*d^
3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e - 450*d^7*e^2 +
855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d^2*e^7 + 5*(125
*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8
 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e^4 - 655*d^4*e^5
+ 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (625*d^8*e - 250*d^7*
e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3*e^6 + 432*d^2*e^7
- 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^2 - 165*d^6*e^3 + 667*d
^5*e^4 - 393*d^4*e^5 + 459*d^3*e^6 - 135*d^2*e^7 + 81*d*e^8)*x)
```

**mupad** [B] time = 4.94, size = 887, normalized size = 2.15

$$\ln(d+e\,x)\left(\frac{\frac{41\,d}{5}+\frac{29\,e}{5}}{\left(5\,d^2-2\,d\,e+3\,e^2\right)^2}+\frac{168\,e^4\,(458\,d-7\,e)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^4}-\frac{2\,e^2\,(12610\,d+1329\,e)}{125\left(5\,d^2-2\,d\,e+3\,e^2\right)^3}\right)-\frac{\frac{840\,d^6+5525\,d^5\,e-837\,d^4\,e^2}{28\,e\left(125\,d^6-150\,d^5\,e+285\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

```
[Out] log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(4
58*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 252*
e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d^5*e
 - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(42
39*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(125*d^6
- 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2))
 + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4 - 15403
*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 +
171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195*d^5*e + 5
60*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/(28*e*(125*
d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e
^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2 + x^3*(10*d*
e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1
/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*e^2*((17511*14^
(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i) - d*e^4*((17247*
14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i/2)))/(d^8*625i -
 d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^
4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*
(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(1/2))/784 - 21i/2) + d^
3*e^2*((17511*14^(1/2))/392 - 423i) + d^2*e^3*((21429*14^(1/2))/392 + 198i)
 - d*e^4*((17247*14^(1/2))/784 - 57i/2) - d^4*e*((74017*14^(1/2))/784 + 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e
^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
[Out] Timed out
```

## 3.318 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=171

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$

```
[Out] 1/980000*(83065*d-126009*e)*e^2*x+2/125*e^3*x^2-1/7000*(1367+423*x)*(e*x+d)
^3/(5*x^2+2*x+3)^2+1/196000*(e*x+d)^2*(34347*d-6315*e+(11015*d+49177*e)*x)/
(5*x^2+2*x+3)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(5*x^2+2*x+3)+3/68600000*
(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*arctan(1/14*(1+5*x)*14^(1/
2))*14^(1/2)
```

**Rubi [A]** time = 0.34, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1628, 634, 618, 204, 628}

$$\frac{3e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)}{6250}+\frac{3\left(-855175d^2e+353125d^3+74085de^2+556349e^3\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((83065*d - 126009*e)*e^2*x)/980000 + (2*e^3*x^2)/125 - ((1367 + 423*x)*(d
+ e*x)^3)/(7000*(3 + 2*x + 5*x^2)^2) + ((d + e*x)^2*(3*(11449*d - 2105*e) +
 (11015*d + 49177*e)*x))/(196000*(3 + 2*x + 5*x^2)) + (3*(353125*d^3 - 8551
75*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(4900000*S
qrt[14]) + (3*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/6250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{(d+ex)^2\left(\frac{6}{125}(1089d+1367e)\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)^2(3(11449d-2105e)+(11015}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)^2(3(11449d-2105e)+(11015}{196000\left(3+2x+5x^2\right)}$$

$$= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+}{}$$

$$= \frac{(83065d-126009e)e^2x}{980000} + \frac{2e^3x^2}{125} - \frac{(1367+423x)(d+ex)^3}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+}{}$$

**Mathematica [A]** time = 0.20, size = 209, normalized size = 1.22

$$\frac{164640e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right) - \frac{392\left(125d^3(423x+1367)+75d^2e(5989x-1269)-15de^2(18323x+17967)+e^3(54\right.}{\left(5x^2+2x+3\right)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]

[Out] (548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x)
 + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 + 1
8323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3*(3
4347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647195*
```

```
x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245*d*e +
 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

**fricas [B]** time = 0.84, size = 441, normalized size = 2.58

$$\frac{27440000\,e^3x^6 + 2744000\left(60\,de^2 - 41\,e^3\right)x^5 + 8780800\left(15\,de^2 - 8\,e^3\right)x^4 + 70\left(275375\,d^3 + 2726475\,d^2e + 1257\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(1
5*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 304
5929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 17863398*e
^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^2 +
3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^4 +
 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178125*d^
3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 855175*d^
2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e + 74085
*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(749125*d^3 +
1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d^2*e - 245
*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 + 900*d^2*e
- 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^2 + 12*(100*
d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9)
```

**giac [A]** time = 0.21, size = 201, normalized size = 1.18

$$\frac{2}{125}\,x^2e^3+\frac{12}{125}\,dxe^2+\frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3 - 855175\,d^2e + 74085\,de^2 + 556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 2/125*x^2*e^3 + 12/125*d*x*e^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d
^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 49/625*x
*e^3 + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/490
0000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 161
9125*d^3 + (4844125*d^3 + 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2
```

```
 - 1464975*d^2*e + 3*(749125*d^3 + 1444025*d^2*e - 3046875*d*e^2 - 170563*e
^3)*x - 5773275*d*e^2 + 1275957*e^3)/(5*x^2 + 2*x + 3)^2
```

**maple [A]** time = 0.02, size = 267, normalized size = 1.56

$$\frac{2e^3x^2}{125}+\frac{339\sqrt{14}\, d^3 \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}-\frac{102621\sqrt{14}\, d^2e \arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{2744000}+\frac{6d^2e\ln\left(5x^2+2x+3\right)}{125}+\frac{12d\,e^2x}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/1568
*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840*d
^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/7840*
d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d^2*e
-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+6/125*d^2*e*ln(5*x^2
+2*x+3)-147/1250*d*e^2*ln(5*x^2+2*x+3)+141/6250*e^3*ln(5*x^2+2*x+3)+339/219
52*14^(1/2)*d^3*arctan(1/28*(10*x+2)*14^(1/2))-102621/2744000*14^(1/2)*d^2*
e*arctan(1/28*(10*x+2)*14^(1/2))+44451/13720000*14^(1/2)*d*e^2*arctan(1/28*
(10*x+2)*14^(1/2))+1669047/68600000*14^(1/2)*e^3*arctan(1/28*(10*x+2)*14^(1
/2))
```

**maxima [A]** time = 0.97, size = 222, normalized size = 1.30

$$\frac{2}{125}\,e^3x^2+\frac{3}{68600000}\,\sqrt{14}\left(353125\,d^3-855175\,d^2e+74085\,de^2+556349\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{625}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e^
2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^3)
*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49000
00*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 16191
25*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 + 21230
25*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444025*d^2*
e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

**mupad [B]** time = 0.15, size = 299, normalized size = 1.75

$$x\left(\frac{e^2\,\left(12\,d-5\,e\right)}{125}-\frac{24\,e^3}{625}\right)-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\,\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}}{15625\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (292995
*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^3)
/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^3*(
(1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (621801*e
^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*d^3)/
1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x^4 + 5
625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141*e^3)/6
250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2 - 85517
5*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(74085*d*e^2 -
 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2)/980000 -
 (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000))*(74085*d*
e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

**sympy [C]** time = 8.08, size = 469, normalized size = 2.74

$$\frac{2e^3x^2}{125}+x\left(\frac{12de^2}{125}-\frac{49e^3}{625}\right)+\left(\frac{3e\left(100d^2-245de+47e^2\right)}{6250}-\frac{3\sqrt{14}\,i\left(353125d^3-855175d^2e+74085de^2+556349e^3\right.}{137200000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d*
e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**
2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271395
*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sqrt(1
4)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1059375
*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d**2 -
245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085
*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3
271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 + 3*
sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(1
059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (1619125*d*
*3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(1376875*d**3 +
13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(4844125*d**3 + 212
3025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d**3 + 4332075*d
**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 98000000*x**3 + 16
6600000*x**2 + 58800000*x + 44100000)
```

# 3.319 $\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=134

$$\frac{\left(211875d^2-342070de+14817e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\left(5x^2+2x+3\right)}-\frac{(423x+136}{7000\left(5x^2\right.}$$

```
[Out] 4/125*x*e^2-1/7000*(1367+423*x)*(e*x+d)^2/(5*x^2+2*x+3)^2+1/196000*(e*x+d)*
(34347*d-6413*e+5*(2203*d+8553*e)*x)/(5*x^2+2*x+3)+1/1250*(40*d-49*e)*e*ln(
5*x^2+2*x+3)+1/13720000*(211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1+5*
x)*14^(1/2))*14^(1/2)
```

**Rubi** [A] time = 0.24, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1644, 1657, 634, 618, 204, 628}

$$\frac{\left(211875d^2-342070de+14817e^2\right)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{980000\sqrt{14}}+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{196000\left(5x^2+2x+3\right)}-\frac{(423x+136}{7000\left(5x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] (4*e^2*x)/125 - ((1367 + 423*x)*(d + e*x)^2)/(7000*(3 + 2*x + 5*x^2)^2) + (
(d + e*x)*(34347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(196000*(3 + 2*x + 5*
x^2)) + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(980000*Sqrt[14]) + ((40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/1250
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^2 \left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{(d+ex)\left(\frac{2}{125}(3267d+2734e)-\right.}{}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+}{196000\left(3+2x+5x^2\right)}$$

$$= -\frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(2203d+}{196000\left(3+2x+5x^2\right)}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(220}{196000\left(3+2x+5x\right.}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(220}{196000\left(3+2x+5x\right.}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(220}{196000\left(3+2x+5x\right.}$$

$$= \frac{4e^2x}{125} - \frac{(1367+423x)(d+ex)^2}{7000\left(3+2x+5x^2\right)^2} + \frac{(d+ex)(34347d-6413e+5(220}{196000\left(3+2x+5x\right.}$$

**Mathematica [A]** time = 0.18, size = 146, normalized size = 1.09

$$\frac{70\left(\frac{5\left(5d^2\left(11015x^3+38753x^2+17979x+12953\right)+2de\left(181765x^3+28307x^2+57761x-19533\right)+e^2\left(156800x^5+125440x^4+83809x^3-138345x^2-65427x-\right.\right.}{\left(5x^2+2x+3\right)^2}\right.}{68600000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3
,x]
```

```
[Out] (5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]
] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-19533
 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^2 +
 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d - 4
9*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

**fricas [B]** time = 0.83, size = 302, normalized size = 2.25

$$\frac{10976000\,e^2x^5 + 8780800\,e^2x^4 + 70\left(55075\,d^2 + 363530\,de + 83809\,e^2\right)x^3 + 70\left(193765\,d^2 + 56614\,de - 138345\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*e
 + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(14)
*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070*d*
e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 1906875
*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*e^2)*
x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388390*e^
2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e - 49*e^2)
*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*e - 441*
e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x^3 + 34*x
^2 + 12*x + 9)
```

**giac [A]** time = 0.16, size = 144, normalized size = 1.07

$$\frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2 - 342070\,de + 14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{4}{125}\,xe^2+\frac{1}{1250}\left(40\,de - 49\,e^2\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arctan(1/14*sqrt(
14)*(5*x + 1)) + 4/125*x*e^2 + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 + (89895*d^2 + 115522*d*e - 121875*e^2
)*x - 39066*d*e - 76977*e^2)/(5*x^2 + 2*x + 3)^2
```

**maple [A]** time = 0.01, size = 179, normalized size = 1.34

$$\frac{339\sqrt{14}\;d^2\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}-\frac{34207\sqrt{14}\;de\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{1372000}+\frac{4de\ln\left(5x^2+2x+3\right)}{125}+\frac{4e^2x}{125}+\frac{14817\sqrt{14}\;e^2}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(38753
/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/19600
*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5*x^
2+2*x+3)^2+4/125*d*e*ln(5*x^2+2*x+3)-49/1250*e^2*ln(5*x^2+2*x+3)+339/21952*
14^(1/2)*d^2*arctan(1/28*(10*x+2)*14^(1/2))-34207/1372000*14^(1/2)*d*e*arct
an(1/28*(10*x+2)*14^(1/2))+14817/13720000*14^(1/2)*e^2*arctan(1/28*(10*x+2)
*14^(1/2))
```

**maxima** [A] time = 0.96, size = 155, normalized size = 1.16

$$\frac{4}{125}\,e^2x+\frac{1}{13720000}\,\sqrt{14}\left(211875\,d^2-342070\,de+14817\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{1}{1250}\left(40\,de-49\,e^2\right)\text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*arc
tan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x + 3
) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 + 566
14*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d^2 +
 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

**mupad** [B] time = 4.21, size = 203, normalized size = 1.51

$$\frac{x^3\left(\frac{55075\,d^2}{1568}+\frac{181765\,d\,e}{784}-\frac{129439\,e^2}{1568}\right)+x^2\left(\frac{193765\,d^2}{1568}+\frac{28307\,d\,e}{784}-\frac{213609\,e^2}{1568}\right)-\frac{19533\,d\,e}{784}+x\left(\frac{89895\,d^2}{1568}+\frac{57761\,d\,e}{784}-\frac{121875}{1568}\right.}{3125\,x^4+2500\,x^3+4250\,x^2+1500\,x+1125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((2830
7*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x*(
(57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/1568
- (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) + (4*e
^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(1/2)*
atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14^(1/2)
*x*(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 - (34207
*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817*e^2))/1
3720000
```

**sympy [C]** time = 3.96, size = 304, normalized size = 2.27

$$\frac{4e^2x}{125}+\left(\frac{e\left(40d-49e\right)}{1250}-\frac{\sqrt{14}\,i\left(211875d^2-342070de+14817e^2\right)}{27440000}\right)\log\left(x+\frac{42375d^2-244030de+218093e^2+21}{211875d^2-34}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] 4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*e
 + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 + 2
1952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)
/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 + sqrt
(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (42375*d**
2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(211875
*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)
) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 363530*d*e -
129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(89895*d**2
 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 +
 2352000*x + 1764000)
```

# 3.320 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

**Optimal**. Leaf size=103

$$-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000\left(5x^2+2x+3\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log\left(5\right.$$

```
[Out] -1/7000*(1367+423*x)*(e*x+d)/(5*x^2+2*x+3)^2+1/196000*(34347*d-6511*e+(1101
5*d+36353*e)*x)/(5*x^2+2*x+3)+2/125*e*ln(5*x^2+2*x+3)+1/2744000*(42375*d-34
207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1644, 1660, 634, 618, 204, 628}

$$-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}+\frac{x(11015d+36353e)+34347d-6511e}{196000\left(5x^2+2x+3\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log\left(5\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -((1367 + 423*x)*(d + e*x))/(7000*(3 + 2*x + 5*x^2)^2) + (34347*d - 6511*e
+ (11015*d + 36353*e)*x)/(196000*(3 + 2*x + 5*x^2)) + ((42375*d - 34207*e)*
ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3 + 2*x + 5*x^2])/
125
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}dx = -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{2}{125}(3267d+1367e)-\frac{12}{25}(308d-$$

$$(3+2x$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+$$

$$= -\frac{(1367+423x)(d+ex)}{7000\left(3+2x+5x^2\right)^2}+\frac{34347d-6511e+(11015d+36353e)x}{196000\left(3+2x+5x^2\right)}+$$

**Mathematica [A]** time = 0.09, size = 107, normalized size = 1.04

$$\frac{-2115dx-6835d-5989ex+1269e}{35000\left(5x^2+2x+3\right)^2}+\frac{55075dx+171735d+181765ex-44399e}{980000\left(5x^2+2x+3\right)}+\frac{(42375d-34207e)\tan^{-1}\left(\frac{5x}{\sqrt{}}\right.}{196000\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x
]
```

```
[Out] (-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (171
735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((42
375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Log[3
 + 2*x + 5*x^2])/125
```

**fricas [A]** time = 0.76, size = 172, normalized size = 1.67

$$\frac{70\,(11015\,d+36353\,e)x^3+14\,(193765\,d+28307\,e)x^2+\sqrt{14}\left(25\,(42375\,d-34207\,e)x^4+20\,(42375\,d-34207\,e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt(
14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*d
- 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(1/1
4*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20*e*x
^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462*e)/(
25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

**giac [A]** time = 0.19, size = 97, normalized size = 0.94

$$\frac{1}{2744000}\sqrt{14}\,(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2*x
+ 3)^2
```

**maple [A]** time = 0.01, size = 102, normalized size = 0.99

$$\frac{339\sqrt{14}\,d\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}-\frac{34207\sqrt{14}\,e\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{2744000}+\frac{2e\ln\left(5x^2+2x+3\right)}{125}+\frac{25\left(\frac{2203d}{196000}+\frac{36353e}{980000}\right)x^3+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^2
+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^2+
2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+339/21952*14^(1/2)*d*arctan(1/28*(10*x+2)*
14^(1/2))-34207/2744000*14^(1/2)*e*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima [A]** time = 0.96, size = 101, normalized size = 0.98

$$\frac{1}{2744000}\sqrt{14}\,(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)+\frac{5\,(11015\,d+36353\,e)x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2/
125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (193765*
d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 + 20*
x^3 + 34*x^2 + 12*x + 9)
```

**mupad [B]** time = 0.12, size = 125, normalized size = 1.21

$$\frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)\,x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)\,x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)\,x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}+\frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125}+\frac{\sqrt{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] ((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200)
 + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761*e
)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2 + 3
))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(1/2)
*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42375*d
 - 34207*e))/2744000
```

**sympy [C]** time = 2.31, size = 163, normalized size = 1.58

$$\left(\frac{2e}{125}-\frac{\sqrt{14}\,i\,(42375d-34207e)}{5488000}\right)\log\left(x+\frac{8475d-\frac{34207e}{5}-\frac{\sqrt{14}\,i(42375d-34207e)}{5}}{42375d-34207e}\right)+\left(\frac{2e}{125}+\frac{\sqrt{14}\,i\,(42375d-3420}{5488000}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*
e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 + s
qrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqrt(1
4)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e + x**
3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 57761*e))
/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

## 3.321 $\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3} dx$

**Optimal**. Leaf size=64

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

```
[Out] 1/7000*(-1367-423*x)/(5*x^2+2*x+3)^2+1/196000*(34347+11015*x)/(5*x^2+2*x+3)
+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

**Rubi [A]** time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1660, 12, 618, 204}

$$-\frac{423x+1367}{7000\left(5x^2+2x+3\right)^2}+\frac{11015x+34347}{196000\left(5x^2+2x+3\right)}+\frac{339\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] -(1367 + 423*x)/(7000*(3 + 2*x + 5*x^2)^2) + (34347 + 11015*x)/(196000*(3 +
 2*x + 5*x^2)) + (339*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt[14])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-5x^3+4x^4}{\left(3+2x+5x^2\right)^3}\,dx &= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{\frac{6534}{125}-\frac{3696x}{25}+\frac{448x^2}{5}}{\left(3+2x+5x^2\right)^2}\,dx\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{\int \frac{1356}{3+2x+5x^2}\,dx}{6272}\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\int \frac{1}{3+2x+5x^2}\,dx}{1568}\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}-\frac{339}{784}\,\mathrm{Subst}\left(\int \frac{1}{-56-x^2}\,d\right.\\
&= -\frac{1367+423x}{7000\left(3+2x+5x^2\right)^2}+\frac{34347+11015x}{196000\left(3+2x+5x^2\right)}+\frac{339\tan^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 53, normalized size = 0.83

$$\frac{\frac{14\left(11015x^3+38753x^2+17979x+12953\right)}{\left(5x^2+2x+3\right)^2}+8475\sqrt{14}\tan^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{548800}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

```
[Out] ((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475*
Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

**fricas [A]** time = 0.73, size = 75, normalized size = 1.17

$$\frac{154210\,x^3+8475\,\sqrt{14}\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+542542\,x^2+251706\,x+181342}{548800\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)*
arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

**giac [A]** time = 0.16, size = 46, normalized size = 0.72

$$\frac{339}{21952}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\left(5\,x^2+2\,x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

**maple [A]** time = 0.01, size = 47, normalized size = 0.73

$$\frac{339\sqrt{14}\;\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}+\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{\left(5x^2+2x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2*
x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

**maxima [A]** time = 0.96, size = 56, normalized size = 0.88

$$\frac{339}{21952}\,\sqrt{14}\,\arctan\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{11015\,x^3+38753\,x^2+17979\,x+12953}{39200\left(25\,x^4+20\,x^3+34\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

```
[Out] 339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 + 3
8753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

**mupad [B]** time = 0.05, size = 55, normalized size = 0.86

$$\frac{339\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}+\frac{\frac{2203\,x^3}{196000}+\frac{38753\,x^2}{980000}+\frac{17979\,x}{980000}+\frac{12953}{980000}}{x^4+\frac{4\,x^3}{5}+\frac{34\,x^2}{25}+\frac{12\,x}{25}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

```
[Out] (339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/980
000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 + (
34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

**sympy [A]** time = 0.20, size = 61, normalized size = 0.95

$$\frac{11015x^3+38753x^2+17979x+12953}{980000x^4+784000x^3+1332800x^2+470400x+352800}+\frac{339\sqrt{14}\,\operatorname{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

```
[Out] (11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 13
32800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(
14)/14)/21952
```

# 3.322 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3} dx$

**Optimal**. Leaf size=329

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{171735d^3-92989d^2e+25x\left(2203d^3-9033d^2e+3635de^2-1829e^3\right)}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

```
[Out] 1/1400*(-1367*d+293*e-(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^2
+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d^2
*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+e*(4*d^4+5*d
^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^4+5*
d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3+1/21952*
(42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*ar
ctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)
```

**Rubi [A]** time = 0.50, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 800, 634, 618, 204, 628}

$$-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}+\frac{25x\left(-9033d^2e+2203d^3+3635de^2-1829e^3\right)-92989d^2e+171735d^3}{39200\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -(1367*d - 293*e + (423*d - 1367*e)*x)/(1400*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2
*x + 5*x^2)^2) + (171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 + 25*(2
203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(39200*(5*d^2 - 2*d*e + 3*
e^2)^2*(3 + 2*x + 5*x^2)) + ((42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 560
58*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/(1568*Sqrt
[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (e*(4*d^4 + 5*d^3*e + 3*d
^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{1}{112}\int \frac{\frac{2\left(3267d^2-2843de+2800e^2\right)}{25\left(5d^2-2de+3e^2\right)} - \frac{6(308}{2}}{(d+ex)\left(3+2\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

$$= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2} + \frac{171735d^3-92989d^2e+36207de^2+}{39200\left(5d\right.}$$

**Mathematica [A]** time = 0.31, size = 282, normalized size = 0.86

$$\frac{\frac{392\left(5d^2-2de+3e^2\right)^2(e(1367x+293)-d(423x+1367))}{\left(5x^2+2x+3\right)^2} + \frac{14\left(5d^2-2de+3e^2\right)\left(5d^3(11015x+34347)-d^2e(225825x+92989)+de^2(90875x+36207)+e^3(1831-4\right.}{5x^2+2x+3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x
]
```

```
[Out] ((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(
3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) + 5*
```

```
d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*x))
)/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2
- 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] + 5488
00*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274400*e*
(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(548800
*(5*d^2 - 2*d*e + 3*e^2)^3)
```

**fricas [B]** time = 1.13, size = 1052, normalized size = 3.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fri
cas")
```

```
[Out] 1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2 -
43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d^4*
e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 + 5*sq
rt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 + 28629
9*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d
^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e + 58530*
d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d^5 - 166
43*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^2 + 12
*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8
623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 828175*d^4*e
 + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x + 109760*(
36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^4*e + 5*d^3
*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*log(e*x + d
) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*d^
4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e^2
+ 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*
e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x)*
log(5*x^2 + 2*x + 3))/(1125*d^6 - 1350*d^5*e + 2565*d^4*e^2 - 1692*d^3*e^3
+ 1539*d^2*e^4 - 486*d*e^5 + 243*e^6 + 25*(125*d^6 - 150*d^5*e + 285*d^4*e^
2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^4 + 20*(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*x^3 +
34*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^
5 + 27*e^6)*x^2 + 12*(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171
*d^2*e^4 - 54*d*e^5 + 27*e^6)*x)
```

**giac [A]** time = 0.25, size = 460, normalized size = 1.40

$$\frac{\sqrt{14}\left(42375\,d^{5}-16643\,d^{4}e+58530\,d^{3}e^{2}-56058\,d^{2}e^{3}+31811\,de^{4}-8623\,e^{5}\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\left(125\,d^{6}-150\,d^{5}e+285\,d^{4}e^{2}-188\,d^{3}e^{3}+171\,d^{2}e^{4}-54\,de^{5}+27\,e^{6}\right)}-\frac{(\ldots}{2\left(125\,d\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gia
c")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^
6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6
) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(x*e + d))/(
125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^
6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 25*(11015*d^5 - 49571*d^4
*e + 42850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 236238*d
^3*e^2 + (968825*d^5 - 1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 2
50589*d*e^4 - 49377*e^5)*x^2 - 57686*d^2*e^3 + (449475*d^5 - 828175*d^4*e +
 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*x - 25611*d*e^
4 + 18063*e^5)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

**maple [B]** time = 0.02, size = 1437, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x)
```

```
[Out] 5*e^2/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d^3+3*e^3/(5*d^2-2*d*e+3*e^2)^3*ln(e*
x+d)*d^2-e^4/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)*d+193765/1568/(5*d^2-2*d*e+3*e
^2)^3/(5*x^2+2*x+3)^2*x^2*d^5-49377/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3
)^2*x^2*e^5+89895/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^5-11211/78
40/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*e^5-58185/1568/(5*d^2-2*d*e+3*e^
2)^3/(5*x^2+2*x+3)^2*d^4*e+118119/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^
2*d^3*e^2-28843/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d^2*e^3-25611/78
40/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d*e^4-8623/21952/(5*d^2-2*d*e+3*e^
2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*e^5+42375/21952/(5*d^2-2*d*e+3
*e^2)^3*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^5-5/2/(5*d^2-2*d*e+3*e^2)
^3*ln(5*x^2+2*x+3)*d^3*e^2-3/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d^2*e^
3+1/2/(5*d^2-2*d*e+3*e^2)^3*ln(5*x^2+2*x+3)*d*e^4-2/(5*d^2-2*d*e+3*e^2)^3*l
n(5*x^2+2*x+3)*d^4*e+55075/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d
```

```
^5-27435/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*e^5+4*e/(5*d^2-2*d*
e+3*e^2)^3*ln(e*x+d)*d^4+64765/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*d
^5+18063/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*e^5-1/(5*d^2-2*d*e+3*e^
2)^3*ln(5*x^2+2*x+3)*e^5+2*e^5/(5*d^2-2*d*e+3*e^2)^3*ln(e*x+d)-260825/1568/
(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2*d^4*e+655359/3920/(5*d^2-2*d*e+3*
e^2)^3/(5*x^2+2*x+3)^2*x^2*d^3*e^2-388683/3920/(5*d^2-2*d*e+3*e^2)^3/(5*x^2
+2*x+3)^2*x^2*d^2*e^3+250589/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^2
*d*e^4-165635/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^4*e+380997/392
0/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x*d^3*e^2-250449/3920/(5*d^2-2*d*e+
3*e^2)^3/(5*x^2+2*x+3)^2*x*d^2*e^3+147247/7840/(5*d^2-2*d*e+3*e^2)^3/(5*x^2
+2*x+3)^2*x*d*e^4-16643/21952/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/28*(1
0*x+2)*14^(1/2))*d^4*e+29265/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*arctan(1/
28*(10*x+2)*14^(1/2))*d^3*e^2-28029/10976/(5*d^2-2*d*e+3*e^2)^3*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2))*d^2*e^3+31811/21952/(5*d^2-2*d*e+3*e^2)^3*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^4-247855/1568/(5*d^2-2*d*e+3*e^2)^3
/(5*x^2+2*x+3)^2*x^3*d^4*e-108785/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2
*x^3*d^2*e^3+107125/784/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d^3*e^2+7
2815/1568/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)^2*x^3*d*e^4
```

**maxima [A]** time = 1.03, size = 571, normalized size = 1.74

$$\frac{\sqrt{14}\left(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5\right)\arctan\left(\frac{1}{14}\sqrt{14}\,(5\,x+1)\right)}{21952\left(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6\right)}+\frac{}{125\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="max
ima")
```

```
[Out] 1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 +
 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5
*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d^4*
e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*d^5*
e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(4*d
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(125*d^6
 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
 + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 + 64765*d
^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^2*e + 725
57*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e^2 - 3737*
e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^4 + 225*d
^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4 - 20*d^3*e +
 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e
^4)*x)
```

**mupad [B]** time = 4.79, size = 641, normalized size = 1.95

$$\frac{\frac{x\,(89895\,d^3-129677\,d^2\,e+46591\,d\,e^2-3737\,e^3)}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}-\frac{-64765\,d^3+32279\,d^2\,e+4523\,d\,e^2-6021\,e^3}{7840\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{5\,x^3\,(2203\,d^3-9033\,d^2\,e+3635\,d\,e^2-1829\,e^3)}{1568\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x^2\,(1}{78}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] ((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 20
*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 6476
5*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)
) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d^4 -
 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 183319*d^
2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4
+ 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)*((4*e)
/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 - 2*d*e +
3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - (log(x -
 (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (42375*14^(1/2)
*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*((29265*14^
(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*e^4*((31811*
14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^
2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 + 1/5)
*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43904 + d^2*e^3*(
(28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/21952 + 5i/2) +
d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/2))/43904 - 1i/2)
))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*18
8i + d^4*e^2*285i)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

# 3.323 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=443

$$-\frac{x\left(423d^2-2734de+293e^2\right)+1367d^2-586de-703e^2}{280\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)^2}+\frac{171735d^4-117284d^3e-200502d^2e^2+5x\left(11015d^4\right.}{7840\left(5x^2\right.}$$

```
[Out] -e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+1/28
0*(-1367*d^2+586*d*e+703*e^2-(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*e
^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+104428
*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-3589*e^
4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+e*(40*d^5+83*d^4*e+12*d^3*e^2-76*
d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*e*(40*d^5+83*d^
4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^
2)^4+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^
2*e^4-49586*d*e^5-43695*e^6)*arctan(1/14*(1+5*x)*14^(1/2))/(5*d^2-2*d*e+3*e
^2)^4*14^(1/2)
```

**Rubi [A]** time = 0.89, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1646, 1628, 634, 618, 204, 628}

$$\frac{5x\left(34698d^2e^2-85924d^3e+11015d^4+10348de^3-3589e^4\right)-200502d^2e^2-117284d^3e+171735d^4+104428d}{7840\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

```
[Out] -((e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/((5*d^2 - 2*d*e + 3*e^2
)^3*(d + e*x))) - (1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293
*e^2)*x)/(280*(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + (171735*d^4
- 117284*d^3*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 -
 85924*d^3*e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(7840*(5*d^2 - 2*
d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) + ((211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(1568*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(40*d^5 +
83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x])/(5*d^2
 - 2*d*e + 3*e^2)^4 - (e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*
d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(2*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{1}{112}\int \frac{2\left(3267d^4-56\right.}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-11728}{}$$

$$= -\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}+\frac{171735d^4-11728}{}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2\right.}$$

$$= -\frac{e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}-\frac{1367d^2-586de-703e^2+\left(423d^2-2\right.}{280\left(5d^2-2de+3e^2\right)^2\left(3+2\right.}$$

**Mathematica [A]** time = 0.53, size = 389, normalized size = 0.88

$$-\frac{392\left(5d^2-2de+3e^2\right)^2\left(d^2(423x+1367)-2de(1367x+293)+e^2(293x-703)\right)}{\left(5x^2+2x+3\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(5d^4(11015x+34347)-4d^3e(107405x+29321)+6d^2e^2(2\right.}{5x^2+2x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3)
,x]
```

```
[Out] ((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) + d^
2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d^2
- 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) - e^4
*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 107405
*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 209039*d^4*
e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*ArcTan[(1
+ 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 +
46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 -
76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d^2 - 2*d*e
 + 3*e^2)^4)
```

**<u>fricas</u> [B]** time = 1.56, size = 1734, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="f
ricas")
```

```
[Out] 1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(10172
5*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e^5
- 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 474395*d^
5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*e^6 -
 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 - 27958
1*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 394767*e^7)*
x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 8292996*d^4*e
^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*d^6*e + 3
070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 - 49586*d*e^
6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5*e^2 - 3771
064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6 - 174780*e^7
)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 5660777*d^4*e^3
- 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815*e^7)*x^3 + 2*
(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^4*e^3 + 941893*
d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2 + 3*(847500*d^7
 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 1241848*d^3*e^4 + 9435
19*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1))
+ 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325*d^4*e^3 - 14261*d^
3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x + 21952*(360*d^6*e + 74
7*d^5*e^2 + 108*d^4*e^3 - 684*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5
*e^2 + 83*d^4*e^3 + 12*d^3*e^4 - 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(20
0*d^6*e + 575*d^5*e^2 + 392*d^4*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6
- 36*e^7)*x^4 + 2*(400*d^6*e + 1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 -
832*d^2*e^5 + 692*d*e^6 - 153*e^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*
```

```
d^4*e^3 - 1220*d^3*e^4 + 326*d^2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6
*e + 452*d^5*e^2 + 297*d^4*e^3 - 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*
e^7)*x)*log(e*x + d) - 10976*(360*d^6*e + 747*d^5*e^2 + 108*d^4*e^3 - 684*d
^3*e^4 + 414*d^2*e^5 - 81*d*e^6 + 25*(40*d^5*e^2 + 83*d^4*e^3 + 12*d^3*e^4
- 76*d^2*e^5 + 46*d*e^6 - 9*e^7)*x^5 + 5*(200*d^6*e + 575*d^5*e^2 + 392*d^4
*e^3 - 332*d^3*e^4 - 74*d^2*e^5 + 139*d*e^6 - 36*e^7)*x^4 + 2*(400*d^6*e +
1510*d^5*e^2 + 1531*d^4*e^3 - 556*d^3*e^4 - 832*d^2*e^5 + 692*d*e^6 - 153*e
^7)*x^3 + 2*(680*d^6*e + 1651*d^5*e^2 + 702*d^4*e^3 - 1220*d^3*e^4 + 326*d^
2*e^5 + 123*d*e^6 - 54*e^7)*x^2 + 3*(160*d^6*e + 452*d^5*e^2 + 297*d^4*e^3
- 268*d^3*e^4 - 44*d^2*e^5 + 102*d*e^6 - 27*e^7)*x)*log(5*x^2 + 2*x + 3))/(
5625*d^9 - 9000*d^8*e + 18900*d^7*e^2 - 17640*d^6*e^3 + 18774*d^5*e^4 - 105
84*d^4*e^5 + 6804*d^3*e^6 - 1944*d^2*e^7 + 729*d*e^8 + 25*(625*d^8*e - 1000
*d^7*e^2 + 2100*d^6*e^3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*
d^2*e^7 - 216*d*e^8 + 81*e^9)*x^5 + 5*(3125*d^9 - 2500*d^8*e + 6500*d^7*e^2
 - 1400*d^6*e^3 + 2590*d^5*e^4 + 2464*d^4*e^5 - 924*d^3*e^6 + 1944*d^2*e^7
- 459*d*e^8 + 324*e^9)*x^4 + 2*(6250*d^9 + 625*d^8*e + 4000*d^7*e^2 + 16100
*d^6*e^3 - 12460*d^5*e^4 + 23702*d^4*e^5 - 12432*d^3*e^6 + 10692*d^2*e^7 -
2862*d*e^8 + 1377*e^9)*x^3 + 2*(10625*d^9 - 13250*d^8*e + 29700*d^7*e^2 - 2
0720*d^6*e^3 + 23702*d^5*e^4 - 7476*d^4*e^5 + 5796*d^3*e^6 + 864*d^2*e^7 +
81*d*e^8 + 486*e^9)*x^2 + 3*(2500*d^9 - 2125*d^8*e + 5400*d^7*e^2 - 1540*d^
6*e^3 + 2464*d^5*e^4 + 1554*d^4*e^5 - 504*d^3*e^6 + 1404*d^2*e^7 - 324*d*e^
8 + 243*e^9)*x)
```

**giac [A]** time = 0.26, size = 762, normalized size = 1.72

$$\frac{\sqrt{14}\left(211875\,d^6e^2+3070\,d^5e^3+209039\,d^4e^4-921444\,d^3e^5+380621\,d^2e^6-49586\,de^7-43695\,e^8\right)\arctan\left(\frac{1}{14}\right.}{21952\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="g
iac")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d
^3*e^5 + 380621*d^2*e^6 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*
d - 5*d^2/(x*e + d) + 2*d*e/(x*e + d) - 3*e^2/(x*e + d) - e)*e^(-1))*e^(-2)
/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*
d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 +
12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(-10*d/(x*e + d) + 5*d^2/(x*
e + d)^2 + 2*e/(x*e + d) - 2*d*e/(x*e + d)^2 + 3*e^2/(x*e + d)^2 + 5)/(625*
d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^
5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (4*d^4*e^7/(x*e + d) + 5*d^3*e^8/(x
*e + d) + 3*d^2*e^9/(x*e + d) - d*e^10/(x*e + d) + 2*e^11/(x*e + d))/(125*d
^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 188*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11
```

```
 + 27*e^12) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e^3 + 18
35350*d^2*e^4 - 734925*d*e^5 - 5*(165225*d^6*e^2 - 1997830*d^5*e^3 + 121842
1*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d*e^7 + 7963*e^8)*e^(-
1)/(x*e + d) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449735*d^5*e^5 + 82111
75*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947*d*e^9 - 67267*e^10)
*e^(-2)/(x*e + d)^2 - (275375*d^8*e^4 - 3975600*d^7*e^5 + 3752280*d^6*e^6 +
 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 - 1453312*d^2*e^10 - 1
97784*d*e^11 + 66483*e^12)*e^(-3)/(x*e + d)^3 + 17525*e^6)/((5*d^2 - 2*d*e
+ 3*e^2)^4*(10*d/(x*e + d) - 5*d^2/(x*e + d)^2 - 2*e/(x*e + d) + 2*d*e/(x*e
 + d)^2 - 3*e^2/(x*e + d)^2 - 5)^2)
```

**maple [B]** time = 0.03, size = 1850, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x)
```

```
[Out] 99045/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d*e^5-161395/784/(5*d^2-2*d
*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^5*e-379131/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+
2*x+3)^2*d^4*e^2+116869/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^3*e^3-2
0/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^5*e-83/2/(5*d^2-2*d*e+3*e^2)^4*ln
(5*x^2+2*x+3)*d^4*e^2-6/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^3*e^3+38/(5
*d^2-2*d*e+3*e^2)^4*ln(5*x^2+2*x+3)*d^2*e^4-23/(5*d^2-2*d*e+3*e^2)^4*ln(5*x
^2+2*x+3)*d*e^5+211875/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10
*x+2)*14^(1/2))*d^6-43695/21952/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*
(10*x+2)*14^(1/2))*e^6+40*e/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^5+83*e^2/(5*d
^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^4+12*e^3/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^3-
76*e^4/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)*d^2+46*e^5/(5*d^2-2*d*e+3*e^2)^4*ln(
e*x+d)*d-4*e/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^4-5*e^2/(5*d^2-2*d*e+3*e^2)^3/
(e*x+d)*d^3-3*e^3/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)*d^2+e^4/(5*d^2-2*d*e+3*e^2)
^3/(e*x+d)*d+968825/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^6+2753
75/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^6+449475/1568/(5*d^2-2*
d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6*x-53835/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+
2*x+3)^2*x^3*e^6-91101/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*e^6-7
4895/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*e^6-530209/1568/(5*d^2-2*
d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^2*e^4-648385/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^
2+2*x+3)^2*x*d^5*e+218053/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d*e
^5-795401/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^2*e^4+95555/784/
(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d*e^5-916595/784/(5*d^2-2*d*e+3*e
^2)^4/(5*x^2+2*x+3)^2*x^2*d^5*e+504029/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*
x+3)^2*x^2*d^4*e^2+5109/392/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^2*d^3*e
^3+1891915/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^4*e^2-344285/39
2/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^3*e^3+327265/1568/(5*d^2-2*d*
e+3*e^2)^4/(5*x^2+2*x+3)^2*x^3*d^2*e^4-1129125/784/(5*d^2-2*d*e+3*e^2)^4/(5
```

```
*x^2+2*x+3)^2*x^3*d^5*e-434995/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x
*d^2*e^4+323825/1568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*d^6-6309/1568/(5
*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*e^6+9/2/(5*d^2-2*d*e+3*e^2)^4*ln(5*x^2+
2*x+3)*e^6-9*e^6/(5*d^2-2*d*e+3*e^2)^4*ln(e*x+d)-2*e^5/(5*d^2-2*d*e+3*e^2)^
3/(e*x+d)+208007/784/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d*e^5+606287/1
568/(5*d^2-2*d*e+3*e^2)^4/(5*x^2+2*x+3)^2*x*d^4*e^2-3993/392/(5*d^2-2*d*e+3
*e^2)^4/(5*x^2+2*x+3)^2*x*d^3*e^3+1535/10976/(5*d^2-2*d*e+3*e^2)^4*14^(1/2)
*arctan(1/28*(10*x+2)*14^(1/2))*d^5*e+209039/21952/(5*d^2-2*d*e+3*e^2)^4*14
^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^4*e^2-230361/5488/(5*d^2-2*d*e+3*e^
2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^3*e^3+380621/21952/(5*d^2-2*
d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d^2*e^4-24793/10976/(5
*d^2-2*d*e+3*e^2)^4*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))*d*e^5
```

**maxima [B]** time = 1.13, size = 916, normalized size = 2.07

$$\frac{\sqrt{14}\left(211875\,d^6+3070\,d^5e+209039\,d^4e^2-921444\,d^3e^3+380621\,d^2e^4-49586\,de^5-43695\,e^6\right)\arctan\left(\frac{1}{14}\sqrt{14}\right.}{21952\left(625\,d^8-1000\,d^7e+2100\,d^6e^2-1960\,d^5e^3+2086\,d^4e^4-1176\,d^3e^5+756\,d^2e^6-216\,de^7+81\,e^8\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="m
axima")
```

```
[Out] 1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^3
 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x + 1)
)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176
*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 + 12*
d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*d^7*e
 + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6
- 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^
4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6
*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7
 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 22292*d^2*e
^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 - 11178*d^2*
e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e - 272442*d^3
*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (193765*d^5 - 41248
5*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131589*e^5)*x^2 +
 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3 + 19493*d*e^4 -
 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692*d^4*e^3 + 1539*
d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^5*e^2 + 285*d^4*e
^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 + 5*(625*d^7 - 250*
d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^2*e^5 - 81*d*e^6 +
108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2 + 2965*d^4*e^3 - 1486*
d^3*e^4 + 2367*d^2*e^5 - 648*d*e^6 + 459*e^7)*x^3 + 2*(2125*d^7 - 1800*d^6*
e + 3945*d^5*e^2 - 1486*d^4*e^3 + 1779*d^3*e^4 + 108*d^2*e^5 + 135*d*e^6 +
```

```
162*e^7)*x^2 + 3*(500*d^7 - 225*d^6*e + 690*d^5*e^2 + 103*d^4*e^3 + 120*d^3
*e^4 + 297*d^2*e^5 - 54*d*e^6 + 81*e^7)*x)
```

**mupad [B]** time = 4.99, size = 965, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

```
[Out] log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (6
*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))/(
5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965*d^
5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e
- 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (12009*d*
e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^3*e^2)
/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 - 11178*d^
2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 138539*d*e^
4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1568*(125*d^
6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2
)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e^5 + 56850*d^
2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 1
71*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d + 12*e) + x^4*(2
5*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)) + (log(x - (14
^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43
904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^4*e^2*((209039*14^
(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904 + 38i) + d^5*e*((1
535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))/21952 + 23i)))/(d^8*62
5i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^
4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^(1/2)*1i)/5 + 1
/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((43695*14^(1/2))/43904 + 9i/2) - d^
3*e^3*((230361*14^(1/2))/10976 - 6i) + d^4*e^2*((209039*14^(1/2))/43904 + 8
3i/2) + d^2*e^4*((380621*14^(1/2))/43904 - 38i) + d^5*e*((1535*14^(1/2))/21
952 + 20i) - d*e^5*((24793*14^(1/2))/21952 - 23i)))/(d^8*625i - d^7*e*1000i
 - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*e^5*1176i + d^4*e^4*2086i - d^
5*e^3*1960i + d^6*e^2*2100i)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

```
[Out] Timed out
```

# 3.324 $\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx$

**Optimal**. Leaf size=143

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+1005}{7}$$

```
[Out] 11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2)
+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)-1/71680*(1005757+295276*x)*(2*x^2-x+3)^(
3/2)-1183005/131072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-51435/32768*(1-4
*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4-\frac{823\left(2x^2-x+3\right)^{3/2}(2x+5)^3}{1344}+\frac{11433\left(2x^2-x+3\right)^{3/2}(2x+5)^2}{4480}-\frac{(295276x+1005}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-51435*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/32768 + (11433*(5 + 2*x)^2*(3 - x +
2*x^2)^(3/2))/4480 - (823*(5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))/1344 + (5*(5 +
 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 - ((1005757 + 295276*x)*(3 - x + 2*x^2)^
(3/2))/71680 - (1183005*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}+\frac{1}{224}\int (5+2x)\sqrt{3-x+$$

$$= -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2\right.$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+\right.}{1344}$$

$$= \frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}-\frac{823(5+2x)^3\left(3-x+\right.}{1344}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

$$= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+\right.}{4480}$$

**Mathematica [A]** time = 0.15, size = 70, normalized size = 0.49

$$\frac{4\sqrt{2x^2-x+3}\left(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117\right) -}{13762560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
+ 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]])/13762560
```

**fricas [A]** time = 0.81, size = 83, normalized size = 0.58

$$\frac{1}{3440640}\left(4915200\,x^6+12984320\,x^5+1390592\,x^4+20304768\,x^3+11357024\,x^2+14742332\,x+6231117\right)\sqrt{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 113570
24*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(2)
*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**giac [A]** time = 0.20, size = 78, normalized size = 0.55

$$\frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120\,x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2\,x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*x
 + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log(-
2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.02, size = 115, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x^4}{7}+\frac{377\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{168}+\frac{283\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{1120}-\frac{5179\left(2x^2-x+3\right)^{\frac{3}{2}}x}{17920}+\frac{1183005\sqrt{2}\ \mathrm{arcsi}}{13107}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/7*x^4*(2*x^2-x+3)^(3/2)+377/168*x^3*(2*x^2-x+3)^(3/2)+283/1120*x^2*(2*x^2
-x+3)^(3/2)-5179/17920*x*(2*x^2-x+3)^(3/2)+1183005/131072*2^(1/2)*arcsinh(4
/23*23^(1/2)*(x-1/4))+51435/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+242329/215040*(
2*x^2-x+3)^(3/2)
```

**maxima [A]** time = 0.97, size = 126, normalized size = 0.88

$$\frac{5}{7}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{377}{168}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{283}{1120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{5179}{17920}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{242329}{215040}\left(2\,x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/112
0*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329/2
15040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005/13
```

```
1072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2 - x
+ 3)
```

**mupad [B]** time = 1.72, size = 170, normalized size = 1.19

$$\frac{283\,x^2\,{\left(2\,x^2-x+3\right)}^{3/2}}{1120}+\frac{377\,x^3\,{\left(2\,x^2-x+3\right)}^{3/2}}{168}+\frac{5\,x^4\,{\left(2\,x^2-x+3\right)}^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\right.}{2}\right.}{573440}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
+ (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^(1
/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x + 3
)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/3440640
 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^2 - x
 + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+5)\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x
)
```

## 3.325 $\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx$

**Optimal**. Leaf size=124

$$\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-x+3}}{16384}+\frac{5}{12}\left(2x^2-x+\right.$$

```
[Out] 287/5120*(2*x^2-x+3)^(3/2)-71/1280*x*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)
^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-106007/65536*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)
```

**Rubi** [A] time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{3/2}x^2-\frac{71\left(2x^2-x+3\right)^{3/2}x}{1280}+\frac{287\left(2x^2-x+3\right)^{3/2}}{5120}-\frac{4609(1-4x)\sqrt{2x^2-}}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-4609*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 + (287*(3 - x + 2*x^2)^(3/2))/5
120 - (71*x*(3 - x + 2*x^2)^(3/2))/1280 + (7*x^2*(3 - x + 2*x^2)^(3/2))/80
+ (5*x^3*(3 - x + 2*x^2)^(3/2))/12 - (106007*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
32768*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{12}\int \sqrt{3-x+2x^2}\left(24+12x-9x^2+\right.$$

$$=\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}+\frac{1}{120}\int\left(240+5\right.$$

$$=-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2\right.$$

$$=\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x+2x^2\right)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right.$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$

$$=-\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287\left(3-x+2x^2\right)^{3/2}}{5120}-\frac{71x\left(3-x\right.}{12}$$

**Mathematica [A]** time = 0.09, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807\right)-1590105\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{983040}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 59392
*x^4 + 204800*x^5) - 1590105*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/983040
```

**fricas [A]** time = 0.85, size = 78, normalized size = 0.63

$$\frac{1}{245760}\left(204800\,x^5-59392\,x^4+258432\,x^3+105696\,x^2+221868\,x-27807\right)\sqrt{2\,x^2-x+3}+\frac{106007}{131072}\sqrt{2}\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 278
07)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**giac [A]** time = 0.20, size = 73, normalized size = 0.59

$$\frac{1}{245760}\,(4\,(8\,(4\,(16\,(100\,x-29)x+2019)x+3303)x+55467)x-27807)\sqrt{2\,x^2-x+3}-\frac{106007}{65536}\sqrt{2}\log\Big(-2\,\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 27807
)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.01, size = 98, normalized size = 0.79

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x^3}{12}+\frac{7\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{80}-\frac{71\left(2x^2-x+3\right)^{\frac{3}{2}}x}{1280}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}+\frac{287\left(2x^2-x\right.}{5120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(3/2)*x^3+7/80*(2*x^2-x+3)^(3/2)*x^2-71/1280*(2*x^2-x+3)^(
3/2)*x+287/5120*(2*x^2-x+3)^(3/2)+4609/16384*(4*x-1)*(2*x^2-x+3)^(1/2)+1060
07/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 0.95, size = 109, normalized size = 0.88

$$\frac{5}{12}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x+\frac{287}{5120}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2\,x^2-x+3}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*(
2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(2*
x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46
09/16384*sqrt(2*x^2 - x + 3)
```

**mupad [B]** time = 0.77, size = 153, normalized size = 1.23

$$\frac{7\,x^2\left(2\,x^2-x+3\right)^{3/2}}{80}+\frac{5\,x^3\left(2\,x^2-x+3\right)^{3/2}}{12}+\frac{63779\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\left(2\,x-\frac{1}{2}\right)}{2}\right)}{40960}+\frac{2773\,\left(\frac{x}{2}-\frac{1}{8}\right)\sqrt{2\,x^2-}}{1280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] (7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (6377
9*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (27
73*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*(32
*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^(1/2
)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

## 3.326 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

**Optimal**. Leaf size=149

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x}}{4096}$$

```
[Out] 4535/768*(2*x^2-x+3)^(3/2)-127/128*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^2
*(2*x^2-x+3)^(3/2)+5627989/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-110
01/32*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/4096*(489
587-80844*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2-\frac{127}{128}\left(2x^2-x+3\right)^{3/2}(2x+5)+\frac{4535}{768}\left(2x^2-x+3\right)^{3/2}+\frac{(489587-80844x)\sqrt{2x^2-x}}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((489587 - 80844*x)*Sqrt[3 - x + 2*x^2])/4096 + (4535*(3 - x + 2*x^2)^(3/2)
)/768 - (127*(5 + 2*x)*(3 - x + 2*x^2)^(3/2))/128 + ((5 + 2*x)^2*(3 - x + 2
*x^2)^(3/2))/16 + (5627989*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8192*Sqrt[2]) - (1
1001*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}+\frac{1}{160}\int \frac{\sqrt{3-x+2x^2}\left(-805-6\right.}{5+}$$

$$= -\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}+$$

$$= \frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{12}{12}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{12}{12}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{12}{12}$$

$$= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}-\frac{12}{12}$$

**Mathematica [A]** time = 0.15, size = 91, normalized size = 0.61

$$\frac{-16897536\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+4\sqrt{2x^2-x+3}\left(6144x^4-21120x^3+79840x^2-300404x+1561161\right)+}{49152}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*x
^4) + 16883967*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 16897536*Sqrt[2]*ArcTa
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/49152
```

**fricas [A]** time = 0.89, size = 125, normalized size = 0.84

$$\frac{1}{12288}\left(6144\,x^4-21120\,x^3+79840\,x^2-300404\,x+1561161\right)\sqrt{2\,x^2-x+3}+\frac{5627989}{32768}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

**giac [A]** time = 0.22, size = 129, normalized size = 0.87

$$\frac{1}{12288}\left(4\left(8\left(12\left(16x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2x^2-x+3}+\frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
- x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2
- x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*
x^2 - x + 3)))
```

**maple [A]** time = 0.01, size = 127, normalized size = 0.85

$$\frac{\left(2x^2-x+3\right)^{\frac{3}{2}}x^2}{4}-\frac{47\left(2x^2-x+3\right)^{\frac{3}{2}}x}{64}-\frac{5627989\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384}-\frac{11001\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x)
```

```
[Out] 1/4*(2*x^2-x+3)^(3/2)*x^2-47/64*(2*x^2-x+3)^(3/2)*x+1925/768*(2*x^2-x+3)^(3
/2)-20211/4096*(4*x-1)*(2*x^2-x+3)^(1/2)-5627989/16384*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))+3667/32*(2*(x+5/2)^2-11*x-19/2)^(1/2)-11001/32*2^(1/2)*a
rctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

**maxima [A]** time = 1.01, size = 128, normalized size = 0.86

$$\frac{1}{4}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{47}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{1925}{768}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{20211}{1024}\sqrt{2x^2-x+3}\,x-\frac{5627989}{16384}\sqrt{2}\ \operatorname{arsinh}\left(\frac{4}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(2
*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(22/
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096*sqr
t(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{2\,x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5), x
)
```

## 3.327 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx$

**Optimal**. Leaf size=149

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+\frac{23}{}$$

```
[Out] -541/384*(2*x^2-x+3)^(3/2)-3667/576*(2*x^2-x+3)^(3/2)/(5+2*x)+5/64*(5+2*x)*
(2*x^2-x+3)^(3/2)-2551847/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+23920
1/768*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/18432*(19
96953-333380*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.24, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{64}(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}-\frac{541}{384}\left(2x^2-x+3\right)^{3/2}-\frac{(1996953-333380x)\sqrt{2x^2-x+3}}{18432}+\frac{23}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -((1996953 - 333380*x)*Sqrt[3 - x + 2*x^2])/18432 - (541*(3 - x + 2*x^2)^(3
/2))/384 - (3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 -
 x + 2*x^2)^(3/2))/64 - (2551847*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2]
) + (239201*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(384*Sqr
t[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\sqrt{3-x+2x^2}\left(\frac{19341}{16}-\frac{6313x}{2}+4\ldots\right.}{5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{\sqrt{3-x+\ldots}}{\ldots}}{\ldots}$$

$$= -\frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{576(5+2x)} + \frac{5}{64}(5+2x)\left(3-\ldots\right.$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3\ldots}{\ldots}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3\ldots}{\ldots}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3\ldots}{\ldots}$$

$$= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432} - \frac{541}{384}\left(3-x+2x^2\right)^{3/2} - \frac{3\ldots}{\ldots}$$

**Mathematica [A]** time = 0.16, size = 98, normalized size = 0.66

$$\frac{7654432\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{4\sqrt{2x^2-x+3}\left(3840x^4-17344x^3+94936x^2-728410x-3539439\right)}{2x+5}-7655541\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right.}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x
]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 3840
*x^4))/(5 + 2*x) - 7655541*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 7654432*Sq
rt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/24576
```

**fricas [A]** time = 0.92, size = 143, normalized size = 0.96

$$\frac{7655541\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+7654432\sqrt{2}(2x+5)\log\left(\frac{24\sqrt{2}}{}\right.}{49152(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)
) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^2 -
x + 3))/(2*x + 5)
```

**giac [B]** time = 0.53, size = 531, normalized size = 3.56

$$\frac{1}{24576}\sqrt{2}\left(7654432\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)+7655541\log\left(\left|\sqrt{-\frac{}{2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5) +
```

```
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x + 5
)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5)) + 2*
(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*sgn(1/
(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 36/(2*x +
5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 31921920*(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 24458
13*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)
) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
6/(2*x + 5))^2 - 1)^4)
```

**maple [A]** time = 0.01, size = 152, normalized size = 1.02

$$\frac{5\left(2x^2-x+3\right)^{\frac{3}{2}}x}{32}+\frac{2551847\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}+\frac{239201\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{768}-\frac{391\left(2x^2-x\right.}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x)
```

```
[Out] 5/32*(2*x^2-x+3)^(3/2)*x-391/384*(2*x^2-x+3)^(3/2)+6001/2048*(4*x-1)*(2*x^2
-x+3)^(1/2)+2551847/8192*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(
x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-239201/2304*(-11*x+2*(x+5/2)^2-19/2)^
(1/2)+239201/768*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)
^2-19/2)^(1/2))+3667/2304*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 1.00, size = 132, normalized size = 0.89

$$\frac{5}{32}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{391}{384}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{6001}{512}\sqrt{2\,x^2-x+3}\,x+\frac{2551847}{8192}\sqrt{2}\ \operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{23}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sqr
t(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sq
rt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
```

```
sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^{2} - x + 3} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**2
, x)
```

## 3.328 $\int \frac{\sqrt{3-x+2x^2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

**Optimal**. Leaf size=151

$$\frac{357391\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}+\frac{5}{48}\left(2x^2-x+3\right)^{3/2}+\frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}-\frac{12670}{}$$

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-3667/1152*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391/82944*(
2*x^2-x+3)^(3/2)/(5+2*x)+117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
-12670805/110592*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+
5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.23, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{357391\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}+\frac{5}{48}\left(2x^2-x+3\right)^{3/2}+\frac{5(661065-110099x)\sqrt{2x^2-x+3}}{82944}-\frac{12670}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] (5*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2])/82944 + (5*(3 - x + 2*x^2)^(3/2
))/48 - (3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + (357391*(3 - x +
2*x^2)^(3/2))/(82944*(5 + 2*x)) + (117315*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512
*Sqrt[2]) - (12670805*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])]
)/(55296*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2}-\frac{1}{144}\int \frac{\sqrt{3-x+2x^2}\left(\frac{27681}{16}-\frac{14251x}{4}+\right.}{(5+2x)^2}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2}+\frac{357391\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)}+\frac{\int \frac{\sqrt{3-x+2x^2}\,(}{}}{}$$

$$= \frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{3667\left(3-x+2x^2\right)^{3/2}}{1152(5+2x)^2}+\frac{357391\left(3-x+2\right.}{82944(5+2x}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{366}{1}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{366}{1}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{366}{1}$$

$$= \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2}-\frac{366}{1}$$

**Mathematica [A]** time = 0.15, size = 98, normalized size = 0.65

$$\frac{-12670805\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\sqrt{2x^2-x+3}\left(3840x^4-25632x^3+272520x^2+2959330x+4880551\right)}{(2x+5)^2}+12670020\sqrt{2}\sinh^{-1}}{110592}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 38
40*x^4))/(5 + 2*x)^2 + 12670020*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 12670
805*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/110592
```

**fricas [A]** time = 0.84, size = 159, normalized size = 1.05

$$\frac{12670020\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+12670805\sqrt{2}\left(4x^2+2\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x + 25
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 11
53)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959330*
x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

**giac [B]** time = 0.24, size = 258, normalized size = 1.71

$$\frac{1}{768}(4(40x-467)x+19695)\sqrt{2x^2-x+3}+\frac{117315}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{12670805}{110592}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)*
log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*sqr
```

```
t(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805/11
0592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 7989
5946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10
*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

**maple [A]** time = 0.02, size = 158, normalized size = 1.05

$$-\frac{117315\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1024}-\frac{12670805\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{110592}+\frac{5\left(2x^{2}-x+3\right)^{\frac{3}{2}}}{48}-\frac{149\left(4x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(3/2)-149/256*(4*x-1)*(2*x^2-x+3)^(1/2)-117315/1024*2^(1/2
)*arcsinh(4/23*23^(1/2)*(x-1/4))+357391/165888/(x+5/2)*(-11*x+2*(x+5/2)^2-1
9/2)^(3/2)+12670805/331776*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-12670805/110592*2
^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-35
7391/331776*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/4608/(x+5/2)^2*(-11
*x+2*(x+5/2)^2-19/2)^(3/2)
```

**maxima [A]** time = 1.00, size = 143, normalized size = 0.95

$$\frac{5}{48}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}-\frac{149}{64}\sqrt{2\,x^{2}-x+3}\,x-\frac{117315}{1024}\sqrt{2}\ \operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{12670805}{110592}\sqrt{2}\ \operatorname{arsinh}\left(\frac{22}{23\,|2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*arc
sinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877/14
4*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25)
 + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\,\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**3
, x)
```

## 3.329 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

**Optimal**. Leaf size=158

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+}}{5971968}$$

```
[Out] -3667/1728*(2*x^2-x+3)^(3/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2*
x)^2-6467659/5971968*(2*x^2-x+3)^(3/2)/(5+2*x)-10939/512*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{6467659\left(2x^2-x+3\right)^{3/2}}{5971968(2x+5)}+\frac{158527\left(2x^2-x+3\right)^{3/2}}{82944(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}-\frac{(44378877-7400779x)\sqrt{2x^2-x+}}{5971968}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -((44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2])/5971968 - (3667*(3 - x + 2*x^
2)^(3/2))/(1728*(5 + 2*x)^3) + (158527*(3 - x + 2*x^2)^(3/2))/(82944*(5 + 2
*x)^2) - (6467659*(3 - x + 2*x^2)^(3/2))/(5971968*(5 + 2*x)) - (10939*ArcSi
nh[(1 - 4*x)/Sqrt[23]])/(256*Sqrt[2]) + (170114729*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3981312*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} - \frac{1}{216}\int \frac{\sqrt{3-x+2x^2}\left(\frac{36021}{16}-3969x+\right.}{(5+2x)^3}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} + \frac{\int \frac{\sqrt{3-x+2x^2}\,(}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3} + \frac{158527\left(3-x+2x^2\right)^{3/2}}{82944(5+2x)^2} - \frac{6467659\,(3}{597196}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$

$$= -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{1728(5+2x)^3}$$

**Mathematica [A]** time = 0.16, size = 98, normalized size = 0.62

$$\frac{170114729\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\sqrt{2x^2-x+3}\left(414720x^4-5453568x^3-97682900x^2-329667508x-327735797\right)}{(2x+5)^3}-170123328\sqrt{}}{7962624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 5453568
*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170123328*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[
23]] + 170114729*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/7
962624
```

**fricas [A]** time = 0.66, size = 173, normalized size = 1.09

$$170123328\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+17011472$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17
) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 54535
68*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/(8*x^
3 + 60*x^2 + 150*x + 125)
```

**giac [B]** time = 0.26, size = 304, normalized size = 1.92

$$\frac{1}{128}\sqrt{2x^2-x+3}\,(20x-413)-\frac{10939}{512}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs(-
2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552*sqr
t(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 9206213116*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 4948195
2947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^3
```

**maple [A]** time = 0.01, size = 165, normalized size = 1.04

$$\frac{10939\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{512}+\frac{170114729\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{7962624}+\frac{5\left(4x-1\right)\sqrt{2x^{2}-x+3}}{128}-\frac{64676}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x)
```

```
[Out] 5/128*(4*x-1)*(2*x^2-x+3)^(1/2)+10939/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-
1/4))-6467659/11943936/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-170114729/238
87872*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+170114729/7962624*2^(1/2)*arctanh(1/12
*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+6467659/23887872*(4*x
-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+158527/331776/(x+5/2)^2*(-11*x+2*(x+5/2)
^2-19/2)^(3/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

**maxima [A]** time = 0.98, size = 160, normalized size = 1.01

$$\frac{5}{32}\sqrt{2x^{2}-x+3}\,x+\frac{10939}{512}\sqrt{2}\ \operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{170114729}{7962624}\sqrt{2}\ \operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\left|2x+5\right|}-\frac{17\sqrt{23}}{23\left|2x+5\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x +
 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) - 36
67/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/82944
*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2 - x
+ 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^{2}-x+3}\ \left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{\left(2x+5\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\,\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**4,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**4
, x)
```

# 3.330 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

**Optimal**. Leaf size=165

$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x}}{95551488(2x+5)}$$

```
[Out] -3667/2304*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+2
*x)^3-9363383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+7/95551488*(52836655+9616196*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)
```

**Rubi [A]** time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}


$$-\frac{9363383\left(2x^2-x+3\right)^{3/2}}{23887872(2x+5)^2}+\frac{593771\left(2x^2-x+3\right)^{3/2}}{497664(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}+\frac{7(9616196x+52836655)\sqrt{2x^2-x}}{95551488(2x+5)}$$


Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] (7*(52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(95551488*(5 + 2*x)) - (3667
*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + (593771*(3 - x + 2*x^2)^(3/2))
/(497664*(5 + 2*x)^3) - (9363383*(3 - x + 2*x^2)^(3/2))/(23887872*(5 + 2*x)
^2) + (259*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) - (4640586097*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1146617856*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\sqrt{3-x+2x^2}\left(\frac{44361}{16}-\frac{17501x}{4}+\right.}{(5+2x)^4}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4} + \frac{593771\left(3-x+2x^2\right)^{3/2}}{497664(5+2x)^3} - \frac{9363383\left(3\right.}{2388787}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4}$$

$$= \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{2304(5+2x)^4}$$

**Mathematica [A]** time = 0.18, size = 98, normalized size = 0.59

$$\frac{-4640586097\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(238878720x^4+6105343976x^3+31323229164x^2+62847867486x+44676885233\right)}{(2x+5)^4}}{2293235712}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 + 6
105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640219136*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] - 4640586097*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x
 + 4*x^2])])/2293235712
```

**fricas [A]** time = 0.84, size = 189, normalized size = 1.15

$$\frac{4640219136\,\sqrt{2}\left(16\,x^{4}+160\,x^{3}+600\,x^{2}+1000\,x+625\right)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(4\,x-1)-32\,x^{2}+16\,x-25\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625
)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 46405
86097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*
sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 62847867486*
x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x
+ 625)
```

**giac [B]** time = 0.37, size = 327, normalized size = 1.98

$$-\frac{1}{2293235712}\,\sqrt{2}\left(4640586097\,\log\left(12\,\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^{2}}+1}+\frac{72}{2\,x+5}-11\right)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)+46402191\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^
2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4640
219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))
*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 835793
*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) - 46456
9597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 179159040
*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x +
5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(
2*x + 5))^2 - 1))
```

**maple [A]** time = 0.01, size = 167, normalized size = 1.01

$$-\frac{259\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}-\frac{4640586097\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{2293235712}+\frac{4640586097\sqrt{-11x+2\left(x+\frac{5}{2}\right.}}{6879707136}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x)
```

```
[Out] 4640586097/6879707136*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+593771/3981312/(x+5/2)
^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-1
9/2)^(3/2)-9363383/95551488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-201573
155/6879707136*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+201573155/3439853568/
(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-4640586097/2293235712*2^(1/2)*arctan
h(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-259/128*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 1.02, size = 181, normalized size = 1.10

$$-\frac{259}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{4640586097}{2293235712}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{16828343}{47775744}\sqrt{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] -259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/2293
235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2
*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*x^2
- x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^2 - x
 + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x + 3)/(2
*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\,\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**5,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**5
, x)
```

## 3.331 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

**Optimal**. Leaf size=165

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x}}{6879707136(2x+5)^2}$$

```
[Out] -3667/2880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+2
*x)^4-38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^(1/
2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/6879707136*(4583087983+3174439702*x)*(2*x^2
-x+3)^(1/2)/(5+2*x)^2
```

**Rubi [A]** time = 0.23, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{38732321\left(2x^2-x+3\right)^{3/2}}{179159040(2x+5)^3}+\frac{711961\left(2x^2-x+3\right)^{3/2}}{829440(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}-\frac{(3174439702x+4583087983)\sqrt{2x}}{6879707136(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(6879707136*(5 + 2*x)^2)
 - (3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + (711961*(3 - x + 2*x^2
)^(3/2))/(829440*(5 + 2*x)^4) - (38732321*(3 - x + 2*x^2)^(3/2))/(179159040
*(5 + 2*x)^3) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2]) + (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(82556485632*Sqrt[2
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\sqrt{3-x+2x^2}\left(\frac{52701}{16}-\frac{9563x}{2}+\right.}{(5+2x)^5}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} + \frac{\int \frac{\sqrt{3-x+2x^2}\left(\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{2880(5+2x)^5} + \frac{711961\left(3-x+2x^2\right)^{3/2}}{829440(5+2x)^4} - \frac{38732321\left(3\right.}{1791590}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

$$= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667\left(3-x+2\right.}{2880(5+2x}$$

**Mathematica [A]** time = 0.20, size = 98, normalized size = 0.59

$$\frac{64477987315\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}\left(186470433136x^4+1285267446304x^3+3919478861832x^2+5608297138216x+311\right.}{(2x+5)^5}}{825564856320}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832*
x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64497254400*Sqrt
[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 64477987315*Sqrt[2]*ArcTanh[(17 - 22*x)/(
12*Sqrt[6 - 2*x + 4*x^2])])/825564856320
```

**fricas [A]** time = 0.89, size = 203, normalized size = 1.23

$$\frac{64497254400 \sqrt{2} \left(32 x^5 + 400 x^4 + 2000 x^3 + 5000 x^2 + 6250 x + 3125\right) \log \left(-4 \sqrt{2} \sqrt{2 x^2 - x + 3} (4 x - 1) - 32\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 1285267446304
*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^2 - x
+ 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

**giac [B]** time = 0.28, size = 387, normalized size = 2.35

$$-\frac{5}{64} \sqrt{2} \log \left(-2 \sqrt{2} \left(\sqrt{2} x - \sqrt{2 x^2 - x + 3}\right) + 1\right) + \frac{12895597463}{165112971264} \sqrt{2} \log \left(\left|-2 \sqrt{2} x + \sqrt{2} + 2 \sqrt{2 x^2 - x + 3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] -5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 12895
597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt
(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 37631893001870
16*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 2507056315485
767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^5
```

**maple [A]** time = 0.01, size = 188, normalized size = 1.14

$$\frac{5\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}+\frac{12895597463\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{165112971264}-\frac{12895597463\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}}}{495338913792}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x)
```

```
[Out] -12895597463/495338913792*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/92160/(x+5/2)
^5*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-38732321/1433272320/(x+5/2)^3*(-11*x+2*(x
+5/2)^2-19/2)^(3/2)+711961/13271040/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2
)+46569601/6879707136/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+562688629/49
5338913792*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-562688629/247669456896/(x
+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+12895597463/165112971264*2^(1/2)*arcta
nh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+5/64*2^(1/2)*a
rcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 1.04, size = 222, normalized size = 1.35

$$\frac{5}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{12895597463}{165112971264}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{46569601}{3439853568}\sqrt{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] 5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/1651129
71264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*
x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x +
3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711961/
829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) -
38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 4
6569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 562688629/68
79707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\ \left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\,\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**6,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**6
, x)
```

## 3.332 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

**Optimal**. Leaf size=169

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{1172725(\cdots}{33022\cdots}$$

```
[Out] -3667/3456*(2*x^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2*
x)^5-5703277/39813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*x^
2-x+3)^(3/2)/(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(1/2
)/(2*x^2-x+3)^(1/2))*2^(1/2)-1172725/330225942528*(17-22*x)*(2*x^2-x+3)^(1/
2)/(5+2*x)^2
```

**Rubi [A]** time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1650, 806, 720, 724, 206}

$$\frac{87677717\left(2x^2-x+3\right)^{3/2}}{8599633920(2x+5)^3}-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{39813120(2x+5)^4}+\frac{92239\left(2x^2-x+3\right)^{3/2}}{138240(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}-\frac{1172725(\cdots}{33022\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] (-1172725*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(330225942528*(5 + 2*x)^2) - (36
67*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + (92239*(3 - x + 2*x^2)^(3/2)
)/(138240*(5 + 2*x)^5) - (5703277*(3 - x + 2*x^2)^(3/2))/(39813120*(5 + 2*x
)^4) + (87677717*(3 - x + 2*x^2)^(3/2))/(8599633920*(5 + 2*x)^3) - (2697267
5*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3962711310336*Sqr
t[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
```

```
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\,\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{\sqrt{3-x+2x^2}\left(\frac{61041}{16}-\frac{20751x}{4}+\right.}{(5+2x)^6}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} + \int \frac{\sqrt{3-x+2x^2}\,(\ldots}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\,(3-}{39813120}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{92239\left(3-x+2x^2\right)^{3/2}}{138240(5+2x)^5} - \frac{5703277\,(3-}{39813120}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{922}{}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{922}{}$$

$$= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{3456(5+2x)^6} + \frac{922}{}$$

**Mathematica [A]** time = 0.17, size = 91, normalized size = 0.54

$$\frac{24\sqrt{2x^2-x+3}\,\left(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-\right.}{39627113103360(2x+5)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2 +
 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5) - 134863375*Sqrt[2]
*(5 + 2*x)^6*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/(396271131033
60*(5 + 2*x)^6)
```

**fricas [A]** time = 0.82, size = 156, normalized size = 0.92

$$\frac{134863375\sqrt{2}\left(64x^6+960x^5+6000x^4+20000x^3+37500x^2+37500x+15625\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)}{4x^2+20}\right.}{79254226206720\left(64x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^
3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x
 - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(271409942624*
x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 27245373694*x
 - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*
x^3 + 37500*x^2 + 37500*x + 15625)
```

**giac [B]** time = 0.26, size = 405, normalized size = 2.40

$$-\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] -26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sq
rt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(16506981498400*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^10 + 2263923918689840*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 90221
2326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519861840*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 35
6306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 82348353128195465*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1
1)^6
```

**maple [A]** time = 0.02, size = 195, normalized size = 1.15

$$-\frac{26972675\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{7925422620672}+\frac{26972675\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{23776267862016}-\frac{3667\left(-11x+2\left(x+\frac{5}{2}\right)^2\right.}{221184\left(x+\frac{5}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x)
```

```
[Out] 26972675/23776267862016*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/221184/(x+5/2)^
6*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+92239/4423680/(x+5/2)^5*(-11*x+2*(x+5/2)^2
-19/2)^(3/2)+87677717/68797071360/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-
5703277/637009920/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-1172725/33022594
2528/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+12899975/23776267862016*(4*x-
1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-12899975/11888133931008/(x+5/2)*(-11*x+2*
(x+5/2)^2-19/2)^(3/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(-11*x+17
/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 1.05, size = 250, normalized size = 1.48

$$\frac{26972675}{7925422620672}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}-\frac{}{3456\left(64\,x^6+960\,x^5+6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/2
3*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 3667/
3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500
*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*
x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 - x +
3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/8599633920*
(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556485632*
(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*sqrt(2*x^
2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

```
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 - x + 3}\,\left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**7,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**7
, x)
```

## 3.333 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx$

**Optimal**. Leaf size=194

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-36$$

```
[Out] -3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^7+948341/1741824*(2*x^2-x+3)^(3/2)/(5+
2*x)^6-1464037/13934592*(2*x^2-x+3)^(3/2)/(5+2*x)^5+19414831/4013162496*(2*
x^2-x+3)^(3/2)/(5+2*x)^4+246159769/866843099136*(2*x^2-x+3)^(3/2)/(5+2*x)^3
-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2)
)*2^(1/2)-12568315/23776267862016*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2
```

**Rubi [A]** time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1650, 834, 806, 720, 724, 206}

$$\frac{246159769\left(2x^2-x+3\right)^{3/2}}{866843099136(2x+5)^3}+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{4013162496(2x+5)^4}-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{13934592(2x+5)^5}+\frac{948341\left(2x^2-x+3\right)^{3/2}}{1741824(2x+5)^6}-36$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] (-12568315*(17 - 22*x)*Sqrt[3 - x + 2*x^2])/(23776267862016*(5 + 2*x)^2) -
(3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + (948341*(3 - x + 2*x^2)^(
3/2))/(1741824*(5 + 2*x)^6) - (1464037*(3 - x + 2*x^2)^(3/2))/(13934592*(5
+ 2*x)^5) + (19414831*(3 - x + 2*x^2)^(3/2))/(4013162496*(5 + 2*x)^4) + (24
6159769*(3 - x + 2*x^2)^(3/2))/(866843099136*(5 + 2*x)^3) - (289071245*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(285315214344192*Sqrt[2]
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
```

```
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\sqrt{3-x+2x^2}\left(\frac{69381}{16}-5594x+\right.}{(5+2x)^7}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} + \frac{\int \frac{\sqrt{3-x+2x^2}\,(}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\,(3}{1393459}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\,(3}{1393459}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{948341\left(3-x+2x^2\right)^{3/2}}{1741824(5+2x)^6} - \frac{1464037\,(3}{1393459}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{9}{}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{9}{}$$

$$= -\frac{12568315(17-22x)\sqrt{3-x+2x^2}}{23776267862016(5+2x)^2} - \frac{3667\left(3-x+2x^2\right)^{3/2}}{4032(5+2x)^7} + \frac{9}{}$$

**Mathematica [A]** time = 0.20, size = 96, normalized size = 0.49

$$\frac{24\sqrt{2x^2-x+3}\left(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+1471668\right.}{3994413000818688(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 14716683780036*
x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574342
277056*x^6) - 2023498715*Sqrt[2]*(5 + 2*x)^7*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/(3994413000818688*(5 + 2*x)^7)
```

**fricas [A]** time = 0.71, size = 171, normalized size = 0.88

$$\frac{2023498715\,\sqrt{2}\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125\right)\log\left(-\right.}{7988}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 70
000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)
) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 410580
10262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*sqrt(2
*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 2
62500*x^2 + 218750*x + 78125)
```

**giac [B]** time = 0.29, size = 456, normalized size = 2.35

$$-\frac{289071245}{570630428688384}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)+\frac{289071245}{570630428688384}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950391
7760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(sqrt(
2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 7961
35370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 2531523139171005408*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
- 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 162047
0696060604668100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3196254593191113265*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)/(2*(sqrt(2)*x
```

```
- sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^7
```

**maple [A]** time = 0.02, size = 216, normalized size = 1.11

$$-\frac{289071245\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{570630428688384}+\frac{289071245\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}{1711891286065152}-\frac{3667\left(-11x+2\left(x+\frac{5}{2}\right)^{2}\right.}{516096\left(x+\frac{5}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x)
```

```
[Out] 289071245/1711891286065152*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/516096/(x+5/
2)^7*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+948341/111476736/(x+5/2)^6*(-11*x+2*(x+
5/2)^2-19/2)^(3/2)-1464037/445906944/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(3/
2)+246159769/6934744793088/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+1941483
1/64210599936/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-12568315/23776267862
016/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+138251465/1711891286065152*(4*
x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-138251465/855945643032576/(x+5/2)*(-11*
x+2*(x+5/2)^2-19/2)^(3/2)-289071245/570630428688384*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 1.04, size = 301, normalized size = 1.55

$$\frac{289071245}{570630428688384}\sqrt{2}\ \operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{12568315}{11888133931008}\sqrt{2\,x^{2}-x+3}-\frac{}{4032\left(128\,x^{7}+224\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)*(2*x^2-x+3)^(1/2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] 289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3) -
 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^
4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2 - x
 + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x
+ 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*
x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3)^(3/2)
/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/866843099136*(2*x^
2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066965504*(2
```

```
*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/23776267862016*sqrt(2*x
^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{(2\,x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
[Out] int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2-x+3}\,\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)*(2*x**2-x+3)**(1/2)/(5+2*x)**8,x)
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**8
, x)
```

## 3.334 $\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$

**Optimal**. Leaf size=166

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+66$$

```
[Out] -92727/131072*(1-4*x)*(2*x^2-x+3)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^(
5/2)-1121/2304*(5+2*x)^3*(2*x^2-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5/2
)-3/143360*(661397+215900*x)*(2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh(1/
23*(1-4*x)*23^(1/2))*2^(1/2)-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1653, 779, 612, 619, 215}

$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4-\frac{1121\left(2x^2-x+3\right)^{5/2}(2x+5)^3}{2304}+\frac{69415\left(2x^2-x+3\right)^{5/2}(2x+5)^2}{32256}-\frac{3(215900x+66$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-6398163*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (92727*(1 - 4*x)*(3 - x
+ 2*x^2)^(3/2))/131072 + (69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/32256 -
(1121*(5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))/2304 + (5*(5 + 2*x)^4*(3 - x + 2*x
^2)^(5/2))/144 - (3*(661397 + 215900*x)*(3 - x + 2*x^2)^(5/2))/143360 - (14
7157749*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2}+\frac{1}{288}\int (5+2x)\left(3-x\right.$$

$$= -\frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304}+\frac{5}{144}(5+2x)^4\left(3-x\right.$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3-\right.}{2304}$$

$$= \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256}-\frac{1121(5+2x)^3\left(3-\right.}{2304}$$

$$= -\frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072}+\frac{69415(5+2x)^2\left(3-\right.}{32256}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-\right.}{131072}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-\right.}{131072}$$

$$= -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{92727(1-4x)\left(3-\right.}{131072}$$

**Mathematica [A]** time = 0.18, size = 80, normalized size = 0.48

$$\frac{4\sqrt{2x^2-x+3}\left(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290\right.}{2642411520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 12669
290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 2926837760*
x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/26
42411520
```

**fricas [A]** time = 0.81, size = 93, normalized size = 0.56

$$\frac{1}{660602880}\left(1468006400\,x^8+2926837760\,x^7+1033175040\,x^6+12117893120\,x^5+379086848\,x^4+126692901\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] 1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 12117893120
*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x + 1
592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**giac [A]** time = 0.19, size = 88, normalized size = 0.53

$$\frac{1}{660602880}\left(4\left(8\left(4\left(16\left(20\left(8\left(28\left(160\,x+319\right)x+3153\right)x+295847\right)x+185101\right)x+98978829\right)x+152207433\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] 1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x +
 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqrt(
2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.02, size = 134, normalized size = 0.81

$$\frac{5\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{4}}{9}+\frac{479\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{3}}{288}+\frac{2005\left(2x^{2}-x+3\right)^{\frac{5}{2}}x^{2}}{8064}+\frac{5645\left(2x^{2}-x+3\right)^{\frac{5}{2}}x}{21504}+\frac{147157749\sqrt{2}\ \text{ar}}{838}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 120809/143360*(2*x^2-x+3)^(5/2)+5/9*x^4*(2*x^2-x+3)^(5/2)+479/288*x^3*(2*x^
2-x+3)^(5/2)+2005/8064*x^2*(2*x^2-x+3)^(5/2)+5645/21504*x*(2*x^2-x+3)^(5/2)
+92727/131072*(4*x-1)*(2*x^2-x+3)^(3/2)+147157749/8388608*2^(1/2)*arcsinh(4
/23*23^(1/2)*(x-1/4))+6398163/2097152*(4*x-1)*(2*x^2-x+3)^(1/2)
```

**maxima [A]** time = 0.98, size = 155, normalized size = 0.93

$$\frac{5}{9}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x^{4}+\frac{479}{288}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x^{3}+\frac{2005}{8064}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x^{2}+\frac{5645}{21504}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}x+\frac{120809}{143360}\left(2\,x^{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/80
64*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 120809/
143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 92727/
131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 14715
7749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/2097152*sqr
t(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

```
[Out] int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

```
[Out] Integral((2*x + 5)*(2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)
, x)
```

# 3.335 $\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx$

**Optimal**. Leaf size=147

$$\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x^2-x+3\right)^{3/2}}{65536}-\frac{593193(}{}$$

```
[Out] -8597/65536*(1-4*x)*(2*x^2-x+3)^(3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/3584
*x*(2*x^2-x+3)^(5/2)+23/448*x^2*(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2
)-13643439/4194304*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-593193/1048576*(1
-4*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{23}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{125\left(2x^2-x+3\right)^{5/2}x}{3584}+\frac{1167\left(2x^2-x+3\right)^{5/2}}{14336}-\frac{8597(1-4x)\left(2x\right.}{6553}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (-593193*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 - (8597*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/65536 + (1167*(3 - x + 2*x^2)^(5/2))/14336 + (125*x*(3 - x +
2*x^2)^(5/2))/3584 + (23*x^2*(3 - x + 2*x^2)^(5/2))/448 + (5*x^3*(3 - x + 2
*x^2)^(5/2))/16 - (13643439*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)dx = \frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(32+16x+3x\right.$$

$$= \frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int \left(3-\right.$$

$$= \frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{5}{16}x^3\left(3-x\right.$$

$$= \frac{1167\left(3-x+2x^2\right)^{5/2}}{14336}+\frac{125x\left(3-x+2x^2\right)^{5/2}}{3584}+\frac{23}{448}x^2\left(3-x\right.$$

$$= -\frac{8597(1-4x)\left(3-x+2x^2\right)^{3/2}}{65536}+\frac{1167\left(3-x+2x^2\right)^{5/2}}{14336}+\frac{125}{}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3}}{65536}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3}}{65536}$$

$$= -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576}-\frac{8597(1-4x)\left(3-x+2x^2\right)^{3}}{65536}$$

**Mathematica [A]** time = 0.12, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}\left(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x\right.}{29360128}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
- 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/29360128
```

**fricas [A]** time = 0.82, size = 88, normalized size = 0.60

$$\frac{1}{7340032}\left(9175040\,x^7-7667712\,x^6+29335552\,x^5-7497728\,x^4+27023744\,x^3+3845856\,x^2+27845612\,x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

```
[Out] 1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 2702374
4*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 13643439/
8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*
x - 25)
```

**giac [A]** time = 0.33, size = 83, normalized size = 0.56

$$\frac{1}{7340032}\left(4\left(8\left(4\left(16\left(4\left(8\left(140\,x-117\right)x+3581\right)x-3661\right)x+211123\right)x+120183\right)x+6961403\right)x-1663407\right)\sqrt{2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

```
[Out] 1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)*
x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/419430
4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.00, size = 117, normalized size = 0.80

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x^3}{16}+\frac{23\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{448}+\frac{125\left(2x^2-x+3\right)^{\frac{5}{2}}x}{3584}+\frac{13643439\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{1167\left(2x^2\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

```
[Out] 1167/14336*(2*x^2-x+3)^(5/2)+5/16*(2*x^2-x+3)^(5/2)*x^3+23/448*(2*x^2-x+3)^
(5/2)*x^2+125/3584*(2*x^2-x+3)^(5/2)*x+8597/65536*(4*x-1)*(2*x^2-x+3)^(3/2)
+13643439/4194304*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+593193/1048576*(4*
x-1)*(2*x^2-x+3)^(1/2)
```

**maxima [A]** time = 1.00, size = 138, normalized size = 0.94

$$\frac{5}{16}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2\,x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/358
4*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384*(
2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/262144*s
```

```
qrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x
- 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
[Out] int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2), x)
```

# 3.336 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

**Optimal**. Leaf size=172

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-x-\right.}{12288}$$

```
[Out] 1/12288*(500141-123060*x)*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311/
448*(5+2*x)*(2*x^2-x+3)^(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+1622009981/
262144*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/65536*(141051019-23482924*x)*(2*x^2
-x+3)^(1/2)
```

**Rubi [A]** time = 0.27, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}-\frac{311}{448}(2x+5)\left(2x^2-x+3\right)^{5/2}+\frac{3505}{896}\left(2x^2-x+3\right)^{5/2}+\frac{(500141-123060x)\left(2x^2-x-\right.}{12288}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

```
[Out] ((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/65536 + ((500141 - 123060*x)
*(3 - x + 2*x^2)^(3/2))/12288 + (3505*(3 - x + 2*x^2)^(5/2))/896 - (311*(5
+ 2*x)*(3 - x + 2*x^2)^(5/2))/448 + (5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/1
12 + (1622009981*ArcSinh[(1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2]) - (99009*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(8*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx = \frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(573-\right.}{}$$

$$= -\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2}+\frac{5}{112}(5+2x)^2\left(3-x+2x^2\right)^{5/2}$$

$$= \frac{3505}{896}\left(3-x+2x^2\right)^{5/2}-\frac{311}{448}(5+2x)\left(3-x+2x^2\right)^{5/2}+\frac{5}{112}(5$$

$$= \frac{(500141-123060x)\left(3-x+2x^2\right)^{3/2}}{12288}+\frac{3505}{896}\left(3-x+2x^2\right)^{5/2}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x}{1228}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x}{1228}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x}{1228}$$

$$= \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536}+\frac{(500141-123060x}{1228}$$

**Mathematica [A]** time = 0.18, size = 101, normalized size = 0.59

$$\frac{-34065432576\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+4\sqrt{2x^2-x+3}\left(983040x^6-3710976x^5+14493696x^4-46476672x^3\right.}{5505024}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x
]

[Out] (4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 46476672
*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 34062209601*Sqrt[2]*ArcSi
```

```
nh[(1 - 4*x)/Sqrt[23]] - 34065432576*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6
 - 2*x + 4*x^2])])/5505024
```

**fricas [A]** time = 0.62, size = 135, normalized size = 0.78

$$\frac{1}{1376256}\left(983040\,x^6-3710976\,x^5+14493696\,x^4-46476672\,x^3+159973408\,x^2-609499532\,x+314940325\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fri
cas")
```

```
[Out] 1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 1599734
08*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/524288*
sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) +
 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x
^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

**giac [A]** time = 0.23, size = 139, normalized size = 0.81

$$\frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40\,x-151\right)x+2359\right)x-121033\right)x+4999169\right)x-152374883\right)x+3149403255\right)\sqrt{2\,x^2-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gia
c")
```

```
[Out] 1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)*
x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sqrt
(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(2)*
log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqrt(2)
*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

**maple [A]** time = 0.01, size = 183, normalized size = 1.06

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x^2}{28}-\frac{111\left(2x^2-x+3\right)^{\frac{5}{2}}x}{224}-\frac{1622009981\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{262144}-\frac{99009\sqrt{2}\;\operatorname{arctanh}\left(\frac{\left(-11x\right.}{12\sqrt{-11x}}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

```
[Out] 5/28*(2*x^2-x+3)^(5/2)*x^2-111/224*(2*x^2-x+3)^(5/2)*x+1395/896*(2*x^2-x+3)
^(5/2)-10255/4096*(4*x-1)*(2*x^2-x+3)^(3/2)-707595/65536*(4*x-1)*(2*x^2-x+3
)^(1/2)-1622009981/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+3667/96*(-
11*x+2*(x+5/2)^2-19/2)^(3/2)-40337/512*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/
2)+33003/16*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-99009/16*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 1.00, size = 157, normalized size = 0.91

$$\frac{5}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{111}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{10255}{1024}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{500141}{12288}\left(2x^2-x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="max
ima")
```

```
[Out] 5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/896
*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/12288*
(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981/26
2144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2)*ar
csinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 141051
019/65536*sqrt(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
, x)
```

## 3.337 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

**Optimal**. Leaf size=172

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-$$

```
[Out] -1/18432*(909513-226052*x)*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-3667
/576*(2*x^2-x+3)^(5/2)/(5+2*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/131
072*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-1/32768*(85448933-14243732*x)*(2*x^2-x
+3)^(1/2)
```

**Rubi [A]** time = 0.28, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{5}{96}(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}-\frac{839}{960}\left(2x^2-x+3\right)^{5/2}-\frac{(909513-226052x)\left(2x^2-x+3\right)^{3/2}}{18432}-$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

```
[Out] -((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/32768 - ((909513 - 226052*x)
*(3 - x + 2*x^2)^(3/2))/18432 - (839*(3 - x + 2*x^2)^(5/2))/960 - (3667*(3
- x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + (5*(5 + 2*x)*(3 - x + 2*x^2)^(5/2))/9
6 - (982669459*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2]) + (959625*ArcTa
nh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(64*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^2}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} - \frac{1}{72}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{26675}{16} - 499\right.}{5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)\left(3-x+2x^2\right)^{5/2} - \frac{\int \frac{(3-}{}}{}$$

$$= -\frac{839}{960}\left(3-x+2x^2\right)^{5/2} - \frac{3667\left(3-x+2x^2\right)^{5/2}}{576(5+2x)} + \frac{5}{96}(5+2x)$$

$$= -\frac{(909513-226052x)\left(3-x+2x^2\right)^{3/2}}{18432} - \frac{839}{960}\left(3-x+2x^2\right)^{5/2}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-22605}{18}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-22605}{18}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-22605}{18}$$

$$= -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768} - \frac{(909513-22605}{18}$$

**Mathematica [A]** time = 0.21, size = 108, normalized size = 0.63

$$\frac{14739840000\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{4\sqrt{2x^2-x+3}\left(409600x^6-1798144x^5+8283904x^4-35369408x^3+182033816x^2-1404323114x-6\right.}{2x+5}}{1966080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 35369
408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 14740041885*
Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 14739840000*Sqrt[2]*ArcTanh[(17 - 22*
x)/(12*Sqrt[6 - 2*x + 4*x^2])])/1966080
```

**fricas [A]** time = 0.88, size = 153, normalized size = 0.89

$$\frac{14740041885\sqrt{2}\,(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+14739840000\sqrt{2}\,(2x+5)\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="f
ricas")
```

```
[Out] 1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 +
20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3 + 18
2033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

**giac [B]** time = 0.45, size = 707, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="g
iac")
```

```
[Out] 1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147400
41885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*
sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(
```

```
1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) + 331789824
00*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn(1/(2*x +
5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))
^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1
) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 171324610560*(sqrt(-11
/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 600592
81615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x
 + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(2*x + 5)))/((sqrt(-
11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^6)
```

**maple [A]** time = 0.01, size = 208, normalized size = 1.21

$$\frac{5\left(2x^2-x+3\right)^{\frac{5}{2}}x}{48}+\frac{982669459\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}+\frac{959625\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{128}-\frac{589\left(2x^2\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

```
[Out] 5/48*(2*x^2-x+3)^(5/2)*x-589/960*(2*x^2-x+3)^(5/2)+9059/6144*(4*x-1)*(2*x^2
-x+3)^(3/2)+208357/32768*(4*x-1)*(2*x^2-x+3)^(1/2)+982669459/131072*2^(1/2)
*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^
(5/2)-106625/2304*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+1637/16*(4*x-1)*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)-319875/128*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+959625/128*2
^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+36
67/2304*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

**maxima [A]** time = 1.01, size = 161, normalized size = 0.94

$$\frac{5}{48}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x-\frac{589}{960}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{9059}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{185827}{6144}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{3560933}{8192}\sqrt{2\,x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="m
axima")
```

```
[Out] 5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(2
*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*sq
rt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 1
7/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 3667/32*
(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**2, x)
```

# 3.338 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx$

**Optimal**. Leaf size=174

$$\frac{438065\left(2x^2-x+3\right)^{5/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}\left(2x^2-x+3\right)^{5/2}+\frac{(2154633-534617x)\left(2x^2-x+3\right)^{3/2}}{82944}+\frac{(3$$

```
[Out] 1/82944*(2154633-534617*x)*(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/11
52*(2*x^2-x+3)^(5/2)/(5+2*x)^2+438065/82944*(2*x^2-x+3)^(5/2)/(5+2*x)+12934
2063/32768*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24576*(33741483-5623292*x)*
(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 1653, 814, 843, 619, 215, 724, 206}

$$\frac{438065\left(2x^2-x+3\right)^{5/2}}{82944(2x+5)}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}+\frac{1}{16}\left(2x^2-x+3\right)^{5/2}+\frac{(2154633-534617x)\left(2x^2-x+3\right)^{3/2}}{82944}+\frac{(3$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

```
[Out] ((33741483 - 5623292*x)*Sqrt[3 - x + 2*x^2])/24576 + ((2154633 - 534617*x)*
(3 - x + 2*x^2)^(3/2))/82944 + (3 - x + 2*x^2)^(5/2)/16 - (3667*(3 - x + 2*
x^2)^(5/2))/(1152*(5 + 2*x)^2) + (438065*(3 - x + 2*x^2)^(5/2))/(82944*(5 +
 2*x)) + (129342063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2]) - (8083915
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1024*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^3}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2}-\frac{1}{144}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{35015}{16}-\frac{215\cdots}{\cdots}\right.}{(5+2x\cdots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2}+\frac{438065\left(3-x+2x^2\right)^{5/2}}{82944(5+2x)}+\frac{\int \frac{(3-x+2\cdots}{\cdots}}{\cdots}$$

$$= \frac{1}{16}\left(3-x+2x^2\right)^{5/2}-\frac{3667\left(3-x+2x^2\right)^{5/2}}{1152(5+2x)^2}+\frac{438065\left(3-x\cdots\right.}{82944(5+\cdots}$$

$$= \frac{(2154633-534617x)\left(3-x+2x^2\right)^{3/2}}{82944}+\frac{1}{16}\left(3-x+2x^2\right)^{5/2}\cdots$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617\cdots}{829\cdots}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617\cdots}{829\cdots}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617\cdots}{829\cdots}$$

$$= \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576}+\frac{(2154633-534617\cdots}{829\cdots}$$

**Mathematica** [A] time = 0.22, size = 108, normalized size = 0.62

$$\frac{-129342640\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{4\sqrt{2x^2-x+3}\left(8192x^6-43520x^5+253312x^4-1620944x^3+16667188x^2+181223072x+298966737\right)}{(2x+5)^2}}{32768}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3
,x]
```

```
[Out] ((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*x
^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 129342063*Sqrt[2]*Ar
cSinh[(1 - 4*x)/Sqrt[23]] - 129342640*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[
6 - 2*x + 4*x^2])])/32768
```

**fricas** [A] time = 0.92, size = 169, normalized size = 0.97

$$\frac{129342063\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+129342640\sqrt{2}\left(4x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="f
ricas")
```

```
[Out] 1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620944*
x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*x^2 +
 20*x + 25)
```

**giac** [A] time = 0.30, size = 268, normalized size = 1.54

$$\frac{1}{8192}(4(8(4(16x-165)x+4879)x-263469)x+8460377)\sqrt{2x^2-x+3}+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="g
iac")
```

```
[Out] 1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
- x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 170
996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) -
11)^2
```

**maple [A]** time = 0.02, size = 214, normalized size = 1.23

$$-\frac{129342063\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}-\frac{8083915\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{2048}+\frac{\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{16}+\frac{8083915}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

```
[Out] 1/16*(2*x^2-x+3)^(5/2)+8083915/6144*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+8083915/
331776*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/4608/(x+5/2)^2*(-11*x+2*(x+5/2)^
2-19/2)^(5/2)-438065/331776*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+438065/1
65888/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-343745/6144*(4*x-1)*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)-8083915/2048*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)
/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-149/512*(4*x-1)*(2*x^2-x+3)^(3/2)-12934206
3/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-10281/8192*(4*x-1)*(2*x^2-x+
3)^(1/2)
```

**maxima [A]** time = 1.00, size = 172, normalized size = 0.99

$$\frac{1}{16}\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}-\frac{149}{128}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}x+\frac{46691}{4608}\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2\,x^{2}-x+3\right)^{\frac{5}{2}}}{1152\left(4\,x^{2}+20\,x+25\right)}-\frac{1405823}{6144}\sqrt{2\,x^{2}-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="m
axima")
```

```
[Out] 1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*(
2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25)
- 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4/23
*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(23)*
```

```
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x^2 -
x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**3, x)
```

## 3.339 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx$

**Optimal**. Leaf size=181

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2\right.}{17915904}$$

```
[Out] -1/17915904*(138006843-34265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)^
(5/2)/(5+2*x)^3+556255/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365/17915904
*(2*x^2-x+3)^(5/2)/(5+2*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^
(1/2)+517762327/442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^
(1/2)-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {1650, 814, 843, 619, 215, 724, 206}

$$-\frac{32865365\left(2x^2-x+3\right)^{5/2}}{17915904(2x+5)}+\frac{556255\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^2}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}-\frac{(138006843-34265045x)\left(2x^2\right.}{17915904}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

```
[Out] -((135068604 - 22512089*x)*Sqrt[3 - x + 2*x^2])/331776 - ((138006843 - 3426
5045*x)*(3 - x + 2*x^2)^(3/2))/17915904 - (3667*(3 - x + 2*x^2)^(5/2))/(172
8*(5 + 2*x)^3) + (556255*(3 - x + 2*x^2)^(5/2))/(248832*(5 + 2*x)^2) - (328
65365*(3 - x + 2*x^2)^(5/2))/(17915904*(5 + 2*x)) - (19176431*ArcSinh[(1 -
4*x)/Sqrt[23]])/(8192*Sqrt[2]) + (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]
*Sqrt[3 - x + 2*x^2])])/(221184*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^4}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3}-\frac{1}{216}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{43355}{16}-\frac{116}{}\right.}{(5+2x}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3}+\frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2}+\frac{\int \frac{(3-x+2}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{1728(5+2x)^3}+\frac{556255\left(3-x+2x^2\right)^{5/2}}{248832(5+2x)^2}-\frac{328653}{179}$$

$$= -\frac{(138006843-34265045x)\left(3-x+2x^2\right)^{3/2}}{17915904}-\frac{3667\left(3-x+\right.}{1728(5+}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776}-\frac{(138006843-3}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776}-\frac{(138006843-3}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776}-\frac{(138006843-3}{}$$

$$= -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776}-\frac{(138006843-3}{}$$

**Mathematica [A]** time = 0.22, size = 108, normalized size = 0.60

$$\frac{517762327\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{12\sqrt{2x^2-x+3}\left(46080x^6-315648x^5+2626848x^4-33595416x^3-594798908x^2-2006873194x-1994\right.}{(2x+5)^3}}{442368}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4
,x]
```

```
[Out] ((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 3359
5416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 517763637*S
qrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 517762327*Sqrt[2]*ArcTanh[(17 - 22*x)/
(12*Sqrt[6 - 2*x + 4*x^2])])/442368
```

**fricas [A]** time = 0.97, size = 183, normalized size = 1.01

$$\frac{517763637\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+517762327}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="f
ricas")
```

```
[Out] 1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8*x
^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
- 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 315648*x
^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 1994650739
)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

**giac [B]** time = 0.27, size = 314, normalized size = 1.73

$$\frac{1}{4096}\left(4\left(8\left(20x-287\right)x+23341\right)x-1004633\right)\sqrt{2x^2-x+3}-\frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="g
iac")
```

```
[Out] 1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 191
76431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sq
rt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 2031
4214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

**maple [A]** time = 0.02, size = 221, normalized size = 1.22

$$\frac{19176431\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16384}+\frac{517762327\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{442368}-\frac{517762327\sqrt{-11x+2\left(x-\right.}}{1327104}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

```
[Out] -517762327/1327104*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-517762327/71663616*(-11*x
+2*(x+5/2)^2-19/2)^(3/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2
)+556255/995328/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+32865365/71663616*
(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-32865365/35831808/(x+5/2)*(-11*x+2*(
x+5/2)^2-19/2)^(5/2)+22400309/1327104*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2
)+517762327/442368*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/
2)^2-19/2)^(1/2))+5/256*(4*x-1)*(2*x^2-x+3)^(3/2)+19176431/16384*2^(1/2)*ar
csinh(4/23*23^(1/2)*(x-1/4))+345/4096*(4*x-1)*(2*x^2-x+3)^(1/2)
```

**maxima [A]** time = 1.04, size = 189, normalized size = 1.04

$$\frac{5}{64}\left(2x^{2}-x+3\right)^{\frac{3}{2}}x-\frac{1094743}{497664}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{1728\left(8x^{3}+60x^{2}+150x+125\right)}+\frac{556255\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{248832\left(4x^{2}+20x+25\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="m
axima")
```

```
[Out] 5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667/
1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832*(
2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 - x +
 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 5
17762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995328*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2x^{2}-x+3\right)^{3/2}\left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{\left(2x+5\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^4 - x^3 + 3x^2 + x + 2\right)}{(2x + 5)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**4, x)
```

## 3.340 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx$

**Optimal**. Leaf size=188

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x^2\right.}{95551488(2x+5)}$$

```
[Out] 1/95551488*(762984903+67865260*x)*(2*x^2-x+3)^(3/2)/(5+2*x)-3667/2304*(2*x^
2-x+3)^(5/2)/(5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995/2
3887872*(2*x^2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)-8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^
(1/2))*2^(1/2)+1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.27, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 812, 814, 843, 619, 215, 724, 206}

$$-\frac{14477995\left(2x^2-x+3\right)^{5/2}}{23887872(2x+5)^2}+\frac{224815\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^3}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}+\frac{(67865260x+762984903)\left(2x^2\right.}{95551488(2x+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

```
[Out] ((2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2])/31850496 + ((762984903 + 6
7865260*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)) - (3667*(3 - x + 2*x
^2)^(5/2))/(2304*(5 + 2*x)^4) + (224815*(3 - x + 2*x^2)^(5/2))/(165888*(5 +
 2*x)^3) - (14477995*(3 - x + 2*x^2)^(5/2))/(23887872*(5 + 2*x)^2) + (43256
5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqrt[2]) - (8969688643*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(21233664*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^5}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{51695}{16}-\frac{2483}{4}\right.}{(5+2x)}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} + \frac{\int \frac{\left(3-x+2x^2\right.}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2304(5+2x)^4} + \frac{224815\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^3} - \frac{14477995}{23887}$$

$$= \frac{(762984903+67865260x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)} - \frac{3667\left(3-x+2x\right.}{2304(5+2x)}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+6}{955}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+6}{955}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+6}{955}$$

$$= \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+6}{955}$$

**Mathematica [A]** time = 0.23, size = 108, normalized size = 0.57

$$\frac{-8969688643\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(2949120x^6-29270016x^5+468043776x^4+11761910072x^3+60528581892x^2+121\right.}{(2x+5)^4}}{42467328}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^4
```

```
+ 8969667840*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 8969688643*Sqrt[2]*ArcTa
nh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/42467328
```

**fricas [A]** time = 0.96, size = 199, normalized size = 1.06

$$\frac{8969667840\,\sqrt{2}\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="f
ricas")
```

```
[Out] 1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8969688
643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 2
5)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3 + 60
528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(16*x^4
+ 160*x^3 + 600*x^2 + 1000*x + 625)
```

**giac [B]** time = 0.42, size = 503, normalized size = 2.68

$$-\frac{1}{42467328}\,\sqrt{2}\left(8969688643\,\log\left(12\,\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}+\frac{72}{2\,x+5}-11\right)\mathrm{sgn}\left(\frac{1}{2\,x+5}\right)+8969667840\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="g
iac")
```

```
[Out] -1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896966
7840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*s
gn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 42907*sg
n(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54131203
9*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824*(80624
```

```
1*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5
)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn
(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
 + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5) + 36/(2*x +
 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x + 5)))/((sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3)
```

**maple** [A] time = 0.02, size = 204, normalized size = 1.09

$$-\frac{432565\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}-\frac{8969688643\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{42467328}+\frac{8969688643\sqrt{-11x+2\left(x\right.}}{127401984}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

```
[Out] 8969688643/127401984*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+8969688643/6879707136*(
-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/36864/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)
^(5/2)+224815/1327104/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-14477995/955
51488/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-593321753/6879707136*(4*x-1)
*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+593321753/3439853568/(x+5/2)*(-11*x+2*(x+5/
2)^2-19/2)^(5/2)-389975609/127401984*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
-8969688643/42467328*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+
5/2)^2-19/2)^(1/2))-432565/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima** [A] time = 1.04, size = 210, normalized size = 1.12

$$\frac{16966315}{47775744}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2304\left(16x^{4}+160x^{3}+600x^{2}+1000x+625\right)}+\frac{224815\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{165888\left(8x^{3}+60x^{2}+150x+125\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="m
axima")
```

```
[Out] 16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/(
16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3)^
(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)^(5
/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 432565
/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/4246732
8*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
```

```
5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2*x^2 -
x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**5, x)
```

# 3.341 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx$

**Optimal**. Leaf size=195

$$-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{11943936(2x+5)^3}+\frac{158527\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)\left(2x^2-x\right.}{95551488(2x+5)^2}$$

```
[Out] 1/95551488*(246012435+44773976*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2*
x^2-x+3)^(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-3730507/
11943936*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+
3)^(1/2))*2^(1/2)-1/127401984*(5658774871+1028823716*x)*(2*x^2-x+3)^(1/2)/(
5+2*x)
```

**Rubi [A]** time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 812, 843, 619, 215, 724, 206}

$$-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{11943936(2x+5)^3}+\frac{158527\left(2x^2-x+3\right)^{5/2}}{165888(2x+5)^4}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}+\frac{(44773976x+246012435)\left(2x^2-x\right.}{95551488(2x+5)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

```
[Out] -((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(127401984*(5 + 2*x)) +
((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(95551488*(5 + 2*x)^2) - (
3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + (158527*(3 - x + 2*x^2)^(5
/2))/(165888*(5 + 2*x)^4) - (3730507*(3 - x + 2*x^2)^(5/2))/(11943936*(5 +
2*x)^3) - (23775*ArcSinh[(1 - 4*x)/Sqrt[23]])/(512*Sqrt[2]) + (70991525167*
ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1528823808*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^6}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} - \frac{1}{360}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{60035}{16} - 661\ldots\right.}{(5+2x)\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} + \int \frac{\left(3-x+2x^2\ldots\right.}{\ldots}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{2880(5+2x)^5} + \frac{158527\left(3-x+2x^2\right)^{5/2}}{165888(5+2x)^4} - \frac{3730507\ldots}{119439\ldots}$$

$$= \frac{(246012435+44773976x)\left(3-x+2x^2\right)^{3/2}}{95551488(5+2x)^2} - \frac{3667\left(3-x+2x\ldots\right.}{2880(5+2x)\ldots}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+\ldots}{95\ldots}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+\ldots}{95\ldots}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+\ldots}{95\ldots}$$

$$= -\frac{(5658774871+1028823716x)\sqrt{3-x+2x^2}}{127401984(5+2x)} + \frac{(246012435+\ldots}{95\ldots}$$

**Mathematica [A]** time = 0.24, size = 108, normalized size = 0.55

$$\frac{354957625835\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(1592524800x^6-30496849920x^5-1023534029552x^4-7117092892448x^3-2159\ldots\right.}{(2x+5)^5}}{15288238080}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 215904397970
64*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 15925248
```

```
00*x^6))/(5 + 2*x)^5 - 354958848000*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 3
54957625835*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/152882
38080
```

**fricas [A]** time = 1.03, size = 213, normalized size = 1.09

$$\frac{354958848000\sqrt{2}\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="f
ricas")
```

```
[Out] 1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 1
6*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 +
6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 +
 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 30496849920*x^5
 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872393829
992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 +
 5000*x^2 + 6250*x + 3125)
```

**giac [B]** time = 0.32, size = 406, normalized size = 2.08

$$\frac{1}{256}\sqrt{2x^2-x+3}(20x-633)-\frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="g
iac")
```

```
[Out] 1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*log
(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057647
616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1
/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270334576
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 700
60241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 97730658088823
```

```
880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

**maple [A]** time = 0.02, size = 225, normalized size = 1.15

$$\frac{23775\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{1024}+\frac{70991525167\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{3057647616}-\frac{70991525167\sqrt{-11x+2\left(x+\right.}}{9172942848}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

```
[Out] -70991525167/9172942848*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-70991525167/49533891
3792*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/92160/(x+5/2)^5*(-11*x+2*(x+5/2)^2
-19/2)^(5/2)+158527/2654208/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-373050
7/95551488/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+134077495/6879707136/(x
+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+4698578717/495338913792*(4*x-1)*(-11
*x+2*(x+5/2)^2-19/2)^(3/2)-4698578717/247669456896/(x+5/2)*(-11*x+2*(x+5/2)
^2-19/2)^(5/2)+3086715581/9172942848*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
+70991525167/3057647616*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*
(x+5/2)^2-19/2)^(1/2))+23775/1024*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 1.06, size = 251, normalized size = 1.29

$$-\frac{134077495}{3439853568}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{2880\left(32x^{5}+400x^{4}+2000x^{3}+5000x^{2}+6250x+3125\right)}+\frac{15852}{165888\left(16x^{4}+16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="m
axima")
```

```
[Out] -134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5/
2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/165888
*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373050
7/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 134077495
/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/22932357
12*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2
*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(2*x^2 -
x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x+5\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**6, x)
```

# 3.342 $\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^7}\,dx$

**Optimal**. Leaf size=195

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(2x\right.}{13759414272(2x+5)^3}$$

```
[Out] -1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-3667/3
456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)^5-1
4087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)+1/55037657088*(151764102421+27596573612*x)*(2*x
^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1650, 810, 812, 843, 619, 215, 724, 206}

$$-\frac{14087245\left(2x^2-x+3\right)^{5/2}}{71663616(2x+5)^4}+\frac{182165\left(2x^2-x+3\right)^{5/2}}{248832(2x+5)^5}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}-\frac{(6793718806x+9802984711)\left(2x\right.}{13759414272(2x+5)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

```
[Out] ((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(55037657088*(5 + 2*x)
) - ((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(13759414272*(5 + 2
*x)^3) - (3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + (182165*(3 - x +
 2*x^2)^(5/2))/(248832*(5 + 2*x)^5) - (14087245*(3 - x + 2*x^2)^(5/2))/(716
63616*(5 + 2*x)^4) + (369*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2]) - (190
3976002333*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(66045188
5056*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} - \frac{1}{432}\int \frac{(3-x+2x^2)^{3/2}\left(\frac{68375}{16} - \frac{28\cdots}{}\right.}{(5+2\cdots}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} + \frac{\int \frac{(3-x+2\cdots}{}}{}$$

$$= -\frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{1408724\cdots}{7166\cdots}$$

$$= -\frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667(3-\cdots}{3456(\cdots}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(98029847\cdots}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(98029847\cdots}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(98029847\cdots}{}$$

$$= \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(98029847\cdots}{}$$

**Mathematica [A]** time = 0.25, size = 108, normalized size = 0.55

$$\frac{-1903976002333\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + \frac{24\sqrt{2x^2-x+3}\left(275188285440x^6+11854023276320x^5+103803827945872x^4+422554114\cdots\right.}{(2x+5)^6}}{1320903770112}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7
,x]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 9102568424
73992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^5
```

```
+ 275188285440*x^6))/(5 + 2*x)^6 + 1903958949888*Sqrt[2]*ArcSinh[(1 - 4*x)/
Sqrt[23]] - 1903976002333*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*
x^2])])/1320903770112
```

**fricas [A]** time = 1.06, size = 229, normalized size = 1.17

$$\frac{1903958949888\sqrt{2}\left(64x^{6}+960x^{5}+6000x^{4}+20000x^{3}+37500x^{2}+37500x+15625\right)\log\left(4\sqrt{2}\sqrt{2x^{2}-x+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="f
ricas")
```

```
[Out] 1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000
*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6000
*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2
 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48
*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255411485
6528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 458411625354581)*sqrt
(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625)
```

**giac [B]** time = 0.34, size = 452, normalized size = 2.32

$$\frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^{2}-x+3}\right)+1\right)-\frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^{2}-x+3}\right|\right)+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="g
iac")
```

```
[Out] 369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 190
3976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x -
 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/110075
314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^1
1 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268408334
24*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^8 + 88313821135911024*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6
```

```
 - 397211131697032056*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 1170889
7232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 1280348486072849113 8*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 1259303319786757723 4*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3)) + 589526263249780195)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

**maple [A]** time = 0.02, size = 246, normalized size = 1.26

$$-\frac{369\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}-\frac{1903976002333\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{1320903770112}+\frac{1903976002333\sqrt{-11x+}}{39627113103}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

```
[Out] 1903976002333/3962711310336*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+1903976002333/21
3986410758144*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/221184/(x+5/2)^6*(-11*x+2
*(x+5/2)^2-19/2)^(5/2)+182165/7962624/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(5
/2)-14087245/1146617856/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+149610673/
41278242816/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-3607708597/29720334827
52/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-125860542215/213986410758144*(4
*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(3/2)+125860542215/106993205379072/(x+5/2)*(
-11*x+2*(x+5/2)^2-19/2)^(5/2)-82772668391/3962711310336*(4*x-1)*(-11*x+2*(x
+5/2)^2-19/2)^(1/2)-1903976002333/1320903770112*2^(1/2)*arctanh(1/12*(-11*x
+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-369/256*2^(1/2)*arcsinh(4/23
*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 1.07, size = 297, normalized size = 1.52

$$\frac{3607708597}{1486016741376}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{3456\left(64x^{6}+960x^{5}+6000x^{4}+20000x^{3}+37500x^{2}+37500x+15625\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="m
axima")
```

```
[Out] 3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)^
(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 1562
5) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 500
0*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^4 +
```

```
160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x + 3)^(5
/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 - x + 3
)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 - x + 3)*
x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 190397600233
3/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3) + 1258605
42215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2} - x + 3\right)^{\frac{3}{2}} \left(5x^{4} - x^{3} + 3x^{2} + x + 2\right)}{\left(2x + 5\right)^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**7, x)
```

# 3.343 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

**Optimal**. Leaf size=195

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2\right.}{2293235712(2x+5)^4}$$

```
[Out] -1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/4032
*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-1930
441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^
2-x+3)^(1/2))*2^(1/2)-1/440301256704*(146583836191+101679102454*x)*(2*x^2-x
+3)^(1/2)/(5+2*x)^2
```

**Rubi [A]** time = 0.26, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 810, 843, 619, 215, 724, 206}

$$-\frac{1930441\left(2x^2-x+3\right)^{5/2}}{13934592(2x+5)^5}+\frac{114335\left(2x^2-x+3\right)^{5/2}}{193536(2x+5)^6}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}-\frac{(411822458x+463558457)\left(2x^2\right.}{2293235712(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

```
[Out] -((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(440301256704*(5 + 2
*x)^2) - ((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(2293235712*(5 +
 2*x)^4) - (3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + (114335*(3 - x
 + 2*x^2)^(5/2))/(193536*(5 + 2*x)^6) - (1930441*(3 - x + 2*x^2)^(5/2))/(13
934592*(5 + 2*x)^5) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2]) + (41276
0561351*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(52836150804
48*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)}{(5+2x)^8}\,dx = -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} - \frac{1}{504}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(\frac{76715}{16}-\frac{148}{}\right.}{(5+2}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} + \frac{\int \frac{(3-x+2}{}}{}$$

$$= -\frac{3667\left(3-x+2x^2\right)^{5/2}}{4032(5+2x)^7} + \frac{114335\left(3-x+2x^2\right)^{5/2}}{193536(5+2x)^6} - \frac{1930441}{13934}$$

$$= -\frac{(463558457+411822458x)\left(3-x+2x^2\right)^{3/2}}{2293235712(5+2x)^4} - \frac{3667\left(3-x\right.}{4032(5+}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558}{}$$

$$= -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} - \frac{(463558}{}$$

**Mathematica [A]** time = 0.26, size = 108, normalized size = 0.55

$$\frac{2889323929457\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) - \frac{24\sqrt{2x^2-x+3}\left(38463671680832x^6+402255822731712x^5+2069947287085104x^4+596632\right.}{(2x+5)^7}}{73970611126227}$$

Antiderivative was successfully verified.

```
[In] Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8
,x]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 99760653
67498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 40225582273171
2*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889476997120*Sqrt[2]*ArcSinh[(1
 - 4*x)/Sqrt[23]] + 2889323929457*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 -
2*x + 4*x^2])])/73970611126272
```

**fricas [A]** time = 0.91, size = 243, normalized size = 1.25

$$\frac{2889476997120\,\sqrt{2}\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="f
ricas")
```

```
[Out] 1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*(12
8*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750
*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 10
36*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582273171
2*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 9976065367498188*x^2
+ 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*x^7 + 22
40*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125
)
```

**giac [B]** time = 0.33, size = 489, normalized size = 2.51

$$-\frac{5}{128}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{412760561351}{10567230160896}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="g
iac")
```

```
[Out] -5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 4127
60561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x -
11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(11218973984
12224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 4826029630377670 4*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2)*x -
```

```
sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 -
 17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130120947372
46112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 106515880136064432096*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 9229208
0735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 1516171609382750
1349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(
2*x^2 - x + 3)) - 11)^7
```

**maple [A]** time = 0.02, size = 267, normalized size = 1.37

$$\frac{5\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{412760561351\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{10567230160896}-\frac{412760561351\sqrt{-11x+2\left(x+\right.}}{31701690482688}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

```
[Out] -412760561351/31701690482688*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-412760561351/17
11891286065152*(-11*x+2*(x+5/2)^2-19/2)^(3/2)-3667/516096/(x+5/2)^7*(-11*x+
2*(x+5/2)^2-19/2)^(5/2)+114335/12386304/(x+5/2)^6*(-11*x+2*(x+5/2)^2-19/2)^
(5/2)-1930441/445906944/(x+5/2)^5*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+7861079/91
72942848/(x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(5/2)-32967491/330225942528/(x+
5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(5/2)+769352975/23776267862016/(x+5/2)^2*(-
11*x+2*(x+5/2)^2-19/2)^(5/2)+27452157541/1711891286065152*(4*x-1)*(-11*x+2*
(x+5/2)^2-19/2)^(3/2)-27452157541/855945643032576/(x+5/2)*(-11*x+2*(x+5/2)^
2-19/2)^(5/2)+17957520133/31701690482688*(4*x-1)*(-11*x+2*(x+5/2)^2-19/2)^(
1/2)+412760561351/10567230160896*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/
(-11*x+2*(x+5/2)^2-19/2)^(1/2))+5/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)
```

**maxima [B]** time = 1.07, size = 348, normalized size = 1.78

$$-\frac{769352975}{11888133931008}\left(2x^{2}-x+3\right)^{\frac{3}{2}}-\frac{3667\left(2x^{2}-x+3\right)^{\frac{5}{2}}}{4032\left(128x^{7}+2240x^{6}+16800x^{5}+70000x^{4}+175000x^{3}+262500x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="m
axima")
```

```
[Out] -769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3)
^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^
2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 960*x
^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/13934592
*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3
125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2
+ 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^
2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 2
0*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/128*sqrt(2)
*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230160896*sqr
t(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) -
 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23776267862016
*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(2\,x^2-x+3\right)^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x+5\right)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

```
[Out] int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(2x^{2}-x+3\right)^{\frac{3}{2}}\left(5x^{4}-x^{3}+3x^{2}+x+2\right)}{\left(2x+5\right)^{8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)
**8, x)
```

# 3.344 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=120

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}\,(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}$$

```
[Out] -85429/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+761/256*(5+2*x)^2*(2*x^2
-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/16*(5+2*x)^4*(2*x^2-x+3)^
(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1653, 779, 619, 215}

$$\frac{1}{16}\sqrt{2x^2-x+3}\,(2x+5)^4-\frac{105}{128}\sqrt{2x^2-x+3}\,(2x+5)^3+\frac{761}{256}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{(4676x+19227)\sqrt{2x^2-x+3}}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/256 - (105*(5 + 2*x)^3*Sqrt[3 - x + 2
*x^2])/128 + ((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 - ((19227 + 4676*x)*Sqrt[
3 - x + 2*x^2])/2048 - (85429*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4096*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
```

```
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{1}{160}\int \frac{(5+2x)\left(-5055-4390x-5580x\right.}{\sqrt{3-x+2x^2}}$$

$$= -\frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} + \frac{\int \frac{(5+2x)(32}{}}{}$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

$$= \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x$$

**Mathematica [A]** time = 0.12, size = 60, normalized size = 0.50

$$\frac{4\sqrt{2x^2-x+3}\left(2048x^4+7040x^3+352x^2-6916x+2973\right)-85429\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 85
429*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/8192
```

**fricas [A]** time = 0.88, size = 73, normalized size = 0.61

$$\frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3}+\frac{85429}{16384}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
+ 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
 + 16*x - 25)
```

**giac [A]** time = 0.19, size = 68, normalized size = 0.57

$$\frac{1}{2048}\,(4\,(8\,(4\,(16\,x+55)x+11)x-1729)x+2973)\sqrt{2\,x^2-x+3}-\frac{85429}{8192}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3)
 - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.01, size = 95, normalized size = 0.79

$$\sqrt{2x^2-x+3}\;x^4+\frac{55\sqrt{2x^2-x+3}\;x^3}{16}+\frac{11\sqrt{2x^2-x+3}\;x^2}{64}-\frac{1729\sqrt{2x^2-x+3}\;x}{512}+\frac{85429\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right.}{8192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] x^4*(2*x^2-x+3)^(1/2)+55/16*x^3*(2*x^2-x+3)^(1/2)+11/64*x^2*(2*x^2-x+3)^(1/
2)-1729/512*x*(2*x^2-x+3)^(1/2)+2973/2048*(2*x^2-x+3)^(1/2)+85429/8192*2^(1
/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima** [A] time = 0.97, size = 96, normalized size = 0.80

$$\sqrt{2x^2-x+3}\,x^4+\frac{55}{16}\sqrt{2x^2-x+3}\,x^3+\frac{11}{64}\sqrt{2x^2-x+3}\,x^2-\frac{1729}{512}\sqrt{2x^2-x+3}\,x+\frac{85429}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}\,(4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
- x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsinh(
1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x
)
```

## 3.345 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{5}{8}\sqrt{2x^2-x+3}\,x^3-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] -6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-505/1024*(2*x^2-x+3)^(1/2
)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x+3)^(1/2)+5/8*x^3*(2*x^2-x+
3)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x^3+\frac{19}{96}\sqrt{2x^2-x+3}\,x^2-\frac{409}{768}\sqrt{2x^2-x+3}\,x-\frac{505\sqrt{2x^2-x+3}}{1024}-\frac{6863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-505*Sqrt[3 - x + 2*x^2])/1024 - (409*x*Sqrt[3 - x + 2*x^2])/768 + (19*x^2
*Sqrt[3 - x + 2*x^2])/96 + (5*x^3*Sqrt[3 - x + 2*x^2])/8 - (6863*ArcSinh[(1
 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{8}\int \frac{16+8x-21x^2+\frac{19x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{48}\int \frac{96-9x-\frac{409x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2x^2} + \frac{1}{192}\int \frac{\frac{2763}{4}-}{\sqrt{3-x}}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2}$$

$$= -\frac{505\sqrt{3-x+2x^2}}{1024} - \frac{409}{768}x\sqrt{3-x+2x^2} + \frac{19}{96}x^2\sqrt{3-x+2x^2} + \frac{5}{8}x^3\sqrt{3-x+2}$$

**Mathematica [A]** time = 0.07, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\left(1920x^3+608x^2-1636x-1515\right)-20589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[2
]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

**fricas [A]** time = 0.78, size = 68, normalized size = 0.67

$$\frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3}+\frac{6863}{8192}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/8192
*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**giac [A]** time = 0.20, size = 63, normalized size = 0.62

$$\frac{1}{3072}\,(4\,(8\,(60\,x+19)x-409)x-1515)\sqrt{2\,x^2-x+3}-\frac{6863}{4096}\,\sqrt{2}\,\log\left(-2\,\sqrt{2}\left(\sqrt{2}\,x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/4096
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

**maple [A]** time = 0.01, size = 79, normalized size = 0.78

$$\frac{5\sqrt{2x^2-x+3}\;x^3}{8}+\frac{19\sqrt{2x^2-x+3}\;x^2}{96}-\frac{409\sqrt{2x^2-x+3}\;x}{768}+\frac{6863\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{505\sqrt{2x^2-x+3}}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*(2*x^2-x+3)^(1/2)*x^3+19/96*(2*x^2-x+3)^(1/2)*x^2-409/768*(2*x^2-x+3)^(
1/2)*x-505/1024*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))
```

**maxima [A]** time = 0.96, size = 80, normalized size = 0.79

$$\frac{5}{8}\,\sqrt{2\,x^2-x+3}\,x^3+\frac{19}{96}\,\sqrt{2\,x^2-x+3}\,x^2-\frac{409}{768}\,\sqrt{2\,x^2-x+3}\,x+\frac{6863}{4096}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)-\frac{505}{1024}\,\sqrt{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt(
2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 505
/1024*sqrt(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/sqrt(2*x**2 - x + 3), x)
```

# 3.346 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=126

$$\frac{5}{48}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}\,(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657\,\mathrm{s}}{2}$$

```
[Out] 9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1669/128*(2*x^2-x+3)^(1/2)-337/192*
(5+2*x)*(2*x^2-x+3)^(1/2)+5/48*(5+2*x)^2*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1653, 843, 619, 215, 724, 206}

$$\frac{5}{48}\sqrt{2x^2-x+3}\,(2x+5)^2-\frac{337}{192}\sqrt{2x^2-x+3}\,(2x+5)+\frac{1669}{128}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{96\sqrt{2}}+\frac{9657\,\mathrm{s}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (1669*Sqrt[3 - x + 2*x^2])/128 - (337*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/192 +
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + (9657*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])
])/(96*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{1}{96}\int \frac{-2183-3054x-4092x^2-2696x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} + \frac{\int \frac{24504+128736x+16022}{(5+2x)\sqrt{3-x+2x^2}}}{3072}$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} +$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} -$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} -$$

$$= \frac{1669}{128}\sqrt{3-x+2x^2} - \frac{337}{192}(5+2x)\sqrt{3-x+2x^2} + \frac{5}{48}(5+2x)^2\sqrt{3-x+2x^2} +$$

**Mathematica [A]** time = 0.10, size = 81, normalized size = 0.64

$$\frac{4\sqrt{2x^2-x+3}\left(160x^2-548x+2637\right)-29336\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+28971\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 28971*Sqrt[2]*ArcSinh[(1
- 4*x)/Sqrt[23]] - 29336*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x
^2])])/1536
```

**fricas [A]** time = 0.74, size = 115, normalized size = 0.91

$$\frac{1}{384}\left(160\,x^2-548\,x+2637\right)\sqrt{2\,x^2-x+3}+\frac{9657}{1024}\,\sqrt{2}\,\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+\frac{36}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log(
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*sq
rt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25))
```

**giac [A]** time = 0.23, size = 119, normalized size = 0.94

$$\frac{1}{384}\left(4\left(40\,x-137\right)x+2637\right)\sqrt{2\,x^2-x+3}+\frac{9657}{512}\,\sqrt{2}\,\log\left(-4\,\sqrt{2}\,x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right)-\frac{3667}{192}\,\sqrt{2}\,\log\left(\left|-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gia
c")
```

```
[Out] 1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log(
-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(abs(
-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(abs
(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

**maple [A]** time = 0.01, size = 92, normalized size = 0.73

$$\frac{5\sqrt{2x^2-x+3}\;x^2}{12}-\frac{137\sqrt{2x^2-x+3}\;x}{96}-\frac{9657\sqrt{2}\;\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{512}-\frac{3667\sqrt{2}\;\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/12*(2*x^2-x+3)^(1/2)*x^2-137/96*(2*x^2-x+3)^(1/2)*x+879/128*(2*x^2-x+3)^(
1/2)-9657/512*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/192*2^(1/2)*arcta
nh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 0.98, size = 99, normalized size = 0.79

$$\frac{5}{12}\,\sqrt{2\,x^2-x+3}\,x^2-\frac{137}{96}\,\sqrt{2\,x^2-x+3}\,x-\frac{9657}{512}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\,\sqrt{23}\,x-\frac{1}{23}\,\sqrt{23}\right)+\frac{3667}{192}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\,\sqrt{23}\,x}{23\,\left|2\,x+5\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="max
ima")
```

```
[Out] 5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqrt
(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(22/
```

```
23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt(2*
x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)\sqrt{2x^2 - x + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3)),
 x)
```

## 3.347 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=126

$$\frac{5}{32}\sqrt{2x^2-x+3}\,(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-1}\left(\right.}{128\sqrt{2}}$$

```
[Out] -2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824*arctanh(1/24*
(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-243/64*(2*x^2-x+3)^(1/2)-3667/
576*(2*x^2-x+3)^(1/2)/(5+2*x)+5/32*(5+2*x)*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{5}{32}\sqrt{2x^2-x+3}\,(2x+5)-\frac{243}{64}\sqrt{2x^2-x+3}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}+\frac{158527\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6912\sqrt{2}}-\frac{2943\sinh^{-1}\left(\right.}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-243*Sqrt[3 - x + 2*x^2])/64 - (3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x))
+ (5*(5 + 2*x)*Sqrt[3 - x + 2*x^2])/32 - (2943*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(128*Sqrt[2]) + (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2
])])/(6912*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{1}{72}\int \frac{\frac{12007}{16}-1323x+486x^2-180x^3}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{30314-27216x+34992x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{2304}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{\int \frac{417472-}{(5+2x)\sqrt{}}}{18}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{2943}{128}\int$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} + \frac{158527}{576}$$

$$= -\frac{243}{64}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} + \frac{5}{32}(5+2x)\sqrt{3-x+2x^2} - \frac{2943\sinh}{128}$$

**Mathematica [A]** time = 0.12, size = 88, normalized size = 0.70

$$\frac{\frac{48\sqrt{2x^2-x+3}\left(180x^2-1287x-6176\right)}{2x+5}+158527\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-158922\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{13824}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((48*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158922*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 158527*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*S
qrt[6 - 2*x + 4*x^2])])/13824
```

**fricas [A]** time = 0.97, size = 133, normalized size = 1.06

$$\frac{158922\sqrt{2}\,(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+158527\sqrt{2}\,(2x+5)\log\left(\frac{24\sqrt{2}\sqrt{2x^2}}{}\right.}{27648\,(2x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25))
+ 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

**giac [B]** time = 0.40, size = 339, normalized size = 2.69

$$\frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{158922\log\left(\left|\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{6}{2x+5}\right.\right.}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5))
- 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 108*(33
93*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*(sqrt(
-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/(2*x +
5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))
```

**maple [A]** time = 0.01, size = 96, normalized size = 0.76

$$\frac{5\sqrt{2x^2-x+3}\,x}{16}+\frac{2943\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{158527\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{13824}-\frac{193\sqrt{2x^2-x+3}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)*x-193/64*(2*x^2-x+3)^(1/2)+2943/256*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+158
```

```
527/13824*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2
)^(1/2))
```

**maxima [A]** time = 0.98, size = 103, normalized size = 0.82

$$\frac{5}{16}\sqrt{2x^2-x+3}\,x+\frac{2943}{256}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{158527}{13824}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)-\frac{19}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/2
3*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqrt(2
*x^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x + 3
)), x)
```

## 3.348 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=128

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] 149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-1546507/663552*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+5/16*(2*x^2-x+3)^(1/2)-3667/115
2*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239/27648*(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1650, 1653, 843, 619, 215, 724, 206}

$$\frac{92239\sqrt{2x^2-x+3}}{27648(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}+\frac{5}{16}\sqrt{2x^2-x+3}-\frac{1546507\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{331776\sqrt{2}}+\frac{149\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (5*Sqrt[3 - x + 2*x^2])/16 - (3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2)
+ (92239*Sqrt[3 - x + 2*x^2])/(27648*(5 + 2*x)) + (149*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(32*Sqrt[2]) - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
 + 2*x^2])])/(331776*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} - \frac{1}{144}\int \frac{\frac{20347}{16}-\frac{6917x}{4}+972x^2-360x^3}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{647841}{16}-67392x+12960x^2}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{10368}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{\int \frac{\frac{777441}{2}-772416}{(5+2x)\sqrt{3-x+2}}}{82944}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{149}{32}\int \frac{1}{\sqrt{3-x}}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} - \frac{1546507\,\text{Subst}}{}$$

$$= \frac{5}{16}\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2} + \frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)} + \frac{149\sinh^{-1}\left(\frac{1-}{\sqrt{2}}\right.}{32\sqrt{2}}$$

**Mathematica [A]** time = 0.13, size = 88, normalized size = 0.69

$$\frac{\frac{24\sqrt{2x^2-x+3}\left(34560x^2+357278x+589187\right)}{(2x+5)^2} - 1546507\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right) + 1544832\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{663552}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((24*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 154
4832*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - 1546507*Sqrt[2]*ArcTanh[(17 - 22
*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/663552
```

**fricas [A]** time = 0.96, size = 149, normalized size = 1.16

$$\frac{1544832\sqrt{2}\left(4x^2+20x+25\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+1546507\sqrt{2}\left(4x^2+20x\right.}{1327104\left(4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 - x
 + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 25)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2 - x
 + 3))/(4*x^2 + 20*x + 25)
```

**giac [B]** time = 0.26, size = 248, normalized size = 1.94

$$\frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{154650}{66355}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1546
507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645)/(2*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^2
```

**maple [A]** time = 0.01, size = 102, normalized size = 0.80

$$-\frac{149\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{64}-\frac{1546507\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{663552}+\frac{5\sqrt{2x^2-x+3}}{16}+\frac{92239\sqrt{-11x+2}}{55296\left(x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/16*(2*x^2-x+3)^(1/2)-149/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+92239/
55296/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-1546507/663552*2^(1/2)*arctanh
```

```
(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))-3667/4608/(x+5/2
)^2*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 0.98, size = 114, normalized size = 0.89

$$-\frac{149}{64}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)+\frac{1546507}{663552}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{5}{16}\sqrt{2\,x^2-x+3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*s
qrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5))
 + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^3\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x + 3
)), x)
```

## 3.349 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=135

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sinh^{-}}{16}$$

```
[Out] -5/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/2
4*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)-3667/1728*(2*x^2-x+3)^(1/2)/
(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3163415/5971968*(2*x^2-
x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1650, 843, 619, 215, 724, 206}

$$-\frac{3163415\sqrt{2x^2-x+3}}{5971968(2x+5)}+\frac{394907\sqrt{2x^2-x+3}}{248832(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}+\frac{22389491\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{71663616\sqrt{2}}-\frac{5\sinh^{-}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + (394907*Sqrt[3 - x + 2*x^2
])/(248832*(5 + 2*x)^2) - (3163415*Sqrt[3 - x + 2*x^2])/(5971968*(5 + 2*x))
 - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) + (22389491*ArcTanh[(17 - 2
2*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(71663616*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}-\frac{1}{216}\int \frac{\frac{28687}{16}-\frac{4271x}{2}+1458x^2-540x^3}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}+\frac{\int \frac{\frac{1464275}{16}-\frac{413797x}{4}+38880x^2}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{31104}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}-\frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{\int \frac{\frac{111812}{16}}{(5+2x}}{2}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}-\frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}+\frac{5}{16}\int$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}-\frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}+\frac{223894}{}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2}-\frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{5\sinh^{-}}{16}$$

**Mathematica [A]** time = 0.15, size = 88, normalized size = 0.65

$$\frac{-\frac{24\sqrt{2x^2-x+3}\left(12653660x^2+44312764x+44369687\right)}{(2x+5)^3}+22389491\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)-22394880\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{143327232}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] ((-24*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x)
^3 - 22394880*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 22389491*Sqrt[2]*ArcTan
h[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^2])])/143327232
```

**fricas [A]** time = 0.85, size = 163, normalized size = 1.21

$$\frac{22394880\sqrt{2}\left(8x^3+60x^2+150x+125\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+22389491\sqrt{2}}{2866544}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(8*
x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17)
 - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 + 4431
2764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

**giac [B]** time = 0.26, size = 285, normalized size = 2.11

$$-\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)-\frac{22}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] -5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 22389
491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3
))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 27408
02468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

**maple [A]** time = 0.01, size = 109, normalized size = 0.81

$$\frac{5\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32}+\frac{22389491\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{143327232}-\frac{3163415\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{11943936\left(x+\frac{5}{2}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3163415/11943936/(x+5/2)*(-11*x
+2*(x+5/2)^2-19/2)^(1/2)+22389491/143327232*2^(1/2)*arctanh(1/12*(-11*x+17/
```

```
2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+394907/995328/(x+5/2)^2*(-11*x+2
*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3*(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 1.00, size = 131, normalized size = 0.97

$$\frac{5}{32}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{22389491}{143327232}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{3667\sqrt{2\,x^2-x}}{1728\,(8\,x^3+60\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] 5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)
) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 394907/2
48832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*x^2
- x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^4\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x + 3
)), x)
```

# 3.350 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=139

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh^{-1}}{2063912}$$

```
[Out] 2053207/41278242816*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/(5
+2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085/1719926784*(
2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi** [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1650, 806, 724, 206}

$$\frac{26800085\sqrt{2x^2-x+3}}{1719926784(2x+5)}-\frac{16295969\sqrt{2x^2-x+3}}{71663616(2x+5)^2}+\frac{513097\sqrt{2x^2-x+3}}{497664(2x+5)^3}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}+\frac{2053207\tanh^{-1}}{2063912}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```

```
[Out] (-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + (513097*Sqrt[3 - x + 2*x^2
])/(497664*(5 + 2*x)^3) - (16295969*Sqrt[3 - x + 2*x^2])/(71663616*(5 + 2*x
)^2) + (26800085*Sqrt[3 - x + 2*x^2])/(1719926784*(5 + 2*x)) + (2053207*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(20639121408*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} - \frac{1}{288}\int \frac{\frac{37027}{16}-\frac{10167x}{4}+1944x^2-720x^3}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} + \frac{\int \frac{\frac{2607829}{16}-\frac{295607x}{2}+77760x^2}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx}{62208}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} - \frac{\int \frac{1941\cdots}{1\cdots}}{(5+2\cdots}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800\cdots}{1719\cdots}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800\cdots}{1719\cdots}$$

$$= -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4} + \frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2} + \frac{26800\cdots}{1719\cdots}$$

**Mathematica** [A] time = 0.13, size = 81, normalized size = 0.58

$$\frac{2053207\sqrt{2}\,(2x+5)^4\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+24\sqrt{2x^2-x+3}\,\left(214400680x^3+43592076x^2-255525906x-2\right.}{41278242816(2x+5)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x
]
```

```
[Out] (24*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 21440068
0*x^3) + 2053207*Sqrt[2]*(5 + 2*x)^4*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x +
 4*x^2])])/(41278242816*(5 + 2*x)^4)
```

**fricas** [A] time = 0.95, size = 125, normalized size = 0.90

$$\frac{2053207\sqrt{2}\left(16x^4+160x^3+600x^2+1000x+625\right)\log\left(\frac{24\sqrt{2}\sqrt{2x^2-x+3}\,(22x-17)-1060x^2+1036x-1153}{4x^2+20x+25}\right)+48\left(2144\right.}{82556485632\left(16x^4+160x^3+600x^2+1000x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="f
ricas")
```

```
[Out] 1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*
log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)
/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x - 29
8655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

**giac** [A] time = 0.28, size = 164, normalized size = 1.18

$$\frac{1}{41278242816}\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}-\frac{792072}{(2x+5)\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}-\frac{16295969}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}+\frac{26800085}{\mathrm{sgn}\left(\frac{1}{2x+5}\right)}\right)\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
+ 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5) +
26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20532
07*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)/sgn
(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

**maple [A]** time = 0.01, size = 116, normalized size = 0.83

$$\frac{2053207\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{41278242816}+\frac{26800085\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}{3439853568\left(x+\frac{5}{2}\right)}-\frac{16295969\sqrt{-11x+2\left(x+\frac{5}{2}\right)}}{286654464\left(x+\frac{5}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 26800085/3439853568/(x+5/2)*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+2053207/41278242
816*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2
))-16295969/286654464/(x+5/2)^2*(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/36864/(
x+5/2)^4*(-11*x+2*(x+5/2)^2-19/2)^(1/2)+513097/3981312/(x+5/2)^3*(-11*x+2*(
x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 1.02, size = 149, normalized size = 1.07

$$-\frac{2053207}{41278242816}\sqrt{2}\ \operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{3667\sqrt{2\,x^2-x+3}}{2304\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)}+\frac{5}{497664}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3 +
600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2
 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25)
 + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^5\,\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

**sympy** **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^5 \sqrt{2x^2 - x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x + 3
)), x)
```

# 3.351 $\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=124

$$\frac{153}{16}\sqrt{2x^2-x+3}\,x^2+\frac{2645}{128}\sqrt{2x^2-x+3}\,x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}\,x^3+\frac{144217\,\mathrm{si}}{102}$$

```
[Out] 144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-4/23*(346-533*x)/(2*x^2-
x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128*x*(2*x^2-x+3)^(1/2)+153/16*
x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.15, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}\,x^3+\frac{153}{16}\sqrt{2x^2-x+3}\,x^2+\frac{2645}{128}\sqrt{2x^2-x+3}\,x-\frac{13153}{512}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{144217\,\mathrm{si}}{102}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) - (13153*Sqrt[3 - x + 2*x^2])/5
12 + (2645*x*Sqrt[3 - x + 2*x^2])/128 + (153*x^2*Sqrt[3 - x + 2*x^2])/16 +
(5*x^3*Sqrt[3 - x + 2*x^2])/4 + (144217*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*
Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-759-\frac{575x}{2}+805x^2+\frac{1219x^3}{2}+115x^4}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{-6072-2300x+540}{\sqrt{3-x+2}}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{1}{552}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{15}{16}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{15}{16}$$

$$= -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{15}{16}$$

**Mathematica [A]** time = 0.47, size = 74, normalized size = 0.60

$$\frac{3316991\sqrt{4x^2-2x+6}\,\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+4\left(29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165\right)}{47104\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2)
,x]
```

```
[Out] (4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x^5
) + 3316991*Sqrt[6 - 2*x + 4*x^2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/(47104*Sqrt[
3 - x + 2*x^2])
```

**fricas [A]** time = 0.99, size = 102, normalized size = 0.82

$$\frac{3316991\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(4\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(29440\,x^5+210496\,x^4+418\right.}{94208\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3 -
510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

**giac [A]** time = 0.22, size = 72, normalized size = 0.58

$$\frac{144217}{2048}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(8(20x+143)x+2273)x-11099)x+2124123)x-}{11776\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) +
 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 1616
165)/sqrt(2*x^2 - x + 3)
```

**maple [A]** time = 0.02, size = 132, normalized size = 1.06

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}+\frac{144217x}{1024\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}\ \mathrm{arcsin}}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 931255/94208*(4*x-1)/(2*x^2-x+3)^(1/2)-144217/2048*2^(1/2)*arcsinh(4/23*23^
(1/2)*(x-1/4))+5/2*x^5/(2*x^2-x+3)^(1/2)+143/8*x^4/(2*x^2-x+3)^(1/2)+2273/6
4*x^3/(2*x^2-x+3)^(1/2)-11099/256*x^2/(2*x^2-x+3)^(1/2)+144217/1024*x/(2*x^
2-x+3)^(1/2)-521655/4096/(2*x^2-x+3)^(1/2)
```

**maxima [A]** time = 0.97, size = 114, normalized size = 0.92

$$\frac{5x^5}{2\sqrt{2x^2-x+3}}+\frac{143x^4}{8\sqrt{2x^2-x+3}}+\frac{2273x^3}{64\sqrt{2x^2-x+3}}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}-\frac{144217}{2048}\sqrt{2}\ \mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/s
qrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3) -
1616165/11776/sqrt(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3
/2), x)
```

## 3.352 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=103

$$\frac{5}{6}\sqrt{2x^2-x+3}\,x^2+\frac{193}{48}\sqrt{2x^2-x+3}\,x+\frac{33}{64}\sqrt{2x^2-x+3}+\frac{373x-53}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] 3111/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/23*(-53+373*x)/(2*x^2-x+3
)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2-x+3)^(1/2)+5/6*x^2*(2*x^2-x
+3)^(1/2)
```

**Rubi [A]** time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{6}\sqrt{2x^2-x+3}\,x^2+\frac{193}{48}\sqrt{2x^2-x+3}\,x+\frac{33}{64}\sqrt{2x^2-x+3}-\frac{53-373x}{23\sqrt{2x^2-x+3}}+\frac{3111\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] -(53 - 373*x)/(23*Sqrt[3 - x + 2*x^2]) + (33*Sqrt[3 - x + 2*x^2])/64 + (193
*x*Sqrt[3 - x + 2*x^2])/48 + (5*x^2*Sqrt[3 - x + 2*x^2])/6 + (3111*ArcSinh[
(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{575}{4}+161x^2+\frac{115x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{1}{69}\int \frac{-\frac{1725}{2}-345x+\frac{4439x}{4}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{1}{276}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3}$$

$$= -\frac{53-373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3}$$

**Mathematica [A]** time = 0.19, size = 60, normalized size = 0.58

$$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}} - \frac{3111\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x
]
```

```
[Out] (-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*x
^2]) - (3111*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

**fricas [A]** time = 0.86, size = 97, normalized size = 0.94

$$\frac{214659\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(7360x^4+31832x^3-2162\right.}{35328\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

[Out] 1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 12260
7*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)

**giac [A]** time = 0.22, size = 67, normalized size = 0.65

$$\frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(40x+173)x-47)x+122607)x-3345}{4416\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")

[Out] 3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x + 3)

**maple [A]** time = 0.01, size = 115, normalized size = 1.12

$$\frac{5x^4}{3\sqrt{2x^2-x+3}}+\frac{173x^3}{24\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{\frac{10185x}{2944}-}{\sqrt{2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)

[Out] 10185/11776*(4*x-1)/(2*x^2-x+3)^(1/2)-3111/256*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+5/3/(2*x^2-x+3)^(1/2)*x^4+173/24/(2*x^2-x+3)^(1/2)*x^3-47/96/(2*
x^2-x+3)^(1/2)*x^2+3111/128/(2*x^2-x+3)^(1/2)*x+55/512/(2*x^2-x+3)^(1/2)

**maxima [A]** time = 0.97, size = 97, normalized size = 0.94

$$\frac{5x^4}{3\sqrt{2x^2-x+3}}+\frac{173x^3}{24\sqrt{2x^2-x+3}}-\frac{47x^2}{96\sqrt{2x^2-x+3}}-\frac{3111}{256}\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{40869x}{1472\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")

[Out] 5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/sq
rt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 408
69/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2)
, x)
```

# 3.353 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}} dx$

**Optimal**. Leaf size=82

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] 213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+1/92*(89+219*x)/(2*x^2-x+3)^
(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{5}{8}\sqrt{2x^2-x+3}\,x+\frac{27}{32}\sqrt{2x^2-x+3}+\frac{219x+89}{92\sqrt{2x^2-x+3}}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (27*Sqrt[3 - x + 2*x^2])/32 + (5*x*
Sqrt[3 - x + 2*x^2])/8 + (213*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{345}{16}+\frac{69x}{8}+\frac{115x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int \frac{-\frac{345}{2}+\frac{621x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213}{64}\int \frac{1}{\sqrt{3-x+2x^2}}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}-\frac{213\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\right.}{64\sqrt{46}}$$

$$= \frac{89+219x}{92\sqrt{3-x+2x^2}}+\frac{27}{32}\sqrt{3-x+2x^2}+\frac{5}{8}x\sqrt{3-x+2x^2}+\frac{213\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

**Mathematica** [A] time = 0.13, size = 55, normalized size = 0.67

$$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) - (213*ArcSin
h[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

**fricas** [A] time = 0.86, size = 92, normalized size = 1.12

$$\frac{4899\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(920x^3+782x^2+2511x+257\right.}{5888\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(2*
x^2 - x + 3))/(2*x^2 - x + 3)
```

**giac** [A] time = 0.22, size = 62, normalized size = 0.76

$$\frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(20x+17)x+2511)x+2575}{736\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/7
36*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

**maple** [A] time = 0.01, size = 98, normalized size = 1.20

$$\frac{5x^3}{4\sqrt{2x^2-x+3}}+\frac{17x^2}{16\sqrt{2x^2-x+3}}+\frac{213x}{64\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\ \operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{901}{256\sqrt{2x^2-x+3}}+\frac{\frac{123x}{1472}-}{\sqrt{2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4/(2*x^2-x+3)^(1/2)*x^3+17/16/(2*x^2-x+3)^(1/2)*x^2+213/64/(2*x^2-x+3)^(1
/2)*x+901/256/(2*x^2-x+3)^(1/2)+123/5888*(4*x-1)/(2*x^2-x+3)^(1/2)-213/128*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [A]** time = 0.95, size = 80, normalized size = 0.98

$$\frac{5\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{17\,x^2}{16\,\sqrt{2\,x^2-x+3}}-\frac{213}{128}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}\,(4\,x-1)\right)+\frac{2511\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{2575}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt(
2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 2575
/736/sqrt(2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

## 3.354 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

```
[Out] 39/32*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22*
x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/3312*(1191+917*x)/(2*x^2-x+3)^(1/2)
+5/8*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1646, 1653, 843, 619, 215, 724, 206}

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}+\frac{5}{8}\sqrt{2x^2-x+3}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{1728\sqrt{2}}+\frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/8 + (39
*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16*Sqrt[2]) - (3667*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1728*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{6739}{576}+\frac{69x}{8}+\frac{115x^2}{4}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{\frac{3611}{72}-\frac{897x}{2}}{(5+2x)\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{39}{16}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx + \frac{3667}{288}\int \frac{}{(5+}$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} - \frac{3667}{144}\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \frac{17-}{\sqrt{3-x}}\right.$$

$$= \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} - \frac{3667\tanh^{-1}\left(\frac{1}{12\sqrt{2}}\right.}{1728\sqrt{2}}$$

**Mathematica [A]** time = 0.35, size = 86, normalized size = 0.85

$$\frac{\frac{12\left(4140x^2-1153x+7401\right)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}} - 3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) + 3667\log(2x+5) - 4212\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{1728\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x
]
```

```
[Out] ((12*(7401 - 1153*x + 4140*x^2))/(23*Sqrt[3/2 - x/2 + x^2]) - 4212*ArcSinh[
(-1 + 4*x)/Sqrt[23]] + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 -
 2*x + 4*x^2]])/(1728*Sqrt[2])
```

**fricas [A]** time = 1.01, size = 149, normalized size = 1.48

$$\frac{96876\sqrt{2}\left(2x^2-x+3\right)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+84341\sqrt{2}\left(2x^2-x+3\right)\log\left(-\frac{2}{}\right.}{158976\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(
4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2 - x
+ 3)
```

**giac [A]** time = 0.40, size = 118, normalized size = 1.17

$$\frac{39}{32}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right)-\frac{3667}{3456}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3667}{3456}\sqrt{2}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gia
c")
```

```
[Out] 39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/34
56*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/
3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) +
1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

**maple [A]** time = 0.01, size = 148, normalized size = 1.47

$$\frac{5x^2}{4\sqrt{2x^2-x+3}}+\frac{39x}{16\sqrt{2x^2-x+3}}-\frac{39\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32}-\frac{3667\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{3456}-\frac{}{64\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/4/(2*x^2-x+3)^(1/2)*x^2+39/16/(2*x^2-x+3)^(1/2)*x-309/64/(2*x^2-x+3)^(1/2
)-5507/1472*(4*x-1)/(2*x^2-x+3)^(1/2)-39/32*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))+3667/576/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+40337/13248*(4*x-1)/(-11*x+
2*(x+5/2)^2-19/2)^(1/2)-3667/3456*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)
/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 0.98, size = 99, normalized size = 0.98

$$\frac{5x^2}{4\sqrt{2x^2-x+3}}-\frac{39}{32}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{3667}{3456}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right)-\frac{11}{3312\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="max
ima")
```

```
[Out] 5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sqrt(
2*x^2 - x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/2
)), x)
```

# 3.355 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=108

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] -5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24*(17-2
2*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/119232*(9897+2203*x)/(2*x^2-x+3)^
(1/2)-3667/10368*(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1646, 1650, 843, 619, 215, 724, 206}

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{10368(2x+5)}+\frac{25951\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{41472\sqrt{2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(
10368*(5 + 2*x)) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(8*Sqrt[2]) + (25951*Arc
Tanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(41472*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{-\frac{33649}{20736}+\frac{131215x}{10368}+\frac{115x^2}{4}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{1}{828}\int \frac{\frac{100073}{192}-1035x}{(5+2x)\sqrt{3-x+2x^2}}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{5}{8}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx-\frac{25951}{}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}+\frac{25951\,\text{Subst}\left(\int \frac{1}{288-x^2}\,dx,x,\right.}{3456}$$

$$= \frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25951\tanh}{41}$$

**Mathematica [A]** time = 0.32, size = 104, normalized size = 0.96

$$\frac{\frac{8(2203x+9897)}{23\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}-\frac{14668\sqrt{4x^2-2x+6}}{2x+5}+25951\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-25951\log(2x+5)+25920\sinh^{-1}}{41472\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((8*(9897 + 2203*x))/(23*Sqrt[3/2 - x/2 + x^2]) - (14668*Sqrt[6 - 2*x + 4*x
^2])/(5 + 2*x) + 25920*ArcSinh[(-1 + 4*x)/Sqrt[23]] - 25951*Log[5 + 2*x] +
25951*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(41472*Sqrt[2])
```

**fricas [A]** time = 0.96, size = 157, normalized size = 1.45

$$\frac{596160\sqrt{2}\left(4x^3+8x^2+x+15\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+596873\sqrt{2}\left(4x^3+8\right.}{3815424\left(4x^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x^
2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^2
+ x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036
*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt(2*x
^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

**giac [B]** time = 0.42, size = 225, normalized size = 2.08

$$\frac{1}{1907712}\sqrt{2}\left(\frac{12\left(\frac{\frac{315103}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{1012092}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}}{2x+5}-\frac{26645}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}}+\frac{596873\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] 1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1/
(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/(2
*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 7
2/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5) + 36
/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(abs(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x + 5)))
```

**maple [A]** time = 0.01, size = 152, normalized size = 1.41

$$-\frac{5x}{8\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16}+\frac{25951\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{82944}+\frac{99}{32\sqrt{2x^2-x+3}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/8/(2*x^2-x+3)^(1/2)*x+99/32/(2*x^2-x+3)^(1/2)+1529/736*(4*x-1)/(2*x^2-x+
3)^(1/2)+5/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3667/1152/(x+5/2)/(-11
*x+2*(x+5/2)^2-19/2)^(1/2)-25951/13824/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-63749
```

```
3/317952*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+25951/82944*2^(1/2)*arctanh
(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 1.01, size = 116, normalized size = 1.07

$$\frac{5}{16}\sqrt{2}\,\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}\,x-\frac{1}{23}\sqrt{23}\right)-\frac{25951}{82944}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{26645\,x}{79488\sqrt{2\,x^2-x+3}}+\frac{}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2)
*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 266
45/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/576
/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^2\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
3/2)), x)
```

# 3.356 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=112

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

```
[Out] -52631/11943936*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1
/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5+2
*x)^2+115369/1492992*(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}+\frac{115369\sqrt{2x^2-x+3}}{1492992(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{20736(2x+5)^2}-\frac{52631\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])
/(20736*(5 + 2*x)^2) + (115369*Sqrt[3 - x + 2*x^2])/(1492992*(5 + 2*x)) - (
52631*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(5971968*Sqrt[
2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{5168261}{746496}+\frac{3637795x}{186624}+\frac{5620625x^2}{186624}}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} - \frac{\int \frac{\frac{842237}{1296}-\frac{4102487x}{2592}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1656}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} + \frac{52631}{}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{}$$

$$= \frac{65991-8779x}{4292352\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2} + \frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)} - \frac{52631}{}$$

**Mathematica [A]** time = 0.30, size = 84, normalized size = 0.75

$$\frac{-52631\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)+\frac{12\left(3444340x^3+3263288x^2+5842933x+11594283\right)}{23(2x+5)^2\sqrt{x^2-\frac{x}{2}+\frac{3}{2}}}+52631\log(2x+5)}{5971968\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] ((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/(23*(5 + 2*x)^2*Sq
rt[3/2 - x/2 + x^2]) + 52631*Log[5 + 2*x] - 52631*Log[17 - 22*x + 12*Sqrt[6
 - 2*x + 4*x^2]])/(5971968*Sqrt[2])
```

**fricas [A]** time = 0.92, size = 126, normalized size = 1.12

$$\frac{1210513\sqrt{2}\left(8x^4+36x^3+42x^2+35x+75\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)+48\left(3444340x^3\right.}{549421056\left(8x^4+36x^3+42x^2+35x+75\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(24
*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*sqrt
(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

**giac [B]** time = 0.25, size = 220, normalized size = 1.96

$$-\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298598
4*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 19874490*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

**maple [A]** time = 0.01, size = 144, normalized size = 1.29

$$-\frac{52631\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{11943936}-\frac{5}{16\sqrt{2x^2-x+3}}-\frac{149\,(4x-1)}{368\sqrt{2x^2-x+3}}+\frac{196043}{165888\left(x+\frac{5}{2}\right)\sqrt{-11x+2\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(1/2)-149/368*(4*x-1)/(2*x^2-x+3)^(1/2)+196043/165888/(x+
5/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+52631/1990656/(-11*x+2*(x+5/2)^2-19/2)^
(1/2)+19399069/45785088*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-52631/119439
36*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)
)-3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 0.98, size = 149, normalized size = 1.33

$$\frac{52631}{11943936}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{861085\,x}{11446272\sqrt{2x^2-x+3}}-\frac{1163201}{3815424\sqrt{2x^2-x+3}}-\frac{}{1152\left(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(2
3)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/3815424/
sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2
- x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x + 3)*
x + 5*sqrt(2*x^2 - x + 3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^3\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
3/2)), x)
```

# 3.357 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=137

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}\left(\right.}{12899450}$$

```
[Out] -3505819/2579890176*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/
2)+1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)^(
1/2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799/107495424*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}+\frac{430799\sqrt{2x^2-x+3}}{107495424(2x+5)}+\frac{152885\sqrt{2x^2-x+3}}{4478976(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{31104(2x+5)^3}-\frac{3505819\tanh^{-1}\left(\right.}{12899450}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

```
[Out] (369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*
x^2])/(31104*(5 + 2*x)^3) + (152885*Sqrt[3 - x + 2*x^2])/(4478976*(5 + 2*x)
^2) + (430799*Sqrt[3 - x + 2*x^2])/(107495424*(5 + 2*x)) - (3505819*ArcTanh
[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(1289945088*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{348877271}{26873856}+\frac{119871055x}{4478976}+\frac{73960295x^2}{2239488}+\frac{130255}{33592}}{(5+2x)^4\sqrt{3-x+2x^2}}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} - \frac{\int \frac{\frac{79609325}{124416}-\frac{71248733x}{31104}-\frac{1302559x^2}{31104}}{(5+2x)^3\sqrt{3-x+2x^2}}}{2484}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \int$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{43}{1}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{43}{1}$$

$$= \frac{369609-175877x}{154524672\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3} + \frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2} + \frac{43}{1}$$

**Mathematica [A]** time = 0.16, size = 95, normalized size = 0.69

$$\frac{24\left(56754760x^4+572739684x^3+441046842x^2+1257975811x+1873786587\right)-80633837(2x+5)^3\sqrt{4x^2-2}}{59337474048(2x+5)^3\sqrt{2x^2-x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))
,x]
```

```
[Out] (24*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760*x
^4) - 80633837*(5 + 2*x)^3*Sqrt[6 - 2*x + 4*x^2]*ArcTanh[(17 - 22*x)/(12*Sq
rt[6 - 2*x + 4*x^2])])/(59337474048*(5 + 2*x)^3*Sqrt[3 - x + 2*x^2])
```

**fricas [A]** time = 0.88, size = 141, normalized size = 1.03

$$\frac{80633837\sqrt{2}\left(16x^5+112x^4+264x^3+280x^2+325x+375\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)}{118674948096\left(16x^5+112x^4+264x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="f
ricas")
```

```
[Out] 1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 32
5*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 10
36*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4410
46842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 112*x
^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

**giac [B]** time = 0.29, size = 271, normalized size = 1.98

$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)+\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="g
iac")
```

```
[Out] -3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x +
3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2)*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052284471)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^
2 - x + 3)) - 11)^3
```

**maple [A]** time = 0.01, size = 151, normalized size = 1.10

$$-\frac{3505819\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{2579890176}+\frac{\frac{5x}{46}-\frac{5}{184}}{\sqrt{2x^2-x+3}}-\frac{3127169}{35831808\left(x+\frac{5}{2}\right)\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}+\frac{}{429}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 5/184*(4*x-1)/(2*x^2-x+3)^(1/2)-3127169/35831808/(x+5/2)/(-11*x+2*(x+5/2)^2
-19/2)^(1/2)+3505819/429981696/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-261644215/988
9579008*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3505819/2579890176*2^(1/2)*a
```

```
rctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+314233/995
328/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11*x+2*
(x+5/2)^2-19/2)^(1/2)
```

**maxima [A]** time = 1.01, size = 217, normalized size = 1.58

$$\frac{3505819}{2579890176}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}}+\frac{6128291}{824131584\sqrt{2\,x^2-x+3}}-\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="m
axima")
```

```
[Out] 3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291/8
24131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60*sq
rt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x + 3)
) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*x + 2
5*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt
(2*x^2 - x + 3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{(2\,x+5)}^4\,{\left(2\,x^2-x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^{4} - x^{3} + 3x^{2} + x + 2}{\left(2x + 5\right)^{4} \left(2x^{2} - x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
3/2)), x)
```

# 3.358 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx$

**Optimal**. Leaf size=105

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69(2x^2-x+3)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] -4/69*(346-533*x)/(2*x^2-x+3)^(3/2)-1471/64*arcsinh(1/23*(1-4*x)*23^(1/2))*
2^(1/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(1/2)+247/16*(2*x^2-x+3)^(1/2)+5
/4*x*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{4(18982-20383x)}{1587\sqrt{2x^2-x+3}}+\frac{5}{4}x\sqrt{2x^2-x+3}+\frac{247}{16}\sqrt{2x^2-x+3}-\frac{4(346-533x)}{69(2x^2-x+3)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + (4*(18982 - 20383*x))/(1587
*Sqrt[3 - x + 2*x^2]) + (247*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*
x^2])/4 - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)^2\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-145-\frac{1725x}{2}+2415x^2+\frac{3657x^3}{2}+345x}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{33327}{2}+\frac{46023x}{4}+\frac{7935x^2}{4}}{\sqrt{3-x+2x^2}}\,d}{1587}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{5}{4}x\sqrt{3-x+2x^2}+\frac{\int}{}$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

$$= -\frac{4(346-533x)}{69\left(3-x+2x^2\right)^{3/2}}+\frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}}+\frac{247}{16}\sqrt{3-x+2x^2}+\frac{5}{4}x$$

**Mathematica [A]** time = 0.69, size = 65, normalized size = 0.62

$$\frac{126960x^5+1440996x^4-3764360x^3+8639625x^2-6410082x+6663133}{25392\left(2x^2-x+3\right)^{3/2}}-\frac{1471\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2)
,x]
```

```
[Out] (6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^5
)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*ArcSinh[(1 - 4*x)/Sqrt[23]])/(32*Sq
rt[2])
```

**fricas [A]** time = 0.86, size = 122, normalized size = 1.16

$$\frac{2334477\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+8\left(126960x^5\right.}{203136\left(4x^4-4x^3+13x^2-6x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)
*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 1440
996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 - x +
 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

**giac [A]** time = 0.22, size = 71, normalized size = 0.68

$$-\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(1587(20x+227)x-941090)x+8639625)x-641008}{25392\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
25392*((((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 666
3133)/(2*x^2 - x + 3)^(3/2)
```

**maple [B]** time = 0.02, size = 180, normalized size = 1.71

$$\frac{5x^5}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{227x^4}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1471x^3}{48\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{19073x^2}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{32257x}{512\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{1471x}{32\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -162931/50784*(4*x-1)/(2*x^2-x+3)^(1/2)-753223/141312*(4*x-1)/(2*x^2-x+3)^(
3/2)+5*x^5/(2*x^2-x+3)^(3/2)+227/4*x^4/(2*x^2-x+3)^(3/2)-1471/48*x^3/(2*x^2
-x+3)^(3/2)+19073/64*x^2/(2*x^2-x+3)^(3/2)-32257/512*x/(2*x^2-x+3)^(3/2)+14
71/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1471/32/(2*x^2-x+3)^(1/2)*x+57
7397/2048/(2*x^2-x+3)^(3/2)-1471/128/(2*x^2-x+3)^(1/2)
```

**maxima [B]** time = 0.98, size = 219, normalized size = 2.09

$$\frac{5x^5}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{227x^4}{4\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{1471}{50784}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{\left(2x^2-x+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784*
x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x
^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 1
471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x^2 -
 x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)^(3/2
) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2) + 4288
3/138/(2*x^2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)^2\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)^2\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5
/2), x)
```

# 3.359 $\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=86

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}+\frac{373x-53}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

```
[Out] 1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)+5/4*(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1660, 640, 619, 215}

$$\frac{6055-28981x}{3174\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}-\frac{71\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -(53 - 373*x)/(69*(3 - x + 2*x^2)^(3/2)) + (6055 - 28981*x)/(3174*Sqrt[3 -
x + 2*x^2]) + (5*Sqrt[3 - x + 2*x^2])/4 - (71*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(8*Sqrt[2])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{233}{4}+483x^2+\frac{345x^3}{2}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{52371}{16}+\frac{7935x}{8}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71}{8}\int$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}+\frac{71\,\mathrm{Sub}}{}$$

$$= -\frac{53-373x}{69\left(3-x+2x^2\right)^{3/2}}+\frac{6055-28981x}{3174\sqrt{3-x+2x^2}}+\frac{5}{4}\sqrt{3-x+2x^2}-\frac{71\,\mathrm{sinh}}{8}$$

**Mathematica [A]** time = 0.25, size = 60, normalized size = 0.70

$$\frac{31740x^4-147664x^3+185337x^2-199290x+102869}{6348\left(2x^2-x+3\right)^{3/2}}+\frac{71\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x
]
```

```
[Out] (102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2*
x^2)^(3/2)) + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2])
```

**fricas [A]** time = 0.72, size = 117, normalized size = 1.36

$$\frac{112677\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}\,(4x-1)-32x^2+16x-25\right)+8\left(31740x^4\right.}{50784\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 147664*
x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)
```

**giac [A]** time = 0.21, size = 66, normalized size = 0.77

$$-\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(7935x-36916)x+185337)x-199290)x+102869}{6348\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/63
48*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x + 3
)^(3/2)
```

**maple [B]** time = 0.01, size = 163, normalized size = 1.90

$$\frac{5x^4}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71x^3}{12\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{401x^2}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{945x}{128\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71x}{8\sqrt{2x^2-x+3}}+\frac{71\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4}{-}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 643/12696*(4*x-1)/(2*x^2-x+3)^(1/2)-2327/35328*(4*x-1)/(2*x^2-x+3)^(3/2)+5/
(2*x^2-x+3)^(3/2)*x^4-71/12/(2*x^2-x+3)^(3/2)*x^3+401/16/(2*x^2-x+3)^(3/2)*
x^2-945/128/(2*x^2-x+3)^(3/2)*x+71/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)-71/8/(2*x^2-x+3)^(1/2)*x+11749/512/(2*x^2-x+3)^(3/2)-71/32/(2*x^2-x+3)^(1
/2)
```

**maxima [B]** time = 0.97, size = 202, normalized size = 2.35

$$\frac{5\,x^4}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{71}{12696}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*
x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^
(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x + 3
) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 807/92
*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2)
, x)
```

# 3.360 $\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=68

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] 1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/
2)+1/2116*(-1465-2604*x)/(2*x^2-x+3)^(1/2)
```

**Rubi [A]** time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1660, 12, 619, 215}

$$\frac{219x+89}{276\left(2x^2-x+3\right)^{3/2}}-\frac{2604x+1465}{2116\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) - (1465 + 2604*x)/(2116*Sqrt[3 - x
 + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{\left(3-x+2x^2\right)^{5/2}}\,dx &= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{159}{16}+\frac{207x}{8}+\frac{345x^2}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{4\int \frac{7935}{16\sqrt{3-x+2x^2}}\,dx}{1587}\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{4\sqrt{46}}\\
&= \frac{89+219x}{276\left(3-x+2x^2\right)^{3/2}}-\frac{1465+2604x}{2116\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.23, size = 55, normalized size = 0.81

$$
\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{7812x^3+489x^2+7002x+5569}{3174\left(2x^2-x+3\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] -1/3174*(5569 + 7002*x + 489*x^2 + 7812*x^3)/(3 - x + 2*x^2)^(3/2) + (5*Arc
Sinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

**fricas [B]** time = 0.89, size = 112, normalized size = 1.65

$$\frac{7935\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-8\left(7812x^3+48\right.}{25392\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sqr
t(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2 +
7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

**giac [A]** time = 0.30, size = 62, normalized size = 0.91

$$-\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{3\left((2604x+163)x+2334\right)x+5569}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/3174
*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

**maple [B]** time = 0.01, size = 146, normalized size = 2.15

$$-\frac{5x^3}{6\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{x^2}{8\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{47x}{64\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{5x}{4\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}-\frac{}{768\left(2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/6/(2*x^2-x+3)^(3/2)*x^3-1/8/(2*x^2-x+3)^(3/2)*x^2-47/64/(2*x^2-x+3)^(3/2
)*x-271/768/(2*x^2-x+3)^(3/2)+2423/17664*(4*x-1)/(2*x^2-x+3)^(3/2)+173/1587
*(4*x-1)/(2*x^2-x+3)^(1/2)-5/4/(2*x^2-x+3)^(1/2)*x-5/16/(2*x^2-x+3)^(1/2)+5
/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

**maxima [B]** time = 0.98, size = 185, normalized size = 2.72

$$\frac{5}{6348}x\left(\frac{284x}{\sqrt{2x^2-x+3}}-\frac{3174x^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2x^2-x+3}}+\frac{805x}{\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{3243}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\right)+\frac{5}{8}\sqrt{2}\,\operatorname{arsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2 -
x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2) - 18
97/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/(2*x^
2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

# 3.361 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=85

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

```
[Out] 1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-3667/62208*arctanh(1/24*(17-22*x)*2^(
1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/1371168*(-335337-146729*x)/(2*x^2-x+3)^(1
/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1646, 12, 724, 206}

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{146729x+335337}{1371168\sqrt{2x^2-x+3}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) - (335337 + 146729*x)/(1371168*
Sqrt[3 - x + 2*x^2]) - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(31104*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx &= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{1877}{576}+\frac{695x}{18}+\frac{345x^2}{4}}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx \\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{4\int \frac{1939843}{6912(5+2x)\sqrt{3-x+2x^2}}\,dx}{1587} \\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}+\frac{3667\int \frac{1}{(5+2x)\sqrt{3-x+2x^2}}\,dx}{5184} \\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\,\mathrm{Subst}\left(\int \frac{1}{288-x^2}\,dx, x, \ldots\right)}{2592} \\
&= \frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}
\end{aligned}$$

**Mathematica [A]** time = 0.47, size = 80, normalized size = 0.94

$$\frac{-3667\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-\frac{12\sqrt{2}\left(293458x^3+523945x^2-21696x+841653\right)}{529\left(2x^2-x+3\right)^{3/2}}+3667\log(2x+5)}{31104\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x
]
```

```
[Out] ((-12*Sqrt[2]*(841653 - 21696*x + 523945*x^2 + 293458*x^3))/(529*(3 - x + 2
*x^2)^(3/2)) + 3667*Log[5 + 2*x] - 3667*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4
*x^2]])/(31104*Sqrt[2])
```

**fricas [A]** time = 0.84, size = 126, normalized size = 1.48

$$\frac{1939843 \sqrt{2} \left(4 x^4 - 4 x^3 + 13 x^2 - 6 x + 9\right) \log\left(-\frac{24 \sqrt{2} \sqrt{2 x^2 - x + 3} (22 x - 17) + 1060 x^2 - 1036 x + 1153}{4 x^2 + 20 x + 25}\right) - 48 \left(293458 x^3 + 52\right.}{65816064 \left(4 x^4 - 4 x^3 + 13 x^2 - 6 x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sqr
t(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 2
0*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2 - x
 + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

**giac [A]** time = 0.23, size = 92, normalized size = 1.08

$$-\frac{3667}{62208} \sqrt{2} \log\left(\left|-2 \sqrt{2} x + \sqrt{2} + 2 \sqrt{2 x^2 - x + 3}\right|\right) + \frac{3667}{62208} \sqrt{2} \log\left(\left|-2 \sqrt{2} x - 11 \sqrt{2} + 2 \sqrt{2 x^2 - x + 3}\right|\right) - \frac{(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gia
c")
```

```
[Out] -3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2 - x
 + 3)^(3/2)
```

**maple [B]** time = 0.01, size = 190, normalized size = 2.24

$$-\frac{5x^2}{4 \left(2x^2 - x + 3\right)^{\frac{3}{2}}} + \frac{59x}{32 \left(2x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{3667\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x + \frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x + 2\left(x + \frac{5}{2}\right)^2 - \frac{19}{2}}}\right)}{62208} - \frac{1597}{384 \left(2x^2 - x + 3\right)^{\frac{3}{2}}} - \frac{3817}{2944 \left(2x \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/4/(2*x^2-x+3)^(3/2)*x^2+59/32/(2*x^2-x+3)^(3/2)*x-1597/384/(2*x^2-x+3)^(
3/2)-3817/2944*(4*x-1)/(2*x^2-x+3)^(3/2)-3817/4232*(4*x-1)/(2*x^2-x+3)^(1/2
)+3667/1728/(-11*x+2*(x+5/2)^2-19/2)^(3/2)+40337/39744*(4*x-1)/(-11*x+2*(x+
5/2)^2-19/2)^(3/2)+4800103/5484672*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+3
667/10368/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-3667/62208*2^(1/2)*arctanh(1/12*(-
11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))
```

**maxima [A]** time = 0.97, size = 110, normalized size = 1.29

$$\frac{3667}{62208}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}}-\frac{5\,x^2}{4\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{173881}{457056\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="max
ima")
```

```
[Out] 3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/a
bs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x +
3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x + 3)^
(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/2
)), x)
```

## 3.362 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=110

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

```
[Out] 1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-2821/4478976*arctanh(1/24*(17-22*x
)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/24681024*(-1255878+62021*x)/(2*x^2-x
+3)^(1/2)-3667/186624*(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1646, 806, 724, 206}

$$-\frac{1255878-62021x}{24681024\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{186624(2x+5)}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}-\frac{2821\tanh^{-1}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) - (1255878 - 62021*x)/(24681
024*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(186624*(5 + 2*x)) -
(2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(2239488*Sqrt[
2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{\frac{119353}{20736}+\frac{481765x}{10368}+\frac{113983x^2}{1296}}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{10109719}{124416}-\frac{4961491x}{62208}}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}+$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-$$

$$= \frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-$$

**Mathematica [A]** time = 0.41, size = 92, normalized size = 0.84

$$\frac{-2821\log\left(12\sqrt{4x^2-2x+6}-22x+17\right)-\frac{12\sqrt{2}\left(6767036x^4+10350004x^3+63941915x^2-18840090x+79153407\right)}{529(2x+5)\left(2x^2-x+3\right)^{3/2}}+2821\log(2x}{2239488\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((-12*Sqrt[2]*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 676703
6*x^4))/(529*(5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 2821*Log[5 + 2*x] - 2821*Lo
g[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(2239488*Sqrt[2])
```

**fricas [A]** time = 0.85, size = 141, normalized size = 1.28

$$\frac{1492309\sqrt{2}\left(8x^5+12x^4+6x^3+53x^2-12x+45\right)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-17)+1060x^2-1036x+1153}{4x^2+20x+25}\right)-48\left(67670\right.}{4738756608\left(8x^5+12x^4+6x^3+53x^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45)
*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 115
3)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2 - 1
8840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2
 - 12*x + 45)
```

**giac [B]** time = 0.40, size = 206, normalized size = 1.87

$$-\frac{1}{2369378304}\sqrt{2}\left(\frac{1492309\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{12\left(\frac{\frac{\frac{48\left(\frac{23642785}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{52375761}{(2x+5)\operatorname{sgn}\left(\frac{1}{2x+5}\right)}\right)}{2x+5}-\frac{240080}{\operatorname{sgn}\left(\frac{1}{2}\right.}}{2x+5}}{2x+5}\right.}{\left(\frac{11}{2x+5}-\frac{36}{(2x+5)^2}-1\right)\sqrt{-\frac{1}{2x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] -1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x +
 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(1/(
2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/sgn(1
/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) + 36/(
2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

**maple [B]** time = 0.01, size = 194, normalized size = 1.76

$$-\frac{5x}{16\left(2x^2-x+3\right)^{\frac{3}{2}}}-\frac{2821\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{4478976}+\frac{203}{192\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1104}-\frac{3173}{4416}}{\left(2x^2-x+3\right)^{\frac{3}{2}}}+\frac{\frac{3173x}{1587}-\frac{31}{63}}{\sqrt{2x^2-x-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/16/(2*x^2-x+3)^(3/2)*x+203/192/(2*x^2-x+3)^(3/2)+3173/4416*(4*x-1)/(2*x^
2-x+3)^(3/2)+3173/6348*(4*x-1)/(2*x^2-x+3)^(1/2)-3667/1152/(x+5/2)/(-11*x+2
*(x+5/2)^2-19/2)^(3/2)+2821/124416/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-2081161/2
861568*(4*x-1)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-199077743/394896384*(4*x-1)/(
-11*x+2*(x+5/2)^2-19/2)^(1/2)+2821/746496/(-11*x+2*(x+5/2)^2-19/2)^(1/2)-28
21/4478976*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/
2)^(1/2))
```

**maxima [A]** time = 1.01, size = 127, normalized size = 1.15

$$\frac{2821}{4478976}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{1691759\,x}{98724096\sqrt{2\,x^2-x+3}}+\frac{265339}{32908032\sqrt{2\,x^2-x+3}}-\frac{24}{715392\,(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] 2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)
/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032/s
qrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2*(2
*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2 - x
 + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^2\,\left(2\,x^2-x+3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**(
5/2)), x)
```

# 3.363 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=135

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tan}{322}$$

```
[Out] 1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)+774079/644972544*arctanh(1/24*(
17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/592344576*(-4679797+2148263*x
)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979/26873856*
(2*x^2-x+3)^(1/2)/(5+2*x)
```

**Rubi [A]** time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{4679797-2148263x}{592344576\sqrt{2x^2-x+3}}-\frac{45979\sqrt{2x^2-x+3}}{26873856(2x+5)}-\frac{3667\sqrt{2x^2-x+3}}{373248(2x+5)^2}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}+\frac{774079\tan}{322}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) - (4679797 - 2148263*x)/(
592344576*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(373248*(5 + 2*
x)^2) - (45979*Sqrt[3 - x + 2*x^2])/(26873856*(5 + 2*x)) + (774079*ArcTanh[
(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(322486272*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NegQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{11115283}{746496}+\frac{3198845x}{62208}+\frac{605005x^2}{6912}-\frac{8779x^3}{23328}}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{171639869}{2985984}-\frac{1423925}{74649}}{(5+2x)^3\sqrt{3-}}}{1587}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

$$= \frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}} - \frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}$$

**Mathematica [A]** time = 0.31, size = 97, normalized size = 0.72

$$\frac{774079\log\left(12\sqrt{4x^2-2x+6}-22x+17\right) + \frac{12\sqrt{2}\left(217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-895\right.}{529(2x+5)^2\left(2x^2-x+3\right)^{3/2}}}{322486272\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((12*Sqrt[2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 1503926130*x^3
+ 107028732*x^4 + 217883368*x^5))/(529*(5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)) -
 774079*Log[5 + 2*x] + 774079*Log[17 - 22*x + 12*Sqrt[6 - 2*x + 4*x^2]])/(3
22486272*Sqrt[2])
```

**fricas** [A] time = 0.89, size = 155, normalized size = 1.15

$$\frac{409487791\,\sqrt{2}\left(16\,x^{6}+64\,x^{5}+72\,x^{4}+136\,x^{3}+241\,x^{2}+30\,x+225\right)\log\left(\frac{24\,\sqrt{2}\,\sqrt{2\,x^{2}-x+3}\,(22\,x-17)-1060\,x^{2}+1036\,x-115}{4\,x^{2}+20\,x+25}\right.}{682380951552\left(16\,x^{6}+64\,x^{5}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241
*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732*x^
4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2*x^2
 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

**giac** [B] time = 0.27, size = 228, normalized size = 1.69

$$\frac{774079}{644972544}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}\,+2\,\sqrt{2\,x^{2}-x+3}\right|\right)-\frac{774079}{644972544}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}\,+2\,\sqrt{2\,x^{2}-x+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 16812201*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2 + 1
/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x^2 - x
+ 3)^(3/2)
```

**maple** [A] time = 0.01, size = 200, normalized size = 1.48

$$\frac{774079\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^{2}-\frac{19}{2}}}\right)}{644972544}-\frac{5}{48\left(2x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{149\,(4x-1)}{1104\left(2x^{2}-x+3\right)^{\frac{3}{2}}}-\frac{149\,(4x-1)}{1587\sqrt{2x^{2}-x+3}}+\frac{}{16588}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -5/48/(2*x^2-x+3)^(3/2)-149/1104*(4*x-1)/(2*x^2-x+3)^(3/2)-149/1587*(4*x-1)
/(2*x^2-x+3)^(1/2)+115369/165888/(x+5/2)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-774
079/17915904/(-11*x+2*(x+5/2)^2-19/2)^(3/2)+57937675/412065792*(4*x-1)/(-11
*x+2*(x+5/2)^2-19/2)^(3/2)+5366174813/56865079296*(4*x-1)/(-11*x+2*(x+5/2)^
2-19/2)^(1/2)-774079/107495424/(-11*x+2*(x+5/2)^2-19/2)^(1/2)+774079/644972
544*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2
))-3667/4608/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(3/2)
```

**maxima [A]** time = 1.00, size = 178, normalized size = 1.32

$$-\frac{774079}{644972544}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}-\frac{36393601}{4738756608\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqr
t(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 36393601
/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3/2)
 - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25
*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^
2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^3\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{\left(2x + 5\right)^{3} \left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**(
5/2)), x)
```

# 3.364 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=160

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-}{463574016\,(2x}$$

```
[Out] 1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)+4778789/15479341056*arctanh
(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)+1/31986607104*(-27754539
+31190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-8913
7/80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357/1934917632*(2*x^2-x+3)^(1/2)/
(5+2*x)
```

**Rubi [A]** time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1646, 1650, 806, 724, 206}

$$-\frac{27754539-31190998x}{31986607104\sqrt{2x^2-x+3}}+\frac{475357\sqrt{2x^2-x+3}}{1934917632(2x+5)}-\frac{89137\sqrt{2x^2-x+3}}{80621568(2x+5)^2}-\frac{3667\sqrt{2x^2-x+3}}{559872(2x+5)^3}+\frac{369609-}{463574016\,(2x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

```
[Out] (369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) - (27754539 - 3119099
8*x)/(31986607104*Sqrt[3 - x + 2*x^2]) - (3667*Sqrt[3 - x + 2*x^2])/(559872
*(5 + 2*x)^3) - (89137*Sqrt[3 - x + 2*x^2])/(80621568*(5 + 2*x)^2) + (47535
7*Sqrt[3 - x + 2*x^2])/(1934917632*(5 + 2*x)) + (4778789*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(7739670528*Sqrt[2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1650

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{606939313}{26873856}+\frac{727085495x}{13436928}+\frac{186705485x^2}{2239488}-\frac{101\cdots}{3\cdots}}{(5+2x)^4\left(3-x+2x^2\right)^{3\cdots}}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{4811736919}{40310784}-\frac{3\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2\cdots}}{559872(5+2x\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2\cdots}}{559872(5+2x\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2\cdots}}{559872(5+2x\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2\cdots}}{559872(5+2x\cdots}$$

$$= \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2\cdots}}{559872(5+2x\cdots}$$

**Mathematica** [A] time = 0.24, size = 89, normalized size = 0.56

$$\frac{2527979381\sqrt{2}\tanh^{-1}\left(\frac{17-22x}{12\sqrt{4x^2-2x+6}}\right)+\frac{24\left(6664404208x^6+34872810880x^5+46210466520x^4+27484986184x^3-6702882569x^2+736219\cdots\right)}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}}{8188571418624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))
,x]
```

```
[Out] ((24*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 462
10466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2*x
^2)^(3/2)) + 2527979381*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[6 - 2*x + 4*x^
2])])/8188571418624
```

**fricas [A]** time = 0.92, size = 170, normalized size = 1.06

$$\frac{2527979381\,\sqrt{2}\left(32\,x^7+208\,x^6+464\,x^5+632\,x^4+1162\,x^3+1265\,x^2+600\,x+1125\right)\log\left(\frac{24\,\sqrt{2}\,\sqrt{2\,x^2-x+3}\,(22\,x-17}{4\,x^2+20}\right.}{16377142837248}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="f
ricas")
```

```
[Out] 1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
+ 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(
22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440420
8*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 6702882569*x^
2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^6 + 4
64*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

**giac [B]** time = 0.28, size = 279, normalized size = 1.74

$$\frac{4778789}{15479341056}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)-\frac{4778789}{15479341056}\,\sqrt{2}\,\log\left(\left|-2\,\sqrt{2}\,x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="g
iac")
```

```
[Out] 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27298
005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(38030012*s
qrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sqrt(2*
x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 2
154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

**maple [A]** time = 0.01, size = 207, normalized size = 1.29

$$\frac{4778789\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(-11x+\frac{17}{2}\right)\sqrt{2}}{12\sqrt{-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}}}\right)}{15479341056}-\frac{4778789}{429981696\left(-11x+2\left(x+\frac{5}{2}\right)^2-\frac{19}{2}\right)^{\frac{3}{2}}}-\frac{4778789}{2579890176\sqrt{-11x+2\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -4778789/429981696/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-4778789/2579890176/(-11*x
+2*(x+5/2)^2-19/2)^(1/2)-3667/13824/(x+5/2)^3/(-11*x+2*(x+5/2)^2-19/2)^(3/2
)+25951/110592/(x+5/2)^2/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-34861/3981312/(x+5/
2)/(-11*x+2*(x+5/2)^2-19/2)^(3/2)-72646615/9889579008*(4*x-1)/(-11*x+2*(x+5
/2)^2-19/2)^(3/2)+4778789/15479341056*2^(1/2)*arctanh(1/12*(-11*x+17/2)*2^(
1/2)/(-11*x+2*(x+5/2)^2-19/2)^(1/2))+10/1587*(4*x-1)/(2*x^2-x+3)^(1/2)+5/55
2*(4*x-1)/(2*x^2-x+3)^(3/2)-8183108657/1364761903104*(4*x-1)/(-11*x+2*(x+5/
2)^2-19/2)^(1/2)
```

**maxima [A]** time = 1.01, size = 246, normalized size = 1.54

$$-\frac{4778789}{15479341056}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}\,x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{416525263\,x}{341190475776\,\sqrt{2\,x^2-x+3}}-\frac{245375387}{113730158592\,\sqrt{2\,x^2-\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="m
axima")
```

```
[Out] -4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 245
375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 - x
+ 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3)^(3
/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 25951/
27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*(2*x^2
 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x
+ 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^4-x^3+3\,x^2+x+2}{{\left(2\,x+5\right)}^4\,{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
```

```
[Out] int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^4 - x^3 + 3x^2 + x + 2}{(2x+5)^4 \left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**(
5/2)), x)
```

# 3.365 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=354

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+ab^2\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^4
*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/
c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)+j*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(-3*a*i+2*c*g)-b
^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-c*(16*c^4*f-c^3*(-
8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)
/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.38, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)\right.}{3c^3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2*
j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g - 3*a*i) - b^5*j - b^3*c*(c*
h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3*(8*b*
g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a
^2*j))*x))/(3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (j*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f+gx+hx^2+365x^3+jx^4}{\left(a+bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-c\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-c\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-c\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-c\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

$$= -\frac{2\left(c^3\left(bf+\frac{a^2(730c-3bj)}{c^2}-\frac{a\left(365b^2c+2c^3g-bc^2h-b^3j\right)}{c^3}\right)-\left(365b^3c-bc^2(1095a-c\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx\right.}$$

**Mathematica [A]** time = 1.14, size = 316, normalized size = 0.89

$$-\frac{2\left(bc\left(-3a^2j+ac(h+3ix)+c^2(f-gx)\right)+2c^2\left(a^2(i+jx)-ac(g+hx)+c^2fx\right)+b^3(aj-cix)+b^2c(chx-a(i+4jx))+b^4jx\right)}{\left(b^2-4ac\right)(a+x(b+cx))^{3/2}}+\frac{2\left(4bc^2\left(8a^2j+ac(h-3ix)+2c^2(f-gx)\right)+8\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] ((-2*(b^4*j*x + b^3*(a*j - c*i*x) + b*c*(-3*a^2*j + c^2*(f - g*x) + a*c*(h
+ 3*i*x)) + 2*c^2*(c^2*f*x - a*c*(g + h*x) + a^2*(i + j*x)) + b^2*c*(c*h*x
- a*(i + 4*j*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))^(3/2)) + (2*(b^5*j - b^
4*c*(i + 4*j*x) + 2*b^2*c^2*(-2*c*g + 3*a*i + c*h*x + 14*a*j*x) + 4*b*c^2*(
8*a^2*j + 2*c^2*(f - g*x) + a*c*(h - 3*i*x)) + b^3*c*(-10*a*j + c*(h + i*x)
) + 8*c^3*(2*c^2*f*x + a*c*h*x - a^2*(3*i + 4*j*x))))/((b^2 - 4*a*c)^2*Sqrt
[a + x*(b + c*x)]) + 3*Sqrt[c]*j*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]])/(3*c^3)
```

**fricas [B]** time = 88.99, size = 1373, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="frica
s")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 -
 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*
sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b
)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b*c^5*
g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 - 7*a*b
^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 + 4*a*b*
c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2 - (b^3*
c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c - 20*a^3*b
*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5)*f - (b^3
*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt(c*
x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*
b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 +
(b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 1
6*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j
*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt
(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f
 - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*
c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c
^3 + 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c
 - 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^
5)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*
c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*
b^3*c^4 + 16*a^3*b*c^5)*x)]
```

**giac [A]** time = 0.31, size = 465, normalized size = 1.31

$$\frac{2\left(\left(\left(\frac{\left(16\,c^5f-8\,bc^4g+2\,b^2c^3h+8\,ac^4h+b^3c^2i-12\,abc^3i-4\,b^4cj+28\,ab^2c^2j-32\,a^2c^3j\right)x}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\left(8\,bc^4f-4\,b^2c^3g+b^3c^2h+4\,abc^3h-2\,ab^2c^2i-8\,a^2c^3i-b^5j+6\,ab^3c\right.}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right.\right.\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac"
)
```

```
[Out] 2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*b
*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h - 2*
a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 1
6*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g + 4*a*
b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)/(b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a*b^2*c^
2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20*a^3*b*c
*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2) - j*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2)
```

**maple [B]** time = 0.02, size = 1406, normalized size = 3.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] j/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/24*j/c^3*b^4/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x-1/3*j/c^2*b^4/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/4*j/c
^3*b^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*j/c^2*b^3*a/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)+2/3*i/c*b^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/3*h*a/c/(4*
a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+16/3*h*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)
*x-1/2*i/c^2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/12*i/c^2*b^3/(4*a*c-b^
2)/(c*x^2+b*x+a)^(3/2)*x-16/3*g*c*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-8*i
*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-4*i/c*b^2*a/(4*a*c-b^2)^2/(c*x^2+b
*x+a)^(1/2)+1/6*h/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+2/3*f/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*b-i*x^2/c/(c*x^2+b*x+a)^(3/2)+1/24*i/c^3*b^2/(c*x^2+b*
x+a)^(3/2)-2/3*i*a/c^2/(c*x^2+b*x+a)^(3/2)-1/2*h*x/c/(c*x^2+b*x+a)^(3/2)+1/
12*h/c^2*b/(c*x^2+b*x+a)^(3/2)-8/3*g*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-
1/3*j*x^3/c/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^3/(c*x^2+b*x+a)^(3/2)-j/c^2*x/
(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b/(c*x^2+b*x+a)^(1/2)+4*j/c*b^2*a/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*x-i/c*b*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+1/2*j/c^
2*b^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*g*b/(4*a*c-b^2)/(c*x^2+b*x+a)
^(3/2)*x-1/3*g/c*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4/3*f/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*f*c
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/12*h/c^2*b^3/(4*a*c-b^2)/(c*x^2+b*x+
a)^(3/2)+4/3*h*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+2/3*h/c*b^3/(4*a*c-b
^2)^2/(c*x^2+b*x+a)^(1/2)+2/3*h*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+8/3*h*a
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+1/2*j/c^2*b*x^2/(c*x^2+b*x+a)^(3/2)+1/
8*j/c^3*b^2*x/(c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^5/(4*a*c-b^2)/(c*x^2+b*x+a)^
(3/2)-1/6*j/c^3*b^5/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-1/4*i/c^2*b*x/(c*x^2+
b*x+a)^(3/2)+1/24*i/c^3*b^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/3*i/c^2*b^4/(
4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+1/2*j/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
```

```
/2)-1/3*g/c/(c*x^2+b*x+a)^(3/2)+j/c^(5/2)*ln((c*x+1/2*b)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))+1/3*j/c^3*b*a/(c*x^2+b*x+a)^(3/2)
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 zero or nonzero?
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{{\left(c\,x^2+b\,x+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.366 $\int \frac{f+gx+hx^2+ix^3+jx^4}{\left(a+bx-cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=353

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2c\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

```
[Out] 2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4*
f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)/c
^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-j*arctan(1/2*(-2*c*x+b)/c^(1/2)/(-c*x^2
+b*x+a)^(1/2))/c^(5/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*(3*a*i+2*c*g)+b^
5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(16*c^4*f+8*c^3*(-
a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*b*i+b^2*h))*x)/c^3
/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

**Rubi [A]** time = 0.39, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1660, 12, 621, 204}

$$-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+\right.}{3c^3\left(4ac+b^2\right)^2\sqrt{a+bx-cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2*
j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^2*
h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)) -
 (2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3*c*(c*
h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*c^3*(b*
g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i - 16*a^2
*j))*x))/(3*c^3*(b^2 + 4*a*c)^2*Sqrt[a + b*x - c*x^2]) - (j*ArcTan[(b - 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{f + gx + hx^2 + 366x^3 + jx^4}{\left(a + bx - cx^2\right)^{5/2}} dx = -\frac{2\left(c^3\left(bf - \frac{3a^2(244c+bj)}{c^2} - \frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right) - \left(366b^3c + bc^2(1098a + c\right.\right.}{3c^3\left(b^2 + 4ac\right)\left(a + bx - cx\right.}$$

$$= -\frac{2\left(c^3\left(bf - \frac{3a^2(244c+bj)}{c^2} - \frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right) - \left(366b^3c + bc^2(1098a + c\right.\right.}{3c^3\left(b^2 + 4ac\right)\left(a + bx - cx\right.}$$

$$= -\frac{2\left(c^3\left(bf - \frac{3a^2(244c+bj)}{c^2} - \frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right) - \left(366b^3c + bc^2(1098a + c\right.\right.}{3c^3\left(b^2 + 4ac\right)\left(a + bx - cx\right.}$$

$$= -\frac{2\left(c^3\left(bf - \frac{3a^2(244c+bj)}{c^2} - \frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right) - \left(366b^3c + bc^2(1098a + c\right.\right.}{3c^3\left(b^2 + 4ac\right)\left(a + bx - cx\right.}$$

$$= -\frac{2\left(c^3\left(bf - \frac{3a^2(244c+bj)}{c^2} - \frac{a\left(366b^2c+2c^3g+bc^2h+b^3j\right)}{c^3}\right) - \left(366b^3c + bc^2(1098a + c\right.\right.}{3c^3\left(b^2 + 4ac\right)\left(a + bx - cx\right.}$$

**Mathematica [C]** time = 1.11, size = 319, normalized size = 0.90

$$-\frac{2\left(b^3\left(3a^2j + 18acjx^2 + c^2\left(f + 3gx - \left(x^2(3h + ix)\right)\right)\right) + 2b^2c\left(21a^2jx + ac\left(g + x\left(-6h + 3ix - 14jx^2\right)\right) + c^2x(3f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

```
[Out] (-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
+ c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*j*
x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a^3*
j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g - h*x
+ i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*(g +
x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*x))^(3
/2)) + (I*j*Log[(I*(b - 2*c*x))/Sqrt[c] + 2*Sqrt[a + x*(b - c*x)]])/c^(5/2)
```

**fricas [B]** time = 95.78, size = 1385, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

```
[Out] [-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5
+ 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)
*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c*x -
 b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f + 8*b*c
^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^2 + 7*
a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^3 - 4*a
*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*x^2 - (b
^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*c + 20*a^
3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*c^5)*f - (
b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)*x)*sqrt
(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 +
8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^
3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 -
2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^
3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^
2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x + a)*(2*c*x - b)*
sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^
6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b
^4*c^2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^
3*c^3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*
j)*x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^
3*c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a
*c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
(b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*a^
2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 + 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*x)]
```

**giac [A]** time = 0.45, size = 488, normalized size = 1.38

$$-\frac{2\sqrt{-cx^2+bx+a}\left(\left(\left(\frac{\left(16\,c^5f+8\,bc^4g+2\,b^2c^3h-8\,ac^4h-b^3c^2i-12\,abc^3i-4\,b^4cj-28\,ab^2c^2j-32\,a^2c^3j\right)x}{b^4c^2+8\,ab^2c^3+16\,a^2c^4}-\frac{3\left(8\,bc^4f+4\,b^2c^3g+b^3c^2h-4\,abc^3h-2\,ab^2\right.}{b^4c^2+8\,ab^2c^3+16\,a}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

```
[Out] -2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c^
4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*j)
*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b^3*
c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j)/(b
^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b^3*c^
2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^2*b^2*
c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^2*f + 1
2*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i +
3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 -
b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sqrt(-c) + b
))/(sqrt(-c)*c^2)
```

**maple [B]** time = 0.02, size = 1453, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/3*g/c/(-c*x^2+b*x+a)^(3/2)+j/c^(5/2)*arctan(c^(1/2)*(x-1/2/c*b)/(-c*x^2+b
*x+a)^(1/2))+16/3*g*c*b/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+1/12*i/c^2*b^
3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-2/3*i/c*b^3/(-4*a*c-b^2)^2/(-c*x^2+b*
x+a)^(1/2)*x-1/2*i/c^2*b^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-8*i*b*a/(-4*
a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+4*i/c*b^2*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)
^(1/2)-1/6*h/c*b^2/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-1/3*h*a/c/(-4*a*c-b^
2)/(-c*x^2+b*x+a)^(3/2)*b-16/3*h*a*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x+
2*j/c^2*b^3*a/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)+j/c^2*b^2/(-4*a*c-b^2)/(-
c*x^2+b*x+a)^(1/2)*x+1/24*j/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-1/3
*j/c^2*b^4/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-1/4*j/c^3*b^3*a/(-4*a*c-b^
2)/(-c*x^2+b*x+a)^(3/2)+1/2*h*x/c/(-c*x^2+b*x+a)^(3/2)+1/12*h/c^2*b/(-c*x^2
+b*x+a)^(3/2)-8/3*g*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)+2/3*f/(-4*a*c-b
^2)/(-c*x^2+b*x+a)^(3/2)*b+i*x^2/c/(-c*x^2+b*x+a)^(3/2)-1/24*i/c^3*b^2/(-c*
x^2+b*x+a)^(3/2)-2/3*i*a/c^2/(-c*x^2+b*x+a)^(3/2)+1/3*j*x^3/c/(-c*x^2+b*x+a
)^(3/2)-1/48*j/c^4*b^3/(-c*x^2+b*x+a)^(3/2)-j/c^2*x/(-c*x^2+b*x+a)^(1/2)-1/
2*j/c^3*b/(-c*x^2+b*x+a)^(1/2)+i/c*b*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+
1/2*j/c^2*b^2*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x-4*j/c*b^2*a/(-4*a*c-b^2
)^2/(-c*x^2+b*x+a)^(1/2)*x-1/3*j/c^3*b*a/(-c*x^2+b*x+a)^(3/2)-1/2*j/c^3*b^3
/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(1/2)-1/4*i/c^2*b*x/(-c*x^2+b*x+a)^(3/2)-1/24*
i/c^3*b^4/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+1/3*i/c^2*b^4/(-4*a*c-b^2)^2/(-
c*x^2+b*x+a)^(1/2)-4/3*f/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x*c+32/3*f*c^2/(
-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-16/3*f*c/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)
^(1/2)*b-2/3*g*b/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+1/3*g/c*b^2/(-4*a*c-b^
2)/(-c*x^2+b*x+a)^(3/2)+1/12*h/c^2*b^3/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)+4/
3*h*b^2/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*x-2/3*h/c*b^3/(-4*a*c-b^2)^2/(-
c*x^2+b*x+a)^(1/2)+2/3*h*a/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)*x+8/3*h*a/(-4*
```

```
a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)*b+1/2*j/c^2*b*x^2/(-c*x^2+b*x+a)^(3/2)-1/8*
j/c^3*b^2*x/(-c*x^2+b*x+a)^(3/2)-1/48*j/c^4*b^5/(-4*a*c-b^2)/(-c*x^2+b*x+a)
^(3/2)+1/6*j/c^3*b^5/(-4*a*c-b^2)^2/(-c*x^2+b*x+a)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}i\left(\frac{32\,abx}{\sqrt{-cx^2+bx+a}\left(b^2+4\,ac\right)^2}-\frac{16\,ab^2}{\sqrt{-cx^2+bx+a}\left(b^2+4\,ac\right)^2c}+\frac{b^3x}{\left(-cx^2+bx+a\right)^{\frac{3}{2}}\left(b^2+4\,ac\right)c^2}+\frac{2\,(b}{\sqrt{-cx^2+b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] -1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt(
-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^(3/2)*(b^2
 + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^
2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - 3*x^2/((-c*x^2
+ b*x + a)^(3/2)*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*
c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*a/((-c*x
^2 + b*x + a)^(3/2)*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*(b^2 +
4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/((-c*x^2
 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a
*c)*c) + 1/((-c*x^2 + b*x + a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(-c*x^2 + b
*x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) +
 2*c*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 + b*x + a)^(3/
2)*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b*x + a)*(b^2
+ 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) + b^2*x/((-c*x
^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt(-c*x^2 + b*x +
 a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c)) +
j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2*a*c)*x^2 + a^2)*
sqrt(-c*x^2 + b*x + a)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{\left(-c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
```

```
[Out] int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

## 3.367 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) d$

**Optimal**. Leaf size=588

$$\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{m+9}}{e^{11}(m+9)}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{m+8}}{e^{11}(m+8)}+\frac{\left(5d^2-2de+3e^2\right)^3\left(\right.}{}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3+8
6*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6+66
0*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3+m)/
e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3*e^4-
2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6+3150*
d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*(e*x+d)
^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+999*d*e^4
-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2*e^2+686*
d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+3060*d*e^2+4
9*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d)^(9+m)/e^1
1/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11+m)/e^11/(11
+m)
```

**Rubi [A]** time = 0.36, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^3\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^{11}(m+1)}-\frac{\left(5d^2-2de+3e^2\right)^2\left(108d^3e^2-20d^2e^3+169d^4e\right.}{e^{11}(m+2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169
*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m))/(
e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*
e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m))/(e^
11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9
990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m))/(e^11
*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^3 + 1498
5*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m)) - (6*(
21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 - 85*e^5)*(d
 + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*e^2
```

```
 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*(30000*d^3 +
 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8 + m)) + (45*(
500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) - (25*(200*d +
e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11 + m))/(e^11*(11
 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left(\frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3\right.}{e^{10}}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^3 \left(4d^4+5d^3e+3d^2e^2-de^3+2e\right.}{e^{11}(1+m)}$$

**Mathematica [A]** time = 0.38, size = 537, normalized size = 0.91

$$(d+ex)^{m+1}\left(\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^8}{m+9}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^7}{m+8}+\frac{\left(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4\right)}{m+7}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^4*
e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (3*(
5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3 + 5
47*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7 + 105
0*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218
*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e + 53550*
d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^4
)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*
e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e + 21420*d^2*
e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3 + 450*d^2*e
```

```
 + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5*d*e + 17*e^
2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m) + (500*(d +
 e*x)^10)/(11 + m)))/e^11
```

**fricas** [**B**] time = 1.20, size = 4795, normalized size = 8.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (54*d*e^10*m^10 + 500*(e^11*m^10 + 55*e^11*m^9 + 1320*e^11*m^8 + 18150*e^11
*m^7 + 157773*e^11*m^6 + 902055*e^11*m^5 + 3416930*e^11*m^4 + 8409500*e^11*
m^3 + 12753576*e^11*m^2 + 10628640*e^11*m + 3628800*e^11)*x^11 + 1814400000
*d^11 + 99792000*d^10*e + 3392928000*d^9*e^2 + 488980800*d^8*e^3 + 56966976
00*d^7*e^4 - 3392928000*d^6*e^5 + 8853546240*d^5*e^6 - 5728060800*d^4*e^7 +
 6346771200*d^3*e^8 - 2694384000*d^2*e^9 + 2155507200*d*e^10 - 25*(3991680*
e^11 - (20*d*e^10 - e^11)*m^10 - 4*(225*d*e^10 - 14*e^11)*m^9 - 15*(1160*d*
e^10 - 91*e^11)*m^8 - 60*(3150*d*e^10 - 317*e^11)*m^7 - 21*(60260*d*e^10 -
7963*e^11)*m^6 - 84*(64125*d*e^10 - 11492*e^11)*m^5 - 5*(2894720*d*e^10 - 7
37251*e^11)*m^4 - 20*(1172700*d*e^10 - 456659*e^11)*m^3 - 36*(570320*d*e^10
 - 386841*e^11)*m^2 - 144*(50400*d*e^10 - 80939*e^11)*m)*x^10 - 135*(d^2*e^
9 - 26*d*e^10)*m^9 + 5*(678585600*e^11 - (5*d*e^10 - 153*e^11)*m^10 - (1000
*d^2*e^9 + 235*d*e^10 - 8721*e^11)*m^9 - 6*(6000*d^2*e^9 + 785*d*e^10 - 360
06*e^11)*m^8 - 6*(91000*d^2*e^9 + 8785*d*e^10 - 509031*e^11)*m^7 - 105*(432
00*d^2*e^9 + 3445*d*e^10 - 259029*e^11)*m^6 - 21*(1069000*d^2*e^9 + 74815*d
*e^10 - 7560189*e^11)*m^5 - 2*(33642000*d^2*e^9 + 2145620*d*e^10 - 30603656
7*e^11)*m^4 - 4*(29531000*d^2*e^9 + 1761185*d*e^10 - 382172121*e^11)*m^3 -
72*(1522000*d^2*e^9 + 86510*d*e^10 - 32587351*e^11)*m^2 - 1440*(28000*d^2*e
^9 + 1540*d*e^10 - 1370727*e^11)*m)*x^9 + 9*(106*d^3*e^8 - 945*d^2*e^9 + 11
160*d*e^10)*m^8 - (488980800*e^11 - (765*d*e^10 - 98*e^11)*m^10 - (225*d^2*
e^9 + 37485*d*e^10 - 5684*e^11)*m^9 - 3*(15000*d^3*e^8 + 2925*d^2*e^9 + 260
100*d*e^10 - 47726*e^11)*m^8 - 42*(30000*d^3*e^8 + 3375*d^2*e^9 + 214965*d*
e^10 - 48958*e^11)*m^7 - 63*(230000*d^3*e^8 + 19650*d^2*e^9 + 1012095*d*e^1
0 - 294882*e^11)*m^6 - 63*(1400000*d^3*e^8 + 101175*d^2*e^9 + 4503555*d*e^1
0 - 1743812*e^11)*m^5 - (304605000*d^3*e^8 + 19707975*d^2*e^9 + 790573950*d
*e^10 - 428393182*e^11)*m^4 - 4*(147735000*d^3*e^8 + 8860500*d^2*e^9 + 3297
12705*d*e^10 - 270109021*e^11)*m^3 - 36*(16335000*d^3*e^8 + 929925*d^2*e^9
+ 32795550*d*e^10 - 46438966*e^11)*m^2 - 5040*(45000*d^3*e^8 + 2475*d^2*e^9
 + 84150*d*e^10 - 280861*e^11)*m)*x^8 - 6*(574*d^4*e^7 - 9540*d^3*e^8 + 390
15*d^2*e^9 - 277290*d*e^10)*m^7 + (5696697600*e^11 - (98*d*e^10 - 999*e^11)
*m^10 - 3*(2040*d^2*e^9 + 1666*d*e^10 - 19647*e^11)*m^9 - 24*(75*d^3*e^8 +
10710*d^2*e^9 + 4508*d*e^10 - 62937*e^11)*m^8 - 6*(60000*d^4*e^7 + 9600*d^3
*e^8 + 740520*d^2*e^9 + 216482*d*e^10 - 3677319*e^11)*m^7 - 3*(2520000*d^4*
```

e^7 + 243600*d^3*e^8 + 13708800*d^2*e^9 + 3161774*d*e^10 - 67539393*e^11)*m
^6 - 21*(3000000*d^4*e^7 + 228000*d^3*e^8 + 10581480*d^2*e^9 + 2069662*d*e^
10 - 57933009*e^11)*m^5 - 2*(132300000*d^4*e^7 + 8738100*d^3*e^8 + 35715708
0*d^2*e^9 + 62076434*d*e^10 - 2405021571*e^11)*m^4 - 36*(16240000*d^4*e^7 +
 981400*d^3*e^8 + 36788680*d^2*e^9 + 5871278*d*e^10 - 341095341*e^11)*m^3 -
 72*(8820000*d^4*e^7 + 503100*d^3*e^8 + 17778600*d^2*e^9 + 2670010*d*e^10 -
 266622111*e^11)*m^2 - 12960*(20000*d^4*e^7 + 1100*d^3*e^8 + 37400*d^2*e^9
+ 5390*d*e^10 - 1264623*e^11)*m)*x^7 + 6*(4436*d^5*e^6 - 32144*d^4*e^7 + 24
7086*d^3*e^8 - 615195*d^2*e^9 + 2939517*d*e^10)*m^6 + (3392928000*e^11 + 3*
(333*d*e^10 + 170*e^11)*m^10 + (686*d^2*e^9 + 52947*d*e^10 + 30600*e^11)*m^
9 + 6*(7140*d^3*e^8 + 5145*d^2*e^9 + 198801*d*e^10 + 133025*e^11)*m^8 + 6*(
2100*d^4*e^7 + 257040*d^3*e^8 + 95354*d^2*e^9 + 2484513*d*e^10 + 1978800*e^
11)*m^7 + 3*(840000*d^5*e^6 + 109200*d^4*e^7 + 7282800*d^3*e^8 + 1886500*d^
2*e^9 + 37725237*d*e^10 + 37016310*e^11)*m^6 + 3*(12600000*d^5*e^6 + 105000
0*d^4*e^7 + 52264800*d^3*e^8 + 10813418*d^2*e^9 + 179179641*d*e^10 + 226287
000*e^11)*m^5 + 42*(5100000*d^5*e^6 + 348000*d^4*e^7 + 14635980*d^3*e^8 + 2
609495*d^2*e^9 + 37733562*d*e^10 + 64999925*e^11)*m^4 + 4*(141750000*d^5*e^
6 + 8659350*d^4*e^7 + 327983040*d^3*e^8 + 52869334*d^2*e^9 + 692643663*d*e^
10 + 1769460300*e^11)*m^3 + 120*(5754000*d^5*e^6 + 329070*d^4*e^7 + 1165962
0*d^3*e^8 + 1755817*d^2*e^9 + 21444534*d*e^10 + 93454763*e^11)*m^2 + 7200*(
42000*d^5*e^6 + 2310*d^4*e^7 + 78540*d^3*e^8 + 11319*d^2*e^9 + 131868*d*e^1
0 + 1344547*e^11)*m)*x^6 - 3*(20400*d^6*e^5 - 452472*d^5*e^6 + 1526840*d^4*
e^7 - 7212240*d^3*e^8 + 12236805*d^2*e^9 - 41597010*d*e^10)*m^5 + (88535462
40*e^11 + (510*d*e^10 + 1109*e^11)*m^10 - (5994*d^2*e^9 - 28050*d*e^10 - 67
649*e^11)*m^9 - 12*(343*d^3*e^8 + 23976*d^2*e^9 - 54825*d*e^10 - 149715*e^1
1)*m^8 - 6*(42840*d^4*e^7 + 27440*d^3*e^8 + 953046*d^2*e^9 - 1430550*d*e^10
 - 4541355*e^11)*m^7 - 3*(25200*d^5*e^6 + 2656080*d^4*e^7 + 869848*d^3*e^8
+ 20283696*d^2*e^9 - 22710810*d*e^10 - 86713819*e^11)*m^6 - 3*(5040000*d^6*
e^5 + 529200*d^5*e^6 + 30416400*d^4*e^7 + 6969760*d^3*e^8 + 124932942*d^2*e
^9 - 112732950*d*e^10 - 541448179*e^11)*m^5 - 2*(75600000*d^6*e^5 + 5481000
*d^5*e^6 + 242260200*d^4*e^7 + 45047562*d^3*e^8 + 675619704*d^2*e^9 - 51950
1300*d*e^10 - 3335910815*e^11)*m^4 - 4*(132300000*d^6*e^5 + 8221500*d^5*e^6
 + 316416240*d^4*e^7 + 51779280*d^3*e^8 + 688165146*d^2*e^9 - 470707050*d*e
^10 - 4412539105*e^11)*m^3 - 72*(10500000*d^6*e^5 + 602700*d^5*e^6 + 214342
80*d^4*e^7 + 3239978*d^3*e^8 + 39724236*d^2*e^9 - 25005980*d*e^10 - 3955614
47*e^11)*m^2 - 288*(1260000*d^6*e^5 + 69300*d^5*e^6 + 2356200*d^4*e^7 + 339
570*d^3*e^8 + 3956040*d^2*e^9 - 2356200*d*e^10 - 86687203*e^11)*m)*x^5 + 3*
(239760*d^7*e^4 - 918000*d^6*e^5 + 9537400*d^5*e^6 - 19929280*d^4*e^7 + 648
36702*d^3*e^8 - 79518915*d^2*e^9 + 198514620*d*e^10)*m^4 + (5728060800*e^11
 + (1109*d*e^10 + 574*e^11)*m^10 - (2550*d^2*e^9 - 63213*d*e^10 - 35588*e^1
1)*m^9 + 6*(4995*d^3*e^8 - 21675*d^2*e^9 + 257288*d*e^10 + 160433*e^11)*m^8
 + 6*(3430*d^4*e^7 + 219780*d^3*e^8 - 461550*d^2*e^9 + 3512203*d*e^10 + 248
3698*e^11)*m^7 + 15*(85680*d^5*e^6 + 49392*d^4*e^7 + 1554444*d^3*e^8 - 2122
620*d^2*e^9 + 11723239*d*e^10 + 9703470*e^11)*m^6 + 3*(126000*d^6*e^5 + 115
66800*d^5*e^6 + 3361400*d^4*e^7 + 70329600*d^3*e^8 - 71101650*d^2*e^9 + 306

983399*d*e^10 + 310583364*e^11)*m^5 + 2*(37800000*d^7*e^4 + 3213000*d^6*e^5 + 158722200*d^5*e^6 + 32104800*d^4*e^7 + 515019465*d^3*e^8 - 418887225*d^2*e^9 + 1494010421*d*e^10 + 1964946361*e^11)*m^4 + 4*(113400000*d^7*e^4 + 7276500*d^6*e^5 + 288206100*d^5*e^6 + 48409305*d^4*e^7 + 659010330*d^3*e^8 - 460978800*d^2*e^9 + 1424518263*d*e^10 + 2670494533*e^11)*m^3 + 72*(11550000*d^7*e^4 + 666750*d^6*e^5 + 23847600*d^5*e^6 + 3625510*d^4*e^7 + 44710245*d^3*e^8 - 28312225*d^2*e^9 + 79001833*d*e^10 + 245697543*e^11)*m^2 + 720*(630000*d^7*e^4 + 34650*d^6*e^5 + 1178100*d^5*e^6 + 169785*d^4*e^7 + 1978020*d^3*e^8 - 1178100*d^2*e^9 + 3074148*d*e^10 + 22036147*e^11)*m)*x^4 + 12*(41160*d^8*e^3 + 2277720*d^7*e^4 - 4105500*d^6*e^5 + 26582730*d^5*e^6 - 38586863*d^4*e^7 + 91855890*d^3*e^8 - 84312180*d^2*e^9 + 157352130*d*e^10)*m^3 + (6346771200*e^11 + (574*d*e^10 + 477*e^11)*m^10 - (4436*d^2*e^9 - 33866*d*e^10 - 30051*e^11)*m^9 + 24*(425*d^3*e^8 - 9981*d^2*e^9 + 35875*d*e^10 + 34503*e^11)*m^8 - 6*(19980*d^4*e^7 - 81600*d^3*e^8 + 909380*d^2*e^9 - 2053198*d*e^10 - 2183229*e^11)*m^7 - 3*(27440*d^5*e^6 + 1638360*d^4*e^7 - 3202800*d^3*e^8 + 22641344*d^2*e^9 - 36198162*d*e^10 - 43730883*e^11)*m^6 - 3*(1713600*d^6*e^5 + 905520*d^5*e^6 + 26173800*d^4*e^7 - 32844000*d^3*e^8 + 166540748*d^2*e^9 - 201988878*d*e^10 - 288179073*e^11)*m^5 - 2*(756000*d^7*e^4 + 61689600*d^6*e^5 + 16093560*d^5*e^6 + 304195500*d^4*e^7 - 278811900*d^3*e^8 + 1092467028*d^2*e^9 - 1055996410*d*e^10 - 1884673269*e^11)*m^4 - 4*(75600000*d^8*e^3 + 5292000*d^7*e^4 + 224910000*d^6*e^5 + 40069260*d^5*e^6 + 573745680*d^4*e^7 - 419556600*d^3*e^8 + 1349320300*d^2*e^9 - 1086499918*d*e^10 - 2657980899*e^11)*m^3 - 24*(37800000*d^8*e^3 + 2205000*d^7*e^4 + 79682400*d^6*e^5 + 12238240*d^5*e^6 + 152467380*d^4*e^7 - 97540900*d^3*e^8 + 275018692*d^2*e^9 - 193842670*d*e^10 - 763013811*e^11)*m^2 - 4320*(140000*d^8*e^3 + 7700*d^7*e^4 + 261800*d^6*e^5 + 37730*d^5*e^6 + 439560*d^4*e^7 - 261800*d^3*e^8 + 683144*d^2*e^9 - 441980*d*e^10 - 3946963*e^11)*m)*x^3 + 12*(2570400*d^9*e^2 + 1234800*d^8*e^3 + 32307660*d^7*e^4 - 36490500*d^6*e^5 + 165294232*d^5*e^6 - 177258088*d^4*e^7 + 320238402*d^3*e^8 - 224755965*d^2*e^9 + 316309212*d*e^10)*m^2 + 3*(898128000*e^11 + 3*(53*d*e^10 + 15*e^11)*m^10 - (574*d^2*e^9 - 9699*d*e^10 - 2880*e^11)*m^9 + (4436*d^3*e^8 - 32718*d^2*e^9 + 256626*d*e^10 + 80865*e^11)*m^8 - 2*(5100*d^4*e^7 - 115336*d^3*e^8 + 397782*d^2*e^9 - 1926603*d*e^10 - 654210*e^11)*m^7 + (119880*d^5*e^6 - 469200*d^4*e^7 + 4994936*d^3*e^8 - 10728060*d^2*e^9 + 36024471*d*e^10 + 13467195*e^11)*m^6 + (82320*d^6*e^5 + 4675320*d^5*e^6 - 8670000*d^4*e^7 + 57934160*d^3*e^8 - 87138366*d^2*e^9 + 216130131*d*e^10 + 91755720*e^11)*m^5 + (5140800*d^7*e^4 + 2551920*d^6*e^5 + 69170760*d^5*e^6 - 81192000*d^4*e^7 + 383753924*d^3*e^8 - 431689902*d^2*e^9 + 824188584*d*e^10 + 416767635*e^11)*m^4 + 4*(378000*d^8*e^3 + 28274400*d^7*e^4 + 6770820*d^6*e^5 + 117512370*d^5*e^6 - 98809950*d^4*e^7 + 354356552*d^3*e^8 - 312153254*d^2*e^9 + 473899341*d*e^10 + 309068145*e^11)*m^3 + 12*(25200000*d^9*e^2 + 1512000*d^8*e^3 + 56120400*d^7*e^4 + 8842540*d^6*e^5 + 112906980*d^5*e^6 - 73978900*d^4*e^7 + 213535732*d^3*e^8 - 154064470*d^2*e^9 + 192742980*d*e^10 + 188672355*e^11)*m^2 + 2160*(140000*d^9*e^2 + 7700*d^8*e^3 + 261800*d^7*e^4 + 37730*d^6*e^5 + 439560*d^5*e^6 - 261800*d^4*e^7 + 683144*d^3*e^8 - 441980*d^2*e^9 + 489720*d*e^10 + 1

```
047765*e^11)*m)*x^2 + 144*(63000*d^10*e + 4498200*d^9*e^2 + 1025570*d^8*e^3
 + 16893090*d^7*e^4 - 13427450*d^6*e^5 + 45284906*d^5*e^6 - 37254035*d^4*e^
7 + 52296690*d^3*e^8 - 28438425*d^2*e^9 + 30235140*d*e^10)*m + 3*(718502400
*e^11 + 9*(5*d*e^10 + 2*e^11)*m^10 - 3*(106*d^2*e^9 - 945*d*e^10 - 390*e^11
)*m^9 + 2*(574*d^3*e^8 - 9540*d^2*e^9 + 39015*d*e^10 + 16740*e^11)*m^8 - 2*
(4436*d^4*e^7 - 32144*d^3*e^8 + 247086*d^2*e^9 - 615195*d*e^10 - 277290*e^1
1)*m^7 + (20400*d^5*e^6 - 452472*d^4*e^7 + 1526840*d^3*e^8 - 7212240*d^2*e^
9 + 12236805*d*e^10 + 5879034*e^11)*m^6 - (239760*d^6*e^5 - 918000*d^5*e^6
+ 9537400*d^4*e^7 - 19929280*d^3*e^8 + 64836702*d^2*e^9 - 79518915*d*e^10 -
 41597010*e^11)*m^5 - 4*(41160*d^7*e^4 + 2277720*d^6*e^5 - 4105500*d^5*e^6
+ 26582730*d^4*e^7 - 38586863*d^3*e^8 + 91855890*d^2*e^9 - 84312180*d*e^10
- 49628655*e^11)*m^4 - 4*(2570400*d^8*e^3 + 1234800*d^7*e^4 + 32307660*d^6*
e^5 - 36490500*d^5*e^6 + 165294232*d^4*e^7 - 177258088*d^3*e^8 + 320238402*
d^2*e^9 - 224755965*d*e^10 - 157352130*e^11)*m^3 - 48*(63000*d^9*e^2 + 4498
200*d^8*e^3 + 1025570*d^7*e^4 + 16893090*d^6*e^5 - 13427450*d^5*e^6 + 45284
906*d^4*e^7 - 37254035*d^3*e^8 + 52296690*d^2*e^9 - 28438425*d*e^10 - 26359
101*e^11)*m^2 - 8640*(70000*d^10*e + 3850*d^9*e^2 + 130900*d^8*e^3 + 18865*
d^7*e^4 + 219780*d^6*e^5 - 130900*d^5*e^6 + 341572*d^4*e^7 - 220990*d^3*e^8
 + 244860*d^2*e^9 - 103950*d*e^10 - 167973*e^11)*m)*x)*(e*x + d)^m/(e^11*m^
11 + 66*e^11*m^10 + 1925*e^11*m^9 + 32670*e^11*m^8 + 357423*e^11*m^7 + 2637
558*e^11*m^6 + 13339535*e^11*m^5 + 45995730*e^11*m^4 + 105258076*e^11*m^3 +
 150917976*e^11*m^2 + 120543840*e^11*m + 39916800*e^11)
```

**giac [B]** time = 0.62, size = 10960, normalized size = 18.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (500*(x*e + d)^m*m^10*x^11*e^11 + 500*(x*e + d)^m*d*m^10*x^10*e^10 - 25*(x*
e + d)^m*m^10*x^10*e^11 + 27500*(x*e + d)^m*m^9*x^11*e^11 - 25*(x*e + d)^m*
d*m^10*x^9*e^10 + 22500*(x*e + d)^m*d*m^9*x^10*e^10 - 5000*(x*e + d)^m*d^2*
m^9*x^9*e^9 + 765*(x*e + d)^m*m^10*x^9*e^11 - 1400*(x*e + d)^m*m^9*x^10*e^1
1 + 660000*(x*e + d)^m*m^8*x^11*e^11 + 765*(x*e + d)^m*d*m^10*x^8*e^10 - 11
75*(x*e + d)^m*d*m^9*x^9*e^10 + 435000*(x*e + d)^m*d*m^8*x^10*e^10 + 225*(x
*e + d)^m*d^2*m^9*x^8*e^9 - 180000*(x*e + d)^m*d^2*m^8*x^9*e^9 + 45000*(x*e
 + d)^m*d^3*m^8*x^8*e^8 - 98*(x*e + d)^m*m^10*x^8*e^11 + 43605*(x*e + d)^m*
m^9*x^9*e^11 - 34125*(x*e + d)^m*m^8*x^10*e^11 + 9075000*(x*e + d)^m*m^7*x^
11*e^11 - 98*(x*e + d)^m*d*m^10*x^7*e^10 + 37485*(x*e + d)^m*d*m^9*x^8*e^10
 - 23550*(x*e + d)^m*d*m^8*x^9*e^10 + 4725000*(x*e + d)^m*d*m^7*x^10*e^10 -
 6120*(x*e + d)^m*d^2*m^9*x^7*e^9 + 8775*(x*e + d)^m*d^2*m^8*x^8*e^9 - 2730
000*(x*e + d)^m*d^2*m^7*x^9*e^9 - 1800*(x*e + d)^m*d^3*m^8*x^7*e^8 + 126000
0*(x*e + d)^m*d^3*m^7*x^8*e^8 - 360000*(x*e + d)^m*d^4*m^7*x^7*e^7 + 999*(x
```

*e + d)^m*m^10*x^7*e^11 - 5684*(x*e + d)^m*m^9*x^8*e^11 + 1080180*(x*e + d)
^m*m^8*x^9*e^11 - 475500*(x*e + d)^m*m^7*x^10*e^11 + 78886500*(x*e + d)^m*m
^6*x^11*e^11 + 999*(x*e + d)^m*d*m^10*x^6*e^10 - 4998*(x*e + d)^m*d*m^9*x^7
*e^10 + 780300*(x*e + d)^m*d*m^8*x^8*e^10 - 263550*(x*e + d)^m*d*m^7*x^9*e^
10 + 31636500*(x*e + d)^m*d*m^6*x^10*e^10 + 686*(x*e + d)^m*d^2*m^9*x^6*e^9
 - 257040*(x*e + d)^m*d^2*m^8*x^7*e^9 + 141750*(x*e + d)^m*d^2*m^7*x^8*e^9
- 22680000*(x*e + d)^m*d^2*m^6*x^9*e^9 + 42840*(x*e + d)^m*d^3*m^8*x^6*e^8
- 57600*(x*e + d)^m*d^3*m^7*x^7*e^8 + 14490000*(x*e + d)^m*d^3*m^6*x^8*e^8
+ 12600*(x*e + d)^m*d^4*m^7*x^6*e^7 - 7560000*(x*e + d)^m*d^4*m^6*x^7*e^7 +
 2520000*(x*e + d)^m*d^5*m^6*x^6*e^6 + 510*(x*e + d)^m*m^10*x^6*e^11 + 5894
1*(x*e + d)^m*m^9*x^7*e^11 - 143178*(x*e + d)^m*m^8*x^8*e^11 + 15270930*(x*
e + d)^m*m^7*x^9*e^11 - 4180575*(x*e + d)^m*m^6*x^10*e^11 + 451027500*(x*e
+ d)^m*m^5*x^11*e^11 + 510*(x*e + d)^m*d*m^10*x^5*e^10 + 52947*(x*e + d)^m*
d*m^9*x^6*e^10 - 108192*(x*e + d)^m*d*m^8*x^7*e^10 + 9028530*(x*e + d)^m*d*
m^7*x^8*e^10 - 1808625*(x*e + d)^m*d*m^6*x^9*e^10 + 134662500*(x*e + d)^m*d
*m^5*x^10*e^10 - 5994*(x*e + d)^m*d^2*m^9*x^5*e^9 + 30870*(x*e + d)^m*d^2*m
^8*x^6*e^9 - 4443120*(x*e + d)^m*d^2*m^7*x^7*e^9 + 1237950*(x*e + d)^m*d^2*
m^6*x^8*e^9 - 112245000*(x*e + d)^m*d^2*m^5*x^9*e^9 - 4116*(x*e + d)^m*d^3*
m^8*x^5*e^8 + 1542240*(x*e + d)^m*d^3*m^7*x^6*e^8 - 730800*(x*e + d)^m*d^3*
m^6*x^7*e^8 + 88200000*(x*e + d)^m*d^3*m^5*x^8*e^8 - 257040*(x*e + d)^m*d^4
*m^7*x^5*e^7 + 327600*(x*e + d)^m*d^4*m^6*x^6*e^7 - 63000000*(x*e + d)^m*d^
4*m^5*x^7*e^7 - 75600*(x*e + d)^m*d^5*m^6*x^5*e^6 + 37800000*(x*e + d)^m*d^
5*m^5*x^6*e^6 - 15120000*(x*e + d)^m*d^6*m^5*x^5*e^5 + 1109*(x*e + d)^m*m^1
0*x^5*e^11 + 30600*(x*e + d)^m*m^9*x^6*e^11 + 1510488*(x*e + d)^m*m^8*x^7*e
^11 - 2056236*(x*e + d)^m*m^7*x^8*e^11 + 135990225*(x*e + d)^m*m^6*x^9*e^11
 - 24133200*(x*e + d)^m*m^5*x^10*e^11 + 1708465000*(x*e + d)^m*m^4*x^11*e^1
1 + 1109*(x*e + d)^m*d*m^10*x^4*e^10 + 28050*(x*e + d)^m*d*m^9*x^5*e^10 + 1
192806*(x*e + d)^m*d*m^8*x^6*e^10 - 1298892*(x*e + d)^m*d*m^7*x^7*e^10 + 63
761985*(x*e + d)^m*d*m^6*x^8*e^10 - 7855575*(x*e + d)^m*d*m^5*x^9*e^10 + 36
1840000*(x*e + d)^m*d*m^4*x^10*e^10 - 2550*(x*e + d)^m*d^2*m^9*x^4*e^9 - 28
7712*(x*e + d)^m*d^2*m^8*x^5*e^9 + 572124*(x*e + d)^m*d^2*m^7*x^6*e^9 - 411
26400*(x*e + d)^m*d^2*m^6*x^7*e^9 + 6374025*(x*e + d)^m*d^2*m^5*x^8*e^9 - 3
36420000*(x*e + d)^m*d^2*m^4*x^9*e^9 + 29970*(x*e + d)^m*d^3*m^8*x^4*e^8 -
164640*(x*e + d)^m*d^3*m^7*x^5*e^8 + 21848400*(x*e + d)^m*d^3*m^6*x^6*e^8 -
 4788000*(x*e + d)^m*d^3*m^5*x^7*e^8 + 304605000*(x*e + d)^m*d^3*m^4*x^8*e^
8 + 20580*(x*e + d)^m*d^4*m^7*x^4*e^7 - 7968240*(x*e + d)^m*d^4*m^6*x^5*e^7
 + 3150000*(x*e + d)^m*d^4*m^5*x^6*e^7 - 264600000*(x*e + d)^m*d^4*m^4*x^7*
e^7 + 1285200*(x*e + d)^m*d^5*m^6*x^4*e^6 - 1587600*(x*e + d)^m*d^5*m^5*x^5
*e^6 + 214200000*(x*e + d)^m*d^5*m^4*x^6*e^6 + 378000*(x*e + d)^m*d^6*m^5*x
^4*e^5 - 151200000*(x*e + d)^m*d^6*m^4*x^5*e^5 + 75600000*(x*e + d)^m*d^7*m
^4*x^4*e^4 + 574*(x*e + d)^m*m^10*x^4*e^11 + 67649*(x*e + d)^m*m^9*x^5*e^11
 + 798150*(x*e + d)^m*m^8*x^6*e^11 + 22063914*(x*e + d)^m*m^7*x^7*e^11 - 18
577566*(x*e + d)^m*m^6*x^8*e^11 + 793819845*(x*e + d)^m*m^5*x^9*e^11 - 9215
6375*(x*e + d)^m*m^4*x^10*e^11 + 4204750000*(x*e + d)^m*m^3*x^11*e^11 + 574
*(x*e + d)^m*d*m^10*x^3*e^10 + 63213*(x*e + d)^m*d*m^9*x^4*e^10 + 657900*(x

*e + d)^m*d*m^8*x^5*e^10 + 14907078*(x*e + d)^m*d*m^7*x^6*e^10 - 9485322*(x
*e + d)^m*d*m^6*x^7*e^10 + 283723965*(x*e + d)^m*d*m^5*x^8*e^10 - 21456200*
(x*e + d)^m*d*m^4*x^9*e^10 + 586350000*(x*e + d)^m*d*m^3*x^10*e^10 - 4436*(
x*e + d)^m*d^2*m^9*x^3*e^9 - 130050*(x*e + d)^m*d^2*m^8*x^4*e^9 - 5718276*(
x*e + d)^m*d^2*m^7*x^5*e^9 + 5659500*(x*e + d)^m*d^2*m^6*x^6*e^9 - 22221108
0*(x*e + d)^m*d^2*m^5*x^7*e^9 + 19707975*(x*e + d)^m*d^2*m^4*x^8*e^9 - 5906
20000*(x*e + d)^m*d^2*m^3*x^9*e^9 + 10200*(x*e + d)^m*d^3*m^8*x^3*e^8 + 131
8680*(x*e + d)^m*d^3*m^7*x^4*e^8 - 2609544*(x*e + d)^m*d^3*m^6*x^5*e^8 + 15
6794400*(x*e + d)^m*d^3*m^5*x^6*e^8 - 17476200*(x*e + d)^m*d^3*m^4*x^7*e^8
+ 590940000*(x*e + d)^m*d^3*m^3*x^8*e^8 - 119880*(x*e + d)^m*d^4*m^7*x^3*e^
7 + 740880*(x*e + d)^m*d^4*m^6*x^4*e^7 - 91249200*(x*e + d)^m*d^4*m^5*x^5*e
^7 + 14616000*(x*e + d)^m*d^4*m^4*x^6*e^7 - 584640000*(x*e + d)^m*d^4*m^3*x
^7*e^7 - 82320*(x*e + d)^m*d^5*m^6*x^3*e^6 + 34700400*(x*e + d)^m*d^5*m^5*x
^4*e^6 - 10962000*(x*e + d)^m*d^5*m^4*x^5*e^6 + 567000000*(x*e + d)^m*d^5*m
^3*x^6*e^6 - 5140800*(x*e + d)^m*d^6*m^5*x^3*e^5 + 6426000*(x*e + d)^m*d^6*
m^4*x^4*e^5 - 529200000*(x*e + d)^m*d^6*m^3*x^5*e^5 - 1512000*(x*e + d)^m*d
^7*m^4*x^3*e^4 + 453600000*(x*e + d)^m*d^7*m^3*x^4*e^4 - 302400000*(x*e + d
)^m*d^8*m^3*x^3*e^3 + 477*(x*e + d)^m*m^10*x^3*e^11 + 35588*(x*e + d)^m*m^9
*x^4*e^11 + 1796580*(x*e + d)^m*m^8*x^5*e^11 + 11872800*(x*e + d)^m*m^7*x^6
*e^11 + 202618179*(x*e + d)^m*m^6*x^7*e^11 - 109860156*(x*e + d)^m*m^5*x^8*
e^11 + 3060365670*(x*e + d)^m*m^4*x^9*e^11 - 228329500*(x*e + d)^m*m^3*x^10
*e^11 + 6376788000*(x*e + d)^m*m^2*x^11*e^11 + 477*(x*e + d)^m*d*m^10*x^2*e
^10 + 33866*(x*e + d)^m*d*m^9*x^3*e^10 + 1543728*(x*e + d)^m*d*m^8*x^4*e^10
+ 8583300*(x*e + d)^m*d*m^7*x^5*e^10 + 113175711*(x*e + d)^m*d*m^6*x^6*e^1
0 - 43462902*(x*e + d)^m*d*m^5*x^7*e^10 + 790573950*(x*e + d)^m*d*m^4*x^8*e
^10 - 35223700*(x*e + d)^m*d*m^3*x^9*e^10 + 513288000*(x*e + d)^m*d*m^2*x^1
0*e^10 - 1722*(x*e + d)^m*d^2*m^9*x^2*e^9 - 239544*(x*e + d)^m*d^2*m^8*x^3*
e^9 - 2769300*(x*e + d)^m*d^2*m^7*x^4*e^9 - 60851088*(x*e + d)^m*d^2*m^6*x^
5*e^9 + 32440254*(x*e + d)^m*d^2*m^5*x^6*e^9 - 714314160*(x*e + d)^m*d^2*m^
4*x^7*e^9 + 35442000*(x*e + d)^m*d^2*m^3*x^8*e^9 - 547920000*(x*e + d)^m*d^
2*m^2*x^9*e^9 + 13308*(x*e + d)^m*d^3*m^8*x^2*e^8 + 489600*(x*e + d)^m*d^3*
m^7*x^3*e^8 + 23316660*(x*e + d)^m*d^3*m^6*x^4*e^8 - 20909280*(x*e + d)^m*d
^3*m^5*x^5*e^8 + 614711160*(x*e + d)^m*d^3*m^4*x^6*e^8 - 35330400*(x*e + d)
^m*d^3*m^3*x^7*e^8 + 588060000*(x*e + d)^m*d^3*m^2*x^8*e^8 - 30600*(x*e + d
)^m*d^4*m^7*x^2*e^7 - 4915080*(x*e + d)^m*d^4*m^6*x^3*e^7 + 10084200*(x*e +
d)^m*d^4*m^5*x^4*e^7 - 484520400*(x*e + d)^m*d^4*m^4*x^5*e^7 + 34637400*(x
*e + d)^m*d^4*m^3*x^6*e^7 - 635040000*(x*e + d)^m*d^4*m^2*x^7*e^7 + 359640*
(x*e + d)^m*d^5*m^6*x^2*e^6 - 2716560*(x*e + d)^m*d^5*m^5*x^3*e^6 + 3174444
00*(x*e + d)^m*d^5*m^4*x^4*e^6 - 32886000*(x*e + d)^m*d^5*m^3*x^5*e^6 + 690
480000*(x*e + d)^m*d^5*m^2*x^6*e^6 + 246960*(x*e + d)^m*d^6*m^5*x^2*e^5 - 1
23379200*(x*e + d)^m*d^6*m^4*x^3*e^5 + 29106000*(x*e + d)^m*d^6*m^3*x^4*e^5
- 756000000*(x*e + d)^m*d^6*m^2*x^5*e^5 + 15422400*(x*e + d)^m*d^7*m^4*x^2
*e^4 - 21168000*(x*e + d)^m*d^7*m^3*x^3*e^4 + 831600000*(x*e + d)^m*d^7*m^2
*x^4*e^4 + 4536000*(x*e + d)^m*d^8*m^3*x^2*e^3 - 907200000*(x*e + d)^m*d^8*
m^2*x^3*e^3 + 907200000*(x*e + d)^m*d^9*m^2*x^2*e^2 + 135*(x*e + d)^m*m^10*

x^2*e^11 + 30051*(x*e + d)^m*m^9*x^3*e^11 + 962598*(x*e + d)^m*m^8*x^4*e^11
 + 27248130*(x*e + d)^m*m^7*x^5*e^11 + 111048930*(x*e + d)^m*m^6*x^6*e^11 +
 1216593189*(x*e + d)^m*m^5*x^7*e^11 - 428393182*(x*e + d)^m*m^4*x^8*e^11 +
 7643442420*(x*e + d)^m*m^3*x^9*e^11 - 348156900*(x*e + d)^m*m^2*x^10*e^11
 + 5314320000*(x*e + d)^m*m*x^11*e^11 + 135*(x*e + d)^m*d*m^10*x*e^10 + 2909
7*(x*e + d)^m*d*m^9*x^2*e^10 + 861000*(x*e + d)^m*d*m^8*x^3*e^10 + 21073218
*(x*e + d)^m*d*m^7*x^4*e^10 + 68132430*(x*e + d)^m*d*m^6*x^5*e^10 + 5375389
23*(x*e + d)^m*d*m^5*x^6*e^10 - 124152868*(x*e + d)^m*d*m^4*x^7*e^10 + 1318
850820*(x*e + d)^m*d*m^3*x^8*e^10 - 31143600*(x*e + d)^m*d*m^2*x^9*e^10 + 1
81440000*(x*e + d)^m*d*m*x^10*e^10 - 954*(x*e + d)^m*d^2*m^9*x*e^9 - 98154*
(x*e + d)^m*d^2*m^8*x^2*e^9 - 5456280*(x*e + d)^m*d^2*m^7*x^3*e^9 - 3183930
0*(x*e + d)^m*d^2*m^6*x^4*e^9 - 374798826*(x*e + d)^m*d^2*m^5*x^5*e^9 + 109
598790*(x*e + d)^m*d^2*m^4*x^6*e^9 - 1324392480*(x*e + d)^m*d^2*m^3*x^7*e^9
 + 33477300*(x*e + d)^m*d^2*m^2*x^8*e^9 - 201600000*(x*e + d)^m*d^2*m*x^9*e
^9 + 3444*(x*e + d)^m*d^3*m^8*x*e^8 + 692016*(x*e + d)^m*d^3*m^7*x^2*e^8 +
9608400*(x*e + d)^m*d^3*m^6*x^3*e^8 + 210988800*(x*e + d)^m*d^3*m^5*x^4*e^8
 - 90095124*(x*e + d)^m*d^3*m^4*x^5*e^8 + 1311932160*(x*e + d)^m*d^3*m^3*x^
6*e^8 - 36223200*(x*e + d)^m*d^3*m^2*x^7*e^8 + 226800000*(x*e + d)^m*d^3*m*
x^8*e^8 - 26616*(x*e + d)^m*d^4*m^7*x*e^7 - 1407600*(x*e + d)^m*d^4*m^6*x^2
*e^7 - 78521400*(x*e + d)^m*d^4*m^5*x^3*e^7 + 64209600*(x*e + d)^m*d^4*m^4*
x^4*e^7 - 1265664960*(x*e + d)^m*d^4*m^3*x^5*e^7 + 39488400*(x*e + d)^m*d^4
*m^2*x^6*e^7 - 259200000*(x*e + d)^m*d^4*m*x^7*e^7 + 61200*(x*e + d)^m*d^5*
m^6*x*e^6 + 14025960*(x*e + d)^m*d^5*m^5*x^2*e^6 - 32187120*(x*e + d)^m*d^5
*m^4*x^3*e^6 + 1152824400*(x*e + d)^m*d^5*m^3*x^4*e^6 - 43394400*(x*e + d)^
m*d^5*m^2*x^5*e^6 + 302400000*(x*e + d)^m*d^5*m*x^6*e^6 - 719280*(x*e + d)^
m*d^6*m^5*x*e^5 + 7655760*(x*e + d)^m*d^6*m^4*x^2*e^5 - 899640000*(x*e + d)
^m*d^6*m^3*x^3*e^5 + 48006000*(x*e + d)^m*d^6*m^2*x^4*e^5 - 362880000*(x*e
+ d)^m*d^6*m*x^5*e^5 - 493920*(x*e + d)^m*d^7*m^4*x*e^4 + 339292800*(x*e +
d)^m*d^7*m^3*x^2*e^4 - 52920000*(x*e + d)^m*d^7*m^2*x^3*e^4 + 453600000*(x*
e + d)^m*d^7*m*x^4*e^4 - 30844800*(x*e + d)^m*d^8*m^3*x*e^3 + 54432000*(x*e
 + d)^m*d^8*m^2*x^2*e^3 - 604800000*(x*e + d)^m*d^8*m*x^3*e^3 - 9072000*(x*
e + d)^m*d^9*m^2*x*e^2 + 907200000*(x*e + d)^m*d^9*m*x^2*e^2 - 1814400000*(
x*e + d)^m*d^10*m*x*e + 54*(x*e + d)^m*m^10*x*e^11 + 8640*(x*e + d)^m*m^9*x
^2*e^11 + 828072*(x*e + d)^m*m^8*x^3*e^11 + 14902188*(x*e + d)^m*m^7*x^4*e^
11 + 260141457*(x*e + d)^m*m^6*x^5*e^11 + 678861000*(x*e + d)^m*m^5*x^6*e^1
1 + 4810043142*(x*e + d)^m*m^4*x^7*e^11 - 1080436084*(x*e + d)^m*m^3*x^8*e^
11 + 11731446360*(x*e + d)^m*m^2*x^9*e^11 - 291380400*(x*e + d)^m*m*x^10*e^
11 + 1814400000*(x*e + d)^m*x^11*e^11 + 54*(x*e + d)^m*d*m^10*e^10 + 8505*(
x*e + d)^m*d*m^9*x*e^10 + 769878*(x*e + d)^m*d*m^8*x^2*e^10 + 12319188*(x*e
 + d)^m*d*m^7*x^3*e^10 + 175848585*(x*e + d)^m*d*m^6*x^4*e^10 + 338198850*(
x*e + d)^m*d*m^5*x^5*e^10 + 1584809604*(x*e + d)^m*d*m^4*x^6*e^10 - 2113660
08*(x*e + d)^m*d*m^3*x^7*e^10 + 1180639800*(x*e + d)^m*d*m^2*x^8*e^10 - 110
88000*(x*e + d)^m*d*m*x^9*e^10 - 135*(x*e + d)^m*d^2*m^9*e^9 - 57240*(x*e +
 d)^m*d^2*m^8*x*e^9 - 2386692*(x*e + d)^m*d^2*m^7*x^2*e^9 - 67924032*(x*e +
 d)^m*d^2*m^6*x^3*e^9 - 213304950*(x*e + d)^m*d^2*m^5*x^4*e^9 - 1351239408*

(x*e + d)^m*d^2*m^4*x^5*e^9 + 211477336*(x*e + d)^m*d^2*m^3*x^6*e^9 - 12800
59200*(x*e + d)^m*d^2*m^2*x^7*e^9 + 12474000*(x*e + d)^m*d^2*m*x^8*e^9 + 95
4*(x*e + d)^m*d^3*m^8*e^8 + 192864*(x*e + d)^m*d^3*m^7*x*e^8 + 14984808*(x*
e + d)^m*d^3*m^6*x^2*e^8 + 98532000*(x*e + d)^m*d^3*m^5*x^3*e^8 + 103003893
0*(x*e + d)^m*d^3*m^4*x^4*e^8 - 207117120*(x*e + d)^m*d^3*m^3*x^5*e^8 + 139
9154400*(x*e + d)^m*d^3*m^2*x^6*e^8 - 14256000*(x*e + d)^m*d^3*m*x^7*e^8 -
3444*(x*e + d)^m*d^4*m^7*e^7 - 1357416*(x*e + d)^m*d^4*m^6*x*e^7 - 26010000
*(x*e + d)^m*d^4*m^5*x^2*e^7 - 608391000*(x*e + d)^m*d^4*m^4*x^3*e^7 + 1936
37220*(x*e + d)^m*d^4*m^3*x^4*e^7 - 1543268160*(x*e + d)^m*d^4*m^2*x^5*e^7
+ 16632000*(x*e + d)^m*d^4*m*x^6*e^7 + 26616*(x*e + d)^m*d^5*m^6*e^6 + 2754
000*(x*e + d)^m*d^5*m^5*x*e^6 + 207512280*(x*e + d)^m*d^5*m^4*x^2*e^6 - 160
277040*(x*e + d)^m*d^5*m^3*x^3*e^6 + 1717027200*(x*e + d)^m*d^5*m^2*x^4*e^6
 - 19958400*(x*e + d)^m*d^5*m*x^5*e^6 - 61200*(x*e + d)^m*d^6*m^5*e^5 - 273
32640*(x*e + d)^m*d^6*m^4*x*e^5 + 81249840*(x*e + d)^m*d^6*m^3*x^2*e^5 - 19
12377600*(x*e + d)^m*d^6*m^2*x^3*e^5 + 24948000*(x*e + d)^m*d^6*m*x^4*e^5 +
 719280*(x*e + d)^m*d^7*m^4*e^4 - 14817600*(x*e + d)^m*d^7*m^3*x*e^4 + 2020
334400*(x*e + d)^m*d^7*m^2*x^2*e^4 - 33264000*(x*e + d)^m*d^7*m*x^3*e^4 + 4
93920*(x*e + d)^m*d^8*m^3*e^3 - 647740800*(x*e + d)^m*d^8*m^2*x*e^3 + 49896
000*(x*e + d)^m*d^8*m*x^2*e^3 + 30844800*(x*e + d)^m*d^9*m^2*e^2 - 99792000
*(x*e + d)^m*d^9*m*x*e^2 + 9072000*(x*e + d)^m*d^10*m*e + 1814400000*(x*e +
 d)^m*d^11 + 3510*(x*e + d)^m*m^9*x*e^11 + 242595*(x*e + d)^m*m^8*x^2*e^11
+ 13099374*(x*e + d)^m*m^7*x^3*e^11 + 145552050*(x*e + d)^m*m^6*x^4*e^11 +
1624344537*(x*e + d)^m*m^5*x^5*e^11 + 2729996850*(x*e + d)^m*m^4*x^6*e^11 +
 12279432276*(x*e + d)^m*m^3*x^7*e^11 - 1671802776*(x*e + d)^m*m^2*x^8*e^11
 + 9869234400*(x*e + d)^m*m*x^9*e^11 - 99792000*(x*e + d)^m*x^10*e^11 + 351
0*(x*e + d)^m*d*m^9*e^10 + 234090*(x*e + d)^m*d*m^8*x*e^10 + 11559618*(x*e
+ d)^m*d*m^7*x^2*e^10 + 108594486*(x*e + d)^m*d*m^6*x^3*e^10 + 920950197*(x
*e + d)^m*d*m^5*x^4*e^10 + 1039002600*(x*e + d)^m*d*m^4*x^5*e^10 + 27705746
52*(x*e + d)^m*d*m^3*x^6*e^10 - 192240720*(x*e + d)^m*d*m^2*x^7*e^10 + 4241
16000*(x*e + d)^m*d*m*x^8*e^10 - 8505*(x*e + d)^m*d^2*m^8*e^9 - 1482516*(x*
e + d)^m*d^2*m^7*x*e^9 - 32184180*(x*e + d)^m*d^2*m^6*x^2*e^9 - 499622244*(
x*e + d)^m*d^2*m^5*x^3*e^9 - 837774450*(x*e + d)^m*d^2*m^4*x^4*e^9 - 275266
0584*(x*e + d)^m*d^2*m^3*x^5*e^9 + 210698040*(x*e + d)^m*d^2*m^2*x^6*e^9 -
484704000*(x*e + d)^m*d^2*m*x^7*e^9 + 57240*(x*e + d)^m*d^3*m^7*e^8 + 45805
20*(x*e + d)^m*d^3*m^6*x*e^8 + 173802480*(x*e + d)^m*d^3*m^5*x^2*e^8 + 5576
23800*(x*e + d)^m*d^3*m^4*x^3*e^8 + 2636041320*(x*e + d)^m*d^3*m^3*x^4*e^8
- 233278416*(x*e + d)^m*d^3*m^2*x^5*e^8 + 565488000*(x*e + d)^m*d^3*m*x^6*e
^8 - 192864*(x*e + d)^m*d^4*m^6*e^7 - 28612200*(x*e + d)^m*d^4*m^5*x*e^7 -
243576000*(x*e + d)^m*d^4*m^4*x^2*e^7 - 2294982720*(x*e + d)^m*d^4*m^3*x^3*
e^7 + 261036720*(x*e + d)^m*d^4*m^2*x^4*e^7 - 678585600*(x*e + d)^m*d^4*m*x
^5*e^7 + 1357416*(x*e + d)^m*d^5*m^5*e^6 + 49266000*(x*e + d)^m*d^5*m^4*x*e
^6 + 1410148440*(x*e + d)^m*d^5*m^3*x^2*e^6 - 293717760*(x*e + d)^m*d^5*m^2
*x^3*e^6 + 848232000*(x*e + d)^m*d^5*m*x^4*e^6 - 2754000*(x*e + d)^m*d^6*m^
4*e^5 - 387691920*(x*e + d)^m*d^6*m^3*x*e^5 + 318331440*(x*e + d)^m*d^6*m^2
*x^2*e^5 - 1130976000*(x*e + d)^m*d^6*m*x^3*e^5 + 27332640*(x*e + d)^m*d^7*

m^3*e^4 - 147682080*(x*e + d)^m*d^7*m^2*x*e^4 + 1696464000*(x*e + d)^m*d^7*
m*x^2*e^4 + 14817600*(x*e + d)^m*d^8*m^2*e^3 - 3392928000*(x*e + d)^m*d^8*m
*x*e^3 + 647740800*(x*e + d)^m*d^9*m*e^2 + 99792000*(x*e + d)^m*d^10*e + 10
0440*(x*e + d)^m*m^8*x*e^11 + 3925260*(x*e + d)^m*m^7*x^2*e^11 + 131192649*
(x*e + d)^m*m^6*x^3*e^11 + 931750092*(x*e + d)^m*m^5*x^4*e^11 + 6671821630*
(x*e + d)^m*m^4*x^5*e^11 + 7077841200*(x*e + d)^m*m^3*x^6*e^11 + 1919679199
2*(x*e + d)^m*m^2*x^7*e^11 - 1415539440*(x*e + d)^m*m*x^8*e^11 + 3392928000
*(x*e + d)^m*x^9*e^11 + 100440*(x*e + d)^m*d*m^8*e^10 + 3691170*(x*e + d)^m
*d*m^7*x*e^10 + 108073413*(x*e + d)^m*d*m^6*x^2*e^10 + 605966634*(x*e + d)^
m*d*m^5*x^3*e^10 + 2988020842*(x*e + d)^m*d*m^4*x^4*e^10 + 1882828200*(x*e
+ d)^m*d*m^3*x^5*e^10 + 2573344080*(x*e + d)^m*d*m^2*x^6*e^10 - 69854400*(x
*e + d)^m*d*m*x^7*e^10 - 234090*(x*e + d)^m*d^2*m^7*e^9 - 21636720*(x*e + d
)^m*d^2*m^6*x*e^9 - 261415098*(x*e + d)^m*d^2*m^5*x^2*e^9 - 2184934056*(x*e
+ d)^m*d^2*m^4*x^3*e^9 - 1843915200*(x*e + d)^m*d^2*m^3*x^4*e^9 - 28601449
92*(x*e + d)^m*d^2*m^2*x^5*e^9 + 81496800*(x*e + d)^m*d^2*m*x^6*e^9 + 14825
16*(x*e + d)^m*d^3*m^6*e^8 + 59787840*(x*e + d)^m*d^3*m^5*x*e^8 + 115126177
2*(x*e + d)^m*d^3*m^4*x^2*e^8 + 1678226400*(x*e + d)^m*d^3*m^3*x^3*e^8 + 32
19137640*(x*e + d)^m*d^3*m^2*x^4*e^8 - 97796160*(x*e + d)^m*d^3*m*x^5*e^8 -
4580520*(x*e + d)^m*d^4*m^5*e^7 - 318992760*(x*e + d)^m*d^4*m^4*x*e^7 - 11
85719400*(x*e + d)^m*d^4*m^3*x^2*e^7 - 3659217120*(x*e + d)^m*d^4*m^2*x^3*e
^7 + 122245200*(x*e + d)^m*d^4*m*x^4*e^7 + 28612200*(x*e + d)^m*d^5*m^4*e^6
+ 437886000*(x*e + d)^m*d^5*m^3*x*e^6 + 4064651280*(x*e + d)^m*d^5*m^2*x^2
*e^6 - 162993600*(x*e + d)^m*d^5*m*x^3*e^6 - 49266000*(x*e + d)^m*d^6*m^3*e
^5 - 2432604960*(x*e + d)^m*d^6*m^2*x*e^5 + 244490400*(x*e + d)^m*d^6*m*x^2
*e^5 + 387691920*(x*e + d)^m*d^7*m^2*e^4 - 488980800*(x*e + d)^m*d^7*m*x*e^
4 + 147682080*(x*e + d)^m*d^8*m*e^3 + 3392928000*(x*e + d)^m*d^9*e^2 + 1663
740*(x*e + d)^m*m^7*x*e^11 + 40401585*(x*e + d)^m*m^6*x^2*e^11 + 864537219*
(x*e + d)^m*m^5*x^3*e^11 + 3929892722*(x*e + d)^m*m^4*x^4*e^11 + 1765015642
0*(x*e + d)^m*m^3*x^5*e^11 + 11214571560*(x*e + d)^m*m^2*x^6*e^11 + 1638951
4080*(x*e + d)^m*m*x^7*e^11 - 488980800*(x*e + d)^m*x^8*e^11 + 1663740*(x*e
+ d)^m*d*m^7*e^10 + 36710415*(x*e + d)^m*d*m^6*x*e^10 + 648390393*(x*e + d
)^m*d*m^5*x^2*e^10 + 2111992820*(x*e + d)^m*d*m^4*x^3*e^10 + 5698073052*(x*
e + d)^m*d*m^3*x^4*e^10 + 1800430560*(x*e + d)^m*d*m^2*x^5*e^10 + 949449600
*(x*e + d)^m*d*m*x^6*e^10 - 3691170*(x*e + d)^m*d^2*m^6*e^9 - 194510106*(x*
e + d)^m*d^2*m^5*x*e^9 - 1295069706*(x*e + d)^m*d^2*m^4*x^2*e^9 - 539728120
0*(x*e + d)^m*d^2*m^3*x^3*e^9 - 2038480200*(x*e + d)^m*d^2*m^2*x^4*e^9 - 11
39339520*(x*e + d)^m*d^2*m*x^5*e^9 + 21636720*(x*e + d)^m*d^3*m^5*e^8 + 463
042356*(x*e + d)^m*d^3*m^4*x*e^8 + 4252278624*(x*e + d)^m*d^3*m^3*x^2*e^8 +
2340981600*(x*e + d)^m*d^3*m^2*x^3*e^8 + 1424174400*(x*e + d)^m*d^3*m*x^4*
e^8 - 59787840*(x*e + d)^m*d^4*m^4*e^7 - 1983530784*(x*e + d)^m*d^4*m^3*x*e
^7 - 2663240400*(x*e + d)^m*d^4*m^2*x^2*e^7 - 1898899200*(x*e + d)^m*d^4*m*
x^3*e^7 + 318992760*(x*e + d)^m*d^5*m^3*e^6 + 1933552800*(x*e + d)^m*d^5*m^
2*x*e^6 + 2848348800*(x*e + d)^m*d^5*m*x^2*e^6 - 437886000*(x*e + d)^m*d^6*
m^2*e^5 - 5696697600*(x*e + d)^m*d^6*m*x*e^5 + 2432604960*(x*e + d)^m*d^7*m
*e^4 + 488980800*(x*e + d)^m*d^8*e^3 + 17637102*(x*e + d)^m*m^6*x*e^11 + 27

5267160*(x*e + d)^m*m^5*x^2*e^11 + 3769346538*(x*e + d)^m*m^4*x^3*e^11 + 10
681978132*(x*e + d)^m*m^3*x^4*e^11 + 28480424184*(x*e + d)^m*m^2*x^5*e^11 +
 9680738400*(x*e + d)^m*m*x^6*e^11 + 5696697600*(x*e + d)^m*x^7*e^11 + 1763
7102*(x*e + d)^m*d*m^6*e^10 + 238556745*(x*e + d)^m*d*m^5*x*e^10 + 24725657
52*(x*e + d)^m*d*m^4*x^2*e^10 + 4345999672*(x*e + d)^m*d*m^3*x^3*e^10 + 568
8131976*(x*e + d)^m*d*m^2*x^4*e^10 + 678585600*(x*e + d)^m*d*m*x^5*e^10 - 3
6710415*(x*e + d)^m*d^2*m^5*e^9 - 1102270680*(x*e + d)^m*d^2*m^4*x*e^9 - 37
45839048*(x*e + d)^m*d^2*m^3*x^2*e^9 - 6600448608*(x*e + d)^m*d^2*m^2*x^3*e
^9 - 848232000*(x*e + d)^m*d^2*m*x^4*e^9 + 194510106*(x*e + d)^m*d^3*m^4*e^
8 + 2127097056*(x*e + d)^m*d^3*m^3*x*e^8 + 7687286352*(x*e + d)^m*d^3*m^2*x
^2*e^8 + 1130976000*(x*e + d)^m*d^3*m*x^3*e^8 - 463042356*(x*e + d)^m*d^4*m
^3*e^7 - 6521026464*(x*e + d)^m*d^4*m^2*x*e^7 - 1696464000*(x*e + d)^m*d^4*
m*x^2*e^7 + 1983530784*(x*e + d)^m*d^5*m^2*e^6 + 3392928000*(x*e + d)^m*d^5
*m*x*e^6 - 1933552800*(x*e + d)^m*d^6*m*e^5 + 5696697600*(x*e + d)^m*d^7*e^
4 + 124791030*(x*e + d)^m*m^5*x*e^11 + 1250302905*(x*e + d)^m*m^4*x^2*e^11
+ 10631923596*(x*e + d)^m*m^3*x^3*e^11 + 17690223096*(x*e + d)^m*m^2*x^4*e^
11 + 24965914464*(x*e + d)^m*m*x^5*e^11 + 3392928000*(x*e + d)^m*x^6*e^11 +
 124791030*(x*e + d)^m*d*m^5*e^10 + 1011746160*(x*e + d)^m*d*m^4*x*e^10 + 5
686792092*(x*e + d)^m*d*m^3*x^2*e^10 + 4652224080*(x*e + d)^m*d*m^2*x^3*e^1
0 + 2213386560*(x*e + d)^m*d*m*x^4*e^10 - 238556745*(x*e + d)^m*d^2*m^4*e^9
 - 3842860824*(x*e + d)^m*d^2*m^3*x*e^9 - 5546320920*(x*e + d)^m*d^2*m^2*x^
2*e^9 - 2951182080*(x*e + d)^m*d^2*m*x^3*e^9 + 1102270680*(x*e + d)^m*d^3*m
^3*e^8 + 5364581040*(x*e + d)^m*d^3*m^2*x*e^8 + 4426773120*(x*e + d)^m*d^3*
m*x^2*e^8 - 2127097056*(x*e + d)^m*d^4*m^2*e^7 - 8853546240*(x*e + d)^m*d^4
*m*x*e^7 + 6521026464*(x*e + d)^m*d^5*m*e^6 - 3392928000*(x*e + d)^m*d^6*e^
5 + 595543860*(x*e + d)^m*m^4*x*e^11 + 3708817740*(x*e + d)^m*m^3*x^2*e^11
+ 18312331464*(x*e + d)^m*m^2*x^3*e^11 + 15866025840*(x*e + d)^m*m*x^4*e^11
 + 8853546240*(x*e + d)^m*x^5*e^11 + 595543860*(x*e + d)^m*d*m^4*e^10 + 269
7071580*(x*e + d)^m*d*m^3*x*e^10 + 6938747280*(x*e + d)^m*d*m^2*x^2*e^10 +
1909353600*(x*e + d)^m*d*m*x^3*e^10 - 1011746160*(x*e + d)^m*d^2*m^3*e^9 -
7530723360*(x*e + d)^m*d^2*m^2*x*e^9 - 2864030400*(x*e + d)^m*d^2*m*x^2*e^9
 + 3842860824*(x*e + d)^m*d^3*m^2*e^8 + 5728060800*(x*e + d)^m*d^3*m*x*e^8
- 5364581040*(x*e + d)^m*d^4*m*e^7 + 8853546240*(x*e + d)^m*d^5*e^6 + 18882
25560*(x*e + d)^m*m^3*x*e^11 + 6792204780*(x*e + d)^m*m^2*x^2*e^11 + 170508
80160*(x*e + d)^m*m*x^3*e^11 + 5728060800*(x*e + d)^m*x^4*e^11 + 1888225560
*(x*e + d)^m*d*m^3*e^10 + 4095133200*(x*e + d)^m*d*m^2*x*e^10 + 3173385600*
(x*e + d)^m*d*m*x^2*e^10 - 2697071580*(x*e + d)^m*d^2*m^2*e^9 - 6346771200*
(x*e + d)^m*d^2*m*x*e^9 + 7530723360*(x*e + d)^m*d^3*m*e^8 - 5728060800*(x*
e + d)^m*d^4*e^7 + 3795710544*(x*e + d)^m*m^2*x*e^11 + 6789517200*(x*e + d)
^m*m*x^2*e^11 + 6346771200*(x*e + d)^m*x^3*e^11 + 3795710544*(x*e + d)^m*d*
m^2*e^10 + 2694384000*(x*e + d)^m*d*m*x*e^10 - 4095133200*(x*e + d)^m*d^2*m
*e^9 + 6346771200*(x*e + d)^m*d^3*e^8 + 4353860160*(x*e + d)^m*m*x*e^11 + 2
694384000*(x*e + d)^m*x^2*e^11 + 4353860160*(x*e + d)^m*d*m*e^10 - 26943840
00*(x*e + d)^m*d^2*e^9 + 2155507200*(x*e + d)^m*x*e^11 + 2155507200*(x*e +
d)^m*d*e^10)/(m^11*e^11 + 66*m^10*e^11 + 1925*m^9*e^11 + 32670*m^8*e^11 + 3

```
57423*m^7*e^11 + 2637558*m^6*e^11 + 13339535*m^5*e^11 + 45995730*m^4*e^11 +
 105258076*m^3*e^11 + 150917976*m^2*e^11 + 120543840*m*e^11 + 39916800*e^11
)
```

**maple [B]** time = 0.08, size = 5924, normalized size = 10.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] result too large to display
```

**maxima [B]** time = 0.74, size = 2292, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 135*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 5
4*(e*x + d)^(m + 1)/(e*(m + 1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*
d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 -
3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 +
 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x
^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3
*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m^4 + 85*m
^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*
m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2
 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(
e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^
6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*
x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2
 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d
^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5
+ 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 98*((m^7 + 28*m^
6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (
m^7 + 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 -
 7*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^
5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^
2 + 6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)
```

```
*d^6*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*
m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8
) + 765*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 11812
4*m^2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 67
69*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5
 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35
*m^3 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e
^4*x^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2
 - 40320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450
*m^6 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 3628
80)*e^9) - 25*((m^9 + 45*m^8 + 870*m^7 + 9450*m^6 + 63273*m^5 + 269325*m^4
+ 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*e^10*x^10 + (m^9 + 36*m^8
+ 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 118124*m^3 + 109584*m^2 + 40
320*m)*d*e^9*x^9 - 9*(m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m^4 + 13132*
m^3 + 13068*m^2 + 5040*m)*d^2*e^8*x^8 + 72*(m^7 + 21*m^6 + 175*m^5 + 735*m^
4 + 1624*m^3 + 1764*m^2 + 720*m)*d^3*e^7*x^7 - 504*(m^6 + 15*m^5 + 85*m^4 +
 225*m^3 + 274*m^2 + 120*m)*d^4*e^6*x^6 + 3024*(m^5 + 10*m^4 + 35*m^3 + 50*
m^2 + 24*m)*d^5*e^5*x^5 - 15120*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^6*e^4*x^4 +
60480*(m^3 + 3*m^2 + 2*m)*d^7*e^3*x^3 - 181440*(m^2 + m)*d^8*e^2*x^2 + 3628
80*d^9*e*m*x - 362880*d^10)*(e*x + d)^m/((m^10 + 55*m^9 + 1320*m^8 + 18150*
m^7 + 157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 +
10628640*m + 3628800)*e^10) + 500*((m^10 + 55*m^9 + 1320*m^8 + 18150*m^7 +
157773*m^6 + 902055*m^5 + 3416930*m^4 + 8409500*m^3 + 12753576*m^2 + 106286
40*m + 3628800)*e^11*x^11 + (m^10 + 45*m^9 + 870*m^8 + 9450*m^7 + 63273*m^6
 + 269325*m^5 + 723680*m^4 + 1172700*m^3 + 1026576*m^2 + 362880*m)*d*e^10*x
^10 - 10*(m^9 + 36*m^8 + 546*m^7 + 4536*m^6 + 22449*m^5 + 67284*m^4 + 11812
4*m^3 + 109584*m^2 + 40320*m)*d^2*e^9*x^9 + 90*(m^8 + 28*m^7 + 322*m^6 + 19
60*m^5 + 6769*m^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d^3*e^8*x^8 - 720*(m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^4*e^7*x^7 + 5
040*(m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^5*e^6*x^6 - 30240
*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^6*e^5*x^5 + 151200*(m^4 + 6*m^3
+ 11*m^2 + 6*m)*d^7*e^4*x^4 - 604800*(m^3 + 3*m^2 + 2*m)*d^8*e^3*x^3 + 1814
400*(m^2 + m)*d^9*e^2*x^2 - 3628800*d^10*e*m*x + 3628800*d^11)*(e*x + d)^m/
((m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 133395
35*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916
800)*e^11)
```

**mupad [B]** time = 8.39, size = 4341, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] (500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^
4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 362880
0))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*
m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39
916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 1814400000*d^
11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8853546
240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^8*e^3 +
 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m - 5364581
040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432604960*d^
7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*d*e^10*m^2
 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^10*m^5 + 17
637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 3510*d*e^10*m^
9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e^8*m^2 - 2127
097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e^5*m^2 + 38769
1920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m^2 - 1011746160
*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m^3 + 318992760*d
^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 + 493920*d^8*e^3*m
^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 59787840*d^4*e^7*m^4 +
 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7*e^4*m^4 - 36710415
*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^7*m^5 + 1357416*d^5*e^6
*m^5 - 61200*d^6*e^5*m^5 - 3691170*d^2*e^9*m^6 + 1482516*d^3*e^8*m^6 - 1928
64*d^4*e^7*m^6 + 26616*d^5*e^6*m^6 - 234090*d^2*e^9*m^7 + 57240*d^3*e^8*m^7
 - 3444*d^4*e^7*m^7 - 8505*d^2*e^9*m^8 + 954*d^3*e^8*m^8 - 135*d^2*e^9*m^9
+ 4353860160*d*e^10*m + 9072000*d^10*e*m))/(e^11*(120543840*m + 150917976*m
^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7
 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x*(d + e*x)^m*(435
3860160*e^11*m + 2155507200*e^11 + 3795710544*e^11*m^2 + 1888225560*e^11*m^
3 + 595543860*e^11*m^4 + 124791030*e^11*m^5 + 17637102*e^11*m^6 + 1663740*e
^11*m^7 + 100440*e^11*m^8 + 3510*e^11*m^9 + 54*e^11*m^10 - 6346771200*d^2*e
^9*m + 5728060800*d^3*e^8*m - 8853546240*d^4*e^7*m + 3392928000*d^5*e^6*m -
 5696697600*d^6*e^5*m - 488980800*d^7*e^4*m - 3392928000*d^8*e^3*m - 997920
00*d^9*e^2*m + 4095133200*d*e^10*m^2 + 2697071580*d*e^10*m^3 + 1011746160*d
*e^10*m^4 + 238556745*d*e^10*m^5 + 36710415*d*e^10*m^6 + 3691170*d*e^10*m^7
 + 234090*d*e^10*m^8 + 8505*d*e^10*m^9 + 135*d*e^10*m^10 - 7530723360*d^2*e
^9*m^2 + 5364581040*d^3*e^8*m^2 - 6521026464*d^4*e^7*m^2 + 1933552800*d^5*e
^6*m^2 - 2432604960*d^6*e^5*m^2 - 147682080*d^7*e^4*m^2 - 647740800*d^8*e^3
*m^2 - 9072000*d^9*e^2*m^2 - 3842860824*d^2*e^9*m^3 + 2127097056*d^3*e^8*m^
3 - 1983530784*d^4*e^7*m^3 + 437886000*d^5*e^6*m^3 - 387691920*d^6*e^5*m^3
- 14817600*d^7*e^4*m^3 - 30844800*d^8*e^3*m^3 - 1102270680*d^2*e^9*m^4 + 46
3042356*d^3*e^8*m^4 - 318992760*d^4*e^7*m^4 + 49266000*d^5*e^6*m^4 - 273326
40*d^6*e^5*m^4 - 493920*d^7*e^4*m^4 - 194510106*d^2*e^9*m^5 + 59787840*d^3*
e^8*m^5 - 28612200*d^4*e^7*m^5 + 2754000*d^5*e^6*m^5 - 719280*d^6*e^5*m^5 -
 21636720*d^2*e^9*m^6 + 4580520*d^3*e^8*m^6 - 1357416*d^4*e^7*m^6 + 61200*d
^5*e^6*m^6 - 1482516*d^2*e^9*m^7 + 192864*d^3*e^8*m^7 - 26616*d^4*e^7*m^7 -
 57240*d^2*e^9*m^8 + 3444*d^3*e^8*m^8 - 954*d^2*e^9*m^9 + 2694384000*d*e^10
```

*m - 1814400000*d^10*e*m))/(e^11*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^8*(d + e*x)^m*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)*(45000*d^3*m - 29302*e^3*m - 97020*e^3 - 2940*e^3*m^2 - 98*e^3*m^3 + 16065*d*e^2*m^2 + 225*d^2*e*m^2 + 765*d*e^2*m^3 + 84150*d*e^2*m + 2475*d^2*e*m))/(e^3*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (3*x^2*(m + 1)*(d + e*x)^m*(302400000*d^9*m + 1365044400*e^9*m + 898128000*e^9 + 899023860*e^9*m^2 + 337248720*e^9*m^3 + 79518915*e^9*m^4 + 12236805*e^9*m^5 + 1230390*e^9*m^6 + 78030*e^9*m^7 + 2835*e^9*m^8 + 45*e^9*m^9 - 954676800*d^2*e^7*m + 1475591040*d^3*e^6*m - 565488000*d^4*e^5*m + 949449600*d^5*e^4*m + 81496800*d^6*e^3*m + 565488000*d^7*e^2*m + 1255120560*d*e^8*m^2 + 1512000*d^8*e*m^2 + 640476804*d*e^8*m^3 + 183711780*d*e^8*m^4 + 32418351*d*e^8*m^5 + 3606120*d*e^8*m^6 + 247086*d*e^8*m^7 + 9540*d*e^8*m^8 + 159*d*e^8*m^9 - 894096840*d^2*e^7*m^2 + 1086837744*d^3*e^6*m^2 - 322258800*d^4*e^5*m^2 + 405434160*d^5*e^4*m^2 + 24613680*d^6*e^3*m^2 + 107956800*d^7*e^2*m^2 - 354516176*d^2*e^7*m^3 + 330588464*d^3*e^6*m^3 - 72981000*d^4*e^5*m^3 + 64615320*d^5*e^4*m^3 + 2469600*d^6*e^3*m^3 + 5140800*d^7*e^2*m^3 - 77173726*d^2*e^7*m^4 + 53165460*d^3*e^6*m^4 - 8211000*d^4*e^5*m^4 + 4555440*d^5*e^4*m^4 + 82320*d^6*e^3*m^4 - 9964640*d^2*e^7*m^5 + 4768700*d^3*e^6*m^5 - 459000*d^4*e^5*m^5 + 119880*d^5*e^4*m^5 - 763420*d^2*e^7*m^6 + 226236*d^3*e^6*m^6 - 10200*d^4*e^5*m^6 - 32144*d^2*e^7*m^7 + 4436*d^3*e^6*m^7 - 574*d^2*e^7*m^8 + 1057795200*d*e^8*m + 16632000*d^8*e*m))/(e^9*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^6*(d + e*x)^m*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(2520000*d^5*m + 16112940*e^5*m + 28274400*e^5 + 3649050*e^5*m^2 + 410550*e^5*m^3 + 22950*e^5*m^4 + 510*e^5*m^5 + 679140*d^2*e^3*m + 4712400*d^3*e^2*m + 3378618*d*e^4*m^2 + 12600*d^4*e*m^2 + 538461*d*e^4*m^3 + 37962*d*e^4*m^4 + 999*d*e^4*m^5 + 205114*d^2*e^3*m^2 + 899640*d^3*e^2*m^2 + 20580*d^2*e^3*m^3 + 42840*d^3*e^2*m^3 + 686*d^2*e^3*m^4 + 7912080*d*e^4*m + 138600*d^4*e*m))/(e^5*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(3765361680*e^8*m - 302400000*d^8*m + 3173385600*e^8 + 1921430412*e^8*m^2 + 551135340*e^8*m^3 + 97255053*e^8*m^4 + 10818360*e^8*m^5 + 741258*e^8*m^6 + 28620*e^8*m^7 + 477*e^8*m^8 - 1475591040*d^2*e^6*m + 565488000*d^3*e^5*m - 949449600*d^4*e^4*m - 81496800*d^5*e^3*m - 565488000*d^6*e^2*m + 894096840*d*e^7*m^2 - 1512000*d^7*e*m^2 + 354516176*d*e^7*m^3 + 77173726*d*e^7*m^4 + 9964640*d*e^7*m^5 + 763420*d*e^7*m^6 + 32144*d*e^7*m^7 + 574*d*e^7*m^8 - 1086837744*d^2*e^6*m^2 + 322258800*d^3*e^5*m^2 - 405434160*d^4*e^4*m^2 - 24613680*d^5*e^3*m^2 - 107956800*d^6*e^2*m^2 - 330588464*d^2*e^6*m^3 + 72981000*d^3*e^5*m^3 - 64615320*d^4*e^4*m^3 - 2469600*d^5*e^3*m^3 - 5140800*d^6*e^2*m^3 - 53165460*d^2*e^6*m^4 + 8211000*d^3*e^5*m^4 - 4555440*d^4*e^4*m^4 - 82320*d^5*e^3*m^4 - 4768700*d^2*e^6*m^5 + 459000*d^3*e^5*m^

```
5 - 119880*d^4*e^4*m^5 - 226236*d^2*e^6*m^6 + 10200*d^3*e^5*m^6 - 4436*d^2*
e^6*m^7 + 954676800*d*e^7*m - 16632000*d^7*e*m))/(e^8*(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) + (x^4*(d + e*x)
^m*(11*m + 6*m^2 + m^3 + 6)*(75600000*d^7*m + 894096840*e^7*m + 954676800*e
^7 + 354516176*e^7*m^2 + 77173726*e^7*m^3 + 9964640*e^7*m^4 + 763420*e^7*m^
5 + 32144*e^7*m^6 + 574*e^7*m^7 - 141372000*d^2*e^5*m + 237362400*d^3*e^4*m
 + 20374200*d^4*e^3*m + 141372000*d^5*e^2*m + 271709436*d*e^6*m^2 + 378000*
d^6*e*m^2 + 82647116*d*e^6*m^3 + 13291365*d*e^6*m^4 + 1192175*d*e^6*m^5 + 5
6559*d*e^6*m^6 + 1109*d*e^6*m^7 - 80564700*d^2*e^5*m^2 + 101358540*d^3*e^4*
m^2 + 6153420*d^4*e^3*m^2 + 26989200*d^5*e^2*m^2 - 18245250*d^2*e^5*m^3 + 1
6153830*d^3*e^4*m^3 + 617400*d^4*e^3*m^3 + 1285200*d^5*e^2*m^3 - 2052750*d^
2*e^5*m^4 + 1138860*d^3*e^4*m^4 + 20580*d^4*e^3*m^4 - 114750*d^2*e^5*m^5 +
29970*d^3*e^4*m^5 - 2550*d^2*e^5*m^6 + 368897760*d*e^6*m + 4158000*d^6*e*m)
)/(e^7*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395
35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 +
 39916800)) - (25*x^10*(d + e*x)^m*(11*e - 20*d*m + e*m)*(1026576*m + 11727
00*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8
+ m^9 + 362880))/(e*(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)) - (x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 +
 24)*(15120000*d^6*m - 271709436*e^6*m - 368897760*e^6 - 82647116*e^6*m^2 -
 13291365*e^6*m^3 - 1192175*e^6*m^4 - 56559*e^6*m^5 - 1109*e^6*m^6 + 474724
80*d^2*e^4*m + 4074840*d^3*e^3*m + 28274400*d^4*e^2*m - 16112940*d*e^5*m^2
+ 75600*d^5*e*m^2 - 3649050*d*e^5*m^3 - 410550*d*e^5*m^4 - 22950*d*e^5*m^5
- 510*d*e^5*m^6 + 20271708*d^2*e^4*m^2 + 1230684*d^3*e^3*m^2 + 5397840*d^4*
e^2*m^2 + 3230766*d^2*e^4*m^3 + 123480*d^3*e^3*m^3 + 257040*d^4*e^2*m^3 + 2
27772*d^2*e^4*m^4 + 4116*d^3*e^3*m^4 + 5994*d^2*e^4*m^5 - 28274400*d*e^5*m
+ 831600*d^5*e*m))/(e^6*(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599
5730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 +
 66*m^10 + m^11 + 39916800)) - (x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^
3 + 175*m^4 + 21*m^5 + m^6 + 720)*(360000*d^4*m - 3378618*e^4*m - 7912080*e
^4 - 538461*e^4*m^2 - 37962*e^4*m^3 - 999*e^4*m^4 + 673200*d^2*e^2*m + 2930
2*d*e^3*m^2 + 1800*d^3*e*m^2 + 2940*d*e^3*m^3 + 98*d*e^3*m^4 + 128520*d^2*e
^2*m^2 + 6120*d^2*e^2*m^3 + 97020*d*e^3*m + 19800*d^3*e*m))/(e^4*(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800)) - (5*
x^9*(d + e*x)^m*(1000*d^2*m - 3213*e^2*m - 16830*e^2 - 153*e^2*m^2 + 55*d*e
*m + 5*d*e*m^2)*(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
 546*m^6 + 36*m^7 + m^8 + 40320))/(e^2*(120543840*m + 150917976*m^2 + 10525
8076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m
^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

## 3.368 $\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=432

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d\right.}{}$$

```
[Out] (5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e
^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e
^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d^3
*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1575*
d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+m)+(7
000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/(5+m)-
(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*d^2+315
*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e^9/(8+m)
+100*(e*x+d)^(9+m)/e^9/(9+m)
```

**Rubi [A]** time = 0.24, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)^2\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}-\frac{\left(5d^2-2de+3e^2\right)\left(88d^3e^2-4d^2e^3+127d^4e+1\right.}{e^9(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
 + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^
4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^9*(
2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4
 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5 + 157
5*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^(4 + m
))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148
*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 666*d*e^2
 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e + 111*e^
2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^(8 + m))/(
e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \int \left( \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+\right.}{e^8} \right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)^2 \left(4d^4+5d^3e+3d^2e^2-de^3+2e\right.}{e^9(1+m)}$$

**Mathematica [A]** time = 0.24, size = 391, normalized size = 0.91

$$\frac{(d+ex)^{m+1} \left( \frac{\left(2800d^2+315de+111e^2\right)(d+ex)^6}{m+7} - \frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^5}{m+6} + \frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+e\right.}{m+5}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x
]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2
 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e
+ 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((2800*d
^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107
*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d
^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600*d^3 +
945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(8 + m) +
(100*(d + e*x)^8)/(9 + m)))/e^9
```

**fricas [B]** time = 0.97, size = 2796, normalized size = 6.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="f
ricas")
```

```
[Out] (18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 22
449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*x^
```

9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10741
248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 653184
0*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^9)*m^
7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7*(1934
0*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(1210*d*e^
8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^7 - 24*d
*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*e^7 + 675
*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9)*m^6 - 14
*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^7 + 17655*d
*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 326229*e^9)*m^3
- 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(4000*d^2*e^7 + 2
025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7 + 7434*d*e^8)
*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^7 + 1184*d*e^8
- 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e^8 - 5883*e^9)*m
^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 34743*e^9)*m^5 - (476
000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^9)*m^4 - 3*(420000
*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*m^3 - 2*(767200*d^3
*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)*m^2 - 24*(28000*d^3*e^
6 + 14175*d^2*e^7 + 39960*d*e^8 - 248233*e^9)*m)*x^6 - 6*(65*d^4*e^5 - 1391
*d^3*e^6 + 4081*d^2*e^7 - 25872*d*e^8)*m^5 + (10741248*e^9 - 37*(d*e^8 - 4*
e^9)*m^8 - 74*(9*d^2*e^7 + 17*d*e^8 - 80*e^9)*m^7 - 2*(945*d^3*e^6 + 8991*d
^2*e^7 + 8621*d*e^8 - 49580*e^9)*m^6 - 2*(16800*d^4*e^5 + 17955*d^3*e^6 + 9
2241*d^2*e^7 + 61124*d*e^8 - 451400*e^9)*m^5 - (336000*d^4*e^5 + 236250*d^3
*e^6 + 909090*d^2*e^7 + 479113*d*e^8 - 4850404*e^9)*m^4 - 2*(588000*d^4*e^5
 + 344925*d^3*e^6 + 1130202*d^2*e^7 + 513671*d*e^8 - 7804040*e^9)*m^3 - 12*
(140000*d^4*e^5 + 74655*d^3*e^6 + 222444*d^2*e^7 + 91834*d*e^8 - 2422020*e^
9)*m^2 - 144*(5600*d^4*e^5 + 2835*d^3*e^6 + 7992*d^2*e^7 + 3108*d*e^8 - 196
100*e^9)*m)*x^5 + 2*(1776*d^5*e^4 - 6825*d^4*e^5 + 66875*d^3*e^6 - 117810*d
^2*e^7 + 491841*d*e^8)*m^4 + (5896800*e^9 + (148*d*e^8 + 65*e^9)*m^8 + (185
*d^2*e^7 + 5328*d*e^8 + 2665*e^9)*m^7 + 2*(1665*d^3*e^6 + 2775*d^2*e^7 + 38
924*d*e^8 + 22945*e^9)*m^6 + 2*(4725*d^4*e^5 + 38295*d^3*e^6 + 32005*d^2*e^
7 + 295704*d*e^8 + 215345*e^9)*m^5 + (168000*d^5*e^4 + 141750*d^4*e^5 + 616
050*d^3*e^6 + 355200*d^2*e^7 + 2484772*d*e^8 + 2389985*e^9)*m^4 + (1008000*
d^5*e^4 + 614250*d^4*e^5 + 2081250*d^3*e^6 + 974765*d^2*e^7 + 5668992*d*e^8
 + 7946185*e^9)*m^3 + 6*(308000*d^5*e^4 + 165375*d^4*e^5 + 496170*d^3*e^6 +
 206275*d^2*e^7 + 1064712*d*e^8 + 2542410*e^9)*m^2 + 36*(28000*d^5*e^4 + 14
175*d^4*e^5 + 39960*d^3*e^6 + 15540*d^2*e^7 + 74592*d*e^8 + 422435*e^9)*m)*
x^4 + 3*(1480*d^6*e^3 + 35520*d^5*e^4 - 63050*d^4*e^5 + 375570*d^3*e^6 - 44
4059*d^2*e^7 + 1288056*d*e^8)*m^3 + (12942720*e^9 + (65*d*e^8 + 107*e^9)*m^
8 - 2*(296*d^2*e^7 - 1235*d*e^8 - 2247*e^9)*m^7 - 4*(185*d^3*e^6 + 4884*d^2
*e^7 - 9620*d*e^8 - 19902*e^9)*m^6 - 2*(6660*d^4*e^5 + 9990*d^3*e^6 + 12639
2*d^2*e^7 - 157625*d*e^8 - 386163*e^9)*m^5 - (37800*d^5*e^4 + 266400*d^4*e^
5 + 196100*d^3*e^6 + 1607280*d^2*e^7 - 1444235*d*e^8 - 4453233*e^9)*m^4 - 4
*(168000*d^6*e^3 + 113400*d^5*e^4 + 416250*d^4*e^5 + 208125*d^3*e^6 + 12793
12*d^2*e^7 - 903370*d*e^8 - 3864519*e^9)*m^3 - 4*(504000*d^6*e^3 + 274050*d

```
^5*e^4 + 832500*d^4*e^5 + 350390*d^3*e^6 + 1831056*d^2*e^7 - 1103505*d*e^8
- 7764883*e^9)*m^2 - 48*(28000*d^6*e^3 + 14175*d^5*e^4 + 39960*d^4*e^5 + 15
540*d^3*e^6 + 74592*d^2*e^7 - 40950*d*e^8 - 672923*e^9)*m)*x^3 + 2*(39960*d
^7*e^2 + 53280*d^6*e^3 + 594960*d^5*e^4 - 648375*d^4*e^5 + 2629418*d^3*e^6
- 2209977*d^2*e^7 + 4581036*d*e^8)*m^2 + (5987520*e^9 + (107*d*e^8 + 33*e^9
)*m^8 - (195*d^2*e^7 - 4280*d*e^8 - 1419*e^9)*m^7 + 4*(444*d^3*e^6 - 1755*d
^2*e^7 + 17762*d*e^8 + 6468*e^9)*m^6 + 2*(1110*d^4*e^5 + 27528*d^3*e^6 - 50
700*d^2*e^7 + 315115*d*e^8 + 130053*e^9)*m^5 + (39960*d^5*e^4 + 55500*d^4*e
^5 + 648240*d^3*e^6 - 742950*d^2*e^7 + 3192773*d*e^8 + 1567797*e^9)*m^4 + (
113400*d^6*e^3 + 719280*d^5*e^4 + 477300*d^4*e^5 + 3525360*d^3*e^6 - 284680
5*d^2*e^7 + 9072530*d*e^8 + 5752131*e^9)*m^3 + 6*(336000*d^7*e^2 + 189000*d
^6*e^3 + 592740*d^5*e^4 + 257150*d^4*e^5 + 1383504*d^3*e^6 - 857805*d^2*e^7
 + 2152412*d*e^8 + 2062863*e^9)*m^2 + 72*(28000*d^7*e^2 + 14175*d^6*e^3 + 3
9960*d^5*e^4 + 15540*d^4*e^5 + 74592*d^3*e^6 - 40950*d^2*e^7 + 89880*d*e^8
+ 193677*e^9)*m)*x^2 + 12*(18900*d^8*e + 113220*d^7*e^2 + 70670*d^6*e^3 + 4
88400*d^5*e^4 - 366405*d^4*e^5 + 1073852*d^3*e^6 - 663102*d^2*e^7 + 995544*
d*e^8)*m + (6531840*e^9 + 3*(11*d*e^8 + 6*e^9)*m^8 - 2*(107*d^2*e^7 - 693*d
*e^8 - 396*e^9)*m^7 + 6*(65*d^3*e^6 - 1391*d^2*e^7 + 4081*d*e^8 + 2478*e^9)
*m^6 - 2*(1776*d^4*e^5 - 6825*d^3*e^6 + 66875*d^2*e^7 - 117810*d*e^8 - 7761
6*e^9)*m^5 - 3*(1480*d^5*e^4 + 35520*d^4*e^5 - 63050*d^3*e^6 + 375570*d^2*e
^7 - 444059*d*e^8 - 327894*e^9)*m^4 - 2*(39960*d^6*e^3 + 53280*d^5*e^4 + 59
4960*d^4*e^5 - 648375*d^3*e^6 + 2629418*d^2*e^7 - 2209977*d*e^8 - 1932084*e
^9)*m^3 - 12*(18900*d^7*e^2 + 113220*d^6*e^3 + 70670*d^5*e^4 + 488400*d^4*e
^5 - 366405*d^3*e^6 + 1073852*d^2*e^7 - 663102*d*e^8 - 763506*e^9)*m^2 - 14
4*(28000*d^8*e + 14175*d^7*e^2 + 39960*d^6*e^3 + 15540*d^5*e^4 + 74592*d^4*
e^5 - 40950*d^3*e^6 + 89880*d^2*e^7 - 41580*d*e^8 - 82962*e^9)*m)*x)*(e*x +
 d)^m/(e^9*m^9 + 45*e^9*m^8 + 870*e^9*m^7 + 9450*e^9*m^6 + 63273*e^9*m^5 +
269325*e^9*m^4 + 723680*e^9*m^3 + 1172700*e^9*m^2 + 1026576*e^9*m + 362880*
e^9)
```

**giac [B]** time = 0.39, size = 6223, normalized size = 14.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="g
iac")
```

```
[Out] (100*(x*e + d)^m*m^8*x^9*e^9 + 100*(x*e + d)^m*d*m^8*x^8*e^8 - 45*(x*e + d)
^m*m^8*x^8*e^9 + 3600*(x*e + d)^m*m^7*x^9*e^9 - 45*(x*e + d)^m*d*m^8*x^7*e^
8 + 2800*(x*e + d)^m*d*m^7*x^8*e^8 - 800*(x*e + d)^m*d^2*m^7*x^7*e^7 + 111*
(x*e + d)^m*m^8*x^7*e^9 - 1665*(x*e + d)^m*m^7*x^8*e^9 + 54600*(x*e + d)^m*
m^6*x^9*e^9 + 111*(x*e + d)^m*d*m^8*x^6*e^8 - 1350*(x*e + d)^m*d*m^7*x^7*e^
8 + 32200*(x*e + d)^m*d*m^6*x^8*e^8 + 315*(x*e + d)^m*d^2*m^7*x^6*e^7 - 168
00*(x*e + d)^m*d^2*m^6*x^7*e^7 + 5600*(x*e + d)^m*d^3*m^6*x^6*e^6 - 37*(x*e
```

+ d)^m*m^8*x^6*e^9 + 4218*(x*e + d)^m*m^7*x^7*e^9 - 25830*(x*e + d)^m*m^6*x^8*e^9 + 453600*(x*e + d)^m*m^5*x^9*e^9 - 37*(x*e + d)^m*d*m^8*x^5*e^8 + 3552*(x*e + d)^m*d*m^7*x^6*e^8 - 16380*(x*e + d)^m*d*m^6*x^7*e^8 + 196000*(x*e + d)^m*d*m^5*x^8*e^8 - 666*(x*e + d)^m*d^2*m^7*x^5*e^7 + 7560*(x*e + d)^m*d^2*m^6*x^6*e^7 - 140000*(x*e + d)^m*d^2*m^5*x^7*e^7 - 1890*(x*e + d)^m*d^3*m^6*x^5*e^6 + 84000*(x*e + d)^m*d^3*m^5*x^6*e^6 - 33600*(x*e + d)^m*d^4*m^5*x^5*e^5 + 148*(x*e + d)^m*m^8*x^5*e^9 - 1443*(x*e + d)^m*m^7*x^6*e^9 + 67044*(x*e + d)^m*m^6*x^7*e^9 - 218610*(x*e + d)^m*m^5*x^8*e^9 + 2244900*(x*e + d)^m*m^4*x^9*e^9 + 148*(x*e + d)^m*d*m^8*x^4*e^8 - 1258*(x*e + d)^m*d*m^7*x^5*e^8 + 45732*(x*e + d)^m*d*m^6*x^6*e^8 - 103950*(x*e + d)^m*d*m^5*x^7*e^8 + 676900*(x*e + d)^m*d*m^4*x^8*e^8 + 185*(x*e + d)^m*d^2*m^7*x^4*e^7 - 17982*(x*e + d)^m*d^2*m^6*x^5*e^7 + 69300*(x*e + d)^m*d^2*m^5*x^6*e^7 - 588000*(x*e + d)^m*d^2*m^4*x^7*e^7 + 3330*(x*e + d)^m*d^3*m^6*x^4*e^6 - 35910*(x*e + d)^m*d^3*m^5*x^5*e^6 + 476000*(x*e + d)^m*d^3*m^4*x^6*e^6 + 9450*(x*e + d)^m*d^4*m^5*x^4*e^5 - 336000*(x*e + d)^m*d^4*m^4*x^5*e^5 + 168000*(x*e + d)^m*d^5*m^4*x^4*e^4 + 65*(x*e + d)^m*m^8*x^4*e^9 + 5920*(x*e + d)^m*m^7*x^5*e^9 - 23532*(x*e + d)^m*m^6*x^6*e^9 + 579642*(x*e + d)^m*m^5*x^7*e^9 - 1098405*(x*e + d)^m*m^4*x^8*e^9 + 6728400*(x*e + d)^m*m^3*x^9*e^9 + 65*(x*e + d)^m*d*m^8*x^3*e^8 + 5328*(x*e + d)^m*d*m^7*x^4*e^8 - 17242*(x*e + d)^m*d*m^6*x^5*e^8 + 305250*(x*e + d)^m*d*m^5*x^6*e^8 - 370755*(x*e + d)^m*d*m^4*x^7*e^8 + 1313200*(x*e + d)^m*d*m^3*x^8*e^8 - 592*(x*e + d)^m*d^2*m^7*x^3*e^7 + 5550*(x*e + d)^m*d^2*m^6*x^4*e^7 - 184482*(x*e + d)^m*d^2*m^5*x^5*e^7 + 311850*(x*e + d)^m*d^2*m^4*x^6*e^7 - 1299200*(x*e + d)^m*d^2*m^3*x^7*e^7 - 740*(x*e + d)^m*d^3*m^6*x^3*e^6 + 76590*(x*e + d)^m*d^3*m^5*x^4*e^6 - 236250*(x*e + d)^m*d^3*m^4*x^5*e^6 + 1260000*(x*e + d)^m*d^3*m^3*x^6*e^6 - 13320*(x*e + d)^m*d^4*m^5*x^3*e^5 + 141750*(x*e + d)^m*d^4*m^4*x^4*e^5 - 1176000*(x*e + d)^m*d^4*m^3*x^5*e^5 - 37800*(x*e + d)^m*d^5*m^4*x^3*e^4 + 1008000*(x*e + d)^m*d^5*m^3*x^4*e^4 - 672000*(x*e + d)^m*d^6*m^3*x^3*e^3 + 107*(x*e + d)^m*m^8*x^3*e^9 + 2665*(x*e + d)^m*m^7*x^4*e^9 + 99160*(x*e + d)^m*m^6*x^5*e^9 - 208458*(x*e + d)^m*m^5*x^6*e^9 + 2965809*(x*e + d)^m*m^4*x^7*e^9 - 3332385*(x*e + d)^m*m^3*x^8*e^9 + 11812400*(x*e + d)^m*m^2*x^9*e^9 + 107*(x*e + d)^m*d*m^8*x^2*e^8 + 2470*(x*e + d)^m*d*m^7*x^3*e^8 + 77848*(x*e + d)^m*d*m^6*x^4*e^8 - 122248*(x*e + d)^m*d*m^5*x^5*e^8 + 1134309*(x*e + d)^m*d*m^4*x^6*e^8 - 737100*(x*e + d)^m*d*m^3*x^7*e^8 + 1306800*(x*e + d)^m*d*m^2*x^8*e^8 - 195*(x*e + d)^m*d^2*m^7*x^2*e^7 - 19536*(x*e + d)^m*d^2*m^6*x^3*e^7 + 64010*(x*e + d)^m*d^2*m^5*x^4*e^7 - 909090*(x*e + d)^m*d^2*m^4*x^5*e^7 + 724185*(x*e + d)^m*d^2*m^3*x^6*e^7 - 1411200*(x*e + d)^m*d^2*m^2*x^7*e^7 + 1776*(x*e + d)^m*d^3*m^6*x^2*e^6 - 19980*(x*e + d)^m*d^3*m^5*x^3*e^6 + 616050*(x*e + d)^m*d^3*m^4*x^4*e^6 - 689850*(x*e + d)^m*d^3*m^3*x^5*e^6 + 1534400*(x*e + d)^m*d^3*m^2*x^6*e^6 + 2220*(x*e + d)^m*d^4*m^5*x^2*e^5 - 266400*(x*e + d)^m*d^4*m^4*x^3*e^5 + 614250*(x*e + d)^m*d^4*m^3*x^4*e^5 - 1680000*(x*e + d)^m*d^4*m^2*x^5*e^5 + 39960*(x*e + d)^m*d^5*m^4*x^2*e^4 - 453600*(x*e + d)^m*d^5*m^3*x^3*e^4 + 1848000*(x*e + d)^m*d^5*m^2*x^4*e^4 + 113400*(x*e + d)^m*d^6*m^3*x^2*e^3 - 2016000*(x*e + d)^m*d^6*m^2*x^3*e^3 + 2016000*(x*e + d)^m*d^7*m^2*x^2*e^2 + 33*(x*e + d)^m*m^8*x^2*e^9 + 4494*(

x*e + d)^m*m^7*x^3*e^9 + 45890*(x*e + d)^m*m^6*x^4*e^9 + 902800*(x*e + d)^m
*m^5*x^5*e^9 - 1090353*(x*e + d)^m*m^4*x^6*e^9 + 9134412*(x*e + d)^m*m^3*x^
7*e^9 - 5906520*(x*e + d)^m*m^2*x^8*e^9 + 10958400*(x*e + d)^m*m*x^9*e^9 +
33*(x*e + d)^m*d*m^8*x*e^8 + 4280*(x*e + d)^m*d*m^7*x^2*e^8 + 38480*(x*e +
d)^m*d*m^6*x^3*e^8 + 591408*(x*e + d)^m*d*m^5*x^4*e^8 - 479113*(x*e + d)^m*
d*m^4*x^5*e^8 + 2328558*(x*e + d)^m*d*m^3*x^6*e^8 - 746820*(x*e + d)^m*d*m^
2*x^7*e^8 + 504000*(x*e + d)^m*d*m*x^8*e^8 - 214*(x*e + d)^m*d^2*m^7*x*e^7
- 7020*(x*e + d)^m*d^2*m^6*x^2*e^7 - 252784*(x*e + d)^m*d^2*m^5*x^3*e^7 + 3
55200*(x*e + d)^m*d^2*m^4*x^4*e^7 - 2260404*(x*e + d)^m*d^2*m^3*x^5*e^7 + 8
14590*(x*e + d)^m*d^2*m^2*x^6*e^7 - 576000*(x*e + d)^m*d^2*m*x^7*e^7 + 390*
(x*e + d)^m*d^3*m^6*x*e^6 + 55056*(x*e + d)^m*d^3*m^5*x^2*e^6 - 196100*(x*e
 + d)^m*d^3*m^4*x^3*e^6 + 2081250*(x*e + d)^m*d^3*m^3*x^4*e^6 - 895860*(x*e
 + d)^m*d^3*m^2*x^5*e^6 + 672000*(x*e + d)^m*d^3*m*x^6*e^6 - 3552*(x*e + d)
^m*d^4*m^5*x*e^5 + 55500*(x*e + d)^m*d^4*m^4*x^2*e^5 - 1665000*(x*e + d)^m*
d^4*m^3*x^3*e^5 + 992250*(x*e + d)^m*d^4*m^2*x^4*e^5 - 806400*(x*e + d)^m*d
^4*m*x^5*e^5 - 4440*(x*e + d)^m*d^5*m^4*x*e^4 + 719280*(x*e + d)^m*d^5*m^3*
x^2*e^4 - 1096200*(x*e + d)^m*d^5*m^2*x^3*e^4 + 1008000*(x*e + d)^m*d^5*m*x
^4*e^4 - 79920*(x*e + d)^m*d^6*m^3*x*e^3 + 1134000*(x*e + d)^m*d^6*m^2*x^2*
e^3 - 1344000*(x*e + d)^m*d^6*m*x^3*e^3 - 226800*(x*e + d)^m*d^7*m^2*x*e^2
+ 2016000*(x*e + d)^m*d^7*m*x^2*e^2 - 4032000*(x*e + d)^m*d^8*m*x*e + 18*(x
*e + d)^m*m^8*x*e^9 + 1419*(x*e + d)^m*m^7*x^2*e^9 + 79608*(x*e + d)^m*m^6*
x^3*e^9 + 430690*(x*e + d)^m*m^5*x^4*e^9 + 4850404*(x*e + d)^m*m^4*x^5*e^9
- 3422907*(x*e + d)^m*m^3*x^6*e^9 + 16387596*(x*e + d)^m*m^2*x^7*e^9 - 5519
340*(x*e + d)^m*m*x^8*e^9 + 4032000*(x*e + d)^m*x^9*e^9 + 18*(x*e + d)^m*d*
m^8*e^8 + 1386*(x*e + d)^m*d*m^7*x*e^8 + 71048*(x*e + d)^m*d*m^6*x^2*e^8 +
315250*(x*e + d)^m*d*m^5*x^3*e^8 + 2484772*(x*e + d)^m*d*m^4*x^4*e^8 - 1027
342*(x*e + d)^m*d*m^3*x^5*e^8 + 2416248*(x*e + d)^m*d*m^2*x^6*e^8 - 291600*
(x*e + d)^m*d*m*x^7*e^8 - 33*(x*e + d)^m*d^2*m^7*e^7 - 8346*(x*e + d)^m*d^2
*m^6*x*e^7 - 101400*(x*e + d)^m*d^2*m^5*x^2*e^7 - 1607280*(x*e + d)^m*d^2*m
^4*x^3*e^7 + 974765*(x*e + d)^m*d^2*m^3*x^4*e^7 - 2669328*(x*e + d)^m*d^2*m
^2*x^5*e^7 + 340200*(x*e + d)^m*d^2*m*x^6*e^7 + 214*(x*e + d)^m*d^3*m^6*e^6
 + 13650*(x*e + d)^m*d^3*m^5*x*e^6 + 648240*(x*e + d)^m*d^3*m^4*x^2*e^6 - 8
32500*(x*e + d)^m*d^3*m^3*x^3*e^6 + 2977020*(x*e + d)^m*d^3*m^2*x^4*e^6 - 4
08240*(x*e + d)^m*d^3*m*x^5*e^6 - 390*(x*e + d)^m*d^4*m^5*e^5 - 106560*(x*e
 + d)^m*d^4*m^4*x*e^5 + 477300*(x*e + d)^m*d^4*m^3*x^2*e^5 - 3330000*(x*e +
 d)^m*d^4*m^2*x^3*e^5 + 510300*(x*e + d)^m*d^4*m*x^4*e^5 + 3552*(x*e + d)^m
*d^5*m^4*e^4 - 106560*(x*e + d)^m*d^5*m^3*x*e^4 + 3556440*(x*e + d)^m*d^5*m
^2*x^2*e^4 - 680400*(x*e + d)^m*d^5*m*x^3*e^4 + 4440*(x*e + d)^m*d^6*m^3*e^
3 - 1358640*(x*e + d)^m*d^6*m^2*x*e^3 + 1020600*(x*e + d)^m*d^6*m*x^2*e^3 +
 79920*(x*e + d)^m*d^7*m^2*e^2 - 2041200*(x*e + d)^m*d^7*m*x*e^2 + 226800*(
x*e + d)^m*d^8*m*e + 4032000*(x*e + d)^m*d^9 + 792*(x*e + d)^m*m^7*x*e^9 +
25872*(x*e + d)^m*m^6*x^2*e^9 + 772326*(x*e + d)^m*m^5*x^3*e^9 + 2389985*(x
*e + d)^m*m^4*x^4*e^9 + 15608080*(x*e + d)^m*m^3*x^5*e^9 - 6238718*(x*e + d
)^m*m^2*x^6*e^9 + 15456528*(x*e + d)^m*m*x^7*e^9 - 2041200*(x*e + d)^m*x^8*
e^9 + 792*(x*e + d)^m*d*m^7*e^8 + 24486*(x*e + d)^m*d*m^6*x*e^8 + 630230*(x

*e + d)^m*d*m^5*x^2*e^8 + 1444235*(x*e + d)^m*d*m^4*x^3*e^8 + 5668992*(x*e + d)^m*d*m^3*x^4*e^8 - 1102008*(x*e + d)^m*d*m^2*x^5*e^8 + 959040*(x*e + d)^m*d*m*x^6*e^8 - 1386*(x*e + d)^m*d^2*m^6*e^7 - 133750*(x*e + d)^m*d^2*m^5*x*e^7 - 742950*(x*e + d)^m*d^2*m^4*x^2*e^7 - 5117248*(x*e + d)^m*d^2*m^3*x^3*e^7 + 1237650*(x*e + d)^m*d^2*m^2*x^4*e^7 - 1150848*(x*e + d)^m*d^2*m*x^5*e^7 + 8346*(x*e + d)^m*d^3*m^5*e^6 + 189150*(x*e + d)^m*d^3*m^4*x*e^6 + 3525360*(x*e + d)^m*d^3*m^3*x^2*e^6 - 1401560*(x*e + d)^m*d^3*m^2*x^3*e^6 + 1438560*(x*e + d)^m*d^3*m*x^4*e^6 - 13650*(x*e + d)^m*d^4*m^4*e^5 - 1189920*(x*e + d)^m*d^4*m^3*x*e^5 + 1542900*(x*e + d)^m*d^4*m^2*x^2*e^5 - 1918080*(x*e + d)^m*d^4*m*x^3*e^5 + 106560*(x*e + d)^m*d^5*m^3*e^4 - 848040*(x*e + d)^m*d^5*m^2*x*e^4 + 2877120*(x*e + d)^m*d^5*m*x^2*e^4 + 106560*(x*e + d)^m*d^6*m^2*e^3 - 5754240*(x*e + d)^m*d^6*m*x*e^3 + 1358640*(x*e + d)^m*d^7*m*e^2 + 2041200*(x*e + d)^m*d^8*e + 14868*(x*e + d)^m*m^6*x*e^9 + 260106*(x*e + d)^m*m^5*x^2*e^9 + 4453233*(x*e + d)^m*m^4*x^3*e^9 + 7946185*(x*e + d)^m*m^3*x^4*e^9 + 29064240*(x*e + d)^m*m^2*x^5*e^9 - 5957592*(x*e + d)^m*m*x^6*e^9 + 5754240*(x*e + d)^m*x^7*e^9 + 14868*(x*e + d)^m*d*m^6*e^8 + 235620*(x*e + d)^m*d*m^5*x*e^8 + 3192773*(x*e + d)^m*d*m^4*x^2*e^8 + 3613480*(x*e + d)^m*d*m^3*x^3*e^8 + 6388272*(x*e + d)^m*d*m^2*x^4*e^8 - 447552*(x*e + d)^m*d*m*x^5*e^8 - 24486*(x*e + d)^m*d^2*m^5*e^7 - 1126710*(x*e + d)^m*d^2*m^4*x*e^7 - 2846805*(x*e + d)^m*d^2*m^3*x^2*e^7 - 7324224*(x*e + d)^m*d^2*m^2*x^3*e^7 + 559440*(x*e + d)^m*d^2*m*x^4*e^7 + 133750*(x*e + d)^m*d^3*m^4*e^6 + 1296750*(x*e + d)^m*d^3*m^3*x*e^6 + 8301024*(x*e + d)^m*d^3*m^2*x^2*e^6 - 745920*(x*e + d)^m*d^3*m*x^3*e^6 - 189150*(x*e + d)^m*d^4*m^3*e^5 - 5860800*(x*e + d)^m*d^4*m^2*x*e^5 + 1118880*(x*e + d)^m*d^4*m*x^2*e^5 + 1189920*(x*e + d)^m*d^5*m^2*e^4 - 2237760*(x*e + d)^m*d^5*m*x*e^4 + 848040*(x*e + d)^m*d^6*m*e^3 + 5754240*(x*e + d)^m*d^7*e^2 + 155232*(x*e + d)^m*m^5*x*e^9 + 1567797*(x*e + d)^m*m^4*x^2*e^9 + 15458076*(x*e + d)^m*m^3*x^3*e^9 + 15254460*(x*e + d)^m*m^2*x^4*e^9 + 28238400*(x*e + d)^m*m*x^5*e^9 - 2237760*(x*e + d)^m*x^6*e^9 + 155232*(x*e + d)^m*d*m^5*e^8 + 1332177*(x*e + d)^m*d*m^4*x*e^8 + 9072530*(x*e + d)^m*d*m^3*x^2*e^8 + 4414020*(x*e + d)^m*d*m^2*x^3*e^8 + 2685312*(x*e + d)^m*d*m*x^4*e^8 - 235620*(x*e + d)^m*d^2*m^4*e^7 - 5258836*(x*e + d)^m*d^2*m^3*x*e^7 - 5146830*(x*e + d)^m*d^2*m^2*x^2*e^7 - 3580416*(x*e + d)^m*d^2*m*x^3*e^7 + 1126710*(x*e + d)^m*d^3*m^3*e^6 + 4396860*(x*e + d)^m*d^3*m^2*x*e^6 + 5370624*(x*e + d)^m*d^3*m*x^2*e^6 - 1296750*(x*e + d)^m*d^4*m^2*e^5 - 10741248*(x*e + d)^m*d^4*m*x*e^5 + 5860800*(x*e + d)^m*d^5*m*e^4 + 2237760*(x*e + d)^m*d^6*e^3 + 983682*(x*e + d)^m*m^4*x*e^9 + 5752131*(x*e + d)^m*m^3*x^2*e^9 + 31059532*(x*e + d)^m*m^2*x^3*e^9 + 15207660*(x*e + d)^m*m*x^4*e^9 + 10741248*(x*e + d)^m*x^5*e^9 + 983682*(x*e + d)^m*d*m^4*e^8 + 4419954*(x*e + d)^m*d*m^3*x*e^8 + 12914472*(x*e + d)^m*d*m^2*x^2*e^8 + 1965600*(x*e + d)^m*d*m*x^3*e^8 - 1332177*(x*e + d)^m*d^2*m^3*e^7 - 12886224*(x*e + d)^m*d^2*m^2*x*e^7 - 2948400*(x*e + d)^m*d^2*m*x^2*e^7 + 5258836*(x*e + d)^m*d^3*m^2*e^6 + 5896800*(x*e + d)^m*d^3*m*x*e^6 - 4396860*(x*e + d)^m*d^4*m*e^5 + 10741248*(x*e + d)^m*d^5*e^4 + 3864168*(x*e + d)^m*m^3*x*e^9 + 12377178*(x*e + d)^m*m^2*x^2*e^9 + 32300304*(x*e + d)^m*m*x^3*e^9 + 5896800*(x*e + d)^m*x^4*e^9 + 3864168*(x*e + d)^m*d*m^3*e^8 + 795722

```
4*(x*e + d)^m*d*m^2*x*e^8 + 6471360*(x*e + d)^m*d*m*x^2*e^8 - 4419954*(x*e
+ d)^m*d^2*m^2*e^7 - 12942720*(x*e + d)^m*d^2*m*x*e^7 + 12886224*(x*e + d)^
m*d^3*m*e^6 - 5896800*(x*e + d)^m*d^4*e^5 + 9162072*(x*e + d)^m*m^2*x*e^9 +
 13944744*(x*e + d)^m*m*x^2*e^9 + 12942720*(x*e + d)^m*x^3*e^9 + 9162072*(x
*e + d)^m*d*m^2*e^8 + 5987520*(x*e + d)^m*d*m*x*e^8 - 7957224*(x*e + d)^m*d
^2*m*e^7 + 12942720*(x*e + d)^m*d^3*e^6 + 11946528*(x*e + d)^m*m*x*e^9 + 59
87520*(x*e + d)^m*x^2*e^9 + 11946528*(x*e + d)^m*d*m*e^8 - 5987520*(x*e + d
)^m*d^2*e^7 + 6531840*(x*e + d)^m*x*e^9 + 6531840*(x*e + d)^m*d*e^8)/(m^9*e
^9 + 45*m^8*e^9 + 870*m^7*e^9 + 9450*m^6*e^9 + 63273*m^5*e^9 + 269325*m^4*e
^9 + 723680*m^3*e^9 + 1172700*m^2*e^9 + 1026576*m*e^9 + 362880*e^9)
```

**maple [B]** time = 0.04, size = 3222, normalized size = 7.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*e^8*m^7*x^8-800*d*e^7*m^
7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m^6*x^8+315*d*e^7*m^7*x^6-
22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x^6-25830*e^8*m^6*x^7+45360
0*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7*x^5+9450*d*e^7*m^6*x^6-257
600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*x^5+67044*e^8*m^6*x^6-218610
*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*m^6*x^5+117600*d^2*e^6*m^5*x^
6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+114660*d*e^7*m^5*x^6-1568000*d*e^7*
m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-23532*e^8*m^6*x^5+579642*e^8*m^5*x^
6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-33600*d^3*e^5*m^5*x^5+3330*d^2*e^
6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2*e^6*m^4*x^6-592*d*e^7*m^7*x^3+62
90*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727650*d*e^7*m^4*x^6-5415200*d*e^7*m^
3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+99160*e^8*m^6*x^4-208458*e^8*m^5*x^5
+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^7+11812400*e^8*m^2*x^8+9450*d^3*e^5*
m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-41
5800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*
m^6*x^3+86210*d*e^7*m^5*x^4-1831500*d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-105
05600*d*e^7*m^2*x^7+33*e^8*m^8*x+4494*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*
e^8*m^5*x^4-1090353*e^8*m^4*x^5+9134412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+109
58400*e^8*m*x^8+168000*d^4*e^4*m^4*x^4-13320*d^3*e^5*m^5*x^3+179550*d^3*e^5
*m^4*x^4-2856000*d^3*e^5*m^3*x^5+1776*d^2*e^6*m^6*x^2-22200*d^2*e^6*m^5*x^3
+922410*d^2*e^6*m^4*x^4-1871100*d^2*e^6*m^3*x^5+9094400*d^2*e^6*m^2*x^6-214
*d*e^7*m^7*x-7410*d*e^7*m^6*x^2-311392*d*e^7*m^5*x^3+611240*d*e^7*m^4*x^4-6
805854*d*e^7*m^3*x^5+5159700*d*e^7*m^2*x^6-10454400*d*e^7*m*x^7+18*e^8*m^8+
1419*e^8*m^7*x+79608*e^8*m^6*x^2+430690*e^8*m^5*x^3+4850404*e^8*m^4*x^4-342
2907*e^8*m^3*x^5+16387596*e^8*m^2*x^6-5519340*e^8*m*x^7+4032000*e^8*x^8-378
00*d^4*e^4*m^4*x^3+1680000*d^4*e^4*m^3*x^4+2220*d^3*e^5*m^5*x^2-306360*d^3*
e^5*m^4*x^3+1181250*d^3*e^5*m^3*x^4-7560000*d^3*e^5*m^2*x^5+390*d^2*e^6*m^6
```

*x+58608*d^2*e^6*m^5*x^2-256040*d^2*e^6*m^4*x^3+4545450*d^2*e^6*m^3*x^4-434
5110*d^2*e^6*m^2*x^5+9878400*d^2*e^6*m*x^6-33*d*e^7*m^7-8560*d*e^7*m^6*x-11
5440*d*e^7*m^5*x^2-2365632*d*e^7*m^4*x^3+2395565*d*e^7*m^3*x^4-13971348*d*e
^7*m^2*x^5+5227740*d*e^7*m*x^6-4032000*d*e^7*x^7+792*e^8*m^7+25872*e^8*m^6*
x+772326*e^8*m^5*x^2+2389985*e^8*m^4*x^3+15608080*e^8*m^3*x^4-6238718*e^8*m
^2*x^5+15456528*e^8*m*x^6-2041200*e^8*x^7-672000*d^5*e^3*m^3*x^3+39960*d^4*
e^4*m^4*x^2-567000*d^4*e^4*m^3*x^3+5880000*d^4*e^4*m^2*x^4-3552*d^3*e^5*m^5
*x+59940*d^3*e^5*m^4*x^2-2464200*d^3*e^5*m^3*x^3+3449250*d^3*e^5*m^2*x^4-92
06400*d^3*e^5*m*x^5+214*d^2*e^6*m^6+14040*d^2*e^6*m^5*x+758352*d^2*e^6*m^4*
x^2-1420800*d^2*e^6*m^3*x^3+11302020*d^2*e^6*m^2*x^4-4887540*d^2*e^6*m*x^5+
4032000*d^2*e^6*x^6-1386*d*e^7*m^6-142096*d*e^7*m^5*x-945750*d*e^7*m^4*x^2-
9939088*d*e^7*m^3*x^3+5136710*d*e^7*m^2*x^4-14497488*d*e^7*m*x^5+2041200*d*
e^7*x^6+14868*e^8*m^6+260106*e^8*m^5*x+4453233*e^8*m^4*x^2+7946185*e^8*m^3*
x^3+29064240*e^8*m^2*x^4-5957592*e^8*m*x^5+5754240*e^8*x^6+113400*d^5*e^3*m
^3*x^2-4032000*d^5*e^3*m^2*x^3-4440*d^4*e^4*m^4*x+799200*d^4*e^4*m^3*x^2-24
57000*d^4*e^4*m^2*x^3+8400000*d^4*e^4*m*x^4-390*d^3*e^5*m^5-110112*d^3*e^5*
m^4*x+588300*d^3*e^5*m^3*x^2-8325000*d^3*e^5*m^2*x^3+4479300*d^3*e^5*m*x^4-
4032000*d^3*e^5*x^5+8346*d^2*e^6*m^5+202800*d^2*e^6*m^4*x+4821840*d^2*e^6*m
^3*x^2-3899060*d^2*e^6*m^2*x^3+13346640*d^2*e^6*m*x^4-2041200*d^2*e^6*x^5-2
4486*d*e^7*m^5-1260460*d*e^7*m^4*x-4332705*d*e^7*m^3*x^2-22675968*d*e^7*m^2
*x^3+5510040*d*e^7*m*x^4-5754240*d*e^7*x^5+155232*e^8*m^5+1567797*e^8*m^4*x
+15458076*e^8*m^3*x^2+15254460*e^8*m^2*x^3+28238400*e^8*m*x^4-2237760*e^8*x
^5+2016000*d^6*e^2*m^2*x^2-79920*d^5*e^3*m^3*x+1360800*d^5*e^3*m^2*x^2-7392
000*d^5*e^3*m*x^3+3552*d^4*e^4*m^4-111000*d^4*e^4*m^3*x+4995000*d^4*e^4*m^2
*x^2-3969000*d^4*e^4*m*x^3+4032000*d^4*e^4*x^4-13650*d^3*e^5*m^4-1296480*d^
3*e^5*m^3*x+2497500*d^3*e^5*m^2*x^2-11908080*d^3*e^5*m*x^3+2041200*d^3*e^5*
x^4+133750*d^2*e^6*m^4+1485900*d^2*e^6*m^3*x+15351744*d^2*e^6*m^2*x^2-49506
00*d^2*e^6*m*x^3+5754240*d^2*e^6*x^4-235620*d*e^7*m^4-6385546*d*e^7*m^3*x-1
0840440*d*e^7*m^2*x^2-25553088*d*e^7*m*x^3+2237760*d*e^7*x^4+983682*e^8*m^4
+5752131*e^8*m^3*x+31059532*e^8*m^2*x^2+15207660*e^8*m*x^3+10741248*e^8*x^4
-226800*d^6*e^2*m^2*x+6048000*d^6*e^2*m*x^2+4440*d^5*e^3*m^3-1438560*d^5*e^
3*m^2*x+3288600*d^5*e^3*m*x^2-4032000*d^5*e^3*x^3+106560*d^4*e^4*m^3-954600
*d^4*e^4*m^2*x+9990000*d^4*e^4*m*x^2-2041200*d^4*e^4*x^3-189150*d^3*e^5*m^3
-7050720*d^3*e^5*m^2*x+4204680*d^3*e^5*m*x^2-5754240*d^3*e^5*x^3+1126710*d^
2*e^6*m^3+5693610*d^2*e^6*m^2*x+21972672*d^2*e^6*m*x^2-2237760*d^2*e^6*x^3-
1332177*d*e^7*m^3-18145060*d*e^7*m^2*x-13242060*d*e^7*m*x^2-10741248*d*e^7*
x^3+3864168*e^8*m^3+12377178*e^8*m^2*x+32300304*e^8*m*x^2+5896800*e^8*x^3-4
032000*d^7*e*m*x+79920*d^6*e^2*m^2-2268000*d^6*e^2*m*x+4032000*d^6*e^2*x^2+
106560*d^5*e^3*m^2-7112880*d^5*e^3*m*x+2041200*d^5*e^3*x^2+1189920*d^4*e^4*
m^2-3085800*d^4*e^4*m*x+5754240*d^4*e^4*x^2-1296750*d^3*e^5*m^2-16602048*d^
3*e^5*m*x+2237760*d^3*e^5*x^2+5258836*d^2*e^6*m^2+10293660*d^2*e^6*m*x+1074
1248*d^2*e^6*x^2-4419954*d*e^7*m^2-25828944*d*e^7*m*x-5896800*d*e^7*x^2+916
2072*e^8*m^2+13944744*e^8*m*x+12942720*e^8*x^2+226800*d^7*e*m-4032000*d^7*e
*x+1358640*d^6*e^2*m-2041200*d^6*e^2*x+848040*d^5*e^3*m-5754240*d^5*e^3*x+5
860800*d^4*e^4*m-2237760*d^4*e^4*x-4396860*d^3*e^5*m-10741248*d^3*e^5*x+128

```
86224*d^2*e^6*m+5896800*d^2*e^6*x-7957224*d*e^7*m-12942720*d*e^7*x+11946528
*e^8*m+5987520*e^8*x+4032000*d^8+2041200*d^7*e+5754240*d^6*e^2+2237760*d^5*
e^3+10741248*d^4*e^4-5896800*d^3*e^5+12942720*d^2*e^6-5987520*d*e^7+6531840
*e^8)/e^9/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+723680*m^3+1172
700*m^2+1026576*m+362880)
```

**maxima [B]** time = 0.64, size = 1414, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="m
axima")
```

```
[Out] 33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 18
*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d
*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
+ 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 3
5*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
+ (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^
3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 1
5*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 + 85*m^3 +
 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d
*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2
*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x
+ d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) +
 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7
+ (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6
*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e
^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 19
60*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 45*((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^8*x^8 + (m^7
+ 21*m^6 + 175*m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d*e^7*x^7 - 7*(
m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^2*e^6*x^6 + 42*(m^5 +
10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^3*e^5*x^5 - 210*(m^4 + 6*m^3 + 11*m^2 +
6*m)*d^4*e^4*x^4 + 840*(m^3 + 3*m^2 + 2*m)*d^5*e^3*x^3 - 2520*(m^2 + m)*d^6
*e^2*x^2 + 5040*d^7*e*m*x - 5040*d^8)*(e*x + d)^m/((m^8 + 36*m^7 + 546*m^6
+ 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)*e^8) +
100*((m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^
2 + 109584*m + 40320)*e^9*x^9 + (m^8 + 28*m^7 + 322*m^6 + 1960*m^5 + 6769*m
^4 + 13132*m^3 + 13068*m^2 + 5040*m)*d*e^8*x^8 - 8*(m^7 + 21*m^6 + 175*m^5
+ 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*d^2*e^7*x^7 + 56*(m^6 + 15*m^5 + 8
5*m^4 + 225*m^3 + 274*m^2 + 120*m)*d^3*e^6*x^6 - 336*(m^5 + 10*m^4 + 35*m^3
```

```
 + 50*m^2 + 24*m)*d^4*e^5*x^5 + 1680*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^5*e^4*x
^4 - 6720*(m^3 + 3*m^2 + 2*m)*d^6*e^3*x^3 + 20160*(m^2 + m)*d^7*e^2*x^2 - 4
0320*d^8*e*m*x + 40320*d^9)*(e*x + d)^m/((m^9 + 45*m^8 + 870*m^7 + 9450*m^6
 + 63273*m^5 + 269325*m^4 + 723680*m^3 + 1172700*m^2 + 1026576*m + 362880)*
e^9)
```

**mupad [B]** time = 6.05, size = 2625, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^7
 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^3
+ 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4*e^
5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 9162072*d*
e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14868*d
*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 5258836*d^3
*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^3*m^2 +
 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 189150*d^4
*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*m^4 + 133
750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^2*e^7*m^5
+ 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^3*e^6*m^6 -
 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(1026576*m + 117
2700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^
8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*m^2 + 67284*m^
3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/(1026576*m + 11
72700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m
^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531840*e^9 + 9162072
*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*m^5 + 14868*e^9*m^
6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896800*d^3*e^6*m - 107
41248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^3*m - 2041200*d^7*e^2*m
 + 7957224*d*e^8*m^2 + 4419954*d*e^8*m^3 + 1332177*d*e^8*m^4 + 235620*d*e^8
*m^5 + 24486*d*e^8*m^6 + 1386*d*e^8*m^7 + 33*d*e^8*m^8 - 12886224*d^2*e^7*m
^2 + 4396860*d^3*e^6*m^2 - 5860800*d^4*e^5*m^2 - 848040*d^5*e^4*m^2 - 13586
40*d^6*e^3*m^2 - 226800*d^7*e^2*m^2 - 5258836*d^2*e^7*m^3 + 1296750*d^3*e^6
*m^3 - 1189920*d^4*e^5*m^3 - 106560*d^5*e^4*m^3 - 79920*d^6*e^3*m^3 - 11267
10*d^2*e^7*m^4 + 189150*d^3*e^6*m^4 - 106560*d^4*e^5*m^4 - 4440*d^5*e^4*m^4
 - 133750*d^2*e^7*m^5 + 13650*d^3*e^6*m^5 - 3552*d^4*e^5*m^5 - 8346*d^2*e^7
*m^6 + 390*d^3*e^6*m^6 - 214*d^2*e^7*m^7 + 5987520*d*e^8*m - 4032000*d^8*e*
m))/(e^9*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9
450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^5*(d + e*x)^m*(50*m + 35*m
^2 + 10*m^3 + m^4 + 24)*(33600*d^4*m - 244200*e^4*m - 447552*e^4 - 49580*e^
4*m^2 - 4440*e^4*m^3 - 148*e^4*m^4 + 47952*d^2*e^2*m + 7067*d*e^3*m^2 + 189
```

```
0*d^3*e*m^2 + 888*d*e^3*m^3 + 37*d*e^3*m^4 + 11322*d^2*e^2*m^2 + 666*d^2*e^
2*m^3 + 18648*d*e^3*m + 17010*d^3*e*m))/(e^4*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) - (x^7*(d + e*x)^m*(800*d^2*m - 1887*e^2*m - 7992*e^2 - 111*e^2*m^2 +
405*d*e*m + 45*d*e*m^2)*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m
^6 + 720))/(e^2*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*
m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) + (x^6*(d + e*x)^m*(274*
m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)*(5600*d^3*m - 7067*e^3*m - 18648
*e^3 - 888*e^3*m^2 - 37*e^3*m^3 + 1887*d*e^2*m^2 + 315*d^2*e*m^2 + 111*d*e^
2*m^3 + 7992*d*e^2*m + 2835*d^2*e*m))/(e^3*(1026576*m + 1172700*m^2 + 72368
0*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880
)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(168000*d^5*m + 732810*e^5*m
 + 982800*e^5 + 216125*e^5*m^2 + 31525*e^5*m^3 + 2275*e^5*m^4 + 65*e^5*m^5
+ 93240*d^2*e^3*m + 239760*d^3*e^2*m + 244200*d*e^4*m^2 + 9450*d^4*e*m^2 +
49580*d*e^4*m^3 + 4440*d*e^4*m^4 + 148*d*e^4*m^5 + 35335*d^2*e^3*m^2 + 5661
0*d^3*e^2*m^2 + 4440*d^2*e^3*m^3 + 3330*d^3*e^2*m^3 + 185*d^2*e^3*m^4 + 447
552*d*e^4*m + 85050*d^4*e*m))/(e^5*(1026576*m + 1172700*m^2 + 723680*m^3 +
269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (5*
x^8*(d + e*x)^m*(81*e - 20*d*m + 9*e*m)*(13068*m + 13132*m^2 + 6769*m^3 + 1
960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(e*(1026576*m + 1172700*m^2 + 723
680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 3628
80)) + (x^2*(m + 1)*(d + e*x)^m*(2016000*d^7*m + 7957224*e^7*m + 5987520*e^
7 + 4419954*e^7*m^2 + 1332177*e^7*m^3 + 235620*e^7*m^4 + 24486*e^7*m^5 + 13
86*e^7*m^6 + 33*e^7*m^7 - 2948400*d^2*e^5*m + 5370624*d^3*e^4*m + 1118880*d
^4*e^3*m + 2877120*d^5*e^2*m + 6443112*d*e^6*m^2 + 113400*d^6*e*m^2 + 26294
18*d*e^6*m^3 + 563355*d*e^6*m^4 + 66875*d*e^6*m^5 + 4173*d*e^6*m^6 + 107*d*
e^6*m^7 - 2198430*d^2*e^5*m^2 + 2930400*d^3*e^4*m^2 + 424020*d^4*e^3*m^2 +
679320*d^5*e^2*m^2 - 648375*d^2*e^5*m^3 + 594960*d^3*e^4*m^3 + 53280*d^4*e^
3*m^3 + 39960*d^5*e^2*m^3 - 94575*d^2*e^5*m^4 + 53280*d^3*e^4*m^4 + 2220*d^
4*e^3*m^4 - 6825*d^2*e^5*m^5 + 1776*d^3*e^4*m^5 - 195*d^2*e^5*m^6 + 6471360
*d*e^6*m + 1020600*d^6*e*m))/(e^7*(1026576*m + 1172700*m^2 + 723680*m^3 + 2
69325*m^4 + 63273*m^5 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880)) - (x^3
*(d + e*x)^m*(3*m + m^2 + 2)*(672000*d^6*m - 6443112*e^6*m - 6471360*e^6 -
2629418*e^6*m^2 - 563355*e^6*m^3 - 66875*e^6*m^4 - 4173*e^6*m^5 - 107*e^6*m
^6 + 1790208*d^2*e^4*m + 372960*d^3*e^3*m + 959040*d^4*e^2*m - 732810*d*e^5
*m^2 + 37800*d^5*e*m^2 - 216125*d*e^5*m^3 - 31525*d*e^5*m^4 - 2275*d*e^5*m^
5 - 65*d*e^5*m^6 + 976800*d^2*e^4*m^2 + 141340*d^3*e^3*m^2 + 226440*d^4*e^2
*m^2 + 198320*d^2*e^4*m^3 + 17760*d^3*e^3*m^3 + 13320*d^4*e^2*m^3 + 17760*d
^2*e^4*m^4 + 740*d^3*e^3*m^4 + 592*d^2*e^4*m^5 - 982800*d*e^5*m + 340200*d^
5*e*m))/(e^6*(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5
 + 9450*m^6 + 870*m^7 + 45*m^8 + m^9 + 362880))
```

**sympy** [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

# 3.369 $\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right) dx$

**Optimal**. Leaf size=292

$$\frac{\left(300d^2+85de+17e^2\right)(d+ex)^{m+5}}{e^7(m+5)}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d\right.}{e}$$

```
[Out] (5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^7
/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2+m
)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m)/e
^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(300*d
^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e^7/(6
+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

**Rubi** [A] time = 0.19, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {1628}

$$\frac{\left(5d^2-2de+3e^2\right)\left(3d^2e^2+5d^3e+4d^4-de^3+2e^4\right)(d+ex)^{m+1}}{e^7(m+1)}-\frac{\left(68d^3e^2+12d^2e^3+85d^4e+120d^5+42de^4-7\right.}{e^7(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d +
 e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e
^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 170*d
^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)) -
(2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4 + m))
 + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((120*d
+ 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e^7*(7 +
 m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (d+ex)^m \left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \int \left(\frac{\left(20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5\right.}{e^6}\right.$$

$$= \frac{\left(5d^2-2de+3e^2\right)\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d}{e^7(1+m)}$$

**Mathematica** [A] time = 0.17, size = 261, normalized size = 0.89

$$\frac{(d+ex)^{m+1}\left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^4}{m+5}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^3}{m+4}+\frac{\left(300d^4+170d^3e+102d^2e^2+12de^3+21e^4\right)(d+ex)^2}{m+3}+\frac{\left(5d^2-2d\right.}{}\right.}{e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

```
[Out] ((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 +
 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2
 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 34*d*
e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^
4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/(7 + m
)))/e^7
```

**fricas** [B] time = 0.61, size = 1448, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fri
cas")
```

```
[Out] (6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624*
e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5*e
^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280*e^
7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e^6 -
 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)*m^2
- 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (17136*
e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5 - 3*(4
00*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^6 - 3145
*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(240*d^2*e^5
 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 1770*d*e^6)*m
```

```
^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^6 - 96*e^7)
*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4 - (3600*d^3*
e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*e^4 + 1411*d^
2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2*e^5 + 714*d*e
^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2*e^5 + 9990*d*e
^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5 + 84*d*e^6 - 52
5*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 5187*e^7)*m^4 - (240
0*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25599*e^7)*m^3 - 4*(
1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 - 16338*e^7)*m^2 - 4
*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d*e^6 - 19929*e^7)*m)*x^
3 + (408*d^5*e^2 + 432*d^4*e^3 + 7518*d^3*e^4 - 8225*d^2*e^5 + 30624*d*e^6)
*m^2 + (17640*e^7 + 7*(3*d*e^6 + e^7)*m^6 + (12*d^2*e^5 + 483*d*e^6 + 182*e
^7)*m^5 + 3*(68*d^3*e^4 + 76*d^2*e^5 + 1407*d*e^6 + 630*e^7)*m^4 + (1020*d^
4*e^3 + 2856*d^3*e^4 + 1500*d^2*e^5 + 17157*d*e^6 + 9940*e^7)*m^3 + (7200*d
^5*e^2 + 8160*d^4*e^3 + 11220*d^3*e^4 + 3804*d^2*e^5 + 31038*d*e^6 + 27503*
e^7)*m^2 + 6*(1200*d^5*e^2 + 1190*d^4*e^3 + 1428*d^3*e^4 + 420*d^2*e^5 + 29
40*d*e^6 + 6153*e^7)*m)*x^2 + 6*(340*d^6*e + 884*d^5*e^2 + 428*d^4*e^3 + 44
66*d^3*e^4 - 3213*d^2*e^5 + 8028*d*e^6)*m + (30240*e^7 + (7*d*e^6 + 6*e^7)*
m^6 - (42*d^2*e^5 - 175*d*e^6 - 162*e^7)*m^5 - (24*d^3*e^4 + 924*d^2*e^5 -
1715*d*e^6 - 1770*e^7)*m^4 - (408*d^4*e^3 + 432*d^3*e^4 + 7518*d^2*e^5 - 82
25*d*e^6 - 9990*e^7)*m^3 - 6*(340*d^5*e^2 + 884*d^4*e^3 + 428*d^3*e^4 + 446
6*d^2*e^5 - 3213*d*e^6 - 5104*e^7)*m^2 - 24*(600*d^6*e + 595*d^5*e^2 + 714*
d^4*e^3 + 210*d^3*e^4 + 1470*d^2*e^5 - 735*d*e^6 - 2007*e^7)*m)*x)*(e*x + d
)^m/(e^7*m^7 + 28*e^7*m^6 + 322*e^7*m^5 + 1960*e^7*m^4 + 6769*e^7*m^3 + 131
32*e^7*m^2 + 13068*e^7*m + 5040*e^7)
```

**giac [B]** time = 0.27, size = 3098, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gia
c")
```

```
[Out] (20*(x*e + d)^m*m^6*x^7*e^7 + 20*(x*e + d)^m*d*m^6*x^6*e^6 - 17*(x*e + d)^m
*m^6*x^6*e^7 + 420*(x*e + d)^m*m^5*x^7*e^7 - 17*(x*e + d)^m*d*m^6*x^5*e^6 +
 300*(x*e + d)^m*d*m^5*x^6*e^6 - 120*(x*e + d)^m*d^2*m^5*x^5*e^5 + 17*(x*e
+ d)^m*m^6*x^5*e^7 - 374*(x*e + d)^m*m^5*x^6*e^7 + 3500*(x*e + d)^m*m^4*x^7
*e^7 + 17*(x*e + d)^m*d*m^6*x^4*e^6 - 289*(x*e + d)^m*d*m^5*x^5*e^6 + 1700*
(x*e + d)^m*d*m^4*x^6*e^6 + 85*(x*e + d)^m*d^2*m^5*x^4*e^5 - 1200*(x*e + d)
^m*d^2*m^4*x^5*e^5 + 600*(x*e + d)^m*d^3*m^4*x^4*e^4 - 4*(x*e + d)^m*m^6*x^
4*e^7 + 391*(x*e + d)^m*m^5*x^5*e^7 - 3230*(x*e + d)^m*m^4*x^6*e^7 + 14700*
(x*e + d)^m*m^3*x^7*e^7 - 4*(x*e + d)^m*d*m^6*x^3*e^6 + 323*(x*e + d)^m*d*m
^5*x^4*e^6 - 1785*(x*e + d)^m*d*m^4*x^5*e^6 + 4500*(x*e + d)^m*d*m^3*x^6*e^
6 - 68*(x*e + d)^m*d^2*m^5*x^3*e^5 + 1105*(x*e + d)^m*d^2*m^4*x^4*e^5 - 420
```

0*(x*e + d)^m*d^2*m^3*x^5*e^5 - 340*(x*e + d)^m*d^3*m^4*x^3*e^4 + 3600*(x*e + d)^m*d^3*m^3*x^4*e^4 - 2400*(x*e + d)^m*d^4*m^3*x^3*e^3 + 21*(x*e + d)^m *m^6*x^3*e^7 - 96*(x*e + d)^m*m^5*x^4*e^7 + 3519*(x*e + d)^m*m^4*x^5*e^7 - 13940*(x*e + d)^m*m^3*x^6*e^7 + 32480*(x*e + d)^m*m^2*x^7*e^7 + 21*(x*e + d )^m*d*m^6*x^2*e^6 - 84*(x*e + d)^m*d*m^5*x^3*e^6 + 2227*(x*e + d)^m*d*m^4*x ^4*e^6 - 5015*(x*e + d)^m*d*m^3*x^5*e^6 + 5480*(x*e + d)^m*d*m^2*x^6*e^6 + 12*(x*e + d)^m*d^2*m^5*x^2*e^5 - 1088*(x*e + d)^m*d^2*m^4*x^3*e^5 + 4505*(x *e + d)^m*d^2*m^3*x^4*e^5 - 6000*(x*e + d)^m*d^2*m^2*x^5*e^5 + 204*(x*e + d )^m*d^3*m^4*x^2*e^4 - 3400*(x*e + d)^m*d^3*m^3*x^3*e^4 + 6600*(x*e + d)^m*d ^3*m^2*x^4*e^4 + 1020*(x*e + d)^m*d^4*m^3*x^2*e^3 - 7200*(x*e + d)^m*d^4*m^ 2*x^3*e^3 + 7200*(x*e + d)^m*d^5*m^2*x^2*e^2 + 7*(x*e + d)^m*m^6*x^2*e^7 + 525*(x*e + d)^m*m^5*x^3*e^7 - 904*(x*e + d)^m*m^4*x^4*e^7 + 15725*(x*e + d) ^m*m^3*x^5*e^7 - 31433*(x*e + d)^m*m^2*x^6*e^7 + 35280*(x*e + d)^m*m*x^7*e^ 7 + 7*(x*e + d)^m*d*m^6*x*e^6 + 483*(x*e + d)^m*d*m^5*x^2*e^6 - 652*(x*e + d)^m*d*m^4*x^3*e^6 + 6817*(x*e + d)^m*d*m^3*x^4*e^6 - 6358*(x*e + d)^m*d*m^ 2*x^5*e^6 + 2400*(x*e + d)^m*d*m*x^6*e^6 - 42*(x*e + d)^m*d^2*m^5*x*e^5 + 2 28*(x*e + d)^m*d^2*m^4*x^2*e^5 - 5644*(x*e + d)^m*d^2*m^3*x^3*e^5 + 7055*(x *e + d)^m*d^2*m^2*x^4*e^5 - 2880*(x*e + d)^m*d^2*m*x^5*e^5 - 24*(x*e + d)^m *d^3*m^4*x*e^4 + 2856*(x*e + d)^m*d^3*m^3*x^2*e^4 - 7820*(x*e + d)^m*d^3*m^ 2*x^3*e^4 + 3600*(x*e + d)^m*d^3*m*x^4*e^4 - 408*(x*e + d)^m*d^4*m^3*x*e^3 + 8160*(x*e + d)^m*d^4*m^2*x^2*e^3 - 4800*(x*e + d)^m*d^4*m*x^3*e^3 - 2040* (x*e + d)^m*d^5*m^2*x*e^2 + 7200*(x*e + d)^m*d^5*m*x^2*e^2 - 14400*(x*e + d )^m*d^6*m*x*e + 6*(x*e + d)^m*m^6*x*e^7 + 182*(x*e + d)^m*m^5*x^2*e^7 + 518 7*(x*e + d)^m*m^4*x^3*e^7 - 4224*(x*e + d)^m*m^3*x^4*e^7 + 36448*(x*e + d)^ m*m^2*x^5*e^7 - 34646*(x*e + d)^m*m*x^6*e^7 + 14400*(x*e + d)^m*x^7*e^7 + 6 *(x*e + d)^m*d*m^6*e^6 + 175*(x*e + d)^m*d*m^5*x*e^6 + 4221*(x*e + d)^m*d*m ^4*x^2*e^6 - 2268*(x*e + d)^m*d*m^3*x^3*e^6 + 9180*(x*e + d)^m*d*m^2*x^4*e^ 6 - 2856*(x*e + d)^m*d*m*x^5*e^6 - 7*(x*e + d)^m*d^2*m^5*e^5 - 924*(x*e + d )^m*d^2*m^4*x*e^5 + 1500*(x*e + d)^m*d^2*m^3*x^2*e^5 - 10336*(x*e + d)^m*d^ 2*m^2*x^3*e^5 + 3570*(x*e + d)^m*d^2*m*x^4*e^5 + 42*(x*e + d)^m*d^3*m^4*e^4 - 432*(x*e + d)^m*d^3*m^3*x*e^4 + 11220*(x*e + d)^m*d^3*m^2*x^2*e^4 - 4760 *(x*e + d)^m*d^3*m*x^3*e^4 + 24*(x*e + d)^m*d^4*m^3*e^3 - 5304*(x*e + d)^m* d^4*m^2*x*e^3 + 7140*(x*e + d)^m*d^4*m*x^2*e^3 + 408*(x*e + d)^m*d^5*m^2*e^ 2 - 14280*(x*e + d)^m*d^5*m*x*e^2 + 2040*(x*e + d)^m*d^6*m*e + 14400*(x*e + d)^m*d^7 + 162*(x*e + d)^m*m^5*x*e^7 + 1890*(x*e + d)^m*m^4*x^2*e^7 + 2559 9*(x*e + d)^m*m^3*x^3*e^7 - 10180*(x*e + d)^m*m^2*x^4*e^7 + 41004*(x*e + d) ^m*m*x^5*e^7 - 14280*(x*e + d)^m*x^6*e^7 + 162*(x*e + d)^m*d*m^5*e^6 + 1715 *(x*e + d)^m*d*m^4*x*e^6 + 17157*(x*e + d)^m*d*m^3*x^2*e^6 - 3376*(x*e + d) ^m*d*m^2*x^3*e^6 + 4284*(x*e + d)^m*d*m*x^4*e^6 - 175*(x*e + d)^m*d^2*m^4*e ^5 - 7518*(x*e + d)^m*d^2*m^3*x*e^5 + 3804*(x*e + d)^m*d^2*m^2*x^2*e^5 - 57 12*(x*e + d)^m*d^2*m*x^3*e^5 + 924*(x*e + d)^m*d^3*m^3*e^4 - 2568*(x*e + d) ^m*d^3*m^2*x*e^4 + 8568*(x*e + d)^m*d^3*m*x^2*e^4 + 432*(x*e + d)^m*d^4*m^2 *e^3 - 17136*(x*e + d)^m*d^4*m*x*e^3 + 5304*(x*e + d)^m*d^5*m*e^2 + 14280*( x*e + d)^m*d^6*e + 1770*(x*e + d)^m*m^4*x*e^7 + 9940*(x*e + d)^m*m^3*x^2*e^ 7 + 65352*(x*e + d)^m*m^2*x^3*e^7 - 11808*(x*e + d)^m*m*x^4*e^7 + 17136*(x*

```
e + d)^m*x^5*e^7 + 1770*(x*e + d)^m*d*m^4*e^6 + 8225*(x*e + d)^m*d*m^3*x*e^
6 + 31038*(x*e + d)^m*d*m^2*x^2*e^6 - 1680*(x*e + d)^m*d*m*x^3*e^6 - 1715*(
x*e + d)^m*d^2*m^3*e^5 - 26796*(x*e + d)^m*d^2*m^2*x*e^5 + 2520*(x*e + d)^m
*d^2*m*x^2*e^5 + 7518*(x*e + d)^m*d^3*m^2*e^4 - 5040*(x*e + d)^m*d^3*m*x*e^
4 + 2568*(x*e + d)^m*d^4*m*e^3 + 17136*(x*e + d)^m*d^5*e^2 + 9990*(x*e + d)
^m*m^3*x*e^7 + 27503*(x*e + d)^m*m^2*x^2*e^7 + 79716*(x*e + d)^m*m*x^3*e^7
- 5040*(x*e + d)^m*x^4*e^7 + 9990*(x*e + d)^m*d*m^3*e^6 + 19278*(x*e + d)^m
*d*m^2*x*e^6 + 17640*(x*e + d)^m*d*m*x^2*e^6 - 8225*(x*e + d)^m*d^2*m^2*e^5
 - 35280*(x*e + d)^m*d^2*m*x*e^5 + 26796*(x*e + d)^m*d^3*m*e^4 + 5040*(x*e
+ d)^m*d^4*e^3 + 30624*(x*e + d)^m*m^2*x*e^7 + 36918*(x*e + d)^m*m*x^2*e^7
+ 35280*(x*e + d)^m*x^3*e^7 + 30624*(x*e + d)^m*d*m^2*e^6 + 17640*(x*e + d)
^m*d*m*x*e^6 - 19278*(x*e + d)^m*d^2*m*e^5 + 35280*(x*e + d)^m*d^3*e^4 + 48
168*(x*e + d)^m*m*x*e^7 + 17640*(x*e + d)^m*x^2*e^7 + 48168*(x*e + d)^m*d*m
*e^6 - 17640*(x*e + d)^m*d^2*e^5 + 30240*(x*e + d)^m*x*e^7 + 30240*(x*e + d
)^m*d*e^6)/(m^7*e^7 + 28*m^6*e^7 + 322*m^5*e^7 + 1960*m^4*e^7 + 6769*m^3*e^
7 + 13132*m^2*e^7 + 13068*m*e^7 + 5040*e^7)
```

**maple [B]** time = 0.02, size = 1504, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

```
[Out] (e*x+d)^(1+m)*(20*e^6*m^6*x^6-17*e^6*m^6*x^5+420*e^6*m^5*x^6-120*d*e^5*m^5*
x^5+17*e^6*m^6*x^4-374*e^6*m^5*x^5+3500*e^6*m^4*x^6+85*d*e^5*m^5*x^4-1800*d
*e^5*m^4*x^5-4*e^6*m^6*x^3+391*e^6*m^5*x^4-3230*e^6*m^4*x^5+14700*e^6*m^3*x
^6+600*d^2*e^4*m^4*x^4-68*d*e^5*m^5*x^3+1445*d*e^5*m^4*x^4-10200*d*e^5*m^3*
x^5+21*e^6*m^6*x^2-96*e^6*m^5*x^3+3519*e^6*m^4*x^4-13940*e^6*m^3*x^5+32480*
e^6*m^2*x^6-340*d^2*e^4*m^4*x^3+6000*d^2*e^4*m^3*x^4+12*d*e^5*m^5*x^2-1292*
d*e^5*m^4*x^3+8925*d*e^5*m^3*x^4-27000*d*e^5*m^2*x^5+7*e^6*m^6*x+525*e^6*m^
5*x^2-904*e^6*m^4*x^3+15725*e^6*m^3*x^4-31433*e^6*m^2*x^5+35280*e^6*m*x^6-2
400*d^3*e^3*m^3*x^3+204*d^2*e^4*m^4*x^2-4420*d^2*e^4*m^3*x^3+21000*d^2*e^4*
m^2*x^4-42*d*e^5*m^5*x+252*d*e^5*m^4*x^2-8908*d*e^5*m^3*x^3+25075*d*e^5*m^2
*x^4-32880*d*e^5*m*x^5+6*e^6*m^6+182*e^6*m^5*x+5187*e^6*m^4*x^2-4224*e^6*m^
3*x^3+36448*e^6*m^2*x^4-34646*e^6*m*x^5+14400*e^6*x^6+1020*d^3*e^3*m^3*x^2-
14400*d^3*e^3*m^2*x^3-24*d^2*e^4*m^4*x+3264*d^2*e^4*m^3*x^2-18020*d^2*e^4*m
^2*x^3+30000*d^2*e^4*m*x^4-7*d*e^5*m^5-966*d*e^5*m^4*x+1956*d*e^5*m^3*x^2-2
7268*d*e^5*m^2*x^3+31790*d*e^5*m*x^4-14400*d*e^5*x^5+162*e^6*m^5+1890*e^6*m
^4*x+25599*e^6*m^3*x^2-10180*e^6*m^2*x^3+41004*e^6*m*x^4-14280*e^6*x^5+7200
*d^4*e^2*m^2*x^2-408*d^3*e^3*m^3*x+10200*d^3*e^3*m^2*x^2-26400*d^3*e^3*m*x^
3+42*d^2*e^4*m^4-456*d^2*e^4*m^3*x+16932*d^2*e^4*m^2*x^2-28220*d^2*e^4*m*x^
3+14400*d^2*e^4*x^4-175*d*e^5*m^4-8442*d*e^5*m^3*x+6804*d*e^5*m^2*x^2-36720
*d*e^5*m*x^3+14280*d*e^5*x^4+1770*e^6*m^4+9940*e^6*m^3*x+65352*e^6*m^2*x^2-
11808*e^6*m*x^3+17136*e^6*x^4-2040*d^4*e^2*m^2*x+21600*d^4*e^2*m*x^2+24*d^3
```

```
*e^3*m^3-5712*d^3*e^3*m^2*x+23460*d^3*e^3*m*x^2-14400*d^3*e^3*x^3+924*d^2*e
^4*m^3-3000*d^2*e^4*m^2*x+31008*d^2*e^4*m*x^2-14280*d^2*e^4*x^3-1715*d*e^5*
m^3-34314*d*e^5*m^2*x+10128*d*e^5*m*x^2-17136*d*e^5*x^3+9990*e^6*m^3+27503*
e^6*m^2*x+79716*e^6*m*x^2-5040*e^6*x^3-14400*d^5*e*m*x+408*d^4*e^2*m^2-1632
0*d^4*e^2*m*x+14400*d^4*e^2*x^2+432*d^3*e^3*m^2-22440*d^3*e^3*m*x+14280*d^3
*e^3*x^2+7518*d^2*e^4*m^2-7608*d^2*e^4*m*x+17136*d^2*e^4*x^2-8225*d*e^5*m^2
-62076*d*e^5*m*x+5040*d*e^5*x^2+30624*e^6*m^2+36918*e^6*m*x+35280*e^6*x^2+2
040*d^5*e*m-14400*d^5*e*x+5304*d^4*e^2*m-14280*d^4*e^2*x+2568*d^3*e^3*m-171
36*d^3*e^3*x+26796*d^2*e^4*m-5040*d^2*e^4*x-19278*d*e^5*m-35280*d*e^5*x+481
68*e^6*m+17640*e^6*x+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280
*d^2*e^4-17640*d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1
3132*m^2+13068*m+5040)
```

**maxima [B]** time = 0.55, size = 788, normalized size = 2.70

$$\frac{7\left(e^2(m+1)x^2+demx-d^2\right)(ex+d)^m}{\left(m^2+3\,m+2\right)e^2}+\frac{6\,(ex+d)^{m+1}}{e(m+1)}+\frac{21\left(\left(m^2+3\,m+2\right)e^3x^3+\left(m^2+m\right)de^2x^2-2\,d^2emx+2\,d\right.}{\left(m^3+6\,m^2+11\,m+6\right)e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="max
ima")
```

```
[Out] 7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*(
e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^
2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3) - 4
*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 + 35*m^
2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^
4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 1
2*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5 + 15*m^4
 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d*e^5*
x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m^2 + 2*m)*d
^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6)*(e*x + d)^
m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^6) + 20*(
(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*e^7*x^7 + (m^6
 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 - 6*(m^5 + 10*m^4
 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^
3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2 + m)*d^5*e^2*x^2
 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 + 322*m^5 + 1960*m^4
 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

**mupad [B]** time = 5.09, size = 1425, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

```
[Out] ((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 35280
*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4*m
 + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 + 177
0*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3*e^4
*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*e^4*m
^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^5 + 48
168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^
4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m + 1624*m^
2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2 + 6769*m^
3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(35280*d^2*e
^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 - 162*e^7*m^
5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m + 14280*d^5*
e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 175*d*e^6*m^5 -
 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^4*e^3*m^2 + 20
40*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^4*e^3*m^3 + 924
*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^6*m + 14400*d^6*
e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 +
m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4*m - 13398*e^4*m -
 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2856*d^2*e^2*m + 428
*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*m^4 + 884*d^2*e^2*m^2 +
 68*d^2*e^2*m^3 + 840*d*e^3*m + 2380*d^3*e*m))/(e^4*(13068*m + 13132*m^2 +
6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^5*(d + e*x)^m*(5
0*m + 35*m^2 + 10*m^3 + m^4 + 24)*(120*d^2*m - 221*e^2*m - 714*e^2 - 17*e^2
*m^2 + 119*d*e*m + 17*d*e*m^2))/(e^2*(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d + e*x)^m*(11*m + 6*m^2 + m
^3 + 6)*(600*d^3*m - 428*e^3*m - 840*e^3 - 72*e^3*m^2 - 4*e^3*m^3 + 221*d*e
^2*m^2 + 85*d^2*e*m^2 + 17*d*e^2*m^3 + 714*d*e^2*m + 595*d^2*e*m))/(e^3*(13
068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) +
 (x^2*(m + 1)*(d + e*x)^m*(7200*d^5*m + 19278*e^5*m + 17640*e^5 + 8225*e^5*
m^2 + 1715*e^5*m^3 + 175*e^5*m^4 + 7*e^5*m^5 + 2520*d^2*e^3*m + 8568*d^3*e^
2*m + 13398*d*e^4*m^2 + 1020*d^4*e*m^2 + 3759*d*e^4*m^3 + 462*d*e^4*m^4 + 2
1*d*e^4*m^5 + 1284*d^2*e^3*m^2 + 2652*d^3*e^2*m^2 + 216*d^2*e^3*m^3 + 204*d
^3*e^2*m^3 + 12*d^2*e^3*m^4 + 17640*d*e^4*m + 7140*d^4*e*m))/(e^5*(13068*m
+ 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) - (x^6*
(d + e*x)^m*(119*e - 20*d*m + 17*e*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
m^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^
6 + m^7 + 5040))
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

```
[Out] Timed out
```

# 3.370 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

**Optimal**. Leaf size=255

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,{}_2}{3500(m+1)(5id}$$

```
[Out] 1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e*
x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(e*x+d)^(1+m)*hyper
geom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(6412*I+423*14^(1/2))
/(1+m)/(5*I*d-e*(I-14^(1/2)))-1/3500*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m]
,5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(6412*I-423*14^(1/2))/(1+m)/(5*I*d-e*(I+14
^(1/2)))
```

**Rubi [A]** time = 0.48, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1628, 68}

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,{}_2}{3500(m+1)(5id}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

```
[Out] ((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*d
 + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e^3
*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1[1,
 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d - (I
 + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(3
500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \left(\frac{\left(100d^2+165de+81e^2\right)(d+ex)^m}{125e^2} + \frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex)^m}{2-2i\sqrt{14}+10x} +$$

$$= \frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(}{5}$$

$$= \frac{\left(100d^2+165de+81e^2\right)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(}{5}$$

**Mathematica [A]** time = 0.73, size = 221, normalized size = 0.87

$$\frac{(d+ex)^{m+1}\left(\frac{28\left(100d^2+165de+81e^2\right)}{e^3(m+1)} + \frac{2800(d+ex)^2}{e^3(m+3)} - \frac{140(40d+33e)(d+ex)}{e^3(m+2)} - \frac{\left(423\sqrt{14}+6412i\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{(m+1)\left(5id+\left(\sqrt{14}-i\right)e\right)} - \frac{\left(423\sqrt{14}\right.}{}\right.}{3500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x
]
```

```
[Out] ((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140*
(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m)) -
 ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/
(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) - ((-
6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*
d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3500
```

**fricas [F]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{5x^2+2x+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fri
cas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{4} - 5\,x^{3} + 3\,x^{2} + x + 2\right)(ex + d)^{m}}{5\,x^{2} + 2\,x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gia
c")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

**maple [F]** time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(4x^{4} - 5x^{3} + 3x^{2} + x + 2\right)(ex + d)^{m}}{5x^{2} + 2x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(4\,x^{4} - 5\,x^{3} + 3\,x^{2} + x + 2\right)(ex + d)^{m}}{5\,x^{2} + 2\,x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="max
ima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d + e\,x)^{m}\,\left(4\,x^{4} - 5\,x^{3} + 3\,x^{2} + x + 2\right)}{5\,x^{2} + 2\,x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

```
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

```
[Out] Timed out
```

# 3.371 $\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=377

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+4821\right.}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

```
[Out] 4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(1
+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(e*x+d)^(1+m)*hypergeom([1, 1
+m],[2+m],5*(e*x+d)/(5*d-e*(1+I*14^(1/2))))*(80360*d^2-32144*d*e+48216*e^2-
5922*d*e*m+19138*e^2*m-I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))*14^
(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d-e*(1+I*14^(1/2)))+1/19600*(e*x+d)^(1+
m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))*(80360*d^2-3214
4*d*e+48216*e^2-5922*d*e*m+19138*e^2*m+I*(6565*d^2-2*d*e*(1313-3206*m)+e^2*
(3939-98*m))*14^(1/2))/(5*d^2-2*d*e+3*e^2)/(1+m)/(5*d+I*e*(I+14^(1/2)))
```

**Rubi** [A] time = 0.90, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {1648, 1628, 68}

$$\frac{\left(i\sqrt{14}\left(6565d^2-2de(1313-3206m)+e^2(3939-98m)\right)+80360d^2-5922dem-32144de+19138e^2m+4821\right.}{19600(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)\left(5d^2-2de+3e^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

```
[Out] (4*(d + e*x)^(1 + m))/(25*e*(1 + m)) - ((1367*d - 293*e + (423*d - 1367*e)*
x)*(d + e*x)^(1 + m))/(700*(5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((8
0360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 320
6*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(1
9600*(5*d + I*(I + Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m)) + ((80360*
d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3206*m)
+ e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*Hypergeo
metric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e)])/(1960
0*(5*d - (1 + I*Sqrt[14])*e)*(5*d^2 - 2*d*e + 3*e^2)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
```

```
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1648

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*(f*(b*c*d - b^2*e + 2*a*c*e) - a*g*(2*c*d - b*e) + c*(
f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), I
nt[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*
(c*d^2 - b*d*e + a*e^2)*Q + f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m + 2)
- 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m +
b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f*(2
*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x] &
& PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && L
tQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || I
LtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex)^m\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx = -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int \frac{(d+ex)^m\left(\frac{2}{25}(1845\right.}{}}{}$$

$$= -\frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{\int\left(\frac{224}{25}\left(5d^2-2\right.\right.}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} - \frac{(}{}$$

$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} + \frac{(}{}$$

**Mathematica [A]** time = 1.80, size = 441, normalized size = 1.17

$$(d+ex)^{m+1}\left(-\frac{\sqrt{14}\left(\frac{\left(2115d^2+de\left(-846+\left(-6412+423i\sqrt{14}\right)m\right)+e^2\left(1269+\left(98-1367i\sqrt{14}\right)m\right)\right){}_2F_1\left(1,m+1;m+2;\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{5id+\left(\sqrt{14}-i\right)e}-\frac{\left(2115d^2-de\left(846+\left(6412+423i\sqrt{14}\right)\right.\right.}{}\right.}{(m+1)\left(5d^2-2de+3e^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2
,x]

[Out] ((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367*
x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[14]
)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14])
*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[14])*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)
])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 + d*e*(-
846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqrt[14])*
m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[14
])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6412 + (42
3*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hypergeometric
```

```
2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/((5*I)*d -
(I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{25\,x^4+20\,x^3+34\,x^2+12\,x+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="f
ricas")
```

```
[Out] integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34*
x^2 + 12*x + 9), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{\left(5\,x^2+2\,x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="g
iac")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

**maple [F]** time = 0.12, size = 0, normalized size = 0.00

$$\int\frac{\left(4x^4-5x^3+3x^2+x+2\right)(ex+d)^m}{\left(5x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

```
[Out] int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)(ex+d)^m}{\left(5\,x^2+2\,x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="m
axima")
```

```
[Out] integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^m\,\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{{\left(5\,x^2+2\,x+3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

```
[Out] int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

```
[Out] Timed out
```

# 3.372 $\int \frac{d+ex+fx^2+gx^3+hx^4+ix^5}{\left(a+bx+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=528

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci-c^4(6be-4af)+b^5(-i)+12c^5d\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{x\left(c^3\left(2a^2h\right.\right.}{}$$

```
[Out] 1/2*(-a*b^3*c*h-b*c^2*(-3*a^2*h+a*c*f+c^2*d)+a*b^4*i+a*b^2*c*(-4*a*i+c*g)+2
*a*c^2*(a^2*i-a*c*g+c^2*e)-(2*c^5*d-c^4*(2*a*f+b*e)+c^3*(2*a^2*h+3*a*b*g+b^
2*f)-b^5*i+b^3*c*(5*a*i+b*h)-b*c^2*(5*a^2*i+4*a*b*h+b^2*g))*x)/c^4/(-4*a*c+
b^2)/(c*x^2+b*x+a)^2+1/2*(b^5*c*h+b^3*c^2*(-8*a*h+c*f)+2*b*c^3*(11*a^2*h+a*
c*f+3*c^2*d)-b^6*i-b^4*c*(-11*a*i+c*g)-16*a^2*c^3*(-2*a*i+c*g)-b^2*c^2*(39*
a^2*i-5*a*c*g+3*c^2*e)+2*c*(6*c^5*d-c^4*(-2*a*f+3*b*e)+c^3*(-10*a^2*h-3*a*b
*g+b^2*f)+2*b^5*i-b^3*c*(15*a*i+b*h)+a*b*c^2*(25*a*i+8*b*h))*x)/c^4/(-4*a*c
+b^2)^2/(c*x^2+b*x+a)-(12*c^5*d-c^4*(-4*a*f+6*b*e)+2*c^3*(6*a^2*h-3*a*b*g+b
^2*f)-b^5*i+10*a*b^3*c*i-30*a^2*b*c^2*i)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/c^3/(-4*a*c+b^2)^(5/2)+1/2*i*ln(c*x^2+b*x+a)/c^3
```

**Rubi [A]** time = 1.31, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1660, 634, 618, 206, 628}

$$\frac{2cx\left(c^3\left(-10a^2h-3abg+b^2f\right)-b^3c(15ai+bh)-c^4(3be-2af)+abc^2(25ai+8bh)+2b^5i+6c^5d\right)-b^2c^2\left(39a^2i\right.}{2c^4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] -(a*b^3*c*h + b*c^2*(c^2*d + a*c*f - 3*a^2*h) - a*b^4*i - a*b^2*c*(c*g - 4*
a*i) - 2*a*c^2*(c^2*e - a*c*g + a^2*i) + (2*c^5*d - c^4*(b*e + 2*a*f) + c^3
*(b^2*f + 3*a*b*g + 2*a^2*h) - b^5*i + b^3*c*(b*h + 5*a*i) - b*c^2*(b^2*g +
 4*a*b*h + 5*a^2*i))*x)/(2*c^4*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (b^5*c*
h + b^3*c^2*(c*f - 8*a*h) + 2*b*c^3*(3*c^2*d + a*c*f + 11*a^2*h) - b^6*i -
b^4*c*(c*g - 11*a*i) - 16*a^2*c^3*(c*g - 2*a*i) - b^2*c^2*(3*c^2*e - 5*a*c*
g + 39*a^2*i) + 2*c*(6*c^5*d - c^4*(3*b*e - 2*a*f) + c^3*(b^2*f - 3*a*b*g -
 10*a^2*h) + 2*b^5*i - b^3*c*(b*h + 15*a*i) + a*b*c^2*(8*b*h + 25*a*i))*x)/
(2*c^4*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - ((12*c^5*d - c^4*(6*b*e - 4*a*f
) + 2*c^3*(b^2*f - 3*a*b*g + 6*a^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2
*i)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + (i*
Log[a + b*x + c*x^2])/(2*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d + ex + fx^2 + gx^3 + hx^4 + 372x^5}{\left(a + bx + cx^2\right)^3}\,dx = \frac{744a^3c^2 - bc^4d - a^2c\left(1488b^2 + 2c^2g - 3bch\right) + a\left(372b^4 + 2c^4e - bc^3\right.}{}$$

$$= \frac{744a^3c^2 - bc^4d - a^2c\left(1488b^2 + 2c^2g - 3bch\right) + a\left(372b^4 + 2c^4e - bc^3\right.}{}$$

$$= \frac{744a^3c^2 - bc^4d - a^2c\left(1488b^2 + 2c^2g - 3bch\right) + a\left(372b^4 + 2c^4e - bc^3\right.}{}$$

$$= \frac{744a^3c^2 - bc^4d - a^2c\left(1488b^2 + 2c^2g - 3bch\right) + a\left(372b^4 + 2c^4e - bc^3\right.}{}$$

$$= \frac{744a^3c^2 - bc^4d - a^2c\left(1488b^2 + 2c^2g - 3bch\right) + a\left(372b^4 + 2c^4e - bc^3\right.}{}$$

**Mathematica [A]** time = 1.08, size = 488, normalized size = 0.92

$$\frac{\frac{2c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2c^3\left(6a^2h-3abg+b^2f\right)-30a^2bc^2i+10ab^3ci+c^4(4af-6be)+b^5(-i)+12c^5d\right)}{\left(4ac-b^2\right)^{5/2}} + \frac{b^2c\left(-4a^2i+ac(g+4hx)-c^2fx\right)+bc^2\left(a^2(3h+5ix)-ac(f+3g\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x]
```

```
[Out] ((b^5*i*x + b^4*(a*i - c*h*x) + 2*c^2*(a^3*i - c^3*d*x + a*c^2*(e + f*x) -
a^2*c*(g + h*x)) + b^2*c*(-4*a^2*i - c^2*f*x + a*c*(g + 4*h*x)) + b^3*c*(c*
g*x - a*(h + 5*i*x)) + b*c^2*(c^2*(-d + e*x) - a*c*(f + 3*g*x) + a^2*(3*h +
 5*i*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (-(b^6*i) + b^5*c*(h + 4*i*
x) + b^3*c^2*(c*f - 8*a*h - 30*a*i*x) - b^4*c*(-11*a*i + c*(g + 2*h*x)) + 4
*c^3*(8*a^3*i + 3*c^3*d*x + a*c^2*f*x - a^2*c*(4*g + 5*h*x)) + b^2*c^2*(-39
*a^2*i + c^2*(-3*e + 2*f*x) + a*c*(5*g + 16*h*x)) + 2*b*c^3*(3*c^2*(d - e*x
) + a*c*(f - 3*g*x) + a^2*(11*h + 25*i*x)))/((b^2 - 4*a*c)^2*(a + x*(b + c*
x))) + (2*c*(12*c^5*d + c^4*(-6*b*e + 4*a*f) + 2*c^3*(b^2*f - 3*a*b*g + 6*a
^2*h) - b^5*i + 10*a*b^3*c*i - 30*a^2*b*c^2*i)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*i*Log[a + x*(b + c*x)])/(2*c^4)
```

**fricas [B]** time = 0.99, size = 3480, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="fri
cas")
```

```
[Out] [1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^4*c^4 - 2
*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c^2 - 12*a
*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^2 + 85*a^
2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d - 9*(b^4*c^
4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (b^6*c^2 -
 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*c^2 + 30*
a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^
3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - (12*a^2*c^5*d - 6*a^2*b*c^4*e - 6*a^3*b*c
^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a^2*c^5*h +
2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*i)*x^4 +
2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2*(b^3*c^4 +
 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 + (12*(b^2*c
^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b^2*c^4 + 4*
a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2*a^3*c^4)*h
 - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*(a^2*b^2*c^
3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2*(12*a*b*c^
5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^3*c^3 + 2*a
^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/
(c*x^2 + b*x + a)) - (b^5*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3
 + 4*a^2*b^2*c^4 - 32*a^3*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b
^4*c^2 + 4*a^3*b^2*c^3 - 32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a
^4*b*c^3)*h + 3*(a^2*b^6 - 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i +
2*(2*(b^4*c^4 + a*b^2*c^5 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b
*c^5)*e + (5*a*b^4*c^3 - 22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b
^3*c^3 - 20*a^3*b*c^4)*g - (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*
a^4*c^4)*h + (3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*
x + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2
*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3)*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^
4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3
*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 1
28*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b
```

```
*c^6)*x), 1/2*(2*(6*(b^2*c^6 - 4*a*c^7)*d - 3*(b^3*c^5 - 4*a*b*c^6)*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*f - 3*(a*b^3*c^4 - 4*a^2*b*c^5)*g - (b^6*c
^2 - 12*a*b^4*c^3 + 42*a^2*b^2*c^4 - 40*a^3*c^5)*h + (2*b^7*c - 23*a*b^5*c^
2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*i)*x^3 + (18*(b^3*c^5 - 4*a*b*c^6)*d -
9*(b^4*c^4 - 4*a*b^2*c^5)*e + 3*(b^5*c^3 - 2*a*b^3*c^4 - 8*a^2*b*c^5)*f - (
b^6*c^2 - 3*a*b^4*c^3 + 12*a^2*b^2*c^4 - 64*a^3*c^5)*g - (b^7*c - 12*a*b^5*
c^2 + 30*a^2*b^3*c^3 + 8*a^3*b*c^4)*h + (3*b^8 - 31*a*b^6*c + 87*a^2*b^4*c^
2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*i)*x^2 - 2*(12*a^2*c^5*d - 6*a^2*b*c^4*e
- 6*a^3*b*c^3*g + 12*a^4*c^3*h + (12*c^7*d - 6*b*c^6*e - 6*a*b*c^5*g + 12*a
^2*c^5*h + 2*(b^2*c^5 + 2*a*c^6)*f - (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*i)*x^4 + 2*(12*b*c^6*d - 6*b^2*c^5*e - 6*a*b^2*c^4*g + 12*a^2*b*c^4*h + 2
*(b^3*c^4 + 2*a*b*c^5)*f - (b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*i)*x^3 +
 (12*(b^2*c^5 + 2*a*c^6)*d - 6*(b^3*c^4 + 2*a*b*c^5)*e + 2*(b^4*c^3 + 4*a*b
^2*c^4 + 4*a^2*c^5)*f - 6*(a*b^3*c^3 + 2*a^2*b*c^4)*g + 12*(a^2*b^2*c^3 + 2
*a^3*c^4)*h - (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*i)*x^2 + 2*
(a^2*b^2*c^3 + 2*a^3*c^4)*f - (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2)*i + 2
*(12*a*b*c^5*d - 6*a*b^2*c^4*e - 6*a^2*b^2*c^3*g + 12*a^3*b*c^3*h + 2*(a*b^
3*c^3 + 2*a^2*b*c^4)*f - (a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*i)*x)*sqrt
(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^5
*c^3 - 14*a*b^3*c^4 + 40*a^2*b*c^5)*d - (a*b^4*c^3 + 4*a^2*b^2*c^4 - 32*a^3
*c^5)*e + 6*(a^2*b^3*c^3 - 4*a^3*b*c^4)*f - (a^2*b^4*c^2 + 4*a^3*b^2*c^3 -
32*a^4*c^4)*g - (a^2*b^5*c - 14*a^3*b^3*c^2 + 40*a^4*b*c^3)*h + 3*(a^2*b^6
- 11*a^3*b^4*c + 36*a^4*b^2*c^2 - 32*a^5*c^3)*i + 2*(2*(b^4*c^4 + a*b^2*c^5
 - 20*a^2*c^6)*d - (b^5*c^3 + a*b^3*c^4 - 20*a^2*b*c^5)*e + (5*a*b^4*c^3 -
22*a^2*b^2*c^4 + 8*a^3*c^5)*f - (a*b^5*c^2 + a^2*b^3*c^3 - 20*a^3*b*c^4)*g
- (a*b^6*c - 14*a^2*b^4*c^2 + 46*a^3*b^2*c^3 - 24*a^4*c^4)*h + (3*a*b^7 - 3
4*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*i)*x + ((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*i*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*i*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*i*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3)*i*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
*i)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 -
64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 + 2
*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3 -
10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*b^
7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

**giac [A]** time = 0.22, size = 657, normalized size = 1.24

$$\frac{\left(12\,c^{5}di + 2\,b^{2}c^{3}fi + 4\,ac^{4}fi - 6\,abc^{3}gi + 12\,a^{2}c^{3}hi - 6\,bc^{4}ie + b^{5} - 10\,ab^{3}c + 30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{3}i - 8\,ab^{2}c^{4}i + 16\,a^{2}c^{5}i\right)\sqrt{-b^{2}+4\,ac}} + \frac{i\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="gia
c")
```

```
[Out] (12*c^5*d*i + 2*b^2*c^3*f*i + 4*a*c^4*f*i - 6*a*b*c^3*g*i + 12*a^2*c^3*h*i
- 6*b*c^4*i*e + b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b
^2 + 4*a*c))/((b^4*c^3*i - 8*a*b^2*c^4*i + 16*a^2*c^5*i)*sqrt(-b^2 + 4*a*c)
) + 1/2*i*log(c*x^2 + b*x + a)/c^3 - 1/2*(b^3*c^3*d - 10*a*b*c^4*d - 6*a^2*
b*c^3*f + a^2*b^2*c^2*g + 8*a^3*c^3*g + a^2*b^3*c*h - 10*a^3*b*c^2*h - 3*a^
2*b^4*i + 21*a^3*b^2*c*i - 24*a^4*c^2*i + a*b^2*c^3*e + 8*a^2*c^4*e - 2*(6*
c^6*d + b^2*c^4*f + 2*a*c^5*f - 3*a*b*c^4*g - b^4*c^2*h + 8*a*b^2*c^3*h - 1
0*a^2*c^4*h + 2*b^5*c*i - 15*a*b^3*c^2*i + 25*a^2*b*c^3*i - 3*b*c^5*e)*x^3
- (18*b*c^5*d + 3*b^3*c^3*f + 6*a*b*c^4*f - b^4*c^2*g - a*b^2*c^3*g - 16*a^
2*c^4*g - b^5*c*h + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h + 3*b^6*i - 19*a*b^4*c*i
+ 11*a^2*b^2*c^2*i + 32*a^3*c^3*i - 9*b^2*c^4*e)*x^2 - 2*(2*b^2*c^4*d + 10*
a*c^5*d + 5*a*b^2*c^3*f - 2*a^2*c^4*f - a*b^3*c^2*g - 5*a^2*b*c^3*g - a*b^4
*c*h + 10*a^2*b^2*c^2*h - 6*a^3*c^3*h + 3*a*b^5*i - 22*a^2*b^3*c*i + 31*a^3
*b*c^2*i - b^3*c^3*e - 5*a*b*c^4*e)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2
*c^3)
```

**maple [B]** time = 0.02, size = 1244, normalized size = 2.36

$$-\frac{30a^2bi\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c}+\frac{12a^2h\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}}+\frac{10a\,b^3i\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-b^2}\;c^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x)
```

```
[Out] ((25*a^2*b*c^2*i-10*a^2*c^3*h-15*a*b^3*c*i+8*a*b^2*c^2*h-3*a*b*c^3*g+2*a*c^
4*f+2*b^5*i-b^4*c*h+b^2*c^3*f-3*b*c^4*e+6*c^5*d)/c^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^3+1/2*(32*a^3*c^3*i+11*a^2*b^2*c^2*i+2*a^2*b*c^3*h-16*a^2*c^4*g-19*a
*b^4*c*i+8*a*b^3*c^2*h-a*b^2*c^3*g+6*a*b*c^4*f+3*b^6*i-b^5*c*h-b^4*c^2*g+3*
b^3*c^3*f-9*b^2*c^4*e+18*b*c^5*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+(31*a^
3*b*c^2*i-6*a^3*c^3*h-22*a^2*b^3*c*i+10*a^2*b^2*c^2*h-5*a^2*b*c^3*g-2*a^2*c
^4*f+3*a*b^5*i-a*b^4*c*h-a*b^3*c^2*g+5*a*b^2*c^3*f-5*a*b*c^4*e+10*a*c^5*d-b
^3*c^3*e+2*b^2*c^4*d)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+1/2/c^3*(24*a^4*c^2*
i-21*a^3*b^2*c*i+10*a^3*b*c^2*h-8*a^3*c^3*g+3*a^2*b^4*i-a^2*b^3*c*h-a^2*b^2
*c^2*g+6*a^2*b*c^3*f-8*a^2*c^4*e-a*b^2*c^3*e+10*a*b*c^4*d-b^3*c^3*d)/(16*a^
2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+8/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x
^2+b*x+a)*a^2*i-4/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*a*b^2*i+1/
2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^2+b*x+a)*b^4*i-30/c/(16*a^2*c^2-8*a
*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*i+1
2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a^2*h+10/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2
```

```
*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*i-6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g+4*c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+2/(16*a^2*c^
2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*
f-6*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-
b^2)^(1/2))*b*e+12*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))*d-1/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^5*i
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^3,x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 6.17, size = 1027, normalized size = 1.95

$$\frac{\mathrm{atan}\left(\frac{x\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)\left(\cdots\right.}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\cdots}{c^3\,\left(4\,a\,\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + i*x^5)/(a + b*x + c*x^2)^3,x)
```

```
[Out] (atan((x*(32*a^2*c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) -
16*a*b^2*c^4*(4*a*c - b^2)^(5/2)))/(c^2*(4*a*c - b^2)^5) + ((32*a^2*c^5*(4*
a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a*c - b^
2)^(5/2))*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4))/(2*c^5*(4*a*c - b^2)^5*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(12*c^5*d - b^5*i + 2*b^2*c^3*f + 12*a^2*c^3
*h + 4*a*c^4*f - 6*b*c^4*e - 6*a*b*c^3*g + 10*a*b^3*c*i - 30*a^2*b*c^2*i))/
(c^3*(4*a*c - b^2)^(5/2)) - (log(a + b*x + c*x^2)*(b^10*i - 1024*a^5*c^5*i
+ 160*a^2*b^6*c^2*i - 640*a^3*b^4*c^3*i + 1280*a^4*b^2*c^4*i - 20*a*b^8*c*i
))/(2*(1024*a^5*c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b
^4*c^6 - 1280*a^4*b^2*c^7)) - ((8*a^2*c^4*e + b^3*c^3*d + 8*a^3*c^3*g - 3*a
^2*b^4*i - 24*a^4*c^2*i + a^2*b^2*c^2*g - 10*a*b*c^4*d + a*b^2*c^3*e - 6*a^
2*b*c^3*f + a^2*b^3*c*h - 10*a^3*b*c^2*h + 21*a^3*b^2*c*i)/(2*c^3*(b^4 + 16
```

```
*a^2*c^2 - 8*a*b^2*c)) - (x^2*(3*b^6*i - 9*b^2*c^4*e - 16*a^2*c^4*g + 3*b^3
*c^3*f - b^4*c^2*g + 32*a^3*c^3*i + 18*b*c^5*d - b^5*c*h + 11*a^2*b^2*c^2*i
 + 6*a*b*c^4*f - 19*a*b^4*c*i - a*b^2*c^3*g + 8*a*b^3*c^2*h + 2*a^2*b*c^3*h
))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(2*a^2*c^4*f - 2*b^2*c^4*d +
 b^3*c^3*e + 6*a^3*c^3*h - 10*a*c^5*d - 3*a*b^5*i - 10*a^2*b^2*c^2*h + 5*a*
b*c^4*e + a*b^4*c*h - 5*a*b^2*c^3*f + a*b^3*c^2*g + 5*a^2*b*c^3*g + 22*a^2*
b^3*c*i - 31*a^3*b*c^2*i))/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^3*(6*c
^5*d + 2*b^5*i + b^2*c^3*f - 10*a^2*c^3*h + 2*a*c^4*f - 3*b*c^4*e - b^4*c*h
 - 3*a*b*c^3*g - 15*a*b^3*c*i + 8*a*b^2*c^2*h + 25*a^2*b*c^2*i))/(c^2*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x +
2*b*c*x^3)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((i*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**3,x)
```

```
[Out] Timed out
```

# 3.373 $\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx$

**Optimal**. Leaf size=765

$$\frac{x^3\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)}{3c^5}+\frac{x^2\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\left(3a^2m+3a\right.\right.}{}$$

```
[Out] (c^6*f-c^5*(a*h+b*g)+c^4*(a^2*k+2*a*b*j+b^2*h)+b^6*m-b^4*c*(5*a*m+b*l)+b^2*
c^2*(6*a^2*m+4*a*b*l+b^2*k)-c^3*(a^3*m+3*a^2*b*l+3*a*b^2*k+b^3*j))*x/c^7+1/
2*(c^5*g-c^4*(a*j+b*h)+c^3*(a^2*l+2*a*b*k+b^2*j)-b^5*m+b^3*c*(4*a*m+b*l)-b*
c^2*(3*a^2*m+3*a*b*l+b^2*k))*x^2/c^6+1/3*(c^4*h-c^3*(a*k+b*j)+b^4*m-b^2*c*(
3*a*m+b*l)+c^2*(a^2*m+2*a*b*l+b^2*k))*x^3/c^5+1/4*(c^3*j-c^2*(a*l+b*k)-b^3*
m+b*c*(2*a*m+b*l))*x^4/c^4+1/5*(c^2*k+b^2*m-c*(a*m+b*l))*x^5/c^3+1/6*(-b*m+
c*l)*x^6/c^2+1/7*m*x^7/c+1/2*(c^7*e-c^6*(a*g+b*f)+c^5*(a^2*j+2*a*b*h+b^2*g)
-c^4*(a^3*l+3*a^2*b*k+3*a*b^2*j+b^3*h)-b^7*m+b^5*c*(6*a*m+b*l)-b^3*c^2*(10*
a^2*m+5*a*b*l+b^2*k)+b*c^3*(4*a^3*m+6*a^2*b*l+4*a*b^2*k+b^3*j))*ln(c*x^2+b*
x+a)/c^8-(2*c^8*d-c^7*(2*a*f+b*e)+c^6*(2*a^2*h+3*a*b*g+b^2*f)-c^5*(2*a^3*k+
5*a^2*b*j+4*a*b^2*h+b^3*g)+b^8*m-b^6*c*(8*a*m+b*l)+b^4*c^2*(20*a^2*m+7*a*b*
l+b^2*k)-b^2*c^3*(16*a^3*m+14*a^2*b*l+6*a*b^2*k+b^3*j)+c^4*(2*a^4*m+7*a^3*b
*l+9*a^2*b^2*k+5*a*b^3*j+b^4*h))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^8/
(-4*a*c+b^2)^(1/2)
```

**Rubi [A]** time = 5.83, antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^6\left(2a^2h+3abg+b^2f\right)-c^5\left(5a^2bj+2a^3k+4ab^2h+b^3g\right)+c^4\left(9a^2b^2k+7a^3bl+2a^4m+5ab\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x]
```

```
[Out] ((c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b^4*c*(
b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a*b^2*k
 + 3*a^2*b*l + a^3*m))*x)/c^7 + ((c^5*g - c^4*(b*h + a*j) + c^3*(b^2*j + 2*
a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b*l + 3*a
^2*m))*x^2)/(2*c^6) + ((c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l + 3*a*
m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3)/(3*c^5) + ((c^3*j - c^2*(b*k + a*l
) - b^3*m + b*c*(b*l + 2*a*m))*x^4)/(4*c^4) + ((c^2*k + b^2*m - c*(b*l + a*
m))*x^5)/(5*c^3) + ((c*l - b*m)*x^6)/(6*c^2) + (m*x^7)/(7*c) - ((2*c^8*d -
c^7*(b*e + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*
```

```
h + 5*a^2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7
*a*b*l + 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) +
c^4*(b^4*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTanh[(b + 2
*c*x)/Sqrt[b^2 - 4*a*c]])/(c^8*Sqrt[b^2 - 4*a*c]) + ((c^7*e - c^6*(b*f + a*
g) + c^5*(b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a
^3*l) - b^7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m)
+ b*c^3*(b^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + b*x + c*x^2])/(2
*c^8)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex+fx^2+gx^3+hx^4+jx^5+kx^6+lx^7+mx^8}{a+bx+cx^2}\,dx = \int \left( \frac{c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-}{} \right.$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

$$= \frac{\left(c^6f-c^5(bg+ah)+c^4\left(b^2h+2abj+a^2k\right)+b^6m-b^4\right.}{}$$

**Mathematica** [**A**] time = 0.73, size = 754, normalized size = 0.99

$$\frac{140c^3x^3\left(c^2\left(a^2m+2abl+b^2k\right)-b^2c(3am+bl)-c^3(ak+bj)+b^4m+c^4h\right)+210c^2x^2\left(c^3\left(a^2l+2abk+b^2j\right)-bc^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)
/(a + b*x + c*x^2),x]
```

```
[Out] (420*c*(c^6*f - c^5*(b*g + a*h) + c^4*(b^2*h + 2*a*b*j + a^2*k) + b^6*m - b
^4*c*(b*l + 5*a*m) + b^2*c^2*(b^2*k + 4*a*b*l + 6*a^2*m) - c^3*(b^3*j + 3*a
*b^2*k + 3*a^2*b*l + a^3*m))*x + 210*c^2*(c^5*g - c^4*(b*h + a*j) + c^3*(b^
2*j + 2*a*b*k + a^2*l) - b^5*m + b^3*c*(b*l + 4*a*m) - b*c^2*(b^2*k + 3*a*b
*l + 3*a^2*m))*x^2 + 140*c^3*(c^4*h - c^3*(b*j + a*k) + b^4*m - b^2*c*(b*l
+ 3*a*m) + c^2*(b^2*k + 2*a*b*l + a^2*m))*x^3 + 105*c^4*(c^3*j - c^2*(b*k +
 a*l) - b^3*m + b*c*(b*l + 2*a*m))*x^4 + 84*c^5*(c^2*k + b^2*m - c*(b*l + a
*m))*x^5 + 70*c^6*(c*l - b*m)*x^6 + 60*c^7*m*x^7 + (420*(2*c^8*d - c^7*(b*e
 + 2*a*f) + c^6*(b^2*f + 3*a*b*g + 2*a^2*h) - c^5*(b^3*g + 4*a*b^2*h + 5*a^
2*b*j + 2*a^3*k) + b^8*m - b^6*c*(b*l + 8*a*m) + b^4*c^2*(b^2*k + 7*a*b*l +
 20*a^2*m) - b^2*c^3*(b^3*j + 6*a*b^2*k + 14*a^2*b*l + 16*a^3*m) + c^4*(b^4
*h + 5*a*b^3*j + 9*a^2*b^2*k + 7*a^3*b*l + 2*a^4*m))*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 210*(c^7*e - c^6*(b*f + a*g) + c^5*(
b^2*g + 2*a*b*h + a^2*j) - c^4*(b^3*h + 3*a*b^2*j + 3*a^2*b*k + a^3*l) - b^
7*m + b^5*c*(b*l + 6*a*m) - b^3*c^2*(b^2*k + 5*a*b*l + 10*a^2*m) + b*c^3*(b
^3*j + 4*a*b^2*k + 6*a^2*b*l + 4*a^3*m))*Log[a + x*(b + c*x)])/(420*c^8)
```

**fricas [A]** time = 1.84, size = 2643, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="fricas")
```

```
[Out] [1/420*(60*(b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6
 - 4*a*b*c^7)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l
+ (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j -
 (b^3*c^6 - 4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4
 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^
6 - 4*a*b*c^7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b
^3*c^5 + 8*a^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c
^6)*m)*x^3 + 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^
5 - 5*a*b^2*c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k +
(b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5
*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 + 210*(2*c^8*d - b*c^7*e + (b^
2*c^6 - 2*a*c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a
^2*c^6)*h - (b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^
3 + 9*a^2*b^2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 -
7*a^3*b*c^4)*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4
*c^4)*m)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 420*((b^2*c^7 - 4*a*c^8)*f - (
b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*h - (b^5*c^4 -
 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4
*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*l + (
b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*m)*x + 2
10*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7)*f + (b^4*c^5 - 5*a*b^2*c^
6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*h + (b^6*c^3 - 7*a
*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*
b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*
b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*a^2*b^5*c^2 - 44*a^3*b^3*c^
3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*c^8 - 4*a*c^9), 1/420*(60*(
b^2*c^7 - 4*a*c^8)*m*x^7 + 70*((b^2*c^7 - 4*a*c^8)*l - (b^3*c^6 - 4*a*b*c^7
)*m)*x^6 + 84*((b^2*c^7 - 4*a*c^8)*k - (b^3*c^6 - 4*a*b*c^7)*l + (b^4*c^5 -
 5*a*b^2*c^6 + 4*a^2*c^7)*m)*x^5 + 105*((b^2*c^7 - 4*a*c^8)*j - (b^3*c^6 -
4*a*b*c^7)*k + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*l - (b^5*c^4 - 6*a*b^3*c
^5 + 8*a^2*b*c^6)*m)*x^4 + 140*((b^2*c^7 - 4*a*c^8)*h - (b^3*c^6 - 4*a*b*c^
7)*j + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*k - (b^5*c^4 - 6*a*b^3*c^5 + 8*a
^2*b*c^6)*l + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*m)*x^3 +
 210*((b^2*c^7 - 4*a*c^8)*g - (b^3*c^6 - 4*a*b*c^7)*h + (b^4*c^5 - 5*a*b^2*
c^6 + 4*a^2*c^7)*j - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*k + (b^6*c^3 - 7
```

```
*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*l - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^
2*b^3*c^4 - 12*a^3*b*c^5)*m)*x^2 - 420*(2*c^8*d - b*c^7*e + (b^2*c^6 - 2*a*
c^7)*f - (b^3*c^5 - 3*a*b*c^6)*g + (b^4*c^4 - 4*a*b^2*c^5 + 2*a^2*c^6)*h -
(b^5*c^3 - 5*a*b^3*c^4 + 5*a^2*b*c^5)*j + (b^6*c^2 - 6*a*b^4*c^3 + 9*a^2*b^
2*c^4 - 2*a^3*c^5)*k - (b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*l + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*m)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 420
*((b^2*c^7 - 4*a*c^8)*f - (b^3*c^6 - 4*a*b*c^7)*g + (b^4*c^5 - 5*a*b^2*c^6
+ 4*a^2*c^7)*h - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^2*b*c^6)*j + (b^6*c^3 - 7*a*b
^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*k - (b^7*c^2 - 8*a*b^5*c^3 + 19*a^2*b^
3*c^4 - 12*a^3*b*c^5)*l + (b^8*c - 9*a*b^6*c^2 + 26*a^2*b^4*c^3 - 25*a^3*b^
2*c^4 + 4*a^4*c^5)*m)*x + 210*((b^2*c^7 - 4*a*c^8)*e - (b^3*c^6 - 4*a*b*c^7
)*f + (b^4*c^5 - 5*a*b^2*c^6 + 4*a^2*c^7)*g - (b^5*c^4 - 6*a*b^3*c^5 + 8*a^
2*b*c^6)*h + (b^6*c^3 - 7*a*b^4*c^4 + 13*a^2*b^2*c^5 - 4*a^3*c^6)*j - (b^7*
c^2 - 8*a*b^5*c^3 + 19*a^2*b^3*c^4 - 12*a^3*b*c^5)*k + (b^8*c - 9*a*b^6*c^2
 + 26*a^2*b^4*c^3 - 25*a^3*b^2*c^4 + 4*a^4*c^5)*l - (b^9 - 10*a*b^7*c + 34*
a^2*b^5*c^2 - 44*a^3*b^3*c^3 + 16*a^4*b*c^4)*m)*log(c*x^2 + b*x + a))/(b^2*
c^8 - 4*a*c^9)]
```

**giac [A]** time = 0.18, size = 982, normalized size = 1.28

$$\frac{60\,c^6mx^7+70\,c^6lx^6-70\,bc^5mx^6+84\,c^6kx^5-84\,bc^5lx^5+84\,b^2c^4mx^5-84\,ac^5mx^5+105\,c^6jx^4-105\,bc^5kx^4+10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="giac")
```

```
[Out] 1/420*(60*c^6*m*x^7 + 70*c^6*l*x^6 - 70*b*c^5*m*x^6 + 84*c^6*k*x^5 - 84*b*c
^5*l*x^5 + 84*b^2*c^4*m*x^5 - 84*a*c^5*m*x^5 + 105*c^6*j*x^4 - 105*b*c^5*k*
x^4 + 105*b^2*c^4*l*x^4 - 105*a*c^5*l*x^4 - 105*b^3*c^3*m*x^4 + 210*a*b*c^4
*m*x^4 + 140*c^6*h*x^3 - 140*b*c^5*j*x^3 + 140*b^2*c^4*k*x^3 - 140*a*c^5*k*
x^3 - 140*b^3*c^3*l*x^3 + 280*a*b*c^4*l*x^3 + 140*b^4*c^2*m*x^3 - 420*a*b^2
*c^3*m*x^3 + 140*a^2*c^4*m*x^3 + 210*c^6*g*x^2 - 210*b*c^5*h*x^2 + 210*b^2*
c^4*j*x^2 - 210*a*c^5*j*x^2 - 210*b^3*c^3*k*x^2 + 420*a*b*c^4*k*x^2 + 210*b
^4*c^2*l*x^2 - 630*a*b^2*c^3*l*x^2 + 210*a^2*c^4*l*x^2 - 210*b^5*c*m*x^2 +
840*a*b^3*c^2*m*x^2 - 630*a^2*b*c^3*m*x^2 + 420*c^6*f*x - 420*b*c^5*g*x + 4
20*b^2*c^4*h*x - 420*a*c^5*h*x - 420*b^3*c^3*j*x + 840*a*b*c^4*j*x + 420*b^
4*c^2*k*x - 1260*a*b^2*c^3*k*x + 420*a^2*c^4*k*x - 420*b^5*c*l*x + 1680*a*b
^3*c^2*l*x - 1260*a^2*b*c^3*l*x + 420*b^6*m*x - 2100*a*b^4*c*m*x + 2520*a^2
*b^2*c^2*m*x - 420*a^3*c^3*m*x)/c^7 - 1/2*(b*c^6*f - b^2*c^5*g + a*c^6*g +
b^3*c^4*h - 2*a*b*c^5*h - b^4*c^3*j + 3*a*b^2*c^4*j - a^2*c^5*j + b^5*c^2*k
 - 4*a*b^3*c^3*k + 3*a^2*b*c^4*k - b^6*c*l + 5*a*b^4*c^2*l - 6*a^2*b^2*c^3*
l + a^3*c^4*l + b^7*m - 6*a*b^5*c*m + 10*a^2*b^3*c^2*m - 4*a^3*b*c^3*m - c^
```

```
7*e)*log(c*x^2 + b*x + a)/c^8 + (2*c^8*d + b^2*c^6*f - 2*a*c^7*f - b^3*c^5*
g + 3*a*b*c^6*g + b^4*c^4*h - 4*a*b^2*c^5*h + 2*a^2*c^6*h - b^5*c^3*j + 5*a
*b^3*c^4*j - 5*a^2*b*c^5*j + b^6*c^2*k - 6*a*b^4*c^3*k + 9*a^2*b^2*c^4*k -
2*a^3*c^5*k - b^7*c*l + 7*a*b^5*c^2*l - 14*a^2*b^3*c^3*l + 7*a^3*b*c^4*l +
b^8*m - 8*a*b^6*c*m + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 2*a^4*c^4*m - b
*c^7*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^8)
```

**maple [B]** time = 0.01, size = 1960, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x)
```

```
[Out] 1/7*m*x^7/c-1/2/c^2*ln(c*x^2+b*x+a)*b*f-1/2/c^4*ln(c*x^2+b*x+a)*a^3*l+1/2/c
^3*ln(c*x^2+b*x+a)*a^2*j-1/2/c^2*ln(c*x^2+b*x+a)*a*g-1/2/c^8*ln(c*x^2+b*x+a
)*b^7*m+1/2/c^7*ln(c*x^2+b*x+a)*b^6*l-1/6/c^2*x^6*b*m+1/c^7*b^6*m*x-1/4/c^2
*x^4*a*l-1/4/c^4*x^4*b^3*m+1/5/c^3*x^5*b^2*m-1/5/c^2*x^5*b*l-1/5/c^2*x^5*a*
m+1/c^5*b^4*k*x-1/c^4*b^3*j*x+1/c^3*b^2*h*x-1/c^2*b*g*x+1/2/c^5*x^2*b^4*l-1
/2/c^4*x^2*b^3*k-1/2/c^2*x^2*a*j-1/2/c^6*x^2*b^5*m-1/3/c^2*x^3*a*k+1/3/c^5*
x^3*b^4*m-1/3/c^4*x^3*b^3*l+1/3/c^3*x^3*b^2*k-1/3/c^2*x^3*b*j+1/2/c^3*x^2*a
^2*l-1/4/c^2*x^4*b*k+1/3/c^3*x^3*a^2*m+1/4/c^3*x^4*b^2*l+1/2/c^3*x^2*b^2*j-
1/2/c^2*x^2*b*h-1/c^4*a^3*m*x+1/c^3*a^2*k*x-1/c^2*a*h*x-1/c^6*b^5*l*x-1/2/c
^6*ln(c*x^2+b*x+a)*b^5*k+1/2/c^5*ln(c*x^2+b*x+a)*b^4*j-1/2/c^4*ln(c*x^2+b*x
+a)*b^3*h+1/2/c^3*ln(c*x^2+b*x+a)*b^2*g+20/c^6/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^4*m-14/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))*a^2*b^3*l+9/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*a^2*b^2*k-16/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*a^3*b^2*m+7/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*a^3*b*l-5/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*a^2*b*j+7/c^6/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^5*
l-6/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*k+2/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*d+1/2/c*ln(c*x^2+b*x+a)*
e+1/c*f*x+1/2/c*x^2*g+1/3/c*x^3*h+1/6/c*x^6*l+1/5/c*x^5*k+1/4/c*x^4*j+3/c^5
*ln(c*x^2+b*x+a)*a^2*b^2*l-3/2/c^4*ln(c*x^2+b*x+a)*a^2*b*k+3/c^7*ln(c*x^2+b
*x+a)*a*b^5*m-5/2/c^6*ln(c*x^2+b*x+a)*a*b^4*l+2/c^5*ln(c*x^2+b*x+a)*a*b^3*k
-1/c^7/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^7*l+2/c^5*ln
(c*x^2+b*x+a)*a^3*b*m-5/c^6*ln(c*x^2+b*x+a)*a^2*b^3*m-3/2/c^4*ln(c*x^2+b*x+
a)*a*b^2*j+1/c^3*ln(c*x^2+b*x+a)*a*b*h+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*
x+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*e+1/c^8/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*b^8*m-3/c^4*a*b^2*k*x+2/c^3*a*b*j*x+6/c^5*a^2*b^2*m*x-3/c^4*a^2*b*l*x+2/c
^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^4*m-2/c/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*f+1/c^6/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^6*k-1/c^5/(4*a*c-b^2)^(1/2)*arctan((2*
```

```
c*x+b)/(4*a*c-b^2)^(1/2))*b^5*j+1/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*b^4*h-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*b^3*g+1/c^3*x^2*a*b*k-5/c^6*a*b^4*m*x+4/c^5*a*b^3*l*x-3/2/c^4*x^2*a
^2*b*m+2/c^5*x^2*a*b^3*m-3/2/c^4*x^2*a*b^2*l-1/c^4*x^3*a*b^2*m+2/3/c^3*x^3*
a*b*l+1/2/c^3*x^4*a*b*m+5/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2
)^(1/2))*a*b^3*j-4/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*a*b^2*h+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b*g
-8/c^7/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a*b^6*m-2/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^3*k+2/c^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*a^2*h
```

**maxima [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x^8+l*x^7+k*x^6+j*x^5+h*x^4+g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a),x
, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

**mupad [B]** time = 7.26, size = 2779, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*x^2 + g*x^3 + h*x^4 + j*x^5 + k*x^6 + l*x^7 + m*x^8)/(a +
b*x + c*x^2),x)
```

```
[Out] x^6*(l/(6*c) - (b*m)/(6*c^2)) + x*(f/c + (b*((a*(j/c - (a*(l/c - (b*m)/c^2)
)/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c - g/c + (b*(h/c
 - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c +
(a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/c))
/c - (a*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2)
)/c - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c
^2))/c))/c) + x^4*(j/(4*c) - (a*(l/c - (b*m)/c^2))/(4*c) + (b*((b*(l/c - (b
*m)/c^2))/c - k/c + (a*m)/c^2))/(4*c)) - x^2*((a*(j/c - (a*(l/c - (b*m)/c^2
))/c + (b*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(2*c) - g/(2*c)
+ (b*(h/c - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l/c - (b*m)/c^2))/c
 - k/c + (a*m)/c^2))/c))/c + (a*((b*(l/c - (b*m)/c^2))/c - k/c + (a*m)/c^2)
)/c))/(2*c)) + x^3*(h/(3*c) - (b*(j/c - (a*(l/c - (b*m)/c^2))/c + (b*((b*(l
/c - (b*m)/c^2))/c - k/c + (a*m)/c^2))/c))/(3*c) + (a*((b*(l/c - (b*m)/c^2)
```

)/c - k/c + (a*m)/c^2))/(3*c)) - x^5*((b*(l/c - (b*m)/c^2))/(5*c) - k/(5*c) + (a*m)/(5*c^2)) + (log((2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - b^8*m - 2*c^8*d - b^2*c^6*f - 2*a^2*c^6*h + b^3*c^5*g - b^4*c^4*h + 2*a^3*c^5*k + b^5*c^3*j - b^6*c^2*k - 2*a^4*c^4*m + 2*a*c^7*f + b*c^7*e + b^7*c*l + b*c^8*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) - 9*a^2*b^2*c^4*k + 14*a^2*b^3*c^3*l - 20*a^2*b^4*c^2*m + 16*a^3*b^2*c^3*m - 3*a*b*c^6*g + 8*a*b^6*c*m + 4*a*b^2*c^5*h - 5*a*b^3*c^4*j + 5*a^2*b*c^5*j + 6*a*b^4*c^3*k - 7*a*b^5*c^2*l - 7*a^3*b*c^4*l)*(2*c^8*d + b^8*m + 2*c^9*x*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + b*c^8*(-(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l)^2/(c^16*(4*a*c - b^2)))^(1/2) + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l))*(b^9*m - b^2*c^7*e - 4*a^2*c^7*g + b^3*c^6*f - b^4*c^5*g + b^5*c^4*h + 4*a^3*c^6*j - b^6*c^3*j - 4*a^4*c^5*l + b^7*c^2*k + 4*a*c^8*e - b^8*c*l - 13*a^2*b^2*c^5*j + 19*a^2*b^3*c^4*k - 26*a^2*b^4*c^3*l + 25*a^3*b^2*c^4*l + 34*a^2*b^5*c^2*m - 44*a^3*b^3*c^3*m - 4*a*b*c^7*f - 10*a*b^7*c*m + 5*a*b^2*c^6*g - 6*a*b^3*c^5*h + 8*a^2*b*c^6*h + 7*a*b^4*c^4*j - 8*a*b^5*c^3*k - 12*a^3*b*c^5*k + 9*a*b^6*c^2*l + 16*a^4*b*c^4*m))/(2*(4*a*c^9 - b^2*c^8)) + (m*x^7)/(7*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(2*c^8*d + b^8*m + b^2*c^6*f + 2*a^2*c^6*h - b^3*c^5*g + b^4*c^4*h - 2*a^3*c^5*k - b^5*c^3*j + b^6*c^2*k + 2*a^4*c^4*m - 2*a*c^7*f - b*c^7*e - b^7*c*l + 9*a^2*b^2*c^4*k - 14*a^2*b^3*c^3*l + 20*a^2*b^4*c^2*m - 16*a^3*b^2*c^3*m + 3*a*b*c^6*g - 8*a*b^6*c*m - 4*a*b^2*c^5*h + 5*a*b^3*c^4*j - 5*a^2*b*c^5*j - 6*a*b^4*c^3*k + 7*a*b^5*c^2*l + 7*a^3*b*c^4*l))/(c^8*(4*a*c - b^2)^(1/2))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((m*x**8+l*x**7+k*x**6+j*x**5+h*x**4+g*x**3+f*x**2+e*x+d)/(c*x**2+
b*x+a),x)
```

```
[Out] Timed out
```

# 3.374 $\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx$

**Optimal**. Leaf size=208

$$\frac{98060877 \left(5x^2+2x+3\right)^{3/2} x^2}{4375000}+\frac{1045360143 \left(5x^2+2x+3\right)^{3/2} x}{43750000}-\frac{1968340667 \left(5x^2+2x+3\right)^{3/2}}{131250000}-\frac{77159983(5x}{3}$$

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+
3)^(3/2)+98060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x
^2+2*x+3)^(3/2)-888751/105000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^
2+2*x+3)^(3/2)-50519/2250*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-77159983/31
250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.35, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{50}\left(5x^2+2x+3\right)^{3/2} x^7-\frac{50519 \left(5x^2+2x+3\right)^{3/2} x^6}{2250}+\frac{190939 \left(5x^2+2x+3\right)^{3/2} x^5}{3000}-\frac{888751 \left(5x^2+2x+3\right)^{3/2}}{105000}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-77159983*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/31250000 - (1968340667*(3 + 2*x
 + 5*x^2)^(3/2))/131250000 + (1045360143*x*(3 + 2*x + 5*x^2)^(3/2))/4375000
0 + (98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/4375000 - (90960857*x^3*(3 + 2*x
 + 5*x^2)^(3/2))/1575000 - (888751*x^4*(3 + 2*x + 5*x^2)^(3/2))/105000 + (1
90939*x^5*(3 + 2*x + 5*x^2)^(3/2))/3000 - (50519*x^6*(3 + 2*x + 5*x^2)^(3/2
))/2250 - (343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 - (540119881*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(15625000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} + \frac{1}{50}\int \sqrt{3+2x+5x^2}\,\left(100+\right.$$

$$= -\frac{50519x^6\left(3+2x+5x^2\right)^{3/2}}{2250} - \frac{343}{50}x^7\left(3+2x+5x^2\right)^{3/2} +$$

$$= \frac{190939x^5\left(3+2x+5x^2\right)^{3/2}}{3000} - \frac{50519x^6\left(3+2x+5x^2\right)^{3/2}}{2250}$$

$$= -\frac{888751x^4\left(3+2x+5x^2\right)^{3/2}}{105000} + \frac{190939x^5\left(3+2x+5x^2\right)}{3000}$$

$$= -\frac{90960857x^3\left(3+2x+5x^2\right)^{3/2}}{1575000} - \frac{888751x^4\left(3+2x+5x\right.}{105000}$$

$$= \frac{98060877x^2\left(3+2x+5x^2\right)^{3/2}}{4375000} - \frac{90960857x^3\left(3+2x+5\right.}{1575000}$$

$$= \frac{1045360143x\left(3+2x+5x^2\right)^{3/2}}{43750000} + \frac{98060877x^2\left(3+2x+\right.}{4375000}$$

$$= -\frac{1968340667\left(3+2x+5x^2\right)^{3/2}}{131250000} + \frac{1045360143x\left(3+2x\right.}{43750000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667\left(3+2x\right.}{131250000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667\left(3+2x\right.}{131250000}$$

$$= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667\left(3+2x\right.}{131250000}$$

**Mathematica [A]** time = 0.31, size = 85, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3}\,\left(67528125000x^9+248031875000x^8-497593468750x^7-34674656250x^6+225922362500\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(93436408944 - 57768004650*x - 78839046795*x^2 +
17642392275*x^3 + 56757413000*x^4 + 225922362500*x^5 - 34674656250*x^6 - 49
7593468750*x^7 + 248031875000*x^8 + 67528125000*x^9) - 68055105006*Sqrt[5]*
ArcSinh[(1 + 5*x)/Sqrt[14]])/9843750000
```

**fricas** [A] time = 0.86, size = 97, normalized size = 0.47

$$-\frac{1}{1968750000}\left(67528125000\,x^{9} + 248031875000\,x^{8} - 497593468750\,x^{7} - 34674656250\,x^{6} + 225922362500\,x^{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 3467
4656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 788390
46795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/
156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

**giac** [A] time = 0.21, size = 92, normalized size = 0.44

$$-\frac{1}{1968750000}\,(5\,((5\,(10\,(25\,(5\,(49\,(140\,(315\,x + 1157)x - 324959)x - 1109589)x + 36147578)x + 227029652)x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 110
9589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 1155
3600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqrt(5
)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple** [A] time = 0.03, size = 166, normalized size = 0.80

$$-\frac{343\left(5x^{2}+2x+3\right)^{\frac{3}{2}}x^{7}}{50}-\frac{50519\left(5x^{2}+2x+3\right)^{\frac{3}{2}}x^{6}}{2250}+\frac{190939\left(5x^{2}+2x+3\right)^{\frac{3}{2}}x^{5}}{3000}-\frac{888751\left(5x^{2}+2x+3\right)^{\frac{3}{2}}x^{4}}{105000}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -1968340667/131250000*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^(3/2)-50
519/2250*x^6*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^2+2*x+3)^(3/2)-888751
```

```
/105000*x^4*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x^2+2*x+3)^(3/2)+98
060877/4375000*x^2*(5*x^2+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+3)^
(3/2)-540119881/78125000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-77159983/62
500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)
```

**maxima [A]** time = 0.99, size = 177, normalized size = 0.85

$$-\frac{343}{50}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^7-\frac{50519}{2250}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^6+\frac{190939}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^5-\frac{888751}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x^
6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x +
3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4375
000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)^(3/
2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000*sqrt
(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x
 + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 6.31, size = 221, normalized size = 1.06

$$\frac{98060877\,x^2\left(5\,x^2+2\,x+3\right)^{3/2}}{4375000}-\frac{90960857\,x^3\left(5\,x^2+2\,x+3\right)^{3/2}}{1575000}-\frac{888751\,x^4\left(5\,x^2+2\,x+3\right)^{3/2}}{105000}+\frac{190939\,x^5\left(5\,x\right.}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] (98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^2
 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (19093
9*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2))/2
250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x/2 +
1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 + 3)^(1/
2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 + 3)^(3/
2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)
*(10*x + 2))/5))/156250000
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-29x\sqrt{5x^2+2x+3}\right)dx-\int\left(-115x^2\sqrt{5x^2+2x+3}\right)dx-\int 61x^3\sqrt{5x^2+2x+3}\,dx-\int 871x^4\sqrt{5x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5*sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7*sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8*sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

## 3.375 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx$

**Optimal**. Leaf size=166

$$-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{1781669\left(5x^2+2x+3\right)^{3/2}x}{250000}+\frac{198439\left(5x^2+2x+3\right)^{3/2}}{750000}-\frac{2521723(5x+1)\sqrt{5x^2+}}{1250000}$$

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-7750
9/25000*x^2*(5*x^2+2*x+3)^(3/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/200*
x^4*(5*x^2+2*x+3)^(3/2)+49/40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*arcs
inh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2521723/1250000*(1+5*x)*(5*x^2+2*x+3)^(1
/2)
```

**Rubi [A]** time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{3/2}x^4-\frac{25277\left(5x^2+2x+3\right)^{3/2}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{3/2}x^2}{25000}+\frac{178}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-2521723*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/1250000 + (198439*(3 + 2*x + 5*x
^2)^(3/2))/750000 + (1781669*x*(3 + 2*x + 5*x^2)^(3/2))/250000 - (77509*x^2
*(3 + 2*x + 5*x^2)^(3/2))/25000 - (25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/3000
+ (989*x^4*(3 + 2*x + 5*x^2)^(3/2))/200 + (49*x^5*(3 + 2*x + 5*x^2)^(3/2))/
40 - (17652061*ArcSinh[(1 + 5*x)/Sqrt[14]])/(625000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = \frac{49}{40}x^5 \left(3+2x+5x^2\right)^{3/2} + \frac{1}{40} \int \sqrt{3+2x+5x^2} \left(80+840\right.$$

$$= \frac{989}{200}x^4 \left(3+2x+5x^2\right)^{3/2} + \frac{49}{40}x^5 \left(3+2x+5x^2\right)^{3/2} + \frac{\int \sqrt{3}}{}$$

$$= -\frac{25277x^3 \left(3+2x+5x^2\right)^{3/2}}{3000} + \frac{989}{200}x^4 \left(3+2x+5x^2\right)^{3/2} +$$

$$= -\frac{77509x^2 \left(3+2x+5x^2\right)^{3/2}}{25000} - \frac{25277x^3 \left(3+2x+5x^2\right)^{3/2}}{3000}$$

$$= \frac{1781669x \left(3+2x+5x^2\right)^{3/2}}{250000} - \frac{77509x^2 \left(3+2x+5x^2\right)^{3/2}}{25000}$$

$$= \frac{198439 \left(3+2x+5x^2\right)^{3/2}}{750000} + \frac{1781669x \left(3+2x+5x^2\right)^{3/2}}{250000} -$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x\right.}{750000}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x\right.}{750000}$$

$$= -\frac{2521723(1+5x)\sqrt{3+2x+5x^2}}{1250000} + \frac{198439 \left(3+2x+5x\right.}{750000}$$

**Mathematica [A]** time = 0.19, size = 75, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3} \left(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x
^3 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7) - 1059123
66*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

**fricas [A]** time = 0.89, size = 87, normalized size = 0.52

$$\frac{1}{3750000} \left(22968750\, x^7 + 101906250\, x^6 - 107112500\, x^5 - 65693000\, x^4 + 15583725\, x^3 + 23531995\, x^2 + 443336\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 15
583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 1
7652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^
2 - 10*x - 8)
```

**giac [A]** time = 0.20, size = 82, normalized size = 0.49

$$\frac{1}{3750000}(5((5(10(25(15(245x+1087)x-17138)x-262772)x+623349)x+4706399)x+8866730)x-45885$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 6233
49)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 17652061
/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple [A]** time = 0.01, size = 132, normalized size = 0.80

$$\frac{49\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5}{40}+\frac{989\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4}{200}-\frac{25277\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3}{3000}-\frac{77509\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2}{25000}+\frac{1781669}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] 198439/750000*(5*x^2+2*x+3)^(3/2)+49/40*(5*x^2+2*x+3)^(3/2)*x^5+989/200*(5*
x^2+2*x+3)^(3/2)*x^4-25277/3000*(5*x^2+2*x+3)^(3/2)*x^3-77509/25000*(5*x^2+
2*x+3)^(3/2)*x^2+1781669/250000*(5*x^2+2*x+3)^(3/2)*x-17652061/3125000*5^(1
/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-2521723/2500000*(10*x+2)*(5*x^2+2*x+3)^(
1/2)
```

**maxima [A]** time = 0.97, size = 143, normalized size = 0.86

$$\frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5+\frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{178166}{25000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 - 2
5277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/2)
*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 + 2*
x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/3125000*sq
rt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 + 2*x +
 3)
```

**mupad [B]** time = 6.01, size = 187, normalized size = 1.13

$$\frac{989\,x^4\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{200}-\frac{25277\,x^3\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{3000}-\frac{77509\,x^2\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{25000}+\frac{49\,x^5\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{40}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] (989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2))
/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2 +
3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*
x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/250000
 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 + (1781
669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x + 5*x^2
 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+5x+2\right)\sqrt{5x^2+2x+3}\,\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2, x)
```

# 3.376 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$

**Optimal**. Leaf size=124

$$-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x+1)\sqrt{5x^2+2x+3}}{12500}-\frac{7}{30}($$

```
[Out] 7819/7500*(5*x^2+2*x+3)^(3/2)+2149/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(
5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*(1
+5*x)*14^(1/2))*5^(1/2)-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{3/2}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{3/2}x^2+\frac{2149\left(5x^2+2x+3\right)^{3/2}x}{2500}+\frac{7819\left(5x^2+2x+3\right)^{3/2}}{7500}-\frac{4633(5x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-4633*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/12500 + (7819*(3 + 2*x + 5*x^2)^(3/
2))/7500 + (2149*x*(3 + 2*x + 5*x^2)^(3/2))/2500 - (289*x^2*(3 + 2*x + 5*x^
2)^(3/2))/250 - (7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 - (32431*ArcSinh[(1 + 5*
x)/Sqrt[14]])/(6250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{30}\int \sqrt{3+2x+5x^2}\,\left(60+390\right.$$

$$= -\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3\left(3+2x+5x^2\right)^{3/2}+\frac{1}{750}$$

$$= \frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}x^2\left(3+2x+5x^2\right)^{3/2}-\frac{7}{30}x^3$$

$$= \frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+\frac{2149x\left(3+2x+5x^2\right)^{3/2}}{2500}-\frac{289}{250}x^2$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+$$

$$= -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819\left(3+2x+5x^2\right)^{3/2}}{7500}+$$

**Mathematica** [A] time = 0.10, size = 65, normalized size = 0.52

$$\frac{5\sqrt{5x^2+2x+3}\left(-43750x^5-234250x^4+48225x^3+129895x^2+105400x+103386\right)-194586\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{187500}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 2342
50*x^4 - 43750*x^5) - 194586*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/187500
```

**fricas** [A] time = 0.87, size = 77, normalized size = 0.62

$$-\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{62500}\,\sqrt{5}\,\log\left(\sqrt{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*
x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

**giac** [A] time = 0.19, size = 72, normalized size = 0.58

$$-\frac{1}{37500}\,(5\,((5\,(10\,(175\,x+937)x-1929)x-25979)x-21080)x-103386)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{31250}\,\sqrt{5}\,\log\left(-\sqrt{5}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 10338
6)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sq
rt(5*x^2 + 2*x + 3)) - 1)
```

**maple** [A] time = 0.01, size = 98, normalized size = 0.79

$$-\frac{7\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3}{30}-\frac{289\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2}{250}+\frac{2149\left(5x^2+2x+3\right)^{\frac{3}{2}}x}{2500}-\frac{32431\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}+\frac{7819\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/30*(5*x^2+2*x+3)^(3/2)*x^3-289/250*(5*x^2+2*x+3)^(3/2)*x^2+2149/2500*(5*
x^2+2*x+3)^(3/2)*x+7819/7500*(5*x^2+2*x+3)^(3/2)-4633/25000*(10*x+2)*(5*x^2
+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

**maxima [A]** time = 0.96, size = 109, normalized size = 0.88

$$-\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 2
149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) - 46
33/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(14)
*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

**mupad [B]** time = 5.38, size = 153, normalized size = 1.23

$$\frac{7819\sqrt{5x^2+2x+3}\,\left(200x^2+20x+108\right)}{300000}-\frac{7x^3\left(5x^2+2x+3\right)^{3/2}}{30}-\frac{10129\sqrt{5}\,\ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}\,(5x+1}{5}\right.}{62500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] (7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
+ 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1
/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2500
 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3/2))
/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))
/5))/62500
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-13x\sqrt{5x^2+2x+3}\right)dx-\int\left(-7x^2\sqrt{5x^2+2x+3}\right)dx-\int 31x^3\sqrt{5x^2+2x+3}\,dx-\int 7x^4\sqrt{5x^2+2x+3}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2*sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x**
*4*sqrt(5*x**2 + 2*x + 3), x) - Integral(-2*sqrt(5*x**2 + 2*x + 3), x)
```

## 3.377 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2} \, dx$

**Optimal**. Leaf size=187

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)$$

```
[Out] -8233/8575*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1/490*(397+35*x)*(5*x^2+2
*x+3)^(1/2)-3/3773*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1
/2)/(250-34*11^(1/2))^(1/2))*(5467451-1612105*11^(1/2))^(1/2)+3/3773*arctan
h((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/
2))*(5467451+1612105*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.35, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] -((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 - (8233*ArcSinh[(1 + 5*x)/Sqrt[14
]])/(1715*Sqrt[5]) - (3*Sqrt[(497041 - 146555*Sqrt[11])/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/343 + (3*Sqrt[(497041 + 146555*Sqrt[11])/11]*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2
])])/343
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
```

```
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{1}{490}\int \frac{-3442-13408x-16466x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} + \frac{\int \frac{40560+159720x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{3430} - \frac{8233\int}{171}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right)}{3430\sqrt{70}} +$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{(24(14641-50}{}$$

$$= -\frac{1}{490}(397+35x)\sqrt{3+2x+5x^2} - \frac{8233\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}} - \frac{3\sqrt{5467451-16}}{}$$

**Mathematica [A]** time = 0.89, size = 189, normalized size = 1.01

$$\frac{-385\sqrt{5x^2+2x+3}(35x+397) - 75\sqrt{250-34\sqrt{11}}\left(61\sqrt{11}-143\right)\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) + 75\sqrt{250-}}{188650}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-385*(397 + 35*x)*Sqrt[3 + 2*x + 5*x^2] - 181126*Sqrt[5]*ArcSinh[(1 + 5*x)
/Sqrt[14]] - 75*Sqrt[250 - 34*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 -
Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x
^2])] + 75*Sqrt[250 + 34*Sqrt[11]]*(143 + 61*Sqrt[11])*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])
/188650
```

**fricas [B]** time = 0.91, size = 304, normalized size = 1.63

$$\frac{3}{7546}\sqrt{11}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{6\left(\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}\left(87\sqrt{11}-265\right)+6517\sqrt{1\ldots}\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log(6*(sqrt(5*x^2 + 2*x + 3)
*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 265) + 6517*sqrt(11)*(x + 3)
 + 19551*x - 32585)/x) - 3/7546*sqrt(11)*sqrt(146555*sqrt(11) + 497041)*log
(-6*(sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(87*sqrt(11) - 26
5) - 6517*sqrt(11)*(x + 3) - 19551*x + 32585)/x) - 1/15092*sqrt(11)*sqrt(-5
275980*sqrt(11) + 17893476)*log(-(sqrt(5*x^2 + 2*x + 3)*(87*sqrt(11) + 265)
*sqrt(-5275980*sqrt(11) + 17893476) + 39102*sqrt(11)*(x + 3) - 117306*x + 1
95510)/x) + 1/15092*sqrt(11)*sqrt(-5275980*sqrt(11) + 17893476)*log((sqrt(5
*x^2 + 2*x + 3)*(87*sqrt(11) + 265)*sqrt(-5275980*sqrt(11) + 17893476) - 39
102*sqrt(11)*(x + 3) + 117306*x - 195510)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*
(35*x + 397) + 8233/17150*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x +
1) - 25*x^2 - 10*x - 8)
```

**giac [A]** time = 0.27, size = 144, normalized size = 0.77

$$-\frac{1}{490}\sqrt{5x^2+2x+3}\,(35x+397)+\frac{8233}{8575}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right)+2.61475869687464\log\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(5
)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*x
+ sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.09, size = 403, normalized size = 2.16

$$-\frac{\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{25}-\frac{(10x+2)\sqrt{5x^2+2x+3}}{140}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\sqrt{5}\ \operatorname{arcsinh}\left(\frac{\sqrt{5}\left(x+\right.}{\sqrt{\frac{250}{49}-\frac{34\sqrt{11}}{49}-\left(\right.}}\right.}{70}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/140*(10*x+2)*(5*x^2+2*x+3)^(1/2)-1/25*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1
/5))-3/154*(-61+13*11^(1/2))*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7
-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-1
0/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*
11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-3/154*11^(1/2)*(61+
13*11^(1/2))*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*a
rcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x
+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2)
)^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(
1/2))+250+34*11^(1/2))^(1/2)))
```

**maxima [B]** time = 1.17, size = 500, normalized size = 2.67

$$\frac{1}{188650}\sqrt{11}\left(975\sqrt{11}\sqrt{2}\sqrt{17\sqrt{11}+125}\ \operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}\,x}{7\left|14\,x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}\,x}{7\left|14\,x-2\sqrt{11}-4\right|}+\frac{\sqrt{11}\sqrt{7}\sqrt{2}}{7\left|14\,x-2\sqrt{11}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7
*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(
2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2
```

```
*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 1225*sq
rt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7
)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(11) +
250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) -
 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(
11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt(11)*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x -
 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)
+ 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqrt(-34/49*sqrt
(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*sq
rt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 + 2
*x + 3)/(7*x**2 - 4*x - 1), x)
```

# 3.378 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

**Optimal**. Leaf size=199

$$\frac{3\sqrt{5x^2+2x+3}\,(61x+3)}{154\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\ \text{ta}}{}$$

```
[Out] 1/49*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+3/154*(3+61*x)*(5*x^2+2*x+3)^(1
/2)/(-7*x^2+4*x+1)+1/3011932*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2
+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(454056168467-54668425207*11^(1/2))^
(1/2)-1/3011932*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)
/(250-34*11^(1/2))^(1/2))*(454056168467+54668425207*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.25, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3\sqrt{5x^2+2x+3}\,(61x+3)}{154\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{2156}+\frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\ \text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + (Sqrt[5]*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/49 - (Sqrt[(325022311 + 39132731*Sqrt[11])/1397]*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/2156 + (Sqrt[(325022311 - 39132731*Sqrt[11])/1397]
*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/2156
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
```

```
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)} - \frac{1}{308}\int \frac{-948-188x+220x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)} + \frac{\int \frac{6416+436x}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx}{2156} + \frac{5}{49}\int \frac{1}{\sqrt{3+2x+}}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)} + \frac{1}{98}\sqrt{\frac{5}{14}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx, x, 2+10x\right) +$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{(2(1199-11446\sqrt{1}}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\,\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{325022311+39132731}{1397}}}{}$$

**Mathematica [A]** time = 1.28, size = 354, normalized size = 1.78

$$\frac{\frac{56364\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{2772\sqrt{5x^2+2x+3}}{-7x^2+4x+1} + 22892\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3} + (55+17\sqrt{11})x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((2772*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (56364*x*Sqrt[3 + 2*x + 5
*x^2])/(1 + 4*x - 7*x^2) + 968*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 2*Sqrt
[2/(125 - 17*Sqrt[11])]*(-1199 + 11446*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] - 17*x + 5*Sqrt[11]*x)] - 239
8*Sqrt[2/(125 + 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] - 22892*Sqrt[22/(125
+ 17*Sqrt[11])]*Log[2 + Sqrt[11] - 7*x] + 2398*Sqrt[2/(125 + 17*Sqrt[11])]*
Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqr
t[3 + 2*x + 5*x^2]] + 22892*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[1
```

```
1] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]
])/47432
```

**fricas [B]** time = 0.99, size = 378, normalized size = 1.90

$$-\frac{\sqrt{1397}\left(7\,x^2-4\,x-1\right)\sqrt{39132731\,\sqrt{11}+325022311}\,\log\left(-\frac{\sqrt{1397}\,\sqrt{5\,x^2+2\,x+3}\,\sqrt{39132731\,\sqrt{11}+325022311}\left(16943\,\sqrt{11}+2\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] -1/6023864*(sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) + 325022311
)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 325022311
)*(16943*sqrt(11) + 235367) + 26119953475*sqrt(11)*(x + 3) - 78359860425*x
+ 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(39132731*sqrt(11) +
325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(39132731*sqrt(11) + 3
25022311)*(16943*sqrt(11) + 235367) - 26119953475*sqrt(11)*(x + 3) + 783598
60425*x - 130599767375)/x) + sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-39132731*sq
rt(11) + 325022311)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) -
 235367)*sqrt(-39132731*sqrt(11) + 325022311) + 26119953475*sqrt(11)*(x + 3
) + 78359860425*x - 130599767375)/x) - sqrt(1397)*(7*x^2 - 4*x - 1)*sqrt(-3
9132731*sqrt(11) + 325022311)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(16943
*sqrt(11) - 235367)*sqrt(-39132731*sqrt(11) + 325022311) - 26119953475*sqrt
(11)*(x + 3) - 78359860425*x + 130599767375)/x) - 61468*sqrt(5)*(7*x^2 - 4*
x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) +
117348*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}+%%%{-2484
746880,[6]%%%}+%%%{%%{[-5059848192,0]:[1,0,-5]%%},[5]%%%}+%%%{18003120576,[
4]%%%}+%%%{%%{[13432692224,0]:[1,0,-5]%%},[3]%%%}+%%%{-38927701120,[2]%%%}+
%%%{%%{[-9999223808,0]:[1,0,-5]%%},[1]%%%}+%%%{25935486752,[0]%%%} / %%%{24
5,[8]%%%}+%%%{%%{poly1[560,0]:[1,0,-5]%%},[7]%%%}+%%%{-3300,[6]%%%}+%%%{%%{
```

```
poly1[-6720,0]:[1,0,-5]%%},[5]%%%}+%%%{23910,[4]%%%}+%%%{%%{poly1[17840,0]:
[1,0,-5]%%},[3]%%%}+%%%{-51700,[2]%%%}+%%%{%%{poly1[-13280,0]:[1,0,-5]%%},[
1]%%%}+%%%{34445,[0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.03, size = 1084, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] (183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))
*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49
+34/49*11^(1/2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(
250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49
+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(
x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*1
1^(1/2))^(1/2)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arc
sinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1
/5))))+161/484*11^(1/2)*(1/49*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/70*(34/7-10/7*11^(1/2)
)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^
2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(4
9/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-
34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+(183/44-39/44*11^(1/2))*(-1/49/(2
50/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-
10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98*(34/7
-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49
*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/
7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-
10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2)
)^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10
/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250/49-34
/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))
*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11
^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/
2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-161/484*11^(1/2)*(1/49*(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
```

```
4*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49
+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49+34/49*1
1^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/
7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))
^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x^2+2\,x+3}\,\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\sqrt{5\,x^2+2\,x+3}}{\left(-7\,x^2+4\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+5x+2\right)\sqrt{5x^2+2x+3}}{\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

```
[Out] Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

# 3.379 $\int \frac{\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}}{\left(1+4x-7x^2\right)^3}\,dx$

**Optimal**. Leaf size=213

$$-\frac{\sqrt{5x^2+2x+3}\,(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5}{\sqrt{2(125-}}\right.}{491744}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-1/1721104*(272941-81311
3*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-1/686966368*arctanh((23+x*(17-5*11^
(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(906967747026
5753-16595199192187*11^(1/2))^(1/2)+1/686966368*arctanh((23+11^(1/2)+x*(17+
5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(9069677470265753
+16595199192187*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1054, 1060, 1032, 724, 206}

$$-\frac{\sqrt{5x^2+2x+3}\,(272941-813113x)}{1721104\left(-7x^2+4x+1\right)}+\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308\left(-7x^2+4x+1\right)^2}-\frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5}{\sqrt{2(125-}}\right.}{491744}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] (3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) - ((272941 -
 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1721104*(1 + 4*x - 7*x^2)) - (Sqrt[(6492
253020949 - 11879169071*Sqrt[11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sq
rt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/491744 + (
Sqrt[(6492253020949 + 11879169071*Sqrt[11])/1397]*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/
491744
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
```

```
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{1}{616}\int \frac{-3012-1564x-3220x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{\int \frac{4758}{(1+4x}}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{(13919}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} + \frac{(-1391}{}$$

$$= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} - \frac{\sqrt{\frac{64922}{}}}{}$$

**Mathematica [A]** time = 1.40, size = 334, normalized size = 1.57

$$\frac{-\sqrt{\frac{22}{125-17\sqrt{11}}}\left(126542\sqrt{11}-1740003\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}+15\right)+2\sqrt{\frac{22}{125+17\sqrt{11}}}\left(1740003+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((-44*Sqrt[3 + 2*x + 5*x^2]*(31807 - 106279*x - 737577*x^2 + 813113*x^3))/(
1 + 4*x - 7*x^2)^2 - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-1740003 + 126542*Sqrt
[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[1
1] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126
542*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11])]*(-1740
003 + 126542*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(125 - 17*Sqr
t[11])]*(-1740003 + 126542*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11
])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(1740003 + 126542*Sqrt[11])
*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sq
rt[3 + 2*x + 5*x^2]])/10818368
```

**fricas [B]** time = 0.96, size = 390, normalized size = 1.83

$$-\frac{\sqrt{1397}\left(49\,x^4-56\,x^3+2\,x^2+8\,x+1\right)\sqrt{11879169071\,\sqrt{11}+6492253020949}\,\log\left(\frac{\sqrt{1397}\,\sqrt{5\,x^2+2\,x+3}\,\sqrt{1187916907}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1373932736*(sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169
071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11
879169071*sqrt(11) + 6492253020949)*(4822219*sqrt(11) - 37335441) + 5690716
98870455*sqrt(11)*(x + 3) + 1707215096611365*x - 2845358494352275)/x) - sqr
t(1397)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11879169071*sqrt(11) + 649
2253020949)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(11879169071*sqrt(11
) + 6492253020949)*(4822219*sqrt(11) - 37335441) - 569071698870455*sqrt(11)
*(x + 3) - 1707215096611365*x + 2845358494352275)/x) + sqrt(1397)*(49*x^4 -
 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log(
-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-1187
9169071*sqrt(11) + 6492253020949) + 569071698870455*sqrt(11)*(x + 3) - 1707
215096611365*x + 2845358494352275)/x) - sqrt(1397)*(49*x^4 - 56*x^3 + 2*x^2
 + 8*x + 1)*sqrt(-11879169071*sqrt(11) + 6492253020949)*log((sqrt(1397)*sqr
t(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*sqrt(-11879169071*sqrt(11)
 + 6492253020949) - 569071698870455*sqrt(11)*(x + 3) + 1707215096611365*x -
 2845358494352275)/x) + 5588*(813113*x^3 - 737577*x^2 - 106279*x + 31807)*s
qrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

**giac [B]** time = 0.26, size = 378, normalized size = 1.77

$$\frac{6200558\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7-835775\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^6-190947036\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x\right.}{430276\left(7\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] 1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81632
1374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)*x -
 sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) + 76511
1048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.13
9051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000
) - 0.139051039089329*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811
3000) + 0.138209741946100*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

**maple [B]** time = 0.03, size = 2342, normalized size = 11.00

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] -21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/
7*11^(1/2))^2*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250/49+34/49*11^(1/2))^(3/2)-1/1372*(34/7+10/7*11^(1/2))/(250/49+34/
49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(3/2)+1/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11
^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2)
)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1
/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
)+10/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(50
00/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/
49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(250/
49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1
/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/
```

2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*
(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*1
1^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+3535/21296*11^(1/2)*(1/49*(245*(x-2/
7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1
/2))^(1/2)+1/70*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-(250/49-34/49*11^(1/2)
)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
)-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+
1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2))+250/49-34/49*11^(1/2))^(3/2)-1/1372*(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1
/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1
/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/
7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1
/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/
2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*
11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^
(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2
)))+10/(250/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(
5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(25
0/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))+5/686/(25
0/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2
))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^
(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(
1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/
2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34
*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7
+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(-3535/1936+273/1936*11^(1/2))*(-1
/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/98
*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2)
)^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/1
0*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*
(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11
^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2
/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(3
4/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+10/49/(250
/49-34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680
/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*

```
11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-(-3535/1936-273/1936
*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(3/2)+1/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/
7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1
/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*
11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/
2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^
(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2
)))+10/49/(250/49+34/49*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+
(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/20
0*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/
(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))))-3535/2
1296*11^(1/2)*(1/49*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/70*(34/7+10/7*11^(1/2))*5^(1/2)*
arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(
x+1/5))-(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2
))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250+34*11^(1/2))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{5x^2+2x+3}\left(x^2+5x+2\right)}{\left(7x^2-4x-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5x+2\right)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{5x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x**
6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(x**
2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

## 3.380 $\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx$

**Optimal**. Leaf size=231

$$\frac{2173004363\left(5x^2+2x+3\right)^{5/2}x^2}{173250000}+\frac{837379699\left(5x^2+2x+3\right)^{5/2}x}{72187500}-\frac{6133820867\left(5x^2+2x+3\right)^{5/2}}{1203125000}-\frac{22840599(5x$$

```
[Out] -22840599/62500000*(1+5*x)*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^2
+2*x+3)^(5/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/173250000
*x^2*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-796559/1
23750*x^4*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3
300*x^6*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/78125
0000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-479652579/312500000*(1+5*x)*(5*
x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.36, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{343}{60}\left(5x^2+2x+3\right)^{5/2}x^7-\frac{61103\left(5x^2+2x+3\right)^{5/2}x^6}{3300}+\frac{1031177\left(5x^2+2x+3\right)^{5/2}x^5}{20625}-\frac{796559\left(5x^2+2x+3\right)^{5/2}}{123750}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-479652579*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/312500000 - (22840599*(1 + 5*x
)*(3 + 2*x + 5*x^2)^(3/2))/62500000 - (6133820867*(3 + 2*x + 5*x^2)^(5/2))/
1203125000 + (837379699*x*(3 + 2*x + 5*x^2)^(5/2))/72187500 + (2173004363*x
^2*(3 + 2*x + 5*x^2)^(5/2))/173250000 - (190236913*x^3*(3 + 2*x + 5*x^2)^(5
/2))/4950000 - (796559*x^4*(3 + 2*x + 5*x^2)^(5/2))/123750 + (1031177*x^5*(
3 + 2*x + 5*x^2)^(5/2))/20625 - (61103*x^6*(3 + 2*x + 5*x^2)^(5/2))/3300 -
(343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 - (3357568053*ArcSinh[(1 + 5*x)/Sqrt[1
4]])/(156250000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = -\frac{343}{60}x^7 \left(3+2x+5x^2\right)^{5/2} + \frac{1}{60}\int \left(3+2x+5x^2\right)^{3/2} \left(120\right.$$

$$= -\frac{61103x^6 \left(3+2x+5x^2\right)^{5/2}}{3300} - \frac{343}{60}x^7 \left(3+2x+5x^2\right)^{5/2} +$$

$$= \frac{1031177x^5 \left(3+2x+5x^2\right)^{5/2}}{20625} - \frac{61103x^6 \left(3+2x+5x^2\right)^{5/}}{3300}$$

$$= -\frac{796559x^4 \left(3+2x+5x^2\right)^{5/2}}{123750} + \frac{1031177x^5 \left(3+2x+5x\right.}{20625}$$

$$= -\frac{190236913x^3 \left(3+2x+5x^2\right)^{5/2}}{4950000} - \frac{796559x^4 \left(3+2x+\right.}{123750}$$

$$= \frac{2173004363x^2 \left(3+2x+5x^2\right)^{5/2}}{173250000} - \frac{190236913x^3 \left(3+2x\right.}{4950000}$$

$$= \frac{837379699x \left(3+2x+5x^2\right)^{5/2}}{72187500} + \frac{2173004363x^2 \left(3+2x\right.}{173250000}$$

$$= -\frac{6133820867 \left(3+2x+5x^2\right)^{5/2}}{1203125000} + \frac{837379699x \left(3+2x\right.}{72187500}$$

$$= -\frac{22840599(1+5x) \left(3+2x+5x^2\right)^{3/2}}{62500000} - \frac{6133820867 \left(3\right.}{120312}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x}{6250}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x}{6250}$$

$$= -\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} - \frac{22840599(1+5x}{6250}$$

**Mathematica [A]** time = 0.42, size = 95, normalized size = 0.41

$$\frac{-4653589321458\sqrt{5}\,\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) - 5\sqrt{5x^2+2x+3}\,\left(30950390625000x^{11} + 125007421875000x^{10} - 1483937\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(10506617068392 - 6352777129950*x - 1586584440868
5*x^2 - 19041688239675*x^3 - 2573089891000*x^4 + 85130334087500*x^5 + 52106
830406250*x^6 - 72918247281250*x^7 - 30505457500000*x^8 - 148393743750000*x
^9 + 125007421875000*x^10 + 30950390625000*x^11) - 4653589321458*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/1082812500000
```

**fricas [A]** time = 0.88, size = 107, normalized size = 0.46

$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x^{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 1483937437500
00*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 851
30334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 15865844408685*x
^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/1
562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

**giac [A]** time = 0.23, size = 102, normalized size = 0.44

$$-\frac{1}{216562500000}\,(5\,((5\,(10\,(25\,(5\,(7\,(20\,(105\,(875\,(77\,x+311)x-323034)x-6972676)x-333340559)x+1667$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034)
*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 102923595
64)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617068
392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*(sqr
t(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple [A]** time = 0.04, size = 185, normalized size = 0.80

$$-\frac{343\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{7}}{60}-\frac{61103\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{6}}{3300}+\frac{1031177\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{5}}{20625}-\frac{796559\left(5x^{2}+2x+3\right)^{\frac{5}{2}}x^{4}}{123750}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -6133820867/1203125000*(5*x^2+2*x+3)^(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-6
1103/3300*x^6*(5*x^2+2*x+3)^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-796
559/123750*x^4*(5*x^2+2*x+3)^(5/2)-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2
)+2173004363/173250000*x^2*(5*x^2+2*x+3)^(5/2)-3357568053/781250000*5^(1/2)
*arcsinh(5/14*14^(1/2)*(x+1/5))-479652579/625000000*(10*x+2)*(5*x^2+2*x+3)^
(1/2)+837379699/72187500*x*(5*x^2+2*x+3)^(5/2)-22840599/125000000*(10*x+2)*
(5*x^2+2*x+3)^(3/2)
```

**maxima [A]** time = 1.01, size = 206, normalized size = 0.89

$$-\frac{343}{60}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^7-\frac{61103}{3300}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^6+\frac{1031177}{20625}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^5-\frac{796559}{123750}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^4-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x^
6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*x
+ 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004363
/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*x +
3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/12500
000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(3/2) -
 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 + 2*x + 3
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x^2+5x+2\right)\left(5x^2+2x+3\right)^{3/2}\left(-7x^2+4x+1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-91x\sqrt{5x^2+2x+3}\right)dx-\int\left(-413x^2\sqrt{5x^2+2x+3}\right)dx-\int\left(-192x^3\sqrt{5x^2+2x+3}\right)dx-\int 2160x^4\sqrt{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-91*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-413*x**2*sqrt(5*x**2
 + 2*x + 3), x) - Integral(-192*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(
2160*x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(1666*x**5*sqrt(5*x**2 + 2*x
 + 3), x) - Integral(-2094*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-1384
*x**7*sqrt(5*x**2 + 2*x + 3), x) - Integral(-7042*x**8*sqrt(5*x**2 + 2*x +
3), x) - Integral(6321*x**9*sqrt(5*x**2 + 2*x + 3), x) - Integral(1715*x**1
0*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2 + 2*x + 3), x)
```

## 3.381 $\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx$

**Optimal**. Leaf size=189

$$-\frac{219271 \left(5x^2+2x+3\right)^{5/2} x^2}{105000}+\frac{86721 \left(5x^2+2x+3\right)^{5/2} x}{21875}+\frac{505667 \left(5x^2+2x+3\right)^{5/2}}{2187500}-\frac{690561(5x+1) \left(5x^2+2x\right.}{1250000}$$

```
[Out] -690561/1250000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5
/2)+86721/21875*x*(5*x^2+2*x+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)
-18379/3000*x^3*(5*x^2+2*x+3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/50*x
^5*(5*x^2+2*x+3)^(5/2)-101512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^
(1/2)-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.23, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1661, 640, 612, 619, 215}

$$\frac{49}{50} \left(5x^2+2x+3\right)^{5/2} x^5+\frac{581}{150} \left(5x^2+2x+3\right)^{5/2} x^4-\frac{18379 \left(5x^2+2x+3\right)^{5/2} x^3}{3000}-\frac{219271 \left(5x^2+2x+3\right)^{5/2} x^2}{105000}+\frac{867}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-14501781*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/6250000 - (690561*(1 + 5*x)*(3
+ 2*x + 5*x^2)^(3/2))/1250000 + (505667*(3 + 2*x + 5*x^2)^(5/2))/2187500 +
(86721*x*(3 + 2*x + 5*x^2)^(5/2))/21875 - (219271*x^2*(3 + 2*x + 5*x^2)^(5/
2))/105000 - (18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/3000 + (581*x^4*(3 + 2*x +
 5*x^2)^(5/2))/150 + (49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 - (101512467*ArcSi
nh[(1 + 5*x)/Sqrt[14]])/(3125000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx = \frac{49}{50}x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{1}{50} \int \left(3+2x+5x^2\right)^{3/2} \left(100+\right.$$

$$= \frac{581}{150}x^4 \left(3+2x+5x^2\right)^{5/2} + \frac{49}{50}x^5 \left(3+2x+5x^2\right)^{5/2} + \frac{\int \left(3\right.}{}$$

$$= -\frac{18379x^3 \left(3+2x+5x^2\right)^{5/2}}{3000} + \frac{581}{150}x^4 \left(3+2x+5x^2\right)^{5/2} +$$

$$= -\frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000} - \frac{18379x^3 \left(3+2x+5x^2\right)^{5}}{3000}$$

$$= \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} - \frac{219271x^2 \left(3+2x+5x^2\right)^{5/2}}{105000}$$

$$= \frac{505667 \left(3+2x+5x^2\right)^{5/2}}{2187500} + \frac{86721x \left(3+2x+5x^2\right)^{5/2}}{21875} -$$

$$= -\frac{690561(1+5x) \left(3+2x+5x^2\right)^{3/2}}{1250000} + \frac{505667 \left(3+2x+\right.}{2187500}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{125000}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{125000}$$

$$= -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x) \left(3\right.}{125000}$$

**Mathematica [A]** time = 0.25, size = 85, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3} \left(3215625000x^9 + 15281875000x^8 - 5561281250x^7 - 4105593750x^6 - 12554262500x^5 - 32275\right.}{6562500}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 5959
365525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250
*x^7 + 15281875000*x^8 + 3215625000*x^9) - 4263523614*Sqrt[5]*ArcSinh[(1 +
5*x)/Sqrt[14]])/656250000
```

**fricas** [A] time = 0.70, size = 97, normalized size = 0.51

$$\frac{1}{131250000}\left(3215625000\,x^9+15281875000\,x^8-5561281250\,x^7-4105593750\,x^6-12554262500\,x^5-3227597\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 4105593750
*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2 +
 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sqrt(
5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

**giac** [A] time = 0.22, size = 92, normalized size = 0.49

$$\frac{1}{131250000}\,(5\,((5\,(10\,(25\,(5\,(7\,(140\,(105\,x+499)x-25423)x-131379)x-2008682)x-12910388)x+2383746$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)*
x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x -
 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt(5)
*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple** [A] time = 0.01, size = 151, normalized size = 0.80

$$\frac{49\left(5x^2+2x+3\right)^{\frac{5}{2}}x^5}{50}+\frac{581\left(5x^2+2x+3\right)^{\frac{5}{2}}x^4}{150}-\frac{18379\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3}{3000}-\frac{219271\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2}{105000}+\frac{86721}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 505667/2187500*(5*x^2+2*x+3)^(5/2)+49/50*(5*x^2+2*x+3)^(5/2)*x^5+581/150*(5
*x^2+2*x+3)^(5/2)*x^4-18379/3000*(5*x^2+2*x+3)^(5/2)*x^3-219271/105000*(5*x
^2+2*x+3)^(5/2)*x^2-101512467/15625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5
))-14501781/12500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)+86721/21875*(5*x^2+2*x+3)
^(5/2)*x-690561/2500000*(10*x+2)*(5*x^2+2*x+3)^(3/2)
```

**maxima** [A] time = 0.98, size = 172, normalized size = 0.91

$$\frac{49}{50}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^5+\frac{581}{150}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^4-\frac{18379}{3000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^3-\frac{219271}{105000}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^2+\frac{86721}{21875}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 - 1
8379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(5/
2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 + 2*
x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000*(5*
x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 101512467
/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000*sqrt(
5*x^2 + 2*x + 3)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+5\,x+2\right)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^2+5x+2\right)\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(7x^2-4x-1\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2,
x)
```

## 3.382 $\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx$

**Optimal**. Leaf size=147

$$-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-\frac{18397(5x+1)\left(5x^2+2x+3\right.}{150000}$$

```
[Out] -18397/150000*(1+5*x)*(5*x^2+2*x+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)
+2809/5250*x*(5*x^2+2*x+3)^(5/2)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x^3
*(5*x^2+2*x+3)^(5/2)-901453/625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-1
28779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1661, 640, 612, 619, 215}

$$-\frac{7}{40}\left(5x^2+2x+3\right)^{5/2}x^3-\frac{1163\left(5x^2+2x+3\right)^{5/2}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{5/2}x}{5250}+\frac{149509\left(5x^2+2x+3\right)^{5/2}}{262500}-1839$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-128779*(1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/250000 - (18397*(1 + 5*x)*(3 + 2*
x + 5*x^2)^(3/2))/150000 + (149509*(3 + 2*x + 5*x^2)^(5/2))/262500 + (2809*
x*(3 + 2*x + 5*x^2)^(5/2))/5250 - (1163*x^2*(3 + 2*x + 5*x^2)^(5/2))/1400 -
 (7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 - (901453*ArcSinh[(1 + 5*x)/Sqrt[14]])/
(125000*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}dx = -\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+\frac{1}{40}\int \left(3+2x+5x^2\right)^{3/2}\left(80\right.$$

$$= -\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-\frac{7}{40}x^3\left(3+2x+5x^2\right)^{5/2}+$$

$$= \frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-\frac{1163x^2\left(3+2x+5x^2\right)^{5/2}}{1400}-$$

$$= \frac{149509\left(3+2x+5x^2\right)^{5/2}}{262500}+\frac{2809x\left(3+2x+5x^2\right)^{5/2}}{5250}-$$

$$= -\frac{18397(1+5x)\left(3+2x+5x^2\right)^{3/2}}{150000}+\frac{149509\left(3+2x+5\right.}{262500}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2\right.}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2\right.}{150000}$$

$$= -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)\left(3+2\right.}{150000}$$

**Mathematica [A]** time = 0.15, size = 75, normalized size = 0.51

$$\frac{-5\sqrt{5x^2+2x+3}\left(22968750x^7+127406250x^6+48237500x^5+28373000x^4-78608475x^3-86464445x^2-3\right.}{26250000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-22275576 - 36695150*x - 86464445*x^2 - 78608475
*x^3 + 28373000*x^4 + 48237500*x^5 + 127406250*x^6 + 22968750*x^7) - 378610
26*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/26250000
```

**fricas [A]** time = 0.94, size = 87, normalized size = 0.59

$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-36695\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 78
608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3) +
901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

**giac [A]** time = 0.20, size = 82, normalized size = 0.56

$$-\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245\,x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)x-222\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 3144
339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 90145
3/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple [A]** time = 0.01, size = 117, normalized size = 0.80

$$-\frac{7\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3}{40}-\frac{1163\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2}{1400}+\frac{2809\left(5x^2+2x+3\right)^{\frac{5}{2}}x}{5250}-\frac{901453\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}+\frac{149}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 149509/262500*(5*x^2+2*x+3)^(5/2)-7/40*(5*x^2+2*x+3)^(5/2)*x^3-1163/1400*(5
*x^2+2*x+3)^(5/2)*x^2-901453/625000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-
128779/500000*(10*x+2)*(5*x^2+2*x+3)^(1/2)+2809/5250*(5*x^2+2*x+3)^(5/2)*x-
18397/300000*(10*x+2)*(5*x^2+2*x+3)^(3/2)
```

**maxima [A]** time = 0.96, size = 138, normalized size = 0.94

$$-\frac{7}{40}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^3-\frac{1163}{1400}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x^2+\frac{2809}{5250}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}x+\frac{149509}{262500}\left(5\,x^2+2\,x+3\right)^{\frac{5}{2}}-\frac{18397}{30000}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2 +
 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5/2
) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x + 3)^
(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*arcsin
h(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}\,\left(-7\,x^2+4\,x+1\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
```

```
[Out] int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-43x\sqrt{5x^2+2x+3}\right) dx-\int \left(-57x^2\sqrt{5x^2+2x+3}\right) dx-\int 14x^3\sqrt{5x^2+2x+3}\, dx-\int 48x^4\sqrt{5x^2+2x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-43*x*sqrt(5*x**2 + 2*x + 3), x) - Integral(-57*x**2*sqrt(5*x**2
+ 2*x + 3), x) - Integral(14*x**3*sqrt(5*x**2 + 2*x + 3), x) - Integral(48*
x**4*sqrt(5*x**2 + 2*x + 3), x) - Integral(169*x**5*sqrt(5*x**2 + 2*x + 3),
 x) - Integral(35*x**6*sqrt(5*x**2 + 2*x + 3), x) - Integral(-6*sqrt(5*x**2
 + 2*x + 3), x)
```

# 3.383 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

**Optimal**. Leaf size=210

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right.}}{1680}$$

```
[Out] -1/980*(267+35*x)*(5*x^2+2*x+3)^(3/2)-34425687/4201750*arcsinh(1/14*(1+5*x)
*14^(1/2))*5^(1/2)-3/240100*(571621+196105*x)*(5*x^2+2*x+3)^(1/2)-6/184877*
arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2
))^(1/2))*(178175857354-53550689170*11^(1/2))^(1/2)+6/184877*arctanh((23+11
^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(178
175857354+53550689170*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.30, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1066, 1076, 619, 215, 1032, 724, 206}

$$-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}-\frac{3(196105x+571621)\sqrt{5x^2+2x+3}}{240100}-\frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right.}}{1680}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-3*(571621 + 196105*x)*Sqrt[3 + 2*x + 5*x^2])/240100 - ((267 + 35*x)*(3 +
2*x + 5*x^2)^(3/2))/980 - (34425687*ArcSinh[(1 + 5*x)/Sqrt[14]])/(840350*Sq
rt[5]) - (6*Sqrt[(2*(8098902607 - 2434122235*Sqrt[11]))/11]*ArcTanh[(23 - S
qrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/16807 + (6*Sqrt[(2*(8098902607 + 2434122235*Sqrt[11]))/11]*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3
 + 2*x + 5*x^2])])/16807
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = -\frac{1}{980}(267+35x)\left(3+2x+5x^2\right)^{3/2} - \frac{\int \frac{\left(-20358-79272x-100854x^2\right)\sqrt{3+2x+5x^2}}{1+4x-7x^2}}{2940}$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)$$

$$= -\frac{3(571621+196105x)\sqrt{3+2x+5x^2}}{240100} - \frac{1}{980}(267+35x)\left(3+2x+5x^2\right)$$

**Mathematica [A]** time = 0.94, size = 202, normalized size = 0.96

$$-5\left(600\sqrt{1572625-425459\sqrt{11}}\,\left(61\sqrt{11}-143\right)\tanh^{-1}\left(\frac{-5\sqrt{11}\,x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\,\sqrt{5x^2+2x+3}}\right)-600\left(143+61\sqrt{11}\right)\sqrt{157262}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

```
[Out] (-757365114*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] - 5*(77*Sqrt[3 + 2*x + 5*x^
2]*(1911108 + 744870*x + 344225*x^2 + 42875*x^3) + 600*Sqrt[1572625 - 42545
9*Sqrt[11]]*(-143 + 61*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]
*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] - 600*(143 + 61*Sqrt[1
1])*Sqrt[1572625 + 425459*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[1
1])*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/92438500
```

**fricas [B]** time = 0.98, size = 326, normalized size = 1.55

$$\frac{3}{184877}\sqrt{11}\sqrt{2}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{12\left(\sqrt{2}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+8098902\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fric
as")
```

```
[Out] 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(12*(sq
rt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*sq
rt(11) - 24697) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) -
 3/184877*sqrt(11)*sqrt(2)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-12*(
sqrt(2)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(7690*
sqrt(11) - 24697) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
 - 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log(-(sqr
t(5*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23
32483950816) + 486663492*sqrt(11)*(x + 3) - 1459990476*x + 2433317460)/x) +
 1/739508*sqrt(11)*sqrt(-701027203680*sqrt(11) + 2332483950816)*log((sqrt(5
*x^2 + 2*x + 3)*(7690*sqrt(11) + 24697)*sqrt(-701027203680*sqrt(11) + 23324
83950816) - 486663492*sqrt(11)*(x + 3) + 1459990476*x - 2433317460)/x) - 1/
240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x + 3)
+ 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

**giac [A]** time = 0.28, size = 154, normalized size = 0.73

$$-\frac{1}{240100}\left(35\left(35\left(35x+281\right)x+21282\right)x+1911108\right)\sqrt{5x^2+2x+3}+\frac{34425687}{4201750}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="giac
")
```

```
[Out] -1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x + 3
) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*
```

```
x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924
736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2
5295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) -
1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 730, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x)
```

```
[Out] -1/280*(10*x+2)*(5*x^2+2*x+3)^(3/2)-3/200*(10*x+2)*(5*x^2+2*x+3)^(1/2)-21/2
50*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-3/154*(-61+13*11^(1/2))*11^(1/2)*
(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-
2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*
11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1
/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/
2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+4
9*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34
/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7
-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2
))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-3/154*11^(1/2)
*(61+13*11^(1/2))*(1/21*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/
20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1
/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7
*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/
7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7
-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/
2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*1
1^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2
))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(
1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)
)))
```

**maxima [B]** time = 1.31, size = 535, normalized size = 2.55

$$\frac{1}{92438500}\sqrt{11}\left(19500\sqrt{11}\sqrt{2}\left(17\sqrt{11}+125\right)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}x}{7\left|14x-2\sqrt{11}-4\right|}+\frac{\sqrt{11}}{7\left|14x-2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="maxi
ma")
```

```
[Out] 1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 3
00125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sqrt(11
) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(
11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
 + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcsinh(5/7*s
qrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)
*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*s
qrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 15692250*
(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs
(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
 + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*s
qrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^2 + 2*x + 3)^(3/
2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*sqrt(11)*sqrt(5)*
arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 60020205*sqrt(11)*
sqrt(5*x^2 + 2*x + 3))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{-7\,x^2+4\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
```

```
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x**2
```

```
 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3
)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x**2 -
 4*x - 1), x)
```

# 3.384 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

**Optimal**. Leaf size=222

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}}{26411}$$

```
[Out] 3/154*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)+16691/12005*arcsinh(1/14*
(1+5*x)*14^(1/2))*5^(1/2)+1/3773*(5826+3395*x)*(5*x^2+2*x+3)^(1/2)-1/581042
*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/
2))^(1/2))*(1147858806842-289418283682*11^(1/2))^(1/2)-1/581042*arctanh((23
+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(
1147858806842+289418283682*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1054, 1066, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}+\frac{(3395x+5826)\sqrt{5x^2+2x+3}}{3773}-\frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\tanh^{-1}}{26411}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((5826 + 3395*x)*Sqrt[3 + 2*x + 5*x^2])/3773 + (3*(3 + 61*x)*(3 + 2*x + 5*x
^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (16691*ArcSinh[(1 + 5*x)/Sqrt[14]])/(2
401*Sqrt[5]) - (Sqrt[(52175400311 - 13155376531*Sqrt[11])/22]*ArcTanh[(23 -
 Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x
+ 5*x^2])])/26411 - (Sqrt[(52175400311 + 13155376531*Sqrt[11])/22]*ArcTanh[
(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 +
 2*x + 5*x^2])])/26411
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
```

```
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{1}{308}\int \frac{\sqrt{3+2x+5x^2}\,(-912+724x+3}{1+4x-7x^2}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{\int \frac{70020}{(1}}{}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} - \frac{\int \frac{244}{(1+4x}}{}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691}{}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691}{2}$$

$$= \frac{(5826+3395x)\sqrt{3+2x+5x^2}}{3773} + \frac{3(3+61x)(3+2x+5x^2)^{3/2}}{154(1+4x-7x^2)} + \frac{16691}{2}$$

**Mathematica [A]** time = 1.83, size = 354, normalized size = 1.59

$$\frac{5\sqrt{\frac{22}{125-17\sqrt{11}}}\left(743879\sqrt{11}-1701489\right)\log\left(49x^2+14\left(\sqrt{11}-2\right)x-4\sqrt{11}+15\right)-10\sqrt{\frac{22}{125+17\sqrt{11}}}\left(1701489+7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

```
[Out] ((770*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-1
- 4*x + 7*x^2) + 8078444*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]] + 10*Sqrt[22/(
125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqr
t[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] + 10
*Sqrt[22/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[2 + Sqrt[11]
- 7*x] - 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*Sqrt[11])*Log[(-
2 + Sqrt[11] + 7*x)^2] + 5*Sqrt[22/(125 - 17*Sqrt[11])]*(-1701489 + 743879*
```

```
Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2] - 10*Sqrt[22
/(125 + 17*Sqrt[11])]*(1701489 + 743879*Sqrt[11])*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5810
420
```

**fricas [B]** time = 0.90, size = 378, normalized size = 1.70

$$\frac{5\sqrt{11}\left(7x^2-4x-1\right)\sqrt{26310753062\sqrt{11}+104350800622}\log\left(\frac{\sqrt{5x^2+2x+3}\sqrt{26310753062\sqrt{11}+104350800622}\left(16206\sqrt{\right.}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="fr
icas")
```

```
[Out] 1/5810420*(5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 1043508
00622)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 104350800622)
*(16206*sqrt(11) - 68441) + 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 89
75958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(26310753062*sqrt(11) + 104
350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(26310753062*sqrt(11) + 10435080
0622)*(16206*sqrt(11) - 68441) - 1795191685*sqrt(11)*(x + 3) - 5385575055*x
 + 8975958425)/x) - 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*sqrt(11)
 + 104350800622)*log(-(sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)*sqrt(
-26310753062*sqrt(11) + 104350800622) + 1795191685*sqrt(11)*(x + 3) - 53855
75055*x + 8975958425)/x) + 5*sqrt(11)*(7*x^2 - 4*x - 1)*sqrt(-26310753062*s
qrt(11) + 104350800622)*log((sqrt(5*x^2 + 2*x + 3)*(16206*sqrt(11) + 68441)
*sqrt(-26310753062*sqrt(11) + 104350800622) - 1795191685*sqrt(11)*(x + 3) +
 5385575055*x - 8975958425)/x) + 4039222*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqr
t(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 770*(2695*x^3 +
 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4*x - 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7]%%%}+%%%{
-852268179840,[6]%%%}+%%%{%%{[-1735527929856,0]:[1,0,-5]%%},[5]%%%}+%%%{617
5070357568,[4]%%%}+%%%{%%{[4607413432832,0]:[1,0,-5]%%},[3]%%%}+%%%{-133522
```

```
01484160,[2]%%%}+%%%{%%{[-3429733766144,0]:[1,0,-5]%%},[1]%%%}+%%%{88958719
55936,[0]%%%} / %%%{245,[8]%%%}+%%%{%%{poly1[560,0]:[1,0,-5]%%},[7]%%%}+%%%
{-3300,[6]%%%}+%%%{%%{poly1[-6720,0]:[1,0,-5]%%},[5]%%%}+%%%{23910,[4]%%%}+
%%%{%%{poly1[17840,0]:[1,0,-5]%%},[3]%%%}+%%%{-51700,[2]%%%}+%%%{%%{poly1[-
13280,0]:[1,0,-5]%%},[1]%%%}+%%%{34445,[0]%%%} Error: Bad Argument Value
```

**maple [B]** time = 0.02, size = 1828, normalized size = 8.23

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

```
[Out] 161/484*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(1/2
0*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/
2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*
11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7
*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+
1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2
))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11
^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2)
)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))
))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/
2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250
/49-34/49*11^(1/2))^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))
*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2
/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*1
1^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/
2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2
)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10
/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7
*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49
*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/
2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000
/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49
```

```
-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+(183/44+39/4
4*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1
/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7
-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^
(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2)
)^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+
1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1
/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(25
0/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^
(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(
1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/
2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34
*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11^(1/2))*(1/4
0*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*1
1^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(
x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(
1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)
-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-161/484*11^(1/2)*(1/21*(5*(
x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/4
9*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11
^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^
(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsin
h(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)
))+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11
^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(
1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)
))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="ma
xima")
```

[Out] integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{{\left(-7\,x^2+4\,x+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)

[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} + 5x + 2\right)\left(5x^{2} + 2x + 3\right)^{\frac{3}{2}}}{\left(7x^{2} - 4x - 1\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)

[Out] Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2, x)

## 3.385 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx$

**Optimal**. Leaf size=234

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(12}}\right.}{332024}$$

```
[Out] 3/308*(3+61*x)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2-5/343*arcsinh(1/14*(1+5
*x)*14^(1/2))*5^(1/2)-1/23716*(9495-37088*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*
x+1)-1/927675056*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2
)/(250-34*11^(1/2))^(1/2))*(174049987116977774-5826721433301670*11^(1/2))^(
1/2)+1/927675056*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2
)/(250+34*11^(1/2))^(1/2))*(174049987116977774+5826721433301670*11^(1/2))^(
1/2)
```

**Rubi [A]** time = 0.32, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1054, 1076, 619, 215, 1032, 724, 206}

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{23716\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(12}}\right.}{332024}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] -((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(23716*(1 + 4*x - 7*x^2)) + (3*(3
 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) - (5*Sqrt[5]*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/343 - (Sqrt[(62294197250171 - 2085440742055*Sqrt
[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17
*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024 + (Sqrt[(62294197250171 + 20854
40742055*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqr
t[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/332024
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
```

```
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^3}\,dx = \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{1}{616}\int \frac{\sqrt{3+2x+5x^2}\,\left(-2976-652x\right.}{\left(1+4x-7x^2\right)^2}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} + \frac{\int}{}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{\int}{}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{1}{68}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{5}{34}$$

$$= -\frac{(9495-37088x)\sqrt{3+2x+5x^2}}{23716\left(1+4x-7x^2\right)} + \frac{3(3+61x)\left(3+2x+5x^2\right)^{3/2}}{308\left(1+4x-7x^2\right)^2} - \frac{5}{34}$$

**Mathematica [A]** time = 2.24, size = 376, normalized size = 1.61

$$\frac{\frac{88\sqrt{5x^2+2x+3}\,(138372-189161x)}{7x^2-4x-1} + \frac{11616(5028x+655)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} - \sqrt{\frac{22}{125-17\sqrt{11}}}\left(674221\sqrt{11}-7706073\right)\log\left(49x^2+14\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

```
[Out] ((11616*(655 + 5028*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (88*(13
8372 - 189161*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 212960*Sqrt[5]
```

```
*ArcSinh[(1 + 5*x)/Sqrt[14]] - 2*Sqrt[22/(125 - 17*Sqrt[11])]*(-7706073 + 6
74221*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-2
3 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 2*Sqrt[22/(125 + 17*Sqrt[11])]*(770
6073 + 674221*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + Sqrt[22/(125 - 17*Sqrt[11
])]*(-7706073 + 674221*Sqrt[11])*Log[(-2 + Sqrt[11] + 7*x)^2] - Sqrt[22/(12
5 - 17*Sqrt[11])]*(-7706073 + 674221*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2
 + Sqrt[11])*x + 49*x^2] + 2*Sqrt[22/(125 + 17*Sqrt[11])]*(7706073 + 674221
*Sqrt[11])*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]])/14609056
```

**fricas [B]** time = 0.99, size = 447, normalized size = 1.91

$$-\frac{\sqrt{2794}\left(49\,x^4-56\,x^3+2\,x^2+8\,x+1\right)\sqrt{2085440742055\,\sqrt{11}+62294197250171}\,\log\left(\frac{\sqrt{2794}\,\sqrt{5\,x^2+2\,x+3}\,\sqrt{2085440}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="fr
icas")
```

```
[Out] -1/1855350112*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(20854407
42055*sqrt(11) + 62294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt
(2085440742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) +
542667120256069*sqrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345
)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(2085440742055*sq
rt(11) + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(208544
0742055*sqrt(11) + 62294197250171)*(11840590*sqrt(11) - 83479737) - 5426671
202560069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) +
sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11)
 + 62294197250171)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11
) + 83479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) + 542667120256
0069*sqrt(11)*(x + 3) - 16280013607680207*x + 27133356012800345)/x) - sqrt(
2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-2085440742055*sqrt(11) + 62
294197250171)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-2085440742055*sqrt(11) + 62294197250171) - 5426671202560069*s
qrt(11)*(x + 3) + 16280013607680207*x - 27133356012800345)/x) - 13522960*sq
rt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)
*(5*x + 1) - 25*x^2 - 10*x - 8) + 78232*(189161*x^3 - 246464*x^2 - 42767*x
+ 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{-1771684761728,[12]%%%}+%%%{%%{[-6074347754496,0]:[1,0,-5]%%},[11]%%
%}+%%%{18439984254720,[10]%%%}+%%%{%%{[120412580657152,0]:[1,0,-5]%%},[9]%%
%}+%%%{-108578966111616,[8]%%%}+%%%{%%{[-915119084156928,0]:[1,0,-5]%%},[7]
%%%}+%%%{1093279290575360,[6]%%%}+%%%{%%{[2784778529734656,0]:[1,0,-5]%%},[
5]%%%}+%%%{-4014694487954304,[4]%%%}+%%%{%%{[-3629195511796736,0]:[1,0,-5]%
%},[3]%%%}+%%%{5826260235237120,[2]%%%}+%%%{%%{[1708007415539712,0]:[1,0,-5
]%%},[1]%%%}+%%%{-2953429489370752,[0]%%%} / %%%{%%{[1715,0]:[1,0,-5]%%},[1
2]%%%}+%%%{29400,[11]%%%}+%%%{%%{[-17850,0]:[1,0,-5]%%},[10]%%%}+%%%{-58280
0,[9]%%%}+%%%{%%{[105105,0]:[1,0,-5]%%},[8]%%%}+%%%{4429200,[7]%%%}+%%%{%%{
[-1058300,0]:[1,0,-5]%%},[6]%%%}+%%%{-13478400,[5]%%%}+%%%{%%{[3886245,0]:[
1,0,-5]%%},[4]%%%}+%%%{17565400,[3]%%%}+%%%{%%{[-5639850,0]:[1,0,-5]%%},[2]
%%%}+%%%{-8266800,[1]%%%}+%%%{%%{[2858935,0]:[1,0,-5]%%},[0]%%%} Error: Bad
 Argument Value
```

**maple [B]** time = 0.02, size = 3828, normalized size = 16.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

```
[Out] 3535/21296*11^(1/2)*(1/21*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x
-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/14*(34/7-10/7*11^(1/2))*(
1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10
/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-1
0/7*11^(1/2))^2)^(1/2)*(x+1/5)))+1/7*(250/49-34/49*11^(1/2))*(1/7*(245*(x-2
/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(
1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49
*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1
/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7
*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11
^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/
2))))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-
2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1
/7*11^(1/2))+250/49-34/49*11^(1/2))^(5/2)+1/1372*(34/7-10/7*11^(1/2))/(250/
49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2
/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*1
1^(1/2))^(5/2)+3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-
2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*
```

11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/(250/49-34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))+15/686/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))))-(-3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(5/2)+3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))+20/49/(250/49+34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11

^(1/2))+250/49+34/49*11^(1/2))^(3/2)+3/80*(5000/49+680/49*11^(1/2)-(34/7+10
/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*
11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(
1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-(-3535/1936+273/1936*11
^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*1
1^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))
^(5/2)+3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/3*(5*(x-2/7+1/7
*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2
))^(3/2)+1/2*(34/7-10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/20
0*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/
(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49
-34/49*11^(1/2))*(1/7*(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(
x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)+1/10*(34/7-10/7*11^(1/2))*5^(1/2
)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/20*(34/7-10/7*11^(1/2))^2)^(1/2)
*(x+1/5))-7*(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(5
00/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^
(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2))+250-34*11^(1/2))^(1/2))))+20/49/(250/49-34/49*11^(1/2))*(1/40*(1
0*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(3/2)+3/80*(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1
/2))^2)*(1/20*(10*x+2)*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/200*(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49-34/49*11^(1/2)-1/2
0*(34/7-10/7*11^(1/2))^2)^(1/2)*(x+1/5)))))-3535/21296*11^(1/2)*(1/21*(5*(x
-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49
*11^(1/2))^(3/2)+1/14*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^
(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(
1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh
(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))
)+1/7*(250/49+34/49*11^(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/
7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^
(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1
/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*ar
ctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
)/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))))-21/968*(61+13*11^(1/2))*1
1^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(5/2)+1/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250/49+3
4/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(5/2)+3/2*(34/7+10/7*11
^(1/2))/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1/2*(34/7+10/7*1

```
1^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2
)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/
20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^(1/2))*(1/7*(24
5*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+3
4*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(5^(1/2)/(250/49
+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-7*(250/49+34/49
*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2))))+20/(250/49+34/49*11^(1/2))*(1/40*(10*x+2)*(5*(x-2/7-1/7*11^(1/2
))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)
+3/80*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*(1/20*(10*x+2)*(5*(x
-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49
*11^(1/2))^(1/2)+1/200*(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(
1/2)*arcsinh(5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1
/2)*(x+1/5)))))+15/686/(250/49+34/49*11^(1/2))*(1/3*(5*(x-2/7-1/7*11^(1/2))
^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(3/2)+1
/2*(34/7+10/7*11^(1/2))*(1/20*(10*x+2)*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/200*(5000/49
+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)*5^(1/2)*arcsinh(5^(1/2)/(250/49+34
/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5)))+(250/49+34/49*11^
(1/2))*(1/7*(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*
11^(1/2))+250+34*11^(1/2))^(1/2)+1/10*(34/7+10/7*11^(1/2))*5^(1/2)*arcsinh(
5^(1/2)/(250/49+34/49*11^(1/2)-1/20*(34/7+10/7*11^(1/2))^2)^(1/2)*(x+1/5))-
7*(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/4
9*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/
2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250+34*11^(1/2))^(1/2)))))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(x^2+5x+2\right)}{\left(7x^2-4x-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="ma
xima")
```

```
[Out] -integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^2+5\,x+2\right)\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{{\left(-7\,x^2+4\,x+1\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
[Out] int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx-\int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

```
[Out] -Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x*
*3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*x*
*6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(23
*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*x**6
- 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x**
4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x*
*2 - 12*x - 1), x)
```

# 3.386 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=185

$$\frac{40722851\sqrt{5x^2+2x+3}\,x^2}{750000}+\frac{5793077\sqrt{5x^2+2x+3}\,x}{75000}-\frac{16515809\sqrt{5x^2+2x+3}}{156250}-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{}$$

```
[Out] -77513689/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-16515809/156250*(5
*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(
5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*
x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3
)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}\,x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}\,x^5-\frac{47807\sqrt{5x^2+2x+3}\,x^4}{3750}-\frac{5160533\sqrt{5x}}{500}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-16515809*Sqrt[3 + 2*x + 5*x^2])/156250 + (5793077*x*Sqrt[3 + 2*x + 5*x^2]
)/75000 + (40722851*x^2*Sqrt[3 + 2*x + 5*x^2])/750000 - (5160533*x^3*Sqrt[3
 + 2*x + 5*x^2])/50000 - (47807*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 + (26159*x^
5*Sqrt[3 + 2*x + 5*x^2])/300 - (1141*x^6*Sqrt[3 + 2*x + 5*x^2])/40 - (343*x
^7*Sqrt[3 + 2*x + 5*x^2])/40 - (77513689*ArcSinh[(1 + 5*x)/Sqrt[14]])/(6250
00*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{1}{40}\int \frac{80+1160x+4600x^2-2440x^3-34840x^4}{\sqrt{3+2x+5x^2}}$$

$$= -\frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} + \frac{\int \frac{2800+40600x+161000x^2-8}{}}{}$$

$$= \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2}$$

$$= -\frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5}$$

$$= -\frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x}$$

$$= \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+}}{3750}$$

$$= \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3}}{50000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3}}{75000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3}}{75000}$$

$$= -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3}}{75000}$$

**Mathematica [A]** time = 0.25, size = 75, normalized size = 0.41

$$\frac{-5\sqrt{5x^2+2x+3}\left(32156250x^7+106968750x^6-326987500x^5+47807000x^4+387039975x^3-203614255x^2-\right.}{18750000}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(396379416 - 289653850*x - 203614255*x^2 + 387039
975*x^3 + 47807000*x^4 - 326987500*x^5 + 106968750*x^6 + 32156250*x^7) - 46
5082134*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/18750000
```

**fricas [A]** time = 0.81, size = 87, normalized size = 0.47

$$-\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x^2-28\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 + 3
87039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x +
3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) -
 25*x^2 - 10*x - 8)
```

**giac [A]** time = 0.25, size = 82, normalized size = 0.44

$$-\frac{1}{3750000}\left(5\left(\left(5\left(10\left(175\left(15\left(49\,x+163\right)x-7474\right)x+191228\right)x+15481599\right)x-40722851\right)x-57930770\right)x+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 15481
599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 775
13689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple [A]** time = 0.03, size = 147, normalized size = 0.79

$$-\frac{343\sqrt{5x^2+2x+3}\;x^7}{40}-\frac{1141\sqrt{5x^2+2x+3}\;x^6}{40}+\frac{26159\sqrt{5x^2+2x+3}\;x^5}{300}-\frac{47807\sqrt{5x^2+2x+3}\;x^4}{3750}-\frac{5160533}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -16515809/156250*(5*x^2+2*x+3)^(1/2)-77513689/3125000*5^(1/2)*arcsinh(5/14*
14^(1/2)*(x+1/5))-343/40*x^7*(5*x^2+2*x+3)^(1/2)-1141/40*x^6*(5*x^2+2*x+3)^
(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-47807/3750*x^4*(5*x^2+2*x+3)^(1/2)-
5160533/50000*x^3*(5*x^2+2*x+3)^(1/2)+40722851/750000*x^2*(5*x^2+2*x+3)^(1/
2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)
```

**maxima [A]** time = 0.99, size = 148, normalized size = 0.80

$$-\frac{343}{40}\sqrt{5x^2+2x+3}\,x^7-\frac{1141}{40}\sqrt{5x^2+2x+3}\,x^6+\frac{26159}{300}\sqrt{5x^2+2x+3}\,x^5-\frac{47807}{3750}\sqrt{5x^2+2x+3}\,x^4-\frac{5160533}{50000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 261
59/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 - 5
160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*x +
 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt(5)*
arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)^3}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{29x}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{115x^2}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{61x^3}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{871x^4}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{1}{\sqrt{5x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-29*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-115*x**2/sqrt(5*x**2
 + 2*x + 3), x) - Integral(61*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(87
1*x**4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-127*x**5/sqrt(5*x**2 + 2*x +
3), x) - Integral(-2065*x**6/sqrt(5*x**2 + 2*x + 3), x) - Integral(1127*x**
7/sqrt(5*x**2 + 2*x + 3), x) - Integral(343*x**8/sqrt(5*x**2 + 2*x + 3), x)
 - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

# 3.387 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=143

$$-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5x^2+2x+3}\,x}{1875}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+$$

```
[Out] -1719097/156250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-22053/31250*(5*x^2+2
*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/37500*x^2*(5*x^2+2*x+3)
^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x^4*(5*x^2+2*x+3)^(1/2)+
49/30*x^5*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.20, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1661, 640, 619, 215}

$$\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}\,x^4-\frac{33259\sqrt{5x^2+2x+3}\,x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500}+\frac{36073\sqrt{5x^2}}{187}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-22053*Sqrt[3 + 2*x + 5*x^2])/31250 + (36073*x*Sqrt[3 + 2*x + 5*x^2])/1875
 - (207427*x^2*Sqrt[3 + 2*x + 5*x^2])/37500 - (33259*x^3*Sqrt[3 + 2*x + 5*x
^2])/2500 + (5131*x^4*Sqrt[3 + 2*x + 5*x^2])/750 + (49*x^5*Sqrt[3 + 2*x + 5
*x^2])/30 - (1719097*ArcSinh[(1 + 5*x)/Sqrt[14]])/(31250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{30}\int \frac{60+630x+1350x^2-2820x^3-6135x^4+51}{\sqrt{3+2x+5x^2}}$$

$$= \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2} + \frac{1}{750}\int \frac{1500+15750x+}{}$$

$$= -\frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+5x^2} + \frac{49}{30}x^5\sqrt{3+2x+5x^2}$$

$$= -\frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+}$$

$$= \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5}}{2500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

$$= -\frac{22053\sqrt{3+2x+5x^2}}{31250} + \frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5}}{37500}$$

**Mathematica** [A] time = 0.14, size = 65, normalized size = 0.45

$$\frac{5\sqrt{5x^2+2x+3}\left(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318\right)-10314582\sqrt{5}\sin}{937500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (5*Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 +
 1282750*x^4 + 306250*x^5) - 10314582*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/
937500
```

**fricas** [A] time = 0.87, size = 77, normalized size = 0.54

$$\frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{312500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
- 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt(5
*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

**giac** [A] time = 0.39, size = 72, normalized size = 0.50

$$\frac{1}{187500}\,(5\,((5\,(70\,(175\,x+733)x-99777)x-207427)x+721460)x-132318)\sqrt{5\,x^2+2\,x+3}+\frac{1719097}{156250}\,\sqrt{5}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 13
2318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple** [A] time = 0.01, size = 113, normalized size = 0.79

$$\frac{49\sqrt{5x^2+2x+3}\;x^5}{30}+\frac{5131\sqrt{5x^2+2x+3}\;x^4}{750}-\frac{33259\sqrt{5x^2+2x+3}\;x^3}{2500}-\frac{207427\sqrt{5x^2+2x+3}\;x^2}{37500}+\frac{36073\sqrt{5x}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -22053/31250*(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*arcsinh(5/14*14^(1/
2)*(x+1/5))+49/30*(5*x^2+2*x+3)^(1/2)*x^5+5131/750*(5*x^2+2*x+3)^(1/2)*x^4-
33259/2500*(5*x^2+2*x+3)^(1/2)*x^3-207427/37500*(5*x^2+2*x+3)^(1/2)*x^2+360
73/1875*(5*x^2+2*x+3)^(1/2)*x
```

**maxima [A]** time = 0.97, size = 114, normalized size = 0.80

$$\frac{49}{30}\sqrt{5x^2+2x+3}\,x^5+\frac{5131}{750}\sqrt{5x^2+2x+3}\,x^4-\frac{33259}{2500}\sqrt{5x^2+2x+3}\,x^3-\frac{207427}{37500}\sqrt{5x^2+2x+3}\,x^2+\frac{36073}{1875}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 3325
9/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2 +
 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/14*s
qrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5x+2\right)\left(-7x^2+4x+1\right)^2}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2+5x+2\right)\left(7x^2-4x-1\right)^2}{\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/sqrt(5*x**2 + 2*x + 3), x)
```

## 3.388 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

```
[Out] -1901/1250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+463/125*(5*x^2+2*x+3)^(1/
2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2+2*x+3)^(1/2)-7/20*x^3*(5*
x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1661, 640, 619, 215}

$$-\frac{7}{20}\sqrt{5x^2+2x+3}\,x^3-\frac{571}{300}\sqrt{5x^2+2x+3}\,x^2+\frac{59}{30}\sqrt{5x^2+2x+3}\,x+\frac{463}{125}\sqrt{5x^2+2x+3}-\frac{1901\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{250\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (463*Sqrt[3 + 2*x + 5*x^2])/125 + (59*x*Sqrt[3 + 2*x + 5*x^2])/30 - (571*x^
2*Sqrt[3 + 2*x + 5*x^2])/300 - (7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 - (1901*Arc
Sinh[(1 + 5*x)/Sqrt[14]])/(250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{20}\int \frac{40+260x+203x^2-571x^3}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{1}{300}\int \frac{600+7326x+59}{\sqrt{3+2x+5x^2}}$$

$$= \frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3\sqrt{3+2x+5x^2}+\frac{\int \frac{-11}{\sqrt{}}}{}$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3$$

$$= \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2}-\frac{571}{300}x^2\sqrt{3+2x+5x^2}-\frac{7}{20}x^3$$

**Mathematica [A]** time = 0.08, size = 55, normalized size = 0.54

$$\frac{-5\sqrt{5x^2+2x+3}\,(525x^3+2855x^2-2950x-5556)-11406\sqrt{5}\,\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{7500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

```
[Out] (-5*Sqrt[3 + 2*x + 5*x^2]*(-5556 - 2950*x + 2855*x^2 + 525*x^3) - 11406*Sqr
t[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/7500
```

**fricas [A]** time = 0.73, size = 67, normalized size = 0.66

$$-\frac{1}{1500}\left(525\,x^3+2855\,x^2-2950\,x-5556\right)\sqrt{5\,x^2+2\,x+3}+\frac{1901}{2500}\,\sqrt{5}\,\log\left(\sqrt{5}\,\sqrt{5\,x^2+2\,x+3}\,(5\,x+1)-25\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/2
500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8
)
```

**giac [A]** time = 0.22, size = 62, normalized size = 0.61

$$-\frac{1}{1500}\,(5\,((105\,x+571)x-590)x-5556)\sqrt{5\,x^2+2\,x+3}+\frac{1901}{1250}\,\sqrt{5}\,\log\left(-\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/1
250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

**maple [A]** time = 0.01, size = 79, normalized size = 0.78

$$-\frac{7\sqrt{5x^2+2x+3}\;x^3}{20}-\frac{571\sqrt{5x^2+2x+3}\;x^2}{300}+\frac{59\sqrt{5x^2+2x+3}\;x}{30}-\frac{1901\sqrt{5}\;\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}+\frac{463\sqrt{5x^2+}}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -7/20*(5*x^2+2*x+3)^(1/2)*x^3-571/300*(5*x^2+2*x+3)^(1/2)*x^2+59/30*(5*x^2+
2*x+3)^(1/2)*x+463/125*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/2)*arcsinh(5/14*1
4^(1/2)*(x+1/5))
```

**maxima [A]** time = 0.96, size = 80, normalized size = 0.79

$$-\frac{7}{20}\,\sqrt{5\,x^2+2\,x+3}\,x^3-\frac{571}{300}\,\sqrt{5\,x^2+2\,x+3}\,x^2+\frac{59}{30}\,\sqrt{5\,x^2+2\,x+3}\,x-\frac{1901}{1250}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)+\frac{46}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/30
*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1
)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{13x}{\sqrt{5x^2+2x+3}}\right)dx-\int\left(-\frac{7x^2}{\sqrt{5x^2+2x+3}}\right)dx-\int\frac{31x^3}{\sqrt{5x^2+2x+3}}\,dx-\int\frac{7x^4}{\sqrt{5x^2+2x+3}}\,dx-\int\left(-\frac{}{\sqrt{5x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(-13*x/sqrt(5*x**2 + 2*x + 3), x) - Integral(-7*x**2/sqrt(5*x**2 +
 2*x + 3), x) - Integral(31*x**3/sqrt(5*x**2 + 2*x + 3), x) - Integral(7*x*
*4/sqrt(5*x**2 + 2*x + 3), x) - Integral(-2/sqrt(5*x**2 + 2*x + 3), x)
```

# 3.389 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=164

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(125}}\right.$$

```
[Out] -1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*arctanh((23+x*(17-5*11
^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(11430254-29
47670*11^(1/2))^(1/2)+3/39116*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(11430254+2947670*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1076, 619, 215, 1032, 724, 206}

$$-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)+\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{(17}{\sqrt{2(125}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -ArcSinh[(1 + 5*x)/Sqrt[14]]/(7*Sqrt[5]) - (3*Sqrt[(4091 - 1055*Sqrt[11])/2
794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/14 + (3*Sqrt[(4091 + 1055*Sqrt[11])/2794]*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/14
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx = -\left(\frac{1}{7}\int \frac{1}{\sqrt{3+2x+5x^2}}\,dx\right)-\frac{1}{7}\int \frac{-15-39x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{56}}}\,dx,x,2+10x\right)}{14\sqrt{70}}+\frac{1}{77}\left(3\left(143-61\sqrt{11}\right)\right)\int \frac{}{\left(4-2\sqrt{11}\right.}$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}}-\frac{1}{77}\left(6\left(143-61\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{}{2352+112\left(4-2\sqrt{\right.}}\right.$$

$$= -\frac{\sinh^{-1}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}}-\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-\right.}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{}}\right.$$

**Mathematica [A]** time = 0.47, size = 157, normalized size = 0.96

$$-\frac{3\left(\sqrt{4091-1055\sqrt{11}}\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)-\sqrt{4091+1055\sqrt{11}}\tanh^{-1}\left(\frac{5\sqrt{11}x+17x+\sqrt{11}+23}{\sqrt{250+34\sqrt{11}}\sqrt{5x^2+2x+3}}\right)\right)}{14\sqrt{2794}}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] -1/7*ArcSinh[(1 + 5*x)/Sqrt[14]]/Sqrt[5] - (3*(Sqrt[4091 - 1055*Sqrt[11]]*A
rcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqrt[250 - 34*Sqrt[11]]*Sqrt[
3 + 2*x + 5*x^2])] - Sqrt[4091 + 1055*Sqrt[11]]*ArcTanh[(23 + Sqrt[11] + 17
*x + 5*Sqrt[11]*x)/(Sqrt[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])]))/(14*S
qrt[2794])
```

**fricas [B]** time = 0.86, size = 297, normalized size = 1.81

$$-\frac{3}{78232}\sqrt{2794}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{1055\sqrt{11}+4091}\left(172\sqrt{11}-715\right)+1858\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fric
as")
```

```
[Out] -3/78232*sqrt(2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(sqrt(2794)*sqrt(5*x^2
 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 185801*sqrt(1
1)*(x + 3) + 557403*x - 929005)/x) + 3/78232*sqrt(2794)*sqrt(1055*sqrt(11)
+ 4091)*log(-3*(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)
*(172*sqrt(11) - 715) - 185801*sqrt(11)*(x + 3) - 557403*x + 929005)/x) - 1
/78232*sqrt(2794)*sqrt(-9495*sqrt(11) + 36819)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495*sqrt(11) + 36819) + 557403*sqrt(
11)*(x + 3) - 1672209*x + 2787015)/x) + 1/78232*sqrt(2794)*sqrt(-9495*sqrt(
11) + 36819)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqr
t(-9495*sqrt(11) + 36819) - 557403*sqrt(11)*(x + 3) + 1672209*x - 2787015)/
x) + 1/70*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10
*x - 8)
```

**giac [A]** time = 0.27, size = 125, normalized size = 0.76

$$\frac{1}{35}\sqrt{5}\log\left(-5\sqrt{5}x - \sqrt{5} + 5\sqrt{5x^2+2x+3}\right) + 0.353184817631429\log\left(-\sqrt{5}x + \sqrt{5x^2+2x+3} + 4.4192473\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.3531
84817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.
0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000)
 - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113
000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.094112
35400000)
```

**maple [A]** time = 0.02, size = 204, normalized size = 1.24

$$-\frac{\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\ \sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+2}}\right.}{154\sqrt{250-34\sqrt{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(-61+13*11^(1/2))*11^(1/
2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
```

```
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
)+3/154*11^(1/2)*(61+13*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500
/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1
/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*1
1^(1/2))+250+34*11^(1/2))^(1/2))
```

**maxima [B]** time = 1.14, size = 465, normalized size = 2.84

$$-\frac{1}{10780}\sqrt{11}\left(28\sqrt{11}\sqrt{5}\,\operatorname{arsinh}\left(\frac{5}{14}\sqrt{7}\sqrt{2}\,x+\frac{1}{14}\sqrt{7}\sqrt{2}\right)-\frac{1365\sqrt{11}\sqrt{2}\,\operatorname{arsinh}\left(\frac{5\sqrt{11}\sqrt{7}\sqrt{2}\,x}{7\left|14\,x-2\sqrt{11}-4\right|}+\frac{17\sqrt{7}\sqrt{2}}{7\left|14\,x-2\sqrt{1\cdots}\right.}\cdots\right.}{\sqrt{17\sqrt{11}+12\cdots}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/1
4*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqr
t(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) + 390*
sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4)
- 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*
sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt
(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5/7*sqrt(1
1)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/ab
s(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(1
1) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11)
 + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*sq
rt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x +
2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx-\int \frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) -
 sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x + 3)
 - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(7
*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2
*x + 3)), x)
```

# 3.390 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=178

$$-\frac{3\sqrt{5x^2+2x+3}\,(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-1403568}{2794}}}{}$$

```
[Out] -3/5588*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)+1/31225744*arctanh((2
3+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*
(8459955268270-39215692714*11^(1/2))^(1/2)-1/31225744*arctanh((23+x*(17-5*1
1^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(8459955268
270+39215692714*11^(1/2))^(1/2)
```

**Rubi [A]** time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3\sqrt{5x^2+2x+3}\,(40-371x)}{5588\left(-7x^2+4x+1\right)}-\frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\,\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\,\sqrt{5x^2+2x+3}}\right)}{11176}+\frac{\sqrt{\frac{3027900955-1403568}{2794}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) - (Sqrt[(3
027900955 + 14035681*Sqrt[11])/2794]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[
11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176 + (Sqrt
[(3027900955 - 14035681*Sqrt[11])/2794]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sq
rt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/11176
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\int \frac{-52136-29544x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{44704}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(-40623+53005\sqrt{11}\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)}}{61468}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\left(40623-53005\sqrt{11}\right)\operatorname{Subst}\left(\int \frac{}{2352+11}\right.}{}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\left(1+4x-7x^2\right)} - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\tanh^{-1}\left(\frac{23}{\sqrt{2(1}}\right.}{11176}$$

**Mathematica [A]** time = 1.00, size = 313, normalized size = 1.76

$$\frac{\frac{48972\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1}+\frac{5280\sqrt{5x^2+2x+3}}{7x^2-4x-1}+53005\sqrt{\frac{22}{125+17\sqrt{11}}}\log\left(\sqrt{2750+374\sqrt{11}}\sqrt{5x^2+2x+3}+\left(55+17\sqrt{11}\right)x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((48972*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (5280*Sqrt[3 + 2*x + 5
*x^2])/(-1 - 4*x + 7*x^2) + Sqrt[2/(125 - 17*Sqrt[11])]*(-40623 + 53005*Sqr
t[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[
11] + (-17 + 5*Sqrt[11])*x)] - Sqrt[2/(125 + 17*Sqrt[11])]*(40623 + 53005*S
qrt[11])*Log[2 + Sqrt[11] - 7*x] + 40623*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11
 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
2*x + 5*x^2]] + 53005*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (
55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/245
872
```

**fricas [B]** time = 0.87, size = 330, normalized size = 1.85

$$-\frac{\sqrt{2794}\left(7x^2-4x-1\right)\sqrt{14035681\sqrt{11}+3027900955}\log\left(-\frac{\sqrt{2794}\sqrt{5x^2+2x+3}\sqrt{14035681\sqrt{11}+3027900955}\left(71796\sqrt{11}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/62451488*(sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(11) + 30279009
55)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(11) + 3027900
955)*(71796*sqrt(11) + 567523) + 265381033753*sqrt(11)*(x + 3) - 7961431012
59*x + 1326905168765)/x) - sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(14035681*sqrt(
11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(14035681*sqrt(
11) + 3027900955)*(71796*sqrt(11) + 567523) - 265381033753*sqrt(11)*(x + 3)
 + 796143101259*x - 1326905168765)/x) + sqrt(2794)*(7*x^2 - 4*x - 1)*sqrt(-
14035681*sqrt(11) + 3027900955)*log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(7179
6*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955) + 265381033753*s
qrt(11)*(x + 3) + 796143101259*x - 1326905168765)/x) - sqrt(2794)*(7*x^2 -
4*x - 1)*sqrt(-14035681*sqrt(11) + 3027900955)*log(-(sqrt(2794)*sqrt(5*x^2
+ 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-14035681*sqrt(11) + 3027900955)
- 265381033753*sqrt(11)*(x + 3) - 796143101259*x + 1326905168765)/x) + 3352
8*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*x - 1)
```

**giac [B]** time = 0.28, size = 276, normalized size = 1.55

$$\frac{3\left(1231\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^3+1735\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^2-3913\,\sqrt{5}\,x-3989\,\sqrt{5}+391\right.}{2794\left(7\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^2+16\,\sqrt{5}\left(\sqrt{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(5*
x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2
+ 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0924287071106453
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386080346
04765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.092428
7071106453*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0
938608034604765*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 510, normalized size = 2.87

$$-\frac{161\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+250-34\sqrt{11}}}\right)}{484\sqrt{250-34\sqrt{11}}}+161\sqrt{11}\,\operatorname{arctanh}\left(\frac{}{\sqrt{250+34}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x)
```

```
[Out] -161/484*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/
2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*
(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*
11^(1/2))^(1/2))+(183/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-
2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-
2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^
(1/2))^(1/2)))+161/484*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2
))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
(1/2))+250+34*11^(1/2))^(1/2))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49
*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+1/14*(34/7+10/7*11^(1
/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^
(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250+34*11^(1/2))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5\,x+2}{\left(7\,x^2-4\,x-1\right)^2\sqrt{5\,x^2+2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
[Out] Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2), x
)
```

# 3.391 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx$

**Optimal**. Leaf size=227

$$-\frac{7\sqrt{5x^2+2x+3}\,(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\cdots\right)}{124902976\sqrt{22\left(125-1\cdots\right.}}$$

```
[Out] -3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7/62451488*(409769
-1189370*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)-7/124902976*arctanh((23+x*(1
7-5*11^(1/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(39370
231-2538725*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/124902976*arctanh((23+11^
(1/2)+x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(3937
0231+2538725*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

**Rubi** [A] time = 0.27, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{7\sqrt{5x^2+2x+3}\,(409769-1189370x)}{62451488\left(-7x^2+4x+1\right)}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}-\frac{7\left(39370231-2538725\sqrt{11}\right)\tanh^{-1}\left(\cdots\right)}{124902976\sqrt{22\left(125-1\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] (-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) - (7*(4
09769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(62451488*(1 + 4*x - 7*x^2)) - (7
*(39370231 - 2538725*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x
)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(124902976*Sqrt[22*
(125 - 17*Sqrt[11])]) + (7*(39370231 + 2538725*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(124902976*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3 \sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{\int \frac{-130024-81000x-89040x^2}{\left(1+4x-7x^2\right)^2 \sqrt{3+2x+5x^2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)}$$

$$= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\left(1+4x-7x^2\right)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\left(1+4x-7x^2\right)}$$

**Mathematica [A]** time = 1.17, size = 371, normalized size = 1.63

$$\frac{\frac{732651920\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{547311072\sqrt{5x^2+2x+3}\,x}{\left(-7x^2+4x+1\right)^2} - \frac{59009280\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} + \frac{252417704\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 551183234\sqrt{\frac{22}{125+17\sqrt{11}}}\ \mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

```
[Out] ((-59009280*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (547311072*x*Sqrt[
3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (732651920*x*Sqrt[3 + 2*x + 5*x^2])
/(1 + 4*x - 7*x^2) + (252417704*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) +
 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-27925975 + 39370231*Sqrt[11])*ArcTanh[(Sq
rt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqr
t[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(27925975 + 39370231*Sqrt[11])*
Log[2 + Sqrt[11] - 7*x] + 390963650*Sqrt[2/(125 + 17*Sqrt[11])]*Log[11 + 23
*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x +
 5*x^2]] + 551183234*Sqrt[22/(125 + 17*Sqrt[11])]*Log[11 + 23*Sqrt[11] + (5
5 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2]])/5495
730944
```

**fricas [B]** time = 1.10, size = 390, normalized size = 1.72

$$-\frac{\sqrt{2794}\left(49\,x^4-56\,x^3+2\,x^2+8\,x+1\right)\sqrt{1283973697005131\,\sqrt{11}+82616280769148425}\,\log\left(-\frac{\sqrt{2794}\,\sqrt{5\,x^2+2\,x+3}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/697957829888*(sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(128397
3697005131*sqrt(11) + 82616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x
+ 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(358684877*sqrt(11
) + 2940638404) + 723215097220611079 7*sqrt(11)*(x + 3) - 216964529166183323
91*x + 36160754861030553985)/x) - sqrt(2794)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x
 + 1)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*log((sqrt(2794)*s
qrt(5*x^2 + 2*x + 3)*sqrt(1283973697005131*sqrt(11) + 82616280769148425)*(3
58684877*sqrt(11) + 2940638404) - 7232150972206110797*sqrt(11)*(x + 3) + 21
696452916618332391*x - 36160754861030553985)/x) + sqrt(2794)*(49*x^4 - 56*x
^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425)*
log((sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqr
t(-1283973697005131*sqrt(11) + 82616280769148425) + 7232150972206110797*sqr
t(11)*(x + 3) + 21696452916618332391*x - 36160754861030553985)/x) - sqrt(27
94)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1283973697005131*sqrt(11) + 8
2616280769148425)*log(-(sqrt(2794)*sqrt(5*x^2 + 2*x + 3)*(358684877*sqrt(11
) - 2940638404)*sqrt(-1283973697005131*sqrt(11) + 82616280769148425) - 7232
150972206110797*sqrt(11)*(x + 3) - 21696452916618332391*x + 361607548610305
53985)/x) + 11176*(58279130*x^3 - 53381041*x^2 - 3071502*x + 3538943)*sqrt(
5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)
```

**giac [B]** time = 0.32, size = 378, normalized size = 1.67

$$\frac{124397525\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7+26796567\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^6-3595807617\left(\sqrt{5}\,x-\sqrt{5\,x^2+\cdots}\right.}{31225744\left(7\left(\sqrt{5}\,x-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(5*
```

```
x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^
4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 7800623485*
sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.
41924736459000) - 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3)
 + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
 + 3) - 1.02258038113000) + 0.0446437606656958*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 1194, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x)

[Out] -3535/21296*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^
(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(2
45*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-
34*11^(1/2))^(1/2))-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49
*11^(1/2))/(x-2/7+1/7*11^(1/2))^2*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(
1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/1372*(34/7-10/7*1
1^(1/2))/(250/49-34/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^
(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+
250/49-34/49*11^(1/2))^(1/2)+7/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2
))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*1
1^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(
1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
))+5/98/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/4
9-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2
))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^
(1/2))+250-34*11^(1/2))^(1/2)))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/
49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))*(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/
7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)+1/14*(34/7-10
/7*11^(1/2))/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(
500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11
^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/
7*11^(1/2))+250-34*11^(1/2))^(1/2)))+3535/21296*11^(1/2)/(250+34*11^(1/2))^
(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*1
1^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7
*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))-(-3535/1936-273/193
6*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1
```

```
/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1
/2))^(1/2)+1/14*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/
2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+
10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-21/968*(61+13*
11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2*(
5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+3
4/49*11^(1/2))^(1/2)-3/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-
1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))*(5*(x-2/7-1/7*11^(1/2))^2+(3
4/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)+7/2*(3
4/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(
49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250
+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-
2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))+5/98/(250/49+34/49*11^(1/2))/(25
0+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^
2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^3\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,\left(-7x^2+4x+1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x^2\sqrt{5x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

```
[Out] -Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x
 + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) -
 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**
5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(
5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x
 + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x**2 + 2*
x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3)
+ 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*s
qrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

# 3.392 $\int \frac{(1+4x-7x^2)^3 (2+5x+x^2)}{(3+2x+5x^2)^{3/2}} \, dx$

**Optimal**. Leaf size=166

$$-\frac{2583293\sqrt{5x^2+2x+3}\,x^2}{187500}-\frac{3192602\sqrt{5x^2+2x+3}\,x}{46875}+\frac{15715799\sqrt{5x^2+2x+3}}{156250}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}-\frac{34}{15}$$

```
[Out] 50047657/781250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)+16/546875*(6122807-5
338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2*x+3)^(1/2)-3192602/4
6875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+2*x+3)^(1/2)+393659/12
500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*x+3)^(1/2)-343/150*x^5*
(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{343}{150}\sqrt{5x^2+2x+3}\,x^5-\frac{25921\sqrt{5x^2+2x+3}\,x^4}{3750}+\frac{393659\sqrt{5x^2+2x+3}\,x^3}{12500}-\frac{2583293\sqrt{5x^2+2x+3}\,x^2}{187500}-\frac{319260}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + (15715799*Sqrt[
3 + 2*x + 5*x^2])/156250 - (3192602*x*Sqrt[3 + 2*x + 5*x^2])/46875 - (25832
93*x^2*Sqrt[3 + 2*x + 5*x^2])/187500 + (393659*x^3*Sqrt[3 + 2*x + 5*x^2])/1
2500 - (25921*x^4*Sqrt[3 + 2*x + 5*x^2])/3750 - (343*x^5*Sqrt[3 + 2*x + 5*x
^2])/150 + (50047657*ArcSinh[(1 + 5*x)/Sqrt[14]])/(156250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{473724104}{78125}+\frac{94462228x}{15625}-\frac{40822404x^2}{3125}-\frac{12103}{62}}{\sqrt{3+2x+}}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{1}{840}\int \frac{\frac{2842344624}{15625}+\frac{5}{}}{}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x}}{3750}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2}}{12500}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2}}{187500}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x}}{46875}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x}}{46875}$$

$$= \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x}}{46875}$$

**Mathematica [A]** time = 0.43, size = 75, normalized size = 0.45

$$\frac{2102001594\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right) - \frac{5(75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-}{\sqrt{5x^2+2x+3}}}{32812500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((-5*(-3155769618 + 1045703388*x - 2135143465*x^2 + 1795638985*x^3 + 174819
575*x^4 - 897612625*x^5 + 256821250*x^6 + 75031250*x^7))/Sqrt[3 + 2*x + 5*x
^2] + 2102001594*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/32812500
```

**fricas [A]** time = 0.83, size = 112, normalized size = 0.67

$$\frac{1051000797\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)-5\left(75031250x^7+2568\right.}{32812}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6 -
 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457033
88*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.26, size = 81, normalized size = 0.49

$$-\frac{50047657}{781250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)-\frac{(35((5(35(70(175x+599)x-146549)x+998969)x}{656250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x +
51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x +
3)
```

**maple [A]** time = 0.03, size = 166, normalized size = 1.00

$$-\frac{343x^7}{30\sqrt{5x^2+2x+3}}-\frac{29351x^6}{750\sqrt{5x^2+2x+3}}+\frac{1025843x^5}{7500\sqrt{5x^2+2x+3}}-\frac{998969x^4}{37500\sqrt{5x^2+2x+3}}-\frac{51303971x^3}{187500\sqrt{5x^2+2x+3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 50047657/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+176049701/10937500*(
10*x+2)/(5*x^2+2*x+3)^(1/2)-998969/37500*x^4/(5*x^2+2*x+3)^(1/2)-51303971/1
```

```
87500*x^3/(5*x^2+2*x+3)^(1/2)+61004099/187500*x^2/(5*x^2+2*x+3)^(1/2)-50047
657/156250*x/(5*x^2+2*x+3)^(1/2)-343/30*x^7/(5*x^2+2*x+3)^(1/2)-29351/750*x
^6/(5*x^2+2*x+3)^(1/2)+1025843/7500*x^5/(5*x^2+2*x+3)^(1/2)+175268451/39062
5/(5*x^2+2*x+3)^(1/2)
```

**maxima [A]** time = 0.99, size = 148, normalized size = 0.89

$$-\frac{343\,x^7}{30\,\sqrt{5\,x^2+2\,x+3}}-\frac{29351\,x^6}{750\,\sqrt{5\,x^2+2\,x+3}}+\frac{1025843\,x^5}{7500\,\sqrt{5\,x^2+2\,x+3}}-\frac{998969\,x^4}{37500\,\sqrt{5\,x^2+2\,x+3}}-\frac{51303971\,x^3}{187500\,\sqrt{5\,x^2+2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) + 1
025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x +
 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sqrt(
5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1))
 - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*x^2 +
 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)^3}{\left(5\,x^2+2\,x+3\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right) dx-\int \left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
```

```
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
+ 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*s
qrt(5*x**2 + 2*x + 3)), x)
```

# 3.393 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=124

$$\frac{203}{100}\sqrt{5x^2+2x+3}\,x^2-\frac{8749\sqrt{5x^2+2x+3}\,x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}+\frac{49}{100}\sqrt{5x^2+2x+3}\,x^3$$

```
[Out] 89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-8/21875*(12983+136602*x)/
(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1/2)-8749/1250*x*(5*x^2+2*x+3)
^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100*x^3*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{49}{100}\sqrt{5x^2+2x+3}\,x^3+\frac{203}{100}\sqrt{5x^2+2x+3}\,x^2-\frac{8749\sqrt{5x^2+2x+3}\,x}{1250}-\frac{5086\sqrt{5x^2+2x+3}}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) - (5086*Sqrt[3 + 2*x
+ 5*x^2])/3125 - (8749*x*Sqrt[3 + 2*x + 5*x^2])/1250 + (203*x^2*Sqrt[3 + 2*
x + 5*x^2])/100 + (49*x^3*Sqrt[3 + 2*x + 5*x^2])/100 + (89583*ArcSinh[(1 +
5*x)/Sqrt[14]])/(1250*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{1}{28}\int \frac{\frac{4291112}{3125}-\frac{296716x}{625}-\frac{194012x^2}{125}+\frac{23716x^3}{25}+\frac{137}{}}{\sqrt{3+2x+5x^2}}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} + \frac{1}{560}\int \frac{\frac{17164448}{625}-\frac{118686}{125}}{\sqrt{3}}$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} + \frac{49}{100}x^3\sqrt{3+2x+5x^2} +$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} + \frac{203}{100}x^2\sqrt{3+2x+5x^2} +$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} +$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} +$$

$$= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} - \frac{5086\sqrt{3+2x+5x^2}}{3125} - \frac{8749x\sqrt{3+2x+5x^2}}{1250} +$$

**Mathematica [A]** time = 0.27, size = 65, normalized size = 0.52

$$\frac{\frac{5\left(42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536\right)}{\sqrt{5x^2+2x+3}}+1254162\sqrt{5}\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{87500}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] ((5*(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5
))/Sqrt[3 + 2*x + 5*x^2] + 1254162*Sqrt[5]*ArcSinh[(1 + 5*x)/Sqrt[14]])/875
00
```

**fricas [A]** time = 0.69, size = 102, normalized size = 0.82

$$\frac{627081\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)+5\left(42875x^5+194775x^4-51\right.}{87500\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3
)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3 -
 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.25, size = 71, normalized size = 0.57

$$-\frac{89583}{6250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)+\frac{(35((35(35x+159)x-14733)x-8023)x-1298674)x}{17500\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] -89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) +
 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 16853
6)/sqrt(5*x^2 + 2*x + 3)
```

**maple [A]** time = 0.01, size = 132, normalized size = 1.06

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}-\frac{89583x}{1250\sqrt{5x^2+2x+3}}+\frac{89583}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-5564/21875*(10*x+2)/(5*x^
2+2*x+3)^(1/2)+1113/100/(5*x^2+2*x+3)^(1/2)*x^4-14733/500/(5*x^2+2*x+3)^(1/
2)*x^3-8023/500/(5*x^2+2*x+3)^(1/2)*x^2-89583/1250/(5*x^2+2*x+3)^(1/2)*x+49
/20/(5*x^2+2*x+3)^(1/2)*x^5-28506/3125/(5*x^2+2*x+3)^(1/2)
```

**maxima [A]** time = 0.97, size = 114, normalized size = 0.92

$$\frac{49x^5}{20\sqrt{5x^2+2x+3}}+\frac{1113x^4}{100\sqrt{5x^2+2x+3}}-\frac{14733x^3}{500\sqrt{5x^2+2x+3}}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}+\frac{89583}{6250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] 49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 1473
3/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 8958
3/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x^2
+ 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)^2}{\left(5\,x^2+2\,x+3\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2} + 5x + 2\right)\left(7x^{2} - 4x - 1\right)^{2}}{\left(5x^{2} + 2x + 3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
x)
```

# 3.394 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=82

$$-\frac{7}{50}\sqrt{5x^2+2x+3}\,x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

```
[Out] 149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-2/875*(2321+2449*x)/(5*x^2+2
*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*x*(5*x^2+2*x+3)^(1/2)
```

**Rubi [A]** time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1660, 1661, 640, 619, 215}

$$-\frac{7}{50}\sqrt{5x^2+2x+3}\,x-\frac{261}{250}\sqrt{5x^2+2x+3}-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}+\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] (-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) - (261*Sqrt[3 + 2*x + 5*x^
2])/250 - (7*x*Sqrt[3 + 2*x + 5*x^2])/50 + (149*ArcSinh[(1 + 5*x)/Sqrt[14]]
)/(25*Sqrt[5])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(1+4x-7x^2\right)\left(2+5x+x^2\right)}{\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}+\frac{1}{28}\int \frac{\frac{15736}{125}-\frac{3948x}{25}-\frac{196x^2}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{1}{280}\int \frac{\frac{34412}{25}-\frac{7308x}{5}}{\sqrt{3+2x+5x^2}}\,dx$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149}{25}\int$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149\,\mathrm{Sul}}{}$$

$$= -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}}-\frac{261}{250}\sqrt{3+2x+5x^2}-\frac{7}{50}x\sqrt{3+2x+5x^2}+\frac{149\sin}{2}$$

**Mathematica** [A] time = 0.15, size = 55, normalized size = 0.67

$$\frac{149\sinh^{-1}\left(\frac{5x+1}{\sqrt{14}}\right)}{25\sqrt{5}}-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

```
[Out] -1/350*(2953 + 2837*x + 1925*x^2 + 245*x^3)/Sqrt[3 + 2*x + 5*x^2] + (149*Ar
cSinh[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[5])
```

**fricas** [A] time = 0.99, size = 92, normalized size = 1.12

$$\frac{1043\sqrt{5}\left(5x^2+2x+3\right)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}\,(5x+1)-25x^2-10x-8\right)-5\left(245x^3+1925x^2+2837x+\right.}{1750\left(5x^2+2x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(
5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sqrt
(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

**giac** [A] time = 0.22, size = 62, normalized size = 0.76

$$-\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)-\frac{(35(7x+55)x+2837)x+2953}{350\sqrt{5x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] -149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1/
350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

**maple** [A] time = 0.01, size = 98, normalized size = 1.20

$$-\frac{7x^3}{10\sqrt{5x^2+2x+3}}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}-\frac{149x}{25\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\ \operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}-\frac{1001}{125\sqrt{5x^2+2x+3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -7/10/(5*x^2+2*x+3)^(1/2)*x^3-11/2/(5*x^2+2*x+3)^(1/2)*x^2-149/25/(5*x^2+2*
x+3)^(1/2)*x-1001/125/(5*x^2+2*x+3)^(1/2)-751/3500*(10*x+2)/(5*x^2+2*x+3)^(
1/2)+149/125*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

**maxima [A]** time = 0.96, size = 80, normalized size = 0.98

$$-\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}}-\frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}}+\frac{149}{125}\,\sqrt{5}\,\operatorname{arsinh}\left(\frac{1}{14}\,\sqrt{14}\,(5\,x+1)\right)-\frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}}-\frac{2953}{350\,\sqrt{5\,x^2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125*
sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x + 3)
 - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+5\,x+2\right)\,\left(-7\,x^2+4\,x+1\right)}{{\left(5\,x^2+2\,x+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

```
[Out] int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx-\int \left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3)
 + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 + 2
*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Inte
gral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) +
```

```
3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 + 2*x +
 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral
(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*
x**2 + 2*x + 3)), x)
```

# 3.395 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=166

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17\cdots}{\sqrt{2(12\cdots}}\right.}{1016}$$

```
[Out] 1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*arctanh((23+x*(17-5*11^(1
/2))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(393525121-3494
5955*11^(1/2))^(1/2)+3/1419352*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x
^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(393525121+34945955*11^(1/2))^(1/2
)
```

**Rubi [A]** time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17-5\sqrt{11})x-\sqrt{11}+23}{\sqrt{2(125-17\sqrt{11})}\sqrt{5x^2+2x+3}}\right)}{1016}+\frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{(17\cdots}{\sqrt{2(12\cdots}}\right.}{1016}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -(131 - 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) - (3*Sqrt[(281693 - 25015*Sqrt[
11])/1397]*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*
Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016 + (3*Sqrt[(281693 + 25015*Sqrt[11]
)/1397]*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/1016
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\int \frac{13776+14112x}{\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}}\,dx}{28448}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} + \frac{\left(21\left(66-53\sqrt{11}\right)\right)\int \frac{1}{\left(4-2\sqrt{11}-14x\right)\sqrt{3+2x+5x^2}}\,dx}{2794}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{\left(21\left(66-53\sqrt{11}\right)\right)\text{Subst}\left(\int \frac{1}{2352+112\left(4-2\sqrt{11}\right)+}\right.}{1}$$

$$= -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\tanh^{-1}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{1}\right.}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2}}\right.}{1016}$$

**Mathematica [A]** time = 1.13, size = 174, normalized size = 1.05

$$\frac{\frac{2794(605x-131)}{\sqrt{5x^2+2x+3}} - 21\sqrt{127\left(125+17\sqrt{11}\right)}\left(53\sqrt{11}-66\right)\tanh^{-1}\left(\frac{-5\sqrt{11}x+17x-\sqrt{11}+23}{\sqrt{250-34\sqrt{11}}\sqrt{5x^2+2x+3}}\right) + 21\sqrt{127\left(125-17\sqrt{11}\right)}}{9935464}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((2794*(-131 + 605*x))/Sqrt[3 + 2*x + 5*x^2] - 21*Sqrt[127*(125 + 17*Sqrt[1
1])]*(-66 + 53*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + 17*x - 5*Sqrt[11]*x)/(Sqr
t[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])] + 21*Sqrt[127*(125 - 17*Sqrt[1
1])]*(66 + 53*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt
[250 + 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])])/9935464
```

**fricas [B]** time = 0.78, size = 333, normalized size = 2.01

$$-\frac{21\sqrt{1397}\left(5x^2+2x+3\right)\sqrt{25015\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+2359672\right.}{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fric
as")
```

```
[Out] -1/19870928*(21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335
*sqrt(11) - 8173) + 23596727*sqrt(11)*(x + 3) + 70790181*x - 117983635)/x)
- 21*sqrt(1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqr
t(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 23596727*sqrt(11)*(x + 3) - 70790181*x + 117983635)/x) + 7*sqrt(1
397)*(5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log(-(sqrt(1397)*sq
rt(5*x^2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237)
 + 70790181*sqrt(11)*(x + 3) - 212370543*x + 353950905)/x) - 7*sqrt(1397)*(
5*x^2 + 2*x + 3)*sqrt(-225135*sqrt(11) + 2535237)*log((sqrt(1397)*sqrt(5*x^
2 + 2*x + 3)*(1335*sqrt(11) + 8173)*sqrt(-225135*sqrt(11) + 2535237) - 7079
0181*sqrt(11)*(x + 3) + 212370543*x - 353950905)/x) - 5588*sqrt(5*x^2 + 2*x
 + 3)*(605*x - 131))/(5*x^2 + 2*x + 3)
```

**giac [A]** time = 0.25, size = 112, normalized size = 0.67

$$\frac{605\,x-131}{3556\,\sqrt{5\,x^2+2\,x+3}}+0.0477059376663667\,\log\left(-\sqrt{5}\,x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)-0.03521749$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="giac
")
```

```
[Out] 1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376663667*log(-sqrt(5
)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957838020*log(-s
qrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0477059376663667*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0352174957838
020*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 489, normalized size = 2.95

$$-\frac{10x+2}{196\sqrt{5x^2+2x+3}}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(-\frac{\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\;\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+250-34\sqrt{11}}}\right)}{\left(\frac{250}{49}-\frac{34\sqrt{11}}{49}\right)\sqrt{250-34\sqrt{11}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] -1/196*(10*x+2)/(5*x^2+2*x+3)^(1/2)-3/154*(-61+13*11^(1/2))*11^(1/2)*(1/7/(
250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34/7-10/7*11^(1/2))/(250/
49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2
)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/4
9-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-3/154*(61+13*11^(1/2))*11
^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11
^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11
^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/
7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(
1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7
-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))
```

**maxima [B]** time = 1.16, size = 777, normalized size = 4.68

$$-\frac{1}{4312}\sqrt{11}\left(\frac{20\sqrt{11}\,x}{\sqrt{5x^2+2x+3}}-\frac{7890\sqrt{11}\,x}{17\sqrt{11}\sqrt{5x^2+2x+3}+125\sqrt{5x^2+2x+3}}+\frac{7890\sqrt{11}\,x}{17\sqrt{11}\sqrt{5x^2+2x+3}-125\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="maxi
ma")
```

```
[Out] -1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(17
*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(11
)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13377
*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(
11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)
*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^2 + 2*x
 + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x +
 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x +
3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt
(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11
)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*
x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*sqrt(2)*arcsi
nh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)
*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14
```

```
*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/(17
*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) +
 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*sqrt(5*x^2 + 2*x +
 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)
*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11
) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqr
t(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) +
 12678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 12
678/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)))
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+5\,x+2}{{\left(5\,x^2+2\,x+3\right)}^{3/2}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
[Out] -Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5
*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x
**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sqrt(5*
x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x
+ 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

## 3.396 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=215

$$-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right.}}\right.}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}}$$

```
[Out] 1/19870928*(-76567-22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+4
*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*arctanh((23+x*(17-5*11^(1/2))-11^(1/2))
/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(541543-5144*11^(1/2))/(2750-
374*11^(1/2))^(1/2)+7/2838704*arctanh((23+11^(1/2)+x*(17+5*11^(1/2)))/(5*x^
2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(541543+5144*11^(1/2))/(2750+374*11
^(1/2))^(1/2)
```

**Rubi [A]** time = 0.32, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{22755x+76567}{19870928\sqrt{5x^2+2x+3}}-\frac{7\left(541543-5144\sqrt{11}\right)\tanh^{-1}\left(\frac{\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right.}}\right.}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] -(76567 + 22755*x)/(19870928*Sqrt[3 + 2*x + 5*x^2]) - (3*(40 - 371*x))/(558
8*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(541543 - 5144*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 - 17*Sqrt[11])]) + (7*(541543
+ 5144*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(2838704*Sqrt[22*(125 + 17*Sqrt[1
1])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int \frac{-50216-37752x-89040x^2}{\left(1+4x-7x^2\right)\left(3+2x+5x^2\right)^{3/2}}\,dx}{44704}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{\int}{}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} + \frac{(7}{}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{(7}{}$$

$$= -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{5588\left(1+4x-7x^2\right)\sqrt{3+2x+5x^2}} - \frac{7(}{}$$

**Mathematica [A]** time = 1.14, size = 351, normalized size = 1.63

$$\frac{\frac{5084772\sqrt{5x^2+2x+3}\,x}{-7x^2+4x+1} + \frac{24422640x}{7\sqrt{5x^2+2x+3}} + \frac{12968296}{7\sqrt{5x^2+2x+3}} + \frac{1672044\sqrt{5x^2+2x+3}}{7x^2-4x-1} + 7581602\sqrt{\frac{22}{125+17\sqrt{11}}}\,\log\left(\sqrt{2750+374\sqrt{11}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (12968296/(7*Sqrt[3 + 2*x + 5*x^2]) + (24422640*x)/(7*Sqrt[3 + 2*x + 5*x^2]
) + (5084772*x*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (1672044*Sqrt[3 +
 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) + 14*Sqrt[2/(125 - 17*Sqrt[11])]*(-56584
+ 541543*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[11]]*Sqrt[3 + 2*x + 5*x^2])/
(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sqrt[2/(125 + 17*Sqrt[11])]*(
56584 + 541543*Sqrt[11])*Log[2 + Sqrt[11] - 7*x] + 792176*Sqrt[2/(125 + 17*
Sqrt[11])]*Log[11 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sq
rt[11]]*Sqrt[3 + 2*x + 5*x^2]] + 7581602*Sqrt[22/(125 + 17*Sqrt[11])]*Log[1
1 + 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 +
 2*x + 5*x^2]])/124902976
```

**fricas [B]** time = 0.88, size = 392, normalized size = 1.82

$$-\frac{7\sqrt{1397}\left(35x^4-6x^3+8x^2-14x-3\right)\sqrt{4294093814065\sqrt{11}+35653135368317}\log\left(-\frac{\sqrt{1397}\sqrt{5x^2+2x+3}\sqrt{42}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/111038745664*(7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(4294
093814065*sqrt(11) + 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)
*sqrt(4294093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905
) + 2865029444171587*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857
935)/x) - 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(42940938140
65*sqrt(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(42
94093814065*sqrt(11) + 35653135368317)*(5609479*sqrt(11) + 77949905) - 2865
029444171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x)
+ 7*sqrt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt
(11) + 35653135368317)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(
11) - 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) + 2865029444
171587*sqrt(11)*(x + 3) + 8595088332514761*x - 14325147220857935)/x) - 7*sq
rt(1397)*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(-4294093814065*sqrt(11) +
 35653135368317)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) -
 77949905)*sqrt(-4294093814065*sqrt(11) + 35653135368317) - 286502944417158
7*sqrt(11)*(x + 3) - 8595088332514761*x + 14325147220857935)/x) + 5588*(159
285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x^2 + 2*x + 3))/(35*x^4 -
 6*x^3 + 8*x^2 - 14*x - 3)
```

**giac [A]** time = 0.27, size = 295, normalized size = 1.37

$$\frac{25230x+13397}{903224\sqrt{5x^2+2x+3}}+\frac{3\left(42623\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3+77302\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^2-2\right.}{709676\left(7\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)^3-70\left(\sqrt{5}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x
 + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x + 3)
```

```
)/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*
x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276254033*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.0218058276254033*log
(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287436443391
1*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 1214, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 161/484*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34
/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*(34
/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)
-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-
2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))/(2
50-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))
^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))+(1
83/44-39/44*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(
5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-3
4/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(
250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/
7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-3
4/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5
*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34
/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arcta
nh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(
250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*11^(1/2))
*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11
^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^
(1/2))+(183/44+39/44*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11
^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))
+250/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1
/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/
2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/
(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/
2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+
250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1
/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^
```

```
(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*1
1^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(250/49+34/49*
11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2
/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*1
1^(1/2))^(1/2))-161/484*11^(1/2)*(1/7/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7
*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2
))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49
+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10
/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-1/(250/49+34
/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2
/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(
1/2))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5x+2}{\left(7x^2-4x-1\right)^2\left(5x^2+2x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2+5x+2}{\left(5x^2+2x+3\right)^{3/2}\left(-7x^2+4x+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+5x+2}{\left(5x^2+2x+3\right)^{\frac{3}{2}}\left(7x^2-4x-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2)
, x)
```

## 3.397 $\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=250

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x}}$$

```
[Out] -5/222077491328*(461370781+1118731375*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-37
1*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)+1/62451488*(-2701733+9148874*x)/(
-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*arctanh((23+x*(17-5*11^(1/2
))-11^(1/2))/(5*x^2+2*x+3)^(1/2)/(250-34*11^(1/2))^(1/2))*(2792860024-84865
895*11^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*arctanh((23+11^(1/2)+
x*(17+5*11^(1/2)))/(5*x^2+2*x+3)^(1/2)/(250+34*11^(1/2))^(1/2))*(2792860024
+84865895*11^(1/2))/(2750+374*11^(1/2))^(1/2)
```

**Rubi** [A] time = 0.32, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1060, 1032, 724, 206}

$$-\frac{2701733-9148874x}{62451488\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}-\frac{5(1118731375x+461370781)}{222077491328\sqrt{5x^2+2x+3}}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] (-5*(461370781 + 1118731375*x))/(222077491328*Sqrt[3 + 2*x + 5*x^2]) - (3*(
40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) - (2701733 -
 9148874*x)/(62451488*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) - (7*(279286
0024 - 84865895*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(31725355904*Sqrt[22*(12
5 - 17*Sqrt[11])]) + (7*(2792860024 + 84865895*Sqrt[11])*ArcTanh[(23 + Sqrt
[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x
^2])])/(31725355904*Sqrt[22*(125 + 17*Sqrt[11])])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f))))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx = -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{\int \frac{-128104-89208x-178080x^2}{\left(1+4x-7x^2\right)^2\left(3+2x+5x^2\right)^{3/2}}\,dx}{89408}$$

$$= -\frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5x^2}} - \frac{2701733-91488}{62451488\left(1+4x-7x^2\right)\sqrt{}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5}}$$

$$= -\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}} - \frac{3(40-371x)}{11176\left(1+4x-7x^2\right)^2\sqrt{3+2x+5}}$$

**Mathematica [A]** time = 1.59, size = 381, normalized size = 1.52

$$\frac{\frac{44\sqrt{5x^2+2x+3}\,(507770113-1167248019x)}{7x^2-4x-1} + \frac{737616(38521x-12667)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2} + \frac{21296(501205x+1702037)}{7\sqrt{5x^2+2x+3}} - 7\sqrt{\frac{22}{125-17\sqrt{11}}}\,(848658}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

```
[Out] ((21296*(1702037 + 501205*x))/(7*Sqrt[3 + 2*x + 5*x^2]) + (737616*(-12667 +
 38521*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2 + (44*(507770113 - 116
7248019*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 14*Sqrt[22/(125 - 17
*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])*ArcTanh[(Sqrt[250 - 34*Sqrt[1
1]]*Sqrt[3 + 2*x + 5*x^2])/(-23 + Sqrt[11] + (-17 + 5*Sqrt[11])*x)] - 14*Sq
rt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[2 + Sqrt[11
```

```
] - 7*x] + 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024 + 84865895*Sqrt[11])
*Log[(-2 + Sqrt[11] + 7*x)^2] - 7*Sqrt[22/(125 - 17*Sqrt[11])]*(-2792860024
 + 84865895*Sqrt[11])*Log[15 - 4*Sqrt[11] + 14*(-2 + Sqrt[11])*x + 49*x^2]
+ 14*Sqrt[22/(125 + 17*Sqrt[11])]*(2792860024 + 84865895*Sqrt[11])*Log[11 +
 23*Sqrt[11] + (55 + 17*Sqrt[11])*x + Sqrt[2750 + 374*Sqrt[11]]*Sqrt[3 + 2*
x + 5*x^2]])/1395915659776
```

**fricas [B]** time = 1.33, size = 452, normalized size = 1.81

$$-\frac{7\sqrt{1397}\left(245x^6-182x^5+45x^4-124x^3+27x^2+26x+3\right)\sqrt{74693314710639641467\sqrt{11}+896266498377}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="fr
icas")
```

```
[Out] -1/1240969021540864*(7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 2
7*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 89626649837723365785
5)*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11
) + 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) + 75502120
686844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 37751060343
4220275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27
*x^2 + 26*x + 3)*sqrt(74693314710639641467*sqrt(11) + 896266498377233657855
)*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(74693314710639641467*sqrt(11)
+ 896266498377233657855)*(37271563201*sqrt(11) + 407780707037) - 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) + 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log((sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 407780707037
)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) + 7550212068
6844055144479*sqrt(11)*(x + 3) + 226506362060532165433437*x - 3775106034342
20275722395)/x) - 7*sqrt(1397)*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x
^2 + 26*x + 3)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855)
*log(-(sqrt(1397)*sqrt(5*x^2 + 2*x + 3)*(37271563201*sqrt(11) - 40778070703
7)*sqrt(-74693314710639641467*sqrt(11) + 896266498377233657855) - 755021206
86844055144479*sqrt(11)*(x + 3) - 226506362060532165433437*x + 377510603434
220275722395)/x) + 5588*(274089186875*x^5 - 200208943655*x^4 + 109737266678
*x^3 - 148022158802*x^2 + 7828199499*x + 14298727813)*sqrt(5*x^2 + 2*x + 3)
)/(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)
```

**giac [B]** time = 0.32, size = 397, normalized size = 1.59

$$\frac{501205\,x+1702037}{458837792\,\sqrt{5\,x^2+2\,x+3}}+\frac{6871871279\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right)^7+4012856750\,\sqrt{5}\left(\sqrt{5}\,x-\sqrt{5\,x^2+2\,x+3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(6871
871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5)*
x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 12550
97956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*(sqrt
(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 561299654796*s
qrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 +
2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
3)) + 83)^2 + 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4
.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) + 1.25295163054000) - 0.0107382277384513*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^2
 + 2*x + 3) - 2.09411235400000)
```

**maple [B]** time = 0.02, size = 2600, normalized size = 10.40

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x)
```

```
[Out] 3535/21296*11^(1/2)*(1/7/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+
(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/7*
(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1
/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*
(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-1/(250/49-34/49*11^(1/2))
/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/
2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))
-21/968*(-61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49-34/49*11^(1/2))/(x-2/7+1
/7*11^(1/2))^2/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11
^(1/2))+250/49-34/49*11^(1/2))^(1/2)-5/1372*(34/7-10/7*11^(1/2))/(250/49-34
```

/49*11^(1/2))*(-1/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/2*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2))-15/686/(250/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))))-(-3535/1936+273/1936*11^(1/2))*(-1/49/(250/49-34/49*11^(1/2))/(x-2/7+1/7*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-3/98*(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(1/(250/49-34/49*11^(1/2))/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-(34/7-10/7*11^(1/2))/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2)-7/(250/49-34/49*11^(1/2))/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)))-20/49/(250/49-34/49*11^(1/2))*(10*x+2)/(5000/49-680/49*11^(1/2)-(34/7-10/7*11^(1/2))^2)/(5*(x-2/7+1/7*11^(1/2))^2+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250/49-34/49*11^(1/2))^(1/2))-21/968*(61+13*11^(1/2))*11^(1/2)*(-1/686/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))^2/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-5/1372*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(-1/(250/49+34/49*11^(1/2))/(x-2/7-1/7*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-3/2*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arct

```
anh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/
(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))
*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/(250/49+34/49*11^(1/2))*(
10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(
1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1
/2))-15/686/(250/49+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/
7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/
2))^(1/2)-(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+68
0/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*
11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49
*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/
7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-
1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2
))^(1/2))))-(-3535/1936-273/1936*11^(1/2))*(-1/49/(250/49+34/49*11^(1/2))/(
x-2/7-1/7*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-3/98*(34/7+10/7*11^(1/2))/(250/49
+34/49*11^(1/2))*(1/(250/49+34/49*11^(1/2))/(5*(x-2/7-1/7*11^(1/2))^2+(34/7
+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-(34/7+10/
7*11^(1/2))/(250/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7
+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/
7*11^(1/2))+250/49+34/49*11^(1/2))^(1/2)-7/(250/49+34/49*11^(1/2))/(250+34*
11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x
-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*
(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2)))-20/49/(2
50/49+34/49*11^(1/2))*(10*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2)
)^2)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+25
0/49+34/49*11^(1/2))^(1/2))-3535/21296*11^(1/2)*(1/7/(250/49+34/49*11^(1/2)
)/(5*(x-2/7-1/7*11^(1/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/4
9+34/49*11^(1/2))^(1/2)-1/7*(34/7+10/7*11^(1/2))/(250/49+34/49*11^(1/2))*(1
0*x+2)/(5000/49+680/49*11^(1/2)-(34/7+10/7*11^(1/2))^2)/(5*(x-2/7-1/7*11^(1
/2))^2+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250/49+34/49*11^(1/2))^(1/
2)-1/(250/49+34/49*11^(1/2))/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^
(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/
2))+250+34*11^(1/2))^(1/2)))
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2+5\,x+2}{\left(7\,x^2-4\,x-1\right)^3\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="ma
xima")
```

```
[Out] -integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 + 5\,x + 2}{\left(5\,x^2 + 2\,x + 3\right)^{3/2}\,\left(-7\,x^2 + 4\,x + 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
```

```
[Out] int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{5x}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640x^4\sqrt{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

```
[Out] -Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2
*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3)
 + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*
x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 +
 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**
7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(
5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2
+ 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3)
- 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**2 + 2*x +
 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) -
866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**
3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x*
*2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

# 3.398 $\int \left(a+cx^2\right)^p \left(A+Cx^2\right) \left(d+fx^2\right)^q \, dx$

**Optimal**. Leaf size=166

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)
```

**Rubi [A]** time = 0.15, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {531, 430, 429, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+cx^2\right)^p \left(A+Cx^2\right)\left(d+fx^2\right)^q dx &= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q dx + C\int x^2\left(a+cx^2\right)^p\left(d+fx^2\right)^q dx\\ &= \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(d+fx^2\right)^q dx + \left(C\left(a+cx^2\right)^{\cdots}\right.\\ &= \left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}\right)\int\left(1+\frac{cx^2}{a}\right)^p\left(1+\cdots\right.\\ &= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^{\cdots}}{a}\right.\end{aligned}$$

**Mathematica [A]** time = 0.40, size = 242, normalized size = 1.46

$$\frac{1}{3}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{9aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1\left(\cdots\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((9*a*A*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^
2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*
x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2, -((c*x^2)/a), -((f*x^
2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2)/a), -((f*x^2)/d)]))
+ (C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/((1 + (c*x
^2)/a)^p*(1 + (f*x^2)/d)^q)))/3
```

**fricas [F]** time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="fricas")
[Out] integral((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="giac")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**maple [F]** time = 0.08, size = 0, normalized size = 0.00

$$\int\left(C\,x^2+A\right)\left(c\,x^2+a\right)^p\left(f\,x^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
[Out] int((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Cx^2+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(C\,x^2+A\right)\,\left(c\,x^2+a\right)^p\,\left(f\,x^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q,x)
[Out] int((A + C*x^2)*(a + c*x^2)^p*(d + f*x^2)^q, x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+A)*(f*x**2+d)**q,x)
[Out] Timed out
```

# 3.399 $\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx$

**Optimal**. Leaf size=167

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(\cdots}{\cdots}\right.}{\cdots}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)^(1+p)*(f*x^2+d)^q*hypergeom([-q,
 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

**Rubi [A]** time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1010, 430, 429, 444, 70, 69}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{B\left(a+cx^2\right)^{p+1}\left(d+fx^2\right)^q\left(\frac{c(\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p
)*(d + f*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d
- a*f))])/(2*c*(1 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int (A+Bx)\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx &= A\int \left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx+B\int x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\,dx\\ &= \frac{1}{2}B\,\text{Subst}\left(\int (a+cx)^p(d+fx)^q\,dx,x,x^2\right)+\left(A\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\right)\int\\ &= \frac{1}{2}\left(B\left(d+fx^2\right)^q\left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p\left(\frac{cd}{cd-af}+\frac{cfx}{cd-af}\right.\right.\\ &= Ax\left(a+cx^2\right)^p\left(1+\frac{cx^2}{a}\right)^{-p}\left(d+fx^2\right)^q\left(1+\frac{fx^2}{d}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},\right.\end{aligned}$$

**Mathematica [A]** time = 0.32, size = 236, normalized size = 1.41

$$\frac{1}{2}x\left(a+cx^2\right)^p\left(d+fx^2\right)^q\left(\frac{6aAdF_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2};1-p,-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+afqF_1\left(\frac{3}{2};-p,1-q;\frac{5}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)\right)+3adF_1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*x)*(a + c*x^2)^p*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (6*a*A*d*AppellF1[1/2
, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q, 3/
2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5/2,
 -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*x^2
)/a), -((f*x^2)/d)]))))/2
```

**fricas [F]** time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(cx^2+a\right)^p\left(fx^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int (Bx+A)\left(c\,x^2+a\right)^p\left(f\,x^2+d\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

```
[Out] int((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+a)^p*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,x^2 + a\right)^p \left(f\,x^2 + d\right)^q (A + B\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x),x)
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+a)**p*(f*x**2+d)**q,x)
[Out] Timed out
```

# 3.400 $\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right)\left(d+fx^2\right)^q \, dx$

**Optimal**. Leaf size=252

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}$$

```
[Out] A*x*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(1/2,-p,-q,3/2,-c*x^2/a,-f*x^2/d)/((c*x
^2/a+1)^p)/((1+f*x^2/d)^q)+1/3*C*x^3*(c*x^2+a)^p*(f*x^2+d)^q*AppellF1(3/2,-
p,-q,5/2,-c*x^2/a,-f*x^2/d)/((c*x^2/a+1)^p)/((1+f*x^2/d)^q)+1/2*B*(c*x^2+a)
^(1+p)*(f*x^2+d)^q*hypergeom([-q, 1+p],[2+p],-f*(c*x^2+a)/(-a*f+c*d))/c/(1+
p)/((c*(f*x^2+d)/(-a*f+c*d))^q)
```

**Rubi [A]** time = 0.48, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6742, 430, 429, 444, 70, 69, 511, 510}

$$Ax\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}\left(d+fx^2\right)^q\left(\frac{fx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{cx^2}{a},-\frac{fx^2}{d}\right)+\frac{1}{3}Cx^3\left(a+cx^2\right)^p\left(\frac{cx^2}{a}+1\right)^{-p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (A*x*(a + c*x^2)^p*(d + f*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((c*x^2)/a), -
((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (C*x^3*(a + c*x^2)^p*
(d + f*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*(1
 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (B*(a + c*x^2)^(1 + p)*(d + f*x^2)^q*H
ypergeometric2F1[1 + p, -q, 2 + p, -((f*(a + c*x^2))/(c*d - a*f))])/(2*c*(1
 + p)*((c*(d + f*x^2))/(c*d - a*f))^q)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
```

```
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \left(a+cx^2\right)^p \left(A+Bx+Cx^2\right) \left(d+fx^2\right)^q \, dx = \int \left(A\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Bx\left(a+cx^2\right)^p \left(d+fx^2\right)^q + Cx^2 \left(a\right.\right.$$

$$= A\int \left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx + B\int x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \, dx$$

$$= \frac{1}{2}B\,\text{Subst}\left(\int (a+cx)^p(d+fx)^q \, dx, x, x^2\right) + \left(A\left(a+cx^2\right)^p \left(1+\right.\right.$$

$$= \frac{1}{2}\left(B\left(d+fx^2\right)^q \left(\frac{c\left(d+fx^2\right)}{cd-af}\right)^{-q}\right)\text{Subst}\left(\int (a+cx)^p \left(\frac{cd}{cd-af}+\right.\right.$$

$$= Ax\left(a+cx^2\right)^p \left(1+\frac{cx^2}{a}\right)^{-p} \left(d+fx^2\right)^q \left(1+\frac{fx^2}{d}\right)^{-q} F_1\left(\frac{1}{2}; -p, -q;\right.$$

**Mathematica [A]** time = 0.52, size = 302, normalized size = 1.20

$$\frac{1}{6}x\left(a+cx^2\right)^p \left(d+fx^2\right)^q \left(\frac{18aAdF_1\left(\frac{1}{2}; -p, -q; \frac{3}{2}; -\frac{cx^2}{a}, -\frac{fx^2}{d}\right)}{2x^2\left(cdpF_1\left(\frac{3}{2}; 1-p, -q; \frac{5}{2}; -\frac{cx^2}{a}, -\frac{fx^2}{d}\right) + afqF_1\left(\frac{3}{2}; -p, 1-q; \frac{5}{2}; -\frac{cx^2}{a}, -\frac{fx^2}{d}\right)\right) + 3adF_1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2)^p*(A + B*x + C*x^2)*(d + f*x^2)^q,x]
```

```
[Out] (x*(a + c*x^2)^p*(d + f*x^2)^q*((3*B*x*AppellF1[1, -p, -q, 2, -((c*x^2)/a),
 -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q) + (18*a*A*d*AppellF1[
1/2, -p, -q, 3/2, -((c*x^2)/a), -((f*x^2)/d)])/(3*a*d*AppellF1[1/2, -p, -q,
 3/2, -((c*x^2)/a), -((f*x^2)/d)] + 2*x^2*(c*d*p*AppellF1[3/2, 1 - p, -q, 5
/2, -((c*x^2)/a), -((f*x^2)/d)] + a*f*q*AppellF1[3/2, -p, 1 - q, 5/2, -((c*
x^2)/a), -((f*x^2)/d)])) + (2*C*x^2*AppellF1[3/2, -p, -q, 5/2, -((c*x^2)/a)
, -((f*x^2)/d)])/((1 + (c*x^2)/a)^p*(1 + (f*x^2)/d)^q)))/6
```

**fricas [F]** time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Cx^2+Bx+A\right)\left(cx^2+a\right)^p\left(fx^2+d\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \left(C x^2 + Bx + A\right)\left(c x^2 + a\right)^p \left(f x^2 + d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
[Out] int((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Cx^2 + Bx + A\right)\left(cx^2 + a\right)^p \left(fx^2 + d\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^p*(C*x^2+B*x+A)*(f*x^2+d)^q,x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(c*x^2 + a)^p*(f*x^2 + d)^q, x)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c x^2 + a\right)^p \left(f x^2 + d\right)^q \left(C x^2 + B x + A\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2),x)
[Out] int((a + c*x^2)^p*(d + f*x^2)^q*(A + B*x + C*x^2), x)
```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**p*(C*x**2+B*x+A)*(f*x**2+d)**q,x)
```

```
[Out] Timed out
```

# Chapter 4

# Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```